



Research article

A robust algorithmic framework for the evaluation of international cricket batters in ODI format based on q-rung linguistic neutrosophic quantification

Muhammad Saeed^a, Abdul Wahab^a, Jawad Ali^b, Ebenzer Bonyah^{c,d,*}

^a Department of Mathematics, University of Management and Technology Lahore, 54770, Pakistan

^b Department of Mathematics, Quaid-I-Azam University Islamabad, 45320, Pakistan

^c Department of Mathematics Education, Akenen Appiah Menkah University of Skills Training and Entrepreneurial Development, Kumasi, Ghana

^d Department of Mathematics, Faculty of Science and Technology Universitas Airlangga, Surabaya, 60115, Indonesia

ARTICLE INFO

Keywords:

Basic operations

Q-rung linguistic neutrosophic set

Aggregation operators

Decision-making

ABSTRACT

This article presents a novel approach for decision-making problems in which the criteria and alternatives are evaluated under a q-rung linguistic neutrosophic set (QRLNS) environment. QRLN sets are an extension of the traditional linguistic variables, which allow more flexibility and accuracy in modeling complex decision-making situations. We introduce several QRLN weighted aggregation operators, including QRLN weighted averaging operator, QRLN weighted geometric operator, and QRLN weighted hybrid operator, which can be used to aggregate the QRLN information provided by decision-makers. The properties and characteristics of these operators are analyzed, and their performance is compared with other existing aggregation operators. Finally, this system is able to handle the uncertainty and imprecision in the data and provide a more reliable assessment of the performance of cricket players. Our study demonstrates the potential of QRLN-based approaches for ranking assessment in other fields and provides insights for future research in this area.

1. Introduction

In the ever-evolving landscape of technology, the transformative power of machine learning has undeniably come to the forefront, revolutionizing various domains. From the precise identification of objects within images to the intricate comprehension of language nuances, machine learning has spearheaded breakthroughs in image recognition, deep learning, natural language generation, and language detection [1]. Through data-driven analysis and pattern recognition, machine learning has empowered researchers to unravel the intricate complexities of biological processes, fostering new insights and accelerating discoveries in the field of life sciences [2]. Through sophisticated algorithms and neural networks, image recognition has enabled machines to discern and categorize objects with astonishing accuracy [3,4]. Deep learning has propelled the boundaries of artificial intelligence, enabling systems to unravel complex patterns and make informed decisions [5,6]. In the realm of language, the fusion of machine learning has given rise to unparalleled capabilities, where machines can generate text that mirrors human expression, and can even adeptly identify and decipher diverse languages for seamless communication on a global scale [7–9].

* Corresponding author.

E-mail address: ebonyah@aamusted.edu.gh (E. Bonyah).

<https://doi.org/10.1016/j.heliyon.2023.e21429>

Received 15 June 2023; Received in revised form 4 October 2023; Accepted 20 October 2023

Available online 25 October 2023

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Cricket, as one of the most popular sports globally, garners immense attention and excitement from fans around the world [10]. The ranking of international cricket players holds significant importance in evaluating their performances and determining their standing within the sport. The existing ranking systems, such as those employed by the International Cricket Council (ICC), provide valuable insights into players' relative positions across different game formats [11]. However, there is a need for a robust approach that takes into account a wide range of factors to ensure a more accurate and holistic assessment of a player's ranking [12]. In this research article, we propose an algorithmic approach to assessing the ranking of international cricket players. By considering various parameters, including statistical performance, match-winning contributions, consistency, adaptability, and impact in different playing conditions, we aim to enhance the objectivity and effectiveness of player rankings. This robust approach acknowledges the multi-dimensionality of player performance and recognizes that individual contributions can significantly influence team outcomes [13]. The proposed approach seeks to address certain limitations of existing ranking systems, which often rely heavily on traditional statistical metrics such as batting average or bowling average. While these metrics offer valuable insights, they may not capture the full range of a player's abilities or their impact on the game. By incorporating additional parameters such as strike rate, centuries, match-winning innings, dismissal rate, dot ball percentage, and adaptability, we aim to provide a more nuanced and comprehensive evaluation of players' contributions [14]. Furthermore, this research article aims to contribute to the ongoing discourse on player rankings and their implications for team selection, performance evaluation, and strategic decision-making [15]. By employing a data-driven approach and conducting a thorough analysis of historical player data, we can derive valuable insights into the factors that contribute to a player's ranking and their overall value to the team.

A fuzzy set is a concept that extends classical sets by allowing elements to have varying degrees of membership. In contrast to classical sets where elements are either fully inside or outside a set, fuzzy sets assign membership values between 0 and 1 to indicate the degree of belonging [16]. For instance, a fuzzy set "Tall" might assign a person's height a membership value of 0.8 to indicate they are "quite tall." These membership values are determined by functions called membership functions, and they capture the uncertainty or imprecision inherent in certain real-world concepts. Applications of fuzzy sets in the fields of diverse, such as AI, control systems, and decision-making, where situations involve ambiguity or incomplete information. A q-rung fuzzy set constitutes an innovative extension of classical sets, introducing membership degrees characterized by a q-rung function [17]. The essence of the q-rung function lies in its ability to mold membership degrees across diverse temperature ranges, effectively capturing the nuanced transitions between different levels of warmth. This concept proves invaluable in scenarios where conventional fuzzy sets fall short of accurately encapsulating the intricacies of membership degrees. Applications of the q-rung fuzzy set span domains ranging from artificial intelligence to decision analysis, offering a versatile approach to managing imprecision across various fields. A neutrosophic set is a mathematical concept that generalizes the notions of classical sets, fuzzy sets, and intuitionistic fuzzy sets [18]. It was introduced by mathematician Florentin Smarandache in 1998 as an extension of these existing set theories to account for indeterminacy, ambiguity, and incomplete information in a more flexible manner. Neutrosophic sets are particularly useful in situations where the truth membership, indeterminacy membership, and falsity membership of elements are considered simultaneously. In real-world circumstances, due to the growing ambiguities and uncertainties, the issues of decision-making [19] have gotten increasingly complicated across various sectors. Researchers have proposed a variety of fuzzy set types [20], intuitionistic fuzzy [21], hesitant fuzzy [22], and linguistic variables [23], to describe and express the confusing and vague information during making decisions situations in order to address these obstacles. A mathematical framework called fuzzy set theory was developed to cope with uncertainty and ambiguity in data. An element can belong to a set with a positive degree between 0 and 1, instead of a binary value of 0 or 1, in this expansion of traditional set theory. Control systems, AI, and pattern identification have all benefited from the use of fuzzy sets [24]. Fuzzy set theory has been extended to solve the drawbacks of conventional fuzzy sets with the Q-Rung fuzzy set [17]. In it, the idea of q-rung linguistic variables is introduced. These variables are used to characterize the degree to which an element belongs to a set based on linguistic words. The linguistic variables of q-rung offer a more adaptable and precise representation of the ambiguity and uncertainty in data. A Q-rung fuzzy set theory, which is a modification of fuzzy set theory [25] enables the representation of ambiguous and imprecise information in decision-making procedures. A q-rung linguistic variable in Q-rung fuzzy sets represents an element's membership level in the set. The q-rung linguistic variable, which has a maximum value of r and a range of 0 to r , determines an element's level of membership based on a linguistic term. The Q-rung fuzzy set theory has been used in a variety of areas, including pattern detection, decision-making, and image processing. Another addition to the classical set theory that enables the representation of ambiguous or partial information is neutrosophic set theory [26]. The truthness degree, indeterminacy degree, and falsity degree are the three factors that neutrosophic sets utilize to describe an element's membership in a set. The amount to which an element is true is indicated by its degree of truth, and the extent to which an element is false is shown by its degree of falsity. The degree of indeterminacy is a measure of how uncertain an element's truth or falsity is [27]. Numerous disciplines, including artificial intelligence, decision-making, and image processing, have used neutrosophic set theory. Combining q-rung fuzzy set theory with neutrosophic set theory, linguistic neutrosophic fuzzy set theory [28] enables the representation and manipulation of ambiguous, incomplete, and hazy information in decision-making processes. Decision-making, pattern recognition, and image processing are just a few of the areas where linguistic neutrosophic fuzzy set theory has been used [29]. However, these fuzzy sets have difficulties when it comes to handling the ambiguities and uncertainties that emerge during decision-making processes, particularly when the linguistic variables are utilized to convey the decision-makers viewpoints [30]. The idea of neutrosophic sets, which is an advancement on fuzzy sets that can manage ambiguity, inconsistently [31], and incompleteness in decision-making scenarios, was presented to solve these shortcomings. Neutrosophic sets, however, also have limits when it comes to handling difficult decision-making circumstances including language factors. As an extension of neutrosophic sets and linguistic variables, q-rung linguistic neutrosophic fuzzy (QRLNF) sets were proposed to get over these restrictions [32]. When making decisions, the weight is used to express how important an alternative is, whereas the q-rung linguistic phrase represents the

degrees of membership of the linguistic variable. The notions of q-rung linguistic variables and neutrosophic sets are combined to form neutrosophic (q-LNS) fuzzy sets [33]. A linguistic term, a degree of positive membership, and a degree of negative membership are all included in q-rung linguistic words, which are used to represent the membership degrees of elements in q-LNS fuzzy sets. The degree of non-membership describes the degree to which an element does not belong to the linguistic term, whereas the degree of membership indicates the true value of an element belonging to a certain linguistic word. Complex decision-making issues may be handled with the q-LNS fuzzy sets, especially when working with shaky and ambiguous data. They give decision-making processes a more thorough framework for describing and managing ambiguity, vagueness, and indeterminacy [34]. Here is an illustration of a q-LNS set's representation: Think about a decision-making scenario involving the assessment of an employee's performance. Language-based descriptors like "excellent," "good," "fair," and "poor" can be used to rate an employee's performance. We notify a membership and non-membership of each linguistic phrase. Suppose the membership and non-membership for an employee are expressed as follows: Excellent: Truthiness = 0.8, Falsity = 0.1, Good: Truthiness = 0.6, Falsity = 0.3, Fair: Truthiness = 0.4, Falsity = 0.5, Poor: Truthiness = 0.2, Falsity = 0.7. The truthiness and falsity in this example denote the degree of agreement and disagreement with each linguistic phrase, respectively. Decision-makers in a variety of industries, including banking, medical, engineering, and decision support systems, can manage more complex and ambiguous information by employing q-LNS fuzzy sets, which enables them to make more educated judgments [35,36]. QLN operators, such as QLN weighted aggregation [37] operators and QLN hesitant fuzzy linguistic term sets, perform various decision-making tasks in a Q-LNS framework. Q-RLNs have been applied in various fields, including finance, engineering, medicine, and environmental science, to address the challenges of decision-making in uncertain and complex systems. The QRLN approach has also been used in ranking assessment, where it has been shown to outperform traditional ranking methods [38]. In this regard, suggests a robust method for solving decision-making issues in which the alternatives and criteria are assessed in a Q-RLNs environment. The authors introduce several Q-RLNs weighted aggregation operators, including the Q-RLNsWA operator, Q-RLNsWG operator, and Q-RLNs weighted hybrid operator, to aggregate the Q-RLNs information provided by DMs. These operators are designed to take into account the importance of the criteria or alternatives and to handle the uncertainty and ambiguity in the Q-RLNs information.

The Q-Rung Linguistic Neutrosophic Set (QRLNS) is a comprehensive framework that merges linguistic variables, neutrosophic sets [39], and Q-rung theory to address uncertainty in decision-making. It utilizes linguistic variables to qualitatively describe concepts, neutrosophic sets to incorporate degrees of truth, indeterminacy, and falsity, and Q-rung theory to introduce a Q-rung value indicating the truthfulness of intervals [40]. By combining these elements, QRLNS offers a sophisticated approach for expressing and analyzing imprecise and uncertain information, enhancing decision-making processes in fields such as medical diagnosis, risk assessment, and expert systems [41].

The advantages of QRLNS are as follows: QRLNS offers advantages such as enriched uncertainty representation through linguistic variables, incorporation of indeterminacy via neutrosophic sets, and precision enhancement with Q-rung intervals. It enables flexible decision-making, finds applicability in real-world scenarios involving qualitative assessments, and provides an interpretable framework to handle paradoxes and contradictions arising from uncertain data.

The properties and characteristics of these operators are analyzed, and their performance is compared with other existing aggregation operators. Finally, the proposed approach is illustrated through a real-world case study of rankings in an international cricket team of different countries. The results demonstrate the effectiveness and applicability of the suggested method for resolving decision-making issues using Q-RLNs data. Overall, this research provides a valuable contribution to the field of decision-making by introducing a new approach that can handle complex decision-making situations involving Q-RLNs information.

Our main objectives to this paper are as follows:

- Introduce Q-RLNs for precise international cricket player batting ranking evaluation, surpassing conventional methods.
- Showcase Q-RLNs' superiority by demonstrating accurate rankings compared to traditional techniques.
- Advance cricket player rankings using Q-RLNs for enhanced accuracy aligned with actual performance.
- Enhance the cricketing experience by proposing a nuanced ranking system adaptable to existing methodologies.
- Address uncertainty via hybrid linguistic-Q-Rung Neutrosophic Logic, capturing intricate nuances for robust research outcomes.

