Contents lists available at ScienceDirect

Heliyon



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Research article

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Linguistic intuitionistic fuzzy multi-attribute bilateral matching considering satisfaction and fairness degree

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ARTICLE INFO

Keywords: Multi-attribute bilateral matching Linguistic intuitionistic fuzzy number Satisfaction degree Fairness degree Bilateral matching model

ABSTRACT

Aiming at the multi-attribute bilateral matching problem with unknown attribute weights under a linguistic intuitionistic fuzzy environment, a decision method based on TODIM considering satisfaction and fairness degrees is proposed. First, the theories of linguistic intuitionistic fuzzy sets and bilateral matching are given, and the multi-attribute bilateral matching problem under a linguistic intuitionistic fuzzy environment is described. To solve this problem, according to linguistic intuitionistic fuzzy preference matrices, the overall attribute dominances are calculated based on TODIM; considering group consensus, a new method is proposed to calculate attribute weights based on linguistic intuitionistic fuzzy induced ordered weighted averaging (LIFIOWA) operator; then, the overall dominances of bilateral subjects are obtained by aggregating the overall attribute dominances and attribute weights. Furthermore, the overall dominances are standardized to calculate the satisfaction degrees of bilateral subjects; the fairness degrees of bilateral subjects are calculated considering the loss attenuation coefficient. Based on satisfaction degree matrices, fairness degree matrices and bilateral matching matrices, multiple bilateral matching models are established and then solved to obtain the optimal bilateral matching scheme. Finally, an example shows the effectiveness, reliability and accuracy of the proposed method. The research results indicate the following main characteristics of the proposed method: (1) A new method for calculating the unknown attribute weights using LIFIOWA operator is proposed. (2) According to the TODIM idea, a calculation method for fairness degree considering the loss attenuation coefficient is proposed. (3) Considering satisfaction and fairness degrees, multiple bilateral matching models under a linguistic intuitionistic fuzzy environment are established.

1. Introduction

Bilateral matching decision refers to how to use a reasonable matching method to match the bilateral subjects according to their preference information. In real life, there are many bilateral matching problems, such as stable ride-sharing matching [1], person-position matching [2], matching between hospitals and patients [3], venture capital [4], volunteer allocation for emergency tasks [5]. Additionally, scholars have carried out a lot of research on the stability of bilateral matching [6], matching mechanism and related theories [7,8]. Considering that the choice of bilateral matching scheme directly affects the improvement of the satisfaction

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https://doi.org/10.1016/j.heliyon.2024.e27006

Received 7 December 2023; Received in revised form 15 February 2024; Accepted 22 February 2024

Available online 3 March 2024

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degree of the matching subjects, thus the research on bilateral matching theories and methods has important theoretical and practical significance.

In actual decision environment, due to the complexity of the decision problem and the cognitive differences of people, it is difficult for decision makers to express their preferences for objective things with clear and specific values. Therefore, it is inevitable to introducing fuzzy information into the bilateral matching decision. In the process of bilateral matching decision, different types of fuzzy evaluation information are often provided by the bilateral subjects, such as intuitionistic fuzzy sets, hesitant fuzzy sets and linguistic term sets [9–11]. Among them, linguistic intuitionistic fuzzy sets can consider the membership degree, non-membership degree and hesitation degree of linguistic evaluation information at the same time, and reflect the fuzziness of bilateral decision process [12]. Many scholars have applied linguistic intuitionistic fuzzy set theories into problems of group decision, multi-criteria decision and multi-attribute decision [13–15]. Thus, linguistic intuitionistic fuzzy sets are also used to express preferences of subjects in bilateral matching decision.

In bilateral matching decision, there are few studies on psychological behaviors of bilateral subjects. TODIM is a decision method based on the prospect theory [16] considering psychological behaviors of decision makers, which can better express preferences of decision makers by comparing all alternatives. It has been well developed and applied in various fields. For example, Jiang et al. (2017) proposed a new interval TODIM method to solve the I-multi-attribute decision problem, which considers behavioral characteristics of decision makers [17]. Xu et al. (2020) proposed a new method based on PROMETHEE and TODIM to solve the multi-attribute decision problem under the single-valued neutrosophic environment [18]. Thus, TODIM is used to solve the considered linguistic intuitionistic fuzzy bilateral matching decision problem.

In addition, the determination of attribute weights has always been a concern in multi-attribute decision. Many scholars have made some research on the theory and method of multi-attribute decision with the linguistic preference information. For example, Liu et al. (2020) proposed an approach for uncertain multi-attribute group decision making based on linguistic-valued intuitionistic fuzzy preference relations [19]. Garg and Kaur (2018) proposed a new measure to measure the fuzzy degree of intuitionistic fuzzy sets, and applied it into multi-attribute decision making in an intuitionistic fuzzy set environment [20]. Darko et al. (2023) developed a novel decision evaluation model and multi-attribute decision-making with probabilistic linguistic information to identify m-payment usage attributes and utilize these attributes [21]. Yu (2024) established a novel multi-objective decision model for the grade assessment of network security situation based on the two-tuple linguistic operator [22]. From the above reference, it can be known that the linguistic aggregation operator has a great significance on multi-attribute bilateral decision involved in this paper. Especially, in Ref. [22], the two-tuple linguistic operator is proposed and a grade assessment decision model is established, which cannot be directly used to solve the considered problem in this paper. Thus, the determination of attribute weights under a linguistic intuitionistic fuzzy environment needs more attention.

In summary, research motivations of this study are as follows: (1) The application of linguistic intuitionistic fuzzy sets in bilateral matching decision making, utilization of TODIM method that considers the psychological behavior of bilateral subjects is worth more attention. (3) In the linguistic intuitionistic fuzzy bilateral matching environment, the determination for attribute weights is not perfect. (4) The bilateral matching decision model under a linguistic intuitionistic fuzzy environment can express the uncertainty and ambiguity of decision process more flexibly and in detail. Therefore, it is necessary to develop bilateral matching decision models under a linguistic intuitionistic fuzzy environment.

Main contributions of this paper are as follows: (1) A multi-attribute bilateral matching decision method considering satisfaction and fairness degrees under a linguistic intuitionistic fuzzy environment is explored, which provides theoretical and practical references for solving linguistic intuitionistic fuzzy bilateral matching problems. (2) Calculation for satisfaction degree under a linguistic intuitionistic fuzzy environment is developed based on TODIM, which can well reflect subjects' preferences. (3) An unknown attribute weight calculation method is proposed based on the linguistic intuitionistic fuzzy induced ordered weighted averaging (LIFIOWA) operator, which can well reflect the group consistency of subjects. (4) Multiple bilateral matching models under a linguistic intuitionistic fuzzy environment are established, which considers satisfaction and fairness degrees of bilateral subjects.

In view of this, a linguistic intuitionistic fuzzy multi-attribute bilateral matching decision method considering satisfaction and fairness degrees is proposed. According to the TODIM idea, the overall attribute dominances of the bilateral subjects are obtained. Considering the consistency of group opinions, a new method is proposed to calculate the weights of unknown attributes by using LIFIOWA operator. Then the global dominance of bilateral subjects is obtained by aggregating the overall attribute dominance and attribute weight. According to the overall dominances, the satisfaction degrees are calculated, and then the fairness degree considering the fair attenuation coefficient is calculated. Finally, multiple bilateral matching models under a linguistic intuitionistic fuzzy environment are constructed and solved according to satisfaction and fairness degree matrices, and thus the optimal bilateral matching scheme is obtained.

The remainder of this paper is organized as follows: Section 2 explores some concepts of linguistic intuitionistic fuzzy numbers and bilateral matching. Section 3 describes a multi-attribute bilateral matching problem under a linguistic intuitionistic fuzzy environment. Section 4 presents the bilateral matching decision method under a linguistic intuitionistic fuzzy environment based on TODIM. Section 5 gives steps of linguistic intuitionistic fuzzy bilateral matching decision. Section 6 uses a bilateral matching case to reveal the effectiveness and feasibility of the proposed method and to discuss the sensitivity of the proposed method. Section 7 summarizes this paper.

2. Preparatory knowledge

2.1. Theory of linguistic intuitionistic fuzzy numbers

Definition 1. [23] Let $S = \{s_{\theta} | \theta = 0, 1, ..., 2t\}$ be a non-empty discrete uniform linguistic assessment scale set. For linguistic variables $s_i, s_j \in S$, it satisfies: 1) Regularity: if $i \ge j$, then $s_i \ge s_j$; 2) inverse operation: if $neg(s_i) = s_j$, then i + j = 2t. The algorithm of linguistic assessment scale is defined as: 1) $s_i \oplus s_i = s_{i+j}$; 2) $\lambda s_i = s_{\lambda i}$ and $\lambda \in [0, 1]$; 3) if i > j, then $s_i > s_j$.

Definition 2. [24] Let $\mu_a = s_{\mu_a}$, $\nu_a = s_{\nu_a}$; s_{μ_a} , $s_{\nu_a} \in S = \{s_{\theta} | \theta = 0, 1, ..., 2t\}$. If $s_0 \leq s_{\mu_a} \oplus s_{\nu_a} \leq s_{2t}$, then $a = (\mu_a, \nu_a)$ is called a linguistic intuitionistic fuzzy number, where μ_a and ν_a are a linguistic membership degree and a linguistic non-membership degree respectively, and $s_{2t} - \mu_a - \nu_a$ is a linguistic hesitation degree.

This paper studies the linguistic intuitionistic fuzzy numbers. For the convenience of description, *a* and *b* are called the abbreviation of $a = (\mu_a, \nu_a)$ and $b = (\mu_b, \nu_b)$. For example, considering the ordered linguistic term assessment scale set $S = \{s_0: \text{ extremely poor; } s_1: \text{ very poor; } s_2: \text{ poor; } s_3: \text{ middle-lower; } s_4: \text{ middle; } s_5: \text{ middle-upper; } s_6: \text{ good; } s_7: \text{ very good; } s_8: \text{ extremely good}\}$, a linguistic intuitionistic fuzzy evaluation of an enterprise to a candidate under the attribute 'personal quality' is (s_6, s_1) , then the satisfaction degree of enterprise to the candidate is s_6 , and the dissatisfaction degree of enterprise to the candidate is s_1 .

