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Research article

Traveling wave solutions of a coupled Schrödinger-Korteweg-de Vries equation by the generalized coupled trial equation method

Jiaxin Shang, Wenhe Li *, Da Li

School of Mathematics and Statistics, Northeast Petroleum University, 99 Xingzhi Street, Daqing, 163318, Heilongjiang Province, China

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ABSTRACT

The coupled Schrödinger-Korteweg-de Vries equation is a critical system of in nonlinear evolution equations. It describes various processes in dusty plasma, such as Langmuir waves, dust-acoustic waves, and electromagnetic waves. This paper uses the generalized coupled trial equation method to solve the equation. By the complete discrimination system for polynomial, a series of exact traveling wave solutions are obtained, including discontinuous periodic solutions, solitary wave solutions, and Jacobian elliptical function solutions. In addition, to determine the existence of the solutions and understand their properties, we draw three-dimensional images of the modules of the solutions with Mathematica. We obtain more comprehensive and accurate solutions than previous studies, and the results give the system more profound physical significance.

1. Introduction

Soliton theory is an integral part of scientific research today. Because of the characteristic of solitary waves that retain their original shape and velocity after collision, scientists in various fields have successively devoted great enthusiasm and interest to solitons. A more complete and systematic soliton theory has gradually been formed.

The nonlinear Schrödinger (NLS) equation is a necessary part of the solitary optical equations. It has been proved helpful for a deeper understanding of various processes, from nonlinear optics and atomic physics to deep water waves, abnormal surges, plasmas, and other phenomena. Yoshinaga et al. [1] gave a system of equations that can summarise the nonlinear interactions that occur between the two types of long and short waves:

$$\begin{cases} i\dfrac{\partial u}{\partial t} \pm \dfrac{\partial^2 u}{\partial x^2} = uv, \\ \dfrac{\partial v}{\partial t} + \lambda v\dfrac{\partial v}{\partial x} + \sigma\dfrac{\partial^3 v}{\partial x^3} = \dfrac{\partial |u|^2}{\partial x}. \end{cases}$$

When λ and σ are both equal to 0, the long-wave amplitude in this equation is much smaller than the short-wave amplitude, and the equation is of SH (Schrödinger-hyperbolic) type; when λ and σ remain finite, the long-wave amplitude in this equation is of the same order as the short-wave amplitude, and the equation is of S-KdV (Schrödinger-Korteweg-de Vries) type. In this paper, we aim to obtain exact solutions of the following coupled Schrödinger-Korteweg-de Vries (Schrödinger-KdV) equation:

* Corresponding author.
E-mail address: wenheli@nepu.edu.cn (W. Li).

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$$\begin{cases} iu_t - u_{xx} - uv = 0, \\ v_t + 6vv_x + v_{xxx} - (|u|^2)_x = 0. \end{cases}$$

This system governs the long-wave limit of the energy transfer problem along the third non-harmonic medium. Likewise, the system is a significant class of models in plasma physics, describing a variety of processes such as Langmuir waves, dust-acoustic waves, and electromagnetic waves. The coupled system has a number of features, such as the existence of locally resolved solutions to the two classes of equations separated by the known properties of the existence of multiple soliton solutions. Kaya et al. [2] used Adomian's decomposition method to find the exact and approximate solutions. Bai et al. [3] used the finite-element method to study a periodic initial value problem. Filiz et al. [4] used the F-expansion method to obtain some exact solutions. Ullah et al. [5] used the extended optimal homotopy asymptotic method to get the approximate solutions. Numerous other experts and scholars [6–13] have studied this equation and obtained rich research results. However, it is not sufficient to describe the processes in the dust plasma by using the exact solutions obtained in the existing literature. Therefore, finding other ways to solve the problem of too few solutions is necessary.

Many experts and scholars have conducted significant researches on various nonlinear phenomena and have made meaningful discoveries in various fields [14–43], for example, Li et al. [44] studied a magnetic field coupling fractional step lattice Boltzmann, Wang et al. [45] studied an efficient channel prediction method in multiple-input multiple-output systems, and Jin et al. [46,47] investigated the simulation of the chemotactic interactions between one species and two competing attraction-repulsion Keller-Segel system and its asymptotic behavior in one dimension, and a parabolic-elliptic chemotaxis model with density-support by Lyu et al. [48] chemotaxis model with density-suppressed motility and general logistic source, and Ye et al. [49] proposed a state damping control method for the field of rotorcraft UAVs. It is challenging and essential to find the solutions to these equations. At present, there are many methods to obtain the solutions of nonlinear differential equations, such as the first integral method [50], the extended direct algebraic method [51], the Riccati-Bernoulli Sub-ODE method [52], and the modified Kudryashov method [53]. Most of these methods presuppose a solution by substituting the original equation and then solving for the unknown parameters to obtain the final solution. These methods do not start by solving the equation itself, so they are difficult to understand in depth from a theoretical point of view.

In 2005, Liu [54] proposed a simple and effective method called the trial equation method, which is used for solving nonlinear differential equations. The starting point of the trial equation method is to decompose the non-linear operator into the form of a factorial equation, and then derive the specific parameters in the factorial equation from the structure of the equation itself. The exact solutions of the higher order differential equation can then be obtained by solving the factorial equations, which bases the solution of higher order nonlinear differential equations on a rigorous mathematical theory and forms a systematic method of solving them. Compared with other methods, the trial equation method is more straightforward, practical, and all-encompassing. Later, he used it to solve a significant number of equations [55–63] that are famous in many fields, such as Sine-Gördon equation, NNV equation, and a class of generalized Ginzburg-Landau equation. Then, many scholars introduced modified versions based on Liu's method. Some of them are representative: Du [64] extended the trial equation method from the field of rational numbers to the area of irrational numbers and proposed the irrational trial equation method, whose trial equation form is

$$u' = \sum_{i=0}^{k_1} a_i u^i + (\sum_{i=0}^{k_2} b_i u^i)\sqrt{\sum_{i=0}^{k_3} c_i u^i};$$

Gurefe et al. [65] proposed the extended trial equation method, which extends the original trial equation form to

$$(u')^2 = \frac{\sum_{i=1}^{n} a_i u^i}{\sum_{j=1}^{m} b_i u^i};$$

Bulut et al. [66] proposed the modified trial equation method, changing the original trial equation to the following form for use:

$$u' = \frac{\sum_{i=1}^{n} a_i u^i}{\sum_{j=1}^{m} b_i u^i};$$

Li et al. [67] proposed the generalized coupled trial equation method, extended the trial equation method to a system of equations form:

$$(u')^2 = H(u) = \sum_{i=1}^{n} a_i u^i, v = T(u).$$

These modified versions of the trial equation method make the trial equation method more generalizable. By using these trial equation methods, more comprehensive and accurate solutions can be obtained, and the outcomes give the model more profound physical properties.

The aim of this paper is to find new exact solutions of the coupled Schrödinger-KdV equation by using the generalized coupled trial equation method. We thus introduce the generalized coupled trial equation method used in Section 2. In Section 3, the exact solutions of the Schrödinger-KdV equation are found. We draw the three-dimensional (3D) images of the solutions with Mathematica in Section 4. Section 5 provides a brief conclusion.

2. The generalized coupled trial equation method

The generalized coupled trial equation method proceeds as follows.