The following are the main contributions that we made to this paper:

- This research article introduces an innovative algorithmic framework that seamlessly integrates Q-Rung Linguistic Neutrosophic Quantification (Q-RLNQ) with cricket player evaluation in the context of One Day International (ODI) format. This integration enhances the precision and depth of player assessment, revolutionizing the conventional ranking methods.
- The research contributes to the advancement of Q-RLNQ, presenting a novel application in the realm of cricket player evaluation. By merging linguistic quantification with Q-Rung Neutrosophic Logic, the framework accommodates uncertainties and complexities inherent to cricket performances, setting a new standard for accuracy and comprehensiveness.
- One of the central contributions is the demonstration of the superior accuracy and reliability of the proposed framework compared to traditional methods. By harnessing the power of Q-RLNQ, the algorithmic framework offers a more precise representation of players' performances, resulting in a ranking system that aligns more closely with their true capabilities on the field.
- This research introduces refined evaluation metrics specifically tailored for cricket batters in the ODI format. These metrics go beyond conventional measures, embracing the multifaceted nature of the sport. By doing so, the framework delivers a comprehensive analysis that captures both quantitative statistics and qualitative nuances, ensuring a holistic evaluation.

- The research article’s primary contribution lies in its potential to impact the international cricketing community. The algorithmic framework presents a reliable and adaptable ranking system that caters to the needs of players, teams, and enthusiasts alike. The potential to enhance the overall cricketing experience while facilitating fair competition underscores the significance of this contribution.
- The research addresses a critical issue in player evaluation by introducing a solution to manage uncertainty using Q-Rung Linguistic Neutrosophic Quantification. By quantifying linguistic variables within the Q-RLNQ framework, this contribution refines the assessment process, acknowledging the inherent subjectivity in player evaluations.
- We provide a hybrid degree of linguistics with Q-rung Neutrosophic to obtain a generalized structure known as Q-RLNs in order to deal with the problem of uncertainty with positive, indeterminacy, and negative membership degree.

The motivation for conducting a study is grounded in the pressing need to enhance the evaluation and understanding of cricket batters’ performance in the One Day International (ODI) format. Cricket is a widely followed sport with a global fan base, and the ODI format is one of its most popular variants. However, traditional methods of assessing player performance often fall short of capturing the intricacies and nuances of the game. By introducing a robust algorithmic framework based on q-rung linguistic neutrosophic quantification, this research seeks to revolutionize the way we analyze and appreciate the skills of international cricket batters. Such an approach promises to provide a more comprehensive, accurate, and nuanced evaluation, benefiting not only cricket enthusiasts but also coaches, selectors, and stakeholders involved in the sport. In a rapidly evolving cricket landscape, this study aims to contribute valuable insights that can shape strategies, selection processes, and the overall enjoyment of ODI cricket.

This article’s last section is as follows: Sections 2 and 3 discuss their fundamental operations, while Section 4 looks at a few aggregation operators, including Q-RLNs averaging operators, and their fundamental properties. Section 2 discusses fundamental preliminary steps and constructs a unique structure called Q-RLNs. We provide a method for dealing with DMs difficulties in section 5 that is based on Q-rung linguistic neutrosophic operators. In section 6, we evaluate the batting rankings of international players for various nations using the q-rung linguistic neutrosophic. We compute the sensitivity analysis for various values of “q” in section 7 and provide graphs for both arithmetic and geometric data. To prove the new model’s superiority, we compare it to the current model in section 8 of our paper. Give a conclusion and future work in section 9.

2. Preliminaries

In this section, the concepts of q-RLNs and their historical context will be provided. We will define some terms such as fuzzy set with its characteristics, Neutrosophic set with its properties, etc.

To deal with the ambiguous nature of data, fuzzy sets were initially devised in 1965 by Zadeh [42].

Definition 1. (Fuzzy Set) [43] A concept of fuzzy set \tilde{h} defined as $\tilde{h} = \{(c'', C_{\tilde{h}}(c'')) | c'' \in W'\}$ such that $C_{\tilde{h}} : W' \rightarrow I$ where $C_{\tilde{h}}(c'')$ denotes the belonging value of $c'' \in \tilde{h}$.

Definition 2. (Characteristic of fuzzy sets) [44] Let there are two fuzzy sets say \tilde{h} and $\tilde{\ell}$ then $\forall c'' \in W'$, then

- (i) $\tilde{h} \cup \tilde{\ell} = \{(c'', \max\{C_{\tilde{h}}(c''), C_{\tilde{\ell}}(c'')\})\}$
- (ii) $\tilde{h} \cap \tilde{\ell} = \{(c'', \min\{C_{\tilde{h}}(c''), C_{\tilde{\ell}}(c'')\})\}$
- (iii) $\tilde{h}^c = \{(c'', 1 - C_{\tilde{h}}(c'')) | c'' \in W'\}$

Although the degree of membership is a key factor for fuzzy sets when dealing with ambiguous scenarios, there are a number of circumstances where non-membership degrees should be considered to apply fuzzy sets to such situations appropriately.

Definition 3. (Q-rung Orthopair Fuzzy Set) [45] Assume that the set of universal discourse is represented by F' , a q-ROFS on F' is defined below:

$$B'_1 = (\tilde{u}, \{\odot_{B'_1}(\tilde{u}), \uplus_{B'_1}(\tilde{u})\} | \tilde{u} \in F'),$$

$\odot_{B'_1}(\tilde{u})$ in the closed unit interval is referred to as the degree of membership of B'_1 , $\uplus_{B'_1}(\tilde{u})$ is referred as the degree of the non-membership of B'_1 and $\odot_{B'_1}(\tilde{u}), \uplus_{B'_1}(\tilde{u})$ holds the relation: $0 \leq \odot_{B'_1}(\tilde{u})^q + \uplus_{B'_1}(\tilde{u})^q \leq 1$ for $\forall \tilde{u} \in B'_1$. Then indeterminacy degree of \tilde{u} in B'_1 is denoted by $\bar{\pi}_{B'_1}(\tilde{u}) = (\odot_{B'_1}(\tilde{u})^q + \uplus_{B'_1}(\tilde{u})^q - \odot_{B'_1}(\tilde{u})^q \uplus_{B'_1}(\tilde{u})^q)^{1/q}$ [46].

Definition 4. (Neutrosophic set) [47,48] A neutrosophic set (NS) defined on F' is explained here:

$$B_2 = (\tilde{u}, \{\odot_{B_2}(\tilde{u}), \Gamma_{B_2}(\tilde{u}), \uplus_{B_2}(\tilde{u})\} | \tilde{u} \in F'),$$

$\odot_{B_2}(\tilde{u}) \in [0, 1]$ is referred as truth membership degree of B_2 , $\Gamma_{B_2}(\tilde{u}) \in [0, 1]$ is addressed by neutral-membership degree of B_2 and $\uplus_{B_2}(\tilde{u}) \in [0, 3]$ is referred as false membership degree of B_2 and $\odot_{B_2}(\tilde{u}), \Gamma_{B_2}(\tilde{u}), \uplus_{B_2}(\tilde{u})$ holds the following condition: $0 \leq \odot_{B_2}(\tilde{u}) + \Gamma_{B_2}(\tilde{u}) + \uplus_{B_2}(\tilde{u}) \leq 3$ for $\forall \tilde{u} \in B_2$. Then $\pi_{B_2}(\tilde{u}) = 1 - \odot_{B_2}(\tilde{u}) + \Gamma_{B_2}(\tilde{u}) + \uplus_{B_2}(\tilde{u})$ is termed as refusal-membership degree of \tilde{u} in B_2 .

Definition 5. (Property of Neutrosophic set) [49] The NS \hat{T} is present in another NS \hat{L} , showed by $\hat{T} \subseteq \hat{L}$, if and only if $infT\hat{T}(\ddot{u}) \leq infT\hat{L}(\ddot{u})$, $supT\hat{T}(\ddot{u}) \leq supT\hat{L}(\ddot{u})$, $infI\hat{T}(\ddot{u}) \geq infI\hat{L}(\ddot{u})$, $supI\hat{T}(\ddot{u}) \geq supI\hat{L}(\ddot{u})$, $infF\hat{T}(\ddot{u}) \geq infF\hat{L}(\ddot{u})$ and $supF\hat{T}(\ddot{u}) \geq supF\hat{L}(\ddot{u})$ for any $\ddot{u} \in F'$.

Definition 6. (q-rung neutrosophic set) [50] A q-rung neutrosophic set (q-RNS) on F' is explained below:

$$B_3 = (\ddot{u}, \{\odot_{B_3}(\ddot{u}), \Gamma_{B_3}(\ddot{u}), \beth_{B_3}(\ddot{u})\} | \ddot{u} \in F'),$$

$\odot_{B_3}(\ddot{u}) \in [0, 1]$ is referred to as the positive membership degree of B_3 , $\Gamma_{B_3}(\ddot{u}) \in [0, 1]$ is referred to as the indeterminacy degree of B_3 and $\beth_{B_3}(\ddot{u}) \in [0, 1]$ is referred to as the negative membership degree of B_3 and $\odot_{B_3}(\ddot{u}), \Gamma_{B_3}(\ddot{u}), \beth_{B_3}(\ddot{u})$ satisfy the following given condition: $0 \leq \odot_{B_3}(\ddot{u})^q + \Gamma_{B_3}(\ddot{u})^q + \beth_{B_3}(\ddot{u})^q \leq 3$ for $\forall \ddot{u} \in B_3$. Then $\pi_{B_3}(\ddot{u}) = (1 - (\odot_{B_3}(\ddot{u}))^q + (\Gamma_{B_3}(\ddot{u}))^q + (\beth_{B_3}(\ddot{u}))^q)^{1/q}$ is called to as the refusal-membership degree \ddot{u} in B_3 .

Definition 7. [51] Suppose that if we have $\varepsilon = [0,1]$, then the given this mapping $\check{f}: \varepsilon \times \varepsilon \rightarrow \varepsilon$ known as ($t - Norm$) if for $a_0, a_1, a_2 \in \varepsilon$

1. \check{f} is monotonic, associative and also continuous.
2. $\check{f}(a,1) = a$.

Definition 8. [52] Suppose that if we have $\varepsilon = [0,1]$, then the given this mapping $\check{f}: \varepsilon \times \varepsilon \rightarrow \varepsilon$ known as ($t - Conorm$) if for $a_0, a_1, a_2 \in \varepsilon$.

1. \check{f} is monotonic, associative and also continuous.
2. $\check{f}(a,0) = a$.

Definition 9. [53] A t-Norm \check{T} to be known as the Archimedean triangular norm, the given properties must hold:

1. It is continuous.
2. $\check{T}(\check{a}, \check{a}) < \check{a} \forall \check{a} \in (0,1)$.

Definition 10. [54] A t-Conorm \check{f} to be known as the Archimedean triangular-Conorm, the given properties must hold:

1. It is continuous.
2. $\check{f}(\check{a}, \check{a}) > \check{a} \forall \check{a} \in (0,1)$.

3. Basic operations

Definition 11. (q-rung linguistic neutrosophic set) Let us suppose that a q-rung linguistic neutrosophic set (q-RLNS) L on F' is explained below:

$$B_4 = (s'_\alpha(\ddot{u}), \{\odot_{A_4}(\ddot{u}), \Gamma_{A_4}(\ddot{u}), \beth_{A_4}(\ddot{u})\} | \ddot{u} \in F'),$$

here, $s'_\alpha(\ddot{u}) \in \bar{S}$, and satisfy the given condition: $0 \leq \odot_{B_4}(\ddot{u})^q + \Gamma_{B_4}(\ddot{u})^q + \beth_{B_4}(\ddot{u})^q \leq 3$ for $\forall \ddot{u} \in F'$. Then $\pi_{B_4}(\ddot{u}) = (1 - (\odot_{B_4}(\ddot{u}))^q + (\Gamma_{B_4}(\ddot{u}))^q + (\beth_{B_4}(\ddot{u}))^q)^{1/q}$ is referred as refusal-membership degree of \ddot{u} in B_4 . For the purpose of simplicity, $(s'_\alpha(\ddot{u}), \{\odot_{B_4}(\ddot{u}), \Gamma_{B_4}(\ddot{u}), \beth_{B_4}(\ddot{u})\})$ is referred as (q-RLNN) which is denoted as $\zeta = (s'_\alpha, \{\odot, \Gamma, \beth\})$.

The below Scr. and Accuracy functions are the modified version of [55,56].

Definition 12. (Score function) Suppose that $\zeta = (s'_\alpha, \{\odot, \Gamma, \beth\})$ is a q-RLNN, then the term Score-function is

$$Scr(\zeta) = \alpha \times (\odot^q + 1 - \beth^q). \tag{1}$$

Definition 13. (Accuracy function) Suppose that $\zeta = (s'_\alpha, \{\odot, \Gamma, \beth\})$ is a q-RLNN, then the term Accuracy-function is

$$H(\check{\psi}) = \alpha \times (\odot^q + \Gamma^q + \beth^q).$$

Definition 14. [57] Suppose that $\check{\delta}_1 = (s_{\alpha_1}, \{\odot_1, \Gamma_1, \beth_1\})$ and $\check{\delta}_2 = (s_{\alpha_2}, \{\odot_2, \Gamma_2, \beth_2\})$ be any two q-RLNN, $Scr(\check{\delta}_1)$ and $Scr(\check{\delta}_2)$ is a Score-function of $\check{\delta}_1$ and $\check{\delta}_2$, $H(\check{\delta}_1)$ and $H(\check{\delta}_2)$ is an Accuracy-function of $\check{\delta}_1$ and $\check{\delta}_2$.

1. Suppose that if we have $Scr(\check{\delta}_1) > Scr(\check{\delta}_2)$, which showing that $\check{\delta}_1 > \check{\delta}_2$
2. Suppose that if we have $Scr(\check{\delta}_1) = Scr(\check{\delta}_2)$, which showing that $\check{\delta}_1 = \check{\delta}_2$
3. Suppose that if we have $H(\check{\delta}_1) > H(\check{\delta}_2)$, which showing that $\check{\delta}_1 > \check{\delta}_2$
4. Suppose that if we have $H(\check{\delta}_1) = H(\check{\delta}_2)$, which showing that $\check{\delta}_1 = \check{\delta}_2$.