Definition 3. [25] Let $a_1 = (\mu_{a_1}, \nu_{a_1})$ and $a_2 = (\mu_{a_2}, \nu_{a_2})$ be linguistic intuitionistic fuzzy numbers, where $\mu_{a_1} = s_i, \nu_{a_1} = s_j, \mu_{a_2} = s_m, \nu_{a_2} = s_n$, and $s_i, s_j, s_m, s_n \in S$, then the algorithm between a_1 and a_2 is as follows:

1) $a_1 \oplus a_2 = (s_{f(\mu_{a_1})+f(\mu_{a_2})-f(\mu_{a_1})\times f(\mu_{a_2})}, s_{f(v_{a_1})\times f(v_{a_2})}),$ 2) $a_1 \otimes a_2 = (s_{f(\mu_{a_1})\times f(\mu_{a_2})}, s_{f(v_{a_1})+f(v_{a_2})-f(v_{a_1})\times f(v_{a_2})}),$ 3) $\lambda a_1 = (s_{1-(1-f(\mu_{a_1}))^{\lambda}}, s_{f(v_{a_1})^{\lambda}}),$ 4) $a_1^{\lambda} = (s_{f(\mu_{a_1})^{\lambda}}, s_{1-(1-f(v_{a_1}))^{\lambda}}),$

Where
$$f(\mu_{a_1}) = \frac{i}{2t} f(\nu_{a_1}) = \frac{1}{2t} f(\mu_{a_1}) = \frac{m}{2t} f(\nu_{a_1}) = \frac{m}{2t} f(v_{a_1}) = \frac{m}{2t} f(v_{\theta}) = \frac{\theta}{2t} (\theta = 0, 1, ..., 2t)$$
 is a linguistic scale function

Definition 4. [26] Let $S = \{s_{\theta} | \theta = 0, 1, ..., 2t\}$ be a linguistic assessment scale set, and $a_1 = (\mu_{a_1}, \nu_{a_1})$ and $a_2 = (\mu_{a_2}, \nu_{a_2})$ be linguistic intuitionistic fuzzy numbers; then, the distance and the similarity measure between a_1 and a_2 are respectively as follows:

$$D(a_1, a_2) = \frac{1}{2} \left[\left| f(s_{\mu_1}) - f(s_{\mu_2}) \right| + \left| f(s_{\nu_1}) - f(s_{\nu_2}) \right| + \left| f(s_{\pi_1}) - f(s_{\pi_2}) \right| \right], \tag{1}$$

$$P(a_1, a_2) = 1 - \frac{1}{2} \left(\left| s_{\mu_{a_1}} - s_{\mu_{a_2}} \right| + \left| s_{\nu_{a_1}} - s_{\nu_{a_2}} \right| + \left| s_{\pi_{a_1}} - s_{\pi_{a_2}} \right| \right).$$

$$\tag{2}$$

Definition 5. [27] Let $S = \{s_{\theta} | \theta = 0, 1, ..., 2t\}$ be a linguistic assessment scale set, and $a = (\mu_a, \nu_a)$ be a linguistic intuitionistic fuzzy number. The score function and the exact function are respectively as follows:

$$R(a) = f(s_{\mu_a}) - f(s_{\nu_a}),$$
(3)

$$Q(a) = f(s_{\mu_a}) + f(s_{\nu_a}).$$
(4)

For linguistic intuitionistic fuzzy numbers $a_1 = (\mu_{a_1}, \nu_{a_1})$ and $a_2 = (\mu_{a_2}, \nu_{a_2})$, the comparison rules of a_1 and a_2 are as follows:

1) If $R(a_1) > R(a_2)$, then $a_1 > a_2$; 2) if $R(a_1) < R(a_2)$, then $a_1 < a_2$; 3) when $R(a_1) = R(a_2)$, if $Q(a_1) = Q(a_2)$, then $a_1 = a_2$; if $Q(a_1) > Q(a_2)$, then $a_1 > a_2$; if $Q(a_1) < Q(a_2)$, then $a_1 < a_2$.

Definition 6. [28] Let $a_k = (\mu_{a_k}, \nu_{a_k})$ $(k = 1, 2, ..., \sigma)$ be a set of linguistic intuitionistic fuzzy numbers, and $\omega_k (k = 1, 2, ..., \sigma)$ be its corresponding weight, where $0 \le \omega_k \le 1$ and $\sum_{k=1}^{\sigma} \omega_k = 1$. The linguistic intuitionistic fuzzy weighted averaging (LIFWA) operator is calculated as follows:

LIFWA(
$$a_1, a_2, ..., a_\sigma$$
) = $\sum_{k=1}^{\sigma} \omega_k a_k = \left(s_{1-\prod_{k=1}^{\sigma} (1-f(\mu_{a_k}))^{\omega_k}}, s_{1-\prod_{k=1}^{\sigma} f(v_{a_k})^{\omega_k}} \right).$ (5)

Based on the idea of [29], the definition of LIFIOWA operator is given.

Definition 7. Let $a_k = (\mu_k, \nu_k)$ $(k = 1, 2, ..., \sigma)$ be a set of linguistic intuitionistic fuzzy numbers, and $\omega_k (k = 1, 2, ..., \sigma)$ be its corresponding weight, satisfying $0 \le \omega_k \le 1$ and $\sum_{k=1}^s \omega_k = 1$. Let u_k be the induced value of \tilde{a}_k , and $\tilde{a}_k = (\mu_{\tilde{a}_k}, \nu_{\tilde{a}_k})$ $(k = 1, 2, ..., \sigma)$ be the

linguistic intuitionistic fuzzy number pair corresponding to the *k*-th large element u_k in the LIFIOWA operator. Then the LIFIOWA operator is calculated as follow:

$$\text{LIFIOWA}(\langle u_1, \widetilde{a}_1 \rangle, \langle u_2, \widetilde{a}_2 \rangle, ..., \langle u_\sigma, \widetilde{a}_\sigma \rangle) = \sum_{k=1}^{\sigma} \omega_k \widetilde{a}_k = \left(s_{1-\prod_{k=1}^{\sigma} \left(1 - f(\mu_{\widetilde{a}_k})\right)^{\omega_k}}, s_{1-\prod_{k=1}^{\sigma} f(v_{\widetilde{a}_k})^{\omega_k}} \right).$$
(6)

where the weighted vector is set to meet the characteristics from large to small. The smaller the order value u_k is, the greater the role of the decision maker in the group is, and the greater the weight ω_k is.

2.2. Bilateral matching

Let $\partial = (\partial_1, \partial_2, ..., \partial_h)$ be one subject set, and $\ell = (\ell_1, \ell_2, ..., \ell_t)$ be the other subject set, where ∂_i is the *i*-th subject in set ∂ , and ℓ_j is the *j*-th subject in set ℓ , where $H = \{1, 2, ..., k\}$, $T = \{1, 2, ..., t\}$, $h \leq t$.

Definition 8. [30] Assume that $\wp: \partial \cup \ell \rightarrow \partial \cup \ell$ is a mapping. If the mapping \wp meets these conditions: 1) $\wp(\partial_i) \in \ell$; 2) $\wp(\partial_i) \in \partial \cup \{\ell_i\}$; 3) $\wp(\partial_i) = \ell_i$ if and only if $\wp(\ell_i) = \partial_i$, then \wp is called a bilateral matching.

In Definition 8, $\wp(\partial_i) = \ell_j$ (or (∂_i, ℓ_j)) represents ∂_i and ℓ_j are matched and $\wp(\ell_j) = \ell_j$ (or (ℓ_j, ℓ_j)) represents ℓ_j is unmatched (single).

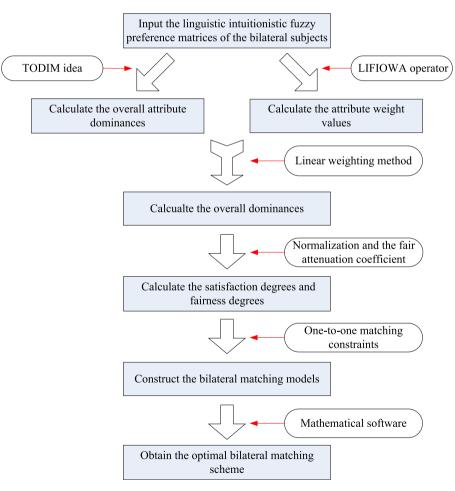


Fig. 1. Solution idea of linguistic intuitionistic fuzzy multi-attribute bilateral matching decision

Note: The calculation process of the unknown attribute weight vectors $\boldsymbol{\omega}^{e} = (\omega_{1}^{c}, \omega_{2}^{c}, ..., \omega_{p}^{c})$ and $\boldsymbol{\omega}^{d} = (\omega_{1}^{d}, \omega_{2}^{d}, ..., \omega_{q}^{d})$ is as follows: According to the linguistic intuitionistic fuzzy matrices $\widetilde{A}^{(\partial_{k})} = [\widetilde{a}_{ij}^{k}]_{t\times p}$ and $\widetilde{B}^{(\ell_{1})} = [\widetilde{b}_{kg}^{i}]_{h\times q}$, the group opinion matrices $R^{(\partial)} = [r_{\bar{a}_{ij}}]_{t\times p}$ and $R^{(\ell)} = [r_{\bar{b}_{ij}}]_{h\times q}$ are calculated by using LIFIOWA operator; and then the group opinion score matrices are obtained by Eqs. (14) and (19); lastly the attribute weights are calculated by Eqs. (15) and (20). The specific calculation method is shown in Section 4.3.

3. Description of multi-attribute bilateral matching problem under a linguistic intuitionistic fuzzy environment

In the problem of multi-attribute bilateral matching under a linguistic intuitionistic fuzzy environment, let $\partial = \{\partial_1, \partial_2, ..., \partial_h\}$ and $\ell = \{\ell_1, \ell_2, ..., \ell_i\}$ be sets of bilateral subjects, where ∂_k represents the k-th subject in set ∂ and ℓ_i represents the i-th subject in set ℓ . Let $C = \{c_1, c_2, ..., c_p\}$ be the attribute set of side ∂ to side ℓ , and $D = \{d_1, d_2, ..., d_q\}$ be the attribute set of side ℓ to side ∂ , where c_i represents the *j*-th attribute in set C and d_g represents the *g*-th attribute in set D. Let $\omega^c = (\omega_1^c, \omega_2^c, ..., \omega_p^c)$ be the weight vector of the attribute set *C*, and $\omega^d = (\omega_1^d, \omega_2^d, ..., \omega_d^d)$ be the weight vector of the attribute set *D*, where the weight vector ω^c and ω^d are unknown. The linguistic assessment scale set $S = \{s_1, s_2, ..., s_r\}$ is considered in this paper. $\widetilde{A}^{(\partial_k)} = [\widetilde{a}_{ij}^k]_{t \times p}$ is set as the multi-attribute linguistic intuitionistic fuzzy matrix of subject ∂_k to subject ℓ_i under the attribute c_j , where the linguistic intuitionistic fuzzy number $\tilde{a}_{ij}^k = (\mu_{ak})^k$ $v_{a_{i}^{k}}$), $\mu_{a_{i}^{k}}$ represents the satisfaction degree of subject ∂_{k} to subject ℓ_{i} under the attribute c_{j} , and $v_{a_{i}^{k}}$ represents the dissatisfaction degree of subject ∂_k to subject ℓ_i under the attribute c_j , $\mu_{\tilde{a}_{ij}}^k$, $v_{\tilde{a}_{ij}}^k \in S$. $\tilde{B}^{(\ell_i)} = [\tilde{b}_{kg}^i]_{h \times q}$ is the linguistic intuitionistic fuzzy matrix of subject ℓ_i to subject ∂_k under the attribute d_g , where the linguistic intuitionistic fuzzy number $\tilde{b}_{kg}^i = (\mu_{\tilde{b}_{kg}^i}, v_{\tilde{b}_{kg}^i}), \mu_{\tilde{b}_{kg}^i}$ represents the satisfaction degree of subject ℓ_i to subject d_k under the attribute d_g , and $v_{b_{k_n}^i}$ represents the dissatisfaction degree of subject ℓ_i to subject d_k under the

attribute d_{σ} .

This paper consider to solve the linguistic intuitionistic fuzzy multi-attribute bilateral matching decision problem according to the linguistic intuitionistic fuzzy matrices $\widetilde{A}^{(d_k)} = [\widetilde{a}_{ij}^k]_{t \times p}$ and $\widetilde{B}^{(\ell_i)} = [\widetilde{b}_{kg}^i]_{h \times q}$, and weight vectors $\boldsymbol{\omega}^c = (\omega_1^c, \omega_2^c, ..., \omega_p^c)$ and $\boldsymbol{\omega}^d = (\omega_1^d, \omega_2^d, ..., \omega_p^d)$ ω_a^d). The procedure of the proposed decision method considering satisfaction and fairness degrees are as follows: First, the overall dominances are calculated based on the TODIM idea, and the attribute weights are calculated by using LIFIOWA operator. Then, the global dominance of bilateral subjects is obtained by aggregating the overall attribute dominance and attribute weight. Third, the satisfaction and fairness degrees are calculated by using the TODIM idea and the fair attenuation coefficients respectively. Finally, multiple bilateral matching models under a linguistic intuitionistic fuzzy environment are constructed and solved to obtain the optimal bilateral matching scheme (see Fig. 1).

4. Multi-attribute bilateral matching decision under a linguistic intuitionistic fuzzy environment

4.1. Calculation of overall attribute dominances

First, the overall attribute dominances of bilateral subjects are calculated by comparing the individual subjects with all subjects in the other side based on the TODIM idea [31].

