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On evaluating the impact of flexibility enhancing strategies on the performance of nurse schedules

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ABSTRACT

Hospitals develop nurse schedules that cover a period of 4–6 weeks and are posted several weeks in advance. Once posted, changes to the schedule require voluntary participation by the nurses, making it difficult for hospitals to respond to changes in nursing needs and availability of nurses. At the same time, nursing needs' forecasts developed several weeks in advance are often wrong. In each hospital setting, there may exist several promising strategies to enhance scheduling flexibility and reduce the mismatch between the nursing needs and the availability of nurses. However, methodologies to evaluate such strategies, before testing them in expensive pilot implementation, do not exist. We demonstrate how such evaluations can be carried out using historical data. Furthermore, we demonstrate the use of our approach by evaluating the benefits of a strategy where nurses are divided into two cohorts and schedules are phase shifted for the two cohorts. Staggering schedules allows nursing unit managers to benefit from more frequent updating of needs' assessments without having to change work rules. Upon applying our approach to data from a large urban hospital, we discovered that in this example staggering did not improve the performance of nurse schedules. We discuss possible reasons for this result, its implications for hospital managers, and other potential uses of our approach.

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1. Introduction

Large hospitals are organized into a variety of specialized nursing units. For example, *telemetry* units house patients under observation or those awaiting a surgical procedure, *medical/surgical* (med/surg) units house patients who are either recovering after a surgery or have a medical need requiring hospitalization, *maternity* units house new mothers and babies, and so on. This allows hospitals to have specialized equipment and nursing staff to care for patients with different care needs. Most nurses are assigned to specific units, although there are some that belong to a float pool—i.e., they serve in different units depending on the

needs. In some situations, nurses that are not in the float pool can be assigned to another unit on an ad hoc basis if the staffing level is higher than the nursing needs (nurse requirements) in their home unit and lower in another unit. However, nurses do not like involuntary floating [7].

Often, nurses' schedules are fixed for a period of 4–6 weeks at a time and posted 4–8 weeks in advance [19,5,13]. We refer to the former length of time as the *review period* and the latter as the *lead time*. For example, for a 4-week review period and a 6-week lead time, a nursing unit manager prepares a schedule that covers weeks seven through ten if the schedule is posted at week zero. This exercise is repeated every 4 weeks. In order to generate this schedule, the nursing unit manager needs a forecast of nursing needs for each shift several weeks in advance. Because such forecasts are invariably incorrect, mismatches between planned staffing levels and actual nursing needs are com-

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mon, which necessitates considerable effort by managers to avoid understaffing in some shifts and overstaffing in others.

Nurses view understaffing as a major factor that leads to poor quality of care and nurse burnout [14]. In a 2008 Safe Nursing Staffing Poll [4], 23.8% of the nurses were considering leaving their jobs at the time of survey, and 42.0% among them reported that the reason was associated with inadequate staffing. This survey reveals one of the key reasons for avoiding understaffing from the nurses' viewpoint. Equally important is the consideration that inadequate staffing relative to the workload is unsafe for patients, being associated with increased instances of hospital-related mortality and failure to rescue [25,3,15]. Nursing unit managers deal with understaffing by using overtime or temporary staff, both of which are costly options for hospitals.

Hospital management is equally concerned about avoiding overstaffing because even with complete flexibility in assigning nurses to different units, it may not be possible to effectively utilize services of all nurses on duty. Note that in reality, only limited assignment flexibility exists in most hospitals. There is also some, but limited, flexibility to require nurses to take involuntary time off. However, these measures can strain relations between the nursing staff and the hospital management.

How can a hospital increase its ability to meet nursing needs in a cost effective manner? Shortening the lengths of the review period and the lead time may help, but such changes are infrequent and perceived to be difficult to realize owing to provisions in nurses' contracts. Hospitals can eliminate specialized nursing units, but that can affect quality of care and lead to lower staff morale [6,24]. Furthermore, general-purpose units exacerbate the nurse staffing problem because the care needs of the patients are more variable. These arguments suggest that for any new strategy or change in policy that is proposed to improve a nursing unit's ability to meet patient-care needs, it is necessary to have a methodology to evaluate its costs and benefits and to subsequently test those strategies/policies that perform

well via pilot implementation studies. We report the first step in this process in which we develop a suite of models that can be used to evaluate a variety of flexibility enhancing strategies.

In addition to describing our methodology, we demonstrate the use of our approach by reporting the results of an experiment in which we evaluate the potential benefits of *schedule staggering*. In particular, we study an instance of staggering with two cohorts—i.e. we divide the nurses assigned to a unit into two cohorts and schedule each cohort with a phase difference. If, for example, the review period is 4 weeks and the lead time is 6 weeks, we develop a schedule for each cohort that is phase shifted by 2 weeks. Each cohort still obtains its schedule every 4 weeks, which is set 6 weeks in advance, but the start of these 4-week periods for the two groups is offset by 2 weeks. A schematic of the staggering strategy is shown in Fig. 1. (In this figure and throughout the remainder of this paper, the review period is denoted by the Greek letter τ .) Clearly, variants of this approach with more than two cohorts are possible. However, the complexity of the evaluation as well as the complexity of the implementation can be significantly higher with more cohorts.