Based on the previous q-rung linguistic picture fuzzy [58], operational rules are expanded to a more generic form in this section. For this, a bijective-function $g : [a, b] \subseteq \mathbb{R}$ in unit interval defined as $g(t) = \frac{t-a_1}{t-a_2}, \forall t \in [a_1, a_2]$, is utilized. Letting $a_1 = 0, a_2 = \xi$, then $g : [0, \xi]$ in unit interval.

Definition 15. Suppose that $\wp = (s_\alpha, \{\ominus, \Gamma, \Delta\})$, $\wp_1 = (s_{\alpha_1}, \{\ominus_1, \Gamma_1, \Delta_1\})$ and $\wp_2 = (s_{\alpha_2}, \{\ominus_2, \Gamma_2, \Delta_2\})$ be any three q-RLNS and \wp be a positive real number, then the following operations are following as:

1. Additive operation:

$$\wp_1 \oplus \wp_2 = \left(s_{\xi^{-1}(\zeta^{-1}(\zeta(g(\alpha_1)) + \zeta(g(\alpha_2)))}, \left\{ (\zeta^{-1}(\zeta(\ominus_1^q) + \zeta(\ominus_2^q)))^{1/q}, (\Delta^{-1}(\Delta(\Gamma_1) + \Delta(\Gamma_2))), (\Delta^{-1}(\Delta(\Delta_1) + \Delta(\Delta_2))) \right\} \right);$$

2. Multiplication:

$$\wp_1 \otimes \wp_2 = \left(s_{\xi^{-1}(\Delta^{-1}(\Delta(g(\alpha_1)) + \Delta(g(\alpha_2))))}, \left\{ (\Delta^{-1}(\Delta(\ominus_1) + \Delta(\ominus_2))), (\Delta^{-1}(\Delta(\Gamma_1) + \Delta(\Gamma_2))), (\zeta^{-1}(\zeta(\Delta_1^q) + (\zeta(\Delta_2^q)))^{1/q} \right\} \right);$$

3. Scalar-multiplication:

$$\beth \wp = \left(s_{\xi^{-1}(\zeta^{-1}(\beth \zeta(g(\alpha))))}, \left\{ (\zeta^{-1}(\beth \zeta(\ominus^q)))^{1/q}, (\Delta^{-1}(\beth \Delta(\Gamma))), (\Delta^{-1}(\beth \Delta(\Delta))) \right\} \right);$$

4. Power operation:

$$\wp^\beth = \left(s_{\xi^{-1}(\Delta^{-1}(\beth \Delta(g(\alpha))))}, \left\{ (\Delta^{-1}(\beth \Delta(\ominus))), (\Delta^{-1}(\beth \Delta(\Gamma))), (\zeta^{-1}(\beth \zeta(\Delta^q)))^{1/q} \right\} \right).$$

Theorem 1. If $\wp = (s_\alpha, \{\ominus, \Gamma, \Delta\})$, $\wp_1 = (s_{\alpha_1}, \{\ominus_1, \Gamma_1, \Delta_1\})$ and $\wp_2 = (s_{\alpha_2}, \{\ominus_2, \Gamma_2, \Delta_2\})$ be any three q-RLNS and $\beth_1, \beth_2, \beth_3 \geq 0$, then the following rules must behold for these three scalars.

1. $\wp_1 \oplus \wp_2 = \wp_2 \oplus \wp_1$
2. $\wp_1 \otimes \wp_2 = \wp_2 \otimes \wp_1$
3. $\beth \circ (\wp_1 \oplus \wp_2) = (\beth \circ \wp_2) \oplus (\beth \circ \wp_1)$
4. $(\wp_1 \otimes \wp_2)^\beth = (\wp_2)^\beth \otimes (\wp_1)^\beth$
5. $(\beth_1 \circ \wp) \oplus (\beth_2 \circ \wp) = (\beth_1 + \beth_2) \circ \wp$
6. $(\wp)^\beth_1 \otimes (\wp)^\beth_2 = (\wp)^\beth_{(1+\beth_2)}$

Theorem 2. Let us suppose that $\wp = (s_\alpha, \{\ominus, \Gamma, \Delta\})$, $\wp_1 = (s_{\alpha_1}, \{\ominus_1, \Gamma_1, \Delta_1\})$ and $\wp_2 = (s_{\alpha_2}, \{\ominus_2, \Gamma_2, \Delta_2\})$ be any three q-RLNS and \beth , thus, for addition and multiplication, the associative rules are:

1. $(\wp_0 \oplus \wp_1) \oplus \wp_2 = \wp_1 \oplus (\wp_0 \oplus \wp_2)$;
2. $\wp_0 \otimes (\wp_1 \otimes \wp_2) = (\wp_1 \otimes \wp_0) \otimes \wp_2$.

4. q-RLN aggregation operators

In this section, we examine the q-rung linguistic neutrosophic operators in terms of specified average arithmetic and geometric operations.

4.1. q-rung linguistic neutrosophic weighted averaging aggregation operators

Definition 16. [59] Suppose we say $\check{\delta}_\ell = (s_{\alpha_\ell}, \{\ominus_\ell, \varepsilon_\ell, \Delta_\ell\})$ ($\ell = 1, 2, 3, \dots, k$) are the q-RLNS, the based upon Archimeadean T-conorm and T-normm, the operator q-RLNWAA defined as follows:

$$q - RLNWAA(\check{\delta}_1, \check{\delta}_2, \check{\delta}_3, \dots, \check{\delta}_k) = \bigoplus_{\ell=1}^k (\aleph_\ell \circ \check{\delta}_\ell),$$

where $\aleph_\ell = (\aleph_1, \aleph_2, \aleph_3, \dots, \aleph_k)^T$ are weighted vectors, such that $\aleph_\ell \in [0, 1]$ and $\sum_{\ell=1}^k \aleph_\ell = 1$.

Theorem 3. Let us suppose that $\delta_\ell = \left(\delta_{\alpha_{ell}}, \{\ominus_\ell, \varepsilon_\ell, \beth_\ell\} \right)$ be an array of q -RLNs. The q -RLNWAA is defined as

$$q - RLNWAA(\delta_1, \delta_2, \delta_3, \dots, \delta_k) = \left(\begin{aligned} & \delta_{\hat{g}^{-1}(\Gamma^{-1}(\sum_{\ell=1}^k \aleph_\ell \Gamma(\hat{g}(\alpha_\ell))))}, \\ & \left\{ \left(\Gamma^{-1}(\sum_{\ell=1}^k \aleph_\ell \Gamma(\ominus_\ell^q)) \right)^{1/q}, \left(\beth^{-1}(\sum_{\ell=1}^k \aleph_\ell \beth(\varepsilon_\ell)) \right), \left(\beth^{-1}(\sum_{\ell=1}^k \aleph_\ell \beth(\beth_\ell)) \right) \right\} \end{aligned} \right). \tag{2}$$

We demonstrate utilizing the mathematical induction approach.

(a) Here, take $k = 2$, then

$$\begin{aligned} q - RLNWAA(\delta_1, \delta_2) &= \aleph_1 \delta_1 \oplus \aleph_2 \delta_2 \\ &= \left(\delta_{\hat{g}^{-1}(\Gamma^{-1}(\aleph_1 \Gamma(\hat{g}(\alpha_1))))}, \left\{ \left(\Gamma^{-1}(\aleph_1 \Gamma(\ominus_1^q)) \right)^{1/q}, \left(\beth^{-1}(\aleph_1 \beth(\varepsilon_1)) \right), \left(\beth^{-1}(\aleph_1 \beth(\beth_1)) \right) \right\} \right) \oplus \\ & \quad \left(\delta_{\hat{g}^{-1}(\Gamma^{-1}(\aleph_2 \Gamma(\hat{g}(\alpha_2))))}, \left\{ \left(\Gamma^{-1}(\aleph_2 \Gamma(\ominus_2^q)) \right)^{1/q}, \left(\beth^{-1}(\aleph_2 \beth(\varepsilon_2)) \right), \left(\beth^{-1}(\aleph_2 \beth(\beth_2)) \right) \right\} \right) \\ &= \left(\delta_{\hat{g}^{-1}(\Gamma^{-1}(\Gamma(\hat{g}(\hat{g}^{-1}(\Gamma^{-1}(\aleph_1 \Gamma(\hat{g}(\alpha_1)))) + \Gamma(\hat{g}(\hat{g}^{-1}(\Gamma^{-1}(\aleph_2 \Gamma(\hat{g}(\alpha_2)))))))))}, \right. \\ & \quad \left\{ \left(\Gamma^{-1}(\Gamma(\Gamma^{-1}(\aleph_1 \Gamma(\ominus_1^q)))^{1/q})^q + \Gamma^{-1}((\aleph_2 \Gamma(\ominus_2^q))^{1/q})^q \right)^{1/q}, \left(\beth^{-1}(\beth(\beth^{-1}(\aleph_1 \beth(\varepsilon_1))) + \beth(\beth^{-1}(\aleph_2 \beth(\varepsilon_2)))) \right) \right. \\ & \quad \left. \left. , \left(\beth^{-1}(\beth(\beth^{-1}(\aleph_1 \beth(\beth_1))) + \beth(\beth^{-1}(\aleph_2 \beth(\beth_2)))) \right) \right\} \right) \\ &= \left(\delta_{\hat{g}^{-1}(\Gamma^{-1}(\sum_{k=1}^2 \aleph_k \Gamma(\hat{g}(\alpha_\ell))))}, \right. \\ & \quad \left. \left\{ \left(\Gamma^{-1}(\sum_{\ell=1}^2 \aleph_\ell \Gamma(\ominus_\ell^q)) \right)^{1/q}, \left(\beth^{-1}(\sum_{\ell=1}^2 \aleph_\ell \beth(\varepsilon_\ell)) \right), \left(\beth^{-1}(\sum_{\ell=1}^2 \aleph_\ell \beth(\beth_\ell)) \right) \right\} \right) \end{aligned}$$

(b) We assume that any value of $k = i$ for which our result holds true,

$$q - RLNFWAA(\delta_1, \delta_2, \delta_3, \dots, \delta_i) = \left(\delta_{\hat{g}^{-1}(\Gamma^{-1}(\sum_{\ell=1}^i \aleph_\ell \Gamma(\hat{g}(\alpha_\ell))))}, \right. \\ \left. \left\{ \left(\Gamma^{-1}(\sum_{\ell=1}^i \aleph_\ell \Gamma(\ominus_\ell^q)) \right)^{1/q}, \left(\beth^{-1}(\sum_{\ell=1}^i \aleph_\ell \beth(\varepsilon_\ell)) \right), \left(\beth^{-1}(\sum_{\ell=1}^i \aleph_\ell \beth(\beth_\ell)) \right) \right\} \right)$$

(c) By utilizing (a) and (b) in this section, we can now demonstrate that our solution holds true for any $k = i + 1$.

$$\begin{aligned} q - RLNWAA(\delta_1, \delta_2, \delta_3, \dots, \delta_i, \delta_{i+1}) &= \bigoplus_{\ell=1}^i (\aleph_\ell \circ \delta_\ell) \oplus (\aleph_{acutet+1} \circ \delta_{i+1}) \\ &= \left(\delta_{\hat{g}^{-1}(\Gamma^{-1}(\sum_{\ell=1}^k \aleph_\ell \Gamma(\hat{g}(\alpha_\ell))))}, \right. \\ & \quad \left. \left\{ \left(\Gamma^{-1}(\sum_{\ell=1}^k \aleph_\ell \Gamma(\ominus_\ell^q)) \right)^{1/q}, \left(\beth^{-1}(\sum_{\ell=1}^k \aleph_\ell \beth(\varepsilon_\ell)) \right), \left(\beth^{-1}(\sum_{\ell=1}^k \aleph_\ell \beth(\beth_\ell)) \right) \right\} \right) \\ &= \left(\delta_{\hat{g}^{-1}(\Gamma^{-1}(\sum_{\ell=1}^i \aleph_\ell \Gamma(\hat{g}(\alpha_\ell))))}, \right. \\ & \quad \left. \left\{ \left(\Gamma^{-1}(\sum_{\ell=1}^i \aleph_\ell \Gamma(\ominus_\ell^q)) \right)^{1/q}, \left(\beth^{-1}(\sum_{\ell=1}^i \aleph_\ell \beth(\varepsilon_\ell)) \right), \left(\beth^{-1}(\sum_{\ell=1}^i \aleph_\ell \beth(\beth_\ell)) \right) \right\} \right) \oplus \left(\delta_{\hat{g}^{-1}(\Gamma^{-1}(\aleph_{i+1} \Gamma(\hat{g}(\alpha_{i+1}))))}, \right. \\ & \quad \left. \left\{ \left(\Gamma^{-1}(\aleph_{i+1} \Gamma(\ominus_{i+1}^q)) \right)^{1/q}, \left(\beth^{-1}(\aleph_{i+1} \beth(\varepsilon_{i+1})) \right), \left(\beth^{-1}(\aleph_{i+1} \beth(\beth_{i+1})) \right) \right\} \right) \\ &= \left(\delta_{\hat{g}^{-1}(\Gamma^{-1}(\sum_{\ell=1}^{i+1} \aleph_\ell \Gamma(\hat{g}(\alpha_\ell))))}, \right. \end{aligned}$$

$$\left\{ \left(\Gamma^{-1} \left(\sum_{\ell=1}^{i+1} \aleph_{\ell} \Gamma(\varrho_{\ell}^q) \right) \right)^{1/q}, \left(\Delta^{-1} \left(\sum_{\ell=1}^{i+1} \aleph_{\ell} \Delta(\varepsilon_{\ell}) \right) \right), \left(\Delta^{-1} \left(\sum_{\ell=1}^{i+1} \aleph_{\ell} \Delta(\Delta_{\ell}) \right) \right) \right\}$$

that is the result finalize satisfied.

