THEORY AND MEASUREMENT OF VISUAL MECHANISMS

III. ΔI As a Function of Area, Intensity, and Wave-length, for Monocular and Binocular Stimulation

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I

An increase in the size (A) of a uniformly illuminated retinal light image usually occasions a decrease in the amount of light ($= I \times t$) required to elicit a threshold response (Aubert, 1865; Riccò, 1877; Abney, 1897; Piper, 1903; Henius, 1909; Fujita, 1909; and Piéron, 1920*a*, *b*). With exposure time as parameter, the intensity of the threshold stimulus (herein labelled ΔI_0) has been found empirically to be, nearly enough, but with certain qualifications and restrictions, a declining power function of the size of the retinal image (Abney, 1897; Abney and Watson, 1916; Wald, 1937–38), at least over a limited range of A and of ΔI_0:

$$\log \Delta I_0 = -z \log A + C; \tag{1}$$

ΔI_0 is the intensity of the threshold stimulus (*i.e.*, for this case, $\tilde{I}_2 - I_1 = \tilde{I}_2 - 0 = \Delta I_0$, in photons); A is the size of the retinal image (in square millimeters, or in degrees visual angle for a symmetrical figure—up to *ca.* 5°); and z and C are constants. The value of z, however, also proves to be a systematic function of particular physiological variables. Equation (1) then assumes the form of a general exponential

$$\Delta I_0 = C' A^{-z} \tag{2}$$

in which z is no longer to be regarded as sensibly constant; instead it is to be treated as a function of one or more dimensions of the retinal light image, so that $z = f$ (I_1, ΔI, A, retinal location, form of test patch, exposure time, ...).

Attempts have been made to account for the properties of threshold data primarily in terms of the assumption that their quantitative features are determined by events at or in the retinal receptors (*cf.* von Kries, 1911; 1929; Hecht, 1937). A variety of considerations leads us to reject this assumption (Crozier, 1936; Crozier and Holway, 1938–39 *b*, *etc.*). Func-

tional properties of the central nervous system provide a more comprehensive basis for the interpretation of the data of intensive discrimination.

Practically all theories of absolute and relative threshold responses have invoked the constant quantity or constant number concept, in some form (*cf.* Weber, 1834; Wald, 1937–38). The assumption involves serious difficulties, and its necessity has never been demonstrated. If used with reference to central nervous properties, the assumption leads to the proposition that the visual minimum perceptible depends upon the eventuation in the nervous system of a definite number of nervous impulses per unit time, or of a certain density of such impulses (*i.e.*, number/unit time/unit volume). The weakness in the traditional mode of using this general conception lies in taking it for granted that this "definite number" is fixed and constant; this leads to the assumptions (*a*) that threshold brilliance increments are equivalent, (*b*) that just discriminable steps in sensorial effect are equal. Over a certain range of intensities this is approximately true, but even the approximation is illusory (Crozier, 1936; Crozier and Holway, 1938–39 *a*, *b*, *etc.*). When appeal is made to "number of excitatory impulses per unit time" we, however, need not and should not be restricted to the notion that for threshold or other intensive discrimination there obtains the requirement of *constancy* in this number at all levels of intensity, or of area, or of retinal location. The statistical mechanism of discriminatory performance definitely forbids the restriction of constancy (Crozier, 1936). The manner in which magnitude of sensory effect E is really related to I, and which determines the nature of ΔE and of ΔI as a function of I, is to be ascertained by means of considerations arising as a consequence of the measured properties of ΔI.

Qualitatively we may assume that the number of central nervous units or elements of excitation (*i.e.*, the number per unit time) increases with increase of area in the retinal image, and in addition their density with increase of intensity, and with increase of exposure time (over a limited range). For present purposes we need not attempt to specify the precise law of this increase. The excitation required for threshold visual effect must, however, be supposed to involve two factors, (*a*) the number of neurons affected and (*b*) the frequency of discharge of impulses in each. The basic supposition here is of course that the rules for activity in single peripheral nerve fibers apply also to central nervous units (Lucas, 1917, *etc.*).

Much has yet to be made out as to the quantitative relations obtaining between the structural elements at various levels in the human visual apparatus (Poljak, 1935; Østerberg, 1935). But it is safe to presume that,

in general, when the number of active central elements is increased by enlarging the size of the light image on the retina, a smaller density of impulses per unit illuminated area of retina should be required to eventuate the frequency of impulses necessary for sensorial discrimination, with other things constant. The relations between critical flash frequency and flash illumination for response to flicker illustrate these considerations (*cf.* Crozier, Wolf, and Zerrahn-Wolf, 1937–38 *b*).

The rôle of the spatial distribution of sensitivities and effectivenesses of the retinal elements and the nerve fibers involved is a separate problem. Whatever the form of this distribution, even if it be one of entire uniformity, the frequency factor must enter, and thus influence the intensity (flux/area) required to produce the necessary number of impulses per unit area. The same type of reasoning applies for the analysis of the time intensity functions (*cf.* data of Graham and Margaria, 1935). Moreover, it is clearly required that ΔI_0 for binocular stimulation must on this basis be *less* than that for monocular presentation. This has sometimes been denied (*cf.* Graham, 1930), but is substantiated by tests in which proper provision has been made to insure binocular accommodated fixation.

The properties of the *minimum discriminable*, ΔI, like those of the *minimum perceptible* (ΔI_0), can be accounted for in the same terms. In general, ΔI (for a given I_1), decreases as the size of the retinal light image is increased (Lasareff, 1911; Heinz and Lippay, 1928; Cobb and Moss, 1928; Steinhardt, 1936–37; Holway and Hurvich, 1938). No *homogeneous* data (Crozier, 1936) have been available, however, for defining the relations between ΔI and A for monocular and binocular excitation at various intensity levels. The present paper contains such data, with exposure time, intensity, and several wave-lengths as parameters. We consider here chiefly measurements made with two observers for which the excitability functions of each of the two eyes are very nearly the same. Results with an observer for whom this is not the case are important for the general theory, and will be treated separately.

II

Apparatus and Procedure

Measurements of ΔI ($= \bar{I}_2 - \mathrm{I}_1$) as a function of the size of the mean visual angle were made at various intensities and wave-lengths for both monocular and binocular stimulation. For monocular excitation a uniformly illuminated, rectangular light image was projected upon the retina, centered at the fovea. Six areas were used. During any single sitting, wave-length (λ), intensity (I_1), and exposure time were parameters. Before each sitting, the observer dark-adapted for 20 minutes. The experiments began with the weakest (photopic) intensity and the smallest area. At the

beginning of a sitting, the observer adapted to the prevailing intensity for about 30 seconds. Then the experimenter added light to I_1 at a constant rate for log I, until a just noticeable increase in brilliance was reported. Five ΔI measurements were taken in this manner for the smallest area in each series. The size of the light image was then doubled, while I_1, λ, and other conditions remained unchanged. When five measurements had been made for this area, the image was again doubled in size. This practice was continued until five measurements had been secured for each of the six areas at the lowest intensity. The same procedure was then employed for an intensity about 10 times as great as the preceding I_1. Thus, with respect to both area (A) and intensity (I_1), the order of securing the measurements was always in the direction of increase.

The method of presentation of the conditions under which judgment of just noticeable intensive differences is to be made has of course a decided influence upon the magnitude of ΔI for a given I_1, as is well known. The procedure adopted for the present experiments was chosen to avoid as far as possible the effect of the presence of a fixed "surround." It is known (Aubert, 1876; Cobb, 1916; Guild, 1932) that the presence of an illuminated area surrounding the adjusted illumination of the test-patch, particularly at higher levels of illumination, reduces ΔI; at minimal and sub-threshold illuminations ΔI is *increased* (unpublished data). The result at high illuminations is not altogether due to the blotting out of internal reflections in the eye-piece of the observing instrument, although this is probably an additional factor in the choice of a surround in certain cases. There remains the difficulty, however, of deciding *how* large and *how* intense a surround to employ, and whether it should be of constant intensity, or equal or proportional to I_1. Decision on these points cannot be based on agreements of the induced properties of ΔI with the requirements of a particular theoretical interpretation of the dependence of ΔI upon I_1; the influence of the surround upon the properties of ΔI is known to be a function of its area and intensity, as well as of the level of I_1. The best solution is not to employ a fixed surround at all.

The same procedure was adopted for the binocular measurements. The observers adapted in the dark for 20 minutes or 45 minutes, depending on the nature of the experiment (*vide infra*). Starting at the lowest value of I_1, and the smallest area, ΔI measurements were taken at all areas for the weakest intensity used for white light or for a given wave-length. Then I_1 was increased tenfold and ΔI values were secured for all areas in order of increasing magnitude.

Only light of a given wave-length composition was used during a single sitting. Hence, for any given wave-length and intensity specifications, the measurements are homogeneous for the determination of the relation between ΔI and A, the angular size of the retinal image.

Fig. 1 gives a simplified plan-view of the apparatus. This instrument, a visual discriminometer, has been described in some detail (Crozier and Holway, 1938–39*a*). S_0 is the primary light source,—in this case a flat ribbon filament, especially designed, operating at 6 volts and 30 amps. The following controls served to eliminate fluctuations in emission due to variations in the line. The current taken from the line passes through a GR Variac (an auto-transformer with continuously adjustable output) to two transformers (capacity = 20 amps., 6 volts) in parallel, and thence through a 30 amp. fuse and a Westinghouse No. 37 ammeter to the lamp. With this arrangement, variations in current as small as 0.5 of 1 per cent can be detected immediately by the experimenter and adjusted at the variable transformer. The lamp was set in operation

for at least 20 minutes before each experiment. (About 15 minutes are required before a steady level of current consumption is reached.) A continuous air blast through the lamp-house insures constant emission after this period of time, provided of course that fluctuations do not occur in the power line. Only twice during our experiments have

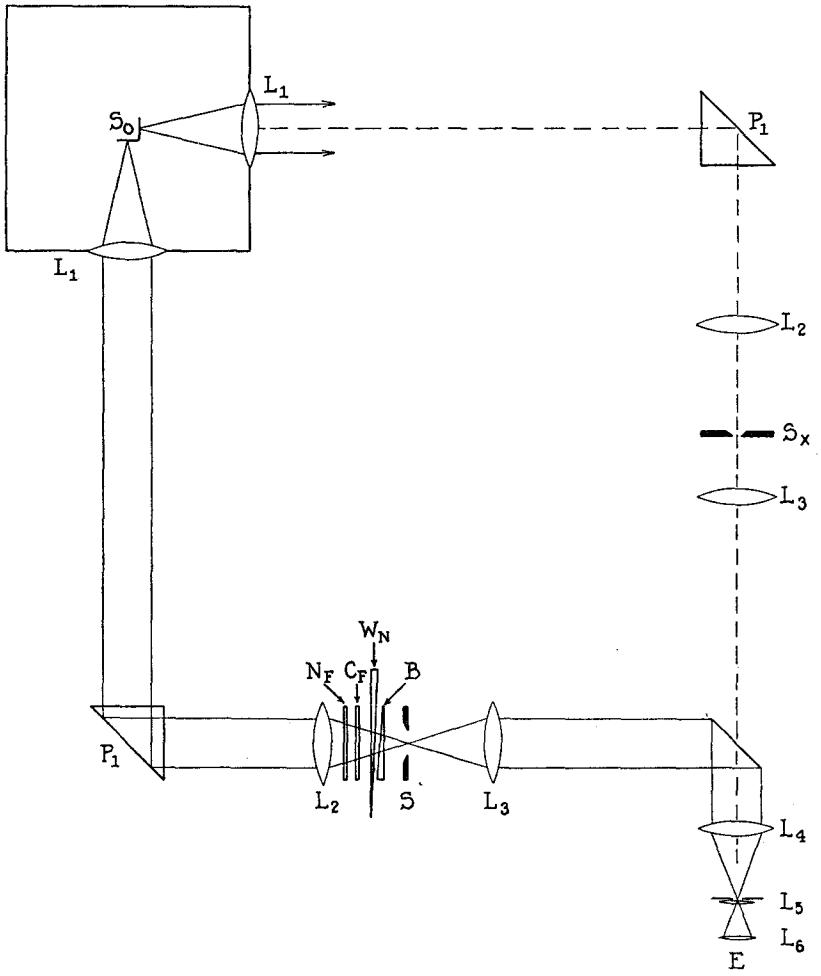

FIG. 1. Schematic plan view of the optical arrangement used in the monocular experiments. For binocular stimulation, a binocular head with matched oculars is substituted for *O*. See text.

we found it necessary to readjust the power supply at the secondary coil of the variable transformer to correct for variations in the line. These fluctuations, however, were of sufficient magnitude to justify our precautions (as a 10 per cent change in current would occasion slightly more than one log unit change in intrinsic brightness). Through-

out the experiments, the current was maintained at 30 amps. The resulting emission gives rise to a brilliant 'snow white' impression.

Light emitted at the constant source S_0 passes through the large quartz condensing lenses, L_1, which are located at a distance equal to their focal lengths from the plane of the flat ribbon filament. Parallel light transmitted along the axis of the collimators is totally reflected 90° by the prisms at P_1. The lenses L_2 place a uniformly illuminated image of S_0 in the plane of the bilateral slits, S. One of the slits contained the fixation point; the other served as variable aperture. Capstan screws regulate the size of the aperture defined by these slit knife-edges. L_3, situated at a distance from S equal to its focal length, sends parallel bundles of light to the front-surfaced mirror (chrom-aluminized by sputter) which deflects the beam 90°. L_4 forms an image of S in the plane of the stop A and the flat surface of the field lens L_5. L_6 is the eye-piece (mag. = 15 ×). With the eye at the exit pupil of the ocular, an observer sees a rectangular field of constant height; the fixation dot lies in the center of the field. All lenses are achromatic. A Bausch and Lomb combination chin-rest and head-support is used at E.

For binocular stimulation, a binocular head was substituted for the ocular, O. Here, right angle prisms divide the beam and two optically identical images of the aperture S are located in the plane of surfaces of the field-lenses in the matched oculars. The proper interocular distance is obtained by adjusting a graduated drum. The slight differences in accommodation which exist for practically all observers are corrected by turning the spiral tube in which one ocular is mounted.

Neutral tint filters inserted at F regulate the fixed values of I_1. The transmission of these filters for white light was determined with a König-Martens polarization photometer. We are under obligation to Dr. C. P. Winsor for his collaboration in these measurements. Four measurements were made in each quadrant,—a total of sixteen in all. Failure to use all quadrants invariably leaves the transmission coefficient with a constant error and thus defeats the chief purpose of the measurements, which is to eliminate constant errors. The probable error of the mean of these measurements is usually less than 0.5 of 1 per cent. Frequently, however, it may be much less; photometric results, even for different observers, may differ only in the fourth place of the mantissa when the transmission, or density, is expressed in logarithmic units.

For wave-length composition as parameter, Wratten color filters Nos. 71A, 74, and 47 were used. The wave-length composition for each filter was determined by a photoelectric spectrophotometer (Hardy, 1929; 1935). These filters transmit frequency "bands," not lines. The spectral distribution of each color filter used is shown in Fig. 2. The transmission coefficients for the light emitted by S_0, however, were measured with a Leeds and Northrup galvanometer (sensitivity = 7 mm./μv at one meter) in series with a large surface Moll thermopile. Even though the lenses, L_1, are quartz, some heat rays can be detected at E (Fig. 1) at highest intensities. It was therefore necessary to use an additional heat filter. A disc of colloidal gold (suspended in glass) inserted in the instrument at L_1 served this purpose adequately. The effectiveness of this arrangement was verified by further measurements made with a rock-salt crystal in front of the thermopile at E. The transmission coefficients of the neutral tint wedges (and filters) were also determined in this manner.