First, consider the coupled equations with constant coefficients:

$$\begin{cases} N_1\left(u, v, \partial u, \partial v, \partial^2 u, \partial^2 v, \cdots, \partial^{l_1} u, \partial^{l_2} v\right)=0, \\ N_2\left(u, v, \partial u, \partial v, \partial^2 u, \partial^2 v, \cdots, \partial^{l_3} u, \partial^{l_4} v\right)=0, \end{cases} \tag{1}$$

where u and v are functions of the independent variables x and t, $\partial^d u(d=1,2,\cdots,\max(l_1,l_3))$ are all d-order partial derivatives of u with respect to independent variables x and t, and $\partial^d v(d=1,2,\cdots,\max(l_2,l_4))$ are all d-order partial derivatives of v with respect to independent variables x and t. Taking the traveling wave transformation

$$u=u(\xi), \quad v=v(\xi), \quad \xi=x+ct,$$

where c is the wave velocity, we can obtain nonlinear ordinary differential equations with constant coefficients

$$\begin{cases} M_1\left(u, v, u', v', u'', v'', \cdots, u^{(l_1)}, v^{(l_2)}\right)=0, \\ M_2\left(u, v, u', v', u'', v'', \cdots, u^{(l_3)}, v^{(l_4)}\right)=0. \end{cases} \tag{2}$$

When Eq. (2) cannot be directly reduced to an integral form, we take the trial equation

$$\begin{cases} (u')^2=H(u)=\sum_{i=1}^{n} a_i u^i, \\ v=T(u), \end{cases}$$

or

$$\begin{cases} (v')^2=H(v)=\sum_{j=1}^{n} b_j u^j, \\ u=T(v), \end{cases}$$

where H and T are two unknown functions. We can substitute the trial equation into the coupled equations to obtain the functions H and T. Integrating Eqs. (5) and (6), we have

$$\pm\left(\xi-\xi_2\right)=\int \frac{du}{\sqrt{H(u)}}, \tag{3}$$

or

$$\pm\left(\xi-\xi_2\right)=\int \frac{dv}{\sqrt{H(v)}}, \tag{4}$$

where ξ_2 is an integration constant.

Finally, we use the complete discrimination system for polynomial to classify the solutions of $H(u)$ or $H(v)$. Thus, Eq. (3) or (4) is solved, and we can obtain the exact traveling wave solutions of Eq. (1).

3. Exact solutions of the coupled Schrödinger-KdV equation

Consider the coupled Schrödinger-KdV equation:

$$\begin{cases} iu_t-u_{xx}-uv=0, \\ v_t+6vv_x+v_{xxx}-(|u|^2)_x=0. \end{cases} \tag{5}$$

Now, we use the following traveling wave transformation:

$$u(x,t)=\mathrm{e}^{i\theta}U(\xi), v(x,t)=V(\xi), \theta=\alpha x+\beta t, \xi=x+ct.$$

Substituting it into Eq. (5) and letting the real and imaginary parts both be 0, we have

$$c=2\alpha,$$

and the coupled nonlinear ordinary differential system

$$\begin{cases} U''+(\beta-\alpha^2)U+UV=0, \\ 2\alpha V'+6VV'+V'''-(U^2)'=0. \end{cases} \tag{6}$$

From the first equation of Eq. (6), we can obtain

$$V = \alpha^2 - \beta - \frac{U''}{U}.$$

Substituting the above equation into the second equation of Eq. (6), we have

$$(2\alpha^3 + 3\alpha^4 - 2\alpha\beta - 6\alpha^2\beta + 3\beta^2)U^3 - U^5 - 2(U')^2U'' + (-2\alpha U'' - 6\alpha^2 U'' + 6\beta U'' - U^{(4)})U^2 + (4(U'')^2 + 2U'U''')U = 0. \tag{7}$$

Now we suppose one takes a trial equation of order n

$$(u')^2 = \sum_{i=1}^{n} a_i u^i.$$

Therefore, we have

$$u'' = \frac{n}{2}a_n u^{n-1} + \frac{n-1}{2}a_{n-1}u^{n-2} + \cdots + \frac{1}{2}a_1,$$
$$u''' = \frac{n(n-1)}{2}a_n u^{n-2}u' + \frac{(n-1)(n-2)}{2}a_{n-1}u^{n-3}u' + \cdots + a_2 a_0,$$
$$u^{(4)} = \frac{2n(n-1)(n-2) + n^2(n-1)}{4}a_n^2 u^{2n-3} + \frac{3n(n-1)(n-2) + n(n-1)^2 + 2(n-1)(n-2)(n-3)}{4}a_n a_{n-1}u^{2n-4} + \cdots + \frac{a_2 a_1}{2},$$

then substituting the trial equation into Eq. (7) and making the highest unknown order in the equation equal to the highest known order, we have

$$n = 3.$$

So we obtain the specific form of the trial equation as follows:

$$(U')^2 = AU^3 + BU^2 + DU + E,$$

where A, B, D, and E are unknown parameters. Substituting the trial equation into Eq. (7) gives

$$r_5U^5 + r_4U^4 + r_3U^3 + r_2U^2 + r_1U + r_0 = 0,$$

where

$$\begin{cases} r_5 = -1 + \frac{9A^2}{2}, \\ r_4 = \frac{15AB}{2} - 3A\alpha - 9A\alpha^2 + 9A\beta, \\ r_3 = 3B^2 + \frac{7AD}{2} - 2B\alpha - 6B\alpha^2 + 2\alpha^3 + 3\alpha^4 + 6B\beta - 2\alpha\beta - 6\alpha^2\beta + 3\beta^2, \\ r_2 = \frac{5BD}{2} - D\alpha - 3D\alpha^2 + 3D\beta, \\ r_1 = 0, \\ r_0 = -DE. \end{cases}$$

Letting $r_i = 0, i = 0, 1, \cdots, 5$, we have the following relationship:

$$\begin{cases} A = \pm\frac{\sqrt{2}}{3}, \\ B = \frac{2}{5}(\alpha + 3\alpha^2 - 3\beta), \\ D = \mp\frac{3\sqrt{2}}{175}(-8\alpha^2 + 2\alpha^3 + 3\alpha^4 - 2\alpha\beta - 6\alpha^2\beta + 3\beta^2), \\ E = 0. \end{cases}$$

To solve the equation, we insert the following transformation into Eq. (7)

$$w = (\pm\frac{\sqrt{2}}{3})^{\frac{1}{3}}U, \xi_1 = (\pm\frac{\sqrt{2}}{3})^{\frac{1}{3}}\xi, \tag{8}$$

and obtain

$$(w')^2 = w^3 + a_2w^2 + a_1w = w(w^2 + a_2w + a_1) = w \cdot F(w), \tag{9}$$

where

$$a_2 = (\frac{\sqrt{2}}{3})^{-\frac{2}{3}}\frac{2}{5}(\alpha + 3\alpha^2 - 3\beta),$$
$$a_1 = -(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}\frac{3\sqrt{2}}{175}(-8\alpha^2 + 2\alpha^3 + 3\alpha^4 - 2\alpha\beta - 6\alpha^2\beta + 3\beta^2).$$

Then, transforming Eq. (9) into elementary integral form, there is

$$\pm(\xi_1 - \xi_2) = \int \frac{\mathrm{d}w}{\sqrt{w \cdot F(w)}},$$

where ξ_2 is an integration constant. The discriminant of $F(w)$ is

$$\Delta=\frac{276}{175}6^{\frac{1}{3}}\alpha^3+\frac{414}{175}6^{\frac{1}{3}}\alpha^4-\frac{276}{175}6^{\frac{1}{3}}\alpha\beta+\frac{414}{175}6^{\frac{1}{3}}\beta^2+\alpha^2(-\frac{54}{175}6^{\frac{1}{3}}-\frac{828}{175}6^{\frac{1}{3}}\beta).$$

According to the discrimination system, we obtain the following three families of exact solutions.

