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Review article

A survey on puncture models and path planning algorithms of bevel-tipped flexible needles

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ABSTRACT

Percutaneous needle insertion is a minimally invasive surgery with broad medical application prospects, such as biopsy and brachytherapy. However, the currently adopted rigid needles have limitations, as they cannot bypass obstacles or correct puncture deviations and can only travel along a straight path. Bevel-tip flexible needles are increasingly being adopted to address these issues, owing to their needle body's ease of deformation and bending. Successful puncture of flexible needles relies on accurate models and path planning, ensuring the needle reaches the target while avoiding vital tissues. This review investigates puncture models and path-planning algorithms by reviewing recent literature, focusing on the path-planning part. According to the literature, puncture models can be divided into three types: mechanical, finite element method (FEM), and kinematic models, while path-planning algorithms are categorized and discussed following the division used for mobile robots, which differs from the conventional approach for flexible needles—an innovation in this review. This review systematically summarizes the following categories: graph theory search, sampling-based, intelligent search, local obstacle avoidance, and other algorithms, including their implementation, advantages, and disadvantages, to further explore the potential to overcome obstacles in path planning for minimally invasive puncture needles. Finally, this study proposes future development trends in path-planning algorithms, providing possible directions for subsequent research for bevel-tipped flexible needles. This research aims to provide a resource for researchers to quickly learn about common path-planning algorithms, their backgrounds, and puncture models.

1. Introduction and motivations

1.1. Introduction

With the rapid development of minimally invasive surgery and localized treatment, puncture surgery has been widely adopted in clinical practice for performing various tasks on most organs in the human body, including biopsy, brachytherapy, drug delivery, and targeted therapy [1]. Thus, this mode of surgery has attracted widespread recognition and is likely to witness future developments. Currently, the needles used for puncture surgery in clinics are rigid. However, these rigid, straight needles can only travel and orient along a straight path, lacking the ability to bypass obstacles such as bones, vessels, and important organs. For example, in functional neurosurgery procedures, which is one of the core areas of minimally invasive, the needle trajectory is also restricted to be linear,

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limiting the target point position and the degree of freedom, which is a difficult problem to break through for some of the major surgeries such as deep brain stimulation [2], spinal cord stimulation [3], and epilepsy surgery [4]. Owing to this demerit, performing minimally invasive surgery deep in the human body or the brain, spinal cord, internal organs, etc., and their vital parts is challenging. Moreover, the needle tip may deviate from the target position because of factors like tissue deformation resulting from needle-tissue interactions and the slight bending of the needle body under force, leading to decreased puncture accuracy. In the event of a deviation or obstacle, the needle tail is either manually adjusted or removed, and puncturing is repeated to forcefully correct the deviation and prevent any damage to important tissues and organs; however, this approach may cause secondary injuries to patients and increase their suffering and postoperative recovery time [5]. For example, during cardiac [6] or lung cancer [7] surgeries, the dynamic nature of a patient's heartbeats or breathing causes the position of organs to be non-static. As a result, the needle tip may deviate slightly from its original position. Correcting this deviation using the above-mentioned method can cause more pain for the patient and even damage other organs. Although some researchers have proposed the idea of "bendable steel needles (rigid needles with the same curvature for both inner and outer needles)" to achieve non-linear punctures, this approach is highly inflexible, challenging to achieve the desired results, prone to errors, and can potentially harm the patient [8].

To address the challenge of correcting needle deviation and bypassing obstacles during penetration, Webster et al. [9] first proposed the concept of an asymmetric bevel-tipped flexible needle made of a nickel–titanium alloy with low stiffness. The asymmetry of the bevel tip produces an uneven resultant transverse force when puncturing into soft tissues, causing the needle tip to deflect and the needle body to bend. Thus, the puncture path of this needle is different from the rigid needles as it can be one or more circular arcs. This allows the puncturing needle to flexibly avoid tissues and precisely target locations that cannot be reached using traditional steel needles. However, owing to the incomplete constraint of the needle tip [9,10] and the presence of sensitive tissues and obstacles (such as bones), obtaining the trajectory to control the needle reaching the desired target while avoiding vital tissues is a key issue in percutaneous surgery. This issue is of great significance for fully using the characteristics of flexible needles in minimally invasive surgery. Therefore, puncture modeling and path planning of flexible needles should be investigated as they are the basis of motion control, a major prerequisite in minimally invasive surgery.

Currently, path-planning algorithms for flexible needle puncture are generally divided into three categories: numerical, inverse solution, and search methods [11]; however, these methods either focus only on shortening the path length or shortening the time, and there is no guarantee that the paths obtained are optimal and not very real-time. To address the shortcomings of existing flexible needle path-planning algorithms, we find that a large number of mature path-planning algorithms have emerged in the field of mobile robot and unmanned aerial vehicle, with tremendous developments in flexibility and safety. Thus, to promote the development of minimally

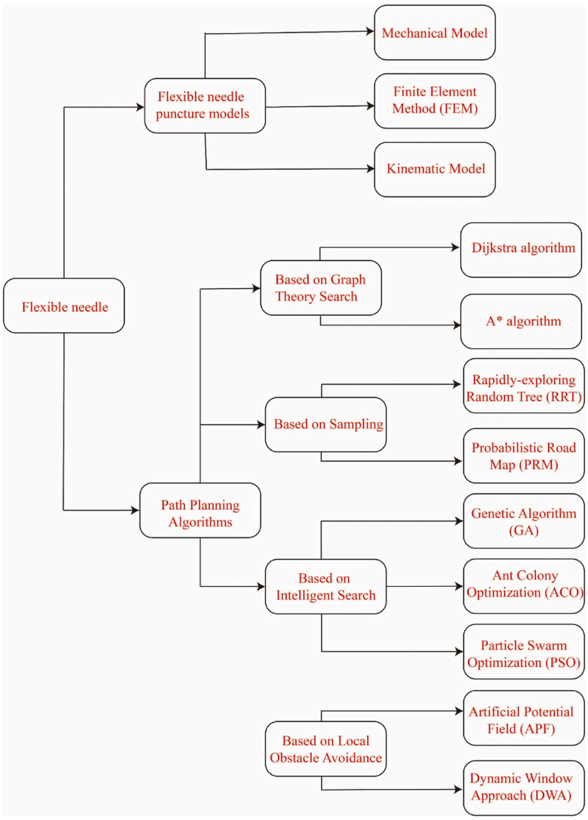


Fig. 1. Classification chart of flexible needle path planning algorithms.

invasive puncture motion planning and to improve minimally invasive interventional medicine, referring to mobile robot path planning, we categorize the needle puncture path-planning algorithms into four classes of algorithms (Fig. 1), which can broaden the field of flexible needle path-planning algorithm and provide new technologies and directions for clinical applications. To our knowledge, this division method for puncture needle path planning is yet to be reported in the related literature.

1.2. Motivations

Since the application field of this study is flexible puncture needle, we will prove the research motivations from the clinical direction, which mainly includes the following aspects.

1.2.1. Functional neurosurgery procedures

In recent years, advances in medical technology and brain science have turned previously unthinkable surgeries into reality, and more and more diseases that were once untreatable have found solutions in functional neurosurgery [12]. For example, deep brain stimulation (DBS) [13,14] can achieve good results in treating movement disorders; chronic neuromodulation with vagus nerve stimulation (VNS) [15] can be used to treat patients with heart failure; and spinal cord stimulation (SCS) [3] for paraplegic patients can significantly improve the quality of life of patients. These new technologies and methods have broadened the scope of functional neurosurgery and opened the door to healing for a wide range of patients. However, most minimally invasive neurosurgical procedures have a linear probe travel trajectory, limiting target flexibility and not conducive to surgical safety. Therefore, research on the core of technology-minimally invasive surgical trajectories is also essential, and it is beneficial to utilize flexible needles to plan out non-linear paths in surgery.

1.2.2. Biopsy and identification of tumors

Puncture biopsy is a commonly used diagnostic method in oncology. It involves using a needle to extract tissue samples for examination. Accurate needle placement is crucial for obtaining reliable specimens. If the specimens are incomplete or even wrong owing to poor accuracy of needle placement, the full picture of disease information cannot be obtained. Puncture modeling is the foundation for path planning and needle steering, while path planning provides the control sequences for needle steering. To further improve the accuracy of needle placement, imaging techniques like ultrasound recognition and magnetic resonance imaging can be used as visual feedback for rapidly replanning the trajectory of flexible needle [16,17].

1.2.3. Brachytherapy

Globally, common cancers in women include breast, colorectal, and lung cancer, while men often face lung, colorectal, and prostate cancer [18]. Brachytherapy is a highly effective local treatment for cancers like breast and prostate cancer, relying on precise radioactive material placement within the patient's body. Accurate needle trajectory simulation through path planning reduces the risk of harming healthy tissue, enhancing procedure safety and aiding physicians during brachytherapy.

1.2.4. Targeted therapy for cancer

Approximately 19–20 million people receive cancer diagnoses yearly, and about 10 million people died from it in 2020 [19]. Targeted therapies are a key focus of cancer treatment, involving the delivery of drugs to specific cancer sites to disrupt proteins that promote tumor growth and spread. Accurate injection is crucial to avoid damage to surrounding organ tissue. Needle path planning provides physicians with a reference for flexible needle trajectories, and real-time planning during surgery can be achieved using ultrasound and CT visual feedback [20,21].

For the above mentioned applications, manual manipulation of the needle is difficult and error-prone for physicians due to problems such as tissue deformation and errors in flexible needle movement. Hence, robot-assisted puncture is combined to precisely manipulate the puncture position and trajectory. Therefore, several studies on robot-assisted flexible puncture needle steering have emerged, and several related reviews have appeared in the literature [22–24]. However, due to technical and ethical factors, most tissues in the study of flexible puncture needles are still gelatin, hydrogel, and other prosthetic tissues, and flexible puncture needles have not been formally used in clinical practice. Puncture modeling and path planning of the flexible needle are crucial before the actual puncture so that the needle tip follows a planned route and the puncture needle reaches the target site smoothly, which reduces the risk of surgical failure.

As the puncture model is the basis of path planning and needle steering, this study first introduces the puncture model for the motion characteristics of the bevel-tipped flexible needle. Then, we generalize the four categories of flexible needle path-planning algorithms and present representative examples of each. These are used to identify the advantages, disadvantages, and avenues of improvement for each algorithm. Finally, this study outlines prospects for future research and predicts future development trends.

2. Bevel-Tipped flexible needle puncture models

Needle insertion into soft tissue generally involves flexible needle bending, soft tissue deformation, and needle–tissue interaction. In this regard, needle–tissue interaction models can be categorized as mechanical, finite element method (FEM), and kinematic models. Accurate modeling of the interaction between the needle and the tissue is essential for understanding the behavior of the flexible needle during penetration and for designing effective needle tips and planners.

2.1. Bevel-tipped flexible needle mechanical model

Flexible needle bending occurs during puncture because of a series of resisting forces tissues provide [25]. The resultant force of these forces determines the bending direction of the needle axis and the radius of curvature, which are critical factors determining the prospective reach of the flexible needle, ranging from the entry point to the target point. Therefore, a well-defined mechanical model for the flexible needle must be established to accurately obtain its bending conditions in tissues, which lays the theoretical and experimental foundation for subsequent research, such as path planning.

