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RESEARCH PAPER

The Time–Frequency Relationship between Oil Price, Stock Returns and Exchange Rate

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Abstract
The concept of time scales is essential for modeling financial decisions. This paper investigates time–frequency relationships across time scales between stock market returns, crude oil prices and exchange rates by applying wavelet analysis technique over the period 1999 to 2021. We find evidence of several strong co-movements between oil price and stock market and between oil price and foreign exchange rate in India. Each of these associations is linked with some important macroeconomic events. This implies economic shocks in developed market have a spillover effect on Indian market. The phase relationships indicate stock returns are in phase with oil prices and exchange rates are in out of phase with oil prices. We find that the impact of volatility at lower scale has a short term effect on the variables. Further, the wavelet coherency at high scale has slower changes with long term effect on the relationship between the variables of our interest. These results are useful for investors aiming specific time horizon of their investment and preferences, for portfolio managers and in risk assessment. Understanding the leading and lagging relationships will also help in business cycle based investing by detecting the subsequent business cycle fluctuations and forecasting the trend.

Keywords Wavelet coherence · Partial wavelet coherence · Macroeconomic events · Business cycle · Structural break

JEL Classification C22 · G1 · G15

1 Introduction

Economic decisions differ significantly at different time horizons. Markets are composed of the actions taken by different participants. Each of these participants operates on different time scales at each moment. For example, a trader operating

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in the market may have a very long view of years; the chartist may operate with a time horizon of days or weeks. The nature of relationship between various economic variables might vary over different time horizons or hold at several time scales from longest to shortest horizon or vice versa. The relationship between two variables is a function of time at different time scale. However, standard measures are not appropriate to distinguish between the short-term and long-term components of risk and co-movement of variables under study. Economic shocks produce distinct effects on the dynamics of time series at different time horizons, suggests that frequency-specific measures of correlation may yield several new insights (Chaudhuri & Lo, 2015). The wavelet transform is an appropriate analytical tool applicable where both the time horizons of economic decisions and the strength of relationships between variables vary with time and frequency (Ramsey, 2002). The wavelet transform decompose a time series into their time scale coefficients, each associated to a specific frequency bands. It captures variations across frequencies and has ability to capture events which are local in time. Wavelet analysis is more detailed and provides more information than the correlation that does not take into account the evolution over time of the relationship (Arnold et al., 2018). The wavelet analysis is a model-free approach of estimating time-varying correlations. This property makes it a very powerful technique in comparison with other standard methods that rely on the parameter estimation method (Vacha & Barunik, 2012). Further, the wavelet analysis is especially relevant to the analysis of non-stationary time series in economics and finance. The wavelet approach can track how the different scales related to the periodic components of the signal changes over time (Cazelles et al., 2008).

In recent, several financial crises have triggered active discussions among academics, practitioners, and policymakers about the relationship between stock market, crude oil prices, and exchange rates. Measuring the dynamic link between stock market, crude oil prices, and exchange rates has prime importance in terms of practical implications in portfolio management, asset allocation, and risk management. The time–frequency analysis provides insights on risk management at different time horizon and a framework for portfolio diversification across these time scales. The existing literature on dynamic links between stock market, crude oil prices, and exchange rates is mostly restricted to standard econometric methodologies with the stationarity assumption. The advantage of wavelet analysis is that it does not require any stationarity assumption. By stationarity we mean time series whose frequency content does not change in time are called stationary series. The purpose of this paper is to focus on the potential use of wavelet technique in the context of time-series analyses in economics and finance. In this context, this study investigates the co-movements between: (a) stock market returns and crude oil prices, and (b) exchange rates and crude oil prices in the time–frequency space and identify the direction (in-phase or out-phase) of correlation. Further, this paper tries to reveals information on the effect–result relationships across time scales and over time.

Crude oil plays a vital role in economy for almost every country in this world. The fluctuation of international crude oil price has effect on the economic stability of a country. In addition, according to Qiang et al. (2019), "Oil has become an important bargaining chip in contemporary international political, military, and diplomatic relations." Further, stock market return in a country reflects health of the economy. There is a strong link between oil price fluctuations and stock markets (Bagirov & Mateus, 2019; Thorbecke, 2019). Similarly, the oil importing or exporting countries get affected because of fluctuating exchange rate in the foreign exchange market. As appreciation of US dollar weakens the purchasing power of oil importing countries, this dependency on crude prices adversely affect any economy and have spillover impact on the monetary policy, consumption and investment behavior of the economy. Crude oil is considered as one of the most important energy commodities. Fluctuation of crude oil prices caused by the dynamics of supply and demand have a direct impact on economic efficiency, energy policy, energy markets and a indirect impact through governmental programs responding to the perceived crisis (Sweeney, 2004). Moreover, supply disruptions in the world oil market and the subsequent business cycle fluctuation has impact on the economy. In the light of recent several financial crises, it becomes more important for countries to understand this relationship with a great extend. India is among the fastest growing countries of the world and a high oil importer. It has a strong financial system and any oil price shocks are followed by an immediate impact on its stock market volatility and foreign exchange market. It is important therefore, for countries like India to study this relationship for preparing a better policy framework.

In order to achieve the objectives, we consider daily returns of S&P Bombay Stock Exchange 500 index (BSE 500), foreign exchange rate of Indian rupees per unit of US dollars, and crude oil prices in US dollars from the period February, 1999 to March 2021. The results show several strong co-movements between oil price and stock market and between oil price and foreign exchange rate. Each of these associations is linked with some important economic or geopolitical issues in the world economy. The phase relationship indicates stock returns are in phase with oil prices and exchange rates are in out of phase with oil prices. We also find that economic shocks in developed market have a spillover effect on Indian market. However, the periods of these impacts are different at different time interval. The impact in volatility has a short term effect with a period less than 128 days. We find the wavelet coherency at high scale has a slower changes and long term effect on the relationship between the variables of our interest. Using wavelet analysis, we have shown that it is a powerful tool for analyzing dynamic and cyclical time series that cannot be analyzed by other classical spectral techniques. The remainder of this paper is organized as follows. In Sect. 2, we review the literature related to the topic. Section 3 describes the wavelet transform methods adopted in this paper. Section 4 describes about data that we use for our analysis. Section 5 depicts experimental results, and discusses about the findings, and Sect. 6 concludes.

