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# The Multiplex Efficiency Index: unveiling the Brazilian air transportation multiplex network—BATMN

Izabela M. Oliveira<sup>1,2</sup>✉, Laura C. Carpi<sup>2,4,5</sup> & A. P. F. Atman<sup>2,3,4,5</sup>

Modern society is increasingly massively connected, reflecting an omnipresent tendency to organize social, economic, and technological structures in complex networks. Recently, with the advent of the so-called multiplex networks, new concepts and tools were necessary to better understand the characteristics of this type of system, as well as to analyze and quantify its performance and efficiency. The concept of diversity in multiplex networks is a striking example of this intrinsically interdisciplinary effort to better understand the nature of complex networks. In this work, we introduce the Multiplex Efficiency Index, which allows quantifying the temporal evolution of connectivity diversity, particularly when the number of layers of the multiplex network varies over time. Using data related to air passenger transportation in Brazil we investigate, through the new index, how the Brazilian air transportation network has been changing over the years due to the privatization processes of airports and mergers of airlines in Brazil. Besides that, we show how the Multiplex Efficiency Index is able to quantify fluctuations in network efficiency in a non-biased way, limiting its values between 0 and 1, taking into account the number of layers in the multiplex structure. We believe that the proposed index is of great value for the evaluation of the performance of any multiplex network, and to analyze, in a quantitative way, its temporal evolution independently of the variation in the number of layers.

Efficiency, according to the Cambridge Dictionary is “the condition or fact of producing the results you want without waste, or a particular way in which this is done” or, by Oxford, “the quality of doing something well with no waste of time or money”. In the context of complex networks, efficiency refers to the navigability of the network, based on the distance from the nodes, so that networks with smaller shortest paths are more efficient<sup>1–3</sup>. In the case of multiplex networks, a special case of multilayer structure in which the same set of nodes is connected by more than one type of relations in a layered structure<sup>4</sup>, it is shown that structural aspects, such as overlapping edges, number of layers and degree correlations between layers, influence the efficient exploration of the network through random walkers<sup>5</sup>. The concept of efficiency in multiplex networks can be related to the level of diversification of the network paths in the different layers. A greater diversification implies in the presence of less overlapping links between layers, reducing the redundancy of the connections, and expanding the possibilities of using different paths to explore the structure in a more efficient way.

In this sense, supported by the definition of diversity of a multiplex network introduced in a recent work<sup>6</sup>, we present the Multiplex Efficiency Index ( $\mathcal{E}$ -index), which is a modified version of the diversity measure capable of quantifying the connectivity patterns of a multiplex network on a normalized scale from 0 to 1, which is independent of the number of layers.

In this way, an  $\mathcal{E}$ -index value close to 1 is indicative of a high paths diversification. On the contrary, if  $\mathcal{E}$ -index is close to 0, connecting paths possess more redundancy. The  $\mathcal{E}$ -index is thought to evaluate temporal evolution

<sup>1</sup>Departamento de Matemática, Centro Federal de Educação Tecnológica de Minas Gerais, CEFET-MG, Av. Amazonas, 7675, Belo Horizonte, MG CEP: 30.510-000, Brazil. <sup>2</sup>Programa de Pós-Graduação em Modelagem Matemática e Computacional, PPGMMC, CEFET-MG, Belo Horizonte, Brazil. <sup>3</sup>Departamento de Física, CEFET-MG, Belo Horizonte, Brazil. <sup>4</sup>Instituto Nacional de Ciência e Tecnologia de Sistemas Complexos, INCT-SC, CEFET-MG, Belo Horizonte, Brazil. <sup>5</sup>These authors contributed equally: Laura C. Carpi and A.P.F. Atman. ✉email: izabelamarques@cefetmg.br

of multiplex networks when the number of layers changes over time. Therefore, the novelty introduced by this measure is the advantage of allowing the comparison between multiple multiplex networks, regardless of the number of layers present in the structure.

There are numerous possibilities of applications of the  $\mathcal{E}$ -index in different areas. For example, to quantify connection differences in social networks<sup>7</sup>, showing user sets that have similar or distinct connections, to analyze networks of criminal relationships<sup>8</sup>, to point out similarities (or differences) between the preferences of a group of people or to verify the protein interactions of the human HIV-1 virus<sup>6</sup>, to check the supply of transportation modalities for a given region<sup>9</sup>, among many others.

In this work we apply the  $\mathcal{E}$ -index to analyze the Brazilian passenger air transportation network. This mode of transportation is fundamental for Brazil, since from the 1950s the Brazilian government has discouraged rail passenger transportation, making the country highly dependent on road transportation<sup>10</sup>. This lack of rail passenger transport linking the different regions of the world's fifth largest country and the high cost of route maintenance which leaves many highways in poor condition, has created a heavy reliance on air transportation for internal mobility. Therefore, the evaluation of the efficiency of the air transportation network is of crucial importance to guarantee a competent performance of the service.

In the first decade of the twentieth century, the Brazilian air market underwent changes in its structure, caused by regulatory policies and other factors. Low-cost companies began operating in the country and large traditional companies closed their activities, increasing the concentration of the air market<sup>11</sup>.