Okamura et al. [26] proposed a mechanical model for needle insertion; in their model, the forces from different sources were separated into three parts (capsule stiffness, friction, and cutting); the puncture force is presented by Equation (1). The model was validated using data from needle insertions into bovine liver, analyzing the impact of needle geometry. The developed mechanical model can be employed for designing virtual environments and robot-assisted needle manipulation simulations for percutaneous therapies.

$$f_{needle}(x) = f_{stiffness}(x) + f_{friction}(x) + f_{cutting}(x), \tag{1}$$

where, $f_{stiffness}(x)$, $f_{friction}(x)$, and $f_{cutting}(x)$ are the capsule stiffness, friction, and cutting forces, respectively, as a function of the tip displacement x. The soft tissue puncture force model for flexible needles based on an improved Karnopp friction model and Dahl friction model is presented by Equations (2) and (3).

$$f_{needle}(x) = \begin{cases} f_{stiffness}(x) = kx \quad 0 \le x \le x_{tip} \\ f_{friction}(x) + f_{cutting}(x) \sin\alpha = b_p v + f_{cutting} \sin\alpha \quad x < x_{tip} \end{cases}, \tag{2}$$

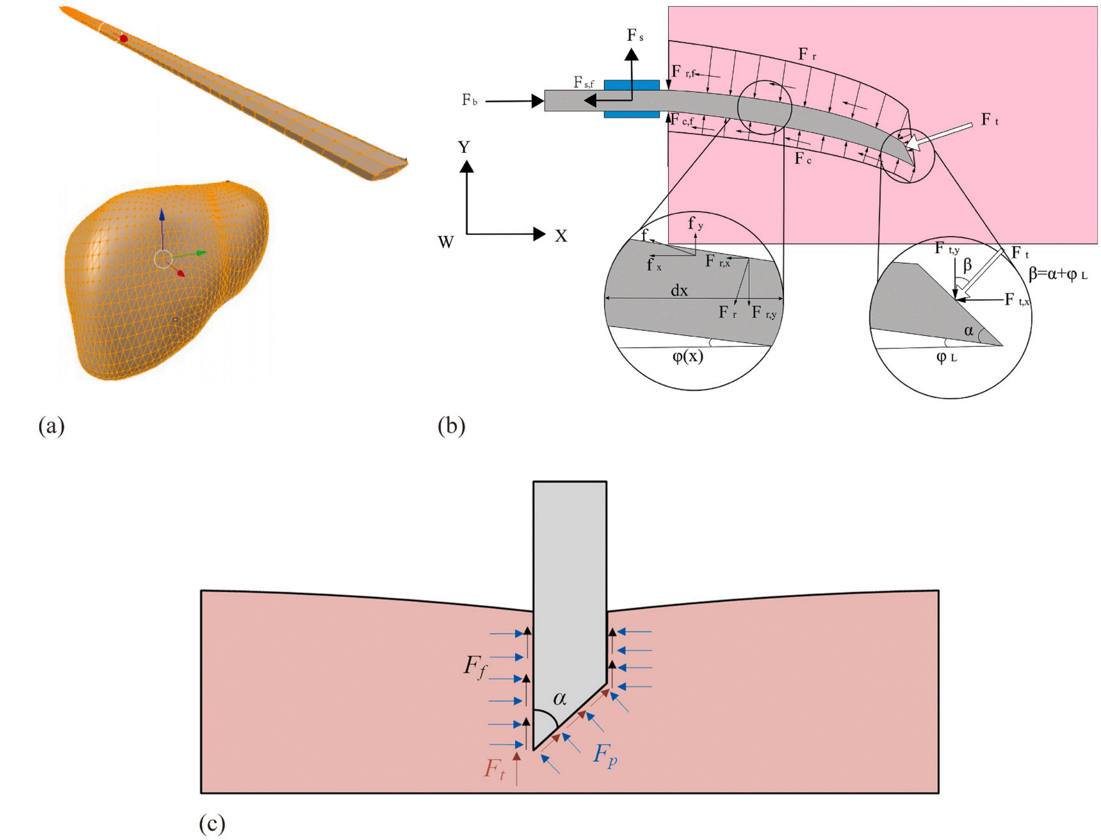


Fig. 2. Mechanical model of a bevel-tipped flexible needle: (a) the geometric model of the flexible needle and liver [31]; (b) mechanical model of a flexible needle puncturing soft tissue [25]; (c) force analysis of the interaction between needle and trachea tissue during puncturing [34].

$$f_{needle}(x) = \begin{cases} f_{stiffness}(x) = kx \quad 0 \le x \le x_{tip} \\ f_{friction}(x) + f_{cutting}(x) sin\ \alpha = f_{friction}(x) + f_{cutting}\ sin\ \alpha \quad x < x_{tip} \\ df_{friction}/dx = \sigma\left(1 - f_{friction}/F_{cutting}\right) \end{cases}, \tag{3}$$

where, k represents the spring elastic coefficient, b_p is the positive damping coefficient, v is the puncture speed, α is the needle tip degree, σ is the material stiffness coefficient, and x_{tip} represents the maximum displacement of the needle penetrating the membrane skin.

Moreover, Kobayashi et al. [27] proposed a friction force model obtained through experiments performed with relative velocities, including hyper-slow velocities between the needle and liver tissue. Furthermore, Mahvash et al. [28] described the mechanics of rupture events by adopting the J integral method and analyzed the effect of insertion velocity on rupture force and tissue deformation based on a non-linear viscoelastic Kelvin model. Their results revealed that the friction forces and rupture forces changed dynamically with the relative velocity between the needle and tissue. However, in the method reported by Kobayashi et al. [27] and Mahvash et al. [28], other forces generated through the needle–tissue interaction, such as extrusion pressure and resistance, were not analyzed.

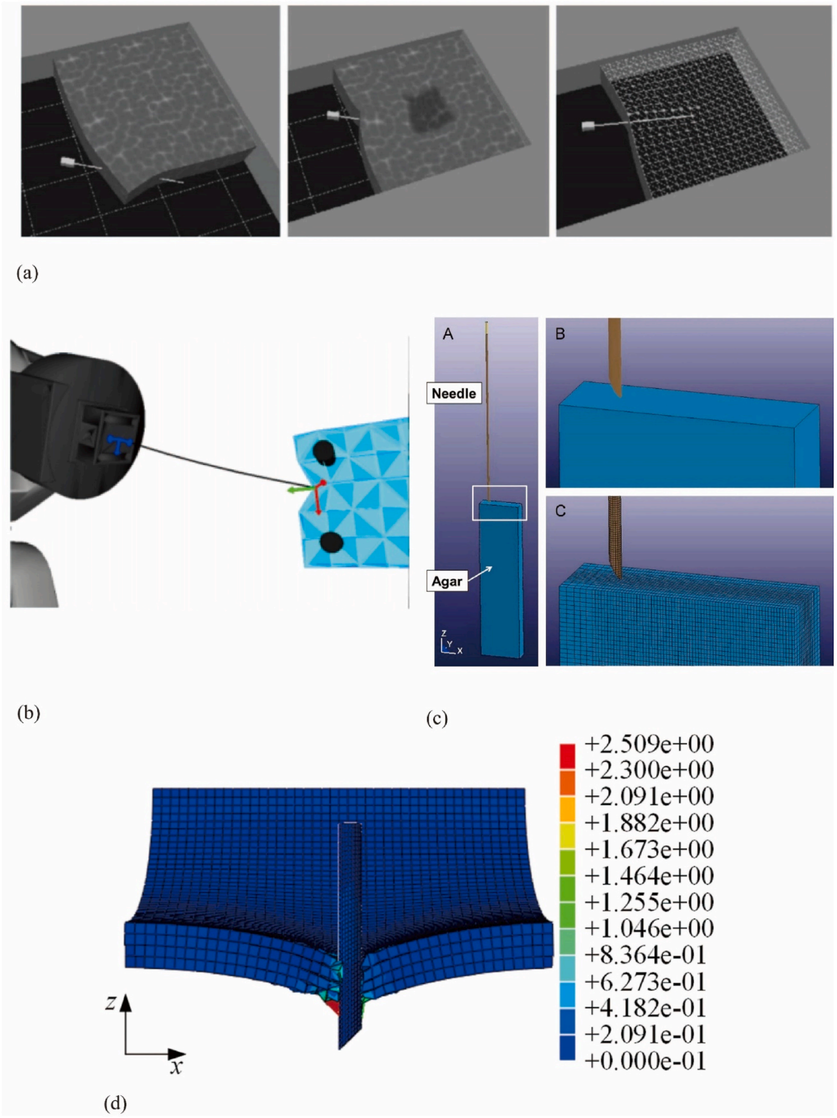


Fig. 3. FEM model of flexible needle puncture: (a) interactive simulation of needle puncturing in a planar environment [35]; (b) finite element simulation of robot–controlled needle insertion into tissue [38]; (c) FEM model for needle insertion analysis [39]; (d) finite element simulation of puncturing process for needle and trachea tissue [34].

Additionally, Misra et al. [29] presented an energy-based mechanics model for predicting needle deflection; they calculated the geometry and material properties of a bevel-tipped needle inserted into a soft elastic medium. Their simulation results match optimally with the results of needle insertion experiments, indicating the efficacy of the energy-based method in predicting needle deflection. Moreover, Zheng et al. [30] proposed a cantilever beam model to predict the needle path according to the laws of needle deflection. Based on the analysis of needle forces, the puncture process was regarded as the quasi-static process, and the flexible needle was divided into a finite number of segments treated as cantilever beams. Subsequently, the path of the needle tip was calculated according to the laws of deflection. However, the findings of Zheng et al. [30] have not been formally implemented in needle and tissue manipulation.

Yang et al. [31] conducted dynamic simulations of flexible needle penetration into the human liver using high-quality tetrahedral meshes integrated into SOFA. They identified and modeled stiffness, friction, and cutting forces based on simulation-derived force data, which is likely more accurate than data obtained from gelatin substitutes. Their research findings indicate that this mechanical model may be suitable for robot-assisted needle steering training. The geometric model of the flexible needle and liver used for these experiments is shown in Fig. 2(a).

Based on the non-linear virtual spring theory, Du et al. [32] considered the needle as a cantilever beam supported by a series of non-linear springs with different spring stiffnesses, established the friction force model and cutting force model of a needle puncturing soft tissue, and verified its accuracy via simulation. Assuming that the deflection of the needle is smaller than the puncture length and using a modified local constraints method, Gao et al. [33] established a new needle–tissue puncture mechanics model and conducted experiments to validate that this model can effectively analogize the deformation of flexible needles during tissue puncture.

Because of the complex forces arising during puncture through a bevel-tipped flexible needle, force analyses in the literature are not comprehensive. To address this challenge, Zhang et al. [25] comprehensively and systematically analyzed the forces on a flexible needle and presented the force model shown in Fig. 2(b) by relating the forces on the needle tip. Notably, sensors used to measure the relevant forces reported favorable data that supported this work as the theoretical foundation for subsequent research focused on modeling flexible needle bending.

Considering that the real tissues utilized in the aforementioned studies were only human or animal liver, Bao et al. [34] created a mechanical model to study how needle tips interact with tracheal tissue during respiratory disease examinations. They conducted in vitro puncture experiments on pig trachea tissue to better understand interactions with various organs. Their findings provide a vital foundation for puncture needle design and improved surgical safety, as shown in Fig. 2(c). The mechanics model simulates the interaction force of the entire axis of the needle and the surrounding soft tissue; however, analyzing the forces on the needle is extremely complex and usually requires simplifying the mechanical model.

2.2. Bevel-tipped flexible needle finite element model

Owing to the recent popularity of puncture surgery, scholars across the globe have focused on puncture modeling. Because of the complexity of needle-tissue interaction and the high cost of manpower and resources involved in puncture experiments, scientists have proposed FEM models for describing puncturing of soft tissue with needles. Finite element analysis (FEA) replaces the complex whole system with smaller finite units, simplifying the complex problem. Currently, it is the most popular approach for modeling such interactions, and several scholars have applied this technique to model the puncturing of soft tissues with flexible needles.

Dimaio et al. [35] developed an experimental system to measure tissue phantom deformation during puncture in the early FEA stages of needles. They estimated contact force information based on a material model with two-dimensional linear elasticity discretized using FEM. This allowed them to create a virtual needle-insertion simulation system based on the estimated forces. The system's validity was confirmed through experimental simulations, as seen in Fig. 3(a). However, real-time execution is challenging due to the substantial computational load associated with FEM, making online implementations impractical.

Oldfield et al. [36] incorporated cohesive elements into a high-resolution FEM model, and the acquired simulation data were used to perform experiments on needle puncturing tissue, which supported the suggested FEM model. Assaad et al. [37] presented an FEA approach for modeling the interaction between a bevel-tipped needle and gel; their approach was based on a damage model featuring linear elastic material behavior. The simulation and experiment result errors for force along the insertion axis and torque did not exceed 5 %. In 2016, Adagolodjo et al. [38] created an inverse FEM model for gelatin puncturing during automatic robot-assisted flexible needle procedures in soft tissue. They used a Newton-type method to control the displacement of needles by the robot, aligning it with the desired trajectory to the best extent. This model was validated through simulations, as depicted in Fig. 3(b). However, as far as we know, this model has not been implemented in an actual robot-assisted puncture system.