The diameter of the eye-ring was less than 2 mm. and served as an effective artificial pupil. Corrections (over-all) were made for the probable losses in transmission suffered through absorption in the ocular media (*cf.* Roggenbau and Wetthauer, 1927; Ludvigh

and McCarthy, 1938). The results, all in energy units, were converted into millilamberts by means of a single binocular match (white light), using a Macbeth illuminometer and standard test-plate. Although these average corrections are imperfect for any given eye, they are in the proper direction and are consequently better than no correction at all. Finally, all intensity values were expressed in terms of retinal illumination, as photons (Troland, 1918).

The shape of the aperture was rectangular at S, and the trace of the light disturbance cut by a plane located at right angles to the optic axis at the retina is similar in form. The angular *height* of this image was constant (= 20.8°). Area was regulated by lateral movement of the horizontal knife-edges at S. The angle subtended by the retinal light image at the principal point of the eye was measured with the aid of an ocular micrometer. The graticule was placed in the plane of the circular stop at the field lens and the screws controlling the lateral width of the slit at S were calibrated so as to read directly in terms of the visual angle. Area measurements can of course be expressed in square millimeters on the retina. For large areas, however, it is usually more convenient to employ angular readings and all our measurements are tabulated in degrees of visual angle.

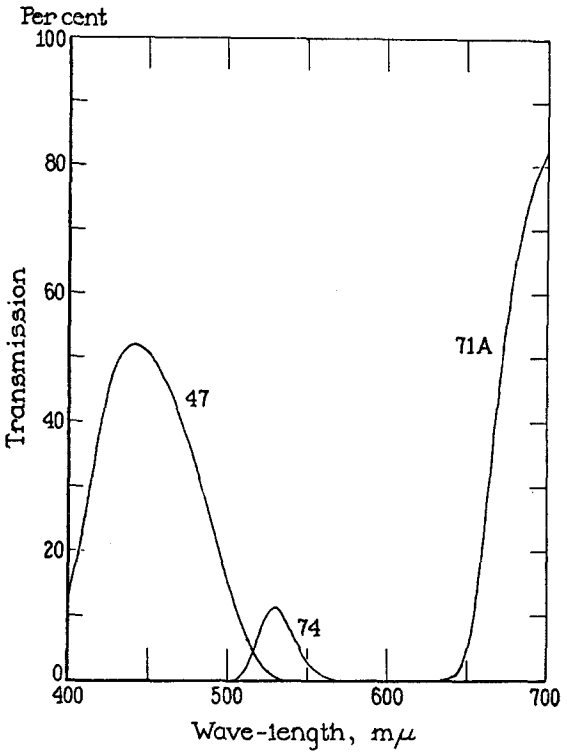

FIG. 2. Showing the wave-length *vs.* intensity distribution of the color filters used in the present experiments. The determination of these curves was made with a photoelectric spectrophotometer (Hardy, 1929; 1935).

The observer was instructed to focus upon the tiny (red) fixation point located in the center of the rectangular light image. The position of this point was controlled at S_x. The stimulus was fixated for about 30 seconds before ΔI was added to I_1.

III

Monocular Measurements, White Light

For white light, monocular measurements were secured at six areas of different size at each of six levels of intensity, I_1. The results for one

observer (A. H. H.) are shown in Table I. At any level of I_1 the data are homogeneous with respect to area. The vertical height of the image was constant at a visual angle of 20.8°, and the A' values in Table I represent the visual angle subtended at the eye by the lateral separation between the vertical edges of the slits in image space. I_1 is the prevailing intensity. I_1 was adjusted in exact steps by suitable adjustments of the wedge W_n (Fig. 1) in terms of the calibrations of the decimal filters N_f. Exposure time (= 30 seconds) and wave-length composition are parameters. Each

TABLE I

White light: homogeneous results for ΔI as a function of A' and I_1; monocular; I_1 is the standard intensity, in photons; A', the angular width of the light image on the retina—the angular height of the image (relaxed accommodation) was constant, = 20.8°. Each ΔI entry is an average of five measurements; $\sigma_{\Delta I}$ is the root-mean-square variation of a single observation. A.H.H.; right eye. See Fig. 3.

Visual Angle, A' Degrees

log I_1, *photons*	0.4°	0.8°	1.6°	3.2°	6.4°	12.8°
$\bar{2}.006$ log ΔI_m	**$\bar{3}.621$**	**$\bar{3}.587$**	**$\bar{3}.445$**	**$\bar{3}.365$**	**$\bar{3}.233$**	**$\bar{3}.204$**
log $\sigma_{1\Delta I}$	($\bar{4}.718$)	($\bar{4}.544$)	($\bar{4}.471$)	($\bar{4}.410$)	($\bar{4}.317$)	($\bar{4}.321$)
$\bar{1}.006$ log ΔI_m	**$\bar{2}.400$**	**$\bar{2}.254$**	**$\bar{2}.203$**	**$\bar{2}.181$**	**$\bar{2}.032$**	**$\bar{3}.999$**
log $\sigma_{1\Delta I}$	($\bar{3}.413$)	($\bar{3}.344$)	($\bar{3}.306$)	($\bar{3}.207$)	($\bar{3}.101$)	($\bar{3}.015$)
0.006 log ΔI_m	**$\bar{1}.282$**	**$\bar{1}.157$**	**$\bar{1}.078$**	**$\bar{1}.063$**	**$\bar{2}.923$**	**$\bar{2}.778$**
log $\sigma_{1\Delta I}$	($\bar{2}.293$)	($\bar{2}.160$)	($\bar{2}.105$)	($\bar{3}.994$)	($\bar{3}.977$)	($\bar{3}.790$)
1.006 log ΔI_m	**0.118**	**$\bar{1}.992$**	**0.048**	**$\bar{1}.999$**	**$\bar{1}.800$**	**$\bar{1}.655$**
log $\sigma_{1\Delta I}$	($\bar{1}.107$)	($\bar{2}.990$)	($\bar{1}.131$)	($\bar{1}.004$)	($\bar{2}.903$)	($\bar{2}.707$)
2.006 log ΔI_m	**1.088**	**0.963**	**0.880**	**0.902**	**0.788**	**0.601**
log $\sigma_{1\Delta I}$	(0.091)	(0.002)	($\bar{1}.913$)	($\bar{1}.966$)	($\bar{1}.810$)	($\bar{1}.675$)
3.006 log ΔI_m	**2.091**	**2.000**	**1.947**	**1.834**	**1.754**	**1.639**
log $\sigma_{1\Delta I}$	(1.135)	(1.011)	(0.923)	(0.914)	(0.799)	(0.708)

ΔI_m entry is an average of five measurements. Associated with each ΔI_m is the measure of dispersion, $\sigma_{1\Delta I}$, the root-mean-square deviation of a single observation. For any fixed value of I_1, both ΔI_m and $\sigma_{1\Delta I}$ are seen to vary inversely with the size of the retinal light image.

These ΔI_m data are plotted in Fig. 3. The coordinates are spaced logarithmically. Units of area (A, square degrees) are used. Each plotted point is an average of five measurements. The solid lines were fitted by the method of averages, and are described by

$$\log \Delta I = -Z \log A + C, \text{ as in (1)} \qquad (3)$$

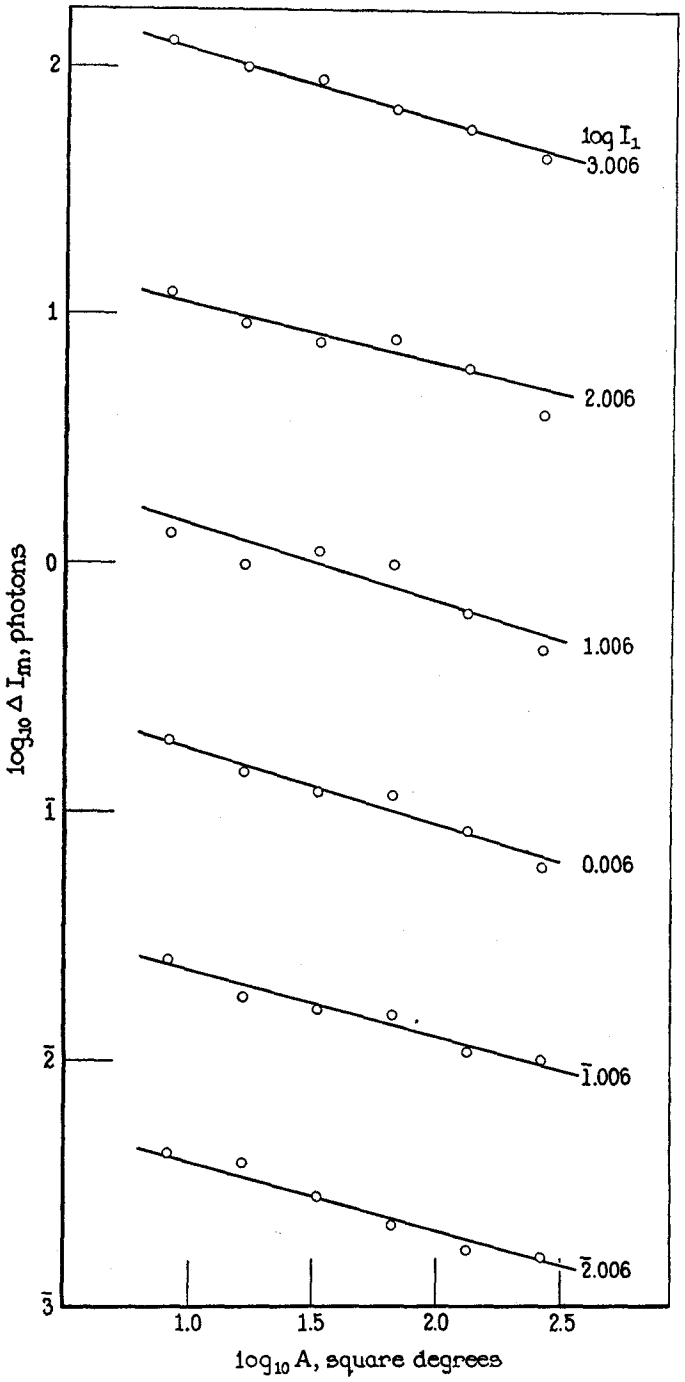

FIG. 3. Dependence of ΔI_m on A; homogeneous data for white light, monocular excitation (Table I). Intensity I_1 as parameter. The lines adjusted are for $\log_{10} \Delta I_m = -Z \log_{10} A - C'$, with $Z = 0.267$; A is in *area* units (square degrees). The maximum departure is less than $4\times \sigma_{\Delta I_m}$.

where ΔI is the added intensity $(\bar{I}_2 - I_1)$ required for the index response. In terms of the measure of sensitivity, $1/\Delta I$ at each level of I_1,

$$\log (1/\Delta I) = Z \log A - C. \tag{4}$$

Fitting by the method of averages, mean $Z = 0.267$ when logarithms of *area* and of ΔI are taken to base 10. If $\log A$ is taken to base 2, the mean slope Z is 0.081. This signifies that, for the conditions considered, an approximately twofold increase in area of the retinal image reduces ΔI_m by a factor of 1.205 arithmetic units. The S.D. of the six values of Z from the slopes in Fig. 3 is only $\sigma_1 = 0.0022$ $\log_{10}$ units. The (monocular) values obtained with colored lights (section V) are also of just this order, although a little smaller. The ratio for doubling area on one retina is therefore definitely *less* than that obtained for the comparison of monocular and binocular thresholds with a *small* fixed area variously located on the retina (Crozier and Holway, 1938–39 *b*), namely 1.4.

IV

The Area Function

The form of equation (4) is the same as that for ΔI_0 in equation (2). Its derivation may be obtained without specific assumptions as to the nature of the threshold of excitability. Such assumptions are to be avoided if possible, since the discovery of the mechanism of threshold effects is one of the objectives of inquiry, and preferably one should not be theoretically committed in advance.

At any level of I_1 a change in the conditions and method of presentation can lead to a change in ΔI. The brilliance increase necessary for the recognition of ΔI can be produced by suitable increase of the excitable area, as both theory and qualitative tests indicate, and our own observations. This implies that, for a given level of I_1, the excitability $(1/\Delta I)$ measures the same kind of a property of the capacity for reacting performance based on recognition of intensive difference as is also measurable by area A. These two measures of the capacity for excitation vary together and directly, although not rectilinearly. The multiplication of the units of one of these measures by the appropriate factor $d(1/\Delta I)/dA$ giving the rate of change of $1/\Delta I$ per unit change of A gives the dimensionally correct expression

$$1/\Delta I = k\, A\, d(1/\Delta I)/dA \tag{5}$$

Rearranging,

$$d(1/\Delta I)/(1/\Delta I) = Z\, dA/A,$$

or

$$\log \Delta I = -Z \log A + C,$$

as already found in empirical equation (1, 4). If over a particular range the factor k, (or $1/Z$), is really constant, and independent of I_1 and of A, we can expect this equation to hold independently of I_1, A, exposure time, and the monocular or binocular mode of presentation; Z should be slightly less for colored light than for white, for reasons considered subsequently (section VII). The dimensional constant C has the meaning of a ΔI times an A.

An equation of the general form of (1) and (5) was derived by Wald (1937–38) for visual thresholds and area, using the conceptions that (1) there is required a constant number of excited elements at the threshold; and (2) that the excitability is statistically distributed in a homogeneous retinal area, in a manner given by a probability summation. In the symbols of our equation (1) and (5), Wald's formula is $-\log \Delta I_0 = k \log (A - n_t) + C$, where n_t is the threshold number of excited elements (*i.e.*, the "active area"). There are certain difficulties with this expression, and its derivation. Obviously, when $A = n_t$ we have

$$\log 1/\Delta I_0 = -\infty, \text{ or } \Delta I_0 = \infty;$$

this contradicts the assumption that ΔI_0 is the intensity required to excite n_t elements. This is the assumption used in deriving the equation, A being really a measure of the probability of finding n_t elements under the conditions of presentation, and essentially in terms of *number* (frequency of encounter) of elements. The argument cannot hold for the application of the equation to the case of $\Delta I = I_2 - I_1$, the differential threshold, for similar reasons. If equation (4) is solved for the integration constant C it is apparent that

$$C = \log C' = \log (\Delta I) (A^Z)$$

and

$$C' = (\Delta I) (A^Z),$$

or

$$C'/A^{(Z-1)} = \Delta I \cdot A.$$

The integration constant has therefore the dimensions of *intensity* times the *area* raised to a power. If we wish to define $A_1 = n_t$ as the *unit* area (in terms of n) capable of being excited in terms of ΔI_0 or of ΔI, then we can write

$$C'/M = C'' = \Delta I_1$$

where M is given by

$$M A_1^Z = A^Z$$

and ΔI_1 is the threshold intensity increment for the response with the elementary "area." Consequently, on the assumption that Z is a constant, the use of the "constant quantity" or "constant number excited" idea really requires that the correction for the elementary, active area (and the threshold for this area) be carried in the terminal constant C of equation (4); it should not be corrected for, as in Wald's treatment, by subtracting A_1 (or its equivalent n_t) from A, but takes the form of a multiplier. The theoretical reason for doing this is of course that if (in terms of the fixed quantity concept) A determines the probability of finding n_t, this probability is *also* measured by ΔI; the dimensional constant C in equation (4) must include both A *and* ΔI. The descriptive power is not improved by subtracting a constant from A. What the equation says is that the threshold excitability per unit area ($1/\Delta I \cdot A$) is proportional (inversely) to a fractional power of the area.

When C is evaluated by taking $A = 10$ units (or any other constant value),

$$C = \log C_1 = \log \Delta I \text{ with } A \text{ fixed},$$

C should follow the same law as $\log \Delta I$ when I_1 is varied; examination of the data (Fig. 3, *etc.*) shows that this is the case; C *vs.* $\log I_1$, with A fixed, gives curves reflecting exactly the behavior of $\log \Delta I$, including the fact that (*cf.* section IX) binocular C is lower than the monocular.