Family 1. When $\Delta=0$, $F(w)=0$ has a double real root, which can be expressed as $F(w)=(w+\frac{a_2}{2})^2$. When $a_2>0$, the solution of Eq. (5) is

$$\begin{cases} u_1(x,t)=\pm(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}\frac{a_2}{2}\tan^2(\frac{\sqrt{a_2}}{2\sqrt{2}}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2))e^{i\theta}, \\ v_1(x,t)=-\frac{1}{5}\alpha^2+\frac{1}{5}\beta-\frac{2}{5}\alpha+\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}\frac{a_2}{2}\tan^2(\frac{\sqrt{a_2}}{2\sqrt{2}}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2))-\frac{\sqrt{2}((\alpha+3\alpha^2-3\beta)^2-25\alpha^2)}{350(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}\frac{a_2}{2}\tan^2(\frac{\sqrt{a_2}}{2\sqrt{2}}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2))}, \end{cases}$$

where $w>0$. This is a singular solution.

When $a_2<0$, the solutions of Eq. (5) are

$$\begin{cases} u_2(x,t)=\mp(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}\frac{a_2}{2}\coth^2(\frac{1}{2}\sqrt{-\frac{a_2}{2}}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2))e^{i\theta}, \\ v_2(x,t)=-\frac{1}{5}\alpha^2+\frac{1}{5}\beta-\frac{2}{5}\alpha-\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}\frac{a_2}{2}\coth^2(\frac{1}{2}\sqrt{-\frac{a_2}{2}}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2)) \\ \qquad +\frac{\sqrt{2}((\alpha+3\alpha^2-3\beta)^2-25\alpha^2)}{350(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}\frac{a_2}{2}\coth^2(\frac{1}{2}\sqrt{-\frac{a_2}{2}}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2))}, \end{cases}$$

$$\begin{cases} u_3(x,t)=\mp(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}\frac{a_2}{2}\tanh^2(\frac{1}{2}\sqrt{-\frac{a_2}{2}}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2))e^{i\theta}, \\ v_3(x,t)=-\frac{1}{5}\alpha^2+\frac{1}{5}\beta-\frac{2}{5}\alpha-\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}\frac{a_2}{2}\tanh^2(\frac{1}{2}\sqrt{-\frac{a_2}{2}}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2)) \\ \qquad +\frac{\sqrt{2}((\alpha+3\alpha^2-3\beta)^2-25\alpha^2)}{350(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}\frac{a_2}{2}\tanh^2(\frac{1}{2}\sqrt{-\frac{a_2}{2}}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2))}, \end{cases}$$

where $w>0$. Both of these are solitary wave solutions.

Family 2. When $\Delta>0$, $F(w)=0$ has two different real roots, which can be expressed as $F(w)=(w-z_1)(w-z_2)$.

When $0>z_1>z_2$, the solutions of Eq. (5) are

$$\begin{cases} u_4(x,t)=\pm(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}(z_2+(z_1-z_2)\mathrm{sn}^2(\frac{\sqrt{-z_2}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2),m))e^{i\theta}, \\ v_4(x,t)=-\frac{1}{5}\alpha^2+\frac{1}{5}\beta-\frac{2}{5}\alpha+\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}(z_2+(z_1-z_2)\mathrm{sn}^2(\frac{\sqrt{-z_2}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2),m)) \\ \qquad -\frac{\sqrt{2}((\alpha+3\alpha^2-3\beta)^2-25\alpha^2)}{350(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}(z_2+(z_1-z_2)\mathrm{sn}^2(\frac{\sqrt{-z_2}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2),m))}, \end{cases}$$

$$\begin{cases} u_5(x,t)=\pm(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}\frac{-z_1\mathrm{sn}^2(\frac{\sqrt{-z_2}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2),m)}{\mathrm{cn}^2(\frac{\sqrt{-z_2}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2),m)}e^{i\theta}, \\ v_5(x,t)=-\frac{1}{5}\alpha^2+\frac{1}{5}\beta-\frac{2}{5}\alpha+\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}\frac{-z_1\mathrm{sn}^2(\frac{\sqrt{-z_2}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2),m)}{\mathrm{cn}^2(\frac{\sqrt{-z_2}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2),m)} \\ \qquad +\frac{\sqrt{2}((\alpha+3\alpha^2-3\beta)^2-25\alpha^2)\mathrm{cn}^2(\frac{\sqrt{-z_2}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2),m)}{350z_1\mathrm{sn}^2(\frac{\sqrt{-z_2}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2),m)}, \end{cases}$$

where $m^2=\frac{z_1-z_2}{-z_2}$. These are double periodic solutions.

When $0 = z_1 > z_2$, the solutions of Eq. (5) are

$$\begin{cases} u_6(x,t) = \pm(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}(-z_2\tanh^2(\frac{\sqrt{-z_2}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2))+z_2)e^{i\theta}, \\ v_6(x,t) = -\frac{1}{5}\alpha^2+\frac{1}{5}\beta-\frac{2}{5}\alpha+\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}(-z_2\tanh^2(\frac{\sqrt{-z_2}}{2}((\frac{\sqrt{2}}{3}^{-\frac{1}{3}}(x+ct)-\xi_2))+z_2) \\ \qquad -\frac{\sqrt{2}((\alpha+3\alpha^2-3\beta)^2-25\alpha^2)}{350(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}(-z_2\tanh^2(\frac{\sqrt{-z_2}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2))+z_2)}. \end{cases}$$

$$\begin{cases} u_7(x,t) = \pm(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}(-z_2\coth^2\frac{\sqrt{-z_2}}{2}((\pm\frac{\sqrt{2}}{3})^{-\frac{1}{3}}(x+ct)-\xi_2)+z_2)e^{i\theta}, \\ v_7(x,t) = -\frac{1}{5}\alpha^2+\frac{1}{5}\beta-\frac{2}{5}\alpha+\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}(-z_2\coth^2\frac{\sqrt{-z_2}}{2}((\pm\frac{\sqrt{2}}{3})^{-\frac{1}{3}}(x+ct)-\xi_2)+z_2) \\ \qquad -\frac{\sqrt{2}((\alpha+3\alpha^2-3\beta)^2-25\alpha^2)}{350(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}(-z_2\coth^2\frac{\sqrt{-z_2}}{2}((\pm\frac{\sqrt{2}}{3})^{-\frac{1}{3}}(x+ct)-\xi_2)+z_2)}. \end{cases}$$

Both of these are solitary wave solutions.

