



# The emergence and resilience of self-organized governance in coupled infrastructure systems

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**Studies of small-scale, self-organized social-ecological systems have contributed to our understanding of successful governance of shared resources. However, the lack of formal analytically tractable models of such coupled infrastructure systems makes it difficult to connect this understanding to such concepts as stability, robustness, and resilience, which are increasingly important in considering such systems. In this paper, we mathematically operationalize a widely used conceptual framework via a stylized dynamical model. The model yields a wide range of system outcomes: sustainability or collapse, infrastructure at full or partial capacity, and social agents seeking outside opportunities or exclusively engaging in the system. The low dimensionality of the model enables us to derive these conditions in clear relationships of biophysical and social factors describing the coupled system. Analysis of the model further reveals regime shifts, trade-offs, and potential pitfalls that one may face in governing these self-organized systems. The intuition and insights derived from the model lay ground for more rigorous treatment of robustness and resilience of self-organized coupled infrastructure systems, which can lead to more effective governance.**

resilience | regime shift | governance challenges | social-ecological systems | coupled natural-human systems

**A** key social dilemma at the core of sustainability challenges stems from the cost and impracticality of having all stakeholders negotiate efficient solutions for externalities imposed on some by others in a connected system (1). This leads to overexploitation of shared resources, be they the carbon-assimilation capacity of the atmosphere, fish in the sea, trees in a forest, or water in a groundwater basin. Solving this social dilemma requires public infrastructure (PI)—i.e., some form of “governance infrastructure” (2, 3). How such infrastructure emerges and is maintained is the focus of this study.

Governance infrastructure is predominantly composed of technologies that monitor a situation, determine which actors may take what actions at what locations and times (i.e., institutions or legal systems), and sanctions violators that deviate from assigned actions. Other classes of PI, such as public security, major water- and energy-distribution works, sanitation, and roads, are essential to operationalize governance infrastructure. Policy solutions to prevent the overexploitation of shared resources rarely make explicit the interplay between these different infrastructures (4). For example, the standard solution of assigning property rights to the shared resource—e.g., catch shares in a fishery—and then allowing market forces to allocate effort presumes the existence of extensive legal, administrative, and monitoring infrastructure to implement the property rights and currency, banking, weights and measures, transportation, and communication networks through which “market forces” act. Top-down regulation, likewise, requires a large and effective “state” to provide monitoring, standards, sanctioning, etc. These considerations lead us to the fundamental challenge all societies face in providing critical PI.

The provision of PI may require large-scale coordination involving task specialization and the emergence of multiple social roles (e.g., refs. 5–8). Models of social-ecological systems

(SEs) that provide insights into various aspects of these systems often imply, but do not explicitly represent, such infrastructure and social roles (e.g., refs. 9–12); to clarify the interplay and feedback among these elements, a model in which they are explicitly represented is needed. In this study, we focus on two such social roles: providers (e.g., elites/administrators) and users (citizens/villagers) of public goods.

The social roles of PI providers (PIPs) and resource users (RUs) may be realized in many ways. When different individuals occupy these roles (e.g., PIPs are professional administrators who do not directly use the resource and who may impose governance arrangements on RUs), PIPs and RUs face very different incentives. Ref. 13 used a stylized dynamic model to capture this situation and study the strategic behavior of PIPs to balance tax extraction and provision of value in PI to maintain economic and social stability [cf. Olson’s “stationary bandit” problem (7)]. Here, we explore this balance in situations where the roles of RUs and PIPs are occupied by the same individuals who all face the same incentives (e.g., small-scale irrigation systems where farmers serve on water-user boards) and in which effort allocation to these roles emerges endogenously. These situations call for a different model formulation, analysis, and result interpretation (*SI Appendix*).

In our model, actors in the PIP role must maintain PIs such as irrigation canal systems to enable RUs to generate income from natural infrastructure (NI), e.g., water to irrigate private land. Outcomes depend on the level of tax/fee that the community

## Significance

**Many small-scale, self-organized systems have persisted for hundreds of years and been the subject of studies seeking to understand effective governance. The lack of formal analytically tractable models of such coupled infrastructure systems hinders linking knowledge gained from those studies to such concepts as resilience and robustness. Here, we develop a stylized, generic model of these systems. The analysis clarifies complex interactions and feedbacks that yield different system outcomes. The boundaries between these outcomes are expressed in clear functions of biophysical and socioeconomic factors and can potentially be used to develop resilience metrics. Such regime boundaries clarify how changes in biophysical and socioeconomic drivers affect the system outcome and what changes in governance may be required to maintain its sustainability.**

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agrees upon: If too high, community members may opt out of the RU role; if too low, RUs' contributions will be insufficient to both support those acting as PIPs and enable them to maintain the infrastructure. If the resource availability becomes too low, community members will opt out of the RU role for outside opportunities, and the system will collapse. Within this general picture, we use the dynamic model to explore the possible dynamically stable regimes that may emerge in networks of coupled social, institutional, built, and natural infrastructures (Fig. 1).