2 Literature

Wavelet filtering provides insights into the dynamics of economic and financial time series which the standard measure is not able to identify (Gençay et al., 2002). There are many applications of wavelet transform in the area of economics and finance where the variables of interest are cyclical in pattern (e.g., see Cazelles et al., 2008; Gencay et al., 2002; Granger & Engle, 1983; Hu & Si, 2016; Ramsey, 2002; Torrence & Compo, 1998). Aguiar-Conraria et al. (2008) uses cross-wavelet tools to show that the relation between monetary policy variables and macroeconomic variables has changed and evolved with time. Gallegati (2012) uses wavelet-based approach to test the contagion effect occurred during the US subprime crisis in G-7 countries plus Brazil and Hong Kong. Evolution of the co-movement of stock market returns is crucial for risk assessment of portfolios. In terms of risk management in portfolio diversification, a higher co-movement among the assets of a given portfolio implies lower gains.

A number of empirical researches exist about the relationship between oil price and stock index and the relationship between oil price and exchange rate. With regard to the effect of oil price on stock market returns, researchers find negative relationship between oil price and stock market returns in different countries (Elian & Kisswani, 2018; Jones & Kaul, 1996; Papapetrou, 2001). Kilian and Park (2009) emphasizes that it is essential to identify the underlying source of supply or demand of the global crude oil prices. While analyzing the influence of oil price on the U.S. real stock returns, Salah and Jamal (2018) examines the effects of oil price shocks on the stock market returns of the Gulf Cooperation Council (GCC) countries. They find that the oil and stock markets are more likely to boom together or crash together. In the same line, Algia and Abdelfatteh (2018) study the impact of oil price changes in the stock market returns of five developed and five emerging countries. They find stock returns for developing markets has effect when the oil prices are up and they have less effect on the emerging market for both up and down prices.

There are several empirical evidences that use wavelet analysis to find co-movement between oil price and stock returns (e.g., Amalia & Purqon, 2019; Arnold et al., 2018; Cai et al., 2017; Huang et al., 2018; Pal & Mitra, 2019). Huang et al. (2018) conclude that the coherence of oil price and stock market nexus is tremendously different in short time scale. Cai et al. (2017) find that oil price and the East Asian stock market move in phase, and oil prices lead to stock returns in the long run. Rua and Nunes (2009) examine the co-movement among international stock markets such as Germany, Japan, UK and US. The co-movement between OPEC (Organization of Petroleum Exporting Countries) oil prices and the six largest African stock markets is relatively low with the exception of emerging stock markets such as South Africa and Egypt (Arnold et al., 2018).

In the context of the relationship between oil price and exchange rate, Qiang et al. (2019) find inconclusive relationship between international crude oil prices and the exchange rates of oil-importing countries. Gençay et al. (2001) investigate the scaling properties of foreign exchange volatility and find that foreign exchange rate volatilities follow different scaling laws at different horizons. Basher et al. (2012) find that oil price shocks tend to depress emerging market stock prices and US dollar exchange rates in the short run. Further, Yang et al. (2017) finds co-movement between the crude oil price and the exchange rates deviates over time. They also find that crude oil price has negative relationship with exchange rates for oil-exporting countries. In the context of Brazil, Russia, India, China, and South Africa (BRICS) countries, Dahir et al. (2018) also find negative relation where stock returns is leading over exchange rates in India. Tiwari et al. (2013) find causal relationships between the oil price and the real effective exchange rate of Indian rupee at higher time scales only.

3 Methodology

3.1 Continuous Wavelet Transform

Wavelet analysis performs the estimation of the spectral characteristics of a time series into time–frequency space. It provides information about how different periodic spectral component occur at what time interval. The wavelet transform has the ability to capture variations across frequencies and capture events which are local in time, with variable resolution to identify periodic features in short time interval. It uses a basis function of small wave (called wavelets) that is stretched and shifted to capture all the information for a specific time horizon and for a specific location in time. A continuous wavelet transform decompose a time series into time–frequency space. The continuous wavelet transform of a time series $x(t)$ with respect to the wavelet ψ is a function of two variables, W_x (τ, s), defined as:

$$W_x(\tau, s) = \int_{-\infty}^{+\infty} x(t)\frac{1}{\sqrt{s}}\Psi^*\left(\frac{t-\tau}{s}\right)dt \tag{1}$$

The transformed series is a function of scale parameters s, and location parameter τ. Ψ* is the transforming function, and it is called the mother wavelet. The location parameter track changes in the time domain, while the scale parameter to determine the resolution by stretching or compressing compacting the wavelet. A stretched wavelet provides good frequency resolution i.e., coarse features of a series by capturing low frequency (high scale). Similarly, a compressed wavelet provides good time resolution i.e., detailed information of a hidden pattern by capturing high frequency (low scale). The scale refers to the width of the wavelet. As the scale increases, the wavelet gets wider, it includes more of the time series and as the scale decreases, the wavelet gets wider and it includes less of the time series. The admissibility condition for wavelet is equivalent to requiring that the wavelet has zero mean and normalized to have unit energy at each scale s.

There are a family of wavelets with different scaling properties to balance between the time and scale (e.g., Daubechies, Morlet, Paul, Mexican hat etc.). A detail characteristic of these wavelets is described in Torrence and Compo (1998). In our analysis we use the Morlet wavelet, defined as:

$$\Psi(t) = \pi^{-1/4}e^{iw_0t}e^{-t^2/2} \tag{2}$$

The Morlet wavelet is composed of a complex exponential multiplied by a Gaussian window where the parameter ω_0 denotes central frequency of the wavelet and t is time. For all wavelets, there is a one-to-one relationship between the scale and period, where period is the inverse of frequency. By choosing $\omega_0 = 6$, the wavelet scale s, is inversely related to the frequency that simplify the interpretation of the wavelet analysis (Aguiar-Conraria & Soares, 2011a). In the Morlet wavelet, the scale can be defined as the distance between oscillations. The Morlet wavelet transform can be intrinsically categorized as a kind of continuous wavelet transform and has both real and imaginary parts. This is convenient in analyzing both amplitude and phase information of a time series. The Morlet wavelet transform yields a finer resolution than discrete wavelet transforms (Schmidbauer et al., 2017).