Some properties are given below.

Theorem 4. (Idempotency) Let $\check{\delta}_{\ell} = (s_{\alpha_{\ell}}, \{\varrho_{\ell}, \varepsilon_{\ell}, \Delta_{\ell}\})$ be the array collection of q -RLNs, here if all $\check{\delta}_{\ell}$ are the equals, i.e., $\check{\delta}_{\ell} = \check{\delta} \forall \ell$, so:

$$q - RLNWAA (\check{\delta}_1, \check{\delta}_2, \dots, \check{\delta}_k) = \check{\delta}.$$

Proof. From above, given that $\check{\delta}_{\ell} = \check{\delta} \forall \ell$, therefore:

$$\begin{aligned} q - RLNWAA (\check{\delta}_1, \check{\delta}_2, \check{\delta}_3, \dots, \check{\delta}_k) &= q - RLNWAA (\check{\delta}, \check{\delta}, \check{\delta}, \dots, \check{\delta}) \\ &= \left(\check{s}_{\check{\delta}^{-1} \left(\Gamma^{-1} \left(\sum_{\ell=1}^k \aleph_{\ell} \Gamma(\check{g}(\alpha_{\ell})) \right) \right)}, \right. \\ &\quad \left. \left\{ \left(\Gamma^{-1} \left(\sum_{\ell=1}^k \aleph_{\ell} \Gamma(\varrho_{\ell}^q) \right) \right)^{1/q}, \left(\Delta^{-1} \left(\sum_{\ell=1}^k \aleph_{\ell} \Delta(\varepsilon_{\ell}) \right) \right), \left(\Delta^{-1} \left(\sum_{\ell=1}^k \aleph_{\ell} \Delta(\Delta_{\ell}) \right) \right) \right\} \right) \\ &= \left(\check{s}_{\alpha}, \left\{ \varrho, \varepsilon, \Delta \right\} \right) = \check{\delta}. \end{aligned}$$

Hence, our result is proved. \square

Theorem 5. (Monotonicity) Let $\check{\delta}_{\ell} = (s_{\alpha_{\ell}}, \{\varrho_{\ell}, \varepsilon_{\ell}, \Delta_{\ell}\})$ and $\wp_{\ell} = (s_{\alpha_{\ell}}, \{\varrho_{\ell}, \varepsilon_{\ell}, \Delta_{\ell}\})$ ($\ell = 1, 2, 3, \dots, k$) are the collections of two q -RLN, if $\check{\delta}_{\ell} \leq \wp_{\ell}$, so:

$$q - RLNWAA (\check{\delta}_1, \check{\delta}_2, \dots, \check{\delta}_{\ell}) \leq q - RLNWAA (\wp_1, \wp_2, \dots, \wp_{\ell}),$$

Proof. Here is given, g' is a monotonically increasing function. So,

$$\begin{aligned} &\left(\check{s}_{\check{g}'^{-1} \left(\Gamma^{-1} \left(\sum_{\ell=1}^k \aleph_{\ell} \Gamma(\check{g}'(\alpha_{\ell})) \right) \right)}, \left\{ \left(\Gamma^{-1} \left(\sum_{\ell=1}^k \aleph_{\ell} \Gamma(\varrho_{\ell}^q) \right) \right)^{1/q}, \left(\Delta^{-1} \left(\sum_{\ell=1}^k \aleph_{\ell} \Delta(\varepsilon_{\ell}) \right) \right), \left(\Delta^{-1} \left(\sum_{\ell=1}^k \aleph_{\ell} \Delta(\Delta_{\ell}) \right) \right) \right\} \right) \leq \\ &\left(\check{s}_{\check{g}'^{-1} \left(\Gamma^{-1} \left(\sum_{\ell=1}^k \aleph_{\ell} \Gamma(\check{g}'(\alpha_{\ell^*})) \right) \right)}, \left\{ \left(\Gamma^{-1} \left(\sum_{\ell=1}^k \aleph_{\ell} \Gamma(\varrho_{\ell^*}^q) \right) \right)^{1/q}, \left(\Delta^{-1} \left(\sum_{\ell=1}^k \aleph_{\ell} \Delta(\varepsilon_{\ell^*}) \right) \right), \left(\Delta^{-1} \left(\sum_{\ell=1}^k \aleph_{\ell} \Delta(\Delta_{\ell^*}) \right) \right) \right\} \right) \\ q - RLNWAA (\check{\delta}_1, \check{\delta}_2, \dots, \check{\delta}_{\ell}) &\leq q - RLNWAA (\wp_1, \wp_2, \dots, \wp_{\ell}). \end{aligned}$$

Hence, our result is proved. \square

Theorem 6. (Boundedness) Suppose that $\check{\delta}_{\ell} = (s_{\alpha_{\ell}}, \{\varrho_{\ell}, \varepsilon_{\ell}, \Delta_{\ell}\})$ ($\ell = 1, 2, 3, \dots, k$) are array of q -RLNs, so:

$$\check{\delta}_{\ell}^{-} \leq q - RLNWAA (\check{\delta}_1, \check{\delta}_2, \dots, \check{\delta}_k) \leq \check{\delta}_{\ell}^{+},$$

Proof. Suppose that $c = \min(\check{\delta}_1, \check{\delta}_2, \dots, \check{\delta}_k)$ and $d = \max(\check{\delta}_1, \check{\delta}_2, \dots, \check{\delta}_k)$, using the theorem 5, we have

$$\begin{aligned} \Rightarrow \min(\check{\delta}_1, \check{\delta}_2, \dots, \check{\delta}_k) &\leq (\check{\delta}_1, \check{\delta}_2, \dots, \check{\delta}_k) \leq \max(\check{\delta}_1, \check{\delta}_2, \dots, \check{\delta}_k) \\ \Rightarrow c &\leq (\check{\delta}_1, \check{\delta}_2, \dots, \check{\delta}_k) \leq d. \end{aligned}$$

Hence, our result is proved. \square

Theorem 7. (Symmetry) Let $\check{\delta}_{\ell} = (s_{\alpha_{\ell}}, \{\varrho_{\ell}, \varepsilon_{\ell}, \Delta_{\ell}\})$ ($\ell = 1, 2, 3, \dots, k$) be the q -RLNs. And, if $\check{\delta}_{\ell'} = (s_{\alpha_{\ell'}}, \{\varrho_{\ell'}, \varepsilon_{\ell'}, \Delta_{\ell'}\})$ ($\ell = 1, 2, 3, \dots, k$) are any Permutation of $\check{\delta}_{\ell} = (s_{\alpha_{\ell}}, \{\varrho_{\ell}, \varepsilon_{\ell}, \Delta_{\ell}\})$, then we have:

$$q - RLNWAA (s_{\alpha_{\ell}}, \{\varrho_{\ell}, \varepsilon_{\ell}, \Delta_{\ell}\}) = q - RLNWAA (s_{\alpha_{\ell'}}, \{\varrho_{\ell'}, \varepsilon_{\ell'}, \Delta_{\ell'}\}).$$

Proof. This result is clear. Therefore, it is skipped. □

In the sections that follow, we look into several applications of the presented Operators.

Case-1: If the $\Gamma(i) = -\log\left((1-i) \text{ and } \Delta(i)\right) = -\log(i)$, then the operator q-RLNWAA is expressed as follows:

$$q - RLNWAA(\delta_1, \delta_2, \dots, \delta_k) = \left(\begin{aligned} & \acute{s} \left(\frac{\acute{s}^{-1} \left(1 - \prod_{\ell=1}^k (1 - \acute{g}(\alpha_{\ell}))^{N_{\ell}} \right)}{\left\{ \left(1 - \prod_{\ell=1}^k (1 - \vartheta_{\ell}^q) \right)^{N_{\ell}} \right\}^{1/q}, \left(\prod_{\ell=1}^k (\epsilon_{\ell})^{N_{\ell}}, \left(\prod_{\ell=1}^k (\Delta_{\ell})^{N_{\ell}} \right) \right\}} \right) \end{aligned} \right)$$

Case-2: If the $\Gamma(i) = \log\left((2 - (1-i))/(1-i)\right)$, and the another $\Delta(i) = \log\left((2-i)/i\right)$, then the operator q-RLNWAA is,

$$E - q - RLNWAA(\delta_1, \delta_2, \dots, \delta_k) = \left(\begin{aligned} & \acute{s} \left(\frac{\acute{s}^{-1} \left(\frac{\prod_{\ell=1}^k (1 + \acute{g}(\alpha_{\ell}))^{N_{\ell}} - \prod_{\ell=1}^k (1 - \acute{g}(\alpha_{\ell}))^{N_{\ell}}}{\prod_{\ell=1}^k (1 + \acute{g}(\alpha_{\ell}))^{N_{\ell}} + \prod_{\ell=1}^k (1 - \acute{g}(\alpha_{\ell}))^{N_{\ell}}} \right)}{\left\{ \left(\frac{\prod_{\ell=1}^k (1 + \vartheta_{\ell}^q) - \prod_{\ell=1}^k (1 - \vartheta_{\ell}^q)}{\prod_{\ell=1}^k (1 + \vartheta_{\ell}^q) + \prod_{\ell=1}^k (1 - \vartheta_{\ell}^q)} \right)^{1/q}, \right.} \\ & \left. \left(\frac{2 \prod_{\ell=1}^k (\epsilon_{\ell})^{N_{\ell}}}{\prod_{\ell=1}^k (2 - \epsilon_{\ell})^{N_{\ell}} + \prod_{\ell=1}^k (\vartheta_{\ell})^{N_{\ell}}}, \left(\frac{2 \prod_{\ell=1}^k (\Delta_{\ell})^{N_{\ell}}}{\prod_{\ell=1}^k (2 - \Delta_{\ell})^{N_{\ell}} + \prod_{\ell=1}^k (\Delta_{\ell})^{N_{\ell}}} \right) \right\}} \right) \end{aligned} \right)$$

Einstein-q-RLNWAA is the name of this Operator [60].

Case-3: Let us suppose, if $\Gamma(i) = \log\left((1 - (1 - \vartheta^{\circ})i)/(1-i)\right)$, and then another $\Delta(i) = \log\left((\vartheta^{\circ} + (1 - \vartheta^{\circ}))/i\right)$, $\vartheta^{\circ} > 0$ then q-RLNWAA operator is represented as:

$$H - q - RLNWAA(\delta_1, \delta_2, \dots, \delta_k) = \left(\begin{aligned} & \acute{s} \left(\frac{\acute{s}^{-1} \left(\frac{\prod_{\ell=1}^k (1 - (1 - \vartheta^{\circ})\acute{g}(\alpha_{\ell}))^{N_{\ell}} - \prod_{\ell=1}^k (1 - \acute{g}(\alpha_{\ell}))^{N_{\ell}}}{\prod_{\ell=1}^k (1 - (1 - \vartheta^{\circ})\acute{g}(\alpha_{\ell}))^{N_{\ell}} - (1 - \vartheta^{\circ}) \prod_{\ell=1}^k (1 - \acute{g}(\alpha_{\ell}))^{N_{\ell}}} \right)}{\left\{ \left(\frac{\prod_{\ell=1}^k (1 - (1 - \vartheta^{\circ})\vartheta_{\ell}^q) - \prod_{\ell=1}^k (1 - \vartheta_{\ell}^q)}{\prod_{\ell=1}^k (1 - (1 - \vartheta^{\circ})\vartheta_{\ell}^q) - (1 - \vartheta^{\circ}) \prod_{\ell=1}^k (1 - \vartheta_{\ell}^q)} \right)^{1/q}, \right.} \\ & \left. \left(\frac{\vartheta^{\circ} \prod_{\ell=1}^k (\epsilon_{\ell})^{N_{\ell}}}{\prod_{\ell=1}^k (1 - (1 - \vartheta^{\circ})(1 - \epsilon_{\ell}))^{N_{\ell}} - (1 - \vartheta^{\circ}) \prod_{\ell=1}^k (\epsilon_{\ell})^{N_{\ell}}}, \right. \right. \\ & \left. \left. \left(\frac{\vartheta^{\circ} \prod_{\ell=1}^k (\Delta_{\ell})^{N_{\ell}}}{\prod_{\ell=1}^k (1 - (1 - \vartheta^{\circ})(1 - \Delta_{\ell}))^{N_{\ell}} - (1 - \vartheta^{\circ}) \prod_{\ell=1}^k (\Delta_{\ell})^{N_{\ell}}} \right) \right\}} \right) \end{aligned} \right)$$

Hammer-q-RLNWAA is the name of this operator [61].