Between 2010 and 2012, eight airlines carried more than 99% of the number of passengers in the country. In 2013, after mergers, that number dropped to six, and since 2014, only four combined companies have already exceeded 99% of passengers carried<sup>12</sup>. Currently, one of these companies is experiencing financial difficulties and have stopped its operations, leading to a dispute for their routes among the other companies<sup>13</sup>. This indicates an even greater concentration of the Brazilian domestic air market in the short term, which may lead to a reduction in the supply of transportation for the population.

Each airline operates with its own strategic business model. Some have a regional profile, others focus on the demand of large cities, or work with many routes to medium-sized city destinations, among other possibilities. According to data released by the Agência Nacional de Aviação Civil (ANAC), the number of domestic flight passengers has grown from around 140 billion in 2010 to over 187 billion in 2018<sup>12</sup>. This means that an increasing number of people have fewer and fewer options of domestic flights in Brazil. The objective of this work is to present the  $\mathcal{E}$ -index and apply it to investigate the efficiency of the coverage of the Brazilian territory by the air passenger service.

Several papers studied the Brazilian air transportation network by means of statistical mechanics tools and different objectives. L.E.C. Rocha, investigates the structure and evolution of the Brazilian airports network (BAN), between 1995 and 2006, concluding that the network has undergone a strong reconnection in accordance with the changes in demand<sup>14</sup>. In addition, the author noted that the size of the network was decreasing (fewer routes), while the number of passengers increased, causing a higher level of occupation of aircraft or the need of larger aircrafts<sup>14</sup>. Oliveira et al.<sup>15</sup> analyze the concentration of the Brazilian air transportation market before and after the period known as the 2006-2007 aerial "big blackout". This period was marked by many flight delays and cancelations, culminating in two major air crashes in the country. These facts gave rise to the crisis of flight controllers, who complained about the lack of public investments in the sector. The authors conclude that the Brazilian air transportation market was already moving toward concentration even before the aforementioned crisis, but this process accelerated especially at the end of that decade.

In another sense, S. Wandelt, X. Sun and J. Zhang analyze and compare the evolution of air networks (including the Brazilian one) during the period of 2002 and 2013<sup>16</sup>. They found a decrease in average degree and attributed it to efforts to consolidate the Brazilian air transportation business and the privatization of airports, which began in 2010. They also found that the Brazilian network underwent many changes in the period, probably attributed to attempts of reorganization of the airlines after the crisis of air traffic controllers between 2006 and 2007, with repercussion up to 2010.

In the context of the global air transportation network, several works have been carried out in an attempt to analyze the network structure, measuring its complexity. We highlight the work of Sales-Pardo et al.<sup>17</sup>, which analyzes the multilevel modular structure of the global network and Verma et al.<sup>18</sup> that verifies the vulnerability of the system through the extraction of network hierarchies. Although both works study network structures in levels, none of them used multiplex approach, which allows us to verify the influence of different airlines in the network and to measure the efficiency of the air service.

We believe that the methodology presented here is useful not only for analyzing air transportation networks from another perspective, but also to quantify multiplex network efficiency in a general unbiased way.

## Methods

**Multiplex Efficiency Index ( $\mathcal{E}$ ).** Next we present the  $\mathcal{E}$ -index, which is based on the concept of multiplexed network diversity<sup>6</sup>. Diversity, as a general concept, can be understood as the level of heterogeneity of a system<sup>19,20</sup>. In the case of multiplex networks, diversity is related to the variety of connectivity patterns present in the structure. Thus, a greater diversity value implies configurations that exhibit more heterogeneous connectivity patterns within the layers.

Formally, a multiplex network with  $M$  layers, each one having the same set of nodes  $N$ , is represented by adjacency matrices  $\mathcal{A} = \{A^{[1]}, A^{[2]}, \dots, A^{[M]}\}$ . From these adjacency matrices it is possible to calculate the distance distribution of nodes (NDD)<sup>6</sup> for each node  $i$  in the layer  $\bar{p}$  ( $\mathcal{N}_i^{\bar{p}}(d)$ ). It corresponds to the fraction of nodes that are at distance  $d$  (shortest path) of node  $i$  in layer  $\bar{p}$ ; and the Transition Matrix ( $\mathcal{T}_i^{\bar{p}}(j)$ ) provides the probability that the node  $j$  in the layer  $\bar{p}$  can be reached, in a single step, by a random walker located in node  $i$

in  $\bar{p}$ . These probability distributions describe the local and global connectivity properties of the  $i$  node in the different layers and can be used to calculate the dissimilarity of the connectivity configurations of the node  $i$  ( $\mathcal{D}_i$ ) in layers  $\bar{p}$  and  $\bar{q}$ :  $\mathcal{D}_i(\bar{p}, \bar{q}) = \frac{\sqrt{\mathcal{J}(A_i^{\bar{p}}, A_i^{\bar{q}})} + \sqrt{\mathcal{J}(T_i^{\bar{p}}, T_i^{\bar{q}})}}{2\sqrt{\log(2)}}$ , where  $\mathcal{J}$  is the Jensen-Shannon (JS) divergence<sup>21</sup> that measures the distance between two probability distributions.  $\mathcal{D}_i(\bar{p}, \bar{q}) = 0$  if node  $i$  has identical connectivity configuration in layers  $\bar{p}$  and  $\bar{q}$ ; while  $\mathcal{D}_i(\bar{p}, \bar{q}) = 1$ , if node  $i$  is not connected in one layer, and connected to all nodes in the other.