Yamaguchi et al. [39] proposed a three-dimensional FEM model based on the Arbitrary Lagrangian–Eulerian (ALE) method for dynamic analysis. They used three types of oblique pointed needles for experiments involving insertion into agar gel, as shown in Fig. 3 (c). The results demonstrate the usefulness of this model for preoperative planning and analysis of large deformations and fractures in biological tissues. Jushiddi et al. [40] introduced a FEM model using the Coupled Eulerian-Lagrangian (CEL) approach to quantify needle deflection forces. They also conducted experiments to validate the effectiveness of this model in optimizing needle tip design and enhancing biopsy needle-cutting efficiency. However, it is worth noting that both methods mentioned require a relatively high level of mesh precision.

In 2021, Perrusi et al. [41] added complementary constraints to present a new FEM model, which reduced the accuracy requirements for meshes, could simulate the forces caused by certain complex behaviors, and was validated through simulations of needle insertion and robotic needle insertion in liver tissue experiments. To obtain more realistic and comprehensive experimental data, Bao et al. [34] used material parameters of porcine tracheal tissue for finite element simulation of the puncture process to obtain the optimal conditions suitable for the puncture procedure (the force on the needle was minimal when the needle's diameter was 22G, and it penetrated vertically into the trachea tissue) and to provide data support for subsequent clinical procedures, as shown in Fig. 3 (d). Based on the viscoelastic constitutive equation of muscle tissue, Jiang et al. [42] defined the fracture characteristics of muscle tissue and puncture needle and established the FEM of puncture muscle tissue. By investigating the deformation of soft tissue in the model, the needle puncture process can be guided to reduce patient trauma.

2.3. *Bevel-tipped flexible needle kinematic model*

Every flexible needle follows a circular trajectory within the tissue as it bends in response to the tissue extrusion pressure during puncture. Generally, flexible needles are influenced by two degrees of freedom: (1) feed movement to manage needle advancement and retraction and (2) rotational motion around the needle axis to alter the bending direction of the needle. The mechanical analysis and experimental data of soft tissue puncture by a bevel-tipped flexible needle indicate that changing the needle feed and rotation controls the trajectory of the needle. This motion is classified into three categories as follows.

1. If only flexible needle feed movement is allowed as the needle is highly flexible, friction and lateral forces exerted by the tissue during puncture will cause the needle axis to bend, forming an arc in the tissue. If the needle tip is rotated by 180°, it produces a second arc, and so forth. Thus, the needle axis can be turned to form multiple arcs in the tissue (Fig. 4(a)) [10,43].
2. When simultaneous rotational and feed movement is allowed with a feed rate considerably higher than the rotational rate, the needle axis forms a spiral line trajectory in the tissue (Fig. 4(b)) [43,44].
3. If the feed rate is much lower than the rotation rate, the needle axis forms a nearly straight-line trajectory in the tissue (Fig. 4(c)) [45,46].

Recently, based on the properties of this movement, scholars across the globe have adopted and improved kinematic models for flexible needles.

As the feed and rotation movement is similar to the moving forward and wheel steering of a bicycle, respectively, Webster et al. [9] described a bicycle model (BM) (Fig. 4(d)) and unicycle model (UM) of bevel-tipped flexible needles based on the Lie group theory—the non-holonomic bicycle model as shown in Fig. 4(d). These models revealed that control of the two degrees of freedom of the flexible needle could allow the needle axis to move along one or more circular arcs with a changeable radius. The UM was validated for puncturing soft tissues using flexible needles in practical settings through experiments based on puncturing agar. However, the modeled and real trajectories exhibit slight differences.

In the non-holonomic unicycle model, the angular velocity ω and linear velocity v can be expressed by Equations (4) and (5).

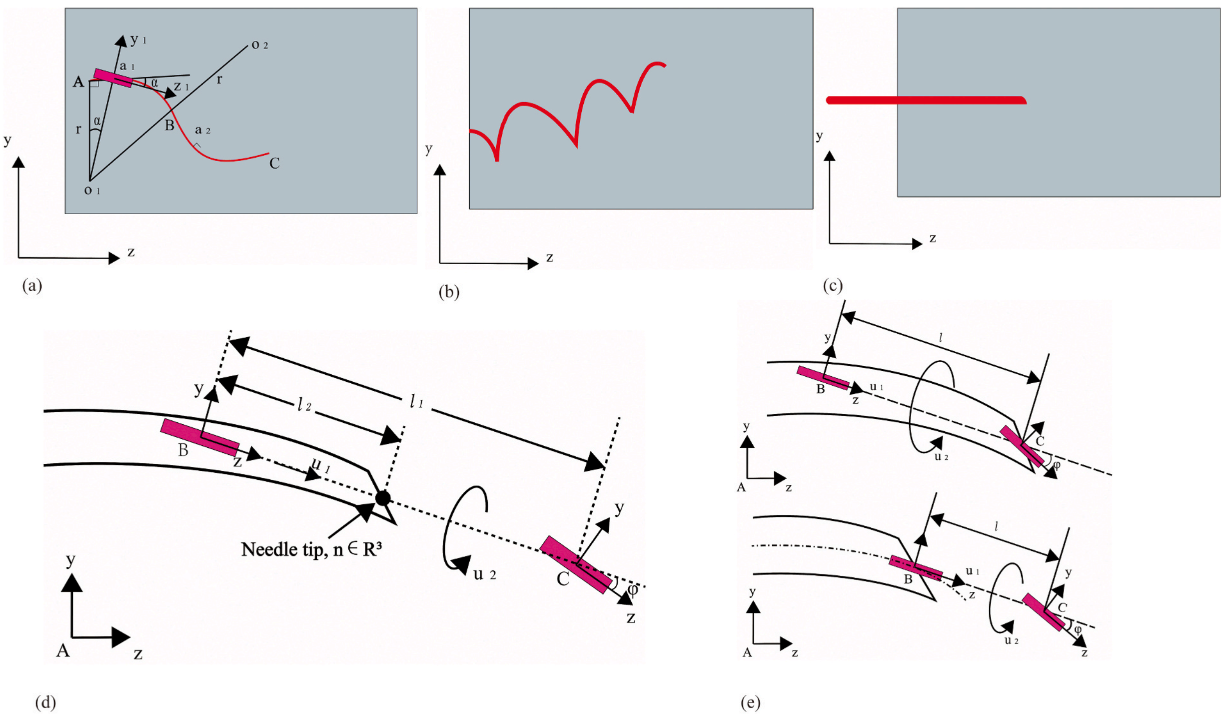


Fig. 4. Flexible needle kinematic model: (a) multiple arcs path; (b) spiral line path; (c) straight-line path; (d) Nonholonomic bicycle model proposed by Webster et al. [9]; (e) Simplified bicycle model [47].

$$\omega = u_1 / r \tag{4}$$

$$v = [0\ 0\ u_1]^T \tag{5}$$

Equation (1) describes that the motion trajectory function and kinematic model can be expressed by Equations (6) and (7).

$$\mathrm{F} = \mathrm{f}(\mathrm{r}, u_1, u_2), \tag{6}$$

Table 1
Three puncture models.

Classification	Models	Advantages and Disadvantages
Mechanical model	Friction model and rupture model proposed by Kobayashi et al. [27] and Mahvash et al. [28], respectively.	**Advantages:** The friction and rupture forces change dynamically with the relative velocity between the needle and tissue, which is beneficial for studying the parameters of the intraoperative information recognition model. **Disadvantages:** The force analysis was not comprehensive; other forces generated through the interaction of needle and tissue, such as extrusion pressure and resistance, were not analyzed.
	The forces on a flexible needle analyzed, and the force model presented comprehensively and systematically by Zhang et al. [25].	**Advantages:** It can relate the forces on the tip of the needle, and favorable data from sensors used to measure the relevant forces support this work as the theoretical foundation for subsequent research modeling of flexible needle bending. **Disadvantages:** This study modeled a specific bevel-tipped flexible needle puncturing into a specific tissue through experiments; thus, the experimental materials are simple and unsuitable for practical applications.
	The mechanical model of the needle tip and trachea tissue established by Bao et al. for the examination of respiratory diseases [34].	**Advantages:** The model reflects the interaction between real trachea tissues and needle tips, which provides a more powerful theoretical basis for the design of puncture needles and for enhancing surgical safety. **Disadvantages:** This model does not consider the deformation of the flexible oblique tip into the trachea tissue and can only be applied to steel needle puncture.
finite element method (FEM)	An inverse FEM model established by Adagolodjo et al. [38].	**Advantages:** The method can control the displacement of the needle (through the robot) by following the desired trajectory to the maximum extent, exhibiting a small error and high accuracy. **Disadvantages:** This model has not been applied to a true robot-assisted puncture system, thereby lacking more convincing experimental data verification.
	The arbitrary Lagrangian–Eulerian (ALE) method proposed by Yamaguchi et al. [39] and the coupled Eulerian–Lagrangian (CEL) FEM model proposed by Jushiddi et al. [40].	**Advantages:** These methods could also be applied pre-operatively to predict needle deflection and thus have the potential to serve as a stepping stone for developing pre-operative surgical planning of needle insertion. **Disadvantages:** These methods require a high mesh accuracy, as the required hardware cost is high, and the simulation time is extremely long.
	A new FEM model with complementary constraints presented by Perrusi et al. [41].	**Advantages:** This model can reduce the accuracy requirements for meshes, can simulate the forces caused by complex behaviors in real-time, and has been applied in multiple needle insertion environments. **Disadvantages:** The simulation results of this study cannot be directly applied to formal percutaneous puncture surgery; thus, it requires robot-assisted puncture experiments to adjust the parameters using a force sensor installed on a robotic system.
kinematic model	Bicycle model (BM) and unicycle model (UM) of bevel-tipped flexible needles described by Webster et al. [9].	**Advantages:** These models are established through the analysis of the needle tip motion, are simple in principle, without the need for complex force analysis, and are widely used. **Disadvantages:** The characteristic of experimental tissue material is different from real tissue, and the real-time performance is poor.
	A simplified BM with retrace proposed by Zhao et al. [47].	**Advantages:** This model is simple in principle, easy to understand, and easy to use without needing complex force analysis. **Disadvantages:** The error of the experimental data is large because the angle of the needle tip cannot be accurately controlled or compensated.

$$P = \begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ u_1 \\ u_1/r \\ 0 \\ u_2 \end{bmatrix}, \tag{7}$$

where u_1 represents the feed movement of the flexible puncture needles, u_2 represents the rotation movement of the flexible puncture needles, P is the movement state of the needle during flexible-needle motion, r represents radius.

The coordinate transformation matrix g_{ab} from the bevel-tipped frame to the world frame is described using Equations (8) and (9).

$$g_{ab} = \begin{bmatrix} R_{ab} & l_{ab} \\ 0 & 1 \end{bmatrix} \in SE(3), \tag{8}$$

$$SE(3) = R_3 \times SO(3), \tag{9}$$

where $R_{ab} \in$SO (3) (SO (3) is a three-dimensional rotation group) and $l_{ab} \in R^3$ are the rotation and location matrices, respectively, of frame B relative to frame A. SE (3) is a Special Euclidean group, also known as a rigid-body transformation group. Therefore, the bevel-tipped needle movement can be described using Equation (10).

$$\hat{P} = \begin{bmatrix} v \\ \omega \end{bmatrix}^{\wedge} = \begin{bmatrix} 0 & -u_2/r & 0 & 0 \\ u_2 & 0 & -u_1/r & 0 \\ 0 & u_1/r & 0 & u_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{10}$$

where ^ is the wedge operator used to form the matrix in se (3). The term se (3) is a Lie algebraic expression of the rigid-body transformation group SE (3), which physically represents the generalized instantaneous rigid-body time velocity.

Zhao et al. [47] simplified the BM as the front- and rear-wheel models and designed a refined BM with retrace, as illustrated in Fig. 4 (e). Experimental data demonstrated that the proposed bicycle front-wheel model with retrace was more accurate than the BM for real applications. However, none of the aforementioned studies illustrated mechanisms for controlling the bending radius of flexible needles. To solve this challenge, Minhas et al. [45] studied the variation in bending curvature when the rotating needle was subjected to different duty cycles during puncturing; they designed a kinematic model based on model parameters that fit the puncture experimental results. To better explore the interaction between the puncture needle and the tissue during actual surgery, Ye et al. [48] derived detailed models of the positive and negative kinematics of a concentric manipulable needle based on modern spiral theory. These models were also demonstrated to be advantageous for controlling manipulable needle robots in neurosurgical experiments.