Our analysis identifies feedback mechanisms by which these “coupled infrastructure systems” (CISs) might collapse and explores subtleties of efficiency–resilience trade-offs associated with different governance regimes. The model suggests that while self-organized governance can be more resilient than the more efficient social-planning solution under certain circumstances, it may lead to a “lose–lose” situation that is both less resilient and less efficient. The model also makes explicit a process by which a cascade of feedback effects can cause a system to crumble quickly, forcing unsuspecting people out of their livelihoods. These insights can help improve the governance of small-scale CISs that feed and provide livelihoods for billions of people as they face simultaneous shocks from globalization and global change.

### Modeling Dynamic CISs

The Robustness of CIS Framework, or “CIS Framework” for short, (14) seeks to capture the essential features of the problems societies face in producing and maintaining critical shared infrastructures. In contrast to the closely related, empirically based Institutional Analysis and Development Framework (15) and SES Framework (16), the CIS Framework focuses on understanding the feedback structures that impact the dynamics and stability of coupled systems in the face of changing exogenous drivers. While it has been used for comparative and qualitative analysis (e.g., refs. 17–20) and to assist the development of models for specific systems (e.g., ref. 21), it has not yet been used to develop more generic models that synthesize understanding from many cases and extract general principles in the same spirit of empirical regularities, such as Ostrom’s institutional design principles (2, 22). We attempt to fill this gap by developing an analytically tractable mathematical model to represent the basic structure of CISs and use it to derive general insights into their behavior in a clear and transparent way. Because the model structure is informed by a large number of empirical cases, the insights derived from the model have direct policy implications and may be leveraged to improve the governance of real systems.

The CIS framework (Fig. 1A) captures relationships among agents acting in the role of RUs who extract resources through use of shared infrastructure, agents acting in the role of PIPs and the infrastructure itself, NI and PI. RUs and PIPs may have access to private infrastructure (PvI) and social infrastructure

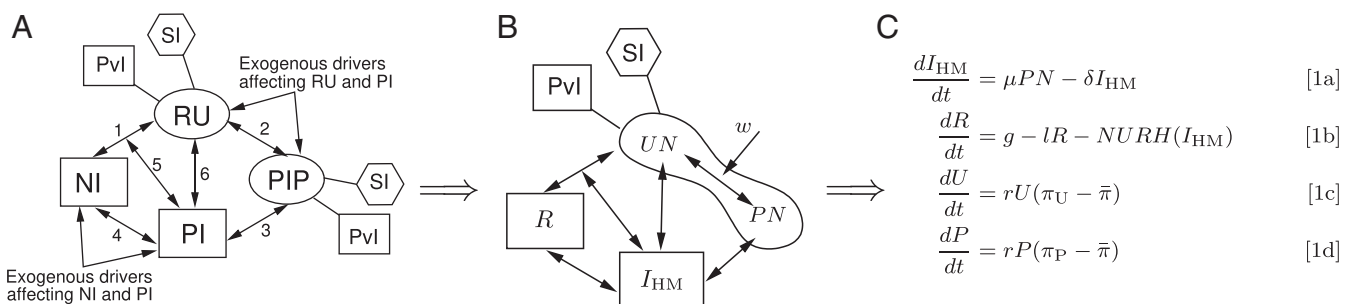
(SI) in the form of networks of interpersonal relationships. We emphasize “acting in the role” because the RU and PIP elements define positions that agents may occupy, rather than characteristics of agents themselves. The details of links 1 to 6 that determine the dynamics of the system cover a huge range of empirical cases, but they share the same general structure: RUs exert effort and extract resource flows along link 1; and PIPs provide PI through link 3, which enhances the productivity of and controls access to NI through links 4 and 5, respectively, and impacts the choice sets of RUs through link 6.