3.2 Wavelet Power Spectrum and Cross Wavelet Spectrum

Because the wavelet transform $W_x(\tau, s)$ is a complex function having real and imaginary part, the (local) wavelet power spectrum of series *x(t)* can be defined as:

$$\text{Wavelet power spectrum} = (\text{WPS})_x = \left|W_x\right|^2 \tag{3}$$

The wavelet power spectrum describes the distribution of energy of a time series at different scales. This provides information about how much each frequency band has contributed to the energy of the series over that time interval. In practice, the distribution of energy within the data array is determined by drawing the wavelet power graphically. By looking for regions within the wavelet power spectrum of large power, it is possible to determine which features of the series are important and which are less important. So, wavelet power spectrum helps to obtain local information in time. The statistical significance of wavelet power can be assessed against a null hypotheses that the series is generated by a stationary process with a given background power spectrum. If a peak in the wavelet power spectrum is significantly above this background spectrum, then it can be assumed to be a true feature with a certain percent confidence (Torrence & Compo, 1998).

The cross wavelet spectrum measure the interaction between two different time series $x(t)$ and $y(t)$. Followed by Maraun and Kurths (2004), the cross wavelet spectrum is defined as expected value of the product of corresponding $W_x(\tau, s)$ and $W_y(\tau, s)$:

$$WCS_{xy}(\tau, s) = \left\langle W_x(\tau, s) W_y(\tau, s)^* \right\rangle \tag{4}$$

where $W_y(\tau, s)^*$ indicates the complex conjugate of $W_y(\tau, s)$. The cross wavelet spectrum measure how two time-series evolve one against the other in a time scale space. The wavelet cross spectrum (WCS) is a complex valued function and can be decomposed into amplitude and phase. It exposes regions with high common power and further reveals information about the phase relationship. The cross-wavelet power which is the modulus of the WCS is interpreted as the wavelet power of two different series in time–frequency domain.

So,

$$Cross - wavelet - power = |WCS(\tau, s)| \tag{5}$$

The cross-wavelet power of two time-series represents the covariance between two time-series at each time and frequency. The complex argument $\arg\{WCS(\tau, s)\}$ can be interpreted as the local relative phase between $x(t)$ and $y(t)$ in time–frequency domain. To quantify the phase relationship, the circular mean of the phase angles can be used.

3.3 Wavelet Coherency and Phase Difference

Cross wavelet power measure how two time series evolves against one another on a time-scale space. Another important measure is the wavelet coherence, defined as the ratio of square of the cross-spectrum to the power spectrum of each series. The wavelet coherency has the advantage over wavelet cross spectrum of being normalized by the power spectrum of the two time series. Further, the wavelet cross spectrum can show strong peaks for the realization of independent processes suggesting the possibility of spurious significance tests (Maraun & Kurths, 2004). Following Aguiar-Conraria et al. (2018), we define the complex wavelet coherence of two time series normalized to the two single WPS as:

$$\rho_{xy} = \frac{S(W_{xy})}{\sqrt{S(\left|W_X\right|^2)S(\left|W_y\right|^2)}} \tag{6}$$

where S is a smoothing operator to ensure that smoothing the squared wavelet cross spectrum gives a value between zero and one. A value close to zero indicates weak correlation, while a value close to one provides evidence of strong correlation. The wavelet coherency is defined as the absolute value of the complex wavelet coherency, i.e. is given by

$$R_{xy} = \left|\rho_{xy}\right| \tag{7}$$

The cross-wavelet spectrum is useful for comparing the frequency contents of two time series, and drawing conclusions about the series' synchronicity at certain frequency or periods and across certain ranges of time. This implies it is possible to determine the variations of frequencies of variables in time and to compare them. The coherence phase describes the delay between the two time series at scale s is defined as:

$$\phi_{xy} = \tan^{-1}\left[\frac{\mathrm{Im}\{s^{-1}W_{xy}\}}{\mathrm{Re}\{s^{-1}W_{xy}\}}\right] \tag{8}$$

Wavelet cross-correlation tools can be used to analyze the lead/lag relationship between two time series for different time scales. If one time series leads the other, then its realizations may be used to forecast the realizations of the lagging time series. Aguiar-Conraria and Soare (2011b) illustrates the range of possible phase differences and their interpretation. In their paper, a phase difference of zero indicates that the time series move together at the specified time–frequency; if $\phi_{xy} \in (0, \pi/2)$, then the series move in phase with time series x leads y; if $\phi_{xy} \in (-\pi/2, 0)$, then time series y is leading; a phase difference of π (or $-\pi$) indicates an anti-phase relation; if $\phi_{xy} \epsilon$ $(\pi/2, \pi)$, then y is leading; if $\phi_{xy} \epsilon$ $(-\pi, -\pi/2)$, time series x is leading.

3.4 Partial Wavelet Coherence

Partial Wavelet Coherence is a technique similar to the partial correlation that helps to find the resulting wavelet coherence between two-time series x and y after eliminating the influence of the time series z. The complex partial wavelet coherence is the quantity given by

$$\rho_{xy,z} = \frac{\rho_{xy} - \rho_{xz}\rho^*_{yz}}{\sqrt{(1 - R^2_{xz})(1 - R^2_{yz})}} \tag{9}$$

The partial wavelet coherence of x and y after controlling for z, denoted by

$$R_{xy,z} = \left|\rho_{xy,z}\right| \tag{10}$$

The partial phase difference of x over y, given z, denoted by $\phi_{xy,z}$ is the phase angle of $\rho_{xy,z}$.

4 Data

In this study, the stock market return data of S&P BSE 500 index, consisting of the top 500 companies listed at Bombay Stock Exchange (BSE) is taken as our stock data.[1] The oil futures contracts play a dominant role in price determination which is traded heavily by speculators and hedgers. Hence the crude oil price data (in US dollar), is taken from Crude Oil WTI (West Texas Intermediate) Futures at the New York Mercantile Exchange (NYMEX). Our exchange rate data in rupees per unit of US dollar comprised of daily data from Reserve Bank of India (RBI) database. We consider daily closing prices of stock data because they appropriately explains the pricing mechanism in a market and more informative. Similarly, daily oil price and exchange rate indicate the rapid trend of fluctuation in the international market. The entire data sample covers 5360 observations from the period February, 1999 to March, 2021. All the returns are computed using logarithm difference of two consecutive prices. The returns for stock market, crude oil prices, and exchange rates are calculated as follows:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right), \tag{11}$$

where P_t represents the closing prices as on date t.

