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Fermatean fuzzy distance and Sugeno-Weber operators-based SPC-MARCOS approach for sustainable supplier evaluation in the healthcare supply chain

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The present work proposes a new decision support tool for assessing the sustainable suppliers in the healthcare supply chain. For this purpose, the classical Measurement of Alternatives and Ranking according to Compromise Solution (MARCOS) model is integrated with the Sugeno-Weber weighted averaging operators, modified symmetry point of criterion (SPC) model, rank sum (RS) tool and Fermatean fuzzy sets (FFSs), and named as the 'FF-SPC-RS-MARCOS' framework. The developed model firstly determines the decision experts' weights through RS model. Second, novel Sugeno-Weber weighted operators are introduced to combine the experts' opinions. Third, a unified weighting procedure is presented based on the combination of modified SPC approach for objective weight and RS method for subjective weight of attributes. To this aim, a novel distance measure is introduced for FFSs and further applied to compute the distance between aggregated Fermatean fuzzy numbers and symmetry point value of an attribute in the modified SPC approach. Further, a hybrid FF-SPC-RS-MARCOS approach is proposed to tackle the decision-making problems on FFSs setting. To elucidate the efficacy of the developed method, it is applied to a case study of sustainable supplier selection problem in the healthcare supply chain. The paper further conducts sensitivity investigation and comparison with existent approaches to test the stability and robustness of the ranking outcomes. This study shows how the proposed MARCOS method in combination with SPC and RS models can be used to prioritize the alternative suppliers in the healthcare supply chain. The introduced work provides a new methodology, which can help the practitioners and academics to evaluate suppliers with uncertain information and can also be employed to other areas facing similar types of decision-making problems.

Keywords Fermatean fuzzy set, Distance measure, Symmetry point criterion, Healthcare supply chain, Sustainable supplier, MARCOS

Due to the development of information technology and continuous globalization, the organizations are adopting new supply chain management (SCM) solutions to improve their competitive advantage. The sustainable supply chain (SC) practices comprised of procurement and sourcing methods, information technology and operations management with the synchronization of material, financial, and information flows. An effective SC recommends on-time product delivery, steadiness of price together with reduction of waste in inventory and distribution channels¹. On account of the globalization and increased economic competition, the operations of SCM have become more difficult and important for the industries. As one of the important service sectors, healthcare has significantly changed due to emergence of digital technologies². Healthcare supply chain (HSC) is an intricate adaptive ecosystem that ensures the supply of medicines, transportation of medical equipment, minimization of

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inventory consumption etc.³. Healthcare organizations need to make tactical alliances with suppliers along with numerous companies involved in operation facilities.

SCs are the essence of healthcare systems to transport equipment to the demand points. The notion of sustainable supplier (SS) selection incorporates social and environmental priorities along with economical features in traditional SCM. The selection of an effective SS is a viable topic due to involvement of multiple alternatives, criteria and experts in unpredictable and complex situations. The choice of an efficient supplier influences the company's status and competitiveness. By selecting suitable suppliers, the firms can certify that their SC operations are environmentally accountable and socially reasonable. This process globally improves their affordability and provides them as the respectful accountable firms. Choosing the right supplier is a critical process in the HSC process, while it is important to achieve the overall sustainability assessment of the hospitals. On the other hand, selecting an inappropriate supplier may result in the loss of money and affects the company's financial and operative stability. Using the dimensions of sustainability can offer implications to the healthcare organizations for encouraging sustainable and green growth by supporting them to prioritize and select the best supplier. To effectually deal such concerns, healthcare enterprises must design strategic framework that provide primary distinguishability in pursuing and drawing uncertain actions and certify the smooth practice of information during the SCM^{4,5}.

Selecting a suitable supplier is a significant multiple-criteria group decision-making (MCGDM) challenge because it could have profound implications. In the process of MCGDM, uncertainty associated to human rational, views and reasoning is the challenging facet. As the general uncertainty notion, a "fuzzy set (FS)"6 has been put forward to express the linguistic assessments and to handle the fuzziness of data. As an alternate method for addressing the vagueness and fuzziness, Atanassov⁷ originated the notion of "intuitionistic fuzzy set (IFS)", which can capture both the membership degree (MD) and non-membership degree (ND). In IFS, the sum of MD and ND cannot exceed one. Afterward, some works on IFSs have been discussed with diverse disciplines⁸⁻¹⁰. To further refine the concept of IFS, Yager¹¹ put forward the notion of "Pythagorean fuzzy set (PFS)" wherein the quadratic sums of MD and ND is limited to one. On account of its capability to handle the uncertainty information, PFS theory has received great attention from many researchers in several regions^{12–14}. In several real-life decision-making situations, an expert may have to handle the problems wherein the quadratic sums of MD and ND of an element are greater than unity. Obviously, the FS, IFS and PFS theories are not able to deal with such situations¹⁵. Further, Senapati and Yager¹⁵ covered the deficiencies of FS, IFS and PFS by introducing the notion of "Fermatean fuzzy set (FFS)", wherein the sum of cubes of MD and ND is bounded to unity. This concept plays a pivotal role in processing uncertain information by providing a larger domain area. The flexibility and efficiency of FFSs in capturing ambiguity have led to the widespread adoption of this concept in numerous domains 16-18. In this work, we employ FFS to express the fuzziness of information during the process of SS selection and implement the developed MCGDM method to a case study of an Indore district, Madhya Pradesh (India).

An MCGDM aims to offer approaches for ranking the options and choosing the optimal option amongst a possible options' set by means of the various attributes^{19–21}. Because of the prevalence of MCGDM problems in our daily life, its theories and applications have been commonly presented in numerous domains^{22–24}. An MCGDM tool on FSs is applied to take decisions under the situations where the information is, inadequate, uncertain or vague²³. Based on the association between options and reference points, Stevic et al.²⁵ introduced notion of "Measurement of Alternatives and Ranking according to COmpromise Solution (MARCOS)" method. It determines the utility degree (UD) from reference point solutions and further presents a procedure to define utility functions with their aggregation. This method measures and prioritizes the options using the compromise solution. It can be applied to the large sets of options and criteria by keeping the steadiness of approach with greater steadiness and accuracy. Due to its simplicity and flexibility, the MARCOS method has been developed and applied from various perspectives^{23,26,27}.

The determination of criteria/attributes weights is one of the important steps of MCGDM problems. According to the existing studies, the attributes' weight-determining methods are classified into objective and subjective weighting models²⁸. In the objective method, the weight of attribute is determined through the data given in the decision-matrix, using mathematical models and scores without considering the DEs' opinions, while in the subjective method, the attributes' weights are obtained with the use of DEs' opinion²¹. As an objective method, the "symmetry point of criterion (SPC)" has been pioneered by Gligorić et al.²⁹, which comprises the notion of symmetry point. Due to its effectiveness and innovation, we use this method to derive the objective weight of each attribute, whereas the subjective weight of each attribute is computed by the "rank sum (RS)" model, which has been firstly presented by Stillwell et al.³⁰. To get the benefits of objective and subjective weight-determination methods, this paper presents a unified weighting process for deriving the criteria weights.

Motivated by the concepts of FFS, MARCOS, SPC and RS model, this study introduces a hybrid FFSs-based methodology, which captures the fuzziness and uncertainty of information more effectively. This approach combines the FF-distance measure (FF-DM), Sugeno–Weber weighted averaging operator, SPC model, RS model and the MARCOS method with Fermatean fuzzy numbers (FFNs), and named as 'FF-SPC-RS-MARCOS'. Up to now, no one has evaluated the SSs using FF-SPC-RS-MARCOS method in the HSC.

The key novelties of present work are given as follows:

- A novel distance measure is introduced to quantify the dissimilarity degree between FFSs.
- Sugeno-Weber weighted operators are presented to aggregate the individual FFNs into single FFN.
- To deal with the SS selection problem, a hybrid FF-SPC-RS-MARCOS method is developed based on the mixture of FF-DM, Sugeno-Weber weighted averaging operator, modified SPC tool, RS model and FFSs.
- In the context of HSC, an empirical study of SSs assessment problem is presented, which confirms the applicability and usefulness of the developed framework.

Other sections are planned as: Section "Literature review" confers inclusive review of literature associated with this work. Section "Proposed distance measure and aggregation operators" firstly discusses fundamental definitions on FFSs and further presents a novel FF-DM with some of its properties. Further, this section develops Sugeno–Weber averaging operators with their certain axioms. Section "An integrated FF-modified SPC-MARCOS method" proposes a Fermatean fuzzy extension of MARCOS model based on the FF-DM, Sugeno–Weber operators, modified SPC and RS model. Section "Case study: sustainable supplier (SS) selection in the HSC" applies the FF-SPC-RS-MARCOS approach for selecting the most suitable SS alternative in the HSC. Sensitivity and comparative assessments are shown to prove the stability and robustness of the findings. Section "Conclusions" discusses the conclusions of this work and recommends the scope for further studies.

Literature review

In this section, we review the research studies related to the FFSs, MARCOS approach and the SS selection in the HSC.

FFSs

The concept of FFS has been effectively utilized into several application settings. For example, Barokab et al. 31 presented a set of Einstein prioritized operators to aggregate the FF-information. Moreover, they have established an algorithm to evaluate the university faculty selection problem under FFSs environment. Zhong et al. 18 introduced a novel Muirhead mean operator for FFSs with its properties. Moreover, an improved algorithm of FMEA method has been developed in the context of FFSs. With the use of FF-similarity metric, Al-Qudah and Ganie³² resolved bidirectional approximation reasoning and pattern recognition problems under FFSs environment. Liu³³ discussed a set of FF-similarity measures and their desirable axioms. Further, they have demonstrated their effectiveness by applying them for pattern recognition and clustering assessment. Golui et al. 34 set forth an extended TOPSIS model with FF-information and correlation coefficient, and applied to opt a suitable electric vehicle. In this regard, they have proposed a novel correlation coefficient measure for FFSs with its certain characteristics. Gao et al. 16 combined the "best worst method (BWM) and VIseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR)" model with FFSs and applied to evaluate the healthcare waste treatment technologies, wherein the information about alternatives and criteria is given in terms of Fermatean fuzzy numbers. Ejegwa et al. 35 firstly pointed out the limitations of existing FF-correlation coefficient measure and further gave a tool to find the correlation coefficient on FFSs. Moreover, they have used their measure to find students' academic performance through clustering assessment procedure. Yu et al. 36 defined a new score function for FFN. Further, a collective "FF-failure mode and effects analysis (FMEA) and combined compromise solution (CoCoSo)"-based algorithm has been introduced for risk evaluation of liquefied natural gas storage tank leakage. Apart from these studies, several different models with their utilities have been discussed using FFSs. Few of them are given by 17,37,38. Till now, there is no work which integrates FFSs with Sugeno-Weber averaging operators, novel distance measure-based SPC weighting model and MARCOS approach and utilizes for SS selection process in the HSC.

Studies related to MARCOS model

The MARCOS model²⁵ includes the association of the reference points, more accurate assessment of UDs and proposes a tool to define utility functions and their aggregation. Table 1 shows the description of existing works on the MARCOS approach.

Decision-making models for SS selection in the HSC

The SC is an indispensable pillar to allow access to comprehensive, reasonable, and on-time access to diagnostic services. A well-functioning health procedure will entail a SC to provide and ease enhanced health result, along with advanced national and regional health security goals⁵⁷. The sustainability of HSC is important to attain the sustainable development goals (SDGs) by falling the carbon footprints of healthcare actions and creating healthcare innovations⁵⁸. To select an effective SS, Stević et al.²⁵ used the classical MARCOS tool with crisp information. Though, their model is not suitable for fuzzy decision-making problem of SS selection. A collective decision support system, integrating fuzzy additive ratio assessment (ARAS) and BWM, has been presented for the assessment of healthcare SSs in the logistics 4.059. Pamucar et al.60 assessed the suppliers during the pandemic of COVID-19. In this regard, they have used a novel rough set-based decision support tool to evaluate the suppliers. An integrated fuzzy group decision tool has been proposed to assess the effective suppliers for a healthcare organization⁶¹. Nayeri et al.⁶² evaluated the sustainable, resilient and responsive suppliers in the medical equipment industry. They have proposed a new multi-stage optimization model using uncertain information. Further, Debnath et al.⁶³ evaluated the suppliers for healthcare testing services using fuzzy decision support system. In the following, they have asked twelve decision experts (DEs) from diverse backgrounds to present their opinions about five suppliers in regard to fourteen attributes. The attributes' weights are ranked through SWARA model, while the suppler alternatives are prioritized via WASPAS approach. However, that approach is unreasonable in the context of uncertain information. Dai et al. suggested an innovative MABAC method to evaluate the healthcare suppliers under the context of belief distribution. Using generalized Dombi operators, Saha et al.⁶⁴ proposed an integrated probabilistic hesitant fuzzy MARCOS method for assessing the suppliers under HSC process. Although, the concept of sustainability is missing in both the articles 1,64 during the evaluation of healthcare suppliers.