In our model, link 2 is “are identical to,” depicted by the RU and PIP elements coalescing into a single entity: The same people can occupy the RU and PIP positions (Fig. 1B). This is a common situation in a small-scale CIS, where villagers (the “agents”) take turns in assuming different roles at different times. Importantly, the RUs and PIPs experience the same exogenous drivers and incentives and mobilize the same assets. In the model—see Table 1 for definitions of its variables and parameters— $N$  agents exert  $UN$  units of effort as RUs to produce  $UNRH(I_{HM})$  resource units from a resource stock  $R$  produced by NI through endogenous dynamics  $g - lR$  (Eq. 1b, see Fig. 1C) and made accessible (harvestable) by PI  $I_{HM}$  via the function  $H(I_{HM})$  described below. RUs convert resource units into revenue at price  $p$ , a proportion  $C$  of which is contributed as a tax/fee to agents in the role of PIPs who are responsible for maintaining  $I_{HM}$  via the function  $\mu PN$  (Eq. 1a, see Fig. 1C).  $I_{HM}$  is subject to endogenous decay,  $-\delta I_{HM}$  (Eq. 1a, see Fig. 1C).  $I_{HM}$  enhances the productivity of the RUs through  $H(I_{HM})$  by way of, say, better information (link 6), enhanced access (link 5), or increasing productivity of NI (link 4; Eq. 2).

These assumptions lead to the dynamical system in Fig. 1C with four state variables: human-made PI ( $I_{HM}$ ), resource ( $R$ ), and fractions of the population who work in the roles of RU ( $U$ ) and PIP ( $P$ ) (Eq. 1, see Fig. 1C; and Table 1). The function  $H(I_{HM})$  maps the state of PI into the ability of RUs to harvest the resource, represented by a piecewise linear function that captures the threshold behavior typical of most PIs (Eq. 2). Behavioral dynamics ( $U$  and  $P$ ) are captured by replicator equations (see, e.g., ref. 23). To simplify the analysis, the model is nondimensionalized, with  $x$  and  $i_{HM}$  being the rescaled versions of  $R$  and  $I_{HM}$ , respectively (Eq. 4 and Table 2). See Materials and Methods for full model details.

### Results and Discussion

The model is rich enough to generate a wide range of outcomes, yet remains analytically tractable. This allows us to summarize the results very compactly and elegantly in a “phase diagram” showing the ranges of tax/fee  $C$  and the relative “potential lucrativeness”  $\phi$  that lead to different qualitative model behavior (Fig. 2). Each region represents different equilibrium levels of effort ( $U^*$ ,  $P^*$ ), resources ( $x^*$ ), and PI ( $i_{HM}^*$ ) (asterisks



**Fig. 1.** Mathematically operationalizing a conceptual framework. (A) The robustness of CIS framework (14) depicts the interaction among PI, PIPs, and how RUs mobilize PvI and SI to interact with NI. (B) The conceptual instantiation of the framework for the model studied here. (C) The formal mathematical representation of the conceptual model. See Materials and Methods and Table 1 for more detailed definitions of variables and parameters in C.

**Table 1. Definitions and dimensions of the variables and parameters**

Symbol	Definition	Unit
<b>Variables</b>		
$I_{HM}$	State of the human-made infrastructure	[ $I$ ]
$R$	Resource level	[ $R$ ]
$U$	Fraction of time a person spends as a RU	[−]
$P$	Fraction of time a person provides PI	[−]
$t$	Time	[ $T$ ]
<b>Parameters</b>		
$\mu$	Maintenance effectiveness	[ $\frac{I}{NT}$ ]
$N$	Number of users	[ $N$ ]
$\delta$	Depreciation rate of infrastructure	[ $\frac{1}{T}$ ]
$g$	Replenishing rate of the resource	[ $\frac{R}{T}$ ]
$l$	Natural loss rate of the resource	[ $\frac{1}{T}$ ]
$r$	Responsiveness of social actors to payoff difference	[ $\frac{1}{S/N}$ ]
$C$	Fraction of user revenue contributed to maintenance	[−]
$\pi_i$	Payoff to a use of strategy $i$ per unit time	[ $\frac{\$}{NT}$ ]
$h$	Maximum per capita harvest rate	[ $\frac{1}{NT}$ ]
$l_0$	Threshold of $I_{HM}$ below which $H$ is zero	[ $I$ ]
$l_m$	Threshold of $I_{HM}$ above which $H$ is maximum	[ $I$ ]
$p$	Conversion factor from resource to revenue	[ $\frac{\$}{R}$ ]
$w$	Per capita revenue from working outside	[ $\frac{\$}{NT}$ ]

$W = 1 - U - P$  is the fraction of time a person works outside.

denote equilibrium values of state variables). Below, we provide intuition for why each different regime emerges.