For the side ∂_t , when subject ∂_k compares subject ℓ_i with the other subject ℓ_m of side ℓ , let $\varphi_m^k(\tilde{a}_{ni}^k;\tilde{a}_{mi}^k)$ be the attribute dominance of subject ℓ_i relative to subject ℓ_m , then $\varphi_{im}^k(\tilde{a}_{ij}^k, \tilde{a}_{mj}^k)$ is calculated as follows:

$$\varphi_{im}^{k}\left(\tilde{a}_{ij}^{k},\tilde{a}_{mj}^{k}\right) = \begin{cases} \sqrt{d\left(\tilde{a}_{ij}^{k},\tilde{a}_{mj}^{k}\right)}, \tilde{a}_{ij}^{k} \ge \tilde{a}_{mj}^{k}, i, m \in T, k \in H; \\ -\frac{1}{\theta}\sqrt{d\left(\tilde{a}_{ij}^{k},\tilde{a}_{mj}^{k}\right)}, \quad \tilde{a}_{ij}^{k} < \tilde{a}_{mj}^{k}, i, m \in T, k \in H. \end{cases}$$

$$\tag{7}$$

In Eq. (7), $d(a_{ij}, a_{ik})$ can be calculated by Eq. (1), and the values of \tilde{a}_{ij}^k and \tilde{a}_{mi}^k can be compared by Eqs. (3) and (4) in Definition 5; θ is the loss attenuation coefficient, indicating the degree of loss avoidance. The smaller θ is, the greater the degree of loss avoidance is. According to reference [32], when $\theta = 2.25$, it is most in line with the decision maker's attitude towards risk.

Let δ_{ii}^{k} be the overall attribute dominance of subject ∂_{k} with respect to subject ℓ_{i} to all other subjects of side ℓ . Then δ_{ii}^{k} is calculated as follows:

$$\delta_{ij}^{k} = \sum_{m=1, i \neq m}^{h} \varphi_{im}^{k} \left(\widetilde{a}_{ij}^{k}, \widetilde{a}_{mj}^{k} \right), i, m \in T, k \in H.$$

$$\tag{8}$$

By Eqs. (7) and (8), the overall attribute dominance matrix $\mathbf{Z}^{(\partial_k)} = [\delta_{ij}^k]_{t \ge 0}$ of subject ∂_k to subject ℓ_i under the attribute C_i is constructed.

Analogously, for side ℓ , when subject ℓ_i compares subject ∂_k with the other subject ∂_n of side ∂ , let $\varphi_{kn}^i(\tilde{b}_{k\sigma}^i, \tilde{b}_{n\sigma}^i)$ be the attribute dominance of subject ∂_k relative to subject ∂_n , then $\varphi_{kn}^i(\tilde{b}_{k\sigma}^i, \tilde{b}_{n\sigma}^i)$ is calculated as follows:

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$$\varphi_{kn}^{i}\left(\tilde{b}_{kg}^{i},\tilde{b}_{ng}^{i}\right) = \begin{cases} \sqrt{d\left(\tilde{b}_{kg}^{i},\tilde{b}_{ng}^{i}\right)}, \quad \tilde{b}_{kg}^{i} \ge \tilde{b}_{ng}^{i}, k, n \in H, i \in T; \\ -\frac{1}{\theta}\sqrt{d\left(\tilde{b}_{kg}^{i},\tilde{b}_{ng}^{i}\right)}, \quad \tilde{b}_{kg}^{i} < \tilde{b}_{ng}^{i}, k, n \in H, i \in T. \end{cases}$$

$$\tag{9}$$

Let δ_{kn}^i be the overall attribute dominance of subject ℓ_i with respect to subject ∂_k to all other subjects of side ∂ , then δ_{kn}^i is calculated as follows:

$$\delta_{kn}^{i} = \sum_{n=1,k\neq n}^{h} \varphi_{kn}^{i} \left(\tilde{b}_{kg}^{i}, \tilde{b}_{ng}^{i} \right), k, n \in H, i \in T.$$

$$\tag{10}$$

By Eqs. (9) and (10), the overall attribute dominance matrix $\mathbf{Z}^{(l_i)} = [\delta_{kn}^i]_{h \times q}$ of subject ℓ_i to subject ∂_k under the attribute d_g is constructed.

4.2. Calculation of attribute weights

To improve the reliability of multi-attribute group opinion aggregation results, some scholars use similarity measures to reflect the overall consistency of individual evaluation and the consistency of each decision scheme under the decision attributes [33]. Therefore, the following method is used to calculate the attribute weights.

According to the linguistic intuitionistic fuzzy matrix $\widetilde{A}^{(\partial_k)} = [\widetilde{a}_{ij}^k]_{t \times p}$, let $p_{ij}^{(kn)}$ be the similarity between $\widetilde{a}_{ij}^k = (\mu_{\widetilde{a}_{ij}^k}, \nu_{\widetilde{a}_{ij}^k})$ and $\widetilde{a}_{ij}^n = (\mu_{\widetilde{a}_{ij}^n}, \nu_{\widetilde{a}_{ij}^n})$, then, $p_{ij}^{(kn)}$ is calculated as follows:

$$p_{ij}^{(kn)} = P\left(\tilde{a}_{ij}^{k}, \tilde{a}_{ij}^{n}\right) = 1 - \frac{1}{2} \left(\left| s_{\mu_{k}} - s_{\mu_{n}} \right|_{\tilde{a}_{ij}} + \left| s_{\nu_{k}} - s_{\nu_{n}} \right|_{\tilde{a}_{ij}} + \left| s_{\pi_{k}} - s_{\pi_{n}} \right|_{\tilde{a}_{ij}} \right).$$
(11)

In Eq. (11), the calculation for similarity is extended by Eq. (2).

Let $p_{ij}^{(k)}$ be the mean similarity between subject ∂_k and other subjects of side ∂ under the attribute c_j , then $p_{ij}^{(k)}$ is calculated as follows:

$$p_{ij}^{(k)} = \frac{1}{h-1} \sum_{n=1,k\neq n}^{h} p_{ij}^{(kn)}.$$
(12)

Then, the mean value of similarity $p_{ij}^{(k)}$ is calculated by Eqs. (11) and (12), and let it be the induced value of LIFIOWA operator; let ω_k be the attribute weight of \tilde{a}_{ij}^k , where ω_k can be calculated according to the method proposed in Ref. [2]. The group opinion evaluation value \tilde{a}_{ij}^{d} can be calculated as follow:

$$\widetilde{a}_{ij}^{d} = \text{LIFIOWA}\left(\langle u_{1}, \widetilde{a}_{ij}^{1} \rangle, \langle u_{2}, \widetilde{a}_{ij}^{2} \rangle, \dots, \langle u_{h}, \widetilde{a}_{ij}^{h} \rangle\right) = \sum_{k=1}^{h} \omega_{k} \widetilde{a}_{ij}^{k} = \left(s_{1-\prod_{k=1}^{h} \left(1 - f\left(\mu_{\widetilde{a}_{ij}}^{k}\right)\right)^{\omega_{k}}, s_{1-\prod_{k=1}^{h} f\left(\nu_{\widetilde{a}_{ij}}^{k}\right)^{\omega_{k}}\right).$$
(13)

In Eq. (13), the calculation for $\tilde{a}_{ij}^{\vartheta}$ is extended by Eqs. (5) and (6). By Eq. (13), $\tilde{a}_{ij}^{\vartheta}$ is obtained by aggregating the evaluation value \tilde{a}_{ii}^k (k = 1, ..., h), and thus a group opinion matrix $\tilde{A} = [\tilde{a}_{ii}^{\vartheta}]_{xxy}$ of side ϑ to ℓ_i under attribute c_j is established.

Furthermore, the group opinion value $\tilde{a}_{ij}^{\vartheta}$ is converted into the score value $r_{\tilde{a}_{ij}}^{\vartheta}$, and thus the group score matrix $\mathbf{R}^{(\vartheta)} = [\mathbf{r}_{\tilde{a}_{ij}}^{\vartheta}]_{t \times p}$ of side ϑ to \mathscr{N}_i is established, where $r_{\tilde{a}_{ij}}^{\vartheta}$ is calculated as follows:

$$r_{\tilde{a}_{ij}} = R\left(\tilde{a}_{ij}^{\vartheta}\right) = f\left(s_{\mu_{\vartheta}}\right) - f\left(s_{\nu_{\vartheta}}\right).$$
(14)

Finally, the score value $r_{a_{p}}^{d}$ is normalized to obtain the attribute weight vector $\boldsymbol{\omega}^{c} = (\omega_{1}^{c}, \omega_{2}^{c}, \cdots, \omega_{p}^{c})$, where ω_{j}^{c} is calculated as follows:

$$\omega_{j}^{c} = \frac{\sum_{i=1}^{r} r_{a_{ij}}^{a}}{\sum_{e=1}^{p} \sum_{i=1}^{t} r_{a_{ij}}^{a}}.$$
(15)

Analogously, according to the linguistic intuitionistic fuzzy matrix $\widetilde{B}^{(\ell_i)} = [\widetilde{b}_{kg}^i]_{h \times q}$, let $p_{kg}^{(im)}$ be the similarity between $\widetilde{b}_{kg}^i = (\mu_{\tilde{b}_{kg}^i}, \nu_{\tilde{b}_{kg}^i})$ and $\widetilde{b}_{kg}^m = (\mu_{\tilde{b}_{kg}^m}, \nu_{\tilde{b}_{kg}^m})$, then $p_{kg}^{(im)}$ is calculated as follows:

$$p_{kg}^{(im)} = P\left(\tilde{b}_{kg}^{i}, \tilde{b}_{kg}^{m}\right) = 1 - \frac{1}{2} \left(\left| s_{\mu_{i}} - s_{\mu_{kg}} - s_{\mu_{kg}} \right| + \left| s_{\nu_{i}} - s_{\nu_{m}} - s_{\mu_{kg}} \right| + \left| s_{\pi_{i}} - s_{\pi_{m}} - s_{\pi_{kg}} \right| \right).$$

$$(16)$$

Let $p_{kg}^{(i)}$ be the mean similarity between subject ℓ_g and the other subjects of side ℓ , then $p_{kg}^{(i)}$ is calculated as follows:

$$p_{kg}^{(i)} = \frac{1}{t-1} \sum_{m=1, i \neq m}^{t} p_{kg}^{(im)}.$$
(17)

Then, the mean value of similarity $p_{kg}^{(i)}$ is calculated by Eqs. (16) and (17), and let it be the induced value of LIFIOWA operator, and ω_i be the corresponding attribute weight of \tilde{b}_{kg}^i . The group opinion evaluation value \tilde{b}_{kg} can be calculated as follow:

$$\widetilde{b}_{kg}^{\prime} = \text{LIFIOWA}\left(\langle u_1, \widetilde{b}_{kg}^1 \rangle, \langle u_2, \widetilde{b}_{kg}^2 \rangle, \dots, \langle u_t, \widetilde{b}_{kg}^t \rangle\right) = \sum_{i=1}^t \omega_i \widetilde{b}_{kg}^i = \left(s_{1-\prod_{i=1}^t \left(1 - f\left(\mu_{\widetilde{b}_{kg}}\right)\right)^{\omega_i}, s_{1-\prod_{i=1}^t f\left(\nu_{\widetilde{b}_{kg}}\right)^{\omega_i}}\right).$$
(18)

By Eq. (18), the group opinion evaluation value \tilde{b}'_{kg} is obtained by aggregating the evaluation value $\tilde{b}^i_{kg}(i = 1,...,t)$, and thus a group opinion matrix $\tilde{B} = [\tilde{b}'_{kg}]_{h \times q}$ of side ℓ to ∂_k under attribute d_g is established.