When $z_1 > 0 > z_2$, the solutions of Eq. (5) are

$$\begin{cases} u_8(x,t) = \pm(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}(-z_2\mathrm{cn}^2(\frac{\sqrt{z_1-z_2}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2),l)e^{i\theta}, \\ v_8(x,t) = -\frac{1}{5}\alpha^2+\frac{1}{5}\beta-\frac{2}{5}\alpha+\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}(-z_2\mathrm{cn}^2(\frac{\sqrt{z_1-z_2}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2),l) \\ \qquad -\frac{\sqrt{2}((\alpha+3\alpha^2-3\beta)^2-25\alpha^2)}{350(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}(-z_2\mathrm{cn}^2(\frac{\sqrt{z_1-z_2}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2),l)}, \end{cases}$$

$$\begin{cases} u_9(x,t) = \pm(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}\frac{z_1}{\mathrm{cn}^2(\frac{\sqrt{z_1-z_2}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2),l)}e^{i\theta}, \\ v_9(x,t) = -\frac{1}{5}\alpha^2+\frac{1}{5}\beta-\frac{2}{5}\alpha+\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}\frac{z_1}{\mathrm{cn}^2(\frac{\sqrt{z_1-z_2}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2),l)} \\ \qquad -\frac{\sqrt{2}((\alpha+3\alpha^2-3\beta)^2-25\alpha^2)\mathrm{cn}^2(\frac{\sqrt{z_1-z_2}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2),l)}{350(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}z_1}, \end{cases}$$

where $l^2 = \frac{-z_2}{z_1-z_2}$. These are double periodic solutions.

When $z_1 > 0 = z_2$, the solutions of Eq. (5) are

$$\begin{cases} u_{10}(x,t) = \pm(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}(z_1+z_1\tan^2(\frac{\sqrt{z_1}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2))e^{i\theta}, \\ v_{10}(x,t) = -\frac{1}{5}\alpha^2+\frac{1}{5}\beta-\frac{2}{5}\alpha+\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}(z_1+z_1\tan^2(\frac{\sqrt{z_1}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2)) \\ \qquad -\frac{\sqrt{2}((\alpha+3\alpha^2-3\beta)^2-25\alpha^2)}{350(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}(z_1+z_1\tan^2(\frac{\sqrt{z_1}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2))}. \end{cases}$$

This is also a singular solution.

When $z_1 > z_2 > 0$, the solutions of Eq. (5) are

$$\begin{cases} u_{11}(x,t) = \pm(\frac{\sqrt{2}}{3})^{-\frac{1}{3}} z_2 \mathrm{sn}^2(\frac{\sqrt{z_1}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2), j)\mathrm{e}^{i\theta}, \\ v_{11}(x,t) = -\frac{1}{5}\alpha^2 + \frac{1}{5}\beta - \frac{2}{5}\alpha + \frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{-\frac{1}{3}} z_2 \mathrm{sn}^2(\frac{\sqrt{z_1}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct) \\ \quad -\xi_2), j) - \frac{\sqrt{2}((\alpha+3\alpha^2-3\beta)^2-25\alpha^2)}{350(\frac{\sqrt{2}}{3})^{-\frac{1}{3}} z_2 \mathrm{sn}^2(\frac{\sqrt{z_1}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2), j)}, \end{cases}$$

$$\begin{cases} u_{12}(x,t) = \pm(\frac{\sqrt{2}}{3})^{-\frac{1}{3}} \frac{-z_2 \mathrm{sn}^2(\frac{\sqrt{z_1}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2), j)+z_1}{\mathrm{cn}^2(\frac{\sqrt{z_1}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2), j)}\mathrm{e}^{i\theta}, \\ v_{12}(x,t) = -\frac{1}{5}\alpha^2 + \frac{1}{5}\beta - \frac{2}{5}\alpha + \frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{-\frac{1}{3}} \frac{-z_2 \mathrm{sn}^2(\frac{\sqrt{z_1}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2), j)+z_1}{\mathrm{cn}^2(\frac{\sqrt{z_1}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2), j)} \\ \quad - \frac{\sqrt{2}((\alpha+3\alpha^2-3\beta)^2-25\alpha^2)\mathrm{cn}^2(\frac{\sqrt{z_1}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2), j)}{350(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}(-z_2 \mathrm{sn}^2(\frac{\sqrt{z_1}}{2}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2), j)+z_1)}, \end{cases}$$

where $j^2 = \frac{z_1}{z_2}$. These are double periodic solutions.

Family 3. When $\Delta < 0$, $F(w) = 0$ has a pair of conjugate complex roots, and then $F(w)$ can be expressed by $F(w) = w^2 + pw + q$, where $p^2 - 4q < 0$. The solution of Eq. (5) is

$$\begin{cases} u_{13}(x,t) = \pm(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}(-\sqrt{q} + \frac{2\sqrt{q}}{1+\mathrm{cn}(q^{\frac{1}{4}}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2), y)})\mathrm{e}^{i\theta}, \\ v_{13}(x,t) = -\frac{1}{5}\alpha^2 + \frac{1}{5}\beta - \frac{2}{5}\alpha + \frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}(-\sqrt{q} + \frac{2\sqrt{q}}{1+\mathrm{cn}(q^{\frac{1}{4}}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2), y)}) \\ \quad - \frac{\sqrt{2}((\alpha+3\alpha^2-3\beta)^2-25\alpha^2)}{350(\frac{\sqrt{2}}{3})^{-\frac{1}{3}}(-\sqrt{q} + \frac{2\sqrt{q}}{1+\mathrm{cn}(q^{\frac{1}{4}}((\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+ct)-\xi_2), y)})}, \end{cases}$$

where $y^2 = \frac{1}{2} - \frac{p}{4\sqrt{q}}$. This is a double periodic solution.

4. Physical realization of exact solutions

In this section, we replace the parameters in the solutions of the system with an arbitrary choice of parameter values within a reasonable range and use Mathematica to plot three-dimensional images of the solutions of the system, where the independent variables belong to $(-5000, 5000)$. The red parts represent u, while the blue parts represent v. There are three kinds of solutions: solitary wave solutions, discontinuous periodic solutions, and Jacobian elliptical function solutions. These images help to understand the spatiotemporal structure of the processes of dusty plasma.

Case 1. Discontinuous periodic solutions

Taking $\alpha = 1, \beta = \frac{4}{3} - \frac{5\sqrt{46}}{69}, \xi_2 = 0$, we get

$$\begin{cases} u_1 = \frac{3\sqrt{23}}{23}\tan^2 46^{-\frac{3}{4}}\sqrt{23}(x+2t)\mathrm{e}^{i(x+(\frac{4}{3}-\frac{5\sqrt{46}}{69})t)}, \\ v_1 = -\frac{1}{3} - \frac{\sqrt{46}}{69} + \frac{3\sqrt{46}}{46}\tan^2 46^{-\frac{3}{4}}\sqrt{23}(x+2t) + \frac{\sqrt{46}}{46\tan^2 46^{-\frac{3}{4}}\sqrt{23}(x+2t)}. \end{cases}$$

The 3D image is shown in Fig. 1.

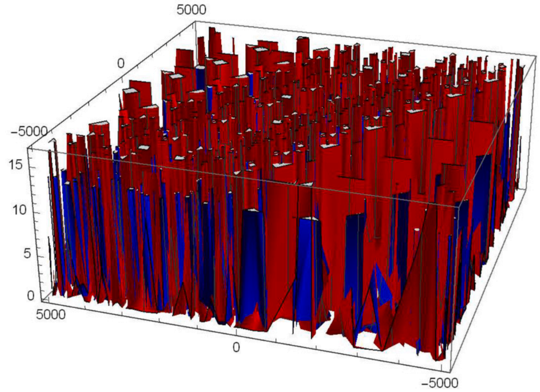

Fig. 1. The modules of $u_1(x,t)$ and $v_1(x,t)$.