**All Remain, Infrastructure at Full Capacity (AF).** When the potential income from the system ( $\phi$ ) is high, everyone may be content with working inside the system ( $U^* + P^* = 1$ ). With an appropriate range of tax/fee (see *SI Appendix* for details)—high enough to keep infrastructure in a great condition, but not so high that people are incentivized to work outside—PI can be maintained at full capacity ( $i_{HM}^* \geq \theta_m$ ).

**All Remain, Infrastructure at Partial Capacity (AP).** If the RUs, however, settle on too low a  $C$ —perhaps due to their inability to solve the social dilemma associated with collecting a sufficiently high tax/fee—the PI can only be maintained at partial capacity, despite the relatively high lucrativeness of the system. Even so, the system is still more attractive than outside wage opportunities. See *SI Appendix* for more details on the range of  $C$  associated with this regime.

**Completely Mixed Strategies, Infrastructure at Partial Capacity (MF).** In this case, the population engages in completely mixed strategies (i.e., allocating effort to all three available strategies) and keeps infrastructure at full capacity. With a high tax/fee relative to the lucrativeness of the system, despite the infrastructure being maintained at full capacity ( $i_{HM}^* \geq \theta_m$ ), the tax burden incentivizes RUs to seek outside employment ( $U^* + P^* < 1$ ).

The upper and lower bounds of tax/fee for this regime depend on the system’s relative lucrativeness  $\phi$  (Fig. 2). From an economic perspective, the community would likely not operate in this regime, as the RUs do not benefit from infrastructure being maintained above the maximum productivity level ( $\theta_m$ ) unless other social factors are operating. For example, underlying social structure may allow for elites to choose to act as PIPs (rather than the whole community spending a proportion of its time, a proportion of the community spends all its time to create an equivalent level of labor person-hours) and use their power to set a higher  $C$ . It is also possible that the natural system is simply not that lucrative ( $\phi < 1 + (\eta(1 - \theta_m))^{-1}$ ; see black star in Fig. 2), and no tax policy exists that keeps everyone inside the system.

**Completely Mixed Strategies, Infrastructure at Partial Capacity (MP).** This regime happens when the tax/fee is relatively low and can only maintain the infrastructure at partial capacity ( $\theta_0 < i_{HM}^* < \theta_m$ ). At partial capacity, the infrastructure does not facilitate the use of the resource with enough efficiency, resulting in lower income, which, in turn, incentivizes people to work outside ( $U^* + P^* < 1$ ). There also exists a range of  $\phi$  in which the CIS can only be sustained in the MP configuration, but this range of  $\phi$  is small and barely visible in Fig. 2.

**Collapse (X).** When  $C$  is too low, infrastructure maintenance is underprovided. When  $C$  is too high, RUs earn too little income. When  $\phi$  is too low ( $< \phi_c$ ), it is simply impossible to maintain the infrastructure. Under these circumstances, the resource system is not economically viable. Working outside becomes a decidedly better option; everyone leaves, and the system collapses. This regime brings into sharp focus how outside wage opportunities may reduce poverty, but have implications for self-sufficiency and food security.

**Social Welfare.** What are the relative levels of social welfare in these different regimes? Here, social welfare is captured by the average income made by the population following the self-organized replicator dynamics, denoted by  $\bar{\pi}$ . Fig. 2 illustrates the interplay between  $\bar{\pi}$ ,  $C$ , and  $\phi$ . When the system collapses, everyone works outside, i.e.,  $\bar{\pi} = w$  or, equivalently,  $\bar{\pi} = 1$ . When all three strategies are adopted (regimes MF and MP), the replicator dynamics imply that, at equilibrium, payoffs to these three strategies are the same, i.e.,  $\bar{\pi}$  is again 1. All agents will spend their effort exclusively in the system (regimes AF and AP) only if  $\bar{\pi} \geq 1$ —this requires both a lucrative resource system ( $\phi \geq \phi_A$ ) and an appropriate policy (intermediate  $C$ ), without either of which some or all effort would be allocated outside the system.