Furthermore, the group opinion value $\tilde{b}_{kg}^{\checkmark}$ is converted into the score value $r_{\tilde{b}_{kg}^{\prime}}$, and thus the group score matrix $\mathbf{R}^{(\checkmark)} = [r_{\tilde{b}_{kg}^{\prime}}]_{h \times q}$ of the side \checkmark to ∂_i is established, where $r_{\tilde{b}_{kg}^{\prime}}$ is calculated as follows:

$$r_{\tilde{b}'_{kg}} = R\left(\tilde{b}'_{kg}\right) = f\left(s_{\mu_{kg}}\right) - f\left(s_{\nu_{kg}}\right).$$
⁽¹⁹⁾

Finally, the score value $r_{\tilde{b}'_{kg}}$ is normalized to obtain the attribute weight vector $\boldsymbol{\omega}^d = (\omega_1^d, \omega_2^d, ..., \omega_q^d)$, where ω_g^d can be calculated as follows:

$$\omega_{g}^{d} = \frac{\sum_{k=1}^{t} r_{\tilde{b}_{kg}}}{\sum_{f=1}^{q} \sum_{k=1}^{t} r_{\tilde{b}_{kg}}}.$$
(20)

4.3. Calculation of overall dominances and satisfaction degrees

First, the overall attribute dominance $\delta_{ij}^k (k = 1, ..., h)$ is aggregated to obtain the overall dominance δ_{ki}^{∂} , and thus the overall dominance matrix $Z^{(\partial)} = [\delta_{ki}^{\partial}]_{h \times t}$ of side ∂ to side ℓ is established, where δ_{ki}^{∂} is calculated as follows:

$$\delta_{ki}^{\partial} = \sum_{j=1}^{p} \omega_j^c \delta_{ij}^k.$$
(21)

Then, the overall dominance ∂_{kl}^{δ} is normalized to obtain the satisfaction degree α_{kl} of ∂_k to ℓ_i , where α_{kl} is calculated as follows:

$$\alpha_{ki} = \frac{\delta_{ki}^{d} - \frac{h+1}{h} \min_{k \in H, i \in T} \left\{\delta_{ki}^{d}\right\}}{\max_{k \in H, i \in T} \left\{\delta_{ki}^{d}\right\}}, k \in H, i \in T.$$
(22)

By Eq. (22), the satisfaction degree matrix $\widetilde{\mathbf{Z}}^{(\partial)} = [a_{ij}]_{h \times t}$ of side ∂ is constructed.

Analogously, the overall attribute dominance $\delta_{kg}^i (i = 1, ..., t)$ is aggregated to obtain the overall dominance δ_{ki}^{ℓ} , and thus the overall dominance matrix $\mathbf{Z}^{(\ell)} = [\delta_{ki}^{\ell}]_{h \times t}$ of side ℓ to side ϑ is established, where δ_{ki}^{ℓ} is calculated as follows:

$$\delta_{ki}^{\prime} = \sum_{g=1}^{q} \omega_g^d \delta_{kg}^i.$$
⁽²³⁾

Furthermore, the overall dominance δ'_{kg} is normalized to obtain the satisfaction degree β_{ki} of ℓ_i to ∂_k , where β_{ki} is calculated as follows:

$$\beta_{ki} = \frac{\delta_{ki}^{\prime} - \frac{t+1}{t} \min_{k \in H, i \in T} \{\delta_{ki}^{\prime}\}}{\max_{k \in H, i \in T} \{\delta_{ki}^{\prime}\} - \frac{t+1}{t} \min_{k \in H, i \in T} \{\delta_{ki}^{\prime}\}}, k \in H, i \in T.$$

$$(24)$$

By Eq. (24), the satisfaction degree matrix $\widetilde{\mathbf{Z}}^{(\ell)} = [\beta_{ki}]_{h \times t}$ of side ℓ is constructed.

4.4. Calculation of fairness degrees

To get the best matching scheme, on the one hand, it is necessary to maximize the satisfaction degree of bilateral subjects, on the other hand, it is also necessary to consider the fairness degree of bilateral subjects; the greater the satisfaction degree of bilateral subjects is, the greater the success rate of matching is, and the ratio of satisfaction degree determines the degree of the fairness degree. When the satisfaction degree ratio of bilateral subjects is closer to 1, the greater the matching fairness degree, the greater the probability of matching success.

Let $\mathscr{B}_{ki}^{\partial}$ be the relative fairness degree of ∂_k to \mathscr{I}_i , $\mathscr{B}_{ki}^{\prime}$ be the relative fairness degree of \mathscr{I}_i to ∂_k , and $\widetilde{\mathscr{B}}_{ki}$ be the relative fairness degree between ∂_k and \mathscr{I}_i . Then, $\mathscr{B}_{ki}^{\partial}$, $\mathscr{B}_{ki}^{\prime}$ are calculated according to the TODIM idea as follows:

$$\begin{split} \boldsymbol{\wp}_{ki}^{\partial} &= \begin{cases} \sqrt{\frac{\alpha_{ki}}{\beta_{ki}}}, \alpha_{ki} > \beta_{ki}, \\ 1, \alpha_{ki} = \beta_{ki}, \\ \frac{1}{\rho} \sqrt{\frac{\alpha_{ki}}{\beta_{ki}}}, \alpha_{ki} < \beta_{ki}; \end{cases} \end{split}$$

$$\begin{aligned} \boldsymbol{\wp}_{ki}^{\prime} &= \begin{cases} \sqrt{\frac{\beta_{ki}}{\alpha_{ki}}}, \beta_{ki} > \alpha_{ki}, \\ 1, \beta_{ki} = \alpha_{ki}, \\ \frac{1}{\rho} \sqrt{\frac{\beta_{ki}}{\alpha_{ki}}}, \beta_{ki} < \alpha_{ki}; \end{cases} \end{aligned}$$

$$\begin{aligned} & (26) \end{aligned}$$

$$\widetilde{\wp}_{ki} = \begin{cases} \frac{\wp_{ki} + \wp_{ki}}{2}, \alpha_{ki} \neq \beta_{ki}, \\ 1, \alpha_{ki} = \beta_{ki}. \end{cases}$$
(27)

In Eqs. (25) and (26), ρ is the fair attenuation coefficient, which indicates the degree of fairness loss aversion. The smaller ρ is, the greater the degree of loss aversion is. Then, the relative fairness degree $\tilde{\wp}_{ki}$ is normalized to obtain the fairness degree \hbar_{ki} of ∂_k and ℓ_i , and \hbar_{ki} can be calculated as follows:

$$\hbar_{ki} = \begin{cases} \widetilde{\wp}_{ki}, \widetilde{\wp}_{ki} \le 1; \\ \frac{1}{\widetilde{\wp}_{ki}}, \widetilde{\wp}_{ki} > 1. \end{cases}$$
(28)

By Eq. (28), the fairness degree matrix $\mathbf{Z} = [\hbar_{ki}]_{h \times t}$ of bilateral subjects is constructed.

4.5. Bilateral matching models based on satisfaction degree

First, the matching matrix $X = [x_{ki}]_{h \times t}$ is introduced, where $x_{ki} = \begin{cases} 1, \gamma(\partial_k) = \ell_i, \\ 0, \gamma(\partial_k) \neq \ell_i. \end{cases}$ Based on the satisfaction degree matrices $\widetilde{Z}^{(\partial)} = 0$

 $[\alpha_{ki}]_{h \times t}$ and $\widetilde{Z}^{(\prime)} = [\beta_{ki}]_{h \times t}$, the model of bilateral matching considering satisfaction degree under the one-to-one matching constraint is established as follows:

$$(M-1) \begin{cases} \max D_1 = \sum_{k=1}^{h} \sum_{i=1}^{t} \alpha_{ki} x_{ki}, \\ \max D_2 = \sum_{k=1}^{h} \sum_{i=1}^{t} \beta_{ki} x_{ki}, \\ \text{s.t. } \sum_{i=1}^{t} x_{ki} = 1, k \in H, \\ \sum_{k=1}^{h} x_{ki} \le 1, i \in T, \\ x_{ki} \in \{0,1\}, k \in H, i \in T, \end{cases}$$

where $\operatorname{Max} D_1 = \sum_{k=1}^h \sum_{i=1}^t \alpha_{ki} x_{ki}$ represents maximizing the satisfaction degree of side ∂ , and $\operatorname{Max} D_2 = \sum_{k=1}^h \sum_{i=1}^t \beta_{ki} x_{ki}$ represents maximizing the satisfaction degree of side ℓ .

For objective functions D_1 and D_2 , considering that α_{ki} and β_{ki} are of the same dimension, the model (M-1) can be transformed into a single objective model (M-2) by the linear weighted method as follows:

$$(\mathbf{M}-2) \begin{cases} \max D = \sum_{k=1}^{n} \sum_{i=1}^{t} [\omega_{1}\alpha_{ki} + \omega_{2}\beta_{ki}]x_{ki}, \\ \text{s.t.} \quad \sum_{i=1}^{t} x_{ki} = 1, k \in H, \\ \sum_{k=1}^{h} x_{ki} \le 1, i \in T, \\ x_{ki} \in \{0, 1\}, k \in H, i \in T, \end{cases}$$

where ω_1 and ω_2 represent the weights of the objective functions D_1 and D_2 respectively. By using mathematical software such as Lingo to solve the model (M-2), the optimal matching scheme considering satisfaction degrees can be obtained.

Similarly, the model (M-1) can be transformed into a single objective model (M-3) by the multiplicative weighted method as follows:

$$(M-3) \begin{cases} \max D = \sum_{k=1}^{h} \sum_{i=1}^{t} [a_{ki}^{\omega_3} \times \beta_{ki}^{\omega_4}] x_{ki}, \\ \text{s.t.} \quad \sum_{i=1}^{t} x_{ki} = 1, k \in H, \\ \sum_{k=1}^{h} x_{ki} \le 1, i \in T, \\ x_{ki} \in \{0, 1\}, k \in H, i \in T, \end{cases}$$

where ω_3 and $\omega_4(0 \le \omega_3, \omega_4 \le 1, \omega_3 + \omega_4 = 1)$ represent the weights of objective function D_1 and D_2 respectively. By solving the model (M-3), the optimal matching scheme considering satisfaction degree can be obtained.

4.6. Bilateral matching models based on satisfaction and fairness degrees

Based on the matching matrix $X = [x_{ki}]_{h \times t}$ and the fairness degree matrix $Z = [\hbar_{ki}]_{h \times t}$, the model of bilateral matching considering fairness degrees under the one-to-one matching constraint is established as follows:

$$(M-4) \begin{cases} Max \quad D_3 = \sum_{k=1}^n \sum_{i=1}^l h_{ki} x_{ki}, \\ s.t. \quad \sum_{i=1}^l x_{ki} = 1, k \in H, \\ \sum_{k=1}^h x_{ki} \le 1, i \in T, \\ x_{ki} \in \{0, 1\}, k \in H, i \in T, \end{cases}$$

By using mathematical software such as Lingo to solve the model (M-4), the optimal matching scheme considering fairness degree can be obtained.

Considering the satisfaction and fairness degrees of bilateral matching, a multi-objective optimization model of bilateral matching is established as follows:

$$(M-5) \begin{cases} \max D_1 = \sum_{k=1}^{h} \sum_{i=1}^{t} \alpha_{ki} x_{ki}, \\ \max D_2 = \sum_{k=1}^{h} \sum_{i=1}^{t} \beta_{ki} x_{ki}, \\ \max D_3 = \sum_{k=1}^{h} \sum_{i=1}^{t} \hbar_{ki} x_{ki}, \\ \text{s.t. } \sum_{i=1}^{t} x_{ki} = 1, k \in H, \\ \sum_{k=1}^{h} x_{ki} \le 1, i \in T, \\ x_{ki} \in \{0, 1\}, k \in H, i \in T, \end{cases}$$

where $\operatorname{Max} D_1 = \sum_{k=1}^{h} \sum_{i=1}^{t} \alpha_{ki} x_{ki}$ represents maximizing the satisfaction degree of side ∂ , $\operatorname{Max} D_2 = \sum_{k=1}^{h} \sum_{i=1}^{t} \beta_{ki} x_{ki}$ represents maximizing the satisfaction degree of side ℓ , and $\operatorname{Max} D_3 = \sum_{k=1}^{h} \sum_{i=1}^{t} h_{ki} x_{ki}$ represents maximizing the fairness degree of bilateral

subjects.