Taking $\alpha=-1, \beta=\frac{1}{3}, \xi_2=0$, we get

$$\begin{cases} u_{10}=(\frac{\sqrt{2}}{3})^{\frac{-1}{3}}(2(\frac{\sqrt{2}}{3})^{\frac{-2}{3}}+2(\frac{\sqrt{2}}{3})^{\frac{-2}{3}}\tan^2(\frac{\sqrt{2}}{2}(x-2t)))e^{i(-x+\frac{1}{3}t)}, \\ v_{10}=\frac{4}{15}+\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{\frac{-1}{3}}(2(\frac{\sqrt{2}}{3})^{\frac{-2}{3}}+2(\frac{\sqrt{2}}{3})\tan^2(\frac{\sqrt{2}}{2}(x-2t)))+\frac{4}{175(\frac{\sqrt{2}}{3})^{\frac{-1}{3}}(2(\frac{\sqrt{2}}{3})^{\frac{-2}{3}}+2(\frac{\sqrt{2}}{3})\tan^2(\frac{\sqrt{2}}{2}(x-2t)))}. \end{cases}$$

The 3D image is shown in Fig. 10. Both u_1 and u_{10} are singular periodic patterns.

Case 2. Solitary wave solutions

Taking $\alpha=-1, \beta=\frac{2}{3}-\frac{5\sqrt{46}}{69}, \xi_2=0$, we get

$$\begin{cases} u_2=3\frac{\sqrt{23}}{23}\coth^2(\frac{1}{2}\sqrt{\frac{\sqrt{46}}{23}}(x-2t))e^{i(-x+(\frac{2}{3}-\frac{5\sqrt{46}}{69})t)}, \\ v_2=\frac{1}{3}-\frac{\sqrt{46}}{69}+3\frac{\sqrt{46}}{46}\coth^2(\frac{1}{2}\sqrt{\frac{\sqrt{46}}{23}}(x-2t))+\frac{\sqrt{2}}{\sqrt{23}\coth^2(\frac{1}{2}\sqrt{\frac{\sqrt{46}}{23}}(x-2t))}. \end{cases}$$

The 3D image is shown in Fig. 2.

Taking $\alpha=-1, \beta=\frac{2}{3}-\frac{5\sqrt{46}}{69}, \xi_2=0$, we get

$$\begin{cases} u_3=3\frac{\sqrt{23}}{23}\tanh^2(\frac{1}{2}\sqrt{\frac{\sqrt{46}}{23}}(x-2t))e^{i(-x+(\frac{2}{3}-\frac{5\sqrt{46}}{69})t)}, \\ v_3=\frac{1}{3}-\frac{\sqrt{46}}{69}+3\frac{\sqrt{46}}{46}\tanh^2(\frac{1}{2}\sqrt{\frac{\sqrt{46}}{23}}(x-2t))+\frac{\sqrt{2}}{\sqrt{23}\tanh^2(\frac{1}{2}\sqrt{\frac{\sqrt{46}}{23}}(x-2t))}. \end{cases}$$

The 3D image is shown in Fig. 3.

Taking $\alpha=1, \beta=-\frac{1}{3}, \xi_2=0$, we get

$$\begin{cases} u_6=(\frac{\sqrt{2}}{3})^{\frac{-1}{3}}(2(\frac{\sqrt{2}}{3})^{\frac{-2}{3}}\tanh^2(\frac{\sqrt{2}}{2}(x+2t))-2(\frac{\sqrt{2}}{3})^{\frac{-2}{3}})e^{i(x-\frac{1}{3}t)}, \\ v_6=\frac{-2}{3}+\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{\frac{-1}{3}}(2(\frac{\sqrt{2}}{3})^{\frac{-2}{3}}\tanh^2(\frac{\sqrt{2}}{2}(x+2t))-2(\frac{\sqrt{2}}{3})^{\frac{-2}{3}}). \end{cases}$$

The 3D image is shown in Fig. 6.

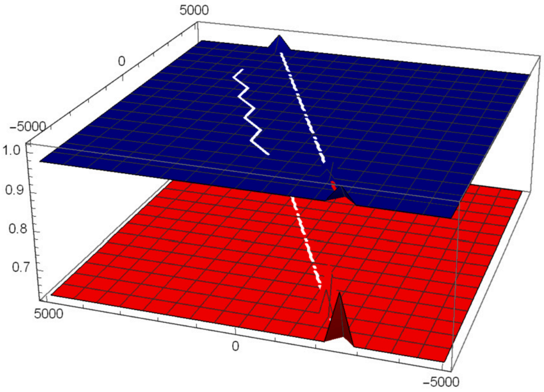

Fig. 2. The modules of $u_2(x,t)$ and $v_2(x,t)$.

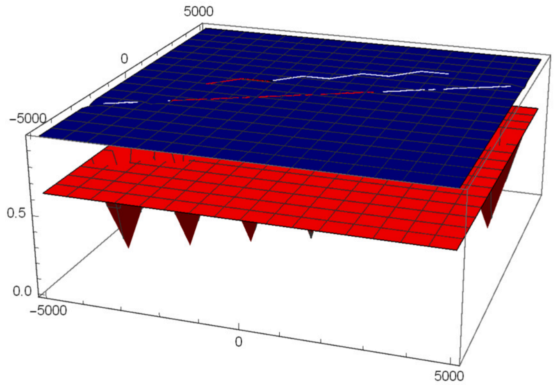

Fig. 3. The modules of $u_3(x,t)$ and $v_3(x,t)$.

Taking $\alpha = 1, \beta = -\frac{1}{3}, \xi_2 = 0$, we get

$$\begin{cases} u_7 = (\frac{\sqrt{2}}{3})^{\frac{-1}{3}}(2(\frac{\sqrt{2}}{3})^{\frac{-2}{3}}\coth^2(\frac{\sqrt{2}}{2}(x+2t)) - 2(\frac{\sqrt{2}}{3})^{\frac{-2}{3}})e^{i(x-\frac{1}{3}t)}, \\ v_7 = \frac{-2}{3} + \frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{\frac{-1}{3}}(2(\frac{\sqrt{2}}{3})^{\frac{-2}{3}}\coth^2(\frac{\sqrt{2}}{2}(x+2t)) - 2(\frac{\sqrt{2}}{3})^{\frac{-2}{3}}). \end{cases}$$

The 3D image is shown in Fig. 7. u_2, u_3, u_6 and u_7 are all solitary wave patterns.

Case 3. Jacobian elliptical function solutions

Taking $\alpha = \frac{\sqrt{31}}{2^{\frac{2}{3}}3^{\frac{1}{6}}}, \beta = \frac{1}{36}(-30\times 6^{\frac{1}{3}} + 93\times 6^{\frac{2}{3}} + 2^{\frac{4}{3}}\times 3^{\frac{5}{6}}\sqrt{31}), \xi_2 = 0$, we get

$$\begin{cases} u_4 = (\frac{\sqrt{2}}{3})^{\frac{-1}{3}}(-2+\mathrm{sn}^2(\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+2^{\frac{1}{3}}\frac{\sqrt{31}}{3^{\frac{1}{6}}}t), \frac{\sqrt{2}}{2}))e^{i(\frac{\sqrt{31}}{2^{\frac{2}{3}}3^{\frac{1}{6}}}x + \frac{1}{36}(-30\times 6^{\frac{1}{3}}+93\times 6^{\frac{2}{3}}+2^{\frac{4}{3}}\times 3^{\frac{5}{6}}\sqrt{31})t)}, \\ v_4 = -31\frac{\sqrt{2}}{20}(\frac{\sqrt{2}}{3})^{\frac{1}{3}} - 2^{\frac{1}{3}}\frac{\sqrt{31}}{6\times 3^{\frac{1}{6}}} + \frac{1}{60}(-10\times 6^{\frac{1}{3}} + 31\times 6^{\frac{2}{3}}) + \frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{\frac{-1}{3}}(-2+\mathrm{sn}^2(\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+2^{\frac{1}{3}}\frac{\sqrt{31}}{3^{\frac{1}{6}}}t), \frac{\sqrt{2}}{2})) \\ \quad - \frac{2^{\frac{1}{6}}}{3^{\frac{1}{3}}(\frac{\sqrt{2}}{3})^{\frac{-1}{3}}(-2+\mathrm{sn}^2(\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+2^{\frac{1}{3}}\frac{\sqrt{31}}{3^{\frac{1}{6}}}t), \frac{\sqrt{2}}{2}))}. \end{cases}$$

The 3D image is shown in Fig. 4.