**Table 2. Definitions and interpretation of dimensionless groups**

Symbol	Definition	Interpretation
$\tau$	$lt$	Rescaled time
$i_{HM}$	$\delta I_{HM} / \mu N$	Rescaled $I_{HM}$
$x$	$lR/g$	Rescaled $R$
$\eta$	$hN/l$	Maximum population-level harvest rate relative to natural loss rate of the resource
$\phi$	$pg/wN$	Relative per-capita lucrativeness of resource system compared to outside wage
$\theta_0$	$\delta l_0 / \mu N$	Infrastructure decay at $l_0$ relative to maximum maintenance
$\theta_m$	$\delta l_m / \mu N$	Infrastructure decay at $l_m$ relative to maximum maintenance
$\beta$	$\delta/l$	Decay rate of infrastructure relative to natural loss rate of the resource
$\omega$	$wr/l$	Responsiveness of population relative to natural loss rate of the resource

**System Trajectories and Regime Shifts.** Fig. 3 illustrates the trajectories of the system, given that it starts in a relatively good state with most people within the system, gravitating toward different regimes at equilibrium (SI Appendix, Table S1). The trajectories exhibit two distinct phases: the initial phase during which the population adapts effort “internally” between  $U$  and  $P$  (trajectories being more or less parallel to the axis linking the  $U$  and  $P$  corners) and the second phase during which they adapt between working inside and outside (trajectories heading toward the  $W$  corner). These patterns can be understood with the help of the following result. Because  $\pi_U = \pi_P$  at a nontrivial equilibrium ( $U^*, P^* > 0$ ), the ratio between effort allocated to acting as RUs and serving as PIPs converges to  $\frac{U^*}{P^*} = \frac{(1-C)}{C}$ . The trajectories initially move quickly toward this ratio. If the outside incentive is still great, people then allocate more efforts to working outside, while still maintaining the ratio between  $U$  and  $P$ , ultimately reaching an equilibrium (either on a blue curve or at the  $W$  corner).

This ratio  $(1 - C)/C$  confirms and quantifies an intuition that lower tax/fee incentivizes people to become users (trajectories moving toward the  $U$  corner). Perhaps less intuitive is the effect of lower  $C$  on how people allocate their efforts between inside and outside, which depends on whether the PI is at full or partial capacity. At full capacity,  $H(I_{HM})$  is insensitive to changes in  $C$ : Lower  $C$  simply means more income for RUs, thereby incentivizing people to spend more time working inside the system (Fig. 3, blue curve above the  $P = \theta_m$  line). At partial capacity, however, lower  $C$  leads to lower capacity, which results in lower income for RUs and thus incentivizes people to allocate more efforts working outside (Fig. 3, blue curve below the  $P = \theta_m$  line).

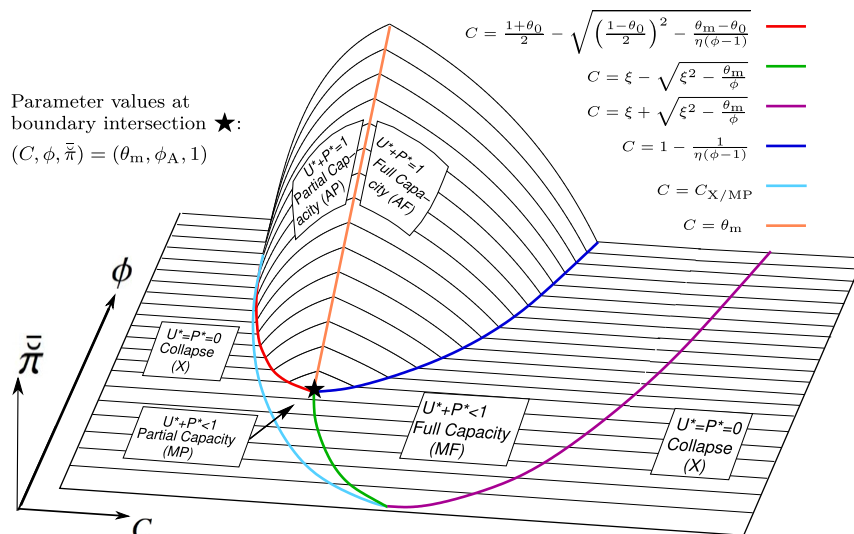
Fig. 3 also exhibits a regime-shift character of the CIS. Note that the lowest value of  $C$  for sustainable self-organized CISs corresponds to  $i_{HM}^*$  significantly  $> \theta_0$  (i.e.,  $I_{HM}^* > I_0$ ). This is a mark of a fold bifurcation. Mathematically speaking, at that lowest end, the nontrivial stable equilibrium collides with another unstable equilibrium and disappears, leaving the collapse regime as the only stable equilibrium. At this point, even though the infrastructure seems to be far above  $\theta_0$ , a small decrease in  $C$ ,

and thus in the infrastructure’s capacity, makes it impossible for the RUs and PIPs to make enough income to match outside opportunities. Consequently, the CIS unravels and devolves toward a collapse, even at  $I_{HM}^* > I_0$ . This should worry anyone who governs a CIS with crumbling infrastructure (24).