Case-4: Supposed $\Gamma(i) = \log\left((\bar{\delta} - 1)/(\bar{\delta}^{1-i} - 1)\right)$, and then another $\Delta(i) = \log\left((\bar{\delta} - 1)/(\bar{\delta}^i - 1)\right)$, $\bar{\delta} > 1$, the q-RLNWAA operator is denoted by:

$$F - q - RLNWAA(\delta_1, \delta_2, \dots, \delta_k) =$$

$$\left(\begin{matrix} \check{s} \\ \check{s}^{-1} \left(1 - \log_{\check{s}} \left(1 + \frac{\prod_{\ell=1}^k (\check{\delta}^{1-\check{g}(\varrho_\ell)} - 1)^{\aleph_\ell} }{\check{\delta} - 1} \right) \right) \end{matrix} \right),$$

$$\left\{ \left(2 - 1 - \log_{\check{s}} \left(2 - 1 + \frac{\prod_{\ell=1}^k (\check{\delta}^{1-\varrho_\ell} - 1)^{\aleph_\ell} }{\check{\delta} - 1} \right) \right)^{1/q}, \left(2 - 1 - \log_{\check{s}} \left(2 - 1 + \frac{\prod_{\ell=1}^k (\check{\delta}^{\check{\varepsilon}_\ell} - 2 + 1)^{\aleph_\ell} }{\check{\delta} - 1} \right) \right), \right.$$

$$\left. \left(2 - 1 - \log_{\check{s}} \left(2 - 1 + \frac{\prod_{\ell=1}^k (\check{\delta}^{\check{\Delta}_\ell} - 2 + 1)^{\aleph_\ell} }{\check{\delta} - 1} \right) \right) \right\}.$$

If $\check{\delta} \rightarrow 1$, so, F-q-RLNWAA denoted as q-RLNWAA.

4.2. q-rung linguistic neutrosophic weighted geometric aggregation operator

Definition 17. [62] Suppose that the $\check{\delta}_\ell = (\check{s}_{\varrho_\ell}, \{\check{\varrho}_\ell, \varepsilon_\ell, \check{\Delta}_\ell\})$ ($\ell = 1, 2, 3, \dots, k$) are arrays of q-RLNs, the q-RLNWGA operator is basing on the Archimedean t-conorm, t-norm and is follow as:

$$q - RLNWGA(\check{\delta}_1, \check{\delta}_2, \check{\delta}_3, \dots, \check{\delta}_k) = \bigotimes_{\ell=1}^k (\check{\delta}_\ell)^{\aleph_\ell}.$$

Theorem 8. Suppose that $\check{\delta}_\ell = (\check{s}_{\varrho_\ell}, \{\check{\varrho}_\ell, \varepsilon_\ell, \check{\Delta}_\ell\})$ ($\ell = 1, 2, 3, \dots, k$) based on the Archimedean norms with its types, we may define the following operator:

$$q - RLNWGA(\check{\delta}_1, \check{\delta}_2, \check{\delta}_3, \dots, \check{\delta}_k) = \left(\begin{matrix} \check{s} \\ \check{s}^{-1} (\check{\Delta}^{-1} (\sum_{\ell=1}^k \aleph_\ell \check{g}(\varrho_\ell))) \end{matrix} \right), \tag{3}$$

$$\left\{ (\check{\Delta}^{-1} (\sum_{\ell=1}^k \aleph_\ell \check{\Delta}(\check{\varrho}_\ell))), (\check{\Delta}^{-1} (\sum_{\ell=1}^k \aleph_\ell \check{\Delta}(\varepsilon_\ell))), (\Gamma^{-1} (\sum_{\ell=1}^k \aleph_\ell \Gamma(\check{\Delta}_\ell^q)))^{1/q} \right\}.$$

Proof. This theorem’s proof is similar to Theorem 3. So, we skipped it because we can easily verify it. □

Similar to the Q-RLNWGA operator, the Q-RLNWGA operator likewise possesses several intriguing characteristics, which are alleged (without evidence) as follows:

Theorem 9. (Idempotency) Let $\check{\delta}_\ell = (\check{s}_{\varrho_\ell}, \{\check{\varrho}_\ell, \varepsilon_\ell, \check{\Delta}_\ell\})$ be the arrays of q-RLNs, if all $\check{\delta}_\ell$ are same, i.e., $\check{\delta}_\ell = \check{\delta} \forall \ell$, so:

$$q - RLNWGA(\check{\delta}_1, \check{\delta}_2, \dots, \check{\delta}_k) = \check{\delta}.$$

Theorem 10. (Monotonicity) Let $\check{\delta}_\ell = (\check{s}_{\varrho_\ell}, \{\check{\varrho}_\ell, \varepsilon_\ell, \check{\Delta}_\ell\})$ and $\wp_\ell = (\check{s}_{\varrho_\ell}, \{\check{\varrho}_\ell, \varepsilon_\ell, \check{\Delta}_\ell\})$ be the q-RLNs, If $\check{\delta}_\ell \leq \wp_\ell$,

$$q - RLNWGA(\check{\delta}_1, \check{\delta}_2, \dots, \check{\delta}_\ell) \leq q - RLNWGA(\wp_1, \wp_2, \dots, \wp_\ell).$$

Theorem 11. (Boundedness) Let $\check{\delta}_\ell = (\check{s}_{\varrho_\ell}, \{\check{\varrho}_\ell, \varepsilon_\ell, \check{\Delta}_\ell\})$ be q-RLNs, so,

$$\check{\delta}_\ell^- \leq q - RLNFPGA(\check{\delta}_1, \check{\delta}_2, \dots, \check{\delta}_k) \leq \check{\delta}_\ell^+.$$

Theorem 12. (Symmetry) Suppose that $\check{\delta}_\ell = (\check{s}_{\varrho_\ell}, \{\check{\varrho}_\ell, \varepsilon_\ell, \check{\Delta}_\ell\})$ be the q-RLNs collections. If $\check{\delta}_{\ell'}$ = $(\check{s}_{\varrho_{\ell'}}, \{\check{\varrho}_{\ell'}, \varepsilon_{\ell'}, \check{\Delta}_{\ell'}\})$ be randomly permutation of $\check{\delta}_\ell = (\check{s}_{\varrho_\ell}, \{\check{\varrho}_\ell, \varepsilon_\ell, \check{\Delta}_\ell\})$, then:

$$q - RLNWGA(\check{s}_{\varrho_\ell}, \{\check{\varrho}_\ell, \varepsilon_\ell, \check{\Delta}_\ell\}) = q - RLNWGA(\check{s}_{\varrho_{\ell'}}, \{\check{\varrho}_{\ell'}, \varepsilon_{\ell'}, \check{\Delta}_{\ell'}\}),$$

The q-RLNWGA operator is shown in several specific examples of various functions in the sections that follow.

Case-1: Let $\Gamma(i) = -\log(1 - i)$ and the another $\check{\Delta}(i) = -\log(i)$, then the q-RLNWGA is denoted by: q-RLNWGA ($\check{\delta}_\ell | \ell = 1, 2, 3, \dots, k$)

$$q - RLNWGA(\check{\delta}_1, \check{\delta}_2, \dots, \check{\delta}_k) =$$

$$\left(\hat{s}_{\hat{g}^{-1}} \left(\prod_{\ell=1}^k (\hat{g}(\varrho_{\ell}))^{N_{\ell}} \right), \left\{ \left(\prod_{\ell=1}^k (\varrho_{\ell})^{N_{\ell}}, \left(\prod_{\ell=1}^k (\varepsilon_{\ell})^{N_{\ell}}, \left(1 - \prod_{\ell=1}^k (1 - \mathfrak{J}_{\ell}^q)^{N_{\ell}} \right)^{1/q} \right) \right\} \right)$$

Case-2: Einstein q-RLNWGA operator is defined in this case, we get

$$E - q - RLNWGA(\check{\delta}_{\ell}, \check{\delta}_2, \dots, \check{\delta}_k) = \tag{4}$$

$$\left(\hat{s}_{\hat{g}^{-1}} \left(\frac{2 \prod_{\ell=1}^k (\hat{g}(\varrho_{\ell}))^{N_{\ell}}}{\prod_{\ell=1}^k (2 - \hat{g}(\varrho_{\ell}))^{N_{\ell}} + \prod_{\ell=1}^k (\hat{g}(\varrho_{\ell}))^{N_{\ell}}} \right), \left\{ \left(\frac{2 \prod_{\ell=1}^k (\varrho_{\ell})^{N_{\ell}}}{\prod_{\ell=1}^k (2 - \varrho_{\ell}^q)^{N_{\ell}} + \prod_{\ell=1}^k (\varrho_{\ell})^{N_{\ell}}}, \left(\frac{2 \prod_{\ell=1}^k (\varepsilon_{\ell})^{N_{\ell}}}{\prod_{\ell=1}^k (2 - \varepsilon_{\ell})^{N_{\ell}} + \prod_{\ell=1}^k (\varrho_{\ell})^{N_{\ell}}}, \left(\frac{\prod_{\ell=1}^k (1 + \mathfrak{J}_{\ell}^q)^{N_{\ell}} - \prod_{\ell=1}^k (1 - \mathfrak{J}_{\ell}^q)^{N_{\ell}}}{\prod_{\ell=1}^k (1 + \mathfrak{J}_{\ell}^q)^{N_{\ell}} + \prod_{\ell=1}^k (1 - \mathfrak{J}_{\ell}^q)^{N_{\ell}}} \right)^{1/q} \right) \right\} \right)$$

This operator is known as Einstein-q-RLNWGA (4).

Case-3: Hammer operator is defined in this case similar to the above-explained case in the previous aggregation operator. We have,

$$H - q - RLNWGA(\check{\delta}_{\ell} | \ell = 1, 2, 3, \dots, k) = \tag{5}$$

$$\left(\hat{s}_{\hat{g}^{-1}} \left(\frac{\prod_{\ell=1}^k (1 - (1 - \vartheta^{\circ}) \hat{g}(\varrho_{\ell}))^{N_{\ell}} - \prod_{\ell=1}^k (1 - \hat{g}(\varrho_{\ell}))^{N_{\ell}}}{\prod_{\ell=1}^k (1 - (1 - \vartheta^{\circ}) \hat{g}(\varrho_{\ell}))^{N_{\ell}} - (1 - \vartheta^{\circ}) \prod_{\ell=1}^k (1 - \hat{g}(\varrho_{\ell}))^{N_{\ell}}} \right), \left\{ \left(\frac{\vartheta^{\circ} \prod_{\ell=1}^k (\varrho_{\ell})^{N_{\ell}}}{\prod_{\ell=1}^k (1 - (1 - \vartheta^{\circ}) (1 - \varrho_{\ell}^q))^{N_{\ell}} - (1 - \vartheta^{\circ}) \prod_{\ell=1}^k (\varrho_{\ell})^{N_{\ell}}}, \left(\frac{\vartheta^{\circ} \prod_{\ell=1}^k (\varepsilon_{\ell})^{N_{\ell}}}{\prod_{\ell=1}^k (1 - (1 - \vartheta^{\circ}) (1 - \varepsilon_{\ell}))^{N_{\ell}} - (1 - \vartheta^{\circ}) \prod_{\ell=1}^k (\varepsilon_{\ell})^{N_{\ell}}}, \left(\frac{\prod_{\ell=1}^k (1 - (1 - \vartheta^{\circ}) \mathfrak{J}_{\ell}^q)^{N_{\ell}} - \prod_{\ell=1}^k (1 - \mathfrak{J}_{\ell}^q)^{N_{\ell}}}{\prod_{\ell=1}^k (1 - (1 - \vartheta^{\circ}) \mathfrak{J}_{\ell}^q)^{N_{\ell}} - (1 - \vartheta^{\circ}) \prod_{\ell=1}^k (1 - \mathfrak{J}_{\ell}^q)^{N_{\ell}}} \right)^{1/q} \right) \right\} \right)$$

Hammer-q-RLNWGA is the name of this operator (5). If $\vartheta^{\circ} = 1$, Hence the operator H-q-RLNWGA may be represented as q-RLNWGA; if $\vartheta^{\circ} = 2$, therefore E-q-RLNWGA can be used to represent H-q-RLNWGA.

Case-4: Similar to above case that is explained already, we have

$$F - q - RLNWGA(\check{\delta}_{\ell} | \ell = 1, 2, 3, \dots, k) = \tag{6}$$

$$\left(\hat{s}_{\hat{g}^{-1}} \left(\log_{\check{\delta}} \left(2 - 1 + \frac{\prod_{\ell=1}^k (\check{\delta} \hat{g}(\varrho_{\ell}) - 1)^{N_{\ell}}}{\check{\delta} - 1} \right) \right), \left\{ \left(\log_{\check{\delta}} \left(2 - 1 + \frac{\prod_{\ell=1}^{\ell} (\check{\delta}^{\varrho_{\ell}} - 1)^{N_{\ell}}}{\check{\delta} - 1} \right) \right), \left(\log_{\check{\delta}} \left(1 + \frac{\prod_{\ell=1}^k (\check{\delta}^{\varepsilon_{\ell}} - 1)^{N_{\ell}}}{\check{\delta} - 1} \right) \right), \left(2 - 1 - \log_{\check{\delta}} \left(1 + \frac{\prod_{\ell=1}^k (\check{\delta}^{2-1-\mathfrak{J}_{\ell}^q} - 1)^{N_{\ell}}}{\check{\delta} - 1} \right) \right)^{1/q} \right\} \right)$$

If $\check{\delta} \rightarrow 1$, then F-q-RLNWGA is similarly written as the q-RLNWGA operator (6).