[1] BSE is Asia's first & the fastest Stock Exchange in the world with a speed of 6 micro seconds and one of India's leading exchange groups. The S&P BSE 500 index covers all major industries in the Indian economy and represent as a broad index for Indian market.

Table 1 Descriptive statistics for daily returns

	Mean	Median	Standard Deviation	Kurtosis	Skewness	Jarque Bera	Min	Max
Stock Returns	0.0005	0.001	0.015	13.992	−0.792	27,550	−0.168	0.146
Return on Crude Oil prices	0.0002	0.001	0.031	324.222	−8.459	231E5	−1.118	0.333
Return on Exchange rate	0.0001	0.000	0.004	10.704	0.212	12,958	−0.030	0.040

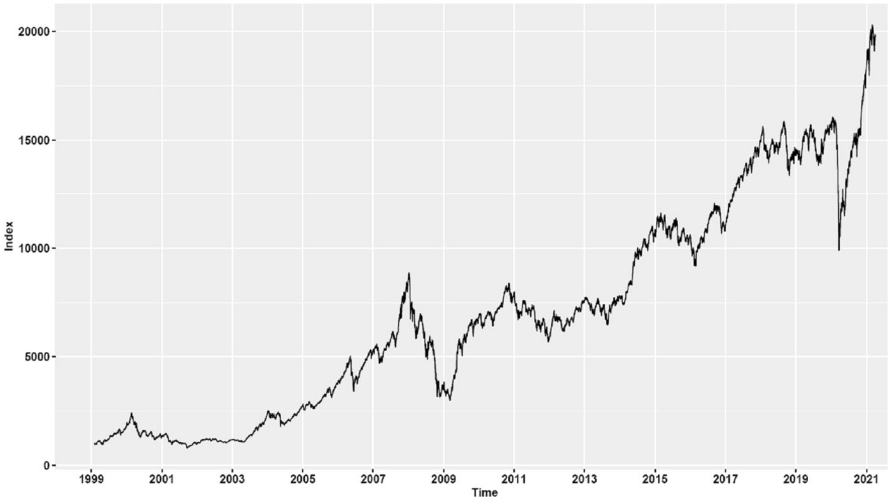

Fig. 1 S&P BSE 500 Stock market index

5 Results and Discussion

5.1 Descriptive Statistics

Table 1 provides the day wise descriptive statistics of the returns on stock market, crude oil price and exchange rate. Total sample period for the variables is from February, 1999 to March, 2021 with 5360 daily observations. The daily standard deviations for oil price and stock return is high compare to exchange rate. The measurement of kurtosis shows heavy tails that means the time series are prone to extreme values on either side. The negative skew for oil price refers to a longer tail on the left side of the distribution. The Jarque–Bera normality test indicates significant test statistics or rejects normality for all the returns.

Figures 1, 2 and 3 shows the time series diagrams of the S&P BSE 500 stock market index, crude oil WTI Futures price in US dollars per barrels and foreign exchange rate in Rupee per US dollar respectively. The diagram shows there is increasing trend for all the three variables from 1999 to 2021. However, the stock

Fig. 2 Crude Oil WTI Future price

Fig. 3 Exchange rate

market index, and crude oil prices are moving in the same direction whereas, the exchange rate is following the opposite direction.

5.2 Wavelet Power Spectrum Plot

Wavelet power spectrum plot shows how the power of the projection varies across the time domain at a given scale. The peaks of the spectrum provides evidence of potentially interesting features or characteristic scale corresponds to one that is

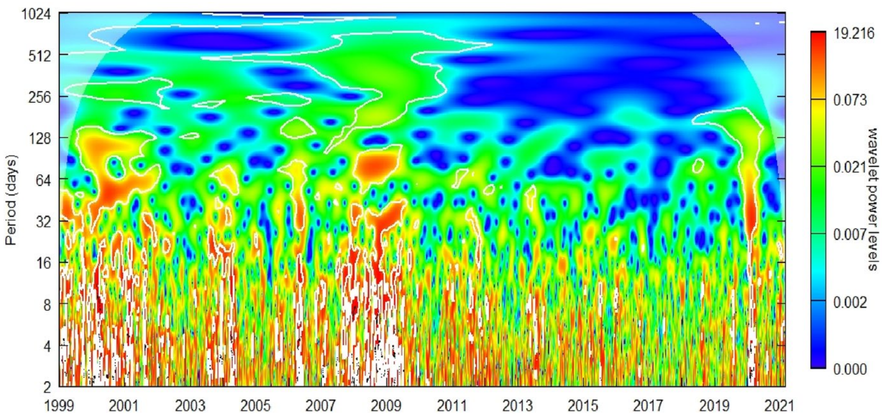

Fig. 4 Wavelet power spectrum for stock market returns. The white contour designates the 5% significance. The cone of influence, which indicates the area affected by edge effects, is the outside region of the black line. The colors code for power indicates level of intensity ranges from dark blue to dark red (colour figure online)

contributing more to the overall variance than scales surrounding it (Keim & Percival, 2010). In wavelet power spectrum plot, time in years is represented by the horizontal axis and the vertical axis is the period where period or scale is inversely related to the frequency. The intensity of variation in power spectrum plot is represented by color code. These colors codes for power ranges from dark blue to dark red indicating weak variation or low intensity to strong variation or high intensity respectively. This essentially tells that there are time intervals where those frequencies are contributing maximum to the energy of the series over that interval of course; whereas, there are other time intervals where the contribution is less and not much activities of the series is happening. The white contour line tests the wavelet power at 5% significance level against the null hypothesis that the data generating process is generated by a stationary process. The statistical significance test of wavelet power answers what part of the results may have been created by pure randomness and what part represents real physical properties of the time series. A peak in the wavelet power spectrum is considered to be significant (i.e. not caused by pure randomness) with 95% confidence if the normalized peak value is larger than the mean power spectrum, while a peak is considered to be insignificant (i.e. caused by randomness) with 95% confidence if otherwise. Continuous wavelet transform applied to a finite length time-series suffers from edge distortions, which increase with scale. The region in which the transform suffers from these edge effects is called the cone of influence (COI). In this area, the results are unreliable and have to be interpreted carefully. The cone of influence lies under a cone which is bordered by a thin black line. Wavelet power spectrum plot is a time–frequency contours of the evolutionary power spectra.