**Social Planning and Resilience–Efficiency Trade-Offs.** Finally, as a point of comparison, we now consider a social planner who can prescribe how the population should allocate their time among the three strategies and aims at maximizing the overall payoff of the population  $\hat{\pi}$ . There is no  $C$  in  $\hat{\pi}$ :  $C$  does not enter the social planner’s consideration (SI Appendix). In this setting, everyone receives the same payoff; the overall income is divided up equally, and everyone receives the amount  $\hat{\pi}$ . There is no social dynamics (i.e., no replicator dynamics) here because  $U$  and  $P$  are prescribed; there is only the biophysical dynamics (Eq. 1, see Fig. 1C) resulting from the prescribed  $U$  and  $P$ . After analyzing all combinations of  $U$  and  $P$ , she instructs the population to allocate  $\hat{U}$  and  $\hat{P}$  of their effort to be RUs and PIPs, respectively, to maximize  $\hat{\pi}$  (SI Appendix).

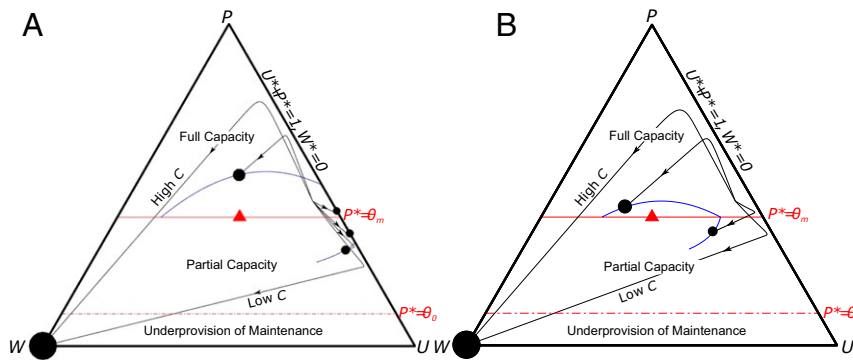
The social planning ( $\hat{U}, \hat{P}$ ) (red triangles in Fig. 3) and self-organized governance ( $U^*, P^*$ ) outcomes differ. The planner will instruct the population to maintain the infrastructure at full capacity or slightly under that, but not more. At this point,  $\hat{\pi} > w$ . This implies that  $\pi_U, \pi_P > w$ , and, if left to their own devices, the population would be incentivized by the greater income to allocate more efforts to working inside the system. Not being allowed to follow such short-term incentives may create social tension in implementing such a social plan (but that is beyond the scope of this model).

Despite higher  $\hat{\pi}$ , social planning introduces a certain vulnerability to the CIS. With  $I_{HM}$  maintained barely at or slightly below  $I_m$ , the CIS is vulnerable to shocks to the infrastructure, e.g., natural disasters in the case of physical infrastructure or political turmoil in the case of governance infrastructure. Although yielding lower  $\hat{\pi}$ , the self-organized governance can lead to outcomes in which infrastructure is maintained at  $I_{HM}^*$  significantly  $> I_m$  (i.e.,  $P^* > \hat{P}$ ), thereby providing a “resilience cushion” against infrastructure shocks (25). This highlights a fundamental



**Fig. 2.** Welfare surface overlaid on phase diagram of qualitative model behavior for different combinations of tax/fee  $C$  and relative potential lucrativeness  $\phi$  of the system. Regimes are AF, AP, MF, MP, or X, where A indicates that all population remain in the system ( $U^* + P^* = 1$ ); M indicates a mixed strategy ( $U^* + P^* < 1$ ), where agents allocate effort to all three available strategies; F or P indicate that infrastructure is maintained at full capacity or partial capacity, respectively; and X indicates system collapse. There exists a critical level of  $\phi$ ,  $\phi_c$ , below which it is impossible to sustain the system. The key shows values of  $C$  at the regime boundaries expressed as functions of  $\phi$ ,  $\eta$ ,  $\theta_0$ ,  $\theta_m$ , and  $\xi$ , where  $\xi := \frac{1}{2}(1 + \frac{\eta\theta_m - 1}{\eta\phi})$  is introduced for brevity. The mathematical expressions highlight the complex dependence of these boundaries on biophysical and social factors. Their derivations, and the analogous surface for the resource level, are provided in SI Appendix. The dimensionless groups used are as follows:  $\eta = 5$ ,  $\theta_0 = 0.1$ , and  $\theta_m = 0.4$ .