For objective functions D_1 , D_2 and D_3 , considering that α_{ki} , β_{ki} and \hbar_{ki} are of the same dimension, the model (M-5) can be transformed into a single objective model (M-6) by the linear weighted method as follows:

$$(M-6) \begin{cases} Max \quad D = \sum_{k=1}^{h} \sum_{i=1}^{t} (w_1 \alpha_{ki} + w_2 \beta_{ki} + w_3 \hbar_{ki}), \\ s.t. \quad \sum_{i=1}^{t} x_{ki} = 1, k \in H, \\ \sum_{k=1}^{h} x_{ki} \le 1, i \in T, \\ x_{ki} \in \{0, 1\}, k \in H, i \in T, \end{cases}$$

where w_1 , w_2 and w_3 ($0 \le w_1$, w_2 , $w_3 \le 1$, $w_1 + w_2 + w_3 = 1$) represent the weights of objective function D_1 , D_2 and D_3 respectively. By solving the model (M-6), the optimal matching scheme considering satisfaction and fairness degrees can be obtained.

Similarly, the model (M-6) can also be transformed into a single objective model (M-7) by using the multiplicative weighting method as follows:

$$(M-7) \begin{cases} Max \quad D = \sum_{k=1}^{h} \sum_{i=1}^{t} (\alpha_{ki}^{v_1} \beta_{ki}^{v_2} \hbar_{ki}^{v_3}) x_{ij}, \\ s.t. \quad \sum_{i=1}^{t} x_{ki} = 1, k \in H, \\ \sum_{k=1}^{h} x_{ki} \le 1, i \in T, \\ x_{ki} \in \{0, 1\}, k \in H, i \in T, \end{cases}$$

where v_1 , v_2 and $v_3(0 \le v_1, v_2, v_3 \le 1, v_1 + v_2 + v_3 = 1)$ represent the weights of objective function D_1 , D_2 and D_3 respectively. By solving the model (M-7), the optimal matching scheme considering the satisfaction and fairness degrees can be obtained.

5. Linguistic intuitionistic fuzzy bilateral matching decision steps

In summary, the steps to solve the multi-attribute bilateral matching decision problem based on linguistic intuitionistic fuzzy information are as follows.

Step 1. By Eqs. (7) and (8), the overall attribute dominance matrix $\mathbf{Z}^{(\partial_k)} = [\delta_{ij}^k]_{t \times p}$ of the subject ∂_k to \mathscr{I}_i under the attribute c_j is constructed. By Eqs. (9) and (10), the overall attribute dominance matrix $\mathbf{Z}^{(\mathcal{I}_i)} = [\delta_{kg}^i]_{h \times q}$ of the subject \mathscr{I}_i to ∂_k under the attribute d_g is constructed.

Step 2. By Eqs. (11)–(13) and (16))-(18), group opinion matrices $\widetilde{A} = [\widetilde{a}_{ij}^{\partial}]_{t\times p}$ and $\widetilde{B} = [\widetilde{b}_{kg}]_{h\times q}$ are calculated; by Eqs. (14) and (19), group score matrices $\mathbf{R}^{(\partial)} = [\mathbf{r}_{\tilde{a}_{ij}}]_{t\times p}$ and $\mathbf{R}^{(\prime)} = [\mathbf{r}_{\tilde{b}_{kg}}]_{h\times q}$ are constructed; by Eqs. (15) and (20), attribute weight vectors $\boldsymbol{\omega}^{c} = (\omega_{1}^{c}, \omega_{2}^{c}, \ldots, \omega_{p}^{c})$ and $\boldsymbol{\omega}^{d} = (\omega_{1}^{d}, \omega_{2}^{d}, \ldots, \omega_{q}^{d})$ are obtained.

Step 3. By Eqs. (21) and (23), overall dominance matrices $\mathbf{Z}^{(\partial)} = [\delta_{ki}^{\partial}]_{h \times t}$ and $\mathbf{Z}^{(\ell)} = [\delta_{ki}^{\ell}]_{h \times t}$ are calculated. Then, by Eqs. (22) and (24), satisfaction degree matrices $\widetilde{\mathbf{Z}}^{(\partial)} = [\alpha_{ki}]_{h \times t}$ of side ∂ and $\widetilde{\mathbf{Z}}^{(\ell)} = [\beta_{ki}]_{h \times t}$ of side ℓ are constructed.

Step 4. By Eqs. (25) and (26), relative fairness degree matrices $\widehat{\mathbf{Z}}^{(\partial)} = [\widetilde{\wp}_{ki}^{\partial}]_{k \times t}$ of side ∂ and $\widehat{\mathbf{Z}}^{(\ell)} = [\widetilde{\wp}_{ki}^{\ell}]_{h \times t}$ of side ℓ are constructed.

By Eqs. (27) and (28), the bilateral relative fairness degree matrix $\widehat{\mathbf{Z}} = [\widetilde{\wp}_{ki}]_{h \times t}$ and the fairness degree matrix $\mathbf{Z} = [\hbar_{ki}]_{h \times t}$ are constructed.

Step 5. A bilateral matching model (M-1) based on satisfaction degrees is constructed and transformed into the single-objective models (M-2) and (M-3); and the models (M-2) and (M-3) are solved to obtain the optimal matching scheme.

Step 6. A bilateral matching model (M-4) based on fairness degrees is constructed; a multi-objective model (M-5) considering satisfaction and fairness degrees is constructed and transformed into models (M-6) and (M-7); and the models (M-6) and (M-7) are solved to obtain the optimal matching scheme.

6. Example analysis

Consider the matching problem between enterprises and job seekers. There are four enterprises $\partial = \{\partial_1, \partial_2, \partial_3, \partial_4\}$ in a campus recruitment meeting that all need to recruit one employee for a certain position, and five undergraduates $\ell = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5\}$ are candidates. The enterprises evaluating the undergraduates mainly considers the following four aspects $C = \{c_1, c_2, c_3, c_4\}$: education, personal quality, professional knowledge and foreign linguistic level, The undergraduates evaluating the enterprises mainly considers

the following four aspects $D = \{d_1, d_2, d_3, d_4\}$: salary status, company prospects, company address and promotion space. Let $\widetilde{A}^{(\partial_k)} = [\widetilde{a}^k_{ij}]_{5\times 4}$ be the linguistic intuitionistic fuzzy preference matrix of subject ∂_k to subject ℓ_i under the attribute c_i ; let $\widetilde{B}^{(\ell_i)} = [\widetilde{B}^i_{kg}]_{4\times 4}$ be the linguistic intuitionistic fuzzy preference matrix of subject ℓ_i to subject ∂_k under the attribute d_g . The considered linguistic assessment scale set is $S = \{s_0: \text{ extremely poor; } s_1: \text{ very poor; } s_2: \text{ poor; } s_3: \text{ middle-lower; } s_4: \text{ middle; } s_5: \text{ middle-upper; } s_6: \text{ good; } s_7: \text{ very good; } s_8: \text{ extremely good}$. Linguistic intuitionistic fuzzy preference matrices $\widetilde{A}^{(\partial_k)} = [\widetilde{a}^k_{ij}]_{5\times 4}$ and $\widetilde{B}^{(\ell_g)} = [\widetilde{b}^k_{ij}]_{4\times 4}$ are shown in Tables 1–9.

6.1. Solution process

Step 1. By Eqs. (7) and (8), the overall attribute dominance matrix $\mathbf{Z}^{(\partial_k)} = [\delta_{ij}^k]_{5\times 4}$ of the subject ∂_k to ℓ_i under the attribute c_j is constructed. By Eqs. (9) and (10), the overall attribute dominance matrix $\mathbf{Z}^{(\ell_i)} = [\delta_{kg}^i]_{4\times 4}$ of the subject ℓ_i to ∂_k under the attribute d_g is constructed, as shown in Tables 10–18:

Step 2. By Eqs. (11)–(13) and (16))-(18), group opinion matrices $\widetilde{A} = [\widetilde{a}_{ij}^{\partial}]_{5\times 4}$ and $\widetilde{B} = [\widetilde{b}_{kg}]_{4\times 4}$ are calculated, as shown in Tables 19 and 20; by Eqs. (14) and (19), group score matrices $\mathbf{R}^{(\partial)} = [\mathbf{r}_{ij}]_{5\times 4}$ and $\mathbf{R}^{(\prime)} = [\mathbf{r}_{ij}]_{4\times 4}$ are constructed, as shown in Tables 21 and 22.

By Eqs. (15) and (20), the attribute weights are calculated, i.e., $\omega^{C} = (0.29, 0.22, 0.25, 0.24), \omega^{D} = (0.26, 0.32, 0.29, 0.13).$

Step 3. By Eqs. (21) and (23), overall dominance matrices $\mathbf{Z}^{(\partial)} = [\delta_{ki}^{\partial}]_{4\times 5}$ and $\mathbf{Z}^{(\ell)} = [\delta_{ki}^{\ell}]_{4\times 5}$ are calculated; by Eqs. (22) and (24), satisfaction degree matrices $\widetilde{\mathbf{Z}}^{(\partial)} = [\alpha_{ki}]_{4\times 5}$ of side ∂ and $\widetilde{\mathbf{Z}}^{(\ell)} = [\beta_{ki}]_{4\times 5}$ of side ℓ are constructed, as shown in Tables 23–26.

Step 4. By Eqs. (25) and (26), relative fairness degree matrices $\widehat{\mathbf{Z}}^{(\partial)} = [\widetilde{\wp}_{ki}^{\partial}]_{4\times 5}$ of side ∂ and $\widehat{\mathbf{Z}}^{(\ell)} = [\widetilde{\wp}_{ki}^{\ell}]_{4\times 5}$ of side ℓ are constructed.

By Eqs. (27) and (28), the bilateral relative fairness degree matrix $\hat{Z} = [\widetilde{\wp}_{ki}]_{4\times 5}$ and the fairness degree matrix $Z = [\hbar_{ki}]_{4\times 5}$ are constructed, as shown in Tables 27–30:

Step 5. A bilateral matching model (M-1) based on satisfaction degrees is constructed, and it can be transformed into the singleobjective models (M-2) and (M-3) by the linear weighting method and the multiplicative weighting method respectively; let $\omega_1 = 0.3$, $\omega_2 = 0.7$, $\omega_3 = 0.3$, $\omega_4 = 0.7$, then the optimal matching schemes are obtained by solving models (M-2) and (M-3), as shown in Tables 31 and 32.

Therefore, the optimal matching scheme based on the model (M-2) is $\{(\partial_1, \ell_5), (\partial_2, \ell_4), (\partial_3, \ell_2), (\partial_4, \ell_1)\}$, which indicates that enterprise ∂_1 is matched with undergraduate ℓ_5 , enterprise ∂_2 is matched with undergraduate ℓ_4 , enterprise ∂_3 is matched with undergraduate ℓ_2 , enterprise ∂_4 is matched with undergraduate ℓ_1 , and undergraduate ℓ_3 is unmatched; the optimal matching scheme based on the model (M-3) is $\{(\partial_1, \ell_1), (\partial_2, \ell_4), (\partial_3, \ell_2), (\partial_4, \ell_5)\}$, which indicates that enterprise ∂_1 is matched with undergraduate ℓ_4 , enterprise ∂_2 is matched with undergraduate ℓ_4 , enterprise ∂_3 is matched with undergraduate ℓ_4 , enterprise ∂_4 is matched with undergraduate ℓ_4 , enterprise ∂_3 is matched with undergraduate ℓ_4 , enterprise ∂_4 is matched with undergraduate ℓ_5 , and undergraduate ℓ_3 is unmatched.