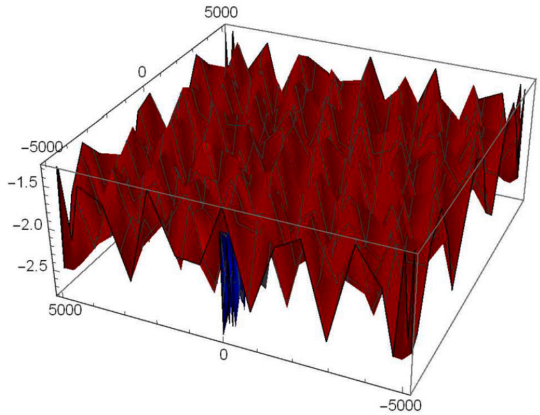

Fig. 4. The modules of $u_4(x,t)$ and $v_4(x,t)$.

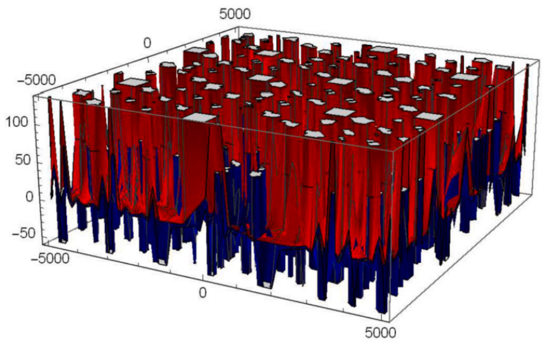

Fig. 5. The modules of $u_5(x,t)$ and $v_5(x,t)$.

Taking $\alpha = \frac{\sqrt{31}}{2^{\frac{2}{3}}3^{\frac{1}{6}}}, \beta = \frac{1}{36}(-30\times 6^{\frac{1}{3}} + 93\times 6^{\frac{2}{3}} + 2^{\frac{4}{3}}\times 3^{\frac{5}{6}}\sqrt{31}), \xi_2 = 0$, we get

$$\begin{cases} u_5 =(\frac{\sqrt{2}}{3})^{\frac{-1}{3}} \dfrac{\mathrm{sn}^2(\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+\frac{2^{\frac{1}{3}}\sqrt{31}}{3^{\frac{1}{6}}}t), \frac{\sqrt{2}}{2})}{\mathrm{cn}^2(\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+\frac{2^{\frac{1}{3}}\sqrt{31}}{3^{\frac{1}{6}}}t), \frac{\sqrt{2}}{2})} \mathrm{e}^{i(\frac{\sqrt{31}}{2^{\frac{2}{3}}3^{\frac{1}{6}}}x+\frac{1}{36}(-30\times 6^{\frac{1}{3}}+93\times 6^{\frac{2}{3}}+2^{\frac{4}{3}}\times 3^{\frac{5}{6}}\sqrt{31})t)}, \\ v_5 = -31\frac{\sqrt{2}}{20} - 2^{\frac{1}{3}}\frac{\sqrt{31}}{6\times 3^{\frac{1}{6}}} + \frac{1}{60}(-10\times 6^{\frac{1}{3}} + 31\times 6^{\frac{2}{3}}) + \frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{\frac{-1}{3}} \\ \quad \times \dfrac{\mathrm{sn}^2(\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+\frac{2^{\frac{1}{3}}\sqrt{31}}{3^{\frac{1}{6}}}t), \frac{\sqrt{2}}{2})}{\mathrm{cn}^2(\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+\frac{2^{\frac{1}{3}}\sqrt{31}}{3^{\frac{1}{6}}}t), \frac{\sqrt{2}}{2})} - \dfrac{2^{\frac{1}{6}}\mathrm{cn}^2(\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+\frac{2^{\frac{1}{3}}\sqrt{31}}{3^{\frac{1}{6}}}t), \frac{\sqrt{2}}{2})}{3^{\frac{1}{3}}\mathrm{sn}^2(\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+\frac{2^{\frac{1}{3}}\sqrt{31}}{3^{\frac{1}{6}}}t), \frac{\sqrt{2}}{2})}. \end{cases}$$

The 3D image is shown in Fig. 5.

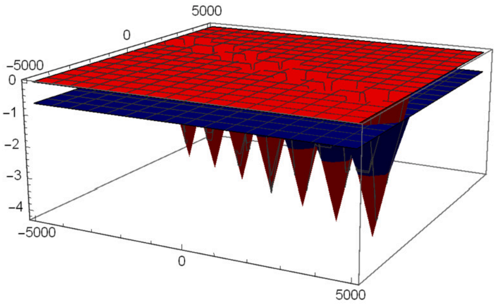

Fig. 6. The modules of $u_6(x,t)$ and $v_6(x,t)$.

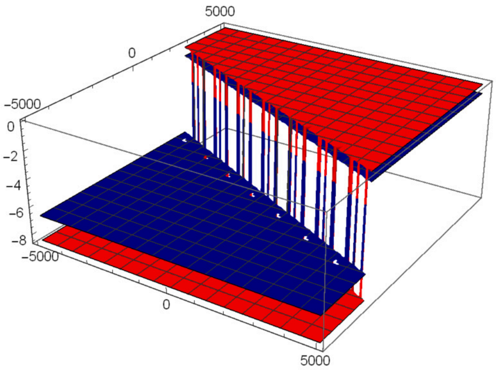

Fig. 7. The modules of $u_7(x,t)$ and $v_7(x,t)$.

Taking $\alpha=1, \beta=-1, \xi_2=0$, we get

$$\begin{cases} u_8 = (\frac{\sqrt{2}}{3})^{\frac{-1}{3}}(\frac{1}{10}(6^{\frac{2}{3}}\sqrt{\frac{359}{7}}+7\times 6^{\frac{2}{3}}))\mathrm{cn}^2(\frac{6^{\frac{1}{3}}(\frac{359}{7})^{\frac{1}{4}}}{2\sqrt{5}}(\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+2t), \frac{2}{6^{\frac{2}{3}}}+\frac{14\sqrt{7}}{6^{\frac{2}{3}}\sqrt{359}})\mathrm{e}^{i(x-t)}, \\ v_8 = \frac{-4}{5}+\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{\frac{-1}{3}}(\frac{1}{10}(6^{\frac{2}{3}}\sqrt{\frac{359}{7}}+7\times 6^{\frac{2}{3}}))\mathrm{cn}^2(\frac{6^{\frac{1}{3}}(\frac{359}{7})^{\frac{1}{4}}}{2\sqrt{5}}(\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x \\ \quad +2t), \frac{2}{6^{\frac{2}{3}}}+\frac{14\sqrt{7}}{6^{\frac{2}{3}}\sqrt{359}}) - \frac{12\sqrt{2}}{175(\frac{\sqrt{2}}{3})^{\frac{-1}{3}}(\frac{1}{10}(6^{\frac{2}{3}}\sqrt{\frac{359}{7}}+7\times 6^{\frac{2}{3}}))} \\ \quad \times \frac{1}{\mathrm{cn}^2(\frac{6^{\frac{1}{3}}(\frac{359}{7})^{\frac{1}{4}}}{2\sqrt{5}}(\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+2t), \frac{2}{6^{\frac{2}{3}}}+\frac{14\sqrt{7}}{6^{\frac{2}{3}}\sqrt{359}})}. \end{cases}$$

The 3D image is shown in Fig. 8.