5. An innovative technique for effective decision-making

In this part, we'll outline a method for solving MADM issues that are based on q-RLN operators according to the flow diagram Fig. 1. Let's say we have $P = \{p_1, p_2, p_3, \dots, p_k\}$ be any finite arrays of k alternative and we have attribute set such as $S = \{s_1, s_2, s_3, \dots, s_{\ell}\}$. Dealing with qualitative entities (variables), such as enormous, extremely large, immense, etc., in DM is sometimes challenging. As a result, these entities must take into account numerical quantities. A linguistic variable, which functions as a type of mapping between a collection of linguistic things to a certain range of real numbers, is used to address such variables. For instance, Chatterjee et al. [63] regarded the "quality of the product" to be a linguistic variable. Using the q-rung linguistic neutrosophic set as a foundation, they will gather data in the form of $\wp = (s_{\wp}, \{\varrho, \Gamma, \mathfrak{J}\})$ Where, s_{\wp} is from Linguistic-Set $S = \{s_0 = \text{Extremely bad}, s_1 = \text{Dreadful}, s_2 = \text{Poor}, s_3 = \text{Unbiased/Fair}, s_4 = \text{Excellent/Outstanding}, s_5 = \text{All right}\}$ and the condition for the quantitative part of \wp is $0 \leq \varrho^q + \Gamma^q + \mathfrak{J}^q \leq 3$.

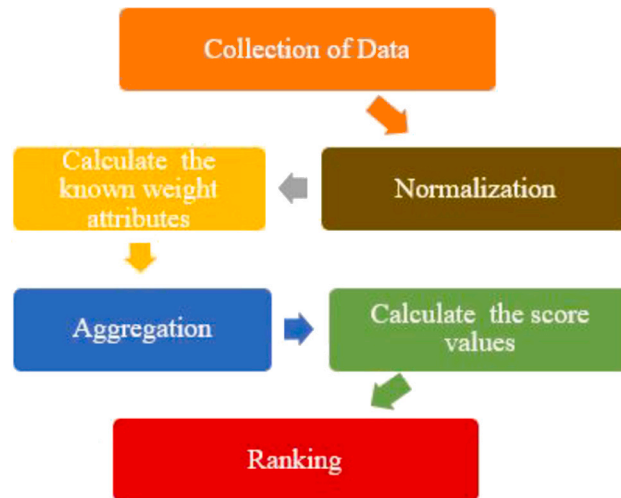


Fig. 1. Flow diagram for the proposed method.

1. Data collection:

Obtain information on the decision-makers' evaluations in the structure of a matrix $G = [P_{nm}]$ as

$$G = \begin{pmatrix} P_{11} & P_{12} & \dots & P_{1m} \\ P_{21} & P_{22} & \dots & P_{2m} \\ \vdots & \vdots & \dots & \vdots \\ P_{n1} & P_{n2} & \dots & P_{nm} \end{pmatrix}$$

2. Normalization:

The decision matrix is used in this stage as $G = [P_{xy}]$ into the normalized matrix transformation $\bar{G} = [\bar{P}_{xy}]$ by the given calculation method:

$$\bar{P}_{xy} = \begin{cases} P_{xy}, & \text{if it belongs to Benefit-attribute} \\ (P_{xy})^c, & \text{if it belongs to Cost-attribute,} \end{cases}$$

here P_{xy}^c is referred as complement of P_{xy} . Worth noting is the fact that for every q-RLN $P = (s_p, \{\ominus, \Gamma, \Delta\})$ its complement can be calculated as

$$B^c = (s_{1-p}, \{\Delta, \Gamma, \ominus\}). \tag{7}$$

3. Aggregation:

Aggregate the q-RLNs $P_{xy} (y = 1, 2, 3, \dots, p)$ for all alternative $P_x (x = 1, 2, 3, \dots, q)$ into the overall worth of preference P by using the q-RLNAA or q-RLNGA operators that have been suggested.

In mathematics, it may be expressed as;

$$P_x = q - RLNsAA_{\delta'}(P_{x1}, P_{x2}, P_{x3}, \dots, P_{xp}),$$

$$P_x = q - RLNsGA_{\delta'}(P_{x1}, P_{x2}, P_{x3}, \dots, P_{xp}),$$

where $\aleph' = (\aleph'_1, \aleph'_2, \dots, \aleph'_n)$ is the possibility vector of attributes.

4. Determine the scoring values:

According to Definition 12, find out the values of score function $Scr (P_x) (x = 1, 2, 3, \dots, p)$ of all q-RLNs $P_x (x = 1, 2, 3, \dots, p)$.

5. Ranking:

Sort the choices to determine which is best. $t_x (x = 1, 2, 3, \dots, p)$ using the score values $Scr (P_x)$.

6. Explanatory example

In this section, an explanatory example regarding the process of ranking men's cricket players is used to elaborate on the implications and practicality of the suggested approach. The ranking of international cricket players is an important measure of their performance in the sport. The rankings are updated regularly by the International Cricket Council (ICC) for all three formats of the game - Test cricket, (ODI) cricket, and International (T20I) cricket. In the context of men's cricket, player rankings are crucial for evaluating the performance and value of batters, bowlers, and all-rounders separately. The ranking system provides valuable insights into the relative standing of players within each category, aiding in team selection, performance evaluation, and strategic

Table 1
Decision-matrix of q-rung linguistic neutrosophic set taken by “D”.

	S_1	S_2	S_3	S_4	S_5
P_1	$\langle s_4, \{0.4, 0.6, 0.9\} \rangle$	$\langle s_3, \{0.1, 0.5, 0.6\} \rangle$	$\langle s_4, \{0.5, 0.4, 0.3\} \rangle$	$\langle s_1, \{0.5, 0.6, 0.8\} \rangle$	$\langle s_3, \{0.2, 0.4, 0.5\} \rangle$
P_2	$\langle s_1, \{0.5, 0.8, 0.9\} \rangle$	$\langle s_4, \{0.4, 0.4, 0.1\} \rangle$	$\langle s_3, \{0.2, 0.5, 0.7\} \rangle$	$\langle s_1, \{0.2, 0.4, 0.9\} \rangle$	$\langle s_3, \{0.4, 0.6, 0.1\} \rangle$
P_3	$\langle s_2, \{0.6, 0.6, 0.6\} \rangle$	$\langle s_3, \{0.4, 0.5, 0.1\} \rangle$	$\langle s_1, \{0.1, 0.6, 0.6\} \rangle$	$\langle s_2, \{0.2, 0.5, 0.7\} \rangle$	$\langle s_4, \{0.4, 0.5, 0.9\} \rangle$
P_4	$\langle s_3, \{0.5, 0.7, 0.2\} \rangle$	$\langle s_4, \{0.5, 0.6, 0.9\} \rangle$	$\langle s_2, \{0.2, 0.7, 0.8\} \rangle$	$\langle s_3, \{0.6, 0.7, 0.8\} \rangle$	$\langle s_4, \{0.6, 0.3, 0.2\} \rangle$
P_5	$\langle s_1, \{0.4, 0.6, 0.7\} \rangle$	$\langle s_4, \{0.5, 0.7, 0.8\} \rangle$	$\langle s_1, \{0.5, 0.6, 0.8\} \rangle$	$\langle s_3, \{0.2, 0.6, 0.4\} \rangle$	$\langle s_2, \{0.8, 0.8, 0.3\} \rangle$

Table 2
Normalized matrix.

	S_1	S_2	S_3	S_4	S_5
P_1	$\langle s_4, \{0.4, 0.6, 0.9\} \rangle$	$\langle s_3, \{0.1, 0.5, 0.6\} \rangle$	$\langle s_4, \{0.5, 0.4, 0.3\} \rangle$	$\langle s_4, \{0.8, 0.6, 0.5\} \rangle$	$\langle s_2, \{0.5, 0.4, 0.2\} \rangle$
P_2	$\langle s_1, \{0.5, 0.8, 0.9\} \rangle$	$\langle s_4, \{0.4, 0.4, 0.1\} \rangle$	$\langle s_3, \{0.2, 0.5, 0.7\} \rangle$	$\langle s_4, \{0.9, 0.4, 0.2\} \rangle$	$\langle s_2, \{0.1, 0.6, 0.4\} \rangle$
P_3	$\langle s_2, \{0.6, 0.6, 0.6\} \rangle$	$\langle s_3, \{0.4, 0.5, 0.1\} \rangle$	$\langle s_1, \{0.1, 0.6, 0.6\} \rangle$	$\langle s_3, \{0.7, 0.5, 0.2\} \rangle$	$\langle s_1, \{0.9, 0.5, 0.4\} \rangle$
P_4	$\langle s_3, \{0.5, 0.7, 0.2\} \rangle$	$\langle s_4, \{0.5, 0.6, 0.9\} \rangle$	$\langle s_2, \{0.2, 0.7, 0.8\} \rangle$	$\langle s_2, \{0.8, 0.7, 0.6\} \rangle$	$\langle s_1, \{0.2, 0.3, 0.6\} \rangle$
P_5	$\langle s_1, \{0.4, 0.6, 0.7\} \rangle$	$\langle s_4, \{0.5, 0.7, 0.8\} \rangle$	$\langle s_1, \{0.5, 0.6, 0.8\} \rangle$	$\langle s_2, \{0.4, 0.6, 0.2\} \rangle$	$\langle s_3, \{0.3, 0.8, 0.8\} \rangle$

decision-making. However, for this example, we only focus on the ranking of batters. The ranking of batters involves a meticulous assessment of various parameters that evaluate their performance and impact on the game. These parameters include some benefits parameters such as batting average, strike rate, centuries and half-centuries, match-winning innings, and consistency, and some cost parameters such as dismissal rate, dot ball percentage, average time spent at the crease, inability to score in crucial conditions, lack of adaptability. It is noted that most hitters are examined using the associated criteria: Batting average (S_1), strike rate (S_2), consistency (S_3), Dismissal rate (S_4) and Inability to score in crucial conditions (S_5). Then, at that point, ICC chooses the batter in which five top rated ranking of batters is following such as, Player-1 (P_1), Player-2 (P_2), Player-3 (P_3), Player-4 (P_4) and Player-5 (P_5). A MCDM issue with five possibilities $\{p_1, p_2, p_3, p_4, p_5\}$ is the batters’ determination interaction, five models $\{s_1, s_2, s_3, s_4, s_5\}$ and specialist d . The optimum arrangement for that moment may then be determined using the created approach. It’s important to note that the specific weightage assigned to these benefit and cost parameters may vary based on the ranking methodology used by the organization responsible for player rankings, such as the International Cricket Council (ICC).

The data used for cricket player assessment within the framework of Q-RLNS encompasses various factors that contribute to a player’s batting performance. These factors are represented using linguistic variables, and neutrosophic sets as defined in QRLNS. Specifically, Linguistic Variables: The linguistic variables define qualitative terms related to batting performance, such as “high,” “medium,” and “low.” These terms describe attributes like batting average, strike rate, centuries scored, etc. Neutrosophic Sets: Neutrosophic sets are employed to account for the uncertainty and indeterminacy in the data. For each linguistic variable, membership degrees for truth (T), indeterminacy (I), and falsity (F) are assigned. For instance, a player’s strike rate could have the memberships of truth, ambiguity, and falsehood, respectively if they consistently maintain a high strike rate.

For linguistic terms, we have used the value of t by taking $t = 6$. The neutrosophic part explains the data according to different parameters regarding different players such as the data collection of P_1 explains the batting average, strike rate, consistency, dismissal rate, and inability to score in crucial conditions in which first, second, and third values Display the memberships of truth, ambiguity, and falsehood, respectively of player-1 and same criteria for also other players. Here, are the criteria for data collection according to the given parameters that, are explained above. Then the linguistics terms and neutrosophic number for each alternative and parameters in the form of the set is as follows:

Step 1: Data collection in the matrix form (For $q = 2$) in Table 1.

Step 2: Normalize the data in accordance (7) to the proposed technique in Table 2.

Step 3: We used aggregation operators in this step (q-RLNWAA and q-RLNWGA) by using known weights $\{0.2413, 0.1972, 0.1793, 0.1973, 0.1849\}$ which we have from the prior step.

We obtained results:

- q-RLNWAA (2):

$$P_1 = (s_{3,5953}, \{0.3847, 0.4913, 0.5770\}), P_2 = (s_{3,0614}, \{0.3748, 0.5304, 0.3715\}),$$

$$P_3 = (s_{2,1610}, \{0.4101, 0.5398, 0.4682\}), P_4 = (s_{2,6898}, \{0.5120, 0.5805, 0.4535\}), \text{ and}$$

$$P_5 = (s_{2,4705}, \{0.5416, 0.6523, 0.5623\}).$$

- q-RLNWGA (3):

$$P_1 = (s_{3,3247}, \{0.2911, 0.4993, 0.7297\}), P_2 = (s_{2,3917}, \{0.3251, 0.5304, 0.7575\}),$$

$$P_3 = (s_{1,8233}, \{0.3427, 0.5398, 0.6835\}), P_4 = (s_{2,2244}, \{0.4548, 0.5805, 0.7196\}), \text{ and}$$

$$P_5 = (s_{1,8464}, \{0.4313, 0.6523, 0.6712\}).$$

Step 4: In this step, we calculated the values of the score for each alternatives in (1).

- q-RLNWAA:

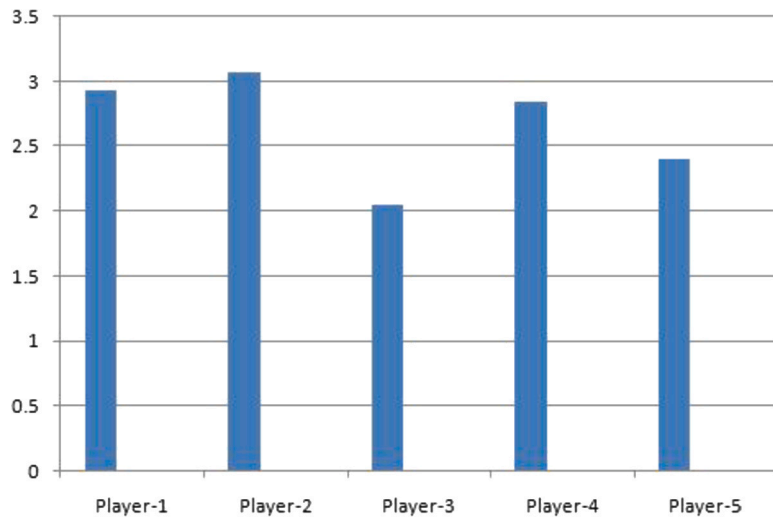


Fig. 2. Using q-RLNWAA, graphically representation of players ranking.