Minimally invasive surgery is expected to have increased applications in clinical medicine [49], particularly in using flexible needles for soft tissue puncture. Puncture models form the basis for studying flexible needle insertions into soft tissue. Among the three mentioned models, the mechanical model can simulate the interaction between the entire needle and tissue, but force analysis is complex and often requires simplification. While the FEM approach offers high accuracy, it demands extensive calculations, leading to lengthy computing times and reduced efficiency. Furthermore, these two models remain theoretical. In contrast, the kinematic model is derived from needle tip motion analysis, with parameters obtained from puncture experiments, avoiding the need for complex force analysis. This model is simple in form and widely used in path planning.

Comparisons of the advantages and disadvantages of these three puncture models are listed in Table 1.

3. Path planning methods and results

We categorize the puncture path planning algorithm into the following categories: graph theory search, sampling, intelligent search, local obstacle avoidance, and other algorithms, drawing inspiration from path planning methods used in mobile robotics. Representative literatures for each algorithm are listed, describing the methods employed in each literature, analyzing their advantages and disadvantages in practical applications, and presenting the results of these literature studies. Among the four types of algorithms mentioned above, the first three types belong to global path planning algorithms, and the fourth type belongs to local path planning algorithms. Global path planning is based on known global roadblock map information for determining an optimal path from the starting point to the destination point, and various such algorithms have been proposed for path optimization. However, this approach has several shortcomings, such as high computational effort, low search efficiency, and slow convergence. The following sections report certain prominent subtypes of global path-planning and high-profile examples of each.

3.1. Path planning algorithms based on graph theory search

The graph theory method primarily considers graphs as the study object. The two most typical graph search path-planning algorithms are described in this subsection.

3.1.1. Dijkstra algorithm

The Dijkstra algorithm was proposed by Edsger Wybe Dijkstra [50]. This algorithm dynamically determines the shortest path from each point in a graph to the starting point and updates these points until the shortest path is found; the time complexity involved is O (n^2). The Dijkstra algorithm is simple and widely used in principle. However, when the edge weights of the graph are negative, the algorithm is not executed, and the shortest path cannot be found. This is a major drawback of the Dijkstra algorithm.

Wang et al. [51] adopted the Dijkstra algorithm and demonstrated its effectiveness in solving the path-planning problem in a maze for a robot using simulation data. Aimed at the path-planning problem of intersection attributes, Zhu et al. [52] proposed and verified a reverse-labeling Dijkstra algorithm (RLDA) to determine the travel time from the origin to the destination. They then compared the convergence rate and computation speed of RLDA with particle swarm optimization (PSO), genetic algorithm (GA), ant colony optimization (ACO), neural-network algorithm (NNA), and optimized path algorithm based on reinforcement learning (OPABRL). Their findings favor the RLDA algorithm over the other five when the network node count is below 350. However, both methods proposed by Wang et al. and Zhu et al. do not provide real-time solutions for the path finding problem.

Recently, studies have focused on improving the performance of the Dijkstra algorithm by combining it with other algorithms. Dirik et al. [53] created an algorithm that blends Rapidly-exploring Random Trees (RRT) with the Dijkstra algorithm, tested it in obstacle-laden environments, and found that it significantly reduced execution time in 2D settings. This combined approach outperformed individual algorithms, enabling effective real-time path tracking. Guo et al. [54] introduced an enhanced Dijkstra algorithm for optimizing the path planning of automated guided vehicles (AGV). Unlike the traditional Dijkstra method, this improved algorithm can identify all equidistant shortest paths. They demonstrated the efficiency and feasibility of their approach in path planning for an automatic storage-retrieval system (AS/RS), albeit without guaranteed efficiency. Çelik et al. [55] further improved the Dijkstra shortest path algorithm by continuously estimating the node distances for two successive steps between two consecutive nodes, greatly improving the computational efficiency and optimal path quality.

To date, the Dijkstra algorithm has been frequently applied in path planning in mobile robotics and vehicles. In a study by Liu et al. [56], it was found that both the Dijkstra and A* algorithms can successfully find paths while avoiding obstacles during needle insertion. However, they noted that these algorithms suffer from excessive search times and low efficiency.

3.1.2. A^* algorithm

The algorithm [57] can solve various path-planning problems with superior performance and accuracy compared with the Dijkstra algorithm. Unlike Dijkstra, which traverses every point in the graph inefficiently, A^* combines Dijkstra's and breadth-first algorithm's features to reduce the number of possible paths. It assesses each search point to find the nearest one to the goal and then searches from that point to the target, avoiding many invalid paths and significantly improving efficiency. However, A^* doesn't explore all feasible solutions and may not always yield the optimal result.

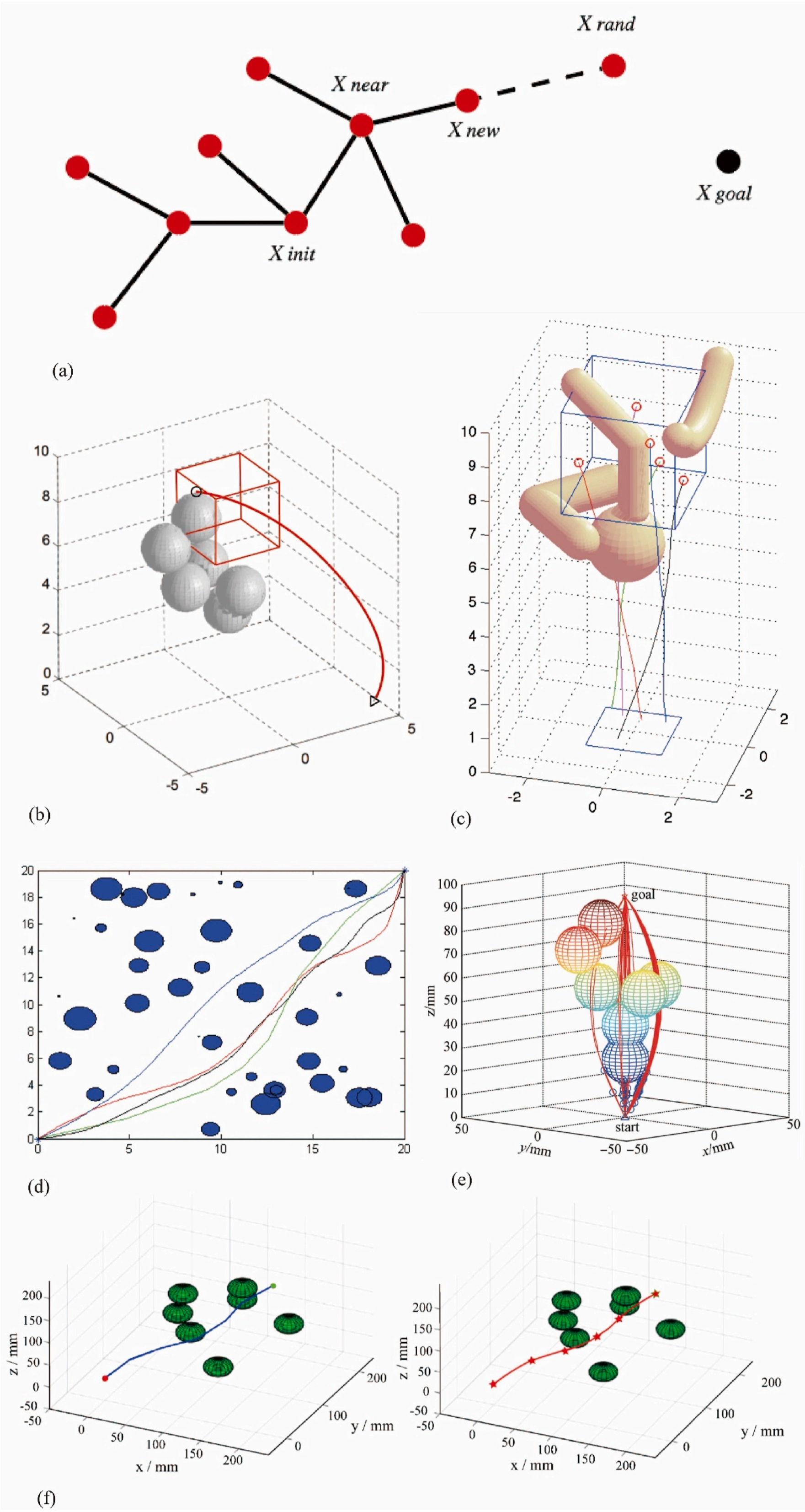
The A^* algorithm embraces the concept of the estimation function, which can be expressed through Equation (11).

$$\mathrm{F}(\mathrm{n}) = \mathrm{g}(\mathrm{n}) + \mathrm{h}(\mathrm{n}) \tag{11}$$

where $\mathrm{f}(\mathrm{n})$ represents the integrated cost from the starting point to the current point n and then to the target point, $\mathrm{g}(\mathrm{n})$ indicates the shortest cost from the starting point to the current point n, and $\mathrm{h}(\mathrm{n})$ is the predicted cost of the best path from current point n to the target point, which is the heuristic function of the algorithm. During the search process, the algorithm always selects the point with the smallest integrated cost as the next traversal point.

The A^* algorithm has been widely adopted and improved in path planning for mobile robot applications. To enhance the time efficiency and space utilization rate of traditional algorithms, Chen et al. [58] proposed the introduction of a direction vector and parallel search. They performed experiments with and without obstacles and reported that time efficiency and space utilization improved by 61.22 % and 41.38 %, respectively. However, the obtained path is not sufficiently smooth, and the quality is poor.

Xiao et al. [59] performed further augmentations to the A^* algorithm. First, they added a steering angle to the cost function to resolve excessive steering angles in mobile robots. Second, they added information from the parent nodes and steering angles to the heuristic function to enhance the time efficiency. Finally, they designed an anti-collision path-planning method that improved the smoothness of the path and involved fewer turns than previous versions. Building upon this research direction, Li et al. [60] introduced a novel heuristic strategy for solving the problems of path length, search efficiency, and turning points of the algorithm in a grid environment. Their experiments showed that in random obstacle environments, this algorithm reduced path length and turning points by 4.3 % and 34.4 %, respectively, while improving search efficiency by 31.8 % and enhancing path quality. Additionally, Guan et al. [61] proposed an improved algorithm, A^*, for path planning of unmanned surface vessels (USVs). This algorithm aids USVs in circumventing static obstacles at sea and reaching the destination smoothly while avoiding falling into local optima.



(caption on next page)

Fig. 5. Path planning based on the RRT algorithm: (a) the search graph of the RRT algorithm; (b) path planning based on back-chaining the RRT algorithm [66]; (c) path planning for multiple target points [67]; (d) path planning based on the I-RRT algorithm [49]; (e) the simulation results combined with greedy heuristic and reachable guidance RRT algorithm [74]; (f) the left image depicts the path before smoothing, and the right image depicts the final path with Bezier Curve Smoothing [78].

Muhammed et al. conducted simulation-based comparisons of the proposed Chaos A^* [62] and Circulation Heuristic Search (CHS) [63] algorithms with the classical A^* algorithm. The results indicate that both Chaos A^* and CHS algorithms generate shorter paths and require less time to complete. Additionally, they implemented an experimental setup to validate the effectiveness of these two new algorithms.

Both the A^* and Dijkstra algorithms are intelligent heuristic search algorithms. However, the paths generated by these algorithms are typically composed of connected line segments, which need further adjustment to be suitable for bevel-tipped flexible needle punctures. Researchers have combined A* with Dijkstra algorithms [64] to address path planning for multiple parallel vehicles in environments with static and dynamic obstacles, optimizing and introducing these approaches to the market.

Owing to the characteristics of graph search, the paths produced by these algorithms, including Dijkstra and A^*, are not directly suitable for needle path planning. This is because these paths often contain numerous suboptimal turning points and path lengths. Thus, further improvements are required, primarily in path smoothing, time and space utilization, and reduction of inflection points. Consequently, when addressing the problem of flexible needle path planning, it may be necessary to combine other algorithms, such as RRT and Artificial Potential Field (APF) algorithms, to find the optimal path, and improved A^* can be potentially applied to needle puncture path planning in the future.