In the process of MCGDM, there are some important steps: (i) computation of involved DEs' weights, (ii) aggregation of the individual opinions of DEs for finding the combined opinion, (iii) determination of the attributes' weights and (iv) computation of candidates' ranking order and the best choice by means of the considered attributes. As per the information given in existing studies, we identify some research gaps of existing methods on SS selection, which are given as follows:

References	Generalized versions of classical MARCOS method	Applications
Stanković et al. ³⁹	FSs-MARCOS method	Road traffic risk estimation
Büyüközkan et al. ⁴⁰	Fuzzy analytic hierarchy process (AHP)-MARCOS model	Analysis of digital transformation strategy
Ecer and Pamucar ⁴¹	IFSs-based MARCOS method	Assessment of insurance companies
Kovac et al. ⁴²	Spherical fuzzy MARCOS model	Evaluation of drone-based city logistics models
Fan et al. ⁴³	Prospect theory and neutrosophic cubic information-based MARCOS approach	MCDM problem
Du et al. ⁴⁴	Fuzzy BWM-MARCOS model	Regional distribution network outage loss evaluation
Peng et al. ⁴⁵	A hybrid MARCOS model with set pair assessment and q-rung orthopair fuzzy set	Assessing the cache placement policy
Tarafdar et al. ⁴⁶	Spherical fuzzy MARCOS model with type-3 logic	Performance-emission optimization
Görçün and Doğan ⁴⁷	FSs-based BWM-MARCOS model	Mobile crane selection
Saha et al. ²³	FF-MARCOS method	Warehouse site selection
Haseli et al. ⁴⁸	MARCOS model with fuzzy Z-numbers	Evaluation of smart jewelry
Altay et al. ⁴⁹	Interval type-2 fuzzy BWM-MARCOS method	Selecting sites for e-scooter sharing stations
Koohathongsumrit and Chankham 50	Fuzzy BWM-MARCOS	Selection of most optimal freight route
Manirathinam et al. ⁵¹	Fermatean neutrosophic fuzzy stratified AHP-MARCOS model	Renewable energy resources evaluation
Majumder et al. ⁵²	Single-valued neutrosophic MARCOS method	Evaluation of indoor sex work risks in the economy
Saha et al. ⁶⁴	Probabilistic hesitant fuzzy MARCOS model	Supplier selection
Wang et al. ⁵³	PFSs-based MARCOS model	Sustainable food suppliers' selection
Rani et al. ⁵⁴	Picture fuzzy extension of MARCOS method model	MCDM problems
Li et al. ⁵⁵	Shannon entropy-based q-rung orthopair fuzzy BWM-MARCOS approach	Assessment of mobile medical app service qualities
Zeng et al. ²⁷	MARCOS model with Pythagorean hesitant fuzzy information	Location assessment of subsea tunnels
Ecer et al. ⁵⁶	Integrated fuzzy-BWM-MARCOS model	Evaluation of cryptocurrency exchanges
Rani et al. ⁵⁴	Similarity measure-based picture fuzzy MARCOS model	Assessment of decision-making problems

Table 1. Some well-known extensions of the classical MARCOS approach from different perspectives.

- Several decision support tools have been developed to evaluate the SSs; however, few of them are focused on the HSC. The selection of SS is a complicated MCGDM problem because of the presence of many attributes and several experts. The decision-making methods by ^{1,25-61,63}, Nayeri et al.⁶² ignored the importance of DEs during the assessment of SSs, which may not be suitable for the realistic situations as it is very difficult for a single DE to provide reliable and consistent results in reality.
- The consideration of objective and subjective weighting approaches cannot only determine the criteria weights based on the quantitative data but also consider the DEs' judgements during the assessment of attribute weights. In the context of SSs in the HSC, most of the existing works have not demonstrated how the amalgamation of objective and subjective weighting techniques impacts the decision results. In addition, few studies assume the direct weights of attributes without considering the objective weight or subjective weight, which may cause information loss.

The limitations of existing studies on SS selection in the HSC motivate us to develop a new MCGDM method which determines the alternatives' rank together with the DEs and criteria significance values during the assessment of SSs. Based on the aforesaid discussions, the purpose of the work is to assess the sustainable suppliers in the HSCs and rank them accordingly under the context of FFSs. To this aim, we need to address the following points:

- What are the attributes/criteria and dimensions to elect the most suitable SS option?
- What are the importance degrees of considered criteria and DEs in order to assess the sustainable suppliers in the HSC?
- Which MCDM method could be considered to assess the SS alternatives?
- What is the rank of the considered SS options under Fermatean fuzzy environment?

Proposed distance measure and aggregation operators

The current part of this study firstly shows fundamental definitions and then, introduces new distance measure for FFSs. In addition, novel aggregation operators (AOs) based on Sugen-Weber triangular norms are developed for combining the individual FFNs.

Preliminaries

Definition 1 Senapati and Yager¹⁵. Let $\Omega = \{h_1, h_2, ..., h_n\}$ be a fixed set. A FFS T on $\Omega = \{h_1, h_2, ..., h_n\}$ is given as.

$$T = \{(h_j, (\mu_T(h_j), \nu_T(h_j))) | h_j \in \Omega\},$$

wherein $\mu_T, \, \nu_T: \Omega \to [0,\,1]$ designate the MD and ND of an element $h_j \in \Omega$ to T, correspondingly, satisfying $0 \leq (\mu_T(h_j))^3 + (\nu_T(h_j))^3 \leq 1$. The degree of indeterminacy is $\pi_T(h_j) = \sqrt[3]{1-\mu_T^3(h_j)-\nu_T^3(h_j)}, \forall h_j \in \Omega$.

The term $(\mu_T(h_i), \nu_T(h_i))$ is defined as "Fermatean fuzzy number (FFN)", and given by $\lambda = (\mu, \nu)$, here $\mu, \nu \in [0, 1]$ and $0 \le \mu^3 + \nu^3 \le 1$.

Definition 2 Senapati and Yager¹⁵. For a FFN $\lambda = (\mu, \nu)$, the score and accuracy values are computed via Eqs. (1) and (2), correspondingly.

$$S(\lambda) = 0.5 \left((\mu)^3 - (\nu)^3 + 1 \right),$$
 (1)

$$A(\lambda) = (\mu)^3 + (\nu)^3, \tag{2}$$

where $S(\lambda) \in [0, 1]$ and $A(\lambda) \in [0, 1]$.

Definition 3 Senapati and Yager¹⁵. For the given two FFNs $\lambda_1 = (\mu_1, \nu_1)$ and $\lambda_2 = (\mu_2, \nu_2)$, some basic operations are presented as follows.

(i)
$$\lambda_i^c = (\nu_j, \mu_j), j = 1, 2,$$

(ii)
$$\lambda_1 \cap \lambda_2 = ((\mu_1 \wedge \mu_2), (\nu_1 \vee \nu_2)),$$

(iii)
$$\lambda_1 \cup \lambda_2 = ((\mu_1 \vee \mu_2), (\nu_1 \wedge \nu_2)),$$

(iv)
$$\lambda_1 \oplus \lambda_2 = \left(\sqrt[3]{\mu_1^3 + \mu_2^3 - \mu_1^3 \mu_2^3}, \nu_1 \nu_2\right),$$

(iv)
$$\lambda_1 \otimes \lambda_2 = \left(\mu_1 \, \mu_2, \, \sqrt[3]{\nu_1^3 + \nu_2^3 - \nu_1^3 \, \nu_2^3} \right),$$

(vi)
$$\varepsilon \lambda = \left(\sqrt[3]{1 - \left(1 - \mu_j^3\right)^{\varepsilon}}, (\nu_j)^{\varepsilon}\right), j = 1, 2, \varepsilon > 0,$$

(vii) $\lambda_j^{\varepsilon} = \left((\mu_j)^{\varepsilon}, \sqrt[3]{1 - \left(1 - \nu_j^3\right)^{\varepsilon}}\right), j = 1, 2, \varepsilon > 0.$

(vii)
$$\lambda_j^{\varepsilon} = \left((\mu_j)^{\varepsilon}, \sqrt[3]{1 - \left(1 - \nu_j^3 \right)^{\varepsilon}} \right), j = 1, 2, \varepsilon > 0.$$

Definition 4 Senapati and Yager¹⁵. To aggregate the Fermatean fuzzy information, Senapati and Yager¹⁵ defined the following operators:

$$FFWA(\lambda_1, \lambda_2, ..., \lambda_n) = \left(\sqrt[3]{1 - \prod_{j=1}^n \left(1 - \mu_j^3\right)^{\varepsilon_j}}, \prod_{j=1}^n (\nu_j)^{\varepsilon_j}\right),\tag{3}$$

$$FFWG(\lambda_1, \lambda_2, ..., \lambda_n) = \left(\prod_{j=1}^{n} (\mu_j)^{\varepsilon_j}, \sqrt[3]{1 - \prod_{j=1}^{n} (1 - \nu_j^3)^{\varepsilon_j}} \right). \tag{4}$$

Here, $\{\varepsilon_1, \varepsilon_2, ..., \varepsilon_n\}$ denotes the weights' set of FFNs $\lambda_1, \lambda_2, ..., \lambda_n$, where ε_j is non-negative and $\varepsilon_1 + \varepsilon_2 + \ldots + \varepsilon_n = 1.$

Definition 5 Deng and Wang⁶⁵. Let $R, S \in FFSs(\Omega)$. A FF-DM is a real-valued function $\Psi: FFSs(\Omega) \times FFSs(\Omega) \rightarrow [0, 1]$ which satisfies (i) $0 \leq \Psi(R, S) \leq 1$, (ii) $\Psi(R, S) = 0$ iff R = S, (iii) $\Psi(R, S) = \Psi(S, R)$, and (iv) If $R \subseteq S \subseteq T$, then $\Psi(R, S) \leq \Psi(R, T)$ and $\Psi(S, T) \leq \Psi(R, T), \forall T \in FFS(\Omega).$

New distance measure for FFSs

Distance measure plays an important role in process of taking reasonable and effective decisions. In the section, we introduce new FFDM to compute the amount of dissimilarity on FFSs.

Theorem 1 For $R, S \in FFSs(\Omega)$, the function, given by Eq. (5),

$$\Psi(R,S) = \frac{1}{n} \sum_{j=1}^{n} \sqrt{\left(\frac{\mu_R^3(h_j) - \nu_R^3(h_j)}{2} - \frac{\mu_S^3(h_j) - \nu_S^3(h_j)}{2}\right)^2 + \frac{1}{3} \left(\frac{\mu_R^3(h_j) + \nu_R^3(h_j)}{2} - \frac{\mu_S^3(h_j) + \nu_S^3(h_j)}{2}\right)^2}$$
(5)

is a distance measure between FFSs R and S.

Proof To prove this theorem, the function, given by Eq. (5), has to satisfy the axioms of Definition 5.

(i) Since $R, S \in FFSs(\Omega)$, therefore, we have $0 \le \mu_R(h_j)$, $\nu_R(h_j) \le 1, 0 \le \mu_S(h_j)$, $\nu_S(h_j) \le 1$, $\mu_R^3(h_j) \leq \mu_S^3(h_j) \leq \mu_T^3(h_j)$ and $\nu_T^3(h_j) \leq \nu_S^3(h_j) \leq \nu_R^3(h_j), \ \forall h_j \in \Omega$. Then, it is evident from Eq. (5) that $\Psi(R,S) \geq 0$. On the other hand, if R and S are two FFSs such that R = (1,0) and S = (0,1), then we obtain $\Psi(R, S) \leq 1$. Hence, we get $0 \leq \Psi(R, S) \leq 1$.