In Fig. 4, the wavelet power spectrum plot of S&P BSE 500 stock index indicates significant high volatility from 1999–2002, in 2004, 2006, from 2008 to mid-2009, 2016, and 2020. The stock market volatility lasts for a short term period of max

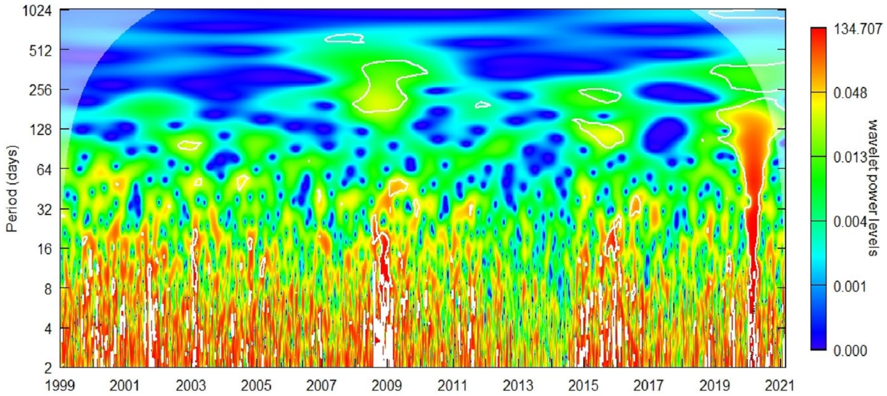

Fig. 5 Wavelet power spectrum for crude oil price returns. The white contour designates the 5% significance. The cone of influence, which indicates the area affected by edge effects, is the outside region of the black line. The colors code for power indicates level of intensity ranges from dark blue to dark red (colour figure online)

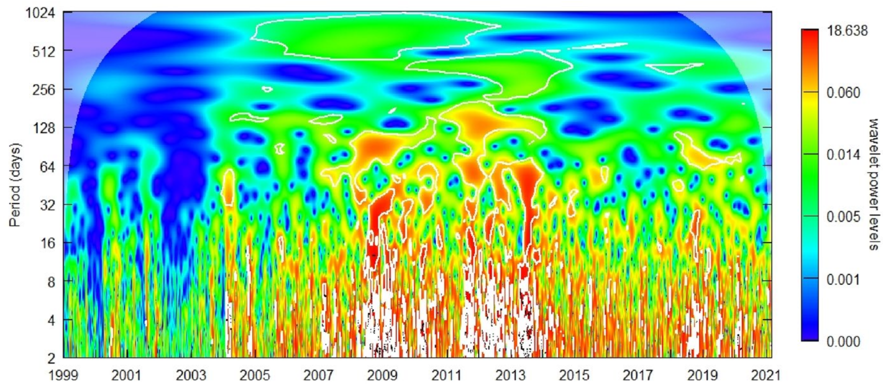

Fig. 6 Wavelet power spectrum for exchange rate returns. The white contour designates the 5% significance. The cone of influence, which indicates the area affected by edge effects, is the outside region of the black line. The colors code for power indicates level of intensity ranges from dark blue to dark red (colour figure online)

128 days (approximately 4 months). We also find a region of low volatility from 2005–2012 at scale more than 128 days. In addition, there are pockets of strong volatilities presents with a short transition periods. Significance testing for power spectrum can also be seen as an example for multiple testing, since one investigates every point in the space–time domain separately. Hence, when repeating a test with $(1-\alpha)\%$ confidence for many independent realizations, around α % of the results spuriously appear to be significant. This effect is referred to as multiple testing. Figure 5 presents the wavelet power spectrum plot of crude oil prices. In the 2003, from 2008–2009, 2015–2016, the high volatility occurred with a maximum period of 64 days and in the 2020s with a period less than 256 days (approximately 1 year).

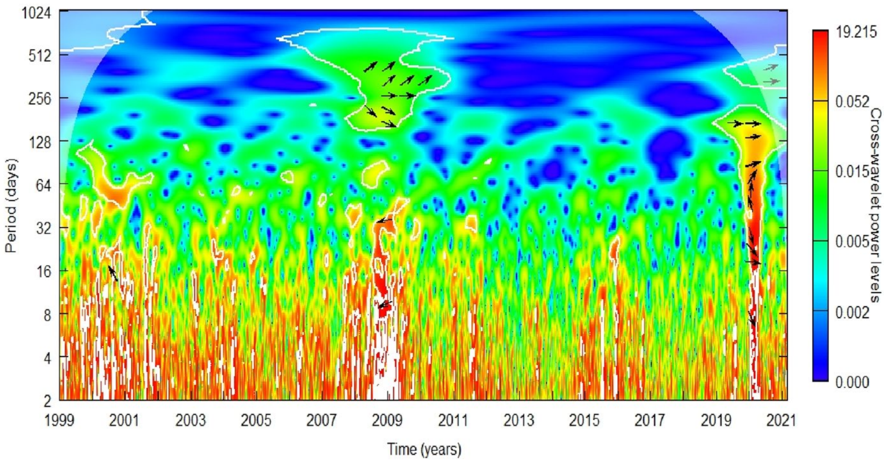

Fig. 7 Cross-wavelet spectrum plot of the stock return and crude oil price returns. The white contour designates the 5% significance. The cone of influence, which indicates the area affected by edge effects, is the outside region of the black line. The colors code for power indicates level of intensity ranges from dark blue to dark red. The phase difference between the two series is indicated by arrows (colour figure online)

Similar to stock market the oil market also has many volatile areas with short transitions less than 64 days period. In Fig. 6, we find significant areas of strong volatility for exchange rate in 2004, from 2008–2010, 2011–2014 and 2018–2020. For all these intervals the period of volatilities varies from 8 to 128 days. We observe long term effect of volatilities from 2008 to 2010 and from 2011 to 2014. Further, there are regions of medium intensities from 2008 to 2010 and 2010–2015 at higher scale around 512 days (approximately 2 years) and 256 days respectively. In wavelet power spectrum plot, each of these significant areas of strong volatilities can be associated with some macroeconomic or geopolitical events. Therefore, it is possible to detect the contagion effect on Indian market by analyzing the power spectrum plot.