Taking $\alpha=1, \beta=-1, \xi_2=0$, we get

$$\begin{cases} u_9 = (\frac{\sqrt{2}}{3})^{\frac{-1}{3}}\frac{(6^{\frac{2}{3}}\sqrt{\frac{359}{7}}-7\times 6^{\frac{2}{3}})}{10\mathrm{cn}^2(\frac{6^{\frac{1}{3}}(\frac{359}{7})^{\frac{1}{4}}}{2\sqrt{5}}(\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+2t), \frac{2}{6^{\frac{2}{3}}}+\frac{14\sqrt{7}}{6^{\frac{2}{3}}\sqrt{359}})}\mathrm{e}^{i(x-t)}, \\ v_9 = \frac{-4}{5}+\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{\frac{-1}{3}}\frac{(6^{\frac{2}{3}}\sqrt{\frac{359}{7}}-7\times 6^{\frac{2}{3}})}{10\mathrm{cn}^2(\frac{6^{\frac{1}{3}}(\frac{359}{7})^{\frac{1}{4}}}{2\sqrt{5}}(\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+2t), \frac{2}{6^{\frac{2}{3}}}+\frac{14\sqrt{7}}{6^{\frac{2}{3}}\sqrt{359}})} - \frac{24\sqrt{2}\mathrm{cn}^2(\frac{6^{\frac{1}{3}}(\frac{359}{7})^{\frac{1}{4}}}{2\sqrt{5}}(\frac{\sqrt{2}}{3})^{\frac{1}{3}}(x+2t), \frac{2}{6^{\frac{2}{3}}}+\frac{14\sqrt{7}}{6^{\frac{2}{3}}\sqrt{359}})}{35(\frac{\sqrt{2}}{3})^{\frac{-1}{3}}(6^{\frac{2}{3}}\sqrt{\frac{359}{7}}-7\times 6^{\frac{2}{3}})}. \end{cases}$$

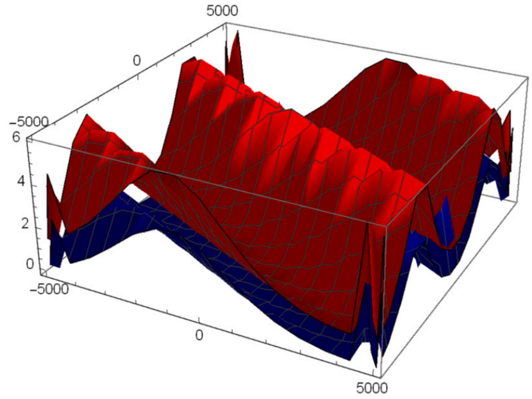

Fig. 8. The modules of $u_8(x,t)$ and $v_8(x,t)$.

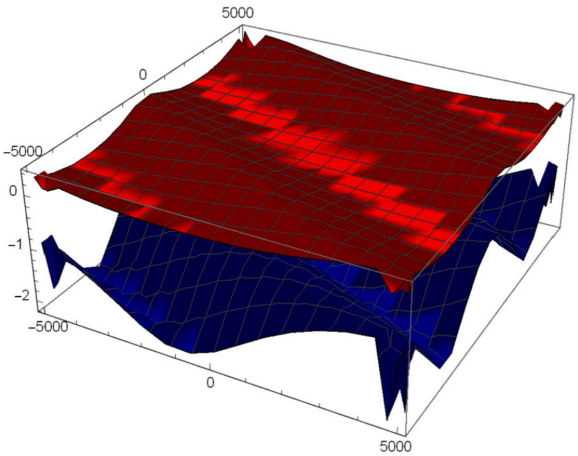

Fig. 9. The modules of $u_9(x,t)$ and $v_9(x,t)$.

The 3D image is shown in Fig. 9.

Taking $\alpha=1,\beta=2,\xi_2=0$, we get

$$\begin{cases} u_{11}=(\frac{\sqrt{2}}{3})^{\frac{-1}{3}}\frac{1}{5}(-2^{\frac{1}{6}}\times 3^{\frac{2}{3}}+6^{\frac{2}{3}})\mathrm{sn}^2(\frac{\sqrt{1+\sqrt{2}}}{2^{\frac{3}{4}}\sqrt{5}}(x+2t),\frac{\sqrt{2}-1}{\sqrt{2}+1})e^{i(x+2t)}, \\ v_{11}=\frac{-1}{5}+\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{\frac{-1}{3}}\frac{1}{5}(-2^{\frac{1}{6}}\times 3^{\frac{2}{3}}+6^{\frac{2}{3}})\mathrm{sn}^2(\frac{\sqrt{1+\sqrt{2}}}{2^{\frac{3}{4}}\sqrt{5}}(x+2t),\frac{\sqrt{2}-1}{\sqrt{2}+1}) \\ \qquad +\frac{3\sqrt{2}}{10(\frac{\sqrt{2}}{3})^{\frac{-1}{3}}\frac{1}{5}(-2^{\frac{1}{6}}\times 3^{\frac{2}{3}}+6^{\frac{2}{3}})\mathrm{sn}^2(\frac{\sqrt{1+\sqrt{2}}}{2^{\frac{3}{4}}\sqrt{5}}(x+2t),\frac{\sqrt{2}-1}{\sqrt{2}+1})}. \end{cases}$$

The 3D image is shown in Fig. 11.

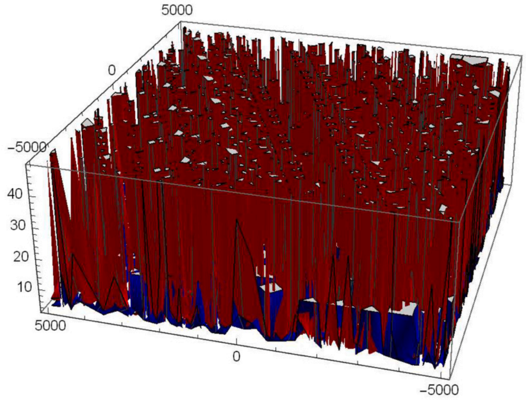

Fig. 10. The modules of $u_{10}(x,t)$ and $v_{10}(x,t)$.

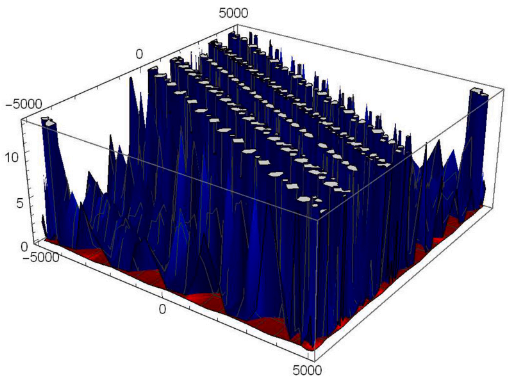

Fig. 11. The modules of $u_{11}(x,t)$ and $v_{11}(x,t)$.