3.2. Path planning algorithms based on sampling

Sampling-based path-planning algorithms include single-query path planning and asymptotically optimal path planning. Single query algorithms focus on finding a solution that emphasizes rapidity. In contrast, asymptotically optimal algorithms focus on optimizing the algorithm to find the optimal solution, emphasizing overall optimality. This section presents two classical single-query methods: RRT and probabilistic road map (PRM). These algorithms are primarily based on random sampling. Through the random sampling of points in space and continuous extension to the blank area with the simultaneous detection of obstacle collisions at the sampled points, a path connecting the starting point and end point can be generated.

3.2.1. Rapidly-exploring random tree

RRT, a widely used sampling-based path-planning algorithm, particularly in the static planning of mobile robots, was first introduced by Lavalle in 1998 [65]. RRT is an incremental search method that efficiently searches for paths in a multi-dimensional space. In the traditional RRT algorithm, the exploration begins by considering an initial point as the root node and continuously expanding outward until one of the leaf nodes in the search tree connects to the target point. The RRT algorithm does not require specific modeling of the environment during path planning and offers excellent adaptability to dynamic scenes. However, the search speed is low, and the path generated is not optimal. A search graph of the algorithm is illustrated in Fig. 5(a). Recently, numerous researchers have applied RRT for the path planning of flexible needles. In 2008, Xu et al. [66], based on the kinematic model of flexible needles, first adopted the RRT algorithm to plan puncture paths in a 3D environment with obstacles. They developed a back-chaining RRT-based motion-planning algorithm that grows backward from the target, as depicted in Fig. 5(b). In the following year [67], they extended the RRT to path planning for multiple needles involving multiple targets, as shown in Fig. 5(c); they revealed through simulation that the search volume increased significantly with a decrease in efficiency. Patil et al. [68] proposed a relatively faster algorithm developed by combining RRT with a reachability-guided sampling heuristic; this strategy greatly enhanced the search efficiency and was also applicable in dynamic trajectory planning. However, traditional RRT's search efficiency lags in regions with complex terrain and narrow passages, and paths generated through random sampling are inadequate. To overcome these drawbacks, researchers have proposed various RRT algorithm variants.

Caborni et al. [69] used flexible probes in their experiments and combined the reachability guidance and goal bias strategies to present a risk-minimizing RGGB-RRT algorithm. When fully exploited, this algorithm can maximally reduce the path length and distance from relevant obstacles. The RGGB-RRT algorithm has numerous applications in bevel-tipped flexible needle-puncture paths [70,71]. However, the search efficiency is low, and the optimization needs to be strengthened further.

In 2015, Xiong et al. [49] introduced an improved RRT algorithm (I-RRT) based on the unicycle model of flexible needles, as illustrated in Fig. 5(d). Compared to the traditional RRT algorithm, I-RRT can generate a continuous planned path and match the requirements corresponding to the motion characteristics of flexible needles. Additionally, it can select the optimal path, considering factors like path length, obstacle avoidance, and arc count. Simulated experiments in a 2D obstacle environment demonstrated the superior accuracy of the I-RRT algorithm compared to RRT. Huang et al. [72] applied The Potential Field-guided Rapidly-exploring Random Trees (PF-RRT) for selecting a candidate path and presented a preoperative path-planning algorithm. The simulation results indicate that the proposed method efficiently determines candidate needle paths. To optimize the RGGB-RRT algorithm, improve search efficiency, and enhance real-time performance, in 2014, Zhao et al. [73] proposed the Reachability and Greedy Heuristic Guided RRT (RGHG-RRT) algorithm, which combines RRT with a reachability-guided and greedy heuristic strategy. They conducted experiments to compare their algorithm with the RGGB-RRT algorithm in a 2D environment. The comparison revealed that the RGHG-RRT algorithm was superior in computational speed, searching capability, and path form. However, the developed algorithm has only been simulated in a 2D environment, and its suitability for 3D environments remains unclear [49,72,73].

In 2017, Zhao et al. [74] conducted 3D experiments using the RGHG-RRTs algorithm and compared it with the RGGB-RRT algorithm. Their findings indicate that the RGHG-RRT algorithm is still an optimal alternative in terms of computational speed, searching capability, and path form and is suitable for clinical operations, as indicated in Fig. 5(e). Wang [75] proposed an RRT algorithm with a point tracking and extreme trend-expansion strategy for path planning of puncturing applications using flexible needles in real-time. In this approach, a static path is initially planned. Subsequently, the path is dynamically adjusted by tracking the positions of obstacles and targets to realize real-time application. However, this approach has not yet been applied to real-world 3D environments. Karaman et al. [76] proposed the RRT^* algorithm, enhancing the RRT algorithm with asymptotic optimal characteristics. Thereafter, Zhang et al. [77] improved the RRT^* algorithm for motion characteristics of flexible needles, which enhanced the smoothness of the generated path and satisfied the movement constraints of the flexible needle. Finally, they applied the APF to the obstacle avoidance strategy of the RRT^* algorithm. The feasibility of this algorithm was verified through 3D experiments. Dong et al. [78] proposed a method that combines Bezier Curve Smoothing techniques to generate shorter and smoother paths than those generated through the method of Zhang et al., as indicated in Fig. 5(f). However, compared with the RRT algorithm, the search time of the method proposed by Dong et al. is larger, and the efficiency is low.

Numerous researchers have enhanced the RRT algorithm, with significant potential for future improvements. For instance, optimizing the tree extension as a binary tree could significantly enhance search efficiency. Additionally, exploring the impact of building a tree in both directions (from the starting and end points) may further improve search speed.

3.2.2. Probabilistic road map

PRM is a sampling-based algorithm for constructing a valid path graph in high-dimensional spaces. It operates by searching for the shortest path through random sampling in a deterministic space, performing collision detection between sampled points, and connecting adjacent sampled points using a graph search algorithm like A^*. Similar to RRT, PRM's efficiency diminishes when the planned path encounters dense obstacles or narrow passages. Additionally, its reliance on random sampling results in suboptimal paths, and as the number of sampling points increases to ensure feasible paths, the time of this algorithm requirement also grows. Therefore, PRM is considered probability-complete but not optimal.

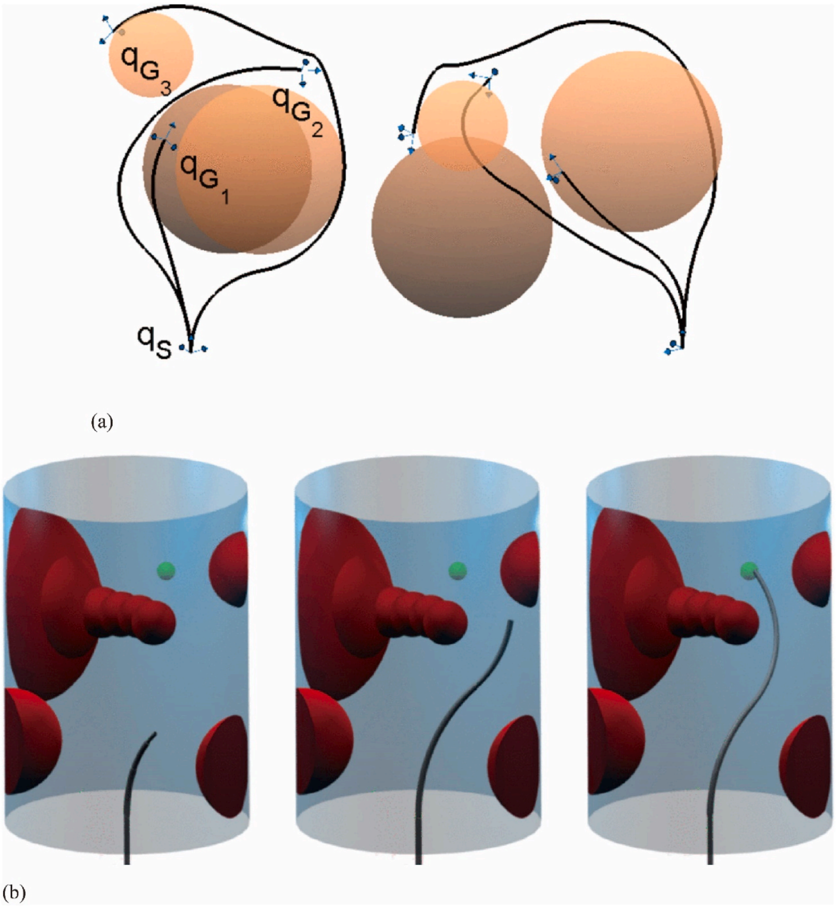


Fig. 6. Path planning based on PRM algorithm: (a) paths planned by Lobaton et al. [80]; (b) path planning based on RRM algorithm [81].

Since Kavraki et al. [79] proposed the PRM algorithm in 1996, this algorithm has been intensively studied and improved. In 2011, Lobaton et al. [80] formed a roadmap by sampling circles of constant curvature, which was used to solve the issue of needle arrival paths to multiple goals, as shown in Fig. 6(a). In the same year, Alterovitz et al. [81] developed a rapidly exploring roadmap (RRM) algorithm based on PRM and RRT algorithms to solve single-query route problems. They simulated the path planning of a puncture probe in a static environment, as illustrated in Fig. 6(b). However, the trajectories planned by these sampling-based algorithms are not smooth and do not fulfill the trajectory of flexible needle motion. In this regard, Sudhakara et al. [82] improved the traditional PRM using the PCHIP interpolation technique for the trajectory planning of flexible needles; they validated the efficiency of the algorithm based on simulations.

Sampling-based path-planning algorithms determine paths by randomly scattering points to build a graph or expand a tree and require only collision detection to sampled points and obstacles without traversing the entire space and boundaries. However, owing to their random nature, the solved path is a sub-optimal solution and does not satisfy the flexible needle non-complete motion constraints, so it must be optimized before applying it to path planning for flexible needle insertion.

3.3. Path planning algorithms based on intelligent search

Intelligent search algorithms, also known as "bionic" algorithms, are inspired by the evolution of species in nature and imitate their behaviors to solve problems. There are numerous path branches when using an intelligent search algorithm to search for a path to the target point. Therefore, the algorithm must be iterated and improved continuously for optimal solutions. Here, we discuss the genetic algorithm (GA), ant colony optimization (ACO), and particle swarm optimization (PSO).

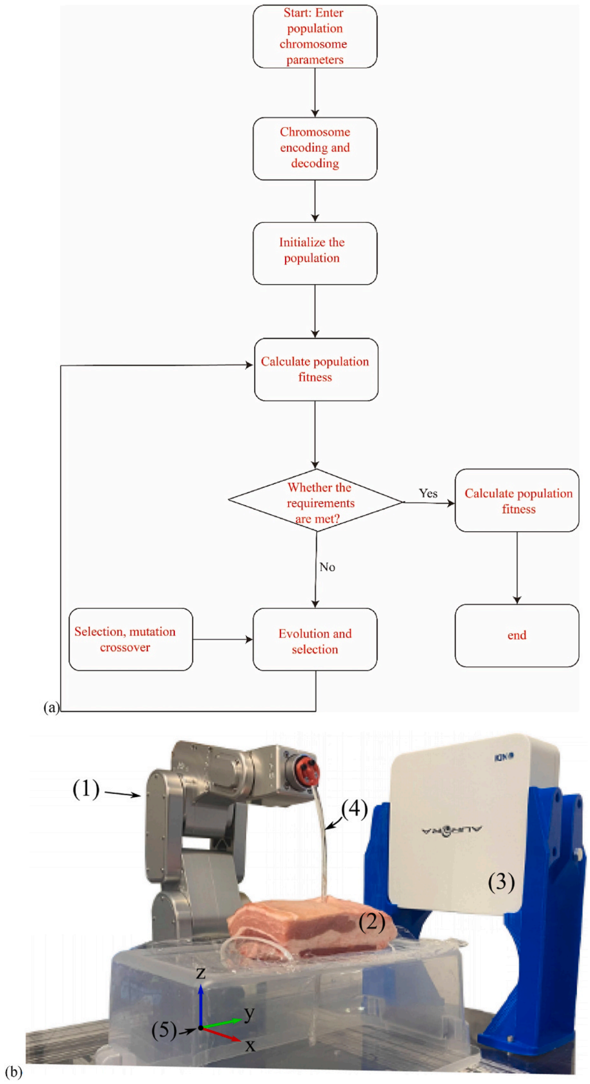


Fig. 7. Genetic Algorithm: (a) the process of the Genetic Algorithm; (b) the experiment of Wilz et al. [86].