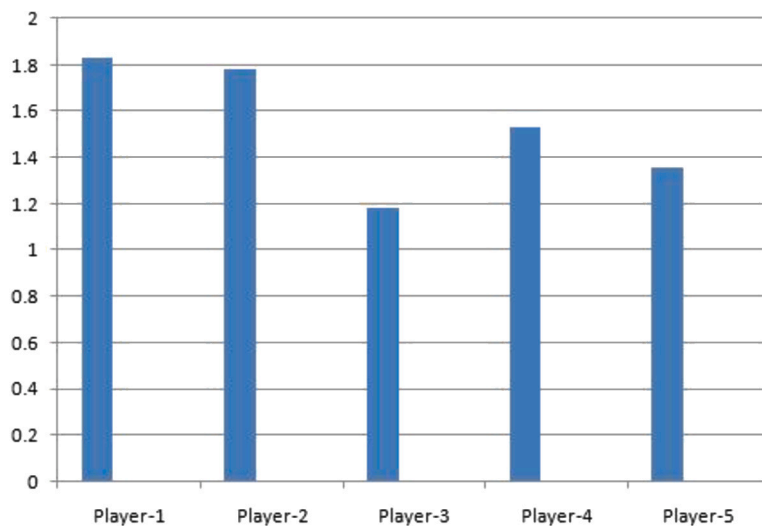


Fig. 3. Using q-RLNWGA, graphically representation of players ranking.

$Sc(P_1) = 2.9304$, $Sc(P_2) = 3.0689$, $Sc(P_3) = 2.0507$, $Sc(P_4) = 2.8417$ and $Sc(P_5) = 2.4107$.

• q-RLNWGA:

$Sc(P_1) = 1.8361$, $Sc(P_2) = 1.7821$, $Sc(P_3) = 1.1856$, $Sc(P_4) = 1.5326$ and $Sc(P_5) = 1.3580$.

Step 5: Then, we assigned a score to each alternative and scored them.

• q-RLNWAA:

$$P_2 > P_1 > P_4 > P_5 > P_3.$$

• q-RLNWGA:

$$P_1 > P_2 > P_4 > P_5 > P_3.$$

The aggregation operators display the finished ranks as a result. Player 2 has the greatest batting ranking among all players according to the q-RLN, which illustrates the supremacy of international cricket players as shown in Fig. 2. q-RLN, however, reveals that player-1 ranking has the greatest batting ranking when compared to other players. The outcomes for both operators are too similar but produce results that are average as shown in Fig. 3.

Table 3
Simply changing the parameter values, a different ranking.

q	Values of scoring function	Ranking
q=2	$Sc(P_1) = 2.9304, Sc(P_2) = 3.0689, Sc(P_3) = 2.0507, Sc(P_4) = 2.8417, Sc(P_5) = 2.4107$	$P_2 > P_1 > P_4 > P_5 > P_3$
q=4	$Sc(P_1) = 3.3060, Sc(P_2) = 3.0818, Sc(P_3) = 2.1496, Sc(P_4) = 2.7868, Sc(P_5) = 2.5187$	$P_1 > P_2 > P_4 > P_5 > P_3$
q=8	$Sc(P_1) = 3.5569, Sc(P_2) = 3.0639, Sc(P_3) = 2.1653, Sc(P_4) = 2.7069, Sc(P_5) = 2.5316$	$P_1 > P_2 > P_4 > P_5 > P_3$
q=10	$Sc(P_1) = 3.5820, Sc(P_2) = 3.0620, Sc(P_3) = 2.1631, Sc(P_4) = 2.6961, Sc(P_5) = 2.5147$	$P_1 > P_2 > P_4 > P_5 > P_3$
q=12	$Sc(P_1) = 3.5907, Sc(P_2) = 3.0615, Sc(P_3) = 2.1619, Sc(P_4) = 2.6921, Sc(P_5) = 2.5005$	$P_1 > P_2 > P_4 > P_5 > P_3$
q=15	$Sc(P_1) = 3.5944, Sc(P_2) = 3.0614, Sc(P_3) = 2.1612, Sc(P_4) = 2.6903, Sc(P_5) = 2.4863$	$P_1 > P_2 > P_4 > P_5 > P_3$

Table 4
Simply changing the parameter values, a different ranking.

q	Values of the scoring function	Ranking
q=2	$Sc(P_1) = 1.8361, Sc(P_2) = 1.7821, Sc(P_3) = 1.1856, Sc(P_4) = 1.5326, Sc(P_5) = 1.3580$	$P_1 > P_2 > P_4 > P_5 > P_3$
q=4	$Sc(P_1) = 2.2475, Sc(P_2) = 1.4524, Sc(P_3) = 1.3625, Sc(P_4) = 1.5719, Sc(P_5) = 1.4717$	$P_1 > P_4 > P_5 > P_2 > P_3$
q=8	$Sc(P_1) = 2.7879, Sc(P_2) = 1.8491, Sc(P_3) = 1.6128, Sc(P_4) = 1.8619, Sc(P_5) = 1.7005$	$P_1 > P_4 > P_2 > P_5 > P_3$
q=10	$Sc(P_1) = 2.9279, Sc(P_2) = 1.9719, Sc(P_3) = 1.6707, Sc(P_4) = 1.9594, Sc(P_5) = 1.7573$	$P_1 > P_2 > P_4 > P_5 > P_3$
q=12	$Sc(P_1) = 3.0249, Sc(P_2) = 2.0628, Sc(P_3) = 1.7086, Sc(P_4) = 2.0287, Sc(P_5) = 1.7916$	$P_1 > P_2 > P_4 > P_5 > P_3$
q=15	$Sc(P_1) = 3.1226, Sc(P_2) = 2.1599, Sc(P_3) = 1.7452, Sc(P_4) = 2.0972, Sc(P_5) = 1.8196$	$P_1 > P_2 > P_4 > P_5 > P_3$

The list of nations’ player caliber in international cricket suggests a hierarchy. The conclusions obtained will, however, only be theoretical without a detailed understanding of the nations these designations designate. It is essential to verify credible sources like the ICC rankings for up-to-date information because player rankings in international cricket might vary. The ICC rankings offer thorough and trustworthy evaluations of player ranks and performances, guaranteeing accurate information for cricket fans.

7. Sensitivity analysis

A type of monetary model called sensitivity analysis impacts changes in input factors on track factors. It is a technique for anticipating a choice’s result given a bunch of significant factors. Sensitivity analysis is used to handle the uncertainty in mathematical models, when the values for the model’s inputs may fluctuate. The two are usually used in combination since it is the analytical technique that goes along with uncertainty analysis. All models constructed and studies done rely on assumptions regarding the accuracy of the inputs used in calculations to obtain results or conclusions for policy decisions. Sensitivity analysis might be useful in various circumstances, including estimating, anticipating, and recognizing regions that need cycle upgrades or changes. However, utilizing historical data might occasionally result in incorrect projections since past occurrences don’t necessarily foretell future ones.

7.1. Analysis of sensitivity with regard to parameter “q”

7.1.1. “q-RLNWAA” operator

Simply employing the q-RLNWAA operator, we do the responsiveness study in this part to examine the effects of various q parameter values on the ranking of the other options in Table 3, and the chart shows that when we increase the values of q, very nothing changes. Furthermore, we have seen that the values of the scoring function for each option became more modest as the values of q increased. The recommended option is the same when q=2, q=4, and q=8, but it changes when q=10, q=12, and q=15 are entered. Additionally, we have observed how each option behaved by looking at the graphic representation of its score values in Fig. 2. There is a very small change appearing in Fig. 3. The parameter q is like a representation of the DM’s attitude. The aggregation operator is appropriate when dealing with critical decision-makers, while the q-RLNWAA operator is helpful when reflecting optimistic decision-makers. Suppose we use the q-RLNWAA operator to collect data for the current cycle. In that case, higher q values indicate that decision-makers have a more negative attitude, while lower values indicate a more positive attitude. Therefore, different DM can select the most appropriate value of q based on their attitude and graphical representation in Fig. 4.

7.1.2. “q-RLNFWGA” operator

In this section, we used q-RLNWGA to adjust the values of the parameter “q” and saw how the ranking of the alternative changed. There is almost no possibility that occurs, similarly to the q-RLNWAA operator, as the worth of the boundary change ought to be self-evident in Table 4. Adding to the conduct of score values, assuming equality, can be noted in Fig. 5, where very little change is occurring when q’s values alter. In order for DMs to make the optimal option, this operator is also very important.

7.2. Analysis of sensitivity in relation to the weights of the characteristics

In order to assess the effects of various weightings given to the qualities under consideration, sensitivity analysis with respect to attribute weights is a method utilized in navigation procedures, notably in multi-criteria decision-making. Multiple traits or criteria are frequently assessed in these decision-making situations, and these attributes may be given varying weights or degrees of relevance.

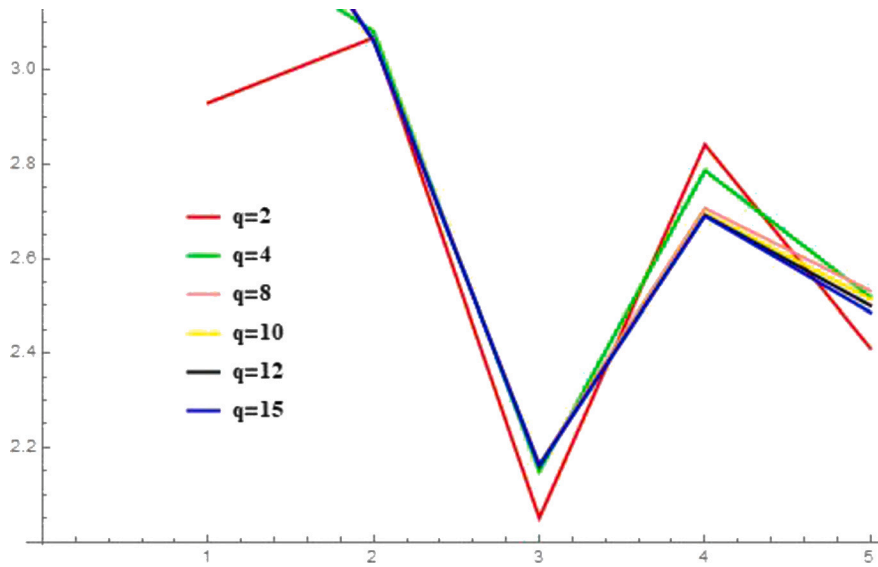


Fig. 4. Graphical illustration of the sensitivity analysis for the parameter q using q-RLNWAA.

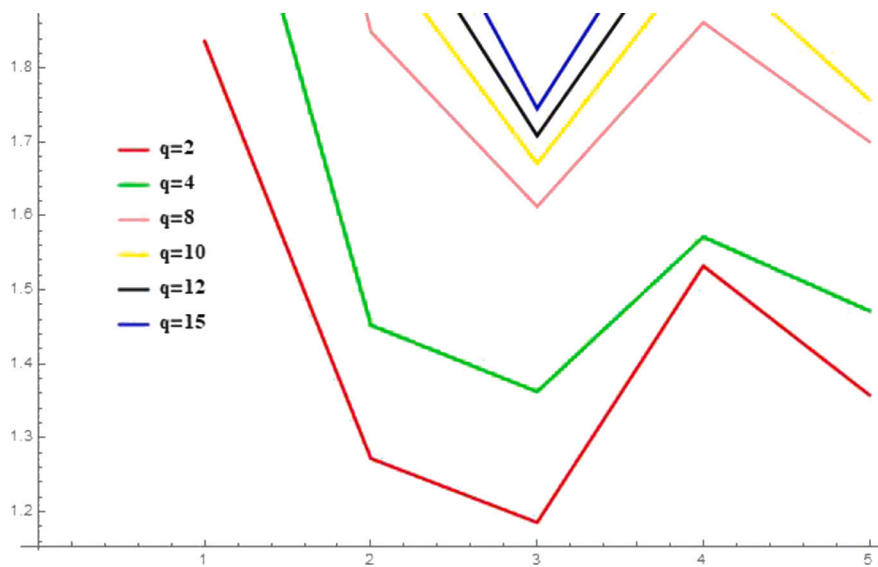


Fig. 5. Graphical illustration of the sensitivity analysis for the parameter q using q-RLNGAA.

The decision-makers preferences, domain expertise, or other considerations are frequently used to apply these weights. Sensitivity analysis for attribute weights is methodically changing the weights given to each attribute and seeing how those changes affect the final judgment or conclusion. This analysis’s objectives are to determine which qualities have the most influence on the choice and how delicate the choice or result is to the loads given to each ascribe. Changing the loads given to every model and afterward analyzing the progressions in the general worldwide cricket rankings comprises responsiveness examination. The ICC might use this data to identify which factors are most important to their choice and change the weights of those factors accordingly. Overall, it is a helpful tool for analyzing decision-making issues and determining how resilient choices are to changes in the relative importance of various traits.

Table 5 shows that We can clearly see that even if we adjust the weights of the qualities, the placement of the options remains the same. This demonstrates the potency of the operator for generalized aggregation. We likewise found, as we alter the weight of qualities, there is essentially no change in the score values. When employing various weight qualities, the order of choices is always $P_1 > P_2 > P_4 > P_5 > P_3$ or a little modification to it.