Step 6. : A bilateral matching model (M-4) based on fairness degrees is constructed; and a multi-objective model (M-5) considering satisfaction and fairness degrees is constructed; it can be transformed into models (M-6) and (M-7) by the linear weighted method and the multiplicative weighted method; let $w_1 = w_2 = w_3 = \frac{1}{3}$, $v_1 = v_2 = v_3 = \frac{1}{3}$, then the optimal matching schemes are obtained by solving models (M-6) and (M-7), as shown in Tables 33–35.

Therefore, the optimal matching scheme based on model (M-4) is $\{(\partial_1, \ell_4), (\partial_2, \ell_3), (\partial_3, \ell_1), (\partial_4, \ell_2)\}$, which indicates that enterprise ∂_1 is matched with undergraduate ℓ_4 , enterprise ∂_2 is matched with undergraduate ℓ_3 , enterprise ∂_3 is matched with undergraduate ℓ_1 , enterprise ∂_4 is matched with undergraduate ℓ_2 , and undergraduate ℓ_5 is unmatched; the optimal matching scheme based on models (M-6) and (M-7) both are $\{(\partial_1, \ell_5), (\partial_2, \ell_4), (\partial_3, \ell_2), (\partial_4, \ell_1)\}$, which indicates that enterprise ∂_1 is matched with undergraduate ℓ_5 , enterprise ∂_2 is matched with undergraduate ℓ_4 , enterprise ∂_3 is matched with undergraduate ℓ_5 , enterprise ∂_2 is matched with undergraduate ℓ_4 , enterprise ∂_3 is matched with undergraduate ℓ_2 , enterprise ∂_4 is matched with undergraduate ℓ_4 , enterprise ∂_3 is matched with undergraduate ℓ_2 , enterprise ∂_4 is matched with undergraduate ℓ_4 , enterprise ∂_3 is matched with undergraduate ℓ_4 , enterprise ∂_3 is matched with undergraduate ℓ_4 , enterprise ∂_4 is matched with undergraduate ℓ_4 , enterprise ∂_3 is matched with undergraduate ℓ_4 , enterprise ∂_3 is matched with undergraduate ℓ_4 , enterprise ∂_3 is matched with undergraduate ℓ_4 , enterprise ∂_4 is matched with undergraduate ℓ_4 , enterprise ∂_3 is matched with undergraduate ℓ_4 , enterprise ∂_4 is matched with undergraduate ℓ_4 , enterprise ∂_4 is matched with undergraduate ℓ_4 , enterprise ∂_4 is matched with undergraduate ℓ_4 , enterprise ∂_4 , enterprise ∂_4 is matched. At the end, the optimal matching scheme is selected as that considering satisfaction and fairness degrees, i.e., $\{(\partial_1, \ell_5), (\partial_2, \ell_4), (\partial_3, \ell_2), (\partial_4, \ell_1)\}$.

6.2. Comparative analysis

To verify the feasibility and effectiveness of the proposed decision method, this paper reconstructs and solves the models based on

	c_1	<i>c</i> ₂	<i>c</i> ₃	C4
ℓ_1	(S_7, S_1)	(S_6, S_2)	(S_4, S_3)	(S_7, S_1)
l ₂	(S_5, S_2)	(S_5, S_2)	(S_6, S_1)	(S_6, S_2)
l3	(S_6, S_1)	(S_5, S_3)	(S_7, S_1)	(S_5, S_2)
14	(S_6, S_1)	(S_7, S_1)	(S_4, S_2)	(S_6, S_1)
15	(S_7, S_1)	(S_6, S_2)	(S_4, S_3)	(S_6, S_1)

Table 1 Linguistic intuitionistic fuzzy preference matrix $\widetilde{A}^{(\partial_1)} = [\widetilde{a}^1_{ij}]_{5\times 4}$ of subject ∂_1 to subject ℓ_i under attribute c_i .

Linguistic intuitionistic fuzzy preference matrix $\widetilde{A}^{(\partial_2)} = [\widetilde{a}_{ij}^2]_{5 \times 4}$ of subject ∂_2 to subject ℓ_i under attribute c_j .

	<i>c</i> ₁	C2	<i>C</i> ₃	C4
ℓ_1	(S_6, S_1)	(S_4, S_4)	(S_5, S_2)	(S_5, S_2)
l ₂	(S_6, S_2)	(S_5, S_1)	(S_6, S_2)	(S_5, S_2)
l ₃	(S_5, S_2)	(S_6, S_1)	(S_6, S_1)	(S_5, S_3)
l ₄	(S_6, S_2)	(S_6, S_1)	(S_6, S_1)	(S_4, S_3)
ľ 5	(S_6, S_1)	(S_5, S_3)	(S_6, S_2)	(S_7,S_1)

Table 3

Linguistic intuitionistic fuzzy preference matrix $\widetilde{A}^{(\partial_3)} = [\widetilde{a}_{ij}^3]_{5 \times 4}$ of subject ∂_3 to subject ℓ_i under attribute c_j .

	C1	c ₂	<i>C</i> ₃	C4
ℓ_1	(S_6, S_1)	(S_5, S_2)	(S_5, S_3)	(S_7, S_1)
12	(S_7, S_1)	(S_6, S_2)	(S_7, S_1)	(S_6, S_1)
13	(S_5, S_3)	(S_5, S_2)	(S_5, S_3)	(S_5, S_3)
14	(S_6, S_2)	(S_6, S_2)	(S_6, S_2)	(S_5, S_3)
ľ 5	(S_5, S_2)	(S_5, S_1)	(S_7, S_1)	(S_6, S_2)

Table 4

Linguistic intuitionistic fuzzy preference matrix $\widetilde{A}^{(\partial_4)} = [\widetilde{a}_{ij}^4]_{5 \times 4}$ of subject ∂_4 to subject ℓ_i under attribute c_j .

	c_1	<i>c</i> ₂	<i>c</i> ₃	<i>c</i> ₄
ℓ_1	(S_5, S_3)	(S_4, S_3)	(S_7, S_1)	(S_6, S_1)
ℓ_2	(S_6, S_1)	(S_7, S_1)	(S_5, S_3)	(S_5, S_3)
l ₃	(S_5, S_1)	(S_4, S_3)	(S_6, S_2)	(S_4, S_3)
l ₄	(S_4, S_3)	(S_5, S_1)	(S_4, S_3)	(S_6, S_2)
15	(S_6,S_2)	(S_5, S_2)	(S_6,S_1)	(S_5,S_3)

Table 5

Linguistic intuitionistic fuzzy preference matrix $\widetilde{B}^{(\ell_1)} = [\widetilde{b}_{kg}^1]_{4 \times 4}$ of subject ℓ_1 to subject ∂_k under attribute d_g .

	d_1	d_2	d_3	d_4
∂_1	(S_6, S_2)	(S_4, S_3)	(S_7, S_1)	(S_5, S_2)
∂_2	(S_5, S_2)	(S_6, S_1)	(S_6, S_2)	(S_7, S_1)
∂_3	(S_5, S_3)	(S_7, S_1)	(S_5, S_1)	(S_4, S_3)
∂_4	(S_7, S_1)	(S_4,S_4)	(S_6,S_1)	(S_4, S_2)

Table 6

Linguistic intuitionistic fuzzy preference matrix $\widetilde{B}^{(\ell_2)} = [\widetilde{b}_{ij}^2]_{4 \times 4}$ of subject ℓ_2 to subject ∂_k under attribute d_g .

	d_1	d_2	d_3	d_4
∂_1	(S_5, S_3)	(S_6, S_2)	(S_6, S_2)	(S_5, S_3)
∂_2	(S_6, S_1)	(S_6, S_1)	(S_5, S_2)	(S_5, S_3)
∂_3	(S_6, S_1)	(S_7, S_1)	(S_5, S_3)	(S_4, S_4)
∂_4	(S_5,S_3)	(S_5,S_2)	(S_7, S_1)	(S_5, S_3)

Table 7

Linguistic intuitionistic fuzzy preference matrix $\widetilde{B}^{(\ell_3)} = [\widetilde{b}_{ij}^3]_{4\times 4}$ of subject ℓ_3 to subject ∂_k under attribute d_g .

	d_1	d_2	d_3	d_4
∂_1	(S_5, S_2)	(S_5, S_2)	(S_7, S_1)	(S_6, S_2)
∂_2	(S_6, S_2)	(S_7, S_1)	(S_6, S_2)	(S_3, S_4)
∂_3	(S_5, S_2)	(S_7, S_1)	(S_6, S_1)	(S_4, S_3)
∂_4	(S_6, S_1)	(S_4, S_3)	(S_6,S_1)	(S_4, S_4)

Linguistic intuitionistic fuzzy preference matrix $\widetilde{B}^{(\ell_4)} = [\widetilde{b}_{ij}^4]_{4\times 4}$ of subject ℓ_4 to subject ∂_k under attribute d_g .

	d_1	d_2	d_3	d_4
∂_1	(S_4, S_4)	(S_7, S_1)	(S_5, S_1)	(S_2, S_4)
∂_2	(S_7, S_1)	(S_6, S_1)	(S_5, S_1)	(S_5, S_2)
∂_3	(S_4, S_3)	(S_6, S_2)	(S_3, S_3)	(S_2, S_5)
∂_4	(S_5, S_1)	(S_4, S_2)	(S_6,S_2)	(S_4, S_3)

Table 9

Linguistic intuitionistic fuzzy preference matrix $\widetilde{B}^{(\ell_5)} = [\widetilde{b}_{ij}^5]_{4\times 4}$ of subject ℓ_5 to subject ∂_k under attribute d_g .

	d_1	d_2	d_3	<i>d</i> ₄
∂_1	(S_5, S_3)	(S_6, S_1)	(S_5, S_3)	(S_4, S_2)
∂_2	(S_6, S_1)	(S_6, S_2)	(S_5, S_3)	(S_5, S_2)
∂_3	(S_6, S_2)	(S_6, S_1)	(S_4, S_3)	(S_3, S_3)
∂_4	(S_4, S_3)	(S_5, S_1)	(S_6, S_2)	(S_5, S_1)

Table 10

Overall attribute dominance matrix $\mathbf{Z}^{(\partial_1)} = [\delta_{ij}^1]_{5 \times 4}$ of subject ∂_1 to \mathcal{N}_i under attribute c_j .

	c_1	<i>C</i> ₂	c ₃	<i>C</i> 4
ℓ_1	1.2071	0.5500	-0.6515	1.5607
l 2	-0.7587	-0.3401	1.3429	-0.1179
13	0.0393	-0.7436	2.1907	-0.6936
14	0.0393	2.2565	0.2127	0.5500
l 5	1.2071	1.3405	-0.6515	0.5500

Table 11

Overall attribute dominance matrix $Z^{(\partial_2)} = [\delta_{ij}^2]_{5 \times 4}$ of subject ∂_2 to ℓ_i under attribute c_j .

	c_1	c_2	c_3	C4
ℓ_1	-0.2480	-0.9736	-0.6285	1.2071
l ₂	0.6964	1.0052	0.0393	0.4849
13	-0.6285	1.4659	1.0607	-0.1829
14	0.6964	1.4659	1.0607	-0.7436
l ₅	1.4142	-0.4052	0.0393	2.1124

Table 12

Overall attribute dominance matrix $Z^{(\partial_3)} = [\delta^3_{ij}]_{5 \times 4}$ of subject ∂_3 to ℓ_i under attribute c_j .

	c_1	c_2	c_3	<i>c</i> ₄
ℓ_1	1.0500	-0.4714	-0.6016	1.7071
12	1.7071	1.4977	1.3536	1.1964
l ₃	-0.7587	-0.5627	-0.6016	-0.6016
14	-0.1179	1.7031	0.3928	-0.6016
l 5	-0.1829	0.0044	1.3536	0.3928

Table 13

Overall attribute dominance matrix $\mathbf{Z}^{(\partial_4)} = [\delta^4_{ij}]_{5 \times 4}$ of subject ∂_4 to ℓ_i under attribute c_j .