The condition for constancy of Z is, from (5), that

$$\frac{d(1/\Delta I)}{dA} = Z/A \cdot \Delta I \tag{6}$$

The product $A \cdot \Delta I$ is directly proportional to the increase of energy (flux per unit time and per unit area, multiplied by area) required to produce recognition of increase of brilliance; this cannot be constant if equation (6) is empirically valid, as it is shown to be as a very fair approximation. It follows from (6) that if Z is constant when A is increased, the added energy required for recognition of increase of brilliance is inversely proportional to the (average) increase of excitability per unit increase of area.

This entirely reasonable proposition can obtain only if the retinal field concerned, and its central representation, is of approximately uniform excitability over its whole extent, at the level of discriminatory response. This condition may well obtain for sufficiently large retinal fields symmetrically centered at the fovea; for an increased area, ΔI_0 and ΔI are then

less than for a smaller area, but the appearance of the smaller area included in the larger is nevertheless of the same brightness as for the rest of the field at the differential threshold for the larger area; the smaller area, of lower intrinsic excitability, when tested by itself, is not seen as such any more than the blind spot is. The larger area as a whole behaves as if of uniform excitability, a fact easily understood on the basis that the measurable properties of intensive experience are determined centrally rather than peripherally at the retina. From this standpoint only the analytical use of "average excitability" has a real physical basis.

TABLE II

Area increased by enlargement toward the fovea, the outer margin of the 20.8° high test patch being kept at 12.8° from the center of the fovea. A.H.H., right eye; white light; 20 minutes dark adaptation, 30 seconds light adaptation; ΔI_m is the mean of five observations; $\sigma_{1_{\Delta I}}$ is the dispersion of these; I_1 is the initial intensity; other conditions as described in the text; A' is the angular breadth of the test patch. Plotted in Figs. 4 and 5.

	A', *degrees*					
log I_1, *photons*	0.4°	0.8°	1.6°	3.2°	6.4°	12.8°
$\bar{1}.680$ log ΔI_m	**$\bar{2}.884$**	**$\bar{2}.530$**	**$\bar{2}.512$**	**$\bar{2}.341$**	**$\bar{2}.227$**	**$\bar{2}.071$**
log $\sigma_{1_{\Delta I}}$	($\bar{3}.921$)	($\bar{3}.665$)	($\bar{3}.416$)	($\bar{3}.305$)	($\bar{3}.015$)	($\bar{3}.122$)
0.680 log ΔI_m	**$\bar{1}.619$**	**$\bar{1}.481$**	**$\bar{1}.340$**	**$\bar{1}.074$**	**$\bar{2}.934$**	**$\bar{2}.687$**
log $\sigma_{1_{\Delta I}}$	($\bar{2}.703$)	($\bar{2}.509$)	($\bar{2}.532$)	($\bar{2}.111$)	($\bar{2}.930$)	($\bar{2}.004$)
1.680 log ΔI_m	**0.611**	**0.533**	**0.159**	**0.163**	**$\bar{1}.921$**	**$\bar{1}.830$**
log $\sigma_{1_{\Delta I}}$	($\bar{1}.628$)	($\bar{1}.480$)	($\bar{1}.062$)	($\bar{1}.205$)	($\bar{1}.005$)	($\bar{1}.070$)
2.680 log ΔI_m	**1.583**	**1.599**	**1.274**	**1.013**	**0.938**	**0.825**
log $\sigma_{1_{\Delta I}}$	(0.605)	(0.591)	(0.288)	($\bar{1}.994$)	(0.107)	($\bar{1}.738$)

Under these conditions Z is found to be nearly independent of A, I_1, and wave-length, and also of exposure time (*cf.* Crozier and Holway, in preparation). So also in certain other types of intensive discrimination, such as that (Crozier, 1935–36) concerned in measurements of visual acuity (Freeman, 1932; 1936), the same formulation is applicable. An arrangement is easily obtained, however, in which Z is constant but has a numerical value different from those already seen. It is not independent of the *shape* of the test patch. The rectangular image as already used (Table I) is in the present test excentrically located, and the edge farthest from the fovea is kept at a fixed position while, for enlargement of the area, the inner edge is moved closer to the fovea. With this arrangement an increase of the area necessarily involves a progressively greater change in the rate of addition

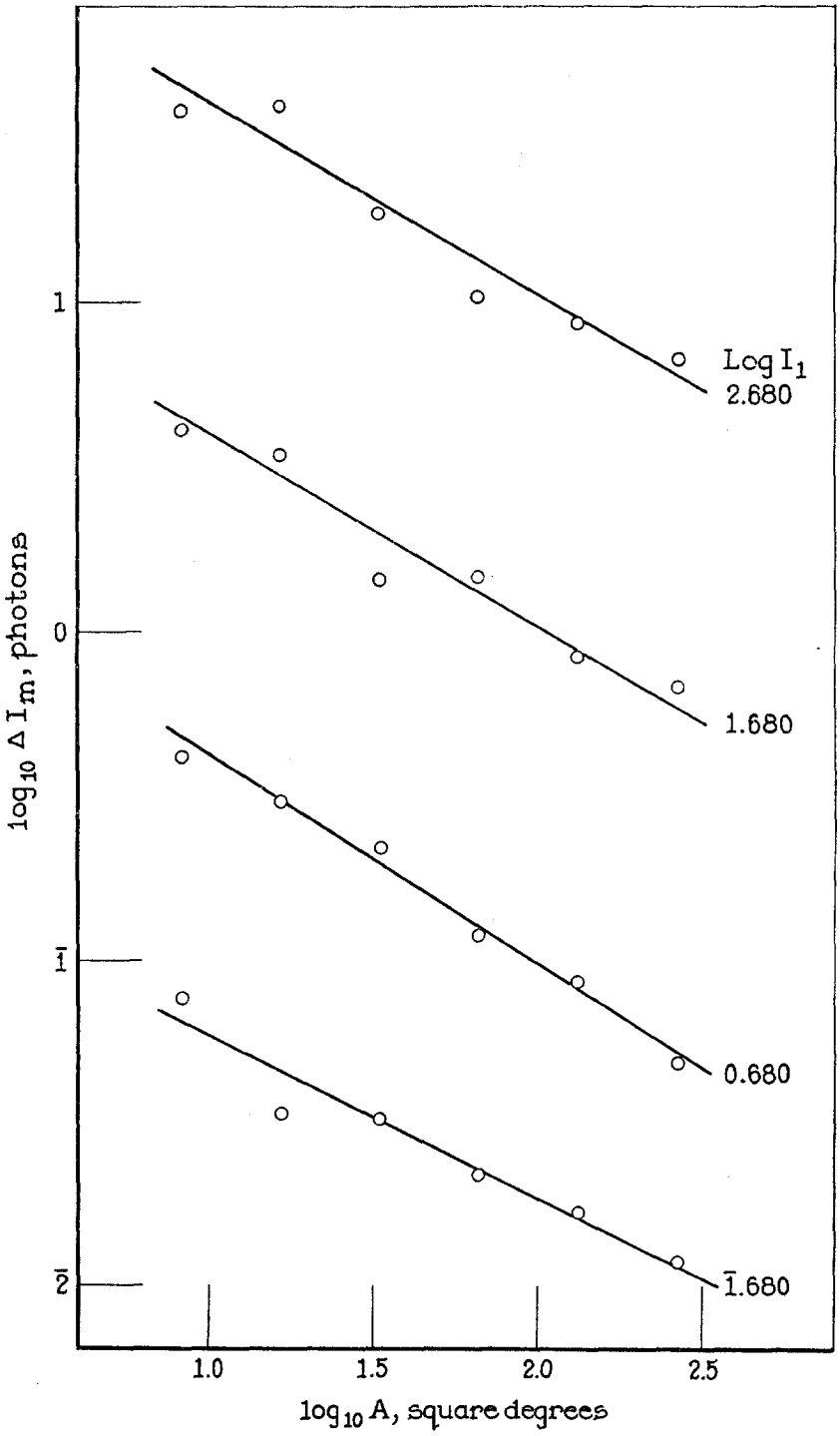

FIG. 4. Data in Table II; see text.

of excitable elements. It is necessary to remember that when area is increased in this manner the edge advancing toward the fovea in successive tests includes at each step regions of the retina for which the absolute excitatory thresholds (ΔI_0) per unit area are successively higher; but each of the doubling steps embraces a larger increment of area; the additional

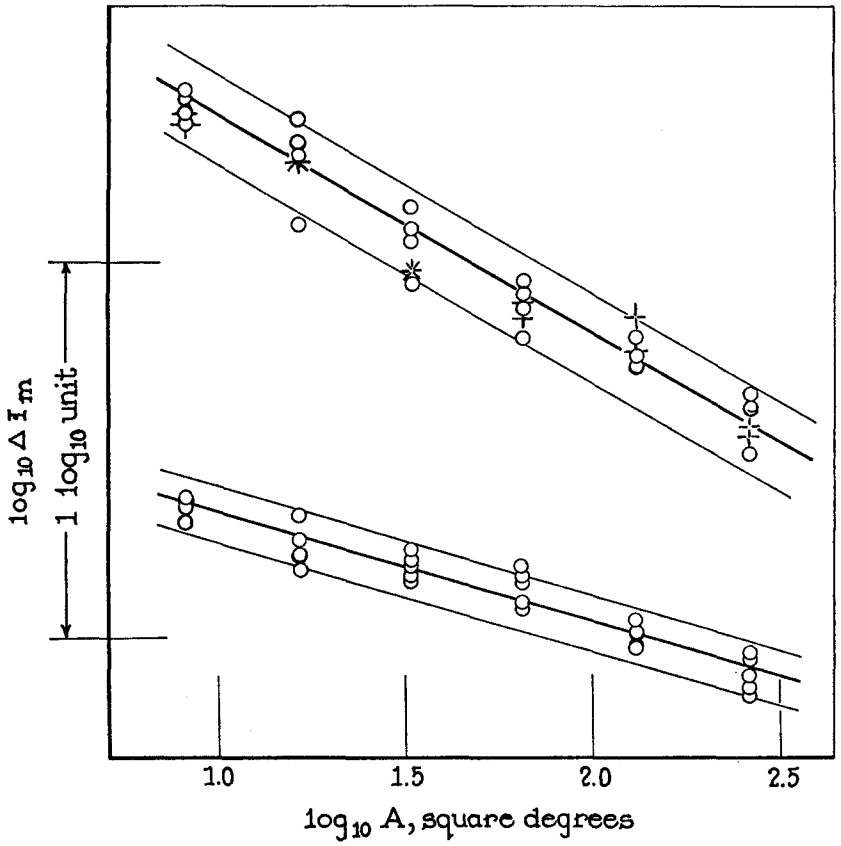

FIG. 5. Data of Fig. 3 (lower plot) and of Fig. 4 (upper plot; with figures from two other sets, not tabulated, given as crosses), brought together at the midpoint for comparison of slopes. The slope (Z in equation (2)) is independent of ΔI_m, and thus of I_1, but is dependent on the manner in which retinal area is increased; see text.

fact that the *differential* excitatory threshold (ΔI, for finite I_1) per unit area declines with approach to the fovea shows that appeal must be made to the changing ratio of numbers of receptor end organs to optic nerve fibers if these two excitability phenomena are to be explained. It must be expected, in terms of the introductory discussion (section I), that in such

an experiment the value of Z, if constant, must be *higher* than for tests of the type considered in Table I. For one of the same observers, data are given in Table II. It is apparent (Fig. 4) that equation (5) gives a very fair description of these data, and that the value of Z is indeed much greater, being 0.591 as compared with 0.267 in Fig. 3. Each doubling of the area on the average reduces ΔI by a factor of 1.52 (in the *same* arithmetic units), rather than by 1.21 as in Table I. Fig. 5 shows that the scatter of the determinations, with all the sets of points brought for comparison to the same log ΔI ordinate at the midpoint, is not too great; it is a little greater, understandably, in the experiment of Table II, since there fixation was outside the test patch. (The series of Table I show (Fig. 5) a certain consistency of bowing upward, which is not apparent in those for Table II and is probably not significant.)

In the discussion of these data it has thus far been accepted that increase of *area* is the significant factor in determining change in $1/\Delta I$ (with I_1 fixed = 0 or some finite value). We have to test this notion. The extent to which it is legitimate will necessarily be restricted by the geometrical pattern of neural organization in the retina. This is in the main on a plan with radial symmetry about the fovea (Østerberg, 1935, *etc.*). When a test patch is used of such form that increase of its area involves radially symmetrical enlargement, as with a circle or square of fixed center, it cannot be told for small ranges of area whether *area* or a linear *visual angle* is the governing feature (*cf.* Abney and Watson, 1916). The exponent Z will of course differ in these two relations, by a factor of 2, since log A will be equal to 2 log D + *const.* (where D is a linear dimension). This was one reason for our selection of rectangular test fields. With constant vertical height of field, Z is of course the same whether log *visual angle* of breadth or log *area* is used. But by employing two or more sets of rectangular test fields of constant but different heights (section VI) we can discover whether *area* or *angular breadth* gives the proper units for analysis.

V

Power functions, such as represented by equation (5), are rather frequently found to be serviceable for the formulation of properties of natural phenomena. And not infrequently the exponent in such a formulation is fractional, not integral, as with our Z.

In dimensional analysis it is recognized that for measurements in which secondary or derived units are used the fundamental units necessarily enter as products of integral *powers* of the primary variables. It has also been recognized (by some at least) that the occurrence of *fractional* powers is

eminently puzzling. If, however, we are dealing with the relation between two indirect, derived ("secondary") measures of the same fundamental attribute, the occurrence of a power function for the relationship between them may well be expected; and there is every reason to find that the exponent is frequently not an integer. In deriving equation (5) we have taken the view that the capacity for excitation is a function of the area A. Presumably this is in some fashion to be expressed in terms of number of potentially excitable central elements and their interconnections. A is then to be taken as a function of products of powers of these elementary dimensions (*cf.* Bridgman, 1922). The deduction of the nature of these elementary dimensions (factors) is of course the essential problem. Since it must be approached indirectly, we are forced to use other, additional modes of measuring the capacity for excitation. We might use exposure time, but ΔI is experimentally more flexible. The same fundamental excitability factors are involved. As different measures of the excitability, we have A and ΔI; for simplicity in illustration we may assume that each is a function of merely the number of potentially excitable elements, raised to a power. The exponents need not be the same. In this case the experimentally found relation of ΔI to A would be given by a power function, and the exponent could have almost any finite value; particularly when it is recognized that the "number of elements" is (in view of the fact of central summation—*cf.* Crozier and Holway, 1938–39) measured by a complex quantity.

When one speaks of a property or capacity of biological performance as being measured in "secondary units" which follow a law determined by the operation of "fundamental" or primary units, a certain vagueness is necessarily introduced by one's inability to write down the list of primary units necessarily involved. This has probably been responsible for the view (Bridgman, 1922, p. 53) that dimensional analysis may not be applicable to the results of most kinds of biological measurements, since in many cases these cannot be described in complete equations without the use of as many dimensional constants as there occur physical variables. This state of affairs arises from the fact that the mechanism whereby a given physical variable influences the biological manifestation considered is not only unknown, but is indeed itself the objective of inquiry. When two kinds of secondary measures of the same performance property are to be correlated, however, this restriction need not enter.

This will depend on the wisdom and understanding exercised in the choice of variables, but in a given instance can be subjected to direct experimental control and test of the result. Such a test is given in the present

case by the determination of several properties of our dimensionless exponent Z,—its demonstrated invariance under certain sets of conditions, and its mode of alteration under other circumstances. There is no valid reason for the occurrence of *properties* found if Z is the outcome of statistical accident. The elementary logic of this situation is of precisely the same kind as that involved in the proof which has been given of the number of excitation elements theory in responses to visual flicker, and of the invariance of certain properties of this quantity (*cf.* Crozier, Wolf, and Zerrahn-Wolf, 1937–38 *a, b, c;* 1938–39; Crozier, 1939.)