5.3 Cross Wavelet Spectrum Plot

Cross wavelet spectrum plot measure how the association between two time series changes in a time scale space. It represents the local covariance between the time series at each scale. It exposes regions with high common power between two timeseries. The phase information is represented by the arrow orientation in the plot. When the arrows points to the right, two times series are in-phase and when the arrows point to left, times series are out of phase. When the arrow is pointing up, the first time series leads the second and when it is pointing down, first time series lags the second.

In Fig. 7, we find strong covariance between stock return and oil prices from the year 2000–2001, 2008–2009 and post 2020. The covariances are observed at scale from 16 to 128 days. We also find a weak interaction from 2006 to 2011 for

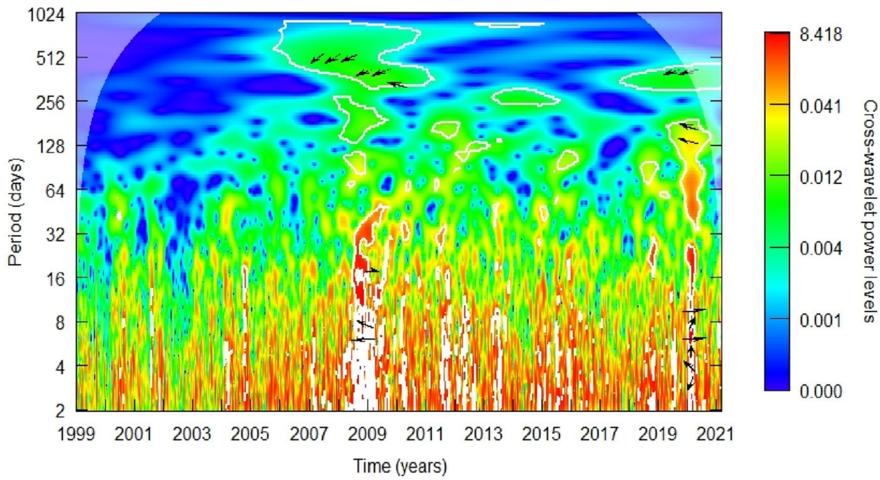

Fig. 8 Cross-wavelet spectrum plot of the exchange rate and crude oil price returns. The white contour designates the 5% significance. The cone of influence, which indicates the area affected by edge effects, is the outside region of the black line. The colors code for power indicates level of intensity ranges from dark blue to dark red. The phase difference between the two series is indicated by arrows (colour figure online)

the longer scale of 128–512 days (approximately 4 months to 2 years). For all the significant areas the stock returns are in phase with oil prices. In Fig. 8, we find strong covariance between exchange rate and oil prices in 2009 and 2020. These are observed at lower scale from 8 days to around 128 days. The exchange rates are in out of phase with the oil prices. The cross-wavelet power spectrum describes the common power of two processes without normalization to the single power spectrum. Because of this, cross-wavelet power spectrum is not suitable for significance testing the relation between two time series (Maraun & Kurths, 2004). Wavelet coherency, which measures the cross-correlation between two time series on scale-by-scale basis provide way to compare two time series.

5.4 Wavelet Coherence Spectrum Plot

Wavelet coherence is a localized correlation coefficient at time frequency space similar to traditional correlation coefficient defined in time domain. Phase difference gives information on the delay, or synchronization. Two time series will have a constant phase difference at the point of perfect coherency. The phase differences are indicated by arrows in the image plot of cross-wavelet power spectrum. The white contour lines indicate coherence significance under the null hypothesis of no linear relation at the 5% level. Wavelet coherence analysis identifies areas where both the series have significant relationship in terms of the degree and direction of correlation. Figure 9 shows strong co-movements between stock return and oil price from the mid-2001–2003, 2006–2011, 2016, and mid-2018–2021. The strong correlations

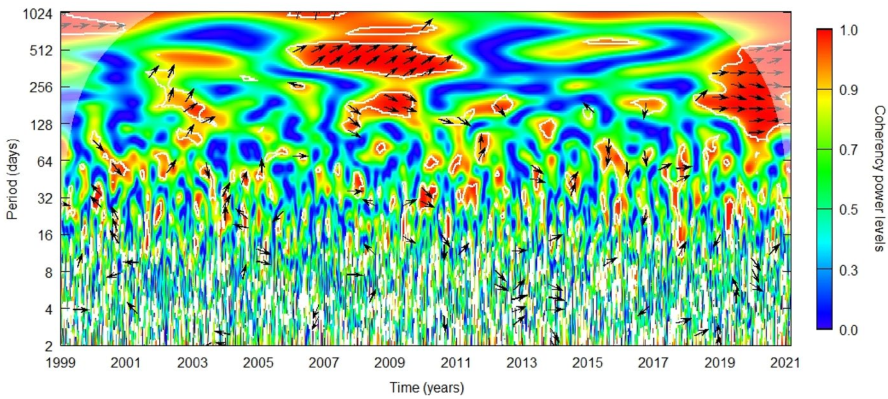

Fig. 9 Wavelet coherence plot between the stock returns and crude oil price returns in India. The color code for power ranges from blue (low coherence) to red (high coherence). A pointwise significance test is performed against an almost process independent background spectrum. The white contour represents a significant region at 95% confidence intervals for the null hypothesis that coherency is zero. The cone of influence is marked by black lines. The phase difference between the two series is indicated by arrows (colour figure online)