3.3.1. Genetic algorithm

The GA was first presented by Bremermann 1958 [83], drawing upon Darwin's theory of biological evolution and Mendel's laws of inheritance to simulate the natural selection process. Through the process of mutation, crossover, replication, and inheritance of DNA, the individual fitness of a population is constantly approaching the optimal solution. However, the GA is limited in its ability to explore new space and is prone to problems such as local convergence, prematurity, low efficiency, and poor stability. The GA process is illustrated in Fig. 7(a). The parameter set information of population chromosomes serves as the first input. In the next step, chromosomes are encoded and decoded, the population is initialized, and the fitness value of population individuals is evaluated and detected. If the demand is satisfied, the optimal solution is directly obtained; otherwise, the algorithm circulates, evolving and selecting new population individuals (unlike the individuals that have been selected before), the fitness value of whom is then re-evaluated until the termination condition is fulfilled.

In recent years, this algorithm has been applied to needle-puncture path-planning problems. Konh et al. [84] optimized a memory alloy needle for auxiliary puncture to reach the target location using a GA based on a FEM model. Moreover, Zaharin et al. [85] used GA to optimize microneedles for drug delivery. The optimization included the shape of the needle, materials, and height of the microneedle, and its effectiveness was experimentally demonstrated. Furthermore, Wilz et al. [86] implemented the non-dominated sorting genetic algorithm II to ensure that a flexible nephroscope accurately reached a stone during a percutaneous nephrolithotomy on ex-vivo porcine tissue. The average error of the experimental results was 2.03 mm, as indicated in Fig. 7(b).

3.3.2. Ant colony optimization

ACO is an intelligent search algorithm using swarm behavior, which emerges from interactions among individuals within a population; this strategy offers new avenues for solving complex path-planning problems. However, ACO is also prone to slow convergence, local optimization, weak optimization ability, and other severe limitations. Italian scholars Dorigo and Maniezzo et al. initially introduced the ACO algorithm in 1991 [87] and was first used to solve the traveling salesman problem (TSP). The basic principles and mathematical models of the ACO algorithm have since been systematically studied, and ACO has progressed greatly in theory and application.

An ant foraging diagram is shown in Fig. 8. A indicates that the ants pass through the path without obstacles, and B implies that given an obstacle on the path, the ant colony randomly chooses a direction to circumvent the obstacle. In this case, the number of ants passing through both sides of the obstacle is equal because there is no pheromone on the path in C. More ants use this route due to less pheromone concentration evaporating and more residual on the short path. D implies that all the ants choose the direction with the highest pheromone concentration, indicating that the shortest path has been determined.

The formulas of the ant colony algorithm can be expressed by Equation 11–16.

$$P_{ij}^{k}(t) = \begin{cases} \dfrac{\tau_{ij}^{a}(t)\eta_{ij}^{\beta}(t)}{\sum\limits_{s\in d_k}\tau_{ij}^{a}(t)\eta_{ij}^{\beta}(t)}, (s \in d_k) \\ 0, (otherwise) \end{cases} \tag{12}$$

$$\eta_{ij} = {}^{1}/_{d_{ij}} \tag{13}$$

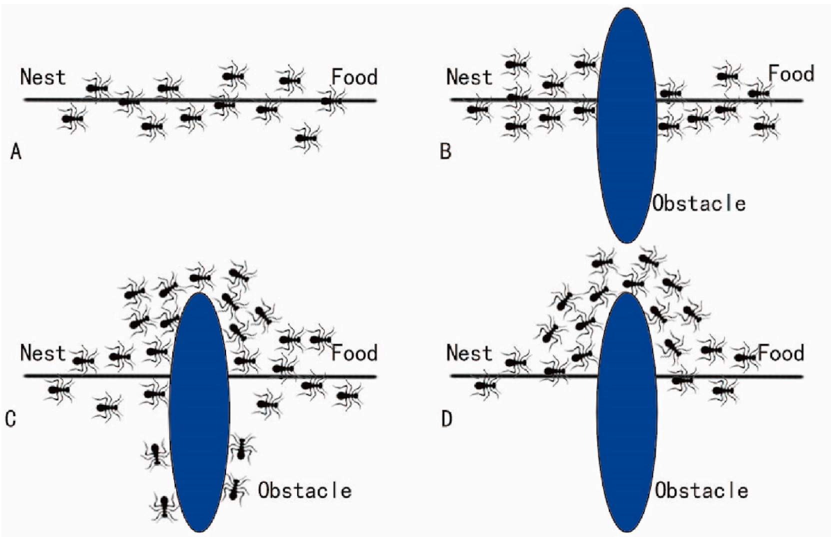


Fig. 8. Ant foraging diagram for ACO.

Herein, P_{ij}^{k} (t) represents the transition probability, $\tau_{ij}^{a}(t)$ represents the pheromone concentration, $\eta_{ij}^{\beta}(t)$ represents the heuristic function, d_k is a collection of access points, and η_{ij} is a heuristic function, usually expressed as the reciprocal of the distance d_{ij} between i and j.

$$\tau_{ij}(t+\Delta t) = (1-\rho)\tau_{ij}(t) + \Delta\tau_{ij}(t)\,, \tag{14}$$

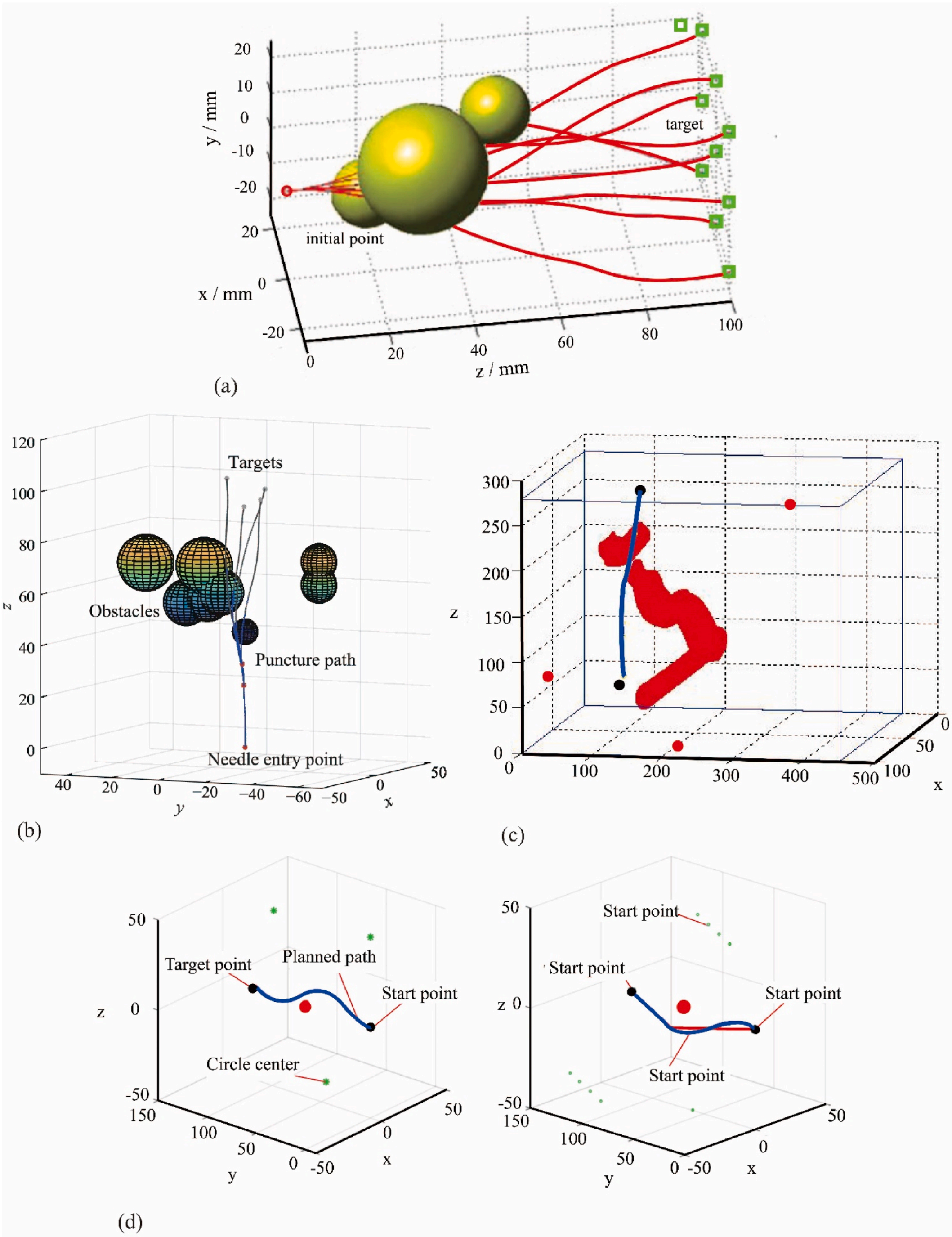


Fig. 9. Path planning based on PSO algorithm: (a) path planning based on MOPSO algorithm [93], (b) path planning based on BFL-PSO algorithm [94], (c) path planning based on PSO algorithm [95], and (d) path planning based on RRT algorithm and PSO algorithm [96].

$$\tau_{ij} = \tau_{ij} + \sum_{k=1}^{M} \Delta\tau_{ij}{}^{k}, \tag{15}$$

$$\Delta\tau_{ij}{}^{k} = \begin{cases} {}^{1}/_{d_{ij}}, Ant\ k\ pass(i,j) \\ 0, otherwise \end{cases}, \tag{16}$$

where, ρ represents the pheromone volatility coefficient, M represents the total number of ants in the ant nest, and $\Delta\tau_{ij}{}^{k}$ represents the pheromone amount released by the kth ant. Notably, the smaller the path length of the ant, the more pheromones will be obtained on each side of the path, and a greater number of ants will choose this side in the subsequent iteration.

ACO is widely used in the static environment of mobile robots. Ajeil et al. [88] proposed an aging-based ACO (ABACO) for the

Table 2
Path planning algorithms based on sampling and intelligent search.

Classification	Methods	Advantages and Disadvantages
Rapidly-exploring Random Tree (RRT)	RRT algorithm represented by Xu and Patil et al. [66–68]	**Advantages**: This strategy enhances path planning efficiency, offers a high searching speed , and is applicable in dynamic trajectory planning. **Disadvantages**: The resolved paths are sub-optimal solutions, and the accuracy of path planning is low. In areas with complex terrain and narrow passages, the search efficiency of the traditional RRT algorithm reduces considerably.
	The reachability guidance and goal bias RGGB-RRT algorithm proposed by Caborni et al. [69–71]	**Advantages**: With complete utilization of this algorithm, the path length and distance from relevant obstacles can be reduced. The path planning offers an acceptable level of obstacle avoidance, which reduces the damage caused to patients. **Disadvantages**: The search efficiency is low, and the optimization needs to be strengthened.
	The Reachability and Greedy Heuristic Guided RRTs (RGHG-RRTs) algorithm proposed by Zhao et al. [74]	**Advantages**: Compared to Caborni's method, this algorithm has a higher search speed and more robustness. This algorithm is suitable for clinical operations involving real-time path planning. **Disadvantages:** The puncture path may present deviation; the accuracy needs to be improved to address this issue.
	The RRT^{*} algorithm represented by Zhang and Dong et al. [77,78]	**Advantages**: The generated path is smooth and satisfies the kinematic constraint of flexible needle, offering acceptable obstacle avoidance. **Disadvantages:** Compared with the RRT algorithm, the search time is larger, and the efficiency is lower.
Probabilistic Road Map (PRM)	A multi-objective algorithm based on PRM proposed by Lobaton et al. [80]	**Advantages**: Path planning offers acceptable obstacle avoidance, thereby reducing the damage caused to the patient, and possesses environmental adaptability. This algorithm can realize path planning involving multiple target points. **Disadvantages:** Path planning has poor target-point positioning accuracy, and it does not consider the impact of uncertainty in the needle motion on the multi-target path planning.
	Rapidly exploring roadmap (RRM) algorithm based on PRM and RRT algorithms proposed by Alterovitz et al. [81]	**Advantages**: The planning process is fast and efficient, with high-quality paths determined through setting weights for exploration and refinement. **Disadvantages:** The generated path curve is not smooth and does not fulfill the trajectory of flexible needle motion. The path cannot be planned online in real-time.
	Enhanced PRM algorithm proposed by Sudhakara et al. [82]	**Advantages**: The generated trajectory is smooth and satisfies the kinematic constraint of flexible needles, which can be effectively applied to the trajectory planning of flexible needles in surgery. **Disadvantages:** The algorithm has a low efficiency and poor robustness, and the path cannot be planned online in real-time.
Genetic Algorithm (GA)	GA applied to puncture needles represented by Konh, Zaharin and Wilz et al. [84–86]	**Advantages**: This method can accurately trace a predefined path with minimal error. **Disadvantages**: The planning time is long, stability is poor, and the path cannot be planned online in real time.
Particle Swarm Optimization (PSO)	Single-objective optimization algorithm based on PSO proposed by Cai et al. [96]	**Advantages**: The path planning has high accuracy, and this algorithm can plan the feasible path of a bevel-tipped flexible needle in a 3D environment. **Disadvantages**: This algorithm readily generates premature convergence and poor local optimization ability when dealing with complex search problems, and the algorithm robustness is poor.
	Multi-objective optimization algorithm based on PSO proposed by Huo and Tan et al. [93,94]	**Advantages**: The path planning has high accuracy, good environmental adaptability, good obstacle avoidance, and a high level of safety. The path planning of multiple target points can be realized. **Disadvantages:** The path planning process is computationally expensive with a low planning efficiency, and the path cannot be planned online in real-time.