The average value of  $\mathcal{D}_i(\bar{p}, \bar{q})$  calculated over all the nodes corresponds to the dissimilarity between the layers  $\bar{p}$  and  $\bar{q}$ , called layer distance (LD):

$$\mathcal{D}(\bar{p}, \bar{q}) = \langle \mathcal{D}_i(\bar{p}, \bar{q}) \rangle_i.$$

If  $\mathcal{D}(\bar{p}, \bar{q}) = 0$ , it implies that the layers  $\bar{p}$  and  $\bar{q}$  are identical; otherwise, if  $\mathcal{D}(\bar{p}, \bar{q}) = 1$ , one of the layers is fully connected, and the other is totally disconnected.

For completeness, the definition of diversity requires quantifying the distance of a set of elements  $S$  for an element  $\bar{g}$  that does not belong to  $S$ , as proposed by Carpi et al.<sup>6</sup>, should be taken as the minimum distance between the element  $\bar{g}$  for any element  $\bar{s}_i$  in the set  $S$ :

$$\mathcal{D}(\bar{g}, S) = \min_{\bar{s}_i \in S} \mathcal{D}(\bar{g}, \bar{s}_i). \quad (1)$$

The diversity function  $U : \tilde{S} \rightarrow \mathbb{R}_+$  is defined recursively as

$$U(S) = \max_{\bar{s}_i \in S} \{U(S \setminus \bar{s}_i) + \mathcal{D}(\bar{s}_i, S \setminus \bar{s}_i)\} \quad (2)$$

for all  $S \in \tilde{S}$  with  $|S| \geq 2$ , where  $|S|$  represents the cardinality of the set, and  $U(S) = 0$ , for all  $S \in \tilde{S}$  such that  $|S| = 1$ .

The Eq. (2) shows that the diversity calculation is obtained by summing  $M - 1$  positive values between 0 and 1. Thus, the value of  $U$  is upper bounded by  $M - 1$ . Thus, it follows that the number of layers considered in the multiplex analysis influences the value of diversity.

The  $\mathcal{E}$ -index corresponds to a weighted diversity value, avoiding then, the intrinsic bias due to diversity definition.  $\mathcal{E}$ -index is defined as the ratio between the diversity value and the number of layers, as:

$$\mathcal{E} = \frac{U}{M - 1} \quad (3)$$

where  $0 \leq \mathcal{E} \leq 1$ .

Thus, it is possible to compare the  $\mathcal{E}$ -index between networks without the influence of the number of layers, both at the level of nodes (local) and at the level of layers (global). For networks where there is a large overlap of links, the value of  $\mathcal{E}$ -index will be close to 0 (low efficiency). On the other hand, when there is a greater diversification of connections between the layers present in the network, the  $\mathcal{E}$ -index value will be closer to 1 (high efficiency). Synthetic examples of the calculation of  $\mathcal{E}$ -index are presented in Note S1 of the Supplementary Information.

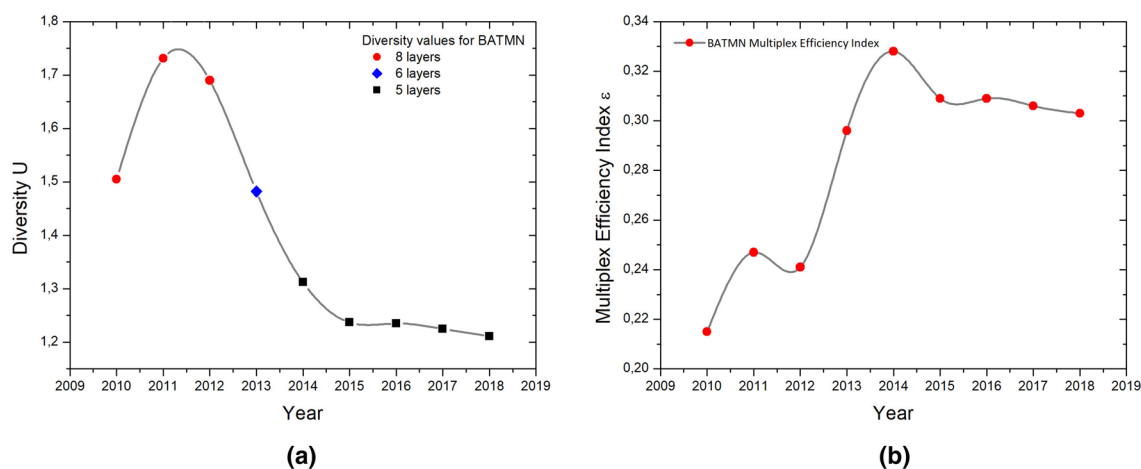
**Brazilian air transportation multiplex network (BATMN).** After the crisis of 2006 and 2007, the Brazilian domestic air network had to reorganize<sup>16,22</sup>. Since then, a notable increase in domestic demand has been observed in the Brazilian air network (BAN), reaching a maximum during two major events: the 2014 FIFA World Cup and the Rio 2016 Olympic Games<sup>12</sup>. The Brazilian government invested in the renovation and expansion of airports in the main cities and, as of 2011, started the project of concession of airports for the private sector<sup>23</sup>. This scenario led to a major reorganization of BAN, which motivated the present analysis in order to evaluate quantitatively the evolution of the domestic BAN between 2010 and 2018 from the multiplex perspective.