Taking $\alpha=1,\beta=2,\xi_2=0$, we get

$$\begin{cases} u_{12}=\left(\dfrac{\sqrt{2}}{3}\right)^{\frac{-1}{3}}\dfrac{\frac{1}{5}\left(-2^{\frac{1}{6}}\times 3^{\frac{2}{3}}-6^{\frac{2}{3}}\right)\left(\mathrm{sn}^2\left(\frac{\sqrt{1+\sqrt{2}}}{2^{\frac{3}{4}}\sqrt{5}}(x+2t),\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)+1\right)}{\mathrm{cn}^2\left(\frac{\sqrt{1+\sqrt{2}}}{2^{\frac{3}{4}}\sqrt{5}}(x+2t),\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)}\mathrm{e}^{i(x+2t)}, \\ v_{12}=\dfrac{-1}{5}+\dfrac{\sqrt{2}}{2}\left(\dfrac{\sqrt{2}}{3}\right)^{\frac{-1}{3}}\dfrac{\frac{1}{5}\left(-2^{\frac{1}{6}}\times 3^{\frac{2}{3}}-6^{\frac{2}{3}}\right)\left(\mathrm{sn}^2\left(\frac{\sqrt{1+\sqrt{2}}}{2^{\frac{3}{4}}\sqrt{5}}(x+2t),\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)+1\right)}{\mathrm{cn}^2\left(\frac{\sqrt{1+\sqrt{2}}}{2^{\frac{3}{4}}\sqrt{5}}(x+2t),\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)} \\ \qquad +\dfrac{3\sqrt{2}\,\mathrm{cn}^2\left(\frac{\sqrt{1+\sqrt{2}}}{2^{\frac{3}{4}}\sqrt{5}}(x+2t),\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)}{50\left(\frac{\sqrt{2}}{3}\right)^{\frac{-1}{3}}\left(\frac{1}{5}\left(-2^{\frac{1}{6}}\times 3^{\frac{2}{3}}-6^{\frac{2}{3}}\right)\left(\mathrm{sn}^2\left(\frac{\sqrt{1+\sqrt{2}}}{2^{\frac{3}{4}}\sqrt{5}}(x+2t),\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)+1\right)\right)}. \end{cases}$$

The 3D image is shown in Fig. 12.

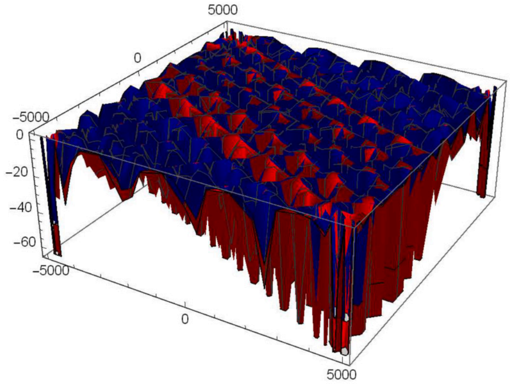

Fig. 12. The modules of $u_{12}(x,t)$ and $v_{12}(x,t)$.

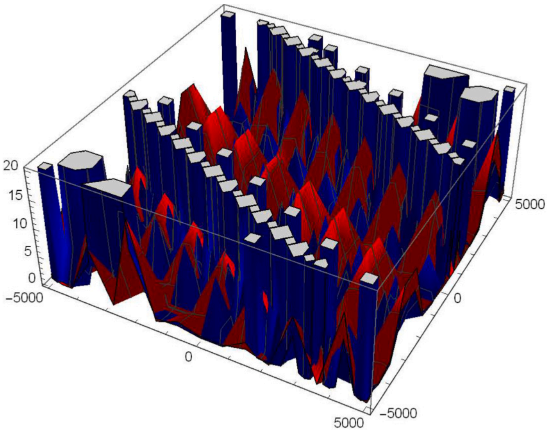

Fig. 13. The modules of $u_{13}(x,t)$ and $v_{13}(x,t)$.

Taking $\alpha = 1, \beta = \frac{4}{3}, \xi_2 = 0$, we get

$$\begin{cases} u_{13} = (\frac{\sqrt{2}}{3})^{\frac{-1}{3}}\left(-\sqrt{\frac{\sqrt{2}}{7}(\frac{\sqrt{2}}{3})^{\frac{-1}{3}}} + \dfrac{2\sqrt{\frac{\sqrt{2}}{7}(\frac{\sqrt{2}}{3})^{\frac{-1}{3}}}}{1+\mathrm{cn}^2((\frac{\sqrt{2}}{7})^{\frac{1}{4}}(\frac{\sqrt{2}}{3})^{\frac{1}{4}}(x+2t), \frac{\sqrt{2}}{2})}\right)e^{i(x+\frac{4}{3}t)}, \\ v_{13} = \frac{-1}{3} + \frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{3})^{\frac{-1}{3}}\left(-\sqrt{\frac{\sqrt{2}}{7}(\frac{\sqrt{2}}{3})^{\frac{-1}{3}}} + \dfrac{2\sqrt{\frac{\sqrt{2}}{7}(\frac{\sqrt{2}}{3})^{\frac{-1}{3}}}}{1+\mathrm{cn}^2((\frac{\sqrt{2}}{7})^{\frac{1}{4}}(\frac{\sqrt{2}}{3})^{\frac{1}{4}}(x+2t), \frac{\sqrt{2}}{2})}\right) \\ \quad + \dfrac{\sqrt{2}}{14(\frac{\sqrt{2}}{3})^{\frac{-1}{3}}\left(-\sqrt{\frac{\sqrt{2}}{7}(\frac{\sqrt{2}}{3})^{\frac{-1}{3}}} + \frac{2\sqrt{\frac{\sqrt{2}}{7}(\frac{\sqrt{2}}{3})^{\frac{-1}{3}}}}{1+\mathrm{cn}^2((\frac{\sqrt{2}}{7})^{\frac{1}{4}}(\frac{\sqrt{2}}{3})^{\frac{1}{4}}(x+2t), \frac{\sqrt{2}}{2})}\right)}. \end{cases}$$

The 3D image is shown in Fig. 13. u_4, u_5, u_8, u_9, u_{11}, u_{12} and u_{13} are all double periodic patterns.

5. Conclusion

This paper uses the coupled trial equation method to solve the coupled Schrödinger-KdV equation, which describes various processes in dusty plasma. By applying the generalized coupled trial equation method and the complete discrimination system for polynomial, a rich variety of exact solutions are obtained, including discontinuous periodic solutions, solitary wave solutions, and Jacobian elliptical function solutions. The physical representations of the three-dimensional images can intuitively express the propagation mode of the process in dusty plasma. Compared to other methods, more comprehensive and accurate solutions are obtained, and the outcomes give the model more profound physical properties. This paper concludes that the generalized coupled trial equation method and the complete discrimination system for polynomial can be well applied to studying nonlinear phenomena and provide practical and effective strategies for existing practical problems.

CRediT authorship contribution statement

Jiaxin Shang: Analyzed and interpreted the data; Wrote the paper.
Wenhe Li: Conceived and designed the analysis.
Da Li: Contributed analysis tools or data.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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