**Fig. 3.** Possible dynamics and long-term outcomes when  $\phi$  is high (A) and  $\phi$  is intermediate (B)—the exact ranges of  $\phi$  are discussed in *SI Appendix*. Black curves are the system’s phase portraits; circles are final outcomes, with sizes being proportional to the long-term resource level; and blue curves show  $U$ – $P$  combinations at equilibrium ( $\pi_U = \pi_P = w$ ) for various values of  $C$  for which  $W^* > 0$ . The phase portraits shown correspond to, from the lowest curve to the highest one,  $C = 0.25, 0.31, 0.35, 0.42, 0.67,$  and  $0.8$  in A and  $C = 0.35, 0.42, 0.67,$  and  $0.8$  in B.  $\phi$  is set to 1.4 and 1.25 in A and B, respectively. The dimensionless groups used in this figure are  $\beta = 1, \omega = 2, \eta = 5, \theta_0 = 0.1,$  and  $\theta_m = 0.4$ . Red triangles represent outcomes prescribed by the social planner.

trade-off between short-term performance and long-term resilience in a CIS, a theme also seen in CISs with a different social structure (13).

Self-organized governance is not free from drawbacks. System outcomes under self-organized governance are more sensitive to changes in the outside incentive  $w$ . Changes in  $w$  can be pictured as the blue curves in Fig. 3 moving up and down; the population’s strategy portfolio would have to move around accordingly, constantly adjusting for these changes. A more serious pitfall is when the population agrees upon a lower  $C$  that results in infrastructure at partial capacity (regimes AP and MP). Under these outcomes, the payoff is lower than that in the social-planning case, and the system is not resilient against infrastructure shocks—it is a lose–lose outcome that should be avoided. Therefore, while self-organized governance holds potential for greater resilience, it could produce a thoroughly inferior result as well.

### Conclusions and Final Remarks

In this paper, we have mathematically operationalized a widely used empirically based conceptual framework to capture the dynamics of self-organized CISs (Fig. 1). The model yields the phase diagram of the system outcomes (Fig. 2) in which boundaries between different regimes are expressed clearly in terms of social and biophysical characteristics. Within each regime, the expressions of the corresponding equilibria—i.e.,  $(i_{HM}^*, x^*, U^*, P^*)$ —are derived (*SI Appendix*). The model is also analyzed under the social-planning setting to enable comparison with the self-organized setting. These results clarify the nature of nonlinear feedbacks and interactions among different components, enabling more rigorous discussion of system properties and their trade-offs.

Underneath the many mathematical results presented here, there are several additional related results that are useful in understanding the system dynamics and generating hypotheses. At equilibrium, the condition  $\pi_U = \pi_P$  leads to the  $U^* : P^*$  ratio of  $(1 - C)/C$ . This ratio, once considered in conjunction with the constraint  $U^* + P^* \leq 1$ , leads to the upper bounds of  $U^*$  and  $P^*$ :  $U^* \leq 1 - C$  and  $P^* \leq C$ , which are realized when the system is relatively lucrative and  $C$  is in an appropriate range (regimes AF and AP). These smaller results can also be seen as predictions, or “hypotheses,” born of the model that are relatively straightforward to test.

The features of a self-organized CIS that the model reveals—a regime shift, fundamental trade-offs, and potential pitfalls—paint a picture of the delicate balance involved in governing this type of CIS. In tackling governance challenges of a CIS, one must be aware of the interplay between these features. As such, it is

important not only to recognize that feedbacks in such a system are complex and nonlinear, but also to make the nature of the complexity and nonlinearity clear. This is the main contribution of our work.

Our model illustrates that the “empty village” problem (26), for example, may be subtler than the simple storyline that farmers find better opportunities in the cities and depopulate rural areas. Rather, some “early adopters” of wage labor may leave, generating a feedback through which infrastructure becomes less productive, generating, in turn, incentives for more farmers to leave out of desperation as the infrastructure declines. Some of those who leave later may, in fact, not have wanted to. Perhaps the worst part of the story is that once everyone is gone, the infrastructure collapses, and there is nothing to go back to, even if wages subsequently drop; many rural people are trapped in poor living conditions in cities with no options. Subtle dynamic and nonlinear interactions behind these and other governance challenges are important to understand systematically if we are to develop policies to address such problems.