Unquestionably considerations of the same type apply to a number of really analogous situations in physics. Their generality from a dynamical standpoint merits brief discussion. For phenomena of sensory discrimination other than visual, homogeneous data on somaesthetic pressure (Holway and Crozier, 1937*b*) show that ΔP, for P constant, is proportional to $A^{1.1}$. A specific biological example among phenomena of a different order is not out of place. Consider the problem of measuring the reproductive performance of mice. Unquestionably the amount of mouse substance produced at a birth is influenced by many factors. Some of these can be ruled out by dealing with first litters borne by mothers of a genetically uniform strain at the same age. The productivity in these litters can be measured in two ways: (1) the number N of young in a litter, (2) the total mass W of the litter. The problem which gives the analogy with the subject of the present paper is that of determining the relation between W and N. Precisely the same reasoning as already used asserts that we should find

$$W = kN\frac{dW}{dN}, \tag{7}$$

and that k should be independent of W, N, size, and speed of growth. This says that $W = W_1N^k$, where W_1 = the weight of a litter of 1. It is found empirically that log W is a rectilinear function of log N, with a fractional exponent, and that k is the same for various kinds of mice and for various other kinds of mammals (Crozier and Enzmann, 1935–36), and k is thus quite independent of the growth rate, size or weight of litter, weight of a litter of 1, or species. The fact that W and N are different quantitative measures of the productive performance, on different kinds of scales, does not interfere with the prediction of the form of their interrelationship. Precisely similar reasoning, involving the use of *formal* equation (7), has been used to predict successfully the relationship between *variations* of performance of visual reactions, when excitability is measured along each of two coordinate axes (Crozier, 1935; Crozier, Wolf, and Zerrahn-Wolf, 1936–37*a, b*; Crozier and Holway, 1938), so that the illustration does not by any means stand alone.

For nonliving physical systems two instances may be conveniently cited for comparison. (1) The maximum (saturation) current (i) in a thermionic tube is determined by the space charge due to the electrons between the electrodes. The charge can be measured indirectly by the current; but it is due to, and also measurable by, the saturating applied voltage V. Consequently we should find the relationship between pairs of values of i, V, to be of the form

$$\log i = n \log V + C.$$

Langmuir's law shows it to be

$$i = kV^{3/2}$$

(Langmuir, 1913). (2) The internal energy of a black body radiator may be measured by observing its temperature (Kelvin); this gives the mean energy of the molecules in the radiator. This energy also determines the energy density in the radiator, and thus the energy density of the emitted radiation with which it is in equilibrium. Hence if T be the temperature and R the external radiation density, we must expect to find R and T interrelated in the form

$$\log R = k \log T + C.$$

It is well known that Stefan's law shows $R = aT^4$.

A further biological instance is not without point. The "hunger drive" of a rat is objectively expressed in the frequency with which (under certain controlled conditions) it devours pellets of food; this "drive" decreases with the number N of equivalent pellets it consumes in series, and is therefore a function of $1/N$; it is also a declining function of the time elapsed during consumption of the N particles (Skinner, 1931 *a*, *b*). Consequently we can write the form of the expected relation between N and t, since $1/N$ and $1/t$ at a given point in an eating series each measures in a different way and on a different scale the magnitude of the "eating potential." Without assumptions of any kind as to the nature of the drive mechanism, but on purely dimensional grounds, as in the cases already discussed, we can say that

$$k\,N = t\,dN/dt, \text{ if } k \text{ is a constant,}$$

or $N^k = Ct$.

This is the relationship found (Skinner, 1931). It says that the dimensional constant C is equal to N^k after the initial unit of elapsed time. It clearly would be incorrect to say that since, in the experimental arrangement used one and only one ($N = 1$) piece can be eaten at a time, we should subtract 1 from N in the equation, yet this is the homologue of the proposed subtraction of n_t in Wald's (1937–38) formula.

VI

In equations (1, 4, and 5) the value of the exponent Z is independent of the units of area and of intensity, but (as we have seen, section IV) it is not independent of the manner in which area of retinal image is increased. This means that it cannot be independent of the form of the image (or of its regional location). Thus for observer A. H. H. there are available tests with circular areas (right eye) foveally fixated at the center (Holway and Hurvich, 1938). These data give $Z = 0.402$, for $\log_{10}A$, to be compared with 0.267 for rectangular areas (section IV) with the same amount of scatter in ΔI_m as in the upper plot of Fig. 5.

When, however, we employ a very narrow band of light, with one end on the fovea, approximating a radius of the centrally fixated circular field, the relation of ΔI_0 to A becomes more complex (Table III and Fig. 6).

Roughly speaking, within the fovea (radius *ca.* 1°), Z has a high value approximating 1 (*cf.* also Abney, 1897). For wider areas the slope (Z) abruptly declines to about half. For the three observers we note in Fig. 6

TABLE III

Visual intensity thresholds (ΔI_0, photons) as a function of area increasing in steps of $\times 2$; white light; exposure time 0.04 second angular height of test patch 2°; each entry is the mean of five measurements; $\sigma_{\Delta I}$ is the root-mean-square deviation of one observation; one edge of the test patch was centrally fixated at the center of the fovea, increasing areas being spread out horizontally on the temporal aspect of the retina. Data on three observers (left eye, A.C.S.H., W.J.C.; right eye, A.H.H.). See Fig. 6.

Width, *degrees*	Obs.: A. H. H.		A. C. S. H.		W. J. C.	
	$\log \Delta I_0$	$\log \sigma_{\Delta I}$	$\log \Delta I_0$	$\log \sigma_{\Delta I}$	$\log \Delta I_0$	$\log \sigma_{\Delta I}$
0.2	$\bar{3}.757$	$\bar{4}.537$	$\bar{2}.214$	$\bar{3}.126$	$\bar{3}.968$	$\bar{4}.891$
	$\bar{3}.888$	$\bar{4}.797$	$\bar{2}.183$	$\bar{3}.135$	$\bar{3}.892$	$\bar{4}.528$
	$\bar{3}.781$	$\bar{4}.537$	$\bar{2}.201$	$\bar{3}.147$	$\bar{3}.915$	$\bar{4}.579$
	$\bar{3}.834$	$\bar{4}.541$	$\bar{2}.199$	$\bar{3}.119$	$\bar{3}.934$	$\bar{4}.941$
0.4	$\bar{3}.451$	$\bar{4}.699$	$\bar{3}.873$	$\bar{4}.704$	$\bar{3}.685$	$\bar{4}.382$
	$\bar{3}.584$	$\bar{4}.817$	$\bar{3}.814$	$\bar{4}.463$	$\bar{3}.654$	$\bar{4}.151$
	$\bar{3}.599$	$\bar{4}.660$	$\bar{3}.927$	$\bar{4}.740$	$\bar{3}.685$	$\bar{4}.461$
	$\bar{3}.590$	$\bar{4}.292$	$\bar{3}.876$	$\bar{4}.798$	$\bar{3}.655$	$\bar{4}.374$
0.8	$\bar{3}.238$	$\bar{4}.662$	$\bar{3}.583$	$\bar{4}.363$	$\bar{3}.367$	$\bar{4}.232$
	$\bar{3}.410$	$\bar{4}.603$	$\bar{3}.608$	$\bar{4}.335$	$\bar{3}.362$	$\bar{4}.363$
	$\bar{3}.389$	$\bar{4}.370$	$\bar{3}.517$	$\bar{4}.241$	$\bar{3}.362$	$\bar{4}.787$
	$\bar{3}.230$	$\bar{4}.351$	$\bar{3}.668$	$\bar{4}.389$	$\bar{3}.373$	$\bar{4}.113$
1.6	$\bar{4}.929$	$\bar{5}.849$	$\bar{3}.453$	$\bar{4}.577$	$\bar{3}.161$	$\bar{4}.063$
	$\bar{3}.053$	$\bar{4}.096$	$\bar{3}.352$	$\bar{4}.247$	$\bar{3}.158$	$\bar{4}.136$
	$\bar{3}.100$	$\bar{4}.170$	$\bar{3}.419$	$\bar{4}.232$	$\bar{3}.137$	$\bar{5}.987$
	$\bar{3}.033$	$\bar{4}.065$	$\bar{3}.398$	$\bar{4}.595$	$\bar{3}.152$	$\bar{5}.916$
3.2	$\bar{4}.754$	$\bar{5}.730$	$\bar{3}.250$	$\bar{4}.0587$	$\bar{4}.981$	$\bar{4}.084$
	$\bar{4}.705$	$\bar{5}.875$	$\bar{3}.193$	$\bar{5}.803$	$\bar{3}.017$	$\bar{4}.173$
	$\bar{4}.717$	$\bar{5}.693$	$\bar{3}.204$	$\bar{4}.222$	$\bar{3}.065$	$\bar{4}.121$
	$\bar{4}.772$	$\bar{5}.668$	$\bar{3}.182$	$\bar{5}.946$	$\bar{4}.966$	$\bar{5}.707$
6.4	$\bar{4}.602$	$\bar{5}.583$	$\bar{3}.004$	$\bar{4}.010$	$\bar{4}.830$	$\bar{5}.804$
	$\bar{4}.623$	$\bar{5}.617$	$\bar{3}.089$	$\bar{5}.937$	$\bar{4}.851$	$\bar{5}.846$
	$\bar{4}.609$	$\bar{5}.554$	$\bar{3}.102$	$\bar{4}.069$	$\bar{4}.803$	$\bar{5}.753$
	$\bar{4}.651$	$\bar{5}.661$	$\bar{3}.018$	$\bar{5}.976$	$\bar{4}.828$	$\bar{5}.835$
9.6	$\bar{4}.483$	$\bar{5}.380$	$\bar{4}.908$	$\bar{5}.531$	$\bar{4}.774$	$\bar{5}.820$
	$\bar{4}.528$	$\bar{5}.486$	$\bar{4}.969$	$\bar{5}.961$	$\bar{4}.680$	$\bar{5}.4137$
	$\bar{4}.558$	$\bar{5}.423$	$\bar{4}.940$	$\bar{5}.740$	$\bar{4}.743$	$\bar{5}.626$
	$\bar{4}.598$	$\bar{5}.471$	$\bar{4}.942$	$\bar{5}.830$	$\bar{4}.666$	$\bar{5}.586$

certain individual differences, which may be directly correlated with already known peculiarities of their individual local excitability thresholds in the same horizontal retinal meridian (Crozier and Holway, 1938–39 *b*). In the

first place the general order of the excitabilities (A.H.H. > W.J.C. > A.C.S.H.) is the same. More interesting is the fact that the ΔI_0 distance

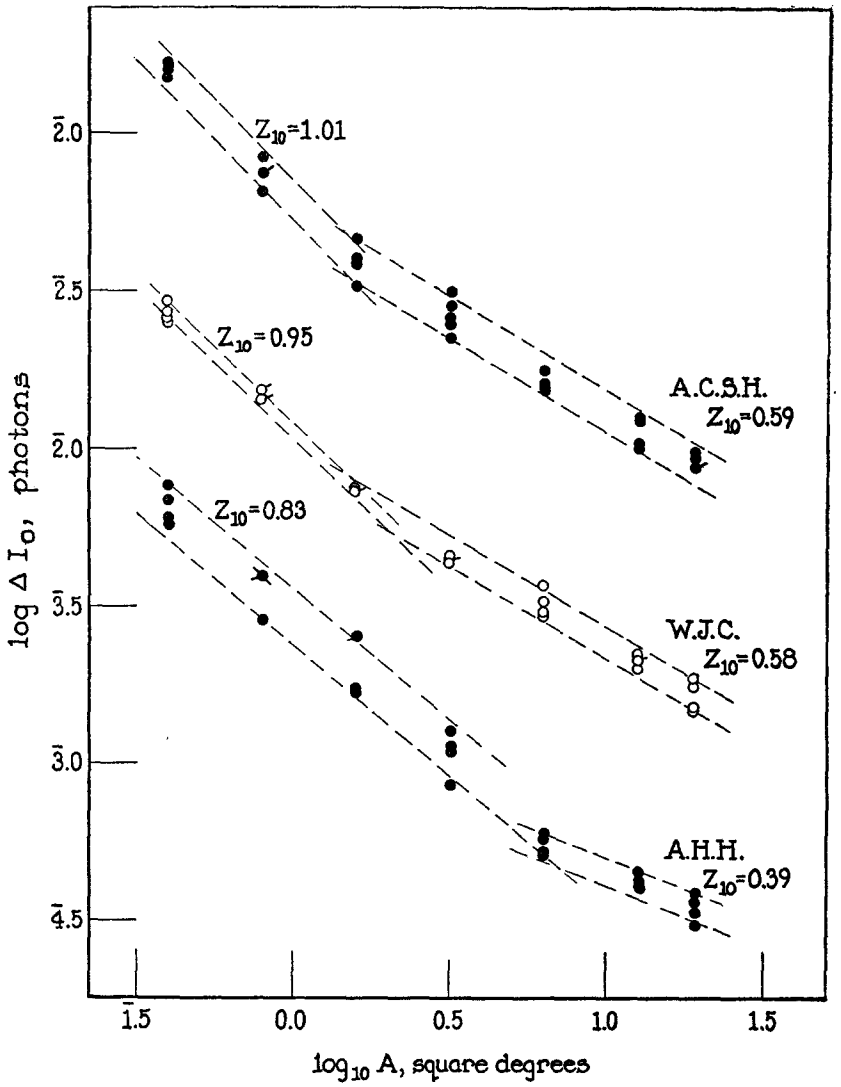

FIG. 6. Dependence of ΔI_m upon A when the image is a narrow bar of increasing length; see text; data in Table III.

between excitabilities at the fovea and in the extra-foveal region is much the greatest for A.H.H., least for A.C.S.H., and intermediate for W.J.C. (*cf.* Crozier and Holway, 1938–39 *b*, Fig. 4 on p. 356). In correlation with

this we find (Fig. 6) that the lower slope for A.H.H. is least, and that for A.H.H. the higher (foveal) slope is continued to a much more extensive area, the order of appearance of the "break" being A.C.S.H. < W.J.C. < A.H.H. This clearly signifies that the intrinsic excitability of each included region contributes to the determination of ΔI_0, but that it is the magnitude

TABLE IV

Visual intensity thresholds ΔI_0, (photons) as a function of area increasing in steps of $\times 2$; white light, exposure time 0.04 second; angular height of test patch 2° at the retina; each entry is the mean of five measurements; $\sigma_{\Delta I}$ is the root-mean-square of one observation; one edge of the test patch was centrally fixated at the center of the fovea; increasing areas A_1, A_2, A_3, A_4 were spread out horizontally on the temporal aspect of the retina; see text. Data on three observers (left eye, W.J.C., A.C.S.H.; right eye, A.H.H.).