(ii) If $\Psi(R, S) = 0$, then Eq. (5) becomes

$$\Psi\left(R,\,S\right)\,=\,\frac{1}{n}\,\sum_{j=1}^{n}\sqrt{\left(\frac{\mu_{R}^{3}(h_{j})\,-\,\nu_{R}^{3}(h_{j})}{2}\,-\,\frac{\mu_{S}^{3}(h_{j})\,-\,\nu_{S}^{3}(h_{j})}{2}\right)^{2}\,+\,\frac{1}{3}\,\left(\frac{\mu_{R}^{3}(h_{j})\,+\,\nu_{R}^{3}(h_{j})}{2}\,-\,\frac{\mu_{S}^{3}(h_{j})\,+\,\nu_{S}^{3}(h_{j})}{2}\right)^{2}}\,=\,0.$$

It implies that $\left(\frac{\mu_R^3(h_j)-\nu_R^3(h_j)}{2}-\frac{\mu_S^3(h_j)-\nu_S^3(h_j)}{2}\right)=0$ and $\left(\frac{\mu_R^3(h_j)+\nu_R^3(h_j)}{2}-\frac{\mu_S^3(h_j)+\nu_S^3(h_j)}{2}\right)=0$, hence, we get $\mu_R(h_j)=\mu_S(h_j)$ and $\nu_R(h_j)=\nu_S(h_j)$, $\forall\,h_j\in\Omega.$ Therefore, R=S. Conversely, if R=S, then obviously $\Psi(R,S)=0$.

Conversely, if R = S, then obviously Ψ (iii) From Eq. (5), we have

$$\Psi\left(R,\,S\right) \,=\, \frac{1}{n}\,\sum_{j=1}^n \sqrt{\left(\frac{\mu_R^3(h_j) -\, \nu_R^3(h_j)}{2} \,-\, \frac{\mu_S^3(h_j) -\, \nu_S^3(h_j)}{2}\right)^2 \,+\, \frac{1}{3}\,\left(\frac{\mu_R^3(h_j) +\, \nu_R^3(h_j)}{2} \,-\, \frac{\mu_S^3(h_j) +\, \nu_S^3(h_j)}{2}\right)^2}.$$

 $\Psi\left(R,\,S\right)$ can also be written as

$$\Psi\left(R,\,S\right) = \frac{1}{n}\,\sum_{j=1}^{n}\,\sqrt{\left(\frac{\mu_{S}^{3}(h_{j}) - \nu_{S}^{3}(h_{j})}{2} - \frac{\mu_{R}^{3}(h_{j}) - \nu_{R}^{3}(h_{j})}{2}\right)^{2} + \frac{1}{3}\left(\frac{\mu_{S}^{3}(h_{j}) + \nu_{S}^{3}(h_{j})}{2} - \frac{\mu_{R}^{3}(h_{j}) + \nu_{R}^{3}(h_{j})}{2}\right)^{2}} = \Psi\left(S,\,R\right)\,.$$

Thus, $\Psi(R, S) = \Psi(S, R)$,

(iv) Let $R, S, T \in FFSs(\Omega)$ and $R \subseteq S \subseteq T$. By definition of FFS, we have $\mu_R^3(h_j) \leq \mu_S^3(h_j) \leq \mu_T^3(h_j)$ and $\nu_T^3(h_j) \leq \nu_S^3(h_j) \leq \nu_R^3(h_j)$, $\forall h_j \in \Omega$.

For R, S, $T \in FFSs(\Omega)$ and from Eq. (5), the distance measures between R & S, S & T and R & T are given as

$$\Psi\left(R,\,S\right)\,=\,\frac{1}{n}\,\sum_{i=1}^{n}\sqrt{\left(\frac{\mu_{R}^{3}(h_{j})\,-\,\nu_{R}^{3}(h_{j})}{2}\,-\,\frac{\mu_{S}^{3}(h_{j})\,-\,\nu_{S}^{3}(h_{j})}{2}\right)^{2}\,+\,\frac{1}{3}\,\left(\frac{\mu_{R}^{3}(h_{j})\,+\,\nu_{R}^{3}(h_{j})}{2}\,-\,\frac{\mu_{S}^{3}(h_{j})\,+\,\nu_{S}^{3}(h_{j})}{2}\right)^{2}},$$

$$\Psi(S,T) = \frac{1}{n} \sum_{j=1}^{n} \sqrt{\left(\frac{\mu_S^3(v_j) - \nu_S^3(h_j)}{2} - \frac{\mu_T^3(h_j) - \nu_T^3(h_j)}{2}\right)^2 + \frac{1}{3} \left(\frac{\mu_S^3(h_j) + \nu_S^3(h_j)}{2} - \frac{\mu_T^3(h_j) + \nu_T^3(h_j)}{2}\right)^2},$$

$$\Psi\left(R,\,T\right) \,=\, \frac{1}{n}\, \sum_{j=1}^n \sqrt{\left(\frac{\mu_R^3(h_j) \,-\, \nu_R^3(h_j)}{2} \,-\, \frac{\mu_T^3(h_j) \,-\, \nu_T^3(h_j)}{2}\right)^2 \,+\, \frac{1}{3}\, \left(\frac{\mu_R^3(h_j) \,+\, \nu_R^3(h_j)}{2} \,-\, \frac{\mu_T^3(h_j) \,+\, \nu_T^3(h_j)}{2}\right)^2},\, \forall\, h_j \,\in\, \Omega.$$

Suppose that

$$f(x, y) = ((x^3 - y^3) - (\iota^3 - \ell^3))^2 + \frac{1}{3}((x^3 + y^3) - (\iota^3 + \ell^3))^2,$$
 (6)

wherein $0 \le x, y, \iota, \ell \le 1$.

Differentiating (6) partially over x and y, respectively, we attain

$$\frac{\partial f(x, y)}{\partial x} = 4x^2 \left(2\left(x^3 - \iota^3\right) + \left(\ell^3 - y^3\right)\right),\tag{7}$$

$$\frac{\partial f(x,y)}{\partial y} = -4y^2 \left(2 \left(y^3 - \ell^3 \right) + \left(x^3 - \iota^3 \right) \right). \tag{8}$$

When $0 \le \iota \le x \le 1$ and $0 \le y \le \ell \le 1$, we acquire $\frac{\partial f(x,y)}{\partial x} \ge 0$ and $\frac{\partial f(x,y)}{\partial y} \le 0$, respectively. As Eq. (6) is similar to Eq. (5), then assume three FFSs R, S and T with $R \subseteq S \subseteq T$, we have $\mu_R^3(h_j) \le \mu_S^3(h_j) \le \mu_T^3(h_j)$ and $\nu_T^3(h_j) \le \nu_S^3(h_j) \le \nu_R^3(h_j)$, $\forall h_j \in \Omega$. Considering the monotonicity of f(x,y) and Eq. (5), we get $\Psi(R,S) \le \Psi(R,T)$. When $0 \le x \le \iota \le 1$ and $0 \le \ell \le y \le 1$, we attain $\frac{\partial f(x,y)}{\partial x} \le 0$ and $\frac{\partial f(x,y)}{\partial y} \ge 0$, respectively. Since Eq. (6) is similar to Eq. (5), then assume three FFSs R, S and T with $R \subseteq S \subseteq T$, we have $\mu_R^3(h_j) \le \mu_S^3(h_j) \le \mu_T^3(h_j)$ and $\nu_T^3(h_j) \le \nu_S^3(h_j) \le \nu_R^3(h_j)$, $\forall h_j \in \Omega$. Considering monotonicity of f(x,y) and Eq. (5), we get $\Psi(S,T) \le \Psi(R,T)$. Thus, the function, given by Eq. (5), is a FFDM. [Proved].

Sugeno-Weber weighted AOs for FFNs

This section firstly proposes some Sugeno-Weber AOs including "FF Sugeno-Weber weighted averaging (FFSWWA)" and "FF Sugeno-Weber ordered weighted averaging (FFSWOWA)", operators for FFNs.

Sugeno-Weber operations on FFNs

Definition 6 ⁶⁶. (For two real numbers δ and ξ , the Sugeno–Weber t-norm (\otimes) and t-conorm (or s-norm) (\oplus) are presented by Eqs. (9) and (10), respectively.

$$T_{SW}(\delta, \xi) = \begin{cases} T_D(\delta, \xi), & \text{if } \lambda = -1, \\ \max\left(0, \frac{\delta + \xi - 1 + \lambda}{1 + \lambda}\right), & \text{if } -1 < \lambda < +\infty, \\ T_P(\delta, \xi), & \text{if } \lambda = +\infty. \end{cases}$$
(9)

$$S_{SW}(\delta, \xi) = \begin{cases} S_D(\delta, \xi), & \text{if } \lambda = -1, \\ \min\left(1, \delta + \xi - \frac{\lambda \delta \xi}{1 + \lambda}\right), & \text{if } -1 < \lambda < +\infty, \\ T_P(\delta, \xi), & \text{if } \lambda = +\infty. \end{cases}$$
 (10)

Corresponding to the Definition 6 and FFNs, we show the subsequent definition.

Definition 7 For given two FFNs $\lambda_1=(\mu_1,\,\nu_1)$ and $\lambda_2=(\mu_2,\,\nu_2)$, some Fermatean fuzzy operations using Sugeno–Weber triangular norms are given as follows.

$$\begin{array}{l} \text{(i)} \quad \lambda_{1} \oplus \lambda_{2} = \left(\sqrt[3]{\mu_{1}^{3} + \mu_{2}^{3} - \frac{\varsigma}{1+\varsigma}} \mu_{1}^{3} \mu_{2}^{3}, \ \sqrt[3]{\frac{\nu_{1}^{3} + \nu_{2}^{3} - 1 + \varsigma}{1+\varsigma}} \frac{\nu_{1}^{3} \nu_{2}^{3}}{1+\varsigma}}\right), \\ \text{(ii)} \quad \lambda_{1} \otimes \lambda_{2} = \left(\sqrt[3]{\frac{\mu_{1}^{3} + \mu_{2}^{3} - 1 + \varsigma}{1+\varsigma}} \frac{\mu_{1}^{3} \mu_{2}^{3}}{1+\varsigma}}, \ \sqrt[3]{\nu_{1}^{3} + \nu_{2}^{3} - \frac{\varsigma}{1+\varsigma}} \nu_{1}^{3} \nu_{2}^{3}}\right), \\ \varepsilon \lambda_{j} = \left(\sqrt[3]{\frac{1+\varsigma}{\varsigma}} \left(1 - \left(1 - \mu_{j}^{3} \left(\frac{\varsigma}{1+\varsigma}\right)\right)^{\varepsilon}\right), \ \sqrt[3]{\frac{1}{\varsigma}} \left(\left(1+\varsigma\right) \left(\frac{\varsigma \nu_{j}^{3} + 1}{1+\varsigma}\right)^{\varepsilon} - 1\right)\right), \ j = 1, \, 2, \, \varepsilon > 0, \\ \lambda_{j}^{\varepsilon} = \left(\sqrt[3]{\frac{1}{\varsigma}} \left(\left(1+\varsigma\right) \left(\frac{\varsigma \mu_{j}^{3} + 1}{1+\varsigma}\right)^{\varepsilon} - 1\right), \ \sqrt[3]{\frac{1+\varsigma}{\varsigma}} \left(1 - \left(1 - \nu_{j}^{3} \left(\frac{\varsigma}{1+\varsigma}\right)\right)^{\varepsilon}\right)\right), \ j = 1, \, 2, \, \varepsilon > 0. \end{array}$$

FFSWWA operator

Definition 8 Let $\lambda_{\ell}=(\mu_{\ell},\,\nu_{\ell})\;(\ell=1,\,2,\,...,\,n)$ be a collection of FFNs and $\varpi=\{\varpi_1,\,\varpi_2,\,...,\,\varpi_n\}$ be the weights' set of $\lambda_{\ell}\,(\ell=1,\,2,\,...,\,n)$, where ϖ_j is positive and $\varpi_1+\varpi_2+...+\varpi_n=1$. The FFSWWA operator is a mapping $FFSWWA:\Lambda^n\to\Lambda$, defined as.

$$FFSWWA(\lambda_1, \lambda_2, ..., \lambda_n) = \bigoplus_{\ell=1}^{n} (\varpi_{\ell} \lambda_{\ell}).$$
(11)

Theorem 2 The aggregated rating with FFSWWA operator is again a FFN and given as.

$$FFSWWA(\lambda_{1}, \lambda_{2}, ..., \lambda_{n}) = \bigoplus_{\ell=1}^{n} (\varpi_{\ell} \lambda_{\ell}) = \varpi_{1} \lambda_{1} \oplus \varpi_{2} \lambda_{2} \oplus ... \oplus \varpi_{n} \lambda_{n}$$

$$= \left(\sqrt[3]{\frac{1+\varsigma}{\varsigma} \left(1 - \prod_{\ell=1}^{n} \left(1 - (\mu_{\ell}^{3}) \frac{\varsigma}{1+\varsigma}\right)^{\varpi_{\ell}}\right)}, \sqrt[3]{\frac{1}{\varsigma} \left((1+\varsigma) \prod_{\ell=1}^{n} \left(\frac{\varsigma(\nu_{\ell}^{3}) + 1}{1+\varsigma}\right)^{\varpi_{\ell}} - 1\right)}\right).$$
(12)

Proof The mathematical induction principle can be applied to prove this theorem.