In this way, we constructed the Brazilian air transportation multiplex network (BATMN), through the database available on the ANAC website<sup>12</sup>. During the study period, the same set of 154 airports was considered each year, representing the network nodes. The links symbolize the existing routes and the layers correspond to the different airlines. As we consider non-directed networks, if a route connecting two airports exists, a link between them is created. According to ANAC, a group of only 8 airlines transported in 2010, more than 99% of passengers, carrying each of them, at least 0.5% of that number. Focusing on the analysis on these companies, we decided to adopt 0.5% of the total number of passengers per year as the lower limit to include an airline as a multiplex network layer. We constructed monthly networks that correspond to a multiplex structure containing all routes existing for each company in that month, and annual networks that correspond to the multiplex structure considering all the routes traveled for each company throughout the year. In view of these criteria, the diversity measure was computed for an 8-layer network between 2010 and 2012, a 6-layer network in 2013 and a 5-layer network in the following years, until 2018. From this, we calculate the  $\mathcal{E}$ -index of BATMN.

As Brazil is a member of the International Civil Aviation Organization (ICAO) their airports and airlines are identified by ICAO codes. Table 1a shows the airlines considered in this work and their respective ICAO code, and Table 1b shows the companies considered as layers of BATMN in each year.

(a)		(b)	
ICAO Code	Airline Names	Year	Airlines considered
AZU	Azul Linhas Aéreas Brasileiras	2010	AZU, GLO, ONE, PTB, TAM, TIB, PTN, WEB
GLO	Gol Linhas Aéreas	2011	AZU, GLO, ONE, PTB, TAM, TIB, PTN, WEB
ONE	Avianca Brasil	2012	AZU, GLO, ONE, PTB, TAM, TIB, PTN, WEB
PTB	Passaredo Transportes Aéreos	2013	AZU, GLO, ONE, PTB, TAM, TIB
PTN	Pantanal Linhas Aéreas	2014	AZU, GLO, ONE, PTB, TAM
TAM	TAM Linhas Aéreas	2015	AZU, GLO, ONE, PTB, TAM
TIB	TRIP Linhas Aéreas	2016	AZU, GLO, ONE, PTB, TAM
WEB	Webjet Linhas Aéreas	2017	AZU, GLO, ONE, PTB, TAM
		2018	AZU, GLO, ONE, PTB, TAM

**Table 1.** List of airlines considered in this study. Panel **a** shows the analysed airlines and their respective ICAO codes, and panel **b** displays the airlines of BATMN considered each year.



**Figure 1.** Analysis of the evolution of BATMN connectivity patterns. **(a)** Left panel: annual diversity values for BATMN, which are influenced by the number of layers present in the network. **(b)** Right panel: annual evolution of the  $\epsilon$ -index for BATMN, which takes into account the number of layers of the network and therefore shows an unbiased result.

## Results and discussions

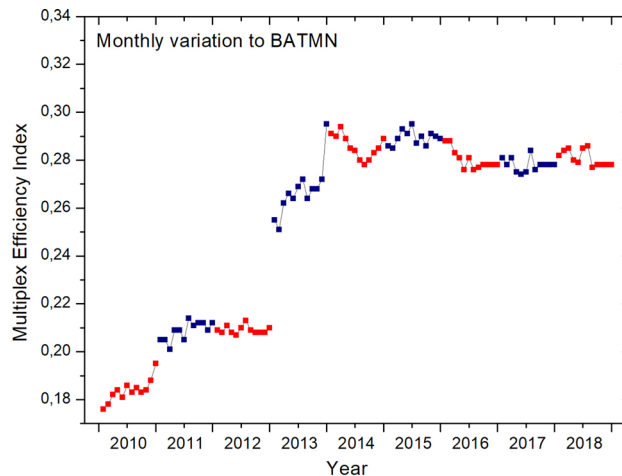
First, we show the evolution of BATMN's annual diversity values calculated between 2010 and 2018—Fig. 1a. Clearly, diversity values strongly correlate with the number of layers in the structure. BATMN is composed of 8 airlines between 2010 and 2012 (red discs), 6 airlines in 2013 (blue disc) and 5 airlines from 2014 to 2018 (black discs).

$\epsilon$ -index displays a completely different scenario when the number of layers is taken into account by Eq. (3). Thus, an unbiased analysis of the efficiency of BATMN can be performed, as shown in Fig. 1b. We can see a local peak of  $\epsilon$ -index in 2011, followed by a decrease in 2012 and a progressive increase, peaking in 2014. After that peak, the  $\epsilon$ -index slightly drops in 2015, remaining stationary until 2018.