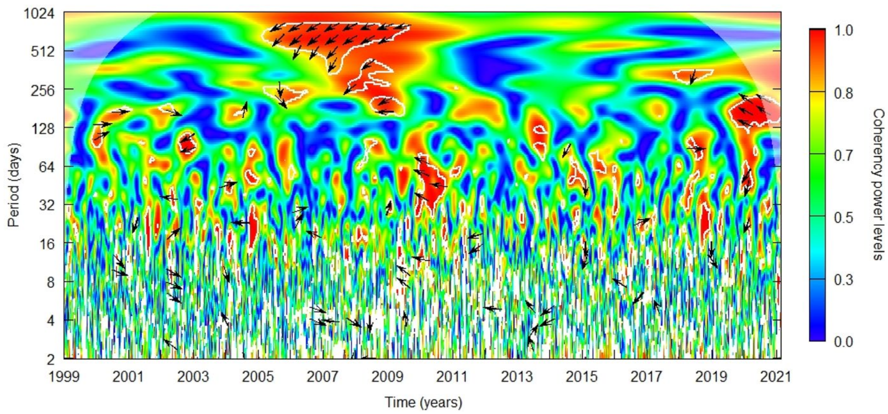

Fig. 10 Wavelet coherence plot between the exchange rate and crude oil price returns in India. The color code for power ranges from blue (low coherence) to red (high coherence). A pointwise significance test is performed against an almost process independent background spectrum. The white contour represents a significant region at 95% confidence intervals for the null hypothesis that coherency is zero. The cone of influence is marked by black lines. The phase difference between the two series is indicated by arrows (colour figure online)

are observed with a period varies from lower scale of 128 days (approximately 4 months) to higher scale of 512 days(approximately 2 years). The phase relationship from the arrow directions indicates that stock returns are in phase with oil prices for all the cases. During the interval 2006–2011, the stock market returns are leading over oil prices at higher scale but it is lagging at lower scales. There are other small areas of strong co-movements observed for short period of time.

Figure 10 shows strong co-movement between oil price and exchange rate from mid-2005–2011 and from 2020 to 2021. There are other small areas of strong coherency observed at lower scale of less than 128 days in 2000, 2002, 2014 and from 2018–2019. The strong coherency from mid-2005–2011 and 2020–2021 occurred at more than 128 days scale. These coherencies at higher scale are slowly changing and have long term effects on the economy. The directions of arrows shows phase relationships are out of phase means the oil price and exchange rate moves in the opposite directions. The left downward angle of the arrow during mid-2005–2011, implies that oil prices lagging to exchange rate and left upward angle of the arrow between 2020 and 2021, implies that oil prices leading to exchange rate.

5.5 Discussion

Time frequency analysis is useful to detect high frequency or rapid changes with short impact, and low frequency or slow changes induced by major impacts in time series. The wavelet power spectrum plot shows several significant areas of strong volatility appear in stock market returns, oil price and exchange rate. Each of these significant regions can be linked with an economic, geopolitical or other important event in the market. These events in the world market as well as in the Indian market have affected oil price, exchange rate and domestic stock market. Between the year 2000 and 2002 the Indian stock market was impacted by the collapse of the post-1999 dotcom bubble and the 9/11 terror attack occurred in the US. In 2003 there was invasion of Iraq and geopolitical tensions in gulf countries that continued from 2003 to 2006. This directly affected both the stock and oil market in India. The sub-prime crisis and global financial recession in 2008, and its impact during and after the U.S. financial crisis, has a major impact on Indian stock, oil and foreign exchange market. For all of these major events we observe evidences of strong volatility in the Indian market. The other major events that occurred are the collapse in oil prices in 2014 that reduced demand for Indian exports and created inflation in India; Chinese markets crash, OPEC oil cut, the US election during 2016. These led to oil price variability and also created high volatility in the exchange market. The volatility in the foreign exchange market continues from 2018 with the recovery in the Indian equity markets, strong demand of the US currency and increase in fuel prices. In recent, the global pandemic due to COVID-19 leads to economic lockdown in several countries including India and slowed down the economic activities from 2020 to 2021. The wavelet power spectrum plot shows strong impact of pandemic on stock and oil market in India but no impact on exchange market. The stock market volatility happens for a short term period of max 128 days. This implies that there is no longer cycle effect on stock prices caused by the major economic events from 1999 to 2020. In case of oil price, the changes are for short term cycle of less than 64 days, except in 2020 with a period less than 256 days. For exchange rate volatility, the period of volatilities varies from 8 to 128 days. Further, the global financial recession in 2008 and the European sovereign-debt crisis in 2011 have longer effect on exchange rate. In aggregate, we observe that all the events have rapid turbulence effects on stock market returns, oil price and exchange rate in India.

Wavelet covariance measure how well two time series are associated with one another at the same time point. Cross-wavelet power spectrum describes the common power of two processes without normalization to the single power spectrum. We find strong covariance between stock return and oil prices between 2000–2001, 2008–2010 and post-2020 at scale less than 128 days; a strong co-variance between exchange rate and oil prices during the year 2009 and in 2020 at scale from 8 days to around 128 days. This implies that the wavelet covariances are positive and appear to be significantly different from zero at 5% level at regions that can be linked with an economic event. However, in cross wavelet spectrum plot, it is difficult to compare wavelet scales because of the differing variability exhibited by them. Wavelet coherence has the advantage of comparing the magnitude of the association across scales. Wavelet coherence is therefore a natural measure used instead of the wavelet covariance. The wavelet coherence also provides a lead/lag relationship on a scale-by-scale basis.

The coherency plot shows strong co-movement between stock return and oil price and between oil price and exchange rate at several time intervals and at different scales. These time intervals are linked with major macroeconomic events occurred across the world. We observe strong correlations with a period vary from lower scale of 32 days to higher scale of 512 days (approximately 2 years). The coherency at higher scale has a long term effect on the economy. The phase relationship indicates stock returns are in phase with oil prices and exchange rates are in out of phase with oil prices. Therefore, whenever stock price is up, oil prices increases as well, and whenever oil prices surge, exchange rate falls and vice versa. During the interval 2006–2011, the stock market returns are leading over oil prices at higher scale but it is lagging at lower scales. The strong correlation between stock market returns and oil prices from 2006 to 2011 at higher scale support the high economic growth rate achieved during 2006–2007 by the Indian economy. This growth was achieved due to rapid increase in investment, effects of reforms, buoyant international economic environment and contribution especially from service sector and industry sector. Relationship at higher scale implies that it has a long term and slowly changing spillover impact on Indian economy. The U.S. financial crisis in 2008 has impacted Indian market at lower scale which implies it has short term effect on the economy. Similarly, the strong correlation between stock market returns and oil prices starting from mid-2018 is linked as the economy was heading for a slowdown. This slowdown originated from the decline in GDP, shrinks in production, falls in consumer confidence, spike in unemployment rates and it further continues with the COVID-19 pandemic. For the relationship between exchange rate and oil prices, we find a strong correlation between exchange rate and oil prices from 2005 to 2010 at higher scale and from 2010 to 2011 at lower scale. Post the 2008 crisis, the macroeconomic policy in India supported growth in Indian economy. The strong coherency in opposite direction in 2010 to 2011 is the reflection of the fact that after recovery the oil prices increases and the exchange rate declines. Between mid-2005–2010, oil prices lagging to exchange rate, and between 2010–2011 and 2020–2021, oil prices leading to exchange rate. As India is a high oil importer, the high economic growth prompted strong co-movement between exchange rate and oil prices. From