For $\ell = 2$, we have

$$FFSWWA(\lambda_1, \lambda_2) = \varpi_1 \lambda_1 \oplus \varpi_2 \lambda_2.$$

Since $\varpi_1\lambda_1$ and $\varpi_2\lambda_2$ are FFNs, thus, from Definition 7, their aggregation $\varpi_1\lambda_1\oplus\varpi_2\lambda_2$ is again a FFN. Using Definition 7, we get

$$FFSWWA(\lambda_{1}, \lambda_{2}) = \left(\sqrt[3]{\frac{1+\varsigma}{\varsigma}} \left(1 - \left(1 - (\mu_{1}^{3}) \frac{\varsigma}{1+\varsigma}\right)^{\varpi_{1}}\right), \sqrt[3]{\frac{1}{\varsigma}} \left((1+\varsigma) \left(\frac{\varsigma(\nu_{1}^{3}) + 1}{1+\varsigma}\right)^{\varpi_{1}} - 1\right)\right) \oplus$$

$$\left(\sqrt[3]{\frac{1+\lambda}{\lambda}} \left(1 - \left(1 - (\mu_{2}^{3}) \frac{\lambda}{1+\lambda}\right)^{\varpi_{2}}\right), \sqrt[3]{\frac{1}{\varsigma}} \left((1+\varsigma) \left(\frac{\varsigma(\nu_{2}^{3}) + 1}{1+\varsigma}\right)^{\varpi_{2}} - 1\right)\right)$$

$$= \left(\sqrt[3]{\frac{1+\varsigma}{\varsigma}} \left(1 - \prod_{\ell=1}^{2} \left(1 - (\mu_{\ell}^{3}) \frac{\varsigma}{1+\varsigma}\right)^{\varpi_{\ell}}\right), \sqrt[3]{\frac{1}{\varsigma}} \left((1+\varsigma) \prod_{\ell=1}^{2} \left(\frac{\varsigma(\nu_{\ell}^{3}) + 1}{1+\varsigma}\right)^{\varpi_{\ell}} - 1\right)\right).$$

Hence, Eq. (12) holds for $\ell=2$. Suppose that Eq. (12) holds for $\ell=p$, i.e.,

$$FFSWWA(\lambda_{1}, \lambda_{2}, ..., \lambda_{p}) = \bigoplus_{\ell=1}^{p} (\varpi_{\ell} \lambda_{\ell}) = \varpi_{1} \lambda_{1} \oplus \varpi_{2} \lambda_{2} \oplus ... \oplus \varpi_{p} \lambda_{p}$$

$$= \left(\sqrt[3]{\frac{1+\varsigma}{\varsigma} \left(1-\prod_{\ell=1}^{p} \left(1-(\mu_{\ell}^{3})\frac{\varsigma}{1+\varsigma}\right)^{\varpi_{\ell}}\right)}, \sqrt[3]{\frac{1}{\varsigma} \left((1+\varsigma)\prod_{\ell=1}^{p} \left(\frac{\varsigma(\nu_{\ell}^{3})+1}{1+\varsigma}\right)^{\varpi_{\ell}}-1\right)}\right).$$

Now, for $\ell = p + 1$, we have

$$FFSWWA(\lambda_{1}, \lambda_{2}, ..., \lambda_{p}, \lambda_{p+1}) = \bigoplus_{\ell=1}^{p+1} (\varpi_{\ell} \lambda_{\ell}) = \varpi_{1} \lambda_{1} \oplus \varpi_{2} \lambda_{2} \oplus ... \oplus \varpi_{p} \lambda_{p} \oplus \varpi_{p+1} \lambda_{p+1}$$

$$= \left(\sqrt[3]{\frac{1+\varsigma}{\varsigma} \left(1 - \prod_{\ell=1}^{p} \left(1 - (\mu_{\ell}^{3}) \frac{\varsigma}{1+\varsigma}\right)^{\varpi_{\ell}}\right)}, \sqrt[3]{\frac{1}{\varsigma} \left((1+\varsigma) \prod_{\ell=1}^{p} \left(\frac{\varsigma(\nu_{\ell}^{3}) + 1}{1+\varsigma}\right)^{\varpi_{\ell}} - 1\right)} \oplus \left(\sqrt[3]{\frac{1+\lambda}{\lambda}} \left(1 - \left(1 - (\mu_{p+1}^{3}) \frac{\lambda}{1+\lambda}\right)^{\varpi_{p+1}}\right), \sqrt[3]{\frac{1}{\varsigma} \left((1+\varsigma) \left(\frac{\varsigma(\nu_{p+1}^{3}) + 1}{1+\varsigma}\right)^{\varpi_{p+1}} - 1\right)}\right)$$

$$= \left(\sqrt[3]{\frac{1+\varsigma}{\varsigma} \left(1 - \prod_{\ell=1}^{p+1} \left(1 - (\mu_{\ell}^{3}) \frac{\varsigma}{1+\varsigma}\right)^{\varpi_{\ell}}\right)}, \sqrt[3]{\frac{1}{\varsigma} \left((1+\varsigma) \prod_{\ell=1}^{p+1} \left(\frac{\varsigma(\nu_{\ell}^{3}) + 1}{1+\varsigma}\right)^{\varpi_{\ell}} - 1\right)}\right).$$

Subsequently, Eq. (12) is true for $\ell=p+1$, and the obtained value is also a FFN. Therefore, Eq. (12) is true for all ℓ . [Proved].

Property 1 (i) Shift invariance: Let $\lambda_\ell = (\mu_\ell, \nu_\ell)$ ($\ell = 1, 2, ..., n$) be the FFNs. For a FFN $\lambda_0 = (\mu_0, \nu_0)$, $(\neq \lambda_\ell)$, we have FFSWWA ($\lambda_0 \oplus \lambda_1, \lambda_0 \oplus \lambda_2, ..., \lambda_0 \oplus \lambda_n$) = $\lambda_0 \oplus FFSWWA$ ($\lambda_1, \lambda_2, ..., \lambda_n$).

- (ii) Idempotency: Let $\lambda_\ell=(\mu_\ell,\,\nu_\ell)\;(\ell=1,\,2,\,...,\,n)$ be the FFNs such that $\lambda_\ell=\lambda_0,\,\forall\,\ell.$ Then $FFSWWA\;(\lambda_1,\,\lambda_2,...,\,\lambda_n)=\lambda_0.$
- (iii) Boundedness: Let $\lambda_{\ell} = (\mu_{\ell}, \nu_{\ell}) \ (\ell = 1, 2, ..., n)$ be FFNs such that $\lambda^- \prec FFSWWA \ (\lambda_1, \lambda_2, ..., \lambda_n) \prec \lambda^+$, where $\lambda^- = \left(\min_{\ell} \mu_{\ell}, \max_{\ell} \nu_{\ell}\right)$ and $\lambda^+ = \left(\max_{\ell} \mu_{\ell}, \min_{\ell} \nu_{\ell}\right)$.
- (iv) Monotonicity: Let $\lambda_\ell = (\mu_\ell, \, \nu_\ell)$ and $\zeta_\ell = (\tilde{\mu}_\ell, \, \tilde{\nu}_\ell)$ $(\ell = 1, \, 2, \, ..., \, n)$ be two FFNs such that $\mu_\ell \leq \tilde{\mu}_\ell$ and $\nu_\ell \geq \tilde{\nu}_\ell$. Then $FFSWWA(\lambda_1, \, \lambda_2, ..., \, \lambda_n) \prec FFSWWA(\zeta_1, \, \zeta_2, ..., \, \zeta_n)$.

FFSWOWA operator

Definition 9 Let $\lambda_{\ell}=(\mu_{\ell},\nu_{\ell})$ $(\ell=1,2,...,n)$ be a collection of FFNs and $\varpi=\{\varpi_1,\varpi_2,...,\varpi_n\}$ be the weights' set of λ_{ℓ} $(\ell=1,2,...,n)$, wherein ϖ_j is positive and $\varpi_1+\varpi_2+...+\varpi_n=1$. The FFSWOWA is a function $FFSWOWA: \Lambda^n \to \Lambda$, given by.

$$FFSWOWA(\lambda_1, \lambda_2, ..., \lambda_n) = \bigoplus_{\ell=1}^n \left(\varpi_\ell \lambda_{\sigma(\ell)} \right).$$
 (13)

Theorem 3 The aggregated value using FFSWOWA operator is again a FFN and defined as.

$$FFSWOWA(\lambda_1, \lambda_2, ..., \lambda_n) = \bigoplus_{\ell=1}^n \left(\varpi_\ell \lambda_{\sigma(\ell)} \right) = \varpi_1 \lambda_{\sigma(1)} \oplus \varpi_2 \lambda_{\sigma(2)} \oplus ... \oplus \varpi_n \lambda_{\sigma(n)}$$

$$= \left(\sqrt[3]{\frac{1+\varsigma}{\varsigma}\left(1-\prod_{\ell=1}^{n}\left(1-\left(\mu_{\sigma(\ell)}^{3}\right)\frac{\varsigma}{1+\varsigma}\right)^{\varpi_{\ell}}\right)}, \sqrt[3]{\frac{1}{\varsigma}\left((1+\varsigma)\prod_{\ell=1}^{n}\left(\frac{\varsigma\left(\nu_{(\ell)}^{3}\right)+1}{1+\varsigma}\right)^{\varpi_{\ell}}-1\right)}\right). \quad (14)$$

Proof Follow as Theorem 2.

- Property 2
- (i) Shift invariance: Let $\lambda_{\ell}=(\mu_{\ell},\,\nu_{\ell})\;(\ell=1,\,2,\,...,\,n)$ be the FFNs. For a FFN $\lambda_{0}=(\mu_{0},\,\nu_{0})\,,(\neq\lambda_{\ell})\,,$ we have $FFSWOWA\,(\lambda_{0}\oplus\lambda_{1},\,\lambda_{0}\oplus\lambda_{2},...,\,\lambda_{0}\oplus\lambda_{n})=\lambda_{0}\oplus FFSWOWA\,(\lambda_{1},\,\lambda_{2},...,\,\lambda_{n})\,.$
- (ii) Idempotency: Let $\lambda_{\ell}=(\mu_{\ell},\,\nu_{\ell})\;(\ell=1,\,2,\,...,\,n)$ be the FFNs such that $\lambda_{\ell}=\lambda_{0},\,\forall\,\ell$. Then $FFSWOWA(\lambda_{1},\,\lambda_{2},...,\,\lambda_{n})=\lambda_{0}.$
- (iii) Boundedness: Let $\lambda_{\ell} = (\mu_{\ell}, \nu_{\ell}) \ (\ell = 1, 2, ..., n)$ be the FFNs such that $\lambda^- \prec FFSWOWA \ (\lambda_1, \lambda_2, ..., \lambda_n) \prec \lambda^+, \quad \text{where} \quad \lambda^- = \left(\min_{\ell} \mu_{\ell}, \max_{\ell} \nu_{\ell}\right) \quad \text{and} \quad \lambda^+ = \left(\max_{\ell} \mu_{\ell}, \min_{\ell} \nu_{\ell}\right).$
- (iv) Monotonicity: Let $\lambda_{\ell} = (\mu_{\ell}, \nu_{\ell})$ and $\zeta_{\ell} = (\tilde{\mu}_{\ell}, \tilde{\nu}_{\ell}) \ (\ell = 1, 2, ..., n)$ be the FFNs such that $\mu_{\ell} \leq \tilde{\mu}_{\ell}$ and $\nu_{\ell} \geq \tilde{\nu}_{\ell}$. Then $FFSWOWA (\lambda_{1}, \lambda_{2}, ..., \lambda_{n}) \prec FFSWOWA (\zeta_{1}, \zeta_{2}, ..., \zeta_{n})$.

An integrated FF-modified SPC-MARCOS method

In the section, we extend the classical MARCOS method into FFS environment along with the Sugeno-Weber weighted AO, RS model-based DEs' weighting tool and criteria weighting model based on FF-SPC and FF-

RS tools. For a FF-information-based MCDM problem, create an expert committee $G = \{g_1, g_2, ..., g_n\}$ to opt a suitable alternative amongst a set of options $F = \{f_1, f_2, ..., f_p\}$ based on the attributes/criteria set $V = \{v_1, v_2, ..., v_q\}$. Each DE presents his/her linguistic opinion regarding the performance of each alternative concerning each criterion. Consequently, we obtain a linguistic assessment matrix (LAM) $X = \left(x_{ij}^{(k)}\right)$, $k = 1, 2, \ldots, n$ and further, we form a FF-decision matrix. The procedure of the developed FF-distance-based SPC-RS-MARCOS framework is presented in the following way (see Fig. 1):

Step 1: Find the weights of DEs.