Obs.: W. J. C.				A. C. S. H.				A. H. H.			
$\log \Delta I_0$	$\log \sigma_{\Delta I_0}$	$\log \Delta I_0$	$\log \sigma_{\Delta I_0}$	$\log \Delta I_0$	$\log \sigma_{\Delta I_0}$	$\log \Delta I_0$	$\log \sigma_{\Delta I_0}$	$\log \Delta I_0$	$\log \sigma_{\Delta I_0}$	$\log \Delta I_0$	$\log \sigma_{\Delta I_0}$
$A_1 = 1.2°$		$(A_2 - A_1) = 1.2°$		$A_1 = 1.2°$		$(A_2 - A_1) = 1.2°$		$A_1 = 1.2°$		$(A_2 - A_1) = 1.2°$	
$\bar{3}.344$	$\bar{4}.543$	$\bar{3}.295$	$\bar{4}.252$	$\bar{3}.485$	$\bar{4}.495$	$\bar{3}.393$	$\bar{4}.415$	$\bar{3}.017$	$\bar{4}.183$	$\bar{4}.899$	$\bar{5}.924$
$\bar{3}.288$	$\bar{4}.525$	$\bar{3}.308$	$\bar{4}.463$	$\bar{3}.492$	$\bar{4}.503$	$\bar{3}.361$	$\bar{4}.451$	$\bar{3}.096$	$\bar{4}.002$	$\bar{4}.876$	$\bar{5}.884$
$\bar{3}.243$	$\bar{4}.428$	$\bar{3}.220$	$\bar{4}.689$	$\bar{3}.470$	$\bar{4}.688$	$\bar{3}.396$	$\bar{4}.300$	$\bar{3}.074$	$\bar{4}.208$	$\bar{4}.955$	$\bar{5}.987$
$\bar{3}.243$	$\bar{4}.613$	$\bar{3}.233$	$\bar{4}.501$	$\bar{3}.459$	$\bar{4}.752$	$\bar{3}.348$	$\bar{4}.702$	$\bar{3}.112$	$\bar{4}.193$	$\bar{4}.899$	$\bar{4}.127$
$A_2 = 2.4°$		$(A_3 - A_2) = 2.4°$		$A_2 = 2.4°$		$(A_3 - A_2) = 2.4°$		$A_2 = 2.4°$		$(A_3 - A_2) = 2.4°$	
$\bar{3}.179$	$\bar{4}.344$	$\bar{3}.173$	$\bar{4}.402$	$\bar{3}.250$	$\bar{4}.397$	$\bar{3}.090$	$\bar{4}.091$	$\bar{4}.803$	$\bar{4}.143$	$\bar{4}.807$	$\bar{4}.001$
$\bar{3}.117$	$\bar{4}.266$	$\bar{3}.185$	$\bar{4}.199$	$\bar{3}.262$	$\bar{4}.353$	$\bar{3}.146$	$\bar{4}.256$	$\bar{4}.870$	$\bar{4}.121$	$\bar{4}.723$	$\bar{5}.983$
$\bar{3}.193$	$\bar{4}.378$	$\bar{3}.152$	$\bar{4}.616$	$\bar{3}.250$	$\bar{4}.411$	$\bar{3}.041$	$\bar{4}.099$	$\bar{4}.856$	$\bar{4}.067$	$\bar{4}.732$	$\bar{5}.939$
$\bar{3}.145$	$\bar{4}.318$	$\bar{3}.158$	$\bar{4}.031$	$\bar{3}.239$	$\bar{4}.295$	$\bar{3}.013$	$\bar{4}.262$	$\bar{4}.831$	$\bar{5}.931$	$\bar{4}.743$	$\bar{5}.778$
$A_3 = 4.8°$		$(A_4 - A_3) = 4.8°$		$A_3 = 4.8°$		$(A_4 - A_3) = 4.8°$		$A_3 = 4.8°$		$(A_4 - A_3) = 4.8°$	
$\bar{3}.029$	$\bar{5}.980$	$\bar{4}.918$	$\bar{4}.001$	$\bar{4}.952$	$\bar{5}.986$	$\bar{4}.958$	$\bar{4}.044$	$\bar{4}.629$	$\bar{4}.004$	$\bar{4}.611$	$\bar{5}.624$
$\bar{3}.061$	$\bar{4}.332$	$\bar{4}.975$	$\bar{4}.503$	$\bar{4}.949$	$\bar{5}.999$	$\bar{4}.896$	$\bar{4}.006$	$\bar{4}.664$	$\bar{5}.580$	$\bar{4}.622$	$\bar{5}.714$
$\bar{3}.021$	$\bar{4}.996$	$\bar{4}.921$	$\bar{4}.286$	$\bar{4}.982$	$\bar{5}.991$	$\bar{4}.947$	$\bar{4}.102$	$\bar{4}.652$	$\bar{5}.579$	$\bar{4}.536$	$\bar{5}.865$
$\bar{3}.041$	$\bar{4}.253$	$\bar{4}.884$	$\bar{5}.689$	$\bar{4}.903$	$\bar{4}.057$	$\bar{4}.902$	$\bar{4}.082$	$\bar{4}.702$	$\bar{5}.900$	$\bar{4}.555$	$\bar{5}.935$
$A_4 = 9.6°$				$A_4 = 9.6°$				$A_4 = 9.6°$			
$\bar{4}.844$	$\bar{4}.232$			$\bar{4}.832$	$\bar{5}.907$			$\bar{4}.619$	$\bar{5}.844$		
$\bar{4}.827$	$\bar{5}.696$			$\bar{4}.817$	$\bar{5}.757$			$\bar{4}.572$	$\bar{5}.724$		
$\bar{4}.831$	$\bar{4}.302$			$\bar{4}.846$	$\bar{4}.087$			$\bar{4}.592$	$\bar{5}.776$		
$\bar{4}.839$	$\bar{5}.860$			$\bar{4}.821$	$\bar{4}.124$			$\bar{4}.600$	$\bar{5}.801$		

of $d(1/\Delta I_0)/dA$, *i.e.* the rate of addition of excitability units as A is increased, which is a governing factor.

The relationship is not simply one of addition, however. It is not our present purpose to discuss the phenomena and theory of "retinal summation," but we are required to demonstrate that there is a sense in which a given stimulated retinal area behaves as a unit, and in which its contribu-

tion to the observed excitability is determined by the concurrent excitation of spatially contiguous regions. An experiment giving such a demonstration is summarized in Table IV. As in the experiment of Table III, one 2° edge of the test patch was centered at the fovea; ΔI_0 was first determined for a bar 1.2° wide (A_1); then for one 2.4° wide (A_2); then for the outer half of A_2, *i.e.* for $A_2 - A_1$; and so on, as indicated in Table IV. It is apparent that under these conditions for all three observers the threshold for a given rectangular area is *less* than for either the near-foveal half or the outer half of the area alone, although for a given area the threshold is higher for the near-foveal half than for the outer half. The plot of log ΔI_0 *vs.* log A for these data shows that Z agrees very well with the values for each observer (Fig. 6) gotten with the rectangular patch 12.8° high (Table III). Hence we have direct support for the view that it is retinal *area* rather than visual angle which is the proper independent variable. It is also apparent in the data of Table IV that the slopes for log ΔI_0 *vs.* A are practically the same for A_1, A_2, A_3, A_4, as for $A_2 - A_1$, $A_3 - A_2$, *etc;* hence these strips behave as if essentially "homogeneous" so far as the resultant relative rate of increase of excitation is concerned.

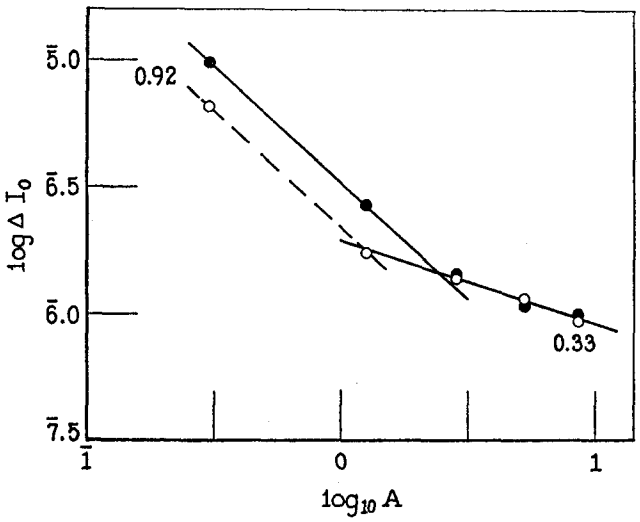

FIG. 6 *bis.* Data from Wald (1937–38, his Table II, p. 272): threshold ΔI_0 (millilamberts, no artificial pupil, *ca.* 1 second exposure) as a function of circular area (A, arbitrary units proportional to square centimeters) for two regions respectively centered 15° above the fovea (open circlets) and 25° above (solid circlets). Within the error of such measurements the slopes, Z = 0.92 and 0.33 as drawn, agree quantitatively with those in our Fig. 6—see particularly the plot for A.H.H.; it is to be noted also that the break in the curve comes in precisely the same zone of absolute area of retinal image.

We desire to avoid any specific discussion of the relationship between the two slopes appearing in Fig. 6 and the properties traditionally assumed for cones and rods respectively. These slopes are of the order of 0.93 and

0.52. (For the data in Fig. 3, Table I, the value of Z is 0.267, but it is to be kept in mind that Z must in all likelihood be a decreasing function of exposure time—*ca.* 30 seconds in the latter case, and 0.40 second in the former.) For small regions 15° and 25° from the fovea, Wald's primary data (1937–38) show (Fig. 6 *bis*) exactly the same sort of break as we have demonstrated in Fig. 6. Piéron (1929) cites foveal measurements which "break" at about the same area ($\log_{10} A = 0.7 \pm$). It is impossible to hold that the high slope is due to the functioning of retinal cones, since at 15° away from the fovea the proportion of cones to rods is about 1:25, while at 25° away, with a smaller number of rods per unit area, the proportion is only slightly less; the assumption that rod thresholds are intrinsically much lower than cone thresholds cannot be used to explain the facts shown in Figs. 6 and 6 *bis*. Obviously, if the low-slope segment, at the larger areas, is due to the threshold activation of rods, there is no reason for supposing that with the smallest areas the decreasing number of high-threshold cones would alone become effective.

VII

Monocular Observations, Colored Light

Monocular determinations of ΔI_m were made for red, green, and blue lights. The method of observation was the same as with white light. The filters and the calibration of intensities are discussed in section II. The data are summarized in Table V.

In Fig. 7 these measurements are plotted in terms of equation (5). The fitted lines give mean $Z = 0.239$; when A is taken logarithmically to base 2, $Z = 0.080$. A twofold increase of area occasions a reduction of ΔI_m by a factor of 1.202. The mean values of Z ($= 0.239$) are less than that obtained with white light ($Z = 0.267$). The reality of the lower slope is supported by the similar difference obtained in the series of binocular measurements (section VIII). The differences between the slopes for the three kinds of colored lights are not certainly significant (Fig. 8), but are reasonably consistent with certain expectations. In Fig. 8 the data for each color are brought together for comparison: in each set the ΔI_m's are multiplied by a constant, as in Fig. 5 for white light.

The discussion in section VI makes it clear, we believe, that if the absolute number of elements open to excitation is less, as result of a given method of presentation, then the slope of the log ΔI *vs.* log A plot must be expected to be less. Specifically, since in energy terms the thresholds for "mono-

chromatic" light are higher than for white, fewer elements of effect are open to arousal; consequently, with a given area, a given percentage increase of

TABLE V

Colored lights: homogeneous results for ΔI and A, A.H.H., right eye. Intensity and exposure time (0.04 second) were parameters. I_1 is the standard intensity, in photons. A, in degrees, is the angular width of the light image on the retina; the height of the image was constant = 20.8°. Each ΔI entry is an average of five measurements; $\sigma_{\Delta I}$ (log, in parentheses) is the root-mean-square variation of a single observation. See Figs. 7 and 8.

log I_1, photons	A = 0.4°	0.8°	1.6°	3.2°	6.4°	12.8°
			λ = 465			
$\bar{1}$.334	$\bar{2}$.093	$\bar{3}$.964	$\bar{3}$.901	$\bar{3}$.802	$\bar{3}$.794	$\bar{3}$.628
	($\bar{3}$.082)	($\bar{4}$.943)	($\bar{4}$.916)	($\bar{4}$.827)	($\bar{4}$.698)	($\bar{4}$.617)
0.334	$\bar{2}$.987	$\bar{2}$.793	$\bar{2}$.841	$\bar{2}$.650	$\bar{2}$.629	$\bar{2}$.601
	($\bar{2}$.914)	($\bar{3}$.816)	($\bar{3}$.873)	($\bar{3}$.779)	($\bar{3}$.593)	($\bar{3}$.615)
1.334	$\bar{1}$.923	$\bar{1}$.688	$\bar{1}$.811	$\bar{1}$.680	$\bar{1}$.627	$\bar{1}$.425
	($\bar{1}$.000)	($\bar{2}$.653)	($\bar{2}$.804)	($\bar{2}$.697)	($\bar{2}$.656)	($\bar{2}$.601)
2.334	0.802	0.703	0.653	0.528	0.521	0.477
	($\bar{1}$.914)	($\bar{1}$.896)	($\bar{1}$.633)	($\bar{1}$.577)	($\bar{1}$.606)	($\bar{1}$.500)
3.334	1.610	1.469	1.583	1.399	1.370	1.294
	(0.573)	(0.458)	(0.600)	(0.404)	(0.380)	(0.305)
			λ = 525			
$\bar{1}$.2	$\bar{2}$.614	$\bar{2}$.563	$\bar{2}$.472	$\bar{2}$.419	$\bar{2}$.343	$\bar{2}$.310
	($\bar{3}$.714)	($\bar{3}$.606)	($\bar{3}$.513)	($\bar{3}$.558)	($\bar{3}$.316)	($\bar{3}$.354)
0.2	$\bar{1}$.501	$\bar{1}$.492	$\bar{1}$.387	$\bar{1}$.369	$\bar{1}$.216	$\bar{1}$.216
	($\bar{2}$.497)	($\bar{2}$.515)	($\bar{2}$.463)	($\bar{2}$.494)	($\bar{2}$.365)	($\bar{2}$.227)
1.2	0.248	0.216	0.076	0.032	0.032	$\bar{1}$.897
	($\bar{1}$.290)	($\bar{1}$.307)	($\bar{1}$.004)	($\bar{1}$.006)	($\bar{2}$.979)	($\bar{2}$.914)
2.2	1.230	1.160	1.056	1.032	0.977	0.915
	(0.293)	0.102	0.125	$\bar{1}$.993	0.043	$\bar{1}$.976
			λ = 680			
$\bar{1}$.49	$\bar{1}$.397	$\bar{1}$.285	$\bar{1}$.246	$\bar{1}$.125	$\bar{1}$.079	$\bar{2}$.958
	($\bar{2}$.388)	($\bar{2}$.209)	($\bar{2}$.271)	($\bar{2}$.135)	($\bar{2}$.092)	($\bar{3}$.896)
0.49	0.250	0.224	0.128	0.053	$\bar{1}$.996	$\bar{1}$.892
	($\bar{1}$.138)	($\bar{1}$.243)	($\bar{1}$.207)	($\bar{1}$.119)	($\bar{2}$.943)	($\bar{2}$.939)
1.49	0.848	0.716	0.699	0.634	0.602	0.544
	($\bar{1}$.794)	($\bar{1}$.806)	($\bar{1}$.703)	($\bar{1}$.619)	($\bar{1}$.654)	($\bar{1}$.582)
2.49	1.799	1.763	1.634	1.544	1.477	1.372
	(0.800)	(0.772)	(0.705)	(0.593)	(0.484)	(0.405)

area, by the same method, should produce a smaller relative decrease of ΔI (for a given I_1). This is what the data of Table V show.