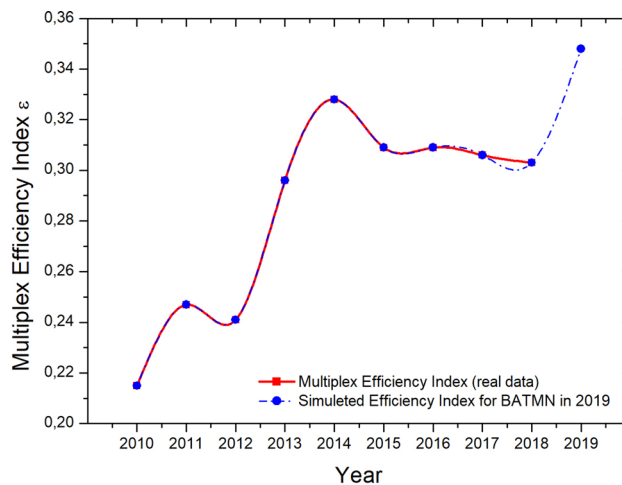
The  $\epsilon$ -index is independent of the number of layers, so just reducing layers in BATMN does not lead to an increase in the  $\epsilon$ -index values. To illustrate this, we show an example in Supplementary Information (Note S1), in which, by excluding a layer from the network, the value of the  $\epsilon$ -index decreases.

In analyzing the BAN panorama, it is possible to describe some factors that help explain the observed changes in the  $\epsilon$ -index values for BATMN. Basically, the merger between companies over the first four years analyzed explains the fluctuations on the efficiency of the network until 2014. Three major merger events were held during this period. In 2010, TAM acquired PTN<sup>24</sup> which was a regional airline operating between Congonhas Airport (located in São Paulo) and medium-sized cities. As of 2011, TAM began to deactivate the PTN routes, until 2013, when the company PTN ceased to operate, with all its routes being incorporated by TAM through ANAC's authorization<sup>25,26</sup>. In 2011, GOL acquired 100% of the shares of WEB<sup>27</sup> and, in 2012, consolidated this merger<sup>28</sup>, definitively closing the airline WEB. Finally, AZU merged with TIB in 2012<sup>29</sup> and started to control 15% of the domestic market. According to the rules established by ANAC, the pre-existing share flights (code-share) between TIB and TAM was gradually shut down by 2014, encouraging competition between AZU and other airlines<sup>30</sup>.

To better analyze the effects of these and other events on the network, we perform the monthly calculation of  $\epsilon$ -index for BATMN, building the network from the routes present in each month, from 2010 to 2018 (see Fig. 2). We found a strong jump in the values of  $\epsilon$ -index from December 2012 to January 2013 and another smaller jump from November to December 2013, which coincides, in the first case, with the exclusion of PTN



**Figure 2.** Variation of the  $\mathcal{E}$ -index for BATMN in a monthly network panorama. The dots represent the values of the  $\mathcal{E}$ -index in consecutive months, and alternating colors delimit different years.



**Figure 3.** Simulation of the annual  $\mathcal{E}$ -index values for BATMN in 2019, maintaining the layer configurations of 2018 and excluding the airline ONE.

and WEB from the network and, in the second, with the exclusion of TIB. It is also possible to observe in Fig. 2 that the highest monthly value of  $\mathcal{E}$ -index corresponds to December 2013 and then there is a stabilization of the  $\mathcal{E}$ -index, with considerably smaller variations. Reorganization of the routes caused by the mergers of the airlines led to an increase in the values of  $\mathcal{E}$ -index, that is, an increase in the diversification of routes between the participating airlines and, subsequently, the consolidation of this group of companies.

An interesting behavior of the  $\mathcal{E}$ -index can be seen in 2014. Monthly values show a pronounced minimum, as they decrease from March to August, increasing in the following months (Fig. 2). This suggests a correlation with the concentration of air routes to meet the growing demand for air transportation to the host cities of the FIFA World Cup games<sup>31</sup>, held in July. In the annual context, shown in Fig. 1b, it can be observed that the highest  $\mathcal{E}$ -index value corresponds to the conclusion of airline mergers in 2014, followed by a stability period in the diversification of routes.

Another reduction in the number of airlines operating in the domestic passenger market in Brazil, occurred in 2019 when ONE ended its operations, following a judicial liquidation process initiated in late 2018<sup>32</sup>. As an exercise to estimate the influence of the removal of this layer from BATMN, we simulated the annual network efficiency in 2019 excluding ONE and maintaining the configurations of remaining layers of 2018—Fig. 3. We can see that the value of  $\mathcal{E}$ -index increases after the exclusion of ONE, which means that the network coverage is similar, however, with fewer layers. ONE has 91 routes (BATMN links) in 2018, of which only 4 are exclusive of their layer, and therefore 95.6% of routes overlap with the other layers. Thus, the increase of  $\mathcal{E}$ -index reflects the exclusion of one layer with high link overlap in relation to the others. This analysis indicates that an increased efficiency does not always translates into benefits for the air transportation user, as in this case there will be less availability of mobility options.

In order to better understand the influence of each airline on the efficiency of the network, we observe the diversity ordering for each year of the period considered. The diversity ordering indicates the elements in order of their contribution to the diversity of the set, and is obtained by calculating a method based on lexicographic distance<sup>6</sup>. The following list shows the ascending sequence of the BATMN layers according to their contribution to the diversity value each year.