path-planning problem in static environments, combining grid-based modeling for static and dynamic environments. They validated the superiority of this algorithm over GA and PSO through simulation. However, the convergence speed is low, and the path smoothness requires optimization. Akka and Khaber [89] proposed an improved ACO algorithm using a new pheromone evaporation rate to improve the convergence speed and enlarge the search space to avoid local optimal solutions. Moreover, Liu et al. [90] proposed a new, improved heuristic mechanism ant colony algorithm (IHMACO) by improving four search mechanisms of the ant colony algorithm. The search efficiency and ability were further improved. Furthermore, Du et al. [91] introduced swarm intelligence algorithms and applied ACO in puncture path planning.

3.3.3. Particle swarm optimization

PSO was introduced by Kennedy and Eberhart in 1995 [92]. Similar to ACO, PSO is a swarm intelligence algorithm. The PSO algorithm works like a swarm of birds searching for food, where each bird represents a candidate solution, and the food represents the optimal solution. This parallel algorithm has the advantages of easy implementation, high precision, and fast convergence. However, problems such as precocious convergence and poor local optimization ability are likely to occur when dealing with complex multi-peak search problems.

Huo et al. [93] proposed a multi-objective PSO (MOPSO) path-planning algorithm by analyzing the flexible needle insertion kinematics model into soft tissue. The superiority of this algorithm was verified through simulation, as illustrated in Fig. 9(a). Tan et al. [94] further optimized the PSO algorithm for the multi-objective path planning problem; they proposed a PSO algorithm based on bee foraging learning (BFL-PSO). Compared with the method reported by Huo et al., The method has higher puncture accuracy, shorter path lengths, and better environmental suitability, as visualized in Fig. 9(b). In 2018, Li [95] improved the RRT and PSO algorithms based on ultrasonic recognition and the puncture mechanical model; the two algorithms were applied to a 3D path-planning task and comparatively studied. The improved RRT algorithm was better suited to practical situations than PSO, as shown in Fig. 9(c).

In 2020, Cai et al. [96] used the PSO algorithm to control the central angle of the arc and the rotation angle of a needle body; with this approach, they planned the feasible path of a bevel-tipped flexible needle in a 3D environment. Experiments show that PSO has higher puncture accuracy than RRT, as shown in Fig. 9(d). However, PSO generally exhibited premature convergence and poor local optimization ability, and the overall robustness was also inadequate. Based on the kinematic model of flexible needle puncture, Tan et al. [97] proposed an adaptive intelligent PSO algorithm with a parameter adjustment mechanism. Compared with other path planning algorithms, this method has better efficiency and accuracy and is well adapted to complex environments.

Intelligent search algorithms are essentially heuristic global optimization algorithms based on natural experience. Heuristic intelligent algorithms can find feasible solutions, but they do not find optimal, predictable solutions. In practical applications, this approach must be combined with the path evaluation function, such as setting the path length, maximum number of iterations, and distance to obstacles, to determine the optimal solution. In flexible needle puncture path planning, the idea of group intelligent search can be used to find a feasible path for the needle, such as finding variable curvature paths by optimizing the parameters of the GA or PSO algorithms or investigating path planning about multiple objective points.

Among these algorithms, the advantages and disadvantages of path planning algorithms based on sampling and intelligent search are compared in Table 2.

3.4. Path planning based on local obstacle avoidance

Path planning based on local obstacle avoidance is part of the local path planning algorithm. Local path planning indicates that trajectories are planned based on partial environmental information, which is generally dynamic (according to environmental changes) and can be optimized in real-time. It has the advantages of simple working principles, high real-time performance, and a high search speed; however, the environmental information obtained through these algorithms is inadequate and easily gets trapped into local optimal solutions. Even if they generate a path, it may not be the optimal one. Hence, these algorithms are generally not used independently and are typically augmented with other algorithms for determining optimal paths. Typical local path planning algorithms include the APF and dynamic window approach (DWA).

3.4.1. Artificial Potential Field

APF was first proposed by Khatib in 1986 and is commonly used for path planning in mobile robotics [98]. The principle of APF is as follows: It generates repulsive forces between the object and obstacles and attractive forces between the object and the goal, guiding the object toward the goal while avoiding obstacles. APF is advantageous as the paths planned by this algorithm are generally smooth and safe; however, if the gravitational and repulsive forces at a point are equal in magnitude but opposite in direction, the object will get easily trapped into a locally optimal solution or oscillation. The algorithm can be expressed by Equation 17–19.

attractive potential :

$$U_{att}(X) = \frac{1}{2} k \bullet d_1^2 \tag{17}$$

repulsive potential :

$$U_{rep}(X) = \begin{cases} \frac{1}{2}m\left(\frac{1}{d_2} - \frac{1}{d_0}\right)^2, d_2 \leq d_0 \\ 0, d_2 > d_0 \end{cases} \tag{18}$$

total potential：

$$U(X) = U_{att}(X) + U_{rep}(X) \tag{19}$$

Herein, $U(X)$, $U_{att}(X)$, and $U_{rep}(X)$ represent the total potential energy field, attractive potential, and repulsive potential, respectively, k denotes the attractive gain coefficient, d_1 represents the Euclidean distance from the object at this time to the target point, d_2 indicates the Euclidean distance from the object at this time to the obstacle, m denotes the repulsive gain coefficient, and d_0 represents the threshold of the distance between the obstacle and the obstacle. If the distance between the obstacle and the obstacle exceeds this threshold, then no repulsion effect will occur.

Jiang et al. [99] and Liu et al. [100] used APF for puncture-needle path planning in a dynamic 3D environment; they combined APF with a dynamic FEA method to obtain the trajectory direction of the puncture needle in each time period, thereby yielding the path in a dynamic environment. In 2022, Jiang et al. [101] introduced a novel robot-assisted puncture path-planning method based on repulsive field theory and a cost map constructed using computer tomography (CT) images to predict the needle trajectory. They demonstrated through experiments that the method can significantly improve the positioning accuracy and safety of path planning compared to path planning unassisted by robots.

In recent years, several researchers have integrated APF with other algorithms for puncture path planning to optimize paths. In 2021, Zhang et al. [77] integrated the APF method with an improved RRT^* algorithm, considering the motion constraints of a flexible needle to mitigate the effects of local tissue motion during puncturing, as shown in Fig. 10(a). However, this method does not achieve path tracking, making it challenging to implement closed-loop motion control during operations. Zhao et al. [102] proposed an intraoperative flexible-needle steering path in a dynamic environment based on APF and curve fitting algorithms, optimized several feasible derived paths, and compared its results with those of the RRT algorithm in 3D simulation experiments, as presented in Fig. 10 (b). The findings demonstrated the superiority of this method over RRT and its ability to enhance tracking accuracy through closed-loop control.

3.4.2. *Dynamic window approach*

DWA is an obstacle avoidance planning method. It is a velocity-based dynamic planning method that can rapidly reach the target point while avoiding collisions with obstacles in space. This algorithm can calculate the optimal speed for an object to reach a target without collision and quickly and safely search for a feasible path. However, traditional DWA has certain limitations. In complex environments, local paths will be locked and cannot be optimized.

In 2016, Henkel et al. [103] proposed a local path planner for mobile robot navigation in dynamic environments. Experiments demonstrated that the new algorithm consumed less energy than the traditional DWA. Additionally, Lai et al. [104] proposed an enhanced DWA algorithm to address issues like poor stability and high energy consumption in mobile robot operations, albeit with lower operational efficiency than traditional DWA. Li et al. [105] proposed an improved DWA algorithm based on a multi-objective particle swarm algorithm to enable the DWA to effectively avoid obstacles in complex environments, which traditional DWA struggles with. They conducted experiments in an environment featuring multiple static obstacles, and the results showed that the algorithm can improve the path and achieve higher speeds in a complex environment compared to traditional DWA. However, their study did not explore path planning in complex environments with multiple dynamic obstacles. In 2018, Daoud et al. [106] used an iterative sliding

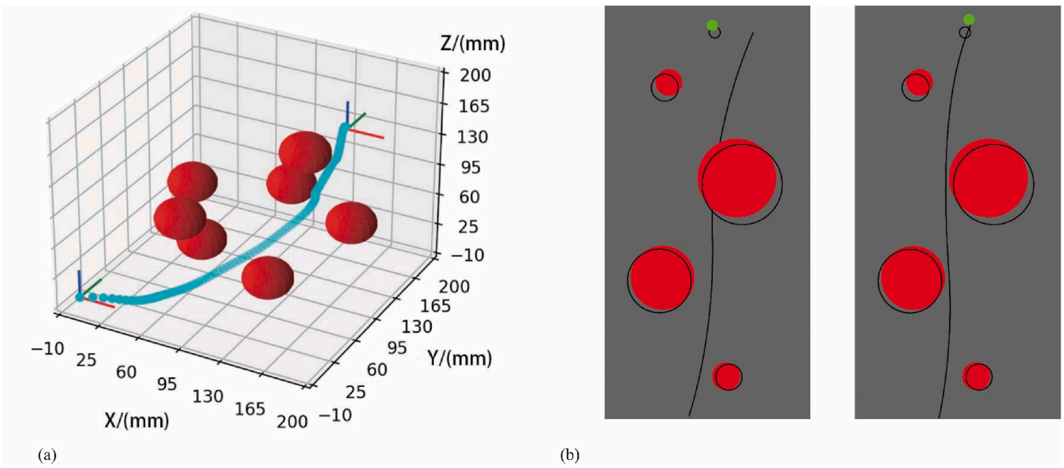


Fig. 10. Path planning based on APF algorithm: **(a)** the path planned by Zhang et al. [77]; **(b)** the path planned by Zhao et al. [102].

window approach to localize the needle tip in an ultrasound imaging-based method for needle detection during punctures. They conducted needle insertion experiments in bovine muscle, achieving results with an error within 1.0 mm. Additionally, in the same year [107], they introduced an alternative method utilizing an acoustic vibrating needle and Doppler ultrasound images to detect needle-induced vibrations and accurately locate the needle.

Because local path planning can recognize dynamic information in real time, it finds applications in intraoperative medical robot navigation. It generates smooth, continuous, and precise robot motion paths, facilitating precise localization and navigation, obstacle avoidance, real-time trajectory adjustments, remote control operation, and autonomous navigation. This ensures that robots can execute tasks accurately and safely within the patient's body, thereby improving the accuracy and success rate of surgical and interventional procedures.

3.5. *Other flexible needle path planning algorithms*

Alterovitz et al. developed a dynamic planning algorithm utilizing the Markov decision process (MDPs) based on the principle of a bevel-tip flexible needle moving along a constant curvature path during puncture [108]. In 2008, they combined the sampling-based roadmap representation of the configuration space (PRM) with Markov theory to introduce the stochastic motion roadmap (SMRM) [109], which maximizes the probability of successfully avoiding obstacles. In the same year, an image-guided approach was used to predict the optimal steering of a needle for path planning. The aforementioned three approaches consider the uncertain motion of the needle, and the planning results are illustrated in Fig. 11 [110].