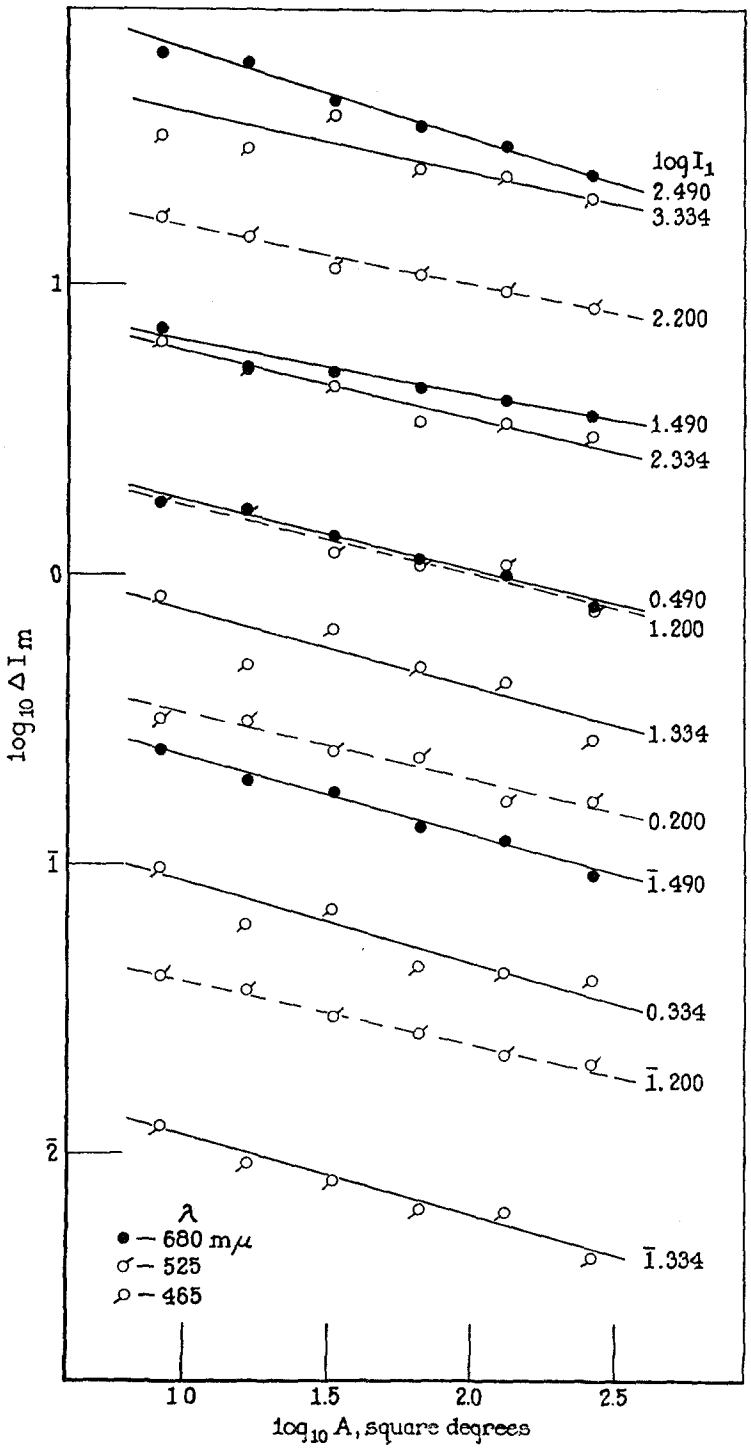

FIG. 7. Monocular excitation, as in Fig. 3, but with I_1 and λ as parameters (Table V).

VIII

Binocular Measurements

The form of the function relating ΔI_m to area for binocular excitation was determined by means of homogeneous data secured under conditions essentially identical with those used in the monocular procedure. The same apparatus, filters, wedges, and observers were used. Matched oculars, instead of a single ocular, were placed in the discriminometer head

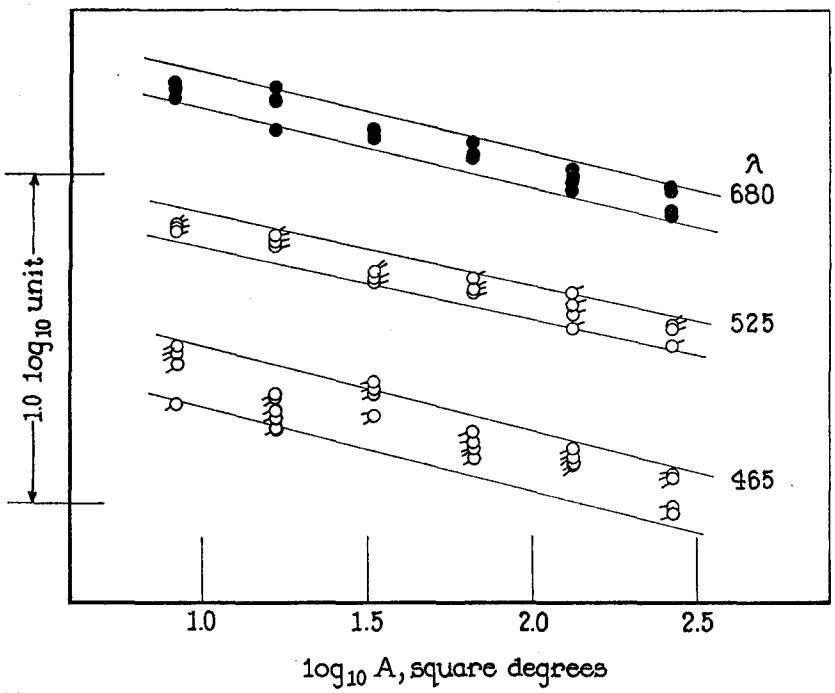

FIG. 8. The plots of Fig. 7 brought together for comparison of slopes (Z) for each of the three colors used; see text.

(section II). Measurements were taken only during times when the observer experienced a perfectly fused, unitary visual impression.

A certain amount of confusion still prevails with respect to the comparison of monocular and binocular thresholds. The pupillary reaction to the effects of photic excitation is consensual. For the commonplace situation, the size of the pupil in either eye is smaller when both eyes are stimulated by a given intensity than when one eye alone is stimulated, as is well known (*cf.* Rea, 1938). This fact clearly requires the use of effective artificial

pupils in experiments designed to test the absolute magnitudes of ΔI for binocular as contrasted with monocular discrimination. In addition, the importance of placing *equivalent* light images at corresponding points on the two retinae necessitates providing conditions which eliminate differences of form, size, and intensity distribution in the images used for both eyes as well as for one eye. This is done by assuring absence of changes in effective

TABLE VI

White light.—homogeneous results for *binocular* ΔI and A. I_1 is standard intensity, in photons. A', in degrees, is the angular width of the light image on the retina; the height of the image was constant, = 20.8°. Each ΔI entry is an average of five measurements; $\sigma_{\Delta I}$ is the root-mean-square variation of a single measurement. Observer: A.H.H. See Figs. 9 and 10.

Colored lights.—homogeneous results for *binocular* ΔI and A'. Intensity and exposure-time were parameters. I_1 is the standard intensity, in photons. A, in degrees, is the angular width of the light image on the retina; the height of the image was constant = 20.8°. Each ΔI entry is an average of five measurements; $\sigma_{\Delta I}$ is the root-mean-square variation of a single observation. See Figs. 9 and 10.

log I, *photons*	A' = 0.4°	0.8°	1.6°	3.2°	6.4°	12.8°
0.0	$\bar{1}$.217	$\bar{1}$.150	$\bar{1}$.021	$\bar{2}$.984	$\bar{2}$.861	$\bar{2}$.777
	($\bar{2}$.039)	($\bar{2}$.215)	($\bar{2}$.296)	($\bar{2}$.051)	($\bar{2}$.120)	($\bar{3}$.753)
1.0	0.153	0.080	$\bar{1}$.964	$\bar{1}$.885	$\bar{1}$.846	$\bar{1}$.730
	($\bar{1}$.217)	($\bar{1}$.193)	($\bar{1}$.045)	($\bar{1}$.126)	($\bar{2}$.803)	($\bar{2}$.821)
2.0	1.041	0.997	0.906	0.785	0.703	0.644
	(0.050)	(0.107)	(0.089)	($\bar{1}$.892)	($\bar{1}$.757)	($\bar{1}$.684)
3.0	2.050	1.903	1.941	1.867	1.680	1.652
	(1.132)	(1.082)	(1.112)	(1.002)	(0.679)	(0.829)
1.334	$\bar{1}$.783	$\bar{1}$.692	$\bar{1}$.625	$\bar{1}$.551	$\bar{1}$.443	$\bar{1}$.300
λ = 465	($\bar{2}$.816)	($\bar{2}$.753)	($\bar{2}$.656)	($\bar{2}$.701)	($\bar{2}$.409)	($\bar{2}$.416)
2.200	1.283	1.210	1.129	1.079	1.044	0.930
λ = 525	(0.288)	(0.312)	(0.065)	(0.207)	($\bar{1}$.993)	($\bar{1}$.942)
1.490	0.813	0.726	0.683	0.650	0.555	0.512
λ = 680	($\bar{1}$.927)	($\bar{1}$.735)	($\bar{1}$.702)	($\bar{1}$.795)	($\bar{1}$.701)	($\bar{1}$.553)

pupil aperture and by maintaining completely relaxed accommodation (Crozier and Holway, 1938–39 *a*, *b*). The former is given by the ocular eye ring in our apparatus (constant and less than 1.8 mm. diameter in the plane of the pupil of the observer's eye), the latter by the presence of a suitable small fixation point sharply defined at the retina only under completely relaxed accommodation (Crozier and Holway, 1938–39 *a*). It appears to us that only under conditions such as these can a real answer be

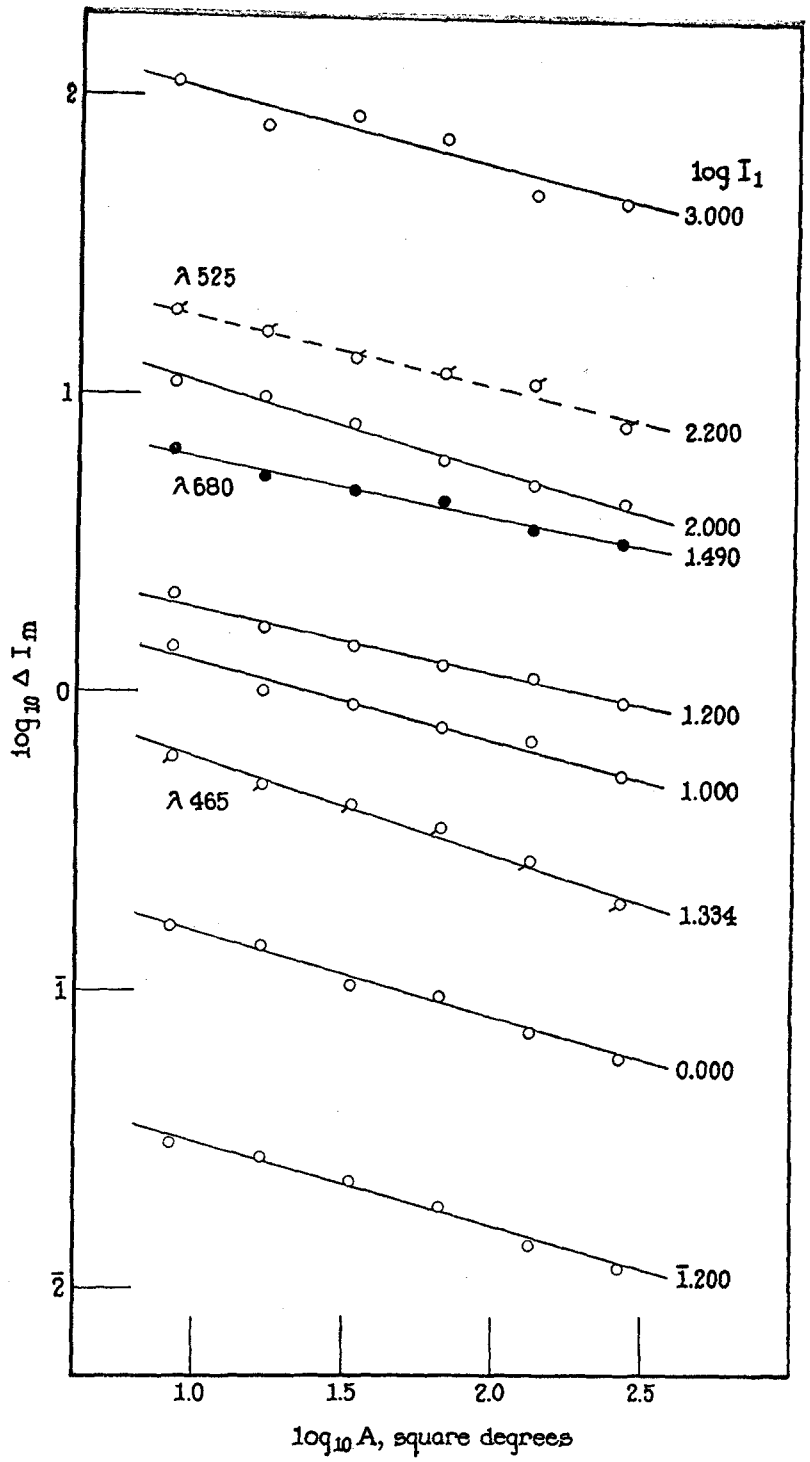

FIG. 9. Dependence of ΔI_m on A for simultaneous excitation of *both* eyes (*i.e.*, "binocular" presentation), at several levels of I_1 and for three colored lights. Table VI.

obtained to the question as to whether a valid difference exists between monocular and binocular thresholds for constant retinal light images.

When these conditions are effectively maintained, binocular absolute thresholds are found to be lower than for either eye separately (*cf.* Crozier and Holway, 1938–39 *b*). There seems to be no possibility of explaining this result through technical error of observation. Failure to maintain

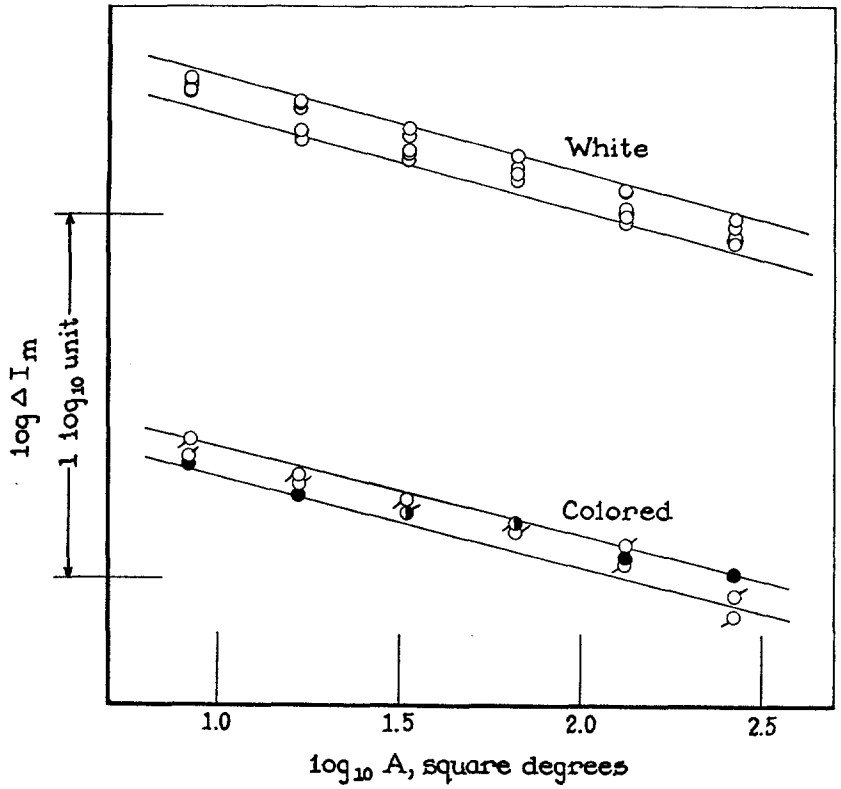

FIG. 10. Data of Fig. 9 brought together for comparison of slopes (Z), white light (above) and colored (below).

such conditions can easily account for data in which the binocular thresholds (absolute or differential) appear either as the average of those for each eye, or as approximately equal to those for the more excitable eye.

Data for white light are given in Table VI and in Fig. 9. In Table VI the visual angle A' (degrees) is the average of the equal angles subtended at each eye. I_1 is the standard intensity (photons). Each I_B entry is the mean of five measurements; $\sigma_{\Delta I_B}$ is the root-mean-square deviation. As with the monocular measurements (section III, *etc.*), both ΔI_B and $\sigma_{\Delta I_B}$

are decreasing functions of A for I_1 as parameter, and increasing functions of I_1 when A is constant. Equation (5) is adequate for these data (Fig. 9) as well as for the monocular observations (Figs. 4 and 5). The mean slope constant $Z = 0.268$ (Fig. 10), and is not significantly higher than the 0.267 for the monocular white data.