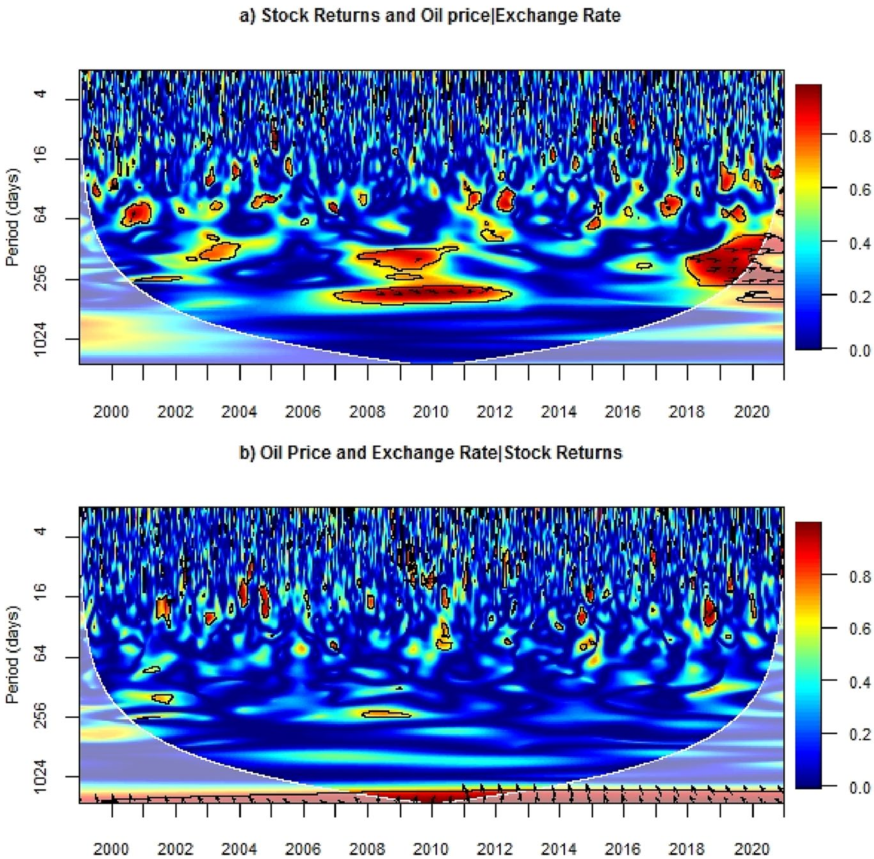

Fig. 11 Partial wavelet coherence plot. The color code for power ranges from blue (low coherence) to red (high coherence). A pointwise significance test is performed against an almost process independent background spectrum. The black contour represents a significant region at 95% confidence intervals for the null hypothesis that coherency is zero. The cone of influence is marked by white lines. The phase difference between the two series is indicated by arrows

the power spectrum plots and phase angle directions, it is observed that there is no regime shift or structural break in the behavior of our time series of interest. The overall analysis implies that there are progressive changes in the time series such as stock returns, exchange rate and oil prices in Indian economy. These changes are compounded with the volatilities caused by macroeconomic events in the market.

The partial coherency estimates the correlation and causal relationship between the two series controlling for the influence of third time series. Figure 11 portraits the partial coherency estimates between (a) stock market returns and oil prices, controlling exchange rate, and (b) oil prices and exchange rate, controlling stock market returns. The pattern of partial coherency in Fig. 11a is quite similar to that in Fig. 9. Similarly, the pattern of partial coherency in Fig. 11b is quite similar to that in Fig. 10, except that we find a strong correlation between exchange rate and oil prices from 2005 to 2010 at the higher scale of 512 days. There is no significant variation in medium and low frequency region. The interpretation of the results given by the partial coherencies support the pattern found in wavelet coherence results. The causality relationship between stock returns, oil prices, and exchange rates are consistent with the baseline results.

6 Conclusions

The co-movement between the time series, their volatility and phase relationship provide lots of insight into data from February, 1999 to March, 2021 on Indian market. We find several strong evidences of co-movements between oil price and stock market and between oil price and foreign exchange rate. Each of these associations is linked with some important economic or geopolitical issues in the world economy. The global financial crisis in 2008 has a strong impact on the Indian stock, oil and foreign exchange market. In recent, the pandemic due to COVID-19 also has created volatility on stock and oil market in India but no major impact on exchange market. The high economic growth rate achieved during 2006–2007 by the Indian economy reflects long term relationship between stock market and oil price and between oil price and foreign exchange rate. From the phase relationship, we find stock returns are in phase with oil prices and exchange rates are in out of phase with oil prices. We also conclude that the economic shocks in developed market have a spillover effect on the Indian market. However, the periods of these impacts are different at different time interval. Each time, the impact of change in volatility at lower scale has a short term effect with a period less than 128 days. The wavelet coherency at higher scale (approximately 2 years) has a slower change and long term effect on the relationship between the variables of our interest.

Using wavelet analysis, we have shown that it is a powerful tool for analyzing time series with different time scales that cannot be provided by other classical spectral techniques. The results found in this paper will motivate further academic research on financial data using the wavelet technique. The results are useful for investors who aims specific time horizon of their investment goals and preferences, for portfolio managers and in risk assessment. Understanding the leading and lagging relationships will help in business cycle based investing by detecting the subsequent business cycle fluctuations and forecast the trend.

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Availability of data and material Data is available upon request.

Code availability All the codes are available upon request.

Declaration

Conflict of interest The author declares no conflict of interest.

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