	c_1	C2	c_3	C4
ℓ_1	-0.2480	-0.6515	1.8195	1.8536
12	1.7071	2.5153	-0.2480	-0.0258
13	0.6206	-0.6218	0.5393	-0.7587
14	-0.8238	1.4179	-0.8737	1.0500
l 5	0.4742	0.0415	1.1964	-0.0258

Overall attribute dominance matrix $\mathbf{Z}^{(\ell_1)} = [\delta_{kg}^1]_{4 \times 4}$ of subject ℓ_1 to ∂_k under attribute d_g .

	d_1	d_2	d_3	d_4
∂_1	0.5500	-0.1408	1.2071	0.4849
∂_2	-0.0258	0.9552	0.1857	1.7247
∂_3	-0.5365	1.5783	-0.6016	-0.5864
∂_4	1.3536	-0.7015	0.5500	-0.0757

Table 15

Overall attribute dominance matrix $\mathbf{Z}^{(\ell_2)} = [\delta_{kg}^2]_{4 \times 4}$ of subject ℓ_2 to ∂_k under attribute d_g .

	d_1	d_2	d_3	d_4
∂_1	-0.4444	0.0393	0.5500	0.3536
∂_2	0.8429	0.5500	-0.0258	0.3536
∂_3	1.3536	1.2071	-0.5365	-0.4714
∂_4	-0.4444	-0.5365	1.3536	0.3536

Table 16

Overall attribute dominance matrix $\mathbf{Z}^{(\ell_3)} = [\delta^3_{kg}]_{4 \times 4}$ of subject ℓ_3 to ∂_k under attribute d_g .

	d_1	d_2	d_3	d_4
∂_1	-0.3143	-0.0909	1.0607	1.6124
∂_2	0.5500	0.2278	-0.4714	-0.5864
∂_3	-0.3143	1.1124	0.1964	-0.0258
∂_4	1.0607	-0.7015	0.1964	-0.0258

Table 17

Overall attribute dominance matrix $\mathbf{Z}^{(\ell_4)} = [\delta_{kg}^4]_{4 \times 4}$ of subject ℓ_4 to ∂_k under attribute d_g .

	d_1	d_2	d_3	d_4
∂_1	-0.7015	1.3195	0.2778	-0.1408
∂_2	1.7247	0.6964	0.2778	1.5783
∂_3	-0.1408	0.1857	-0.7166	-0.6515
∂_4	0.8902	-0.7166	1.6124	0.8429

Table 18

Overall attribute dominance matrix $\mathbf{Z}^{(\ell_5)} = [\delta_{kg}^5]_{4 \times 4}$ of subject ℓ_5 to ∂_k under attribute d_g .

	d_1	d_2	d_3	d_4
∂_1	-0.0258	0.7071	0.1964	0.0393
∂_2	1.3536	0.1857	0.1964	0.6964
∂_3	0.6964	0.7071	-0.5365	-0.6016
∂_4	-0.6016	-0.5365	1.2071	1.2071

Table 19

Group opinion matrix $\widetilde{A} = [\widetilde{a}_{ij}^{\vartheta}]_{5\times 4}$ of side ϑ to ℓ_i under attribute c_j .

	c_1	c_2	c_3	<i>c</i> ₄
ℓ_1	$(S_{6.29}, S_{1.18})$	$(S_{5.16}, S_{2.32})$	$(S_{5.15}, S_{2.25})$	$(S_{6.34}, S_{1.30})$
12	$(S_{6.11}, S_{1.52})$	$(S_{5.71}, S_{1.62})$	$(S_{6.24}, S_{1.64})$	$(S_{5.65}, S_{1.64})$
13	$(S_{5.45}, S_{1.53})$	$(S_{4.92}, S_{2.16})$	$(S_{5.93}, S_{1.57})$	$(S_{4.91}, S_{2.77})$
14	$(S_{5.86}, S_{1.58})$	$(S_{5.98}, S_{1.23})$	$(S_{5.54}, S_{1.58})$	$(S_{5.63}, S_{1.75})$
l ₅	$(S_{6.30}, S_{1.23})$	$(S_{5.34}, S_{2.19})$	$(S_{6.20}, S_{1.56})$	$(S_{5.93}, S_{1.57})$

different fair attenuation coefficients and different weights of bilateral subjects. The results are shown in Tables 36 and 37. The extended method based on Lin et al. [34] and Liu and Wang [35] are compared with the decision method proposed in this paper. The results are shown in Table 38.

It can be seen from Tables 36 and 37 that the optimal matching scheme considering only fairness degree and that considering

	under attribute d_g .

	d_1	d_2	d_3	d_4
∂_1	$(S_{5.21}, S_{2.47})$	$(S_{5.96}, S_{1.50})$	$(S_{6.14}, S_{1.63})$	$(S_{4.61}, S_{2.51})$
∂_2	$(S_{5.78}, S_{1.46})$	$(S_{6.20}, S_{1.02})$	$(S_{5.45}, S_{2.04})$	$(S_{5.02}, S_{2.42})$
∂_3	$(S_{5.44}, S_{2.06})$	$(S_{6.56}, S_{1.17})$	$(S_{5.02}, S_{1.83})$	$(S_{3.67}, S_{3.44})$
∂_4	$(S_{5.60}, S_{1.64})$	$(S_{4.59}, S_{1.95})$	$(S_{6.20}, S_{1.23})$	$(S_{4.56}, S_{2.38})$

Table 21

Group score matrix $\mathbf{R}^{(\partial)} = [\mathbf{r}_{\tilde{a}_{ii}}]_{5 \times 4}$ of side ∂ to \mathcal{I}_i under attribute c_j .

	c_1	<i>c</i> ₂	<i>C</i> ₃	C4
ℓ_1	0.6386	0.3547	0.3624	0.6294
12	0.5745	0.5113	0.5743	0.5005
l ₃	0.4894	0.3449	0.5462	0.2683
14	0.5348	0.5931	0.4949	0.4851
15	0.6334	0.3936	0.5802	0.5462

Table 22

Group score matrix $\mathbf{R}^{(\ell)} = [\mathbf{r}_{\tilde{b}_{kg}}]_{4 \times 4}$ of side ℓ to ∂_k under attribute d_g .

	d_1	d_2	d_3	d_4
∂_1	0.3425	0.5579	0.5638	0.2635
∂_2	0.5399	0.6475	0.4267	0.3253
∂_3	0.4227	0.6740	0.3984	0.0283
∂_4	0.4946	0.3295	0.6208	0.2731

Table 23

Overall dominance matrix $\mathbf{Z}^{(\partial)} = [\delta_{ki}^{\partial}]_{4 \times 5}$ of side ∂ .

	ℓ_1	ℓ_2	l 3	l ₄	ľ 5
∂_1	0.6827	0.0126	0.2290	0.6930	0.6141
∂_2	-0.1536	0.5493	0.3615	0.6112	0.8378
∂_3	0.4601	1.4501	-0.6386	0.2943	0.3806
∂_4	0.6845	0.9802	-0.0041	0.1066	0.4396

Table 24

Overall dominance matrix $\mathbf{Z}^{(\ell)} = [\delta_{ki}^{\ell}]_{4 \times 5}$ of side ℓ .

	ℓ_1	l ₂	ℓ_3	l 4	ľ 5
∂_1	0.5110	0.1025	0.4064	0.3021	0.3228
∂_2	0.5770	0.4336	0.0030	0.9570	0.4141
∂_3	0.1149	0.5213	0.3279	-0.2697	0.2567
∂_4	0.2771	0.1513	0.1049	0.5793	0.2262

Table 25

Satisfaction degree matrix $\widetilde{\mathbf{Z}}^{(\partial)} = [\alpha_{ki}]_{4 \times 5}$ of side ∂ .

	ℓ_1	ℓ_2	ľ 3	l 4	ℓ_5
∂_1	0.6587	0.3606	0.4569	0.6633	0.6282
∂_2	0.2867	0.5994	0.5158	0.6269	0.7277
∂_3	0.5597	1.0000	0.0710	0.4860	0.5243
∂_4	0.6595	0.7910	0.3532	0.4025	0.5505

satisfaction and fairness degrees are different with the variation of fairness attenuation coefficient ρ and weights of objective functions. It shows the necessity of fairness degree constraints in the process of bilateral matching.

It can be seen from Table 38 that the optimal bilateral matching schemes are slightly different due to different forms of preference information, weight assignment, and model construction. The differences are as follows: (1) The linguistic intuitionistic fuzzy numbers

Satisfaction degree matrix $\widetilde{\mathbf{Z}}^{(\mathscr{L})} = [\beta_{ki}]_{4\times 5}$ of side $\mathscr{L}.$

	ℓ_1	ℓ_2	l ₃	l ₄	ℓ_5
∂_1	0.6517	0.3327	0.5701	0.4886	0.5048
∂_2	0.7033	0.5913	0.2550	1.0000	0.5760
∂_3	0.3424	0.6598	0.5087	0.0421	0.4532
∂_4	0.4691	0.3708	0.3346	0.7050	0.4293

Table 27

Relative fairness degree matrix $\widehat{\mathbf{Z}}^{(\partial)} = [\widetilde{\wp}_{ki}^{\partial}]_{4 \times 5}$ of side ∂ .

	ℓ_1	ℓ_2	13	14	l 5
∂_1	1.0053	1.0411	0.3979	1.1651	1.1156
∂_2	0.2838	1.0068	1.4222	0.3519	1.1239
∂_3	1.2785	1.2311	0.1660	3.3967	1.0756
∂_4	1.1857	1.4605	1.0274	0.3358	1.1324

Table 28

Relative fairness degree matrix $\widehat{\mathbf{Z}}^{(\ell)} = [\widetilde{\wp}_{ki}^{\ell}]_{4 \times 5}$ of side ℓ .

	ℓ_1	ℓ_2	l ₃	l ₄	l ₅
∂_1	0.4421	0.4269	1.1170	0.3815	0.3984
∂_2	1.5661	0.4414	0.3125	1.2630	0.3954
∂_3	0.3476	0.3610	2.6766	0.1308	0.4132
∂_4	0.3748	0.3043	0.4326	1.3236	0.3925

Table 29

Bilateral relative fairness degree matrix $\widehat{\mathbf{Z}} = [\widetilde{\wp}_{ki}]_{4 \times 5}$.

	ℓ_1	ℓ_2	l ₃	l ₄	ľ ₅
∂_1	0.7237	0.7340	0.7574	0.7733	0.7570
∂_2	0.9249	0.7241	0.8674	0.8075	0.7597
∂_3	0.8131	0.7961	1.4213	1.7638	0.7444
∂_4	0.7803	0.8824	0.7300	0.8297	0.7624

Table 30

Fairness degree matrix $\mathbf{Z} = [\hbar_{ki}]_{4 \times 5}$.

-						
	ℓ_1	ℓ_2	l ₃	l ₄	l 5	
∂_1	0.7237	0.7340	0.7574	0.7733	0.7570	
∂_2	0.9249	0.7241	0.8674	0.8075	0.7597	
∂_3	0.8131	0.7961	0.7036	0.5670	0.7444	
∂_4	0.7803	0.8824	0.7300	0.8297	0.7624	

Table 31

Optimal matching scheme based on model (M-2).

	ℓ_1	ℓ_2	l ₃	la	l ₅
∂_1	0	0	0	0	1
∂_2	0	0	0	1	0
∂_3	0	1	0	0	0
∂_4	1	0	0	0	0

are used to express preferences of subjects in the proposed method; then unknown attribute weights are calculated by the group opinion matrices; furthermore, multiple bilateral matching models considering satisfaction degrees and fairness degrees are constructed and solved to obtain the optimal scheme. (2) The 2-Tuple linguistic term numbers are used by Lin et al. (2019); then the qualitative evaluations are aggregated by 2TWA to calculate the attribute weights; furthermore, a bi-objective optimization model is constructed and the feedback process is proposed with the expected matching ordinal EMO constraint to obtain the optimal scheme.