Table VI also contains corresponding data for three colored lights (Figs. 9 and 10). The mean value of Z (0.243) is a little greater than that for monocular presentation (sections III and VII), but the difference is probably not significant. For both monocular and binocular measurements Z is *smaller* with monochromatic light than with white. To a very close approximation $\log \Delta I_m$ decreases with respect to increase of $\log A$ at the same rate for monocular and for binocular excitation in each case. If binocular presentation of a given area to each eye actually "doubles" the number of affected elements, then a given increase of area will produce about the proportionate increase in potentially excitable elements in case of two eyes as when one is used, and our Z should not change appreciably.

It will be noticed that in this formulation Z is held to be essentially independent of changes produced in ΔI by means other than alteration of area, except in so far as Z is dependent on λ. The monocular and binocular values of ΔI thus far considered differ at fixed I_1 by a small but definite amount. This has been tested by plotting (from Figs. 3, 7, and 9) the values of $\log \Delta I_m$ at $A = 10$ units against the corresponding values of $\log I_1$. These data, however, are not most suitable for the determination of intrinsic differences between monocular and binocular excitabilities. Series of determinations were therefore made by observing ΔI (single measurement) with one eye, then with the other, and then with both, until five sets of readings were secured at a given A and I_1 (*cf.* Crozier and Holway, 1938–39 *b*).

Homogeneous data secured in this way are given in Table VII. Two of these series, $\log I_1 = \bar{1}.20$ and 1.20, were obtained by the method used in Table I, namely by symmetrical enlargement of the area on either side of the fovea; at another, intermediate intensity ($\log I_1 = 0.80$) the method of Table II was employed, area being enlarged by geometrically progressive steps toward the fovea from a fixed margin 12.8° on the temporal side (*cf.* section IV). It is to be noted that, as in the case of "absolute" threshold measurements with small areas at various retinal positions (Crozier and Holway, 1938–39 *b*), small differences are apparent in the readings with the two eyes. For the observer used in Table VII we have already found these differences to be comparatively minute. This is for certain purposes a distinct advantage. In the series under present consideration the differ-

TABLE VII

Comparable results for binocular and monocular ΔI. Angular breadth, A', intensity, I_1, wave-length composition, and exposure time (0.04 second) are parameters. For the upper and the lower curves (log I_1 = $\bar{1}$.20 and 1.20), all images were *centrally* fixated. A', in degrees, is the angular width of the image on the retina (height, 20.8°). I_1 is the standard intensity. For log I_1 = 0.80 (middle curve) the outer margin of the test patch was at 12.8° from the fovea (*see text*). Each ΔI_m value is an average of five measurements; $\sigma_{\Delta I}$ (in parentheses) is the root-mean-square variation of a single observation. ΔI_B is for binocular regard; ΔI_L, and ΔI_R are the values for the left and right eyes respectively. Observer, A.H.H. For discussion, see text. Fig. 11.

log I_1	log ΔI_L	log ΔI_R	log ΔI_B	
$\bar{1}$.20	$\bar{2}$.641	$\bar{2}$.609	$\bar{2}$.490	A' = 0.4°
	($\bar{3}$.575)	($\bar{3}$.721)	($\bar{3}$.583)	
	$\bar{2}$.559	$\bar{2}$.500	$\bar{2}$.441	A' = 0.8
	($\bar{3}$.560)	($\bar{3}$.612)	($\bar{3}$.603)	
	$\bar{2}$.521	$\bar{2}$.480	$\bar{2}$.362	A' = 1.6
	($\bar{3}$.619)	($\bar{3}$.505)	($\bar{3}$.392)	
	$\bar{2}$.401	$\bar{2}$.396	$\bar{2}$.275	A' = 3.2
	($\bar{3}$.507)	($\bar{3}$.510)	($\bar{3}$.300)	
	$\bar{2}$.325	$\bar{2}$.316	$\bar{2}$.146	A' = 6.4
	($\bar{3}$.367)	($\bar{3}$.355)	($\bar{3}$.099)	
	$\bar{2}$.248	$\bar{2}$.219	$\bar{2}$.068	A' = 12.8
	($\bar{3}$.178)	($\bar{3}$.192)	($\bar{3}$.108)	
1.20	0.482	0.511	0.329	A' = 0.4°
	($\bar{1}$.376)	($\bar{1}$.635)	($\bar{1}$.442)	
	0.449	0.365	0.210	A' = 0.8
	($\bar{1}$.516)	($\bar{1}$.252)	($\bar{1}$.307)	
	0.257	0.271	0.159	A' = 1.6
	($\bar{1}$.379)	($\bar{1}$.302)	($\bar{1}$.162)	
	0.160	0.209	0.097	A' = 3.2
	($\bar{1}$.103)	($\bar{1}$.294)	($\bar{1}$.085)	
	0.281	0.220	0.057	A' = 6.4
	($\bar{1}$.392)	($\bar{1}$.163)	($\bar{1}$.066)	
	0.112	0.100	$\bar{1}$.973	A' = 12.8
	($\bar{1}$.241)	($\bar{1}$.092)	($\bar{1}$.041)	
0.80	$\bar{1}$.905	$\bar{1}$.867	$\bar{1}$.782	A' = 0.4
	($\bar{2}$.882)	($\bar{2}$.953)	($\bar{2}$.891)	
	$\bar{1}$.783	$\bar{1}$.783	$\bar{1}$.640	A' = 0.8
	($\bar{2}$.864)	($\bar{2}$.650)	($\bar{2}$.703)	
	$\bar{1}$.484	$\bar{1}$.499	$\bar{1}$.381	A' = 1.6
	($\bar{2}$.455)	($\bar{2}$.399)	($\bar{2}$.457)	
	$\bar{1}$.407	$\bar{1}$.366	$\bar{1}$.241	A' = 3.2
	($\bar{2}$.512)	($\bar{2}$.402)	($\bar{2}$.286)	
	$\bar{1}$.362	$\bar{1}$.279	$\bar{1}$.115	A' = 6.4
	($\bar{2}$.453)	($\bar{2}$.265)	($\bar{2}$.105)	
	$\bar{1}$.043	$\bar{1}$.075	$\bar{2}$.920	A' = 12.8
	($\bar{2}$.031)	($\bar{2}$.102)	($\bar{1}$.991)	

ences between $\Delta\tilde{I}_L$ and $\Delta\tilde{I}_R$ are probably not significant. For each value of a_1, and for all values of A, $\Delta\tilde{I}_B$ is seen to be smaller than ΔI_L or ΔI_R. Using equation (5) to describe these data (Fig. 11), the mean value of Z is

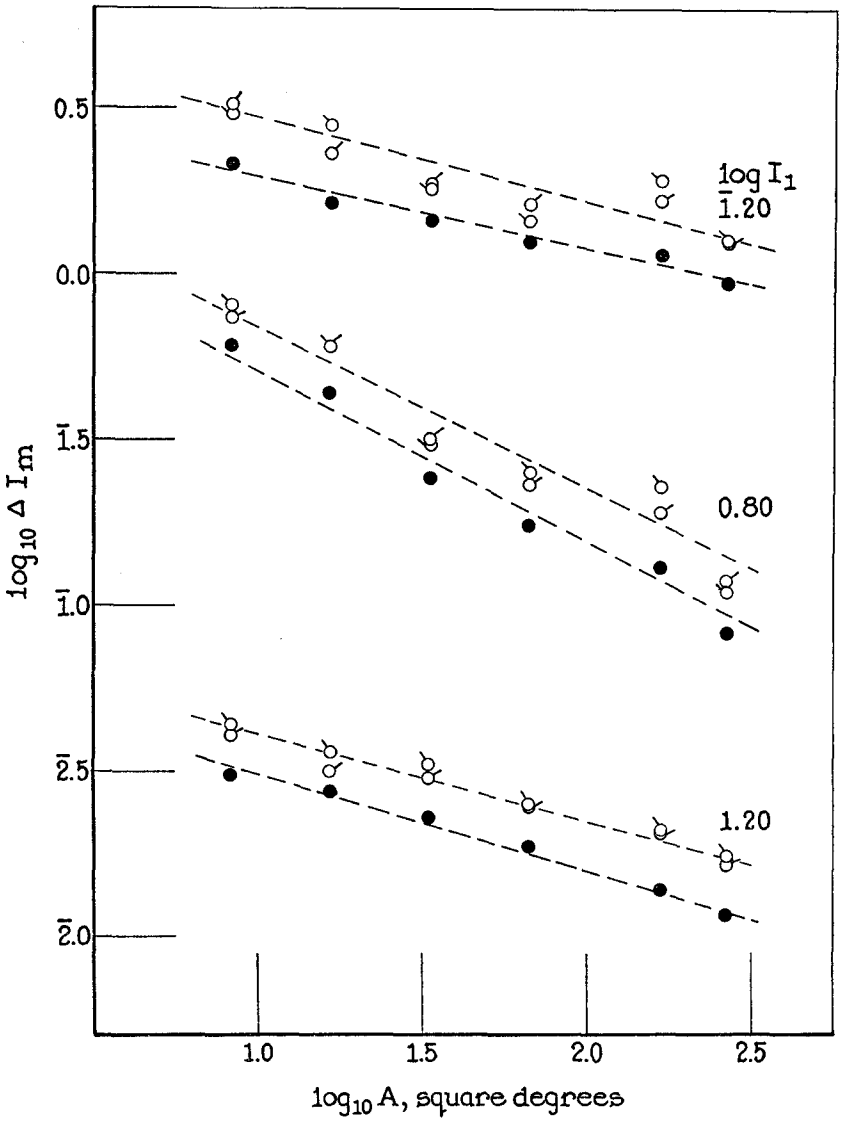

FIG. 11. Data of Table VII, homogeneous for the comparison of monocular with "binocular" differential thresholds; ΔI_m for right eye and left eye are indicated by directions of the tags on the open circlets; for binocular observations, by solid circlets. Central plots, at log $I_1 = 0.80$, are for the method of Table II, the others for the method of Table I (see text).

0.255 when log area A is taken to base 10, for both the monocular and binocular measurements.

The mean difference between $\log\left(\frac{\Delta I_L + \Delta I_B}{2}\right)$ and $\log \Delta I_B = 0.46 \log_{10}$ unit; or 0.14 with A to base 2. The antilog of 0.14 = 1.38. For absolute thresholds (*cf.* also Lythgoe and Phillips, 1938) we have obtained 1.41 for the ratio of monocular to binocular excitatory intensities (Crozier and Holway, 1938–39 *b*), but we noted that for greater than zero values of I_1 the ratio fell below 1.4. The meaning of this ratio clearly is that duplex presentation enlarges the number of potentially excitable elements, and may double it; hence the fineness of statistical discrimination is increased, and the recognizable difference between I_1 and I_2 becomes smaller, together with its σ (*cf.* Crozier, 1936, *etc.*).

The ratio between $(\Delta I_L + \Delta I_B)/2$ and ΔI_B must obviously be in general an individual constant. Consider the case of any individual with defect in one eye, so that $\Delta I'$ is very much greater or less than $\Delta I''$; here the value of the ratio must be *less* than 1.4. It is also a fact (our unpublished data) that a large light image of *subliminal* intensity, placed on the retina of one eye can quite detectably influence the threshold stimulus as applied to the *other* eye.

None of the series in Table VII is homogeneous for the *form* of the area-ΔI function, since they involved successive use of each eye followed by the use of both at each area. We might expect, therefore, to find greater scatter of points in the graphs. However, the slopes of the lines as drawn (Fig. 11) agree well with those gotten respectively for the two types of experiments, in which (1) excited area is symmetrically enlarged on either side of the fovea (Z = 0.267, and 0.255), and (2) by increasing steps toward the fovea (Z = 0.591, and 0.495). For the data and conditions of Table VII, Z is in each case a little lower, which may well be accounted for by the difference in the procedures. This result is fully confirmed by another extensive series of measurements, involving larger areas, data for which are not given in this paper, which also gave identical values of Z for right and left eyes and for both together, but Z here = 0.24.

IX

Organic Variability

One of the first attempts to deal with the form of the function relating (over a limited range) differential sensitivity ($= 1/\Delta I$) and discriminatory precision ($= k/\sigma_{\Delta I}$) was made by G. E. Müller (1879); see also Troland

(1917). Others have been concerned with the relation from the standpoint of correlation methods (Thomson, 1912; Culler, 1927), showing that the two quantities are not independent but are directly related—at least over moderate ranges, and with non-homogeneous data. More recently the

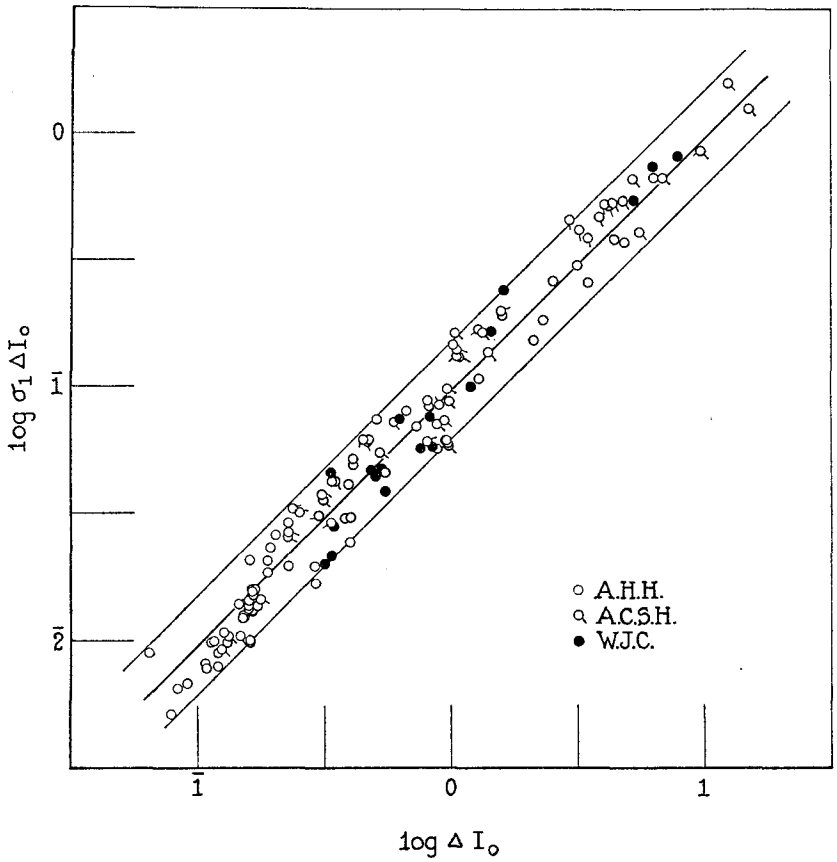

FIG. 12. The relation between ΔI_m and $\sigma_{1_{\Delta I}}$, for absolute thresholds (ΔI_0), for three observers (data in Crozier and Holway, 1938–39 *b*). Proportionality is direct (*i.e.*, the slope of the graph = 1). The line bisecting the area between the margins has 59 points above it, 60 below; in such non-homogeneous data log $\sigma_{\Delta I}$ is symmetrically ("normally") distributed at fixed ΔI; see text.

relation has been determined over wide ranges of intensities for several different sensory modalities (*cf.* Crozier, 1935; 1936; Upton and Crozier, 1936; Crozier and Holway, 1937; 1938; Holway and Crozier, 1937; Crozier, Wolf, and Zerrahn-Wolf, 1936–39). In general, for all these cases, includ-

ing those in which a critical intensity has the meaning of a discriminated ΔI,

$$\sigma_{\Delta I} = k'(\Delta I_m + C),$$

and C is almost always = 0.

Each of the quantities $\sigma_{\Delta I}$ and ΔI_m is probably determined organically. Strictly speaking they are gotten under *identical* experimental conditions, since both depend on the same measurements. Their interrelationship is intrinsically a law of the organism under scrutiny.