Let $g_k = (\mu_k, \nu_k)$, $\bar{k} = 1, 2, ..., n$ be the FFN to describe significance degree of kth DE. To calculate the numeric weight of kth expert, we present the following procedure.

Step 1a: The normalized score rating of FFN associated to the DE is computed as follows:

$$\Phi_k^1 = \frac{\mu_k^3 \left(2 - \mu_k^3 - \nu_k^3\right)}{\sum\limits_{k=1}^n \left(\mu_k^3 \left(2 - \mu_k^3 - \nu_k^3\right)\right)}, \quad k = 1, 2, ..., n.$$
(15)

Step 1b: Prioritize the DEs and find the weight of DE as $n-\rho_k+1$, wherein ρ_k is prioritization of kth expert. The weight of kth DE is estimated as

$$\Phi_k^2 = \frac{n - \rho_k + 1}{\sum\limits_{k=1}^n (n - \rho_k + 1)}, \ k = 1, 2, ..., n.$$
(16)

Step 1c: Compute the kth DE's weight by integrating Eqs. (15) and (16).

$$\Phi_k = \frac{1}{2} \left(\Phi_k^1 + \Phi_k^2 \right), \ k = 1, 2, ..., n.$$
 (17)

Clearly, $\Phi_k \in [0, 1]$ and $\Phi_1 + \Phi_2 + ... + \Phi_n = 1$.

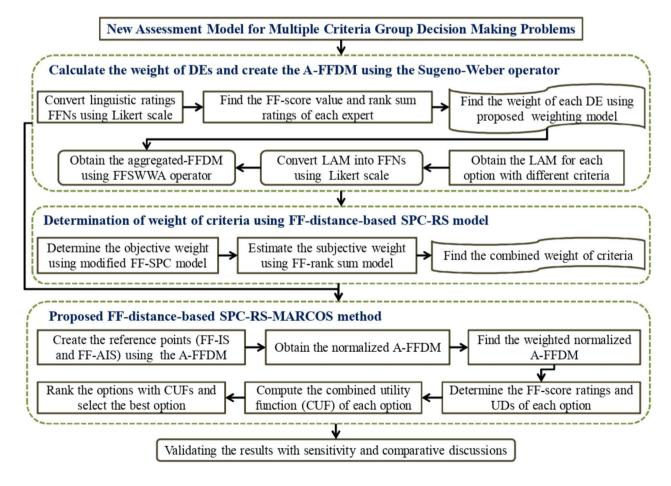


Fig. 1. The proposed group decision-making model.

Step 2: Form an "aggregated FF decision matrix (A-FFDM)".

To obtain the mutual opinion by combining different experts' opinions, we apply the proposed FFSWWA operator (or FFSWOWA operator) to create an A-FFDM $X_A = (\tilde{x}_{ij})_{n \times n}$, where

$$\tilde{x}_{ij} = FFSWWA_{\Phi_k} \left(\tilde{x}_{ij}^{(1)}, \, \tilde{x}_{ij}^{(2)}, ..., \tilde{x}_{ij}^{(n)} \right) \\
= \left(\sqrt[3]{\frac{1+\varsigma}{\varsigma} \left(1 - \prod_{k=1}^{n} \left(1 - \left(\mu_{ij}^{3(k)} \right) \frac{\varsigma}{1+\varsigma} \right)^{\Phi_k} \right)}, \, \sqrt[3]{\frac{1}{\varsigma} \left((1+\varsigma) \prod_{k=1}^{n} \left(\frac{\varsigma \left(\nu_{ij}^{3(k)} \right) + 1}{1+\varsigma} \right)^{\Phi_k} - 1 \right)} \right). \tag{18}$$

Step 3: Determination of attributes' weights by introduced FF-modified SPC-RS model.

In the following, we present a weighting model with the combination of objective weight via FF-modified SPC model and subjective weight by FF-RS tool within the context of FFSs. Let $w=(w_1,\,w_2,\,...,\,w_q)^T$ be the weight vector of factors, where w_j is non-negative and $w_1+w_2+...+w_q=1$. Next, we discuss the procedure for computing the weights of criteria, given as follows:

Case I: Objective weighting through the FF-SPC model.

The FF-modified SPC method comprises the following procedure:

Step 3.1: Computation of symmetry point value (SPV) of criteria.

Let α_{ij} be an aggregated Fermatean fuzzy value of criterion over diverse options. Then the SPV of jth attribute is computed by

$$\alpha_j = (\mu_j, \ \nu_j) = \frac{1}{p} \mathop{\oplus}_{i=1}^p \alpha_{ij}.$$
 (19)

Step 3.2: Estimate the FF-distance matrix.

The FF-distance matrix $D=(d_{ij})_{p\times q}$ is determined by computing the distance between aggregated FFN and SPV of attribute using Eq. (20).

$$d(\alpha_{ij}, \alpha_j) = \sqrt{\left(\frac{\mu_{ij}^3 - \nu_{ij}^3}{2} - \frac{\mu_j^3 - \nu_j^3}{2}\right)^2 + \frac{1}{3}\left(\frac{\mu_{ij}^3 + \nu_{ij}^3}{2} - \frac{\mu_j^3 + \nu_j^3}{2}\right)^2}.$$
 (20)

Step 3.3: Compute the matrix of the modulus of symmetry (MoS).

With the use previous steps, the MoS matrix $X = (x_{ij})_{n \times n}$ is derived as

$$x_{ij} = \frac{\sum_{i=1}^{p} d_{ij}}{p.S\left(\alpha_{ij}\right)}, \forall i, j.$$

$$(21)$$

Step 3.4: Determine the average of matrix of MoS.

Take average of each column of matrix of MoS and obtain an average of matrix of the modulus of symmetry point $C = (c_j)_{1 \times n}$ of jth attribute, which is defined by

$$c_j = \frac{1}{p} \sum_{i=1}^p x_{ij}, \ j = 1, 2, ..., q.$$
 (22)

Step 3.5: Derive the objective weight of attribute using Eq. (23).

$$w_j^o = \frac{c_j}{\sum_{j=1}^q c_j}, \ j = 1, 2, ..., q.$$
 (23)

Case II: Obtain the subjective weight through FF-RS model.

Step 3.6: Using FFSWWA operator, aggregate the Fermatean fuzzy numbers provided by the DEs and obtain an aggregated number.

Step 3.7: Assess the FF-score rating of aggregated FFN using Eq. (1).

Step 3.8: Determine the rating $(q - rank_j + 1)$, of criterion, where $rank_j$ is the ranking position of jth attribute. The subjective weight of jth attribute is estimated as

$$w_j^s = \frac{q - rank_j + 1}{\sum_{j=1}^q (q - rank_j + 1)}, \ j = 1, 2, ..., q.$$
(24)

Case III: Estimation of integrated weight of criteria.

Here, we combine the objective and subjective weights estimated by the FF-modified SPC and the FF-RS models, respectively. The process of determining the overall weight of each criterion is given as

$$w_j = \gamma w_j^o + (1 - \gamma) w_j^s, \ j = 1, 2, ..., q.$$
 (25)

where $\gamma \in [0, 1]$ is a weight strategy coefficient.

Step 4: Find the ideal and anti-ideal ratings from A-FFDM as

$$\alpha_j^+ = \left(\mu_j^+, \nu_j^+\right) = \begin{cases} \left(\max_i \mu_{ij}, \min_i \nu_{ij}\right), & j \in v_b, \\ \left(\min_i \mu_{ij} \max_i \nu_{ij}\right), & j \in v_n. \end{cases}$$
(26)

$$\alpha_j^- = (\mu_j^-, \nu_j^-) = \begin{cases} \left(\min_i \mu_{ij}, \max_i \nu_{ij}\right), & j \in v_b, \\ \left(\max_i \mu_{ij}, \min_i \nu_{ij}\right), & j \in v_n. \end{cases}$$

$$(27)$$

Step 5: Create the normalized A-FFDM. We apply linear normalization to generate the "normalized A-FFDM" $M=(m_{ij})_{v\times a}$ as

$$m_{ij} = (\overline{\mu}_{ij}, \overline{\nu}_{ij}) = \begin{cases} \tilde{x}_{ij} = (\mu_{ij}, \nu_{ij}), & j \in v_b, \\ (\tilde{x}_{ij})^c = (\nu_{ij}, \mu_{ij}), & j \in v_n, \end{cases}$$
(28)

wherein v_h and v_n signify the beneficial and cost categories of attributes.

Step 6: Obtain the weighted normalized A-FFDM (WNA-FFDM).

We obtain the WNA-FFDM $M_w = \left(\widehat{m}_{ij}\right)_{p \times q}$ from normalized A-FFDM by the use of Eq. (29).

$$\widehat{m}_{ij} = \left(\widehat{\mu}_{ij}, \, \widehat{\nu}_{ij}\right) = w_j \oplus m_{ij} = \left(\sqrt[3]{1 - \left(1 - \left(\overline{\mu}_{ij}\right)^3\right)^{w_j}}, \, (\overline{\nu}_{ij})^{w_j}\right). \tag{29}$$

Step 7: Estimation of the score rating of the WNA-FFDM using Eq. (1). Step 8: Obtain the utility degree (UD) of *i*th option through Eq. (30).

$$l_i^- = \frac{O_i}{O_{ais}}$$
 and $l_i^+ = \frac{O_i}{O_{is}}$, (30)

where O_{is} and O_{ais} denote the summation of score ratings of weighted ideal rating α_{jw}^+ and anti-ideal rating α_{jw}^- , correspondingly.

Step 9: Assess the "combined utility function (CUF)".

With the use of Eq. (31), find the CUF of each alternative by means of weighted PIR and NIR.

$$\phi(l_i) = \frac{l_i^+ + l_i^-}{1 + \frac{1 - \phi(l_i^+)}{\phi(l_i^+)} + \frac{1 - \phi(l_i^-)}{\phi(l_i^-)}}, \text{ where } \phi(l_i^+) = \frac{l_i^-}{l_i^- + l_i^+} \text{ and } \phi(l_i^-) = \frac{l_i^+}{l_i^- + l_i^+}, i = 1, 2, ..., p.$$

$$(31)$$

Based on the CUFs, prioritize the options and accordingly choose the appropriate one.

Case study: sustainable supplier (SS) selection in the HSC

This section first shows a case study of SS selection problem and further applies the proposed FF-SPC-RS-MARCOS approach to choose a suitable SS alternative in the HSC. Next, it presents the sensitivity and comparative discussions to approve the superiority and solidity of the developed technique. Lastly, this section discusses the findings of the proposed work.

Background and problem description

To show the performance of introduced FF-modified SPC-RS-MARCOS method, we consider a SS selection for HSC practices of a private healthcare organization, situated in Indore, Madhya Pradesh (India). This organization has been working for more than 15 years and is recognized as a top healthcare leader in Madhya Pradesh because of its well-trained professionals, infrastructure and human resource. To collect the data for assessment and investigation, we conducted in-person meetings with the DEs. Five DEs are agreed to collaborate with us during the preparation of questionnaires and gave their views in making an appropriate decision. In the expert committee, the DEs are having more than 10 years' expertise in the respective fields. Two DEs are from the SCM, one expert is from sustainability and the other two is from healthcare organization. The DEs supported with the scholars during the whole study. They have planned some strategies that can be executed in other MCGDM problems. Based on the literature survey, we have determined 12 criteria, which are denoted as v_1, v_2, \ldots, v_{12} . In this study, we have considered four aspects of criteria including social and environmental, services, economic and logistics for assessing the SSs in the HSC.

After preliminary analysis, the expert committee has selected four SS alternatives, which are SS-1 (f_1) , SS-2 (f_2) , SS-3 (f_3) and SS-4 (f_4) . The SS-1 (f_1) was established for medical equipment distribution including medical

Dimension	Criteria	Sources		
	Training for recycling and reuse (v_1)	Memari et al. ⁶⁷		
	Compliance with environmental protocols (ν_2)	Debnath et al. ⁶³		
Social and environmental	Maintaining safety in operation and material handling (v_3)	Zimmer et al. ⁶⁸ , Luthra et al. ⁶⁹ , Debnath et al. ⁶³		
	Minimize the fuel and energy consumption (v_4)	Stević et al. ²⁵ , Rahman et al. ⁷⁰		
	Appropriate training facility for the workers (ν_5)	Memari et al. ⁶⁷ , Hendiani et al. ⁷¹ , Debnath et al. ⁶³		
Services	Past performance and reputation (v_6)	Baki ⁷² , Debnath et al. ⁶³		
Services	Continuous enhancement and quality control (v_7)	Leong et al. ⁷³ , Hendiani et al. ⁷¹ , Debnath et al. ⁶³		
	Availability of credit system (ν_8)	Debnath et al. ⁶³		
Economic	Flexible size (v_9)	Luthra et al. ⁶⁹		
	Stability of cost (ν_{10})	Leong et al. ⁷³ , Baki ⁷²		
Logistics	Storage management and sustainable inventory (ν_{11})	Fallahpour et al. ⁷⁴		
Logistics	Source of capability and availability degree (v_{12})	Debnath et al. ⁶³		

Table 2. Details of chosen criteria for SS selection in HSCs.