Table 5
Analysis of sensitivity in relation to the weights of the characteristics.

Values of new Weights:	Values of the scoring function	Ranking
{0.2659, 0.1256, 0.1896, 0.1265, 0.2924}	$Sc(P_1) = 2.8236, Sc(P_2) = 2.8151, Sc(P_3) = 1.6574, Sc(P_4) = 2.7521, Sc(P_5) = 2.5295$	$P_1 > P_2 > P_4 > P_5 > P_3$
{0.2856, 0.1596, 0.2651, 0.1986, 0.0911}	$Sc(P_1) = 3.1294, Sc(P_2) = 2.7215, Sc(P_3) = 1.9847, Sc(P_4) = 2.7021, Sc(P_5) = 2.3182$	$P_1 > P_2 > P_4 > P_5 > P_3$
{0.1971, 0.1860, 0.2569, 0.2156, 0.1444}	$Sc(P_1) = 3.0474, Sc(P_2) = 3.0281, Sc(P_3) = 1.9745, Sc(P_4) = 2.6090, Sc(P_5) = 2.2132$	$P_1 > P_2 > P_4 > P_5 > P_3$
{0.2151, 0.1699, 0.2598, 0.1081, 0.2471}	$Sc(P_1) = 1.9623, Sc(P_2) = 1.3800, Sc(P_3) = 0.9106, Sc(P_4) = 1.4170, Sc(P_5) = 1.3772$	$P_1 > P_2 > P_4 > P_5 > P_3$
{0.1616, 0.2659, 0.2830, 0.1591, 0.1304}	$Sc(P_1) = 2.0960, Sc(P_2) = 1.5521, Sc(P_3) = 1.1748, Sc(P_4) = 0.7717, Sc(P_5) = 1.4338$	$P_1 > P_2 > P_5 > P_3 > P_4$

Table 6
Evaluation of existing operators in comparison to current operators [64].

Different Operators :	Score function values	Ranking
m-G-qNWAA [64]	$Sc(P_1) = 0.4425, Sc(P_2) = 0.4718, Sc(P_3) = 0.4449, Sc(P_4) = 0.4556, Sc(P_5) = 0.4204$	$P_2 > P_4 > P_3 > P_1 > P_5$
m-G-qNWGA [64]	$Sc(P_1) = 0.3829, Sc(P_2) = 0.3568, Sc(P_3) = 0.3650, Sc(P_4) = 0.3673, Sc(P_5) = 0.3711$	$P_1 > P_3 > P_4 > P_3 > P_2$
q-RLNWAA (Proposed Operator)	$Sc(P_1) = 2.9304, Sc(P_2) = 3.0689, Sc(P_3) = 2.0507, Sc(P_4) = 2.8417, Sc(P_5) = 2.4107$	$P_2 > P_1 > P_4 > P_3 > P_3$
q-RLNWGA (Proposed Operator)	$Sc(P_1) = 1.8361, Sc(P_2) = 1.2721, Sc(P_3) = 1.1856, Sc(P_4) = 1.5326, Sc(P_5) = 1.3580$	$P_1 > P_4 > P_5 > P_2 > P_3$

Table 7
Evaluation of existing operators in comparison to current operators [65].

Different operators :	Score function values	Ranking
SVNFWA [65]	$Sc(P_1) = 0.4204, Sc(P_2) = 0.4098, Sc(P_3) = 0.3543, Sc(P_4) = 0.3879, Sc(P_5) = 0.3776$	$P_1 > P_2 > P_4 > P_5 > P_3$
SVNFOWA [65]	$Sc(P_1) = 0.3904, Sc(P_2) = 0.4100, Sc(P_3) = 0.3785, Sc(P_4) = 0.3865, Sc(P_5) = 0.3798$	$P_2 > P_1 > P_4 > P_5 > P_3$
q-RLNWAA (Proposed operator)	$Sc(P_1) = 2.9304, Sc(P_2) = 3.0689, Sc(P_3) = 2.0507, Sc(P_4) = 2.8417, Sc(P_5) = 2.4107$	$P_2 > P_1 > P_4 > P_5 > P_3$
q-RLNWGA (Proposed operator)	$Sc(P_1) = 1.8361, Sc(P_2) = 1.2721, Sc(P_3) = 1.1856, Sc(P_4) = 1.5326, Sc(P_5) = 1.3580$	$P_1 > P_4 > P_5 > P_2 > P_3$

8. Comparison analysis

Breaking down at least two related things to perceive how they are like and particular from each other is known as comparative analysis. Individuals might be better ready to see the value in the similitudes and differentiations of numerous things by applying them in various settings and areas. It can help businesses decide wisely on crucial issues. It may be put to good use when combined with scientific data. Logical information will be data that has been accumulated through comparative analysis and will be utilized for a specific reason. When joined with logical information, it exhibits how information is dependable and reliable. It additionally assists researchers with ensuring that their information is dependable and exact. Comparative analysis is fundamental on the off chance that we wish to fathom a subject better or track down answers for significant issues. These are the fundamental objectives that organizations utilize comparative analysis to accomplish. It empowers a definite comprehension of the open doors connected with specific cycles, divisions, or specialty units. This concentrate additionally guarantees that the genuine explanations behind execution holes are being tended to. It is ordinarily utilized since it assists with grasping both the current and past difficulties that a firm has confronted. This method offers objective, verifiable information on performance as well as recommendations for improving it.

We use the following comparison similarities to further illustrate the advantages and benefits of the suggested strategies.

8.1. Comparison between the suggested approach with the existing one proposed by Saha and Smarandache [64]

We utilize Saha and Smarandache [64] to address the aforementioned problem, with visible results in Table 6. In this table, we calculated the values of the score by using existing m-G-qNWAA and m-G-qNWGA operators [64] and contrasted the outcomes with the methodology prompted by this article. The order of significance of each option has not altered considerably. For both methods, the top-ranked alternative is the same. But Saha and Smarandache technique [64]. In our proposed methodology, the weights attribute was already known, making it more reasonable and adaptable. The weights attribute was known, representing that it is extremely simple to determine the ranking of alternatives.

8.2. Comparison between the suggested approach with the existing one proposed by Riaz and Kamaci [65]

Additionally, to demonstrate the suitability and validity of the approach suggested in this work, we compared it with Riaz and Kamaci [65] technique. In Table 7, we solved the above-explanatory example by using SVNFWA and SVNFOWA operators and in this study, the suggested approach outcomes were compared with Riaz and Kamaci technique [65]. We observed that the alternative rankings are essentially the same. In Riaz and Kamaci [65], in the picture fuzzy set, there is just a quantitative portion, but in our work, we have additional information about both the quantitative and qualitative parts, which we call the linguistic parts. We observed that when using [65] it shows that our approach is more effective and instructive.

Table 8
Evaluation of existing operators in comparison to current operators [66].

Different operators :	Score function values	Ranking
q-ROFA-AWA [66]	$Sc(P_1) = 2.6754, Sc(P_2) = 2.4098, Sc(P_3) = 0.3543, Sc(P_4) = 2.3879, Sc(P_5) = 1.3776$	$P_1 > P_2 > P_4 > P_5 > P_3$
q-ROFA-AWG [66]	$Sc(P_1) = 2.3904, Sc(P_2) = 2.4100, Sc(P_3) = 0.3185, Sc(P_4) = 1.3865, Sc(P_5) = 1.3598$	$P_2 > P_1 > P_4 > P_5 > P_3$
q-RLNWAA (Proposed operator)	$Sc(P_1) = 2.9304, Sc(P_2) = 3.0689, Sc(P_3) = 2.0507, Sc(P_4) = 2.8417, Sc(P_5) = 2.4107$	$P_2 > P_1 > P_4 > P_5 > P_3$
q-RLNWGA (Proposed operator)	$Sc(P_1) = 1.8361, Sc(P_2) = 1.2721, Sc(P_3) = 1.1856, Sc(P_4) = 1.5326, Sc(P_5) = 1.3580$	$P_1 > P_4 > P_5 > P_2 > P_3$

Table 9
A review of the study proposal using current and relevant structures [67–72].

Name	Year	Structure	Linguistic	Membership	Indeterminacy	Non membership
Zadeh [16]	1965	Fuzzy Set	×	✓	×	×
Atanassov [67]	1986	Intuitionistic Fuzzy Set	×	✓	×	✓
Yager [68]	2013	Pythagorean Fuzzy Set	×	✓	×	✓
Yager [69]	2016	q-rung orthopair fuzzy set	×	✓	×	✓
Smarandache [70]	2005	Neutrosophic Set	×	✓	✓	✓
Bhowmki [71]	2009	Intuitionistic Neutrosophic Set	×	✓	✓	✓
Jansi [72]	2019	Pythagorean Neutrosophic Set	×	✓	✓	✓
Ali [58]	2022	q-rung linguistic picture fuzzy set	✓	✓	✓	✓
Proposed Technique	2023	q-rung linguistic neutrosophic set	✓	✓	✓	✓

8.3. Comparison between the suggested approach with the existing one by Farid and Riaz [66]

Additionally, to demonstrate the suitability and validity of the approach suggested in this work, we compared it with Farid and Riaz [66] technique. In Table 8, we solved the above-explanatory example by using fuzzy q-rung orthopair Aczel-Alsina operators and contrasted the outcomes of the approach provided in this study with those obtained using the Farid and Riaz technique. We noticed that practically all of the options rank similarly. In Farid and Riaz’s article, in the picture fuzzy set, there is just a quantitative portion, but in our work, we have additional information about both the quantitative and qualitative parts, which we call the linguistic parts. We observed that when using [66] it shows that our approach is more effective and instructive.

The analysis presented above demonstrates the effectiveness of our proposed strategies for solving decision-making (DM) issues, particularly for multiple attribute decision-making (MADM). Compared to other approaches, our methods offer greater flexibility and rationality for addressing MADM challenges. These advantages are largely attributed to the use of q-rung linguistic neutrosophic fuzzy (q-RLNS), which allows DMs to show their opinions more freely while minimizing data loss. Statistical presumptions that decision-makers frequently make subjective decisions and judgments are likewise taken into account by our method, making q-RLNS relevant and sufficient for conveying assessments of various possibilities. Additionally, our MADM technique is based on q-RLNSWAA or q-RLNSWGA, which notify the interrelationships between different attributes or criteria. This makes our method more effective at replicating real-world MADM challenges while giving decision-makers a new instrument for expressing their evaluations. Our method is wider, stronger, and more adaptable than previous tactics, making it an effective solution for addressing MADM issues.

In the following Table 9, the terms linguistic, Membership, indeterminacy, and Non-membership are used to denote these concepts. Structure (GQRPFL) has a constraint in Ali et al. [58]. We talk about the restriction of Ali et al. [58] Structure. It addresses when the existing model exists in the form of q-rung linguistic picture fuzzy set with the condition $0 \leq \vartheta_{B_2}(i) + \Gamma_{B_2}(i) + \lrcorner_{B_2}(i) \leq 1$. However, the Q-RLNS structure we’ve presented deals with when the $0 \leq \vartheta_{B_2}(i) + \Gamma_{B_2}(i) + \lrcorner_{B_2}(i) \leq 3$, then, to change the value in the closed interval between 0 and 3, we generalized it by raising the strength of the memberships of truth, ambiguity and falsehood respectively, up to “q”.

9. Conclusions and future work

The q-rung linguistic neutrosophic weighted aggregation operators proposed in the study provide a flexible framework for effectively aggregating decision information in a complex and uncertain environment. The proposed operators incorporate the linguistic terms, and neutrosophic sets and can handle the decision-making information in a more comprehensive manner than the existing aggregation methods. This method allows for the consideration of multiple criteria, including team performance, player statistics, and match outcomes, while also taking into account linguistic assessments of the criteria by experts in the field. The proposed approach offers several advantages over traditional ranking methods, such as greater transparency in the weighting of criteria and the ability to incorporate linguistic uncertainties in the decision-making process. However, the effectiveness of the q-rung linguistic neutrosophic weighted aggregation operators approach will depend on the quality of data and expert assessments used in the analysis. Additionally, the approach may require significant computational resources and expertise to implement effectively. Overall, our goal is to provide a more accurate and holistic evaluation of player performance. The findings of this study have the potential to contribute to the refinement of existing ranking systems and assist in the identification of players who possess the skills and attributes required for success in the international cricket arena.

In conclusion, the future directions for a comprehensive approach to international cricket player’s ranking assessment using Q-rung linguistic neutrosophic weighted aggregation operators involve q-rung linguistic orthopair bi-polar neutrosophic fuzzy soft set,

q-rung linguistic orthopair cubic neutrosophic fuzzy soft set, q-rung linguistic orthopair interval-valued neutrosophic fuzzy soft set, q-rung linguistic orthopair m-polar neutrosophic fuzzy soft set and q-rung linguistic orthopair neutrosophic fuzzy soft set, etc.

Funding statement

No specific assistance was provided for this research by government, private, or nonprofit organizations.

Ethical approval

This study does not involve the use of human subjects or animals, therefore ethical approval is not required. The research relies exclusively on publicly available data, and no personally identifiable information is being collected or analyzed. All procedures and methodologies strictly adhere to established ethical guidelines and regulations.

CRedit authorship contribution statement

Muhammad Saeed: Conceptualization, Formal analysis, Investigation, Methodology, Supervision. **Abdul Wahab:** Investigation, Writing – original draft. **Jawad Ali:** Investigation, Writing – original draft. **Ebenzer Bonyah:** Funding acquisition, Project administration, Resources, Validation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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