The values of σ here dealt with are of the order of 10 per cent of ΔI_m. Lest these be mistakenly supposed to signify a comparatively large variation, we may point out that we deal with σ_1, the root-mean-square dispersion, not the S.D. of the *mean*. If computed in the more usual manner, giving the P.E. of the mean as a percentage of the mean adjusted intensity, which is of course a function of the number of observations averaged, $\sigma_1 = 0.1\ \Delta I_m$ here corresponds to *ca.* P.E.$_m$ < 0.3 per cent of I_2.

We have already shown that $\sigma_{\Delta I} = k\ \Delta I$ for I_1 as variant (Crozier and Holway, 1937; 1938). If it can now be shown that the same law operates at any fixed level of I_1, but with A as variant, then it cannot be assumed that $\sigma_{\Delta I}(= k\ \sigma_{I_2})$ is determined by the magnitude of the stimulus (intensity). For a given I_1, the *amount of light* entering the eye *increases* with the size of the retinal image. If equation (5) holds, however, $\sigma_{\Delta I}$ should nonetheless *decrease*, as it is found to do.

Fig. 13 shows that this is the case. Each plotted point is an average of five measurements. The outer lines depict statistical limits: $\sigma_{\sigma_{\Delta I}}$ is directly proportional to $\sigma_{\Delta I}$ itself. The solid line is drawn with a slope = 1.0; and, for A and λ as variants, $\sigma_{\Delta I}$ for any value of I_1 is seen to be directly proportional to ΔI. We shall deal in another place with the evidence showing that the same rule holds with exposure time as variant. This fact is consistent with the hypothesis that this relation describes an organic invariant (*cf.* Crozier and Holway, 1937).

In Fig. 12 we also give the variation data from our earlier experiments on absolute thresholds as a function of retinal position (Crozier and Holway, 1938–39 *b*). The data plotted in each assemblage do not form a homogeneous population of σ's; consequently we must expect (*cf.* Holway and Crozier, 1937 *a*) that bisection of the upper and lower limits of log σ, at any (every) level of ΔI_m will divide the band population into numerically equal halves, as is illustrated in the figure. Data which are *homogeneous* show that the σ-width of the band must be divided arithmetically, not the log σ width, to achieve this result (Upton and Crozier, 1936; Holway and Crozier, 1937*a*); this is a consequence of the fact that $\sigma_{\Delta I}$ *determines* ΔI_m. It is noteworthy that when $I_1 = 0$ (*i.e.*, for absolute thresholds) $\sigma_{1\Delta I_0}$ has the

same value for a given level of ΔI_0 as for $\sigma_{1\Delta I}$ with I_1 finite; consequently $\sigma_{1\Delta I}$ cannot be regarded as a constant fraction of I_1 or of I_2. The log σ intercept values at given levels of log ΔI_m for observer A.H.H. (Fig. 12), measuring the proportionality constant in the direct relationship between ΔI_m and $\sigma_{1\Delta I}$ (and thus inversely estimating the precision with which ΔI_m

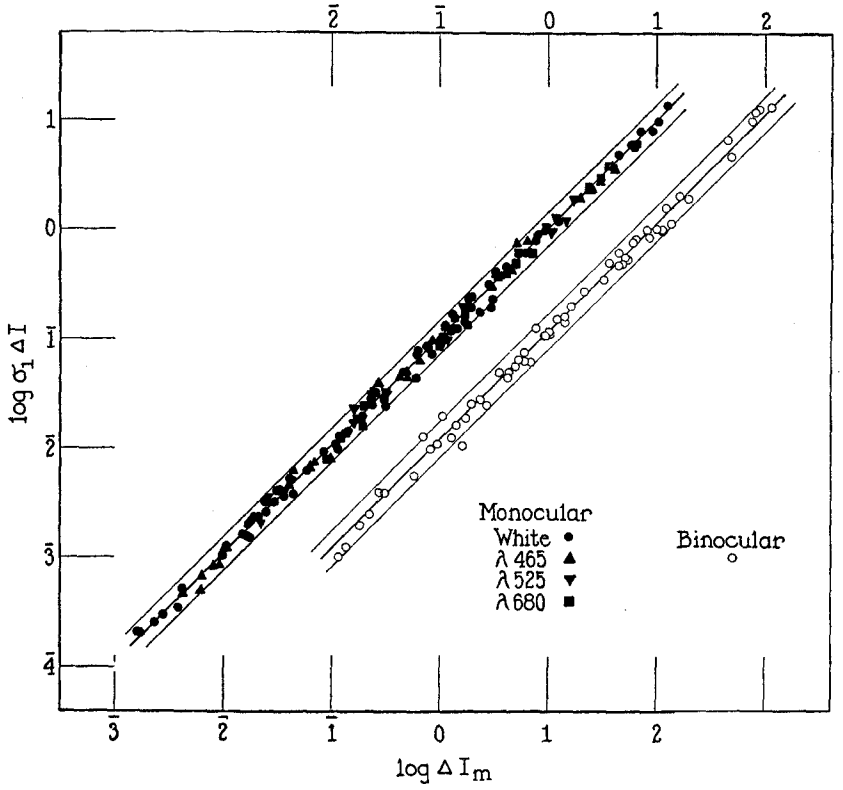

FIG. 13. The relation between ΔI_m and $\sigma_{1\Delta I}$ for ΔI at finite levels of I_1, with area and λ as parameters, is one of constant proportionality for an observer (data of Tables I, II, and V); for binocular observations (log ΔI scale above) the proportionality constant is a little higher.

is obtained), are $\bar{1}.02$ for the main monocular tests and $\bar{1}.07$ for the chief binocular tests discussed in the present paper. For an earlier series of experiments (*cf.* section III) the value $\bar{1}.26$ was obtained. It is of course conceivable that the 15 per cent increased precision in the newer experiments may represent the effect of experience ("practice"); but this is unlikely for several reasons: (1) there is no evidence of drift of precision during any one set of experiments extending over several weeks, and (2)

several series of tests by other methods, before and after this particular series, give higher values of the precision. These latter series have already been briefly considered elsewhere. For ΔI tests involving the method of successive comparison with a *linear* increase of $\tilde{I}_2$ (Holway and Hurvich, 1937), $\log \sigma_{1\Delta I}$ for $\log \Delta I_m = 0$ is $\bar{2}.99$ (*cf.* plot in Crozier and Holway, 1937); for a series involving photometric adjustments with increase of $\tilde{I}_2$ by inverse square of distance (Holway, 1937) $\log \sigma_{1\Delta I}$ is $\bar{1}.15$ (Crozier and Holway, 1937). In our series of absolute threshold (ΔI_0) measurements (white light; Crozier and Holway, 1938–39 *b*) it is $\bar{2}.98$ (Fig. 12); in a further series of relative threshold data it is $\bar{2}.93$; these two sets involved *logarithmic* increase of $\tilde{I}_2$. Thus there is no relationship between the precision and the mode of manipulation of the adjusted intensity, whether by logarithmic optical wedge, linear increase, or by use of the inverse square law, under the conditions of these experiments. It is accordingly difficult to conceive that the law of the scatter of ΔI_1 is determined by the character of the manipulative inaccuracies inherent in different kinds of operating procedure. In these cases cited a given small false adjustment of the manipulated distance of lamp or of travel of the wedge will obviously produce a change of intensity in the logarithmic case proportional to I, in the inverse square case proportional to $I^{3/2}$, and in the linear case an amount which is always constant. Therefore different laws would necessarily be obtained if the properties of $\sigma_{\Delta I}$ were governed by "observational error" of instrumental origin.

It is clear, we believe, that the findings made on the basis of predictions that the variation measured by $\sigma_{\Delta I}$ is organically determined by the tested individual are uniquely consistent with this position. Of these we may refer here to 2: the interrelationships between σ_F and σ_I for responses to visual flicker (Crozier, Wolf, and Zerrahn-Wolf, 1936–39), and the interdependence of $\sigma_{\Delta I}$ and of σ_{I_1} in ΔI measurements (Crozier and Holway, 1938). Further (unpublished) experiments in which, by means of *heterochromatic* matches, "ΔI" has been determined over a range of constant brilliances, show that the law for the behavior of $\sigma_{\Delta I}$ is predictably different depending upon which of the two differently colored lights is adjusted, although their modes of manipulation are identical.

X

SUMMARY

Measurements of ΔI as a function of retinal area illuminated have been obtained at various levels of standard intensity I_1, using "white" light and

light of three modal wave-lengths (λ465, 525, 680), for monocular stimulation and for simultaneous excitation of the two eyes ("binocular"), using several methods of varying (rectangular) area and retinal location, with control of exposure time.

For data homogeneous with respect to method of presentation,

$$\log \Delta I_m = -Z \log A + C,$$

where $\Delta I = \tilde{I}_2 - I_1$, A is area illuminated, and C is a terminal constant ($= \log \Delta I_m$ for $A = 1$ unit) depending on the units in which ΔI and A are expressed, and upon I_1.

The equation is readily deduced on dimensional grounds, without reference to specific theories of the nature of ΔI or of retinal area in terms of its excitable units. Z is independent of the units of I and A. Experimentally it is found to be the same for monocular and binocular excitations, as is to be expected. Also as is expected it is not independent of λ, and it is markedly influenced by the scheme according to which A is varied; it depends directly upon the rate at which potentially excitable elements are added when A is made to increase.

For simultaneous excitation of the two eyes (when of very nearly equivalent excitability), $\Delta\bar{I}_B$ is less than for stimulation of either eye alone, at all levels of I_1, A, and λ. The mean ratio $(\Delta\bar{I}_L + \Delta\bar{I}_R)/2$ to ΔI_B was 1.38. For white light, doubling A on one retina reduces ΔI_m in the ratio 1.21, or a little less than for binocular presentation under the same conditions. These facts are consistent with the view that the properties of ΔI are quantitatively determined by events central to the retina.

The measure $\sigma_{1\Delta I}$ of organic variation in discrimination of intensities and ΔI_m are found to be in simple proportion, independent of I_1, A, λ (and exposure time). Variability ($\sigma_{1\Delta I}$) is not a function of the mode of presentation, save that it may be slightly higher when both retinas are excited, and its magnitude (for a given level of ΔI_m) is independent of the law according to which the adjustable intensity I_2 is instrumentally controlled.

BIBLIOGRAPHY

Abney, W. deW., 1897, *Phil. Tr. Roy. Soc. London, Series A*, **190**, 155.
Abney, W. deW., and Watson, W., 1916, *Phil. Tr. Roy. Soc. London, Series A*, **216**, 91.
Aubert, H., 1865, Physiologie der Netzhaut, Breslau, Morgenstern.
Aubert, H., 1876, Grundzüge der physiologischen Optik, Leipsic, Englemann.
Bridgman, P. W., 1922, Dimensional analysis, New Haven, Yale University Press.
Cobb, P. W., 1916, *Tr. Soc. Ill. Eng.*, **11**, 386.
Cobb, P. W., and Moss, F. K., 1928, *J. Franklin Inst.*, **205**, 831.
Crozier, W. J., 1935, Détérminisme et variabilité, Paris, Hermann et Cie.

Crozier, W. J., 1935–36, *J. Gen. Physiol.*, **19,** 503. 1936, *Proc. Nat. Acad. Sc.*, **22,** 412; 1939, **25,** 78.

Crozier, W. J., and Enzmann, E. V., 1935–36, *J. Gen. Physiol.*, **19,** 249.

Crozier, W. J., and Holway, A. H., 1937, *Proc. Nat. Acad. Sc.*, **23,** 23; 1938, **24,** 130. 1938–39*a*, *J. Gen. Physiol.*, **22,** 341; 1938–39*b*, **22,** 351. 1939*a*, *Proc. Nat. Acad. Sc.*, in press.

Crozier, W. J., Wolf, E., and Zerrahn-Wolf, G., 1936–37*a*, *J. Gen. Physiol.*, **20,** 211; 1936–37*b*, **20,** 363; 1937–38*a*, **21,** 17; 1937–38*b*, **21,** 223; 1937–38*c*, **21,** 313; 1938–39, **22,** 311.

Culler, E., 1927, *Am. J. Psychol.*, **38,** 307.

Freeman, E., 1932, *J. Opt. Soc. America*, **22,** 402; 1936, **26,** 271.

Fujita, T., 1909, *Z. Sinnesphysiol.*, **43,** 243.

Graham, C. H., 1930, *J. Gen. Psychol.* **3,** 494.

Graham, C. H., and Margaria, R., 1935, *Am. J. Physiol.*, **113,** 299.

Guild, J., 1932, Report of joint discussion on vision, Physical and Optical Societies, Cambridge University Press, 167.

Hardy, A. C., 1929, *J. Opt. Soc. America*, **18,** 96; 1935, **25,** 311.

Hecht, S., 1937, *Physiol. Rev.*, **17,** 239.

Heinz, M., and Lippay, F., 1928, *Arch. ges. Physiol.*, **218,** 437.

Henius, K., 1909, *Z. Sinnesphysiol.*, **43,** 99.

Holway, A. H., 1937, *J. Opt. Soc. America*, **27,** 123.

Holway, A. H., and Crozier, W. J., 1937*a*, *Proc. Nat. Acad. Sc.*, **23,** 509. 1937*b*, *Psychol. Rec.*, **1,** 170; 1937 *c*, **1,** 178.

Holway, A. H., and Hurvich, L. M., 1937, *J. Psychol.*, **4,** 309. 1938, *Am. J. Psychol.*, **51,** 687.

König, A., and Brodhun, E., 1889, *Sitzungsber. k. Akad. Wissensch., Berlin,* 641.

v. Kries, J., 1911, *Z. Psychol. u. Physiol. Sinnesorgane, 2. Abt.*, **15,** 327. 1929, in Bethe, A., von Bergman, G., Embden, G., and Ellinger, A., Handbuch der normalen und pathologischen Physiologie, Berlin, J. Springer, **12,** pt. 1, 678.

Langmuir, I., 1913, *Phys. Rev.*, **2,** 453.

Lasareff, P., 1911, *Arch. ges. Physiol.*, **142,** 235.

Lucas, K., 1917, The conduction of the nervous impulse, London, Longmans Green and Co.

Ludvigh, E. J., and McCarthy, E. F., 1938, *J. Ophth.*, **44,** 118.

Lythgoe, R. J., and Phillips, L. R., 1938, *J. Physiol.*, **91,** 427.

Müller, G. E., 1879, Zur Grundlegung der Psychophysik, Berlin, Hoffman (2nd printing).

Østerberg, G., 1935, *Acta Opthalmol.*, **13,** 1.

Piéron, H., 1920*a*, *Compt. rend. Soc. biol.*, **83,** 753; 1920*b*, **83,** 1072. 1929, *Ann. Psychol.*, **30,** 87.

Piper, H., 1903, *Z. Psychol. u. Physiol. Sinnesorgane,* **31,** 161.

Poljak, S., 1935, *Arch. Ophth.*, New York, **15,** 477.

Rea, R. L., 1938, Neuro-ophthalmology, St. Louis, C. V. Mosby Co.

Riccò, D., 1877, *Ann. ottal.*, Pavia, **3,** 373.

Roggenbau, C., and Wetthauer, A., 1927, *Klin. Monatsbl. Augenheilk.*, **78,** 762.

Shaad, D. J., 1934, *Arch. Ophth.*, New York, **12,** 705.

Skinner, B. F., 1931*a*, *J. Gen. Psychol.*, **6,** 22; 1931*b*, **6,** 38.

Steinhardt, J., 1936–37, *J. Gen. Physiol.*, **20**, 185.
Thomson, G. H., 1912, *Brit. J. Psychol.*, **5**, 203.
Troland, L. T., 1917, *J. Exp. Psychol.* **2**, 1. 1918, *Psychol. Rev.*, **25**, 305.
Upton, M., and Crozier, W. J., 1936, *Proc. Nat. Acad. Sc.*, **22**, 417.
Wald, G., 1937–38, *J. Gen. Physiol.*, **21**, 269.
Weber, E. H., 1834, De tactu, Leipsic, C. F. Köhler.