LVs	FFNs
Very very high (VVH)	(0.95, 0.20)
Very high (VH)	(0.80, 0.35)
High (H)	(0.70, 0.45)
Medium high (MH)	(0.60, 0.55)
Average (A)	(0.50, 0.60)
Medium low (ML)	(0.40, 0.70)
Low (L)	(0.30, 0.75)
Very low (VL)	(0.20, 0.85)
Very very low (VVL)	(0.10, 0.95)

Table 3. LVs and corresponding FFNs.

disposables, hospital medical furniture, rehabilitation products etc. The SS-2 (f_2) was formed to monitor, repair and maintenance of medical equipment with an adequate level of service and limiting downtime of medical devices. The SS-3 (f_3) was established to import and distribute medical devices including gauze bandage, disposable syringe, Electro Cardio Graphic (ECG) recording paper, ultrasound gel and paper etc. The SS-4 (f_4) was formed to perform servicing, distribution and maintenance of electronic equipment in the healthcare organization. Table 2 presents the list of criteria obtained from online questionnaire and experts' opinions for SS selection in HSCs. This case study is presented for the demonstration purpose of choosing the suitable suppliers, which proves the applicability of the developed approach. Readers may diminish or add some attributes as per their requirements.

Here, we apply the proposed FF-modified SPC-RS-MARCOS model on the aforesaid case study for selecting the best SS over multiple criteria in the healthcare supply chain. The DEs have assessed the relative significance of chosen alternative by means of each criterion using linguistic values (LVs). Table 3 defines nine-point Likert scale from Mishra and Rani⁷⁵ to assess the SSs in the HSC. Table 4 presents the LAM by five DEs for assessing the performance value of each option f_i over diverse defined criteria.

Implementation process of introduced approach

In this section, we apply the proposed approach for assessing the SSs with respect to considered evaluation criteria. Here, the computational process of the developed methodology is given as below:

Step 1: Based on the expertise and knowledge of the DEs, we firstly provide the linguistic performance value of each DE using Table 3. Further, utilizing Table 3 and Eq. (15)–Eq. (17), the weight of each DE is estimated and mentioned in Table 5.

Step 2: By utilizing Eq. (18) and Tables 3, 4 and 5, the A-FFDM is made and shown in Table 6, in which each aggregated element is a Fermatean fuzzy number.

Step 3: To find the objective criteria weights, we use the FF-modified SPC model given by Eqs. (19)–(23).

Step 3.1: From Eq. (19), we compute the SPV α_j (j=1, 2,..., 12) of each attribute, which is given as α_1 = (0.795, 0.451), α_2 = (0.807, 0.432), α_3 = (0.890, 0.344), α_4 = (0.884, 0.339), α_5 = (0.869, 0.345), α_6 = (0.876, 0.344), α_7 = (0.811, 0.478), α_8 = (0.833, 0.443), α_9 = (0.684, 0.518), α_{10} = (0.710, 0.525), α_{11} = (0.711, 0.549) and α_{12} = (0.856, 0.379).

Step 3.2: With the help of Eq. (20), we compute the distance matrix and given in Table 7.

Step 3.3: Further, we create MoS from SPV of criterion with Eq. (21). For this purpose, we first need to determine the score value of each aggregated FFN of Table 6, and shown in Table 8.

Attributes	f_1	f_2	f_3	f_4
ν_1	H,VVH,H,H,H	A,A,A,H,A	H,VVH,H,VVH,H	A,H,VVH,VVH,VVH
ν_2	A,H,H,A,A	VVH,H,H,VVH,VVH	H,H,VVH,H,VVH	A,VVH,H,H,H
ν_3	VVH,VVH,H,VVH,VH	VVH,VVH,H,VVH,H	VVH,VVH,H,H,H	A,VVH,H,VVH,H
v_4	VVH,VVH,H,VH,VVH	VVH,H,MH,VVH,H	H,H,VVH,VH,VVH	VVH,VVH,VH,VVH,VH
v_5	Н,Н,Н,VН	VVH,VH,VVH,VH,H	VVH,VVH,H,VH,VH	VVH,VVH,VVH,H,H
v_6	H,VVH,H,H,VVH	VVH,H,VVH,VVH,VH	H,H,H,VVH,VVH	VH,VVH,VVH,VH,VVH
v_7	A,VVH,L,A,A	VVH,A,H,A,VVH	VVH,A,H,VVH,L	VVH,H,VVH,ML,A
ν_8	A,VVH,VVH,A,A	VVH,A,VVH,A,A	A,H,VVH,VVH,A	H,VVH,VVH,H,VVH
ν_9	L,A,A,A,H	Н,Н,А,Н	A,A,H,A,H	A,VVH,H,H,H
ν_{10}	A,A,H,A,A	A,H,H,A,A	A,VVH,L,A,A	H,A,VVH,A,VVH
ν ₁₁	A,A,L,A,VVH	A,L,A,A,H	VVH,L,A,VVH,A	H,H,VVH,VVH,A
ν ₁₂	VVH,VH,A,H,VVH	VH,VVH,VVH,VVH,A	Н,Н,А,Н,Н	VVH,VVH,H,VVH,H

Table 4. The LAM created by a set of five DEs.

DEs	LVs	Φ_k^1	ρ_k	Φ_k^2	Φ_k
g_1	VH	0.2636	2	0.2667	0.2652
g_2	VVH	0.3466	1	0.3333	0.3399
g_3	MH	0.1245	4	0.1333	0.1289
g_4	A	0.0739	5	0.0667	0.0703
g ₅	Н	0.1914	3	0.20	0.1957

Table 5. Weight of DEs for the SS selection in the HSC.

Attributes	f_1	$ f_2 $	f_3	$ f_4 $
ν_1	(0.834, 0.389)	(0.522, 0.590)	(0.852, 0.375)	(0.832, 0.426)
ν_2	(0.616, 0.532)	(0.879, 0.349)	(0.829, 0.392)	(0.815, 0.436)
ν_3	(0.913, 0.286)	(0.906, 0.314)	(0.894, 0.332)	(0.836, 0.424)
ν_4	(0.928, 0.267)	(0.827, 0.405)	(0.834, 0.384)	(0.918, 0.265)
ν_5	(0.724, 0.432)	(0.870, 0.331)	(0.905, 0.296)	(0.916, 0.299)
v_6	(0.880, 0.348)	(0.875, 0.341)	(0.812, 0.403)	(0.917, 0.266)
v_7	(0.780, 0.520)	(0.840, 0.440)	(0.787, 0.513)	(0.830, 0.435)
ν_8	(0.835, 0.458)	(0.807, 0.482)	(0.746, 0.491)	(0.904, 0.317)
ν_9	(0.529, 0.612)	(0.691, 0.461)	(0.586, 0.553)	(0.815, 0.436)
ν_{10}	(0.538, 0.582)	(0.616, 0.532)	(0.780, 0.520)	(0.799, 0.463)
ν ₁₁	(0.698, 0.563)	(0.521, 0.623)	(0.770, 0.554)	(0.772, 0.449)
ν_{12}	(0.875, 0.353)	(0.884, 0.358)	(0.682, 0.471)	(0.906, 0.314)

Table 6. The A-FFDM for the SS selection.

Step 3.4: Considering Eq. (22), taking the average of each column of matrix of MoS, we obtain an average of matrix of the modulus of symmetry point and given in the last second column of Table 8.

Step 3.5: Lastly, we find the objective weight of criterion via Eq. (23), presented as $w_1^o = 0.1271$, $w_2^o = 0.0942$, $w_3^o = 0.0358$, $w_4^o = 0.0655$, $w_5^o = 0.0738$, $w_6^o = 0.0381$, $w_7^o = 0.0437$, $w_8^o = 0.0696$, $w_9^o = 0.1335$, $w_{10}^o = 0.1266$, $w_{11}^o = 0.1060$ and $w_{12}^o = 0.0862$. Table 8 presents the required results of FF-modified SPC model. Step 3.6: For the subjective weights, the DEs firstly assign the linguistic assessment ratings of criteria using

Step 3.6: For the subjective weights, the DEs firstly assign the linguistic assessment ratings of criteria using Table 3. On the basis of aggregation operator, we aggregate the individual opinions of DEs and obtain an aggregated Fermatean fuzzy performance value of each criterion.

Step 3.7: By utilizing Eq. (1), we compute the score value of aggregated Fermatean fuzzy performance value of each criterion and accordingly rank the performance value. The highest score value determines a criterion with first rank.

Step 3.8: In accordance with Eq. (24), we determine the subjective weight of each criterion, given as $w_1^s = 0.0513, w_2^s = 0.0769, w_3^s = 0.0385, w_4^s = 0.0128, w_5^s = 0.0897, w_6^s = 0.1538, w_7^s = 0.1410, w_8^s = 0.0641, w_9^s = 0.0256, w_{10}^s = 0.1282, w_{11}^s = 0.1026$ and $w_{12}^s = 0.1154$. Table 9 presents the required outcomes of FF-RS model.

Attributes	Distan	ice mati	rix	
	f_1	f_2	f_3	f_4
v_1	0.056	0.248	0.081	0.046
v_2	0.192	0.102	0.033	0.009
v_3	0.038	0.025	0.006	0.083
v_4	0.068	0.082	0.070	0.054
v_5	0.173	0.004	0.053	0.068
v_6	0.006	0.001	0.087	0.064
v_7	0.046	0.043	0.037	0.033
ν_8	0.005	0.040	0.105	0.112
v_9	0.133	0.027	0.079	0.147
v_{10}	0.134	0.074	0.068	0.104
v_{11}	0.016	0.153	0.055	0.088
ν ₁₂	0.028	0.039	0.195	0.075

Table 7. Resulting matrix of absolute distances.

	Score	values			MoS matrix					
Attributes	f_1	f_2	f_3	f_4	f_1	f_2	f_3	f_4	c_j	w_j^o
ν_1	0.760	0.468	0.783	0.749	0.142	0.230	0.138	0.144	0.163	0.1271
ν_2	0.541	0.819	0.755	0.730	0.155	0.103	0.111	0.115	0.121	0.0942
ν_3	0.869	0.856	0.839	0.754	0.044	0.044	0.045	0.050	0.046	0.0358
v_4	0.890	0.750	0.762	0.878	0.077	0.091	0.090	0.078	0.084	0.0655
v_5	0.649	0.812	0.857	0.870	0.115	0.092	0.087	0.086	0.095	0.0738
v_6	0.820	0.815	0.735	0.876	0.048	0.048	0.054	0.045	0.049	0.0381
v_7	0.667	0.754	0.676	0.745	0.060	0.053	0.059	0.053	0.056	0.0437
ν_8	0.743	0.707	0.648	0.854	0.088	0.092	0.101	0.076	0.089	0.0696
ν_9	0.459	0.616	0.516	0.730	0.210	0.157	0.187	0.132	0.171	0.1335
v ₁₀	0.480	0.541	0.667	0.706	0.198	0.175	0.142	0.135	0.163	0.1266
ν ₁₁	0.581	0.449	0.644	0.685	0.135	0.174	0.122	0.114	0.136	0.1060
ν ₁₂	0.813	0.823	0.607	0.856	0.104	0.102	0.139	0.098	0.111	0.0862

Table 8. Score values, MoS matrix and objective criteria weights.

Step 3.9: By means of Eq. (25), we combine the FF-modified SPC and the FF-RS tools and compute the collective weight ($\gamma=0.5$) of each criterion during the assessment of SSs in the HSC. The collective weights' set of attributes is {0.0892, 0.0855, 0.0372, 0.0392, 0.0818, 0.0959, 0.0923, 0.0668, 0.0795, 0.1274, 0.1043, 0.1008}.