Using models like the one we present here, the conditions of the different long-term system outcomes are expressed in clear relationships of the system’s biophysical and social factors (Fig. 2). The state of infrastructure, resource level, and strategic behaviors of the population are encapsulated in mathematical expressions in terms of these same factors. With such knowledge, we can see more clearly how changes in one factor may influence various system properties. These findings also serve as benchmarks against which to compare the findings from studies or models that include more complex, realistic mechanisms, e.g., migration within a network of CISs and effects of individual heterogeneity (as in agent-based models). We believe that the development of systematic mathematical analysis of CISs such as ours is much needed. It provides intuition and insights into strategic behavior and governance challenges in this important class of systems and lays groundwork for more rigorous treatment of such system properties as robustness, resilience, and trade-offs between them.

### Materials and Methods

This study is based on a stylized model with no empirical data. Detailed derivations of reported results are provided in *SI Appendix*.

**Self-Organized Governance.** We have operationalized the framework in Fig. 1B into the four-dimensional dynamical system in Fig. 1C, where

$$H(I_{HM}) = \begin{cases} 0, & I_{HM} < I_0 \\ h \frac{I_{HM} - I_0}{I_m - I_0}, & I_0 \leq I_{HM} \leq I_m \\ h, & I_{HM} > I_m \end{cases} \quad [2]$$

$\pi_U = (1 - C)pRH(i_{HM})$ ,  $\pi_P = pCR\frac{U}{P}H(i_{HM})$ , and  $\pi_W = w$ , with the population-averaged payoff of

$$\bar{\pi} = U\pi_U + P\pi_P + (1 - U - P)w = pURH(i_{HM}) + (1 - U - P)w. \quad [3]$$

The definitions and dimensions of the variables and parameters are summarized in Table 1. The  $i_{HM}$  dynamics results from the balance between maintenance effort  $\mu PN$  and depreciation/decay  $\delta i_{HM}$ . The  $R$  dynamics includes its natural replenishment  $g$ , natural loss  $lR$ , and human appropriation  $NURH(i_{HM})$ . The strategic behavior of the population (represented by  $U$  and  $P$ ) is modeled by replicator equations (Eq. 1 c and d, see Fig. 1C), capturing the social-learning process in self-organized governance structure.

It should be noted that, as dynamically rich as the system of Eq. 1 (see Fig. 1C) is, it omits some richness in the conceptual framework (Fig. 1A). For example, some social norms (e.g., ostracism) can be very important in these small-scale CISs, but are not currently included in the model. Different sets of equations must be devised to capture these aspects encapsulated in Fig. 1A.

To simplify our analysis, we nondimensionalize the model and rewrite it in terms of dimensionless groups as follows (Table 2):

$$\frac{di_{HM}}{d\tau} = \beta(P - i_{HM}) \quad [4a]$$

$$\frac{dx}{d\tau} = 1 - x - Ux\check{H}(i_{HM}) \quad [4b]$$

$$\frac{dU}{d\tau} = \omega U(\pi_U - \bar{\pi}) \quad [4c]$$

$$\frac{dP}{d\tau} = \omega P(\pi_P - \bar{\pi}), \quad [4d]$$

where

$$\check{H}(i_{HM}) = \begin{cases} 0, & i_{HM} < \theta_0 \\ \eta \frac{i_{HM} - \theta_0}{\theta_m - \theta_0}, & \theta_0 \leq i_{HM} \leq \theta_m \\ \eta, & i_{HM} > \theta_m. \end{cases} \quad [5]$$

$\check{\pi}_U = (1 - C)\phi x \check{H}(i_{HM})$ ,  $\check{\pi}_P = C\phi x \frac{U}{P} \check{H}(i_{HM})$ , and the rescaled population-averaged payoff of

$$\check{\pi} = 1 - U - P + \phi Ux\check{H}(i_{HM}). \quad [6]$$

The system 4 is what our analysis will be performed on.  $U^*$  and  $P^*$  denote values of  $U$  and  $P$  at the equilibrium of this dynamical system.

**Social Planning.** For comparison, we consider the case of social planning, in which a social planner prescribes the levels of  $U$  and  $P$ , denoted by  $\hat{U}$  and  $\hat{P}$ , to maximize  $\bar{\pi}$ , while  $i_{HM}$  and  $x$  respond according to their own dynamics. That is, the model for the governance structure will reduce to the following two-dimensional one:

$$\frac{di_{HM}}{d\tau} = \beta(P - i_{HM}) \quad [7a]$$

$$\frac{dx}{d\tau} = 1 - x - Ux\check{H}(i_{HM}) \quad [7b]$$

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