In 2010, Hu et al. [111] proposed a two-dimensional S-type double-arc algorithm based on geometric principles, but it had certain limitations in terms of path forms. Based on the flexible needle kinematic model, Zhao et al. [112] improved the path using multiple forms and experimentally validated the feasibility of the algorithm. In 2016, based on the Kalman filter algorithms, Zhao et al. used three methods to estimate the needle tip poses in real-time, studying the control method of the needle tip and analyzing the differences between the algorithms [113].

Moreover, Lee et al. [114] used the deep Q-network (DQN) algorithm to perform path planning for needles, constructing a 2D puncture model using CT images, simulating needle–tissue interactions in a stochastic environment, and considering their uncertainty, as depicted in Fig. 12(a). The studies mentioned earlier in this section were conducted only in a 2D environment and were not applied in 3D environments.

In 2005, Park et al. [115] used inverse kinematics to plan the puncture path in a 3D environment by including a probability density function to make the planned path more in line with the real-world situation. In 2010, they successfully applied the path-of-probability (POP) algorithm with and without obstacles; this represents a substantial leap in the field of research on flexible needle path planning, as shown in Fig. 13(a) [116].

Furthermore, Duindam et al. [44] proposed another path-planning method by discretizing the control space with an obstacle 3D environment. This method has high precision, involves simple calculations, and offers high efficiency in path planning. However, the path planned with this method is not globally optimal, as shown in Fig. 13(b). In 2018, Tan et al. [117] proposed a robust navigation planning method based on MDPs with enhanced robustness of flexible needle steering, which solved the uncertainty of flexible needle motion. They performed simulations and experiments to validate the superiority of the method.

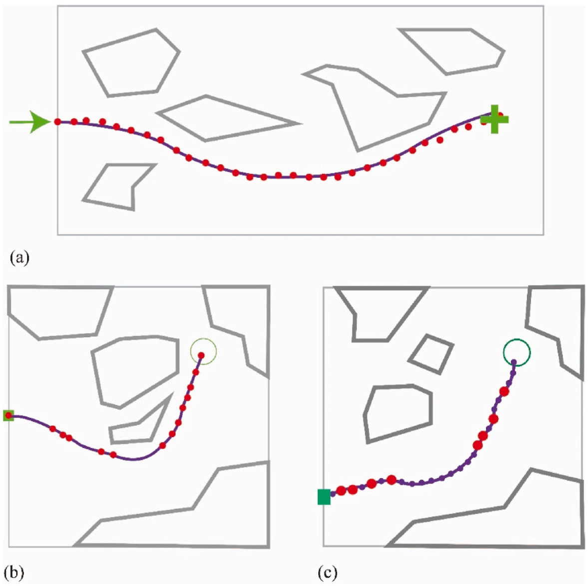


Fig. 11. Paths planned by Alterovitz et al. [110]: (a) Path planning based on MDP method; (b) Path planning based on SMRM algorithm; (c) Path planning based on image-guided method.

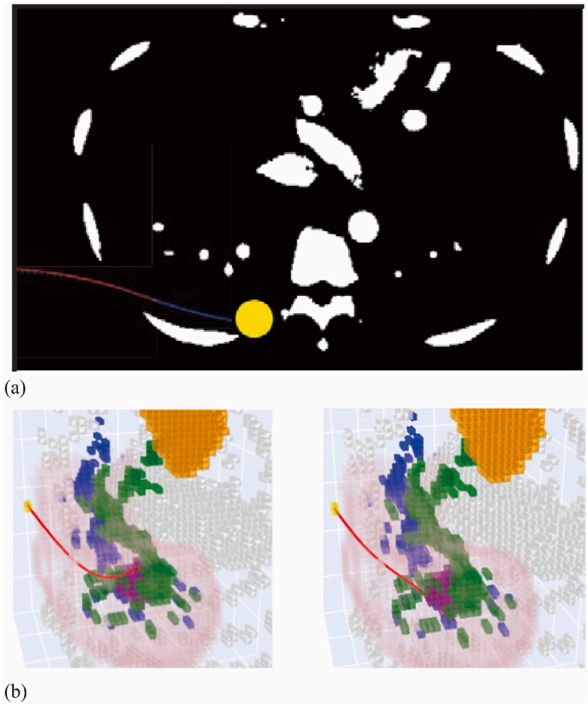


Fig. 12. Path planning based on deep learning: (a) path planning based on the DQN algorithm [114]; (b) path planning based on CT guidance [21].

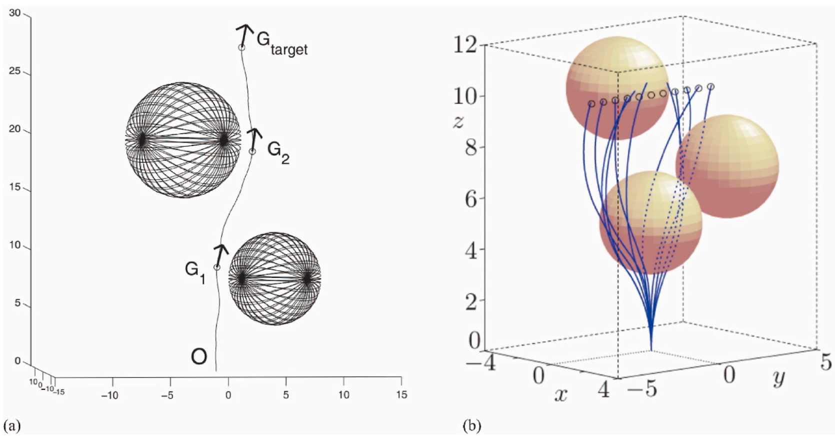


Fig. 13. (a) Path planning based on POP algorithm [116]; (b) Paths planned by Duindam et al. [44].

Additionally, Hu et al. [21] proposed a new method for path planning in needle puncture surgery based on CT guidance; they optimized the surgical path by performing training with the double DQN (DDQN) tool and used 3D reconstructed organ models for path-planning network training, as shown in Fig. 12(b). Notably, this approach can optimize the puncture path for liver tumor surgeries, improving convergence speed and accuracy while preventing local optima. However, it doesn't account for the actual deviation in the path due to a patient's breathing, which may lead to discrepancies between the results and real-life situations, resulting in poorer performance and robustness.

Among these methods, local obstacle avoidance-based and other path-planning methods are classified according to dimension and compared according to their advantages and disadvantages in Table 3.

4. Conclusion and prospect

This paper combines the principles of flexible needle puncture motion, providing an overview of three models for needle-tissue interactions and various path-planning algorithms. It primarily outlines the principles, advantages, disadvantages, and applications

Table 3
Path planning algorithm based on local obstacle avoidance and other algorithms.

Classification	Methods	Advantages and Disadvantages
Two-dimensional path planning	Markov decision dynamic programming algorithm (MDP) proposed by Alterovitz et al. [108–110]	**Advantages**: It maximizes the probability of successfully avoiding obstacles and predicts the optimal steering of a needle for path planning. The path generated by this algorithm is extremely precise, safe, and satisfies the constraints of flexible needle movement. **Disadvantages**: It is only suitable for circular arcs connection in two-dimensional environments, and the path cannot be planned in reality and online in real-time.
	Path planning algorithm combined geometric principles proposed by Hu et al. [111] and Zhao et al. [112]	**Advantages**: Reduced need for control and accuracy of target puncture robot; the algorithm is simple to implement. **Disadvantages**: The problem of angle lag of the needle tip during manipulation remains unsolved; thus, the path planning accuracy is low. The path cannot be planned in reality and online in real-time.
	Based on the deep learning path planning algorithm proposed by Lee et al. [114]	**Advantages:** This method offers high efficiency and accuracy in complex motion planning of bevel-tip flexible needles in human bodies. **Disadvantages:** It requires big data support, large computational volume, and high hardware cost.
Three-dimensional path planning	APF method represented by Zhang, Jiang, Liu, and Zhao et al. [77,99–102]	**Advantages**: The path planning of this method has high positioning accuracy and safety, and the working principle of the algorithm is simple and easy to understand. **Disadvantages**: It is computationally intensive and inefficient, and sometimes, the path cannot be planned, and path planning cannot be planned in real time during surgery.
	Park et al. [115,116] plan the puncture path by including a probability density.	**Advantages**: It can optimally perform path planning in a complex environment with multiple obstacles and has high robustness. **Disadvantages**: The path generated by this method is not smooth enough and has many turning points, and it is not suitable for flexible needle path planning. Further fitting and improvement are needed.
	Improved path planning method proposed by Duindam et al. [44]	**Advantages**: This method has high precision, simple calculation, and high efficiency in path planning. **Disadvantages**: The path planned by this method is not globally optimal and smooth enough, and it cannot be planned online in real-time.
	Combined image-guided path-planning algorithm proposed by Hu et al. [21]	**Advantages:** This method can plan the optimal puncture path for puncture surgery of liver tumors and improves convergence speed and precision. This method does not get trapped in local optima. **Disadvantages:** The method has poor performance and robustness and does not consider the actual situation of path deviation resulting from the patient's breathing.

of each model and algorithm in flexible needle puncture. These models and algorithms are either relevant to this field or show promising applications. While the algorithms and applications are diverse, they are not exhaustive. We have not reported all the references in the field of needle puncture, as doing so would make the article lengthy and significantly increase the workload. We hope that after going through this paper, readers can identify suitable algorithms and work on further improvements and enhancements. However, although much research has focused on flexible needle path planning, the literature remains biased toward the theoretical simulation stage. Therefore, clinical applications may only be achievable in the distant future, and the accuracy and details of theoretical simulations need further improvement. The summarized trends in development are as follows.

(1) Continuous improvement of the algorithm path optimization

After more than a decade of research, with the emergence of more mature path-planning algorithms in other fields, flexible needle path-planning algorithms have been developed with smooth applications in simulations and experiments. However, traditional path planning algorithms may not generate optimal paths for puncture surgery and need to be continuously optimized to meet the planning needs of this specific application. In addition, most research on flexible needle path planning is based on a static ideal environment, which is not representative of the actual environment, and future research on algorithms must consider an actual dynamic environment.

(2) Combining multiple high-quality algorithms

RRT is one of the most adopted algorithms for flexible needle path planning; however, it is limited by the large computational effort involved and low search speed. Evidently, it is difficult for a single algorithm to locate the optimal path required for an actual complex puncture environment; however, a combination of multiple algorithms can bridge the shortcomings of each and improve the overall performance of the algorithms. For example, when the RRT algorithm is applied to flexible needle path-planning problems, incorporating the valuation function of the algorithm based on graph search can prevent the generation of numerous invalid paths, thereby improving the search speed. Therefore, combining several high-quality algorithms is an important research direction.

(3) In-depth research of image-guided path planning

Researchers have conducted considerable research on image-guided dynamic needle puncture path-planning algorithms and have achieved definite results. However, several aspects can be improved, such as enhancing the identification and localization of the needle tip and neurovascular tissues in exploring the 3D dynamic environment and the accurate grasp of the motion information of the target points, needle tips, and organ vessels. Path planning requires preoperative preparation and intraoperative real-time planning, and image-guided path planning is one of the future trends in flexible needle research.

(4) Use of computers, artificial intelligence robots, and other new technologies to assist in control.

Accurate puncturing with complete reliance on manual operation is challenging due to the flexibility and precision of flexible needle puncture. Therefore, numerous recent studies have focused on artificial intelligence robot-assisted flexible needle puncture systems that integrate human experience with the precision of artificial intelligence, thereby realizing puncture control through human–robot interaction and providing a theoretical and technical basis for research focusing on precise manipulation of flexible needles. Such research has significant scientific significance and societal value for developing robotics and advanced medical technology.

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Data availability statement

No data was used for the research described in the article. No data associated with this study has been deposited into a publicly available repository.

CRediT authorship contribution statement

Ye Huang: Conceptualization, Formal analysis, Investigation, Methodology, Validation, Visualization, Writing – original draft, Writing – review & editing. **Longfeng Yu:** Data curation, Formal analysis, Writing – review & editing. **Feifan Zhang:** Funding acquisition, Supervision, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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