Step 4: Apply Eqs. (26), (27) on Table 6 to find the Fermatean fuzzy ideal and anti-ideal ratings during the evaluation of SSs in HSCs, given as $\alpha_j^+ = \{(0.852, 0.375), (0.879, 0.349), (0.913, 0.286), (0.928, 0.267), (0.916, 0.299), (0.917, 0.266), (0.84, 0.44), (0.904, 0.317), (0.815, 0.436), (0.799, 0.463), (0.772, 0.449), (0.906, 0.314)\}$ and $\alpha_j^- = \{(0.522, 0.59), (0.616, 0.532), (0.836, 0.424), (0.827, 0.405), (0.724, 0.432), (0.812, 0.403), (0.78, 0.52), (0.746, 0.491), (0.529, 0.612), (0.538, 0.582), (0.521, 0.623), (0.682, 0.471)\}.$

Steps 5–6: As all the attributes belong to benefit category, thus, it is not needed to form normalized A-FFDM with Eq. (28). Further, using Eq. (29), criteria weights and Table 6, the WNA-FFDM is created and mentioned in Table 10.

Step 7: In accordance with Eq. (1) and Table 10, the FF-score ratings of each alternative, FF-PIR and FF-NIR for SSs evaluation in the HSCs are made and mentioned in Table 11.

Steps 8–9: Based on Eq. (30), we find the UDs of options as $l_1^+ = 0.751$, $l_2^+ = 0.785$, $l_3^+ = 0.776$, $l_4^+ = 0.953$, $l_1^- = 1.257$, $l_2^- = 1.315$, $l_3^- = 1.299$, $l_4^- = 1.596$ and further we determine CUFs using Eq. (31), presented as $\phi(l_1) = 0.614$, $\phi(l_2) = 0.642$, $\phi(l_3) = 0.634$ and $\phi(l_4) = 0.779$. Thus, the ranking order of SS options is $f_4 \succ f_2 \succ f_3 \succ f_1$ and the SS-4 (f_4) is best choice with the maximum CUF for among the other options for SSs evaluation in the HSC.

Sensitivity investigation

In the subsection, we discuss sensitivity investigation to check the impact of variation in attributes' weights on the final results. We study changes in CUF ratings and preferences of sustainable supplier over changing the weights of factors from objective to subjective weights with the "FF-modified SPC-RS" model. The prioritization order of suppliers in the HSC are obtained over the objective, the combined and the subjective weight of criteria with the FF-modified SPC-RS models and are discussed in Table 12 and Fig. 2. This analysis can validate the

Criteria	g_1	g_2	g_3	g_4	g ₅	A-FFNs	Score value	rank _j	$(q-rank_j+1)$	w_j^s
ν_1	Н	MH	VH	A	L	(0.636, 0.546)	0.547	9	4	0.0513
v_2	VH	MH	Н	VL	MH	(0.677, 0.513)	0.588	7	6	0.0769
ν_3	L	Н	VH	Н	A	(0.634, 0.553)	0.543	10	3	0.0385
v_4	VVL	A	Н	Н	VH	(0.615, 0.619)	0.497	12	1	0.0128
v_5	Н	MH	ML	VH	VH	(0.686, 0.497)	0.6	6	7	0.0897
v_6	VVH	VH	МН	A	Н	(0.825, 0.398)	0.749	1	12	0.1538
v_7	ML	VVH	Н	ML	Н	(0.804, 0.486)	0.703	2	11	0.141
ν_8	A	Н	A	VH	MH	(0.632, 0.525)	0.554	8	5	0.0641
v_9	MH	Н	МН	A	ML	(0.612, 0.551)	0.531	11	2	0.0256
v_{10}	MH	MH	Н	VH	VVH	(0.761, 0.475)	0.666	3	10	0.1282
ν ₁₁	VVH	A	Н	MH	A	(0.762, 0.552)	0.638	5	8	0.1026
ν_{12}	VH	Н	ML	МН	VH	(0.731, 0.453)	0.649	4	9	0.1154

Table 9. Results of the FF-RS tool.

Attributes	f_1	f_2	f_3	f_4	α_{jw}^+	α_{jw}^-
v_1	(0.420, 0.919)	(0.239, 0.954)	(0.435, 0.916)	(0.419, 0.927)	(0.435, 0.916)	(0.239, 0.954)
v_2	(0.282, 0.947)	(0.453, 0.914)	(0.412, 0.923)	(0.401, 0.931)	(0.453, 0.914)	(0.282, 0.947)
v_3	(0.373, 0.955)	(0.367, 0.958)	(0.357, 0.960)	(0.318, 0.969)	(0.373, 0.955)	(0.318, 0.969)
v_4	(0.393, 0.950)	(0.318, 0.965)	(0.322, 0.963)	(0.384, 0.949)	(0.393, 0.950)	(0.318, 0.965)
v_5	(0.337, 0.934)	(0.438, 0.914)	(0.471, 0.905)	(0.483, 0.906)	(0.483, 0.906)	(0.337, 0.934)
v_6	(0.470, 0.904)	(0.465, 0.902)	(0.414, 0.916)	(0.509, 0.881)	(0.509, 0.881)	(0.414, 0.917)
v_7	(0.386, 0.941)	(0.431, 0.927)	(0.391, 0.940)	(0.423, 0.926)	(0.430, 0.927)	(0.386, 0.941)
ν_8	(0.384, 0.949)	(0.365, 0.952)	(0.328, 0.954)	(0.441, 0.926)	(0.441, 0.926)	(0.328, 0.954)
v_9	(0.233, 0.962)	(0.315, 0.940)	(0.261, 0.954)	(0.392, 0.936)	(0.392, 0.936)	(0.233, 0.962)
v_{10}	(0.278, 0.933)	(0.322, 0.923)	(0.428, 0.920)	(0.443, 0.906)	(0.443, 0.907)	(0.277, 0.933)
v ₁₁	(0.349, 0.942)	(0.251, 0.952)	(0.395, 0.940)	(0.396, 0.920)	(0.396, 0.920)	(0.251, 0.952)
v ₁₂	(0.473, 0.900)	(0.482, 0.902)	(0.336, 0.927)	(0.504, 0.890)	(0.504, 0.890)	(0.335, 0.927)

Table 10. The WNA-FF-DM for SSs evaluation in the HSC.

Attributes	f_1	f_2	f_3	f_4	α_{jw}^+	α_{jw}^-
ν_1	0.149	0.073	0.157	0.139	0.157	0.073
ν_2	0.086	0.165	0.142	0.128	0.165	0.086
ν_3	0.091	0.085	0.081	0.062	0.091	0.062
v_4	0.102	0.066	0.070	0.101	0.102	0.066
v_5	0.112	0.161	0.181	0.184	0.185	0.112
v_6	0.183	0.184	0.151	0.224	0.224	0.151
v_7	0.112	0.142	0.114	0.141	0.141	0.112
ν_8	0.101	0.092	0.084	0.146	0.146	0.084
ν_9	0.062	0.100	0.075	0.120	0.120	0.062
ν_{10}	0.104	0.124	0.150	0.171	0.171	0.104
ν ₁₁	0.103	0.077	0.115	0.142	0.142	0.077
ν_{12}	0.188	0.189	0.121	0.212	0.212	0.121

Table 11. FF-score ratings of each option for SS selection in HSCs.

superiority and steadiness of the developed technique to assess the SS selection in HSCs. In the following, we present the subsequent cases:

Case-I (Objective weight shuffling): In this case, we consider the FF-modified SPC approach for finding the objective weight of each attribute, i.e., $\gamma = 1.0$ in Eq. (32) during the assessment of SS selection in the HSC. Using the FF-modified SPC, the CUF degree of each option is computed and given as $\phi_1 = 0.616$, $\phi_2 = 0.628$, $\phi_3 = 0.644$ and $\phi_4 = 0.728$. On the basis of increasing values of CUF degrees, the prioritization order of SS

	CUFs: HSCs	for SS s	election		
Models	ϕ_{1}	ϕ_2	ϕ_3	ϕ_{4}	Ranks of SS alternatives
FF-RS for subjective weight ($\gamma = 0.0$)	0.612	0.657	0.628	0.775	$f_4 \succ f_2 \succ f_3 \succ f_1$
y = 0.1	0.612	0.654	0.629	0.776	$f_4 \succ f_2 \succ f_3 \succ f_1$
y=0.2	0.612	0.651	0.630	0.777	$f_4 \succ f_2 \succ f_3 \succ f_1$
y=0.3	0.613	0.648	0.631	0.778	$f_4 \succ f_2 \succ f_3 \succ f_1$
$\gamma = 0.4$	0.613	0.645	0.633	0.779	$f_4 \succ f_2 \succ f_3 \succ f_1$
FF-SPC-RS for Integrated weight ($\gamma = 0.5$)	0.614	0.642	0.634	0.779	$f_4 \succ f_2 \succ f_3 \succ f_1$
$\gamma = 0.6$	0.614	0.639	0.636	0.780	$f_4 \succ f_2 \succ f_3 \succ f_1$
y = 0.7	0.615	0.636	0.638	0.781	$f_4 \succ f_3 \succ f_2 \succ f_1$
$\gamma = 0.8$	0.615	0.633	0.640	0.781	$f_4 \succ f_3 \succ f_2 \succ f_1$
y = 0.9	0.615	0.631	0.642	0.782	$f_4 \succ f_3 \succ f_2 \succ f_1$
FF-SPC for objective weight (γ = 1.0)	0.616	0.628	0.644	0.782	$f_4 \succ f_3 \succ f_2 \succ f_1$

Table 12. The CUFs of SS selection in HSCs with different models.

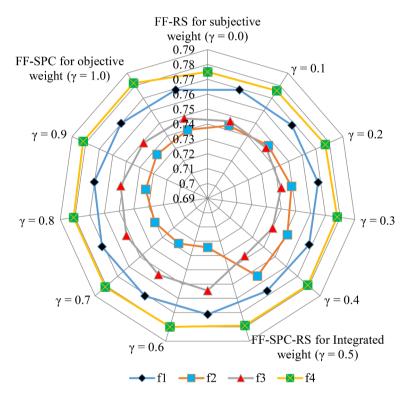


Fig. 2. Changes in CUF ratings of SS selection in HSCs with different models.

selection in HSCs is $f_4 > f_3 > f_2 > f_1$. Hence, the "SS-4 (f_4) " is the most optimal SS in HSCs. Table 12 and Fig. 2 presents the required results of the sensitivity investigation.

Case-II (Subjective weight shuffling): In this case, we consider the FF-rank sum (RS) model to compute the subjective weight of each attribute, i.e., $\gamma = 0.0$ in Eq. (32) during the assessment of SS selection in HSCs. Using the FF-RS, the CUF of each option is computed and given as $\phi_1 = 0.612$, $\phi_2 = 0.657$, $\phi_3 = 0.628$ and $\phi_4 = 0.775$. Based on increasing values of CUFs, the preference order of SS selection in HSCs is $f_4 \succ f_2 \succ f_3 \succ f_1$. Hence, the "SS-4 (f_4)" is the most optimal SS in HSCs.

Case-III (Integrated weight shuffling): Here, we consider an integrated FF-modified SPC-RS procedure to find the final weight of factors, i.e., $\gamma=0.5$ in Eq. (32) during the assessment of SS selection in HSCs. Using the proposed FF-modified-RS model, the CUF of each option is calculated and given as $\phi_1=0.614$, $\phi_2=0.642$, $\phi_3=0.634$ and $\phi_4=0.779$. By means of increasing values of comprehensive degrees, the prioritization order of SS selection in HSCs is $f_4 > f_2 > f_3 > f_1$. Here, we can easily be observed that the "SS-4 (f_4)" is the most optimal SS in HSC. Considering the above-mentioned investigation (see Table 12 and Fig. 2), it can be observed that by changing the values of weighting parameter (γ) will improve the performance of developed FF-distance-based-SPC-RS-MARCOS ranking framework.

Comparative study

In the following, we compare the developed FF-modified SPC-RS-MARCOS and extant MCGDM approaches in the context of FFSs. In this regard, some MCGDM methods are chosen, which are Mishra and Rani's WASPAS⁷⁵, Gül's ARAS⁷⁶, Simić et al.'s CoCoSo⁷⁷ and Senapati and Yager's TOPSIS¹⁵, to deal with the aforementioned case problem.

FF-WASPAS model

The FF-WASPAS technique has been used for choosing most suitable SS in the HSC. After applying this model, the additive relative importance of each alternative is determined as (0.788, 0.453), (0.801, 0.437), (0.801, 0.445) and (0.861, 0.377), respectively, and the corresponding score values are 0.698, 0.715, 0.713 and 0.792. Next, the multiplicative relative importance of each alternative SS is estimated as (0.727, 0.492), (0.736, 0.476), (0.778, 0.469) and (0.848, 0.399), respectively, and the corresponding score values are 0.633, 0.646, 0.684 and 0.773. Further, the total relative importance (or UD) of each candidate SS is computed as 0.6981, 0.7154, 0.7131 and 0.7924, respectively. Therefore, the ranking of option for SS selection in HSCs is $f_4 \succ f_2 \succ f_3 \succ f_1$ and the SS-4 (f_4) is the best one with maximum UD among the others for SSs evaluation in the HSC.