Optimal matching scheme based on model (M-3).

	ℓ_1	ℓ_2	l ₃	l ₄	ľ ₅
∂_1	1	0	0	0	0
∂_2	0	0	0	1	0
∂_3	0	1	0	0	0
∂_4	0	0	0	0	1

Table 33

Optimal matching scheme based on model (M-4).

	ℓ_1	ℓ_2	ℓ_3	l ₄	l 5
∂_1	0	0	0	1	0
∂_2	0	0	1	0	0
∂_3	1	0	0	0	0
∂_4	0	1	0	0	0

Table 34

Optimal matching scheme based on model (M-6).

	ℓ_1	ℓ_2	ℓ_3	l ₄	ľ ₅
∂_1	0	0	0	0	1
∂_2	0	0	0	1	0
∂_3	0	1	0	0	0
∂_4	1	0	0	0	0

Table 35

Optimal matching scheme based on model (M-7).

	ℓ_1	ℓ_2	ℓ_3	l ₄	l ₅
∂_1	0	0	0	0	1
∂_2	0	0	0	1	0
∂_3	0	1	0	0	0
∂_4	1	0	0	0	0

Table 36

Comparison of optimal matching schemes under different fair attenuation coefficients.

Method	Optimal matching scheme
Method based on fairness degree model (M-4) ($\rho = 1.5$)	$\{(\partial_1, \ell_4), (\partial_2, \ell_3), (\partial_3, \ell_1), (\partial_4, \ell_2)\}$
Method based on satisfaction and fairness degree model (M-6) ($\rho = 1.5$)	$\{(\partial_1, \ell_5), (\partial_2, \ell_4), (\partial_3, \ell_2), (\partial_4, \ell_1)\}$
Method based on satisfaction and fairness degree model (M-7) ($\rho = 1.5$)	$\{(\partial_1, \ell_1), (\partial_2, \ell_4), (\partial_3, \ell_2), (\partial_4, \ell_5)\}$
Method based on fairness degree model (M-4) ($\rho = 2.25$)	$\{(\partial_1, \ell_4), (\partial_2, \ell_3), (\partial_3, \ell_1), (\partial_4, \ell_2)\}$
Method based on satisfaction and fairness degree model (M-6) ($\rho = 2.25$)	$\{(\partial_1, \ell_5), (\partial_2, \ell_4), (\partial_3, \ell_2), (\partial_4, \ell_1)\}$
Method based on satisfaction and fairness degree model (M-7) ($\rho = 2.25$)	$\{(\partial_1, \ell_5), (\partial_2, \ell_4), (\partial_3, \ell_2), (\partial_4, \ell_1)\}$
Method based on fairness degree model (M-4) ($\rho = 5$)	$\{(\partial_1, \ell_4), (\partial_2, \ell_3), (\partial_3, \ell_1), (\partial_4, \ell_2)\}$
Method based on satisfaction and fairness degree model (M-6) ($\rho = 5$)	$\{(\partial_1, \ell_5), (\partial_2, \ell_4), (\partial_3, \ell_2), (\partial_4, \ell_1)\}$
Method based on satisfaction and fairness degree model (M-7) ($ ho=5$)	$\{(\partial_1, \mathscr{\ell}_5), (\partial_2, \mathscr{\ell}_4), (\partial_3, \mathscr{\ell}_2), (\partial_4, \mathscr{\ell}_1)\}$

(3) The intuitionistic linguistic numbers are used by Liu and Wang (2021); then the weights of attributes are given by bilateral subjects directly; furthermore, a dual-objective optimization model considering satisfaction degrees is constructed to obtain the optimal matching scheme.

In addition, the methods proposed by Lin et al. (2019) and Liu and Wang (2021) cannot be directly used to solve the problem in this paper, and fail to consider the satisfaction and fairness degrees to construct multiple bilateral matching models, which may lead to inaccurate decision results. Therefore, the optimal matching scheme under a linguistic intuitionistic fuzzy environment obtained in this paper are relatively more reference.

7. Conclusions

A new decision method is proposed for a multi-attribute bilateral matching problem under a linguistic intuitionistic fuzzy environment. In the method, a novel method is proposed to calculate the attribute weights by LIFIOWA operator. Then, the satisfaction

Comparison of optimal matching schemes under different weights.

Method	Optimal matching scheme
Method based on satisfaction degree model (M-2) ($\omega_1 = 0.2, \omega_2 = 0.8$)	$\{(\partial_1, \ell_1), (\partial_2, \ell_4), (\partial_3, \ell_2), (\partial_4, \ell_5)\}$
Method based on satisfaction degree model (M-3) ($\omega_3 = 0.2, \omega_4 = 0.8$)	$\{(\partial_1, \ell_1), (\partial_2, \ell_4), (\partial_3, \ell_2), (\partial_4, \ell_5)\}$
Method based on satisfaction and fairness degree model (M-6) ($w_1 = 0.2, w_2 = 0.3, w_3 = 0.5$)	$\{(\partial_1, \ell_5), (\partial_2, \ell_4), (\partial_3, \ell_2), (\partial_4, \ell_1)\}$
Method based on satisfaction and fairness degree model (M-7) ($\tilde{\omega}_1 = 0.2, \tilde{\omega}_2 = 0.3, \tilde{\omega}_3 = 0.5$)	$\{(\partial_1, \ell_5), (\partial_2, \ell_4), (\partial_3, \ell_2), (\partial_4, \ell_1)\}$
Method based on satisfaction degree model (M-2) ($\omega_1 = 0.5, \omega_2 = 0.5$)	$\{(\partial_1, \ell_5), (\partial_2, \ell_4), (\partial_3, \ell_2), (\partial_4, \ell_1)\}$
Method based on satisfaction degree model (M-3) ($\omega_3 = 0.5, \omega_4 = 0.5$)	$\{(\partial_1, \ell_1), (\partial_2, \ell_4), (\partial_3, \ell_2), (\partial_4, \ell_5)\}$
Method based on satisfaction and fairness degree model (M-6) ($w_1 = \frac{1}{3}$, $w_2 = \frac{1}{3}$, $w_3 = \frac{1}{3}$)	$\{(\partial_1, \ell_5), (\partial_2, \ell_4), (\partial_3, \ell_2), (\partial_4, \ell_1)\}$
Method based on satisfaction and fairness degree model (M-7) ($\widetilde{\omega}_1 = \frac{1}{3}, \widetilde{\omega}_2 = \frac{1}{3}, \widetilde{\omega}_3 = \frac{1}{3}$)	$\{(\partial_1, \ell_5), (\partial_2, \ell_4), (\partial_3, \ell_2), (\partial_4, \ell_1)\}$
Method based on satisfaction degree model (M-2) ($\omega_1 = 0.8, \omega_2 = 0.2$)	$\{(\partial_1, \ell_4), (\partial_2, \ell_5), (\partial_3, \ell_2), (\partial_4, \ell_1)\}$
Method based on satisfaction degree model (M-3) ($\omega_3 = 0.8, \omega_4 = 0.2$)	$\{(\partial_1, \ell_4), (\partial_2, \ell_5), (\partial_3, \ell_2), (\partial_4, \ell_1)\}$
Method based on satisfaction and fairness degree model (M-6) ($w_1 = 0.5, w_2 = 0.3, w_3 = 0.2$)	$\{(\partial_1, \ell_5), (\partial_2, \ell_4), (\partial_3, \ell_2), (\partial_4, \ell_1)\}$
Method based on satisfaction and fairness degree model (M-7) ($\widetilde{\omega}_1=$ 0.5, $\widetilde{\omega}_2=$ 0.3, $\widetilde{\omega}_3=$ 0.2)	$\{(\partial_1, \ell_5), (\partial_2, \ell_4), (\partial_3, \ell_2), (\partial_4, \ell_1)\}$

Table 38

Comparison of optimal matching schemes for different methods.

Method	Optimal matching scheme
Method based on satisfaction degree model (M-2) (Method 1)	$\wp_1 = \{(\partial_1, \ell_5), (\partial_2, \ell_4), (\partial_3, \ell_2), (\partial_4, \ell_1)\}$
Method based on satisfaction degree model (M-3) (Method 2)	$\wp_2 = \{ (\partial_1, \ell_1), (\partial_2, \ell_4), (\partial_3, \ell_2), (\partial_4, \ell_5) \}$
Method based on fairness degree model (M-4) (Method 3)	$\wp_{3} = \{(\partial_{1}, \ell_{4}), (\partial_{2}, \ell_{3}), (\partial_{3}, \ell_{1}), (\partial_{4}, \ell_{2})\}$
Method based on satisfaction and fairness degree model (M-6) (Method 4)	$\mathscr{D}_{4} = \{ (\partial_{1}, \ell_{5}), (\partial_{2}, \ell_{4}), (\partial_{3}, \ell_{2}), (\partial_{4}, \ell_{1}) \}$
Method based on satisfaction and fairness degree model (M-7) (Method 5)	$\wp_5 = \{ (\partial_1, \ell_5), (\partial_2, \ell_4), (\partial_3, \ell_2), (\partial_4, \ell_1) \}$
Extended method based on the idea of Lin et al. (2019) (Method 6)	$\wp_6 = \{ (\partial_1, \ell_4), (\partial_2, \ell_5), (\partial_3, \ell_2), (\partial_4, \ell_1) \}$
Extension method based on the idea of Liu and Wang (2021) (Method 7)	$\wp_7 = \{(\partial_1, \ell_1), (\partial_2, \ell_4), (\partial_3, \ell_2), (\partial_4, \ell_5)\}$

degrees and fairness degrees are calculated using the TODIM idea and the fair attenuation coefficients to construct multiple bilateral matching models. These models can be solved to obtain the optimal bilateral matching scheme.

Compared with the existing methods, the main innovations of this paper are as follows: (1) The proposed method enrich the application of linguistic intuitionistic fuzzy sets in the field of bilateral matching decision, and can provide theoretical references for solving linguistic intuitionistic fuzzy bilateral matching problems. (2) An unknown attribute weight calculation method considering the consistency of group opinions is proposed based on LIFIOWA operator. (3) A fair degree calculation method considering the fair attenuation coefficient is proposed. (4) Multiple bilateral matching models considering satisfaction and fairness degrees under a linguistic intuitionistic fuzzy environment are established.

Limitations of this paper are as follows: (1) The bilateral matching decision problem under a linguistic intuitionistic fuzzy environment is preliminary discussed. The theories of complex types of linguistic preference information needs to be further studied. (2) It is difficult to solve the more complex bilateral matching problem, such as the multi-attribute group decision, multi-attribute multilateral matching and multi-attribute stable matching. (3) A more precise calculation method for fairness degrees considering the fair attenuation coefficient has not been designed.

Future research will mainly focus on the following areas: (1) The multi-attribute bilateral matching problem under a more complex linguistic fuzzy environment needs further study, where the evaluation information of bilateral subjects may be different types of linguistic fuzzy sets. (2) The determination method for attribute weights under other types of linguistic fuzzy environment needs to be discussed. (3) Considering that an unstable bilateral matching scheme may reduce satisfaction degrees of the bilateral subjects, the relevant theories and methods for stable matching under linguistic intuitionistic fuzzy environment will be studied in future. (4) The method for determining the fair attenuation coefficient needs to be explored to calculate fairness degrees of bilateral subjects.

Data availability

No data was used for the research described in this article.

CRediT authorship contribution statement

Shijie Huang: Writing – review & editing, Writing – original draft, Software, Methodology, Conceptualization. Qi Yue: Writing – review & editing, Writing – original draft, Supervision, Software, Methodology, Funding acquisition, Conceptualization. Yuan Tao: Writing – review & editing, Writing – original draft, Supervision, Software.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing

interests: Qi Yue reports financial support was provided by National Natural Science Foundation of China. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

This work was supported by the National Natural Science Foundation of China (Grant Nos. 72371153 and 71861015).

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