- 2010: ONE, WEB, PTN, TAM, AZU, PTB, GLO, TIB;
- 2011: WEB, PTN, TAM, ONE, AZU, PTB, GLO, TIB;
- 2012: PTN, WEB, TAM, ONE, GLO, AZU, PTB, TIB;
- 2013: TAM, ONE, AZU, GLO, PTB, TIB;
- 2014: ONE, TAM, GLO, PTB, AZU;
- 2015: ONE, TAM, GLO, PTB, AZU;
- 2016: ONE, TAM, GLO, PTB, AZU;
- 2017: ONE, TAM, GLO, PTB, AZU;
- 2018: ONE, TAM, GLO, PTB, AZU.

From the list, it is clear that PTN and WEB are among the airlines that less contribute to the value of diversity, and TIB leads this ranking while they are present on the network. In Fig. 4a are depicted all layers with their corresponding links for years 2010, 2014 and 2018, and the Fig. 4b shows the participation of each layer in terms of contribution to the diversity value. These observations are in line with previous discussions made from the analysis of the  $\mathcal{E}$ -index.

Following the merger between AZU and TIB, AZU became the leader in diversity in 2014 (see Fig. 4b). It is worth noting that during the time that the PTN was in the network, there was a progressive decrease of its contribution to the diversity of the multiplex network. Between 2010 and 2012, PTN fell from the 6th to the 8th position in a group of 8 airlines. This observation coincides with the fact that TAM has reallocated PTN routes to meet its business needs<sup>26</sup>, causing a decrease in the diversity of the companies. It should be noted that in 2013 PTN was still operating, however, as it represents less than 0.5% of the total number of passengers, it was not considered in the calculation of diversity.

We see that from 2014 ONE is the layer that contributes less to the diversity of BATMN (see Fig. 4). Carpi et al.<sup>6</sup> mention that when a layer is removed, there is a change in diversity that is at least equal to the value of the distance between the corresponding layer and the remaining set, as can be deduced from the Eq. (1). We then conclude that there is a great overlap of routes between ONE and the other airlines, in agreement with the results already discussed and shown in Fig. 3.

We investigated the influence of the number of network links on the values of the  $\mathcal{E}$ -index, analyzing the number of routes (BATMN links), as shown in Fig. 5a. In 2011 and 2012, TIB was the airline with the largest number of routes and in the following year, TIB and AZU led the number of routes. From 2014, after the merger of these two companies, AZU takes the lead with a significant relative growth. Not surprisingly, TIB was the leader in ordering diversity by 2013 and, from 2014, AZU is leading the network diversity as shown in Fig. 4b and the previous list.

After the merger between TAM and PTN, consolidated in 2013, the growth in the number of TAM routes was not as significant. There was a slight increase in 2014, returning to the same level in 2015. This increase may have been influenced by the FIFA World Cup in 2014. Except for 2014, there is a downward trend in the number of TAM routes.

In the third merger in the system, between WEB and GLO, completed in 2012, the number of GLO routes increases from 2013, outperforming TAM, its closest competitor. Both components had almost the same number of routes in 2014. Thereafter, the number of GLO routes exceeded that of TAM, increasing further in 2016, when the Rio Olympic Games occurred, returning to the level of 2015 in the following years.

The airline ONE did not show significant changes in the number of routes. It increased in 2014 (FIFA World Cup) and declined in 2016, the year of the Rio Olympic Games. Curiously, its number of routes increased in 2018, the year in which the airline opened the process of judicial administration.