FF-ARAS model

The FF-ARAS model has been applied to aforesaid case study. Using this model, the optimal alternative rating is determined as $h_0 = \{(0.852,\ 0.375),\ (0.879,\ 0.349),\ (0.913,\ 0.286),\ (0.928,\ 0.267),\ (0.916,\ 0.299),\ (0.917,\ 0.266),\ (0.84,\ 0.44),\ (0.904,\ 0.317),\ (0.815,\ 0.436),\ (0.799,\ 0.463),\ (0.772,\ 0.449),\ (0.906,\ 0.314)\}.$ Next, the UD of each alternative SS is computed as 0.7507, 0.7854, 0.7760 and 0.9534, respectively. Based on the UD (Q_j) , the prioritization of SS in HSCs is $f_4 \succ f_2 \succ f_3 \succ f_1$ and thus, the SS-4 (f_4) is the ideal one among the others for the SSs evaluation in the HSC.

FF-CoCoSo model

The FF-CoCoSo method has been used to evaluate the SS candidates in the HSC. Firstly, the additive and multiplicative relative importance values of alternatives are determined as FF-WASPAS model⁷⁵. Next, the relative compromise rating of each alternative SS is calculated as $b_1^{(1)} = 0.235$, $b_2^{(1)} = 0.241$, $b_3^{(1)} = 0.247$, $b_4^{(1)} = 0.277$, $b_1^{(2)} = 0.799$, $b_2^{(2)} = 0.817$, $b_3^{(2)} = 0.839$, $b_4^{(2)} = 0.94$, $b_1^{(3)} = 0.881$, $b_2^{(3)} = 0.903$, $b_3^{(3)} = 0.900$ and $b_4^{(3)} = 1.0$. Further, the overall compromise rating of each alternative SS is 0.5938, 0.6077, 0.6165 and 0.6887, respectively. Consistent with the overall compromise ratings, the prioritization order of SS alternatives is $f_4 \succ f_3 \succ f_2 \succ f_1$, and thus, the SS-4 (f_4) is the best supplier over different criteria for SS selection in the HSC.

FF-TOPSIS method

We have implemented the FF-TOPSIS method on the aforesaid case problem. Using this model, the weighted distance between the aggregated FFN and anti-ideal rating is given as 0.123, 0.146, 0.169 and 0.28, respectively. Next, the weighted distance between the aggregated FFN and anti-ideal rating is calculated as 0.18, 0.159, 0.146 and 0.023, respectively. The relative closeness coefficient to the FF-ideal rating is presented as 0.407, 0.48, 0.536 and 0.924. The ranking order of SS alternatives is $f_4 \succ f_3 \succ f_2 \succ f_1$, thus, the SS-4 (f_4) is the best option among the others in the HSC.

Table 13 demonstrates the comparative difference between the developed and existing approaches for determining the ranking order of SSs in the HSCs. The key advantage of present FF-modified SPC-RS-MARCOS methodology is presented as follows (see Fig. 3):

- a) The proposed approach determines the weights of criteria through an integrated FF-modified SPC-RS by integrating the objective and subjective weights, which achieves the weight values of attributes from the most favourable standards, while in the FF-WASPAS, the objective weight of attribute is obtained with FF-similarity measure and improved FF-score degree-based model; in the FF-CoCoSo, only objective weight of attribute is obtained with similarity measure-based model; in the FF-ARAS, only objective weight of attribute is estimated by MEREC tool and in the FF-TOPSIS, criteria weight is chosen randomly.
- b) All the approaches obtain the same optimal option, which is SS-4 (f_4) . The CUFs of the proposed FF-MEREC-SWARA-MARCOS approach have been estimated with the FF-PIR and FF-NIR, while the FF-WASPAS and the FF-CoCoSo models employ the averaging and geometric AOs and the FF-ARAS uses averaging AO and only FF-PIR to find the final rating. Hence, the proposed approach is more comprehensive and effective for assessing the SSs with multiple attributes and DEs. Considering this feature, the proposed approach can be applied more broadly.

Findings and discussion

In this work, we have planned a hybrid MCGDM methodology to evaluate and choose the most suitable SS with respect to twelve criteria including training for recycling and reuse (v_1) compliance with environmental protocols (v_2) , maintaining safety in operation and material handling (v_3) , minimize the fuel and energy consumption (v_4) , appropriate training facility of workers (v_5) , past performance and reputation (v_6) , continuous enhancement and quality control (v_7) , availability of credit system (v_8) , flexible size (v_9) , stability of cost (v_{10}) , storage management and sustainable inventory (v_{11}) and source of capability and availability degree (v_{12}) under four different dimensions. The developed method is based on the distance measure, Sugeno–Weber AOs, modified SPC model, FF-RS model and MARCOS approach with Fermatean fuzzy information. In this method, we have firstly computed the experts' weights based on Eqs. (15)–(17). Further, the proposed AO has been used to aggregate the individual DEs' opinions into a combined opinion. Next, the combined weighting method has been operated to calculate the criteria weights including the FF-modified SPC and the FF-RS models for computing the subjective

Aspects	Gül's ARAS ⁷⁶	Senapati and Yager's TOPSIS ¹⁵	Mishra and Rani's WASPAS ⁷⁵	Simić et al.'s CoCoSo ⁷⁷	Proposed method
Methods	FF-ARAS model	FF-TOPSIS	FF-WASPAS	FF-CoCoSo	FF-modified SPC RS-MARCOS
DEs' weight	Assumed	Assumed	Assumed	Assumed	FF-score and rank sum (Integrated weight)
Criteria weights	Assumed	Assumed	Computed (Similarity measure-based tool)	Computed (FF-MEREC)	Computed using integrated weighting tool (FF-modified SPC-RS)
Aggregation	FFWA	Distance measure	FFWA and EFWG	FFWA and FFWG	FFSWWA and FFSWOWA operators
Normalization	Linear	Vector	Linear	Linear	Linear/vector
Prioritization	$f_4 \succ f_2 \succ f_3 \succ f_1$	$f_4 \succ f_3 \succ f_2 \succ f_1$	$f_4 \succ f_2 \succ f_3 \succ f_1$	$f_4 \succ f_3 \succ f_2 \succ f_1$	$f_4 \succ f_2 \succ f_3 \succ f_1$
Best option	f_4	f_4	f_4	f_4	f_4

Table 13. Comparison of different models for SS selection in HSCs.

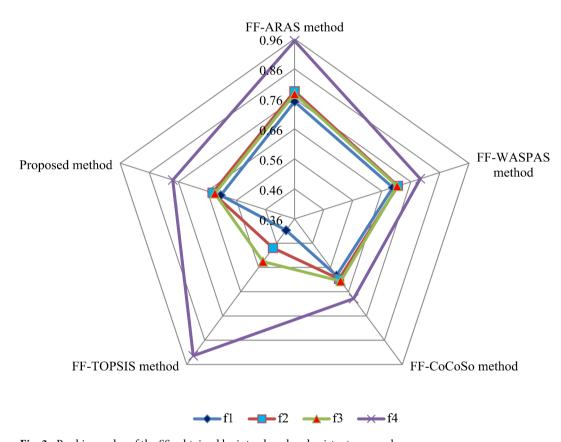


Fig. 3. Ranking order of the SSs obtained by introduced and existent approaches.

and objective weights. As per the obtained outcomes, the criteria weights' set is w = (0.0892, 0.0855, 0.0372, 0.0392, 0.0818, 0.0959, 0.0923, 0.0668, 0.0795, 0.1274, 0.1043, 0.1008). Figure 4 gives the variation of weight of diverse criteria for SS selection in the HSCs. Stability of cost (v_{10}) with weight 0.1274 has come out to be the best indicator, while Storage management and sustainable inventory (v_{11}) with weight 0.1043 is the $2^{\rm nd}$ most suitable indicator. Source of capability and availability degree (v_{12}) has 3rd rank with weight 0.1008, Past performance and reputation (v_6) has 4th rank with weight 0.0959, Continuous enhancement and quality control (v_7) has 5th rank with weight 0.0923, Training for recycling and reuse (v_1) has 6th rank with weight 0.0892, compliance with environmental protocols (v_2) has 7th rank with weight 0.0855, appropriate training facility for the workers (v_5) has 8th rank with weight 0.0818, flexible size (v_9) has come out to be 9th position, availability of credit system (v_8) has obtained 10th rank, minimize the fuel and energy consumption (v_4) has obtained 11th position and maintaining safety in operation and material handling (v_3) has come out to be the last rank.

Further, the developed model has been utilized for SS selection involving four suppliers with respect to twelve criteria. On the basis of computational results, an option 'SS-4 (f_4) ' is the most suitable supplier among the others, while an option "SS-2 (f_2) " is found to be the least desirable choice. The proposed approach does not only present the prioritization order of suppliers but also determines the DEs and criteria significance values in the assessment of SSs. Sensitivity investigation has been conducted to prove the steadiness of the results with respect to diverse values of strategy parameter (γ) . Lastly, we have compared the developed and existent models 15,75–77 to prove the robustness of introduced approach.

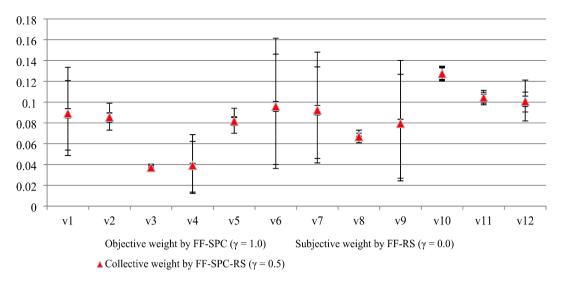


Fig. 4. Weight of the criteria for SS selection in the HSC.

There are some managerial implications of this work.

- The considered aspects of criteria, i.e., social and environmental, services, economic and logistics and their
 related sub-criteria can be applied to another domain that examine the suitable supplier from sustainability
 perspective.
- This work provides an efficient ranking approach to evaluate the sustainable suppliers by means of the
 considered evaluation criteria, which would benefit the healthcare organizations in many ways, including
 eco-friendly regulation compliance, cost effectiveness, sustainable practices and the proper inventory management systems.
- By reading this article, the healthcare managers can understand that only consideration of economic perspective is not enough for the today's competitive market. For a perfect competition environment, they need to consider several other indicators during the assessment of sustainable suppliers. In this work, the most appropriate supplier highlights the significance of training for recycling and reuse, compliance with environmental protocols, maintaining safety in operation & maintenance, minimizing the fuel and energy consumption, providing the appropriate training facility for the workers, past performance, quality control, cost-efficient technique and sustainable logistics practices in the HSC.

Conclusions

This work aims to introduce a collective MCGDM model for assessing the SSs in the HSC. For this purpose, based on literature review and experts' knowledge, 12 criteria were selected from different perspectives including social and environmental, services, economic and logistics. This methodological framework has been incorporated the proposed MARCOS method with FF-DM-based SPC model, RS model and FFNs. In this regard, new FF-DM has been presented to quantify the distance between FFSs. To combine the Fermatean fuzzy data, novel Sugeno-Weber weighted averaging operators are developed with their desirable properties. Next, an integrated criterion weighting model has been provided by considering objective weight through FF-modified SPC tool and subjective weight using FF-RS approach. Further, the developed FF-modified SPC-RS-MARCOS model has been applied to a case study of SS selection problem in the HSC, which illustrates its superiority and efficacy. Sensitivity and comparative investigations have been discussed to test the validity of obtained outcomes. Based on the comparison with existing methods, we found that the presented model is simple, easy-to-use and reliable in order to solve the realistic MCGDM problems. The key novelties of this work are development of novel Sugeno-Weber weighted averaging operators, RS model-based experts' weight-determining tool, a new distance measure-based SPC tool and FF-RS model for estimating weights of attributes and the MARCOS technique with Fermatean fuzzy information. In future, we can extend some new models to elude the weaknesses of proposed work by introducing some correlative MCDM frameworks under FFS environment. In addition, we can propose Sugeno-Weber operators for double hierarchy hesitant fuzzy linguistic term sets, linear Diophantine sets, quadripartitioned single-valued neutrosophic sets and Pythagorean fuzzy hypersoft sets.

Data availability

All data generated or analyzed during this study are included in this published article.

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Author contributions

A.F.A and P.R. wrote the manuscript and introduced the method, A.R.M. worked on the method and computational discussion, A.M.A and F.C. worked on literature review and checked the final version of the manuscript.

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Declarations

Competing interests

The authors declare no competing interests.

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