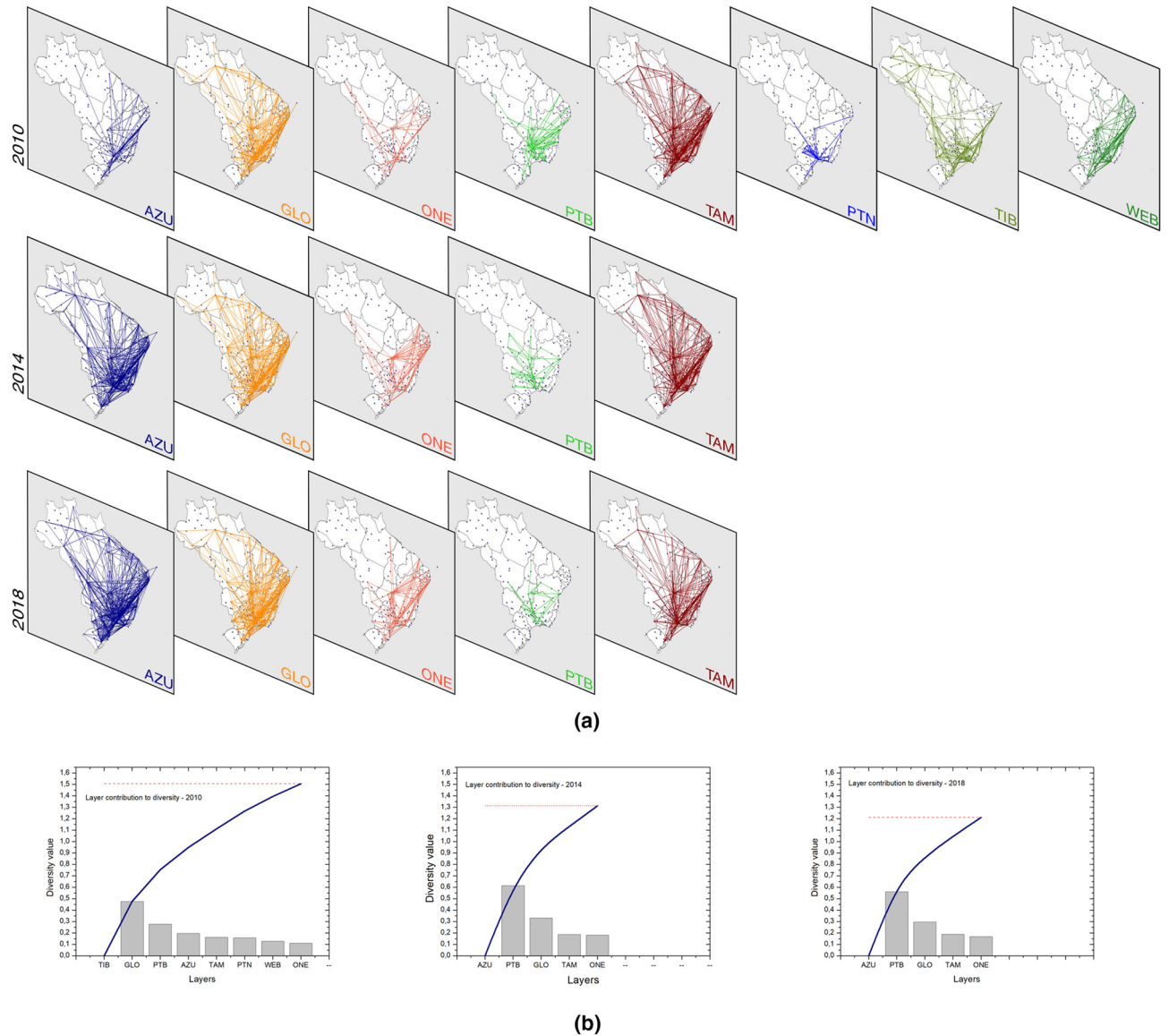
To conclude this analysis, the PTB is the airline that has the lowest number of routes, with a peak in 2011, coinciding with the local peak of the  $\mathcal{E}$ -index value for BATMN, and falling next. From 2013, its number of routes stabilizes.

We can conclude that, in the case of BATMN, as of 2011, the airline that have a greater number of routes is the one that contribute most to the  $\mathcal{E}$ -index of the network, since the surplus of routes cause network diversification.

A remarkable result was obtained by adding the number of routes of the airlines and comparing with the calculation of the diversity. We have verified that there is a high correlation between the total number of routes and the value of the network diversity for the BATMN, as shown in Fig. 5b. We can observe that a larger number of routes occur when there are lower  $\mathcal{E}$ -index values. Hence we realize that higher link density on the network leads to a higher probability of link overlap. In 2016, the number of routes increases significantly, probably driven by the Olympic Games in Rio, while the value of diversity and the  $\mathcal{E}$ -index value increase slightly, which indicates that most new routes correspond to previously existing routes on the network.

All these analyzes demonstrate the utility and importance of the  $\mathcal{E}$ -index to plan and analyze air mobility in BATMN, as well as in other networks of the same kind.

In a final experiment, we look at the evolution of the local  $\mathcal{E}$ -index for the six airports that were privatized before 2017<sup>33</sup>. The corresponding figures are shown in the Supplementary Information (Note S2). An increase in  $\mathcal{E}$ -index value for a (local) node means that there is an increase in the connectivity difference between the layers for that node. That is, different airlines serve different cities linked to the same airport.



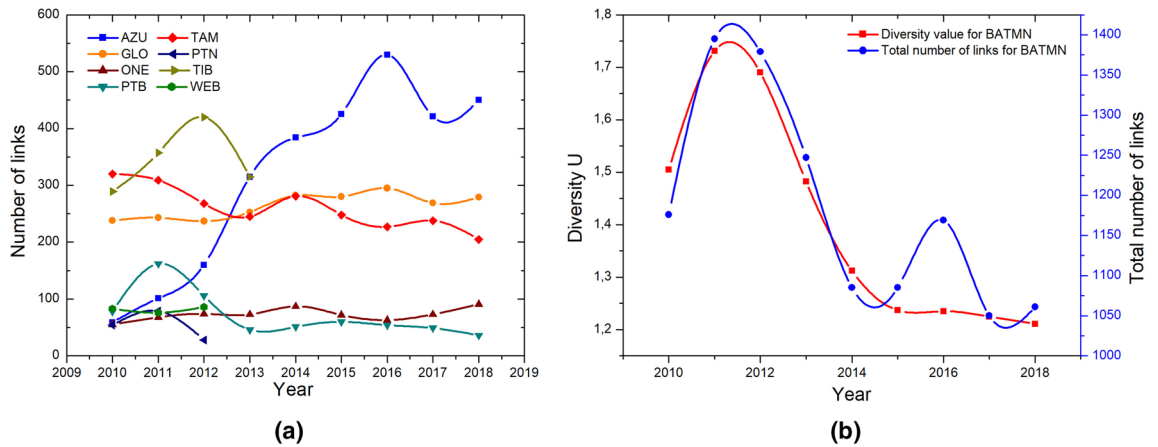
**Figure 4.** BATMN layer evolution for each airline in 2010, 2014 and 2018. **(a)** Illustration of BATMN links in each of the three years. **(b)** Diversity ordering for BATMN in a quantitative way. The columns represent the contribution values of each layer when calculating the network diversity, according to the Eq. (1). The blue curve shows the cumulative values of these distances at each step of the calculation, as given by Eq. (2). In the red line we have the final value of the global diversity for BATMN.

Four airports are granted to the private sector between 2011 and 2012, serving the cities of Natal, Brasília, Campinas and Guarulhos<sup>33</sup>. The value of  $\mathcal{E}$ -index for Campinas (Viracopos International Airport) initially increases and stabilizes after 2015. The other airports show a decrease in the  $\mathcal{E}$ -index value in the first year after privatization, returning to the previous level, and then, stabilizing.

The airports serving the cities of Belo Horizonte (Confins International Airport) and Rio de Janeiro (Galeão International Airport) are privatized in 2013<sup>33</sup>. Confins airport registers a significant increase in the  $\mathcal{E}$ -index in the first year (22%), remaining stable thereafter. Galeão airport increases the value of  $\mathcal{E}$ -index in 2014, with inexpressive oscillations, returning to the previous level.

Among the six privatized airports analyzed, four of them tend to stabilize the  $\mathcal{E}$ -index at a greater or equal value than the observed prior to privatization. However, observing the evolution of  $\mathcal{E}$ -index for an important hub in the country (Congonhas Airport<sup>22</sup>), which has not yet been privatized, we find the same stabilizing effect since 2014. We conclude then, that privatizations of airports do not necessarily imply changes in the  $\mathcal{E}$ -index, but the expansion of airports may contribute to this increase, since large investments occurred in the airports of the country to receive the Fifa World Cup of 2014<sup>34</sup>.

In a universe of 154 airports considered in this work, the privatization of 6 of them did not imply a significant impact on the BATMN  $\mathcal{E}$ -index at the global level.



**Figure 5.** Analysis of the number of routes (links) of BATMN. **(a)** Number of links of the airlines studied from 2010 to 2018. The color lines show the number of routes of each airline, specified in the legend. **(b)** Comparison between the total number of links (blue line) of considered airlines and diversity value for BATMN (red line), for each year from 2010 to 2018.

## Conclusions

Combining the  $\mathcal{E}$ -index and diversity ordering, we have found a way of analyzing the efficiency of multiplex network, regardless of the number of layers. The  $\mathcal{E}$ -index allows quantifying the network connectivity heterogeneity and coverage while the diversity ordering indicates the level of contribution of each layer, to the diversity of the system. The greater the difference in connectivity patterns between the layers, the greater the value of the  $\mathcal{E}$ -index, ranging from 0 to 1.

Applying the  $\mathcal{E}$ -index to the Brazilian domestic passenger air transportation network, we observed that, during the period considered, BATMN exhibited a reorganization of the network in the first four years due to airline mergers. In 2011, regional airlines expanded the number of routes and this fact, combined with the results presented in diversity ordering, explain the local peak observed in annual  $\mathcal{E}$ -index profile. After all the mergers concluded, the annual value of  $\mathcal{E}$ -index reaches its maximum in 2014, decreasing slowly to a plateau from 2015. When considering BATMN with monthly temporal precision during the same period, we notice jumps in the values of  $\mathcal{E}$ -index, which compared to the annual scenario, reflect the fact that the network was reorganizing itself as the new airlines consolidated. The influence of the FIFA World Cup on BATMN is clear when looking at the monthly variation of  $\mathcal{E}$ -index during 2014, when airlines redirected their routes to meet the demand added by the games. By simulating the shutdown of one airline in 2019 while maintaining the same outlook as 2018 for other airlines, we see an increase in annual  $\mathcal{E}$ -index value. This means that the airline excluded from the network operates a large number of routes overlapping with other airlines. In this context of market concentration, there is no significant reduction of routes, but there is a loss in the offer of airlines to the population.

Finally, we conclude that the  $\mathcal{E}$ -index is not influenced by airport privatizations; however, investments in airport expansion can impact it, especially if the offer of routes increases. In addition, fluctuations in the  $\mathcal{E}$ -index of a small fraction of network nodes that do not have a significant impact on the global  $\mathcal{E}$ -index.

## Data availability

The datasets analysed during the current study are available on the ANAC<sup>35</sup> website.

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## Author contributions

I.M.O. planned the experiments, collected data, manipulated data, performed experiments, researched references, interpreted the data and results, constructed figures and tables, and wrote the text. L.C. planned experiments, interpreted data and results and wrote the text. A.P.F.A. planned experiments, interpreted the results and wrote the text. All authors reviewed the manuscript.

## Competing interests

The authors declare no competing interests.

## Additional information

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**Correspondence** and requests for materials should be addressed to I.M.O.

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