The main contribution of this article is in demonstrating how operations management methodology could be used for carrying out preliminary evaluations of strategies for improving nurse schedules and in presenting a detailed example of an application of this approach. Our methodology has two parts; the first part deals with nursing needs' forecasting and the second part with scheduling nurses based on this forecast. The details of our methodology are provided in Section 2, immediately following the literature review, which is presented next.

Broadly speaking, previous papers have neither examined the combined effects of forecasting and scheduling in a single study, nor tested strategic choices that might be available to nurse managers to improve the performance of nurse schedules. We begin with the forecasting literature first. There are many aggregate-level (regional or national)

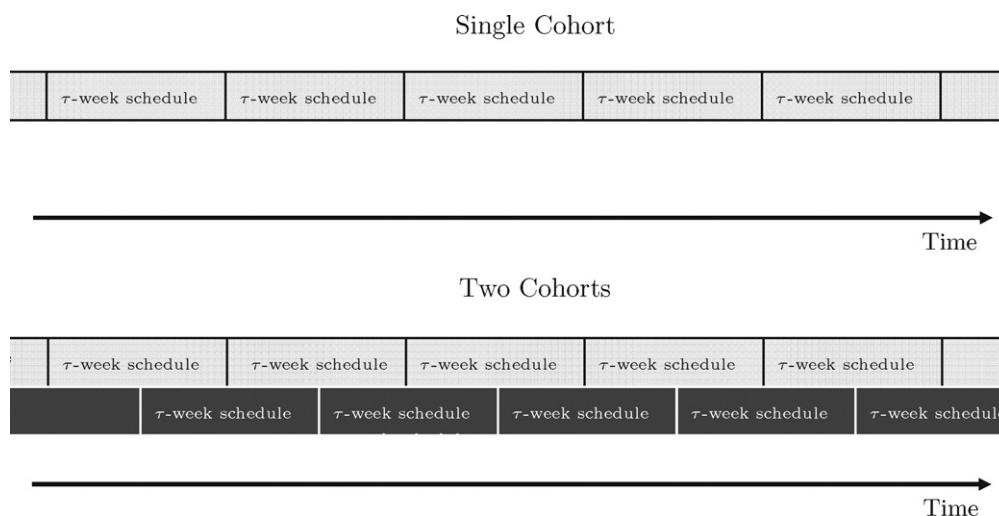


Fig. 1. A schematic showing the two-cohort staggering strategy.

nursing demand forecasting models. A summary of available approaches can be found in O'Brien-Pallas et al. [21]. Aggregate nursing demand forecasting approaches are not suitable for scheduling decisions at the level of a single unit, which require forecasts of medium-term nursing needs. Therefore, in this paper, we focus only on articles that are concerned with forecasts of nursing needs at the nursing-unit level over a 4- to 6-week time horizon.

Côté and Tucker [11] describe four common hospital-level nursing needs' forecasting methods. These are percent adjustment, moving average, trendline, and seasonalized forecasts. The percent-adjustment method is based on the percentage increase or decrease in the past twelve months of historical nurse requirements—if there is $\pm x\%$ change the previous year, then it is estimated that the nursing needs will change by $\pm x\%$ in the following year as well. Trendline methods use linear regression on historical nurse requirements, with time as the explanatory variable, to determine if a trend exists. Both percent adjustment and trendline methods ignore possible seasonalities in nursing requirements. Moving average and seasonalized forecasts are common time-series based models. Moving average can work well when neither the seasonal nor the trend pattern is strong. Seasonalized forecast (with or without trend) is appropriate when repeating patterns are identified in the historical data. The forecast in each season is adjusted by the corresponding seasonal index. These approaches are widely used and discussed in many books; see, for example, Shumway and Stoffer [22].

Kao and Tung [17] studied the use of Autoregressive Integrated Moving Average (ARIMA) time-series models for predicting nurse requirements for inpatient services in a large public health care system. Their monthly nursing requirements forecasting model can help a hospital choose the right level of aggregate capacity but it does not provide information on daily or shift-level nurse requirement fluctuations. Wood [26] used an ARIMA model to forecast shorter term (1-day-ahead) requirements. Earnest et al. [12] also used ARIMA models to predict the number of occupied beds during a SARS (Severe Acute Respiratory Syndrome) outbreak in a tertiary hospital in Singapore.

Cerrito and Pecoraro [9] used the Electronic Medical Record (EMR) data from an emergency department (ED) and a time-series forecasting method to predict the number of nurses needed in the ED. The forecasting model utilizes patient arrival rates, treatment times and diagnoses in an exponential smoothing procedure with time-of-day seasonal factors. However, this model does not take into account the amount of work created by patient movements such as admissions, discharges and transfers in/out (ADT). ADT activity is significant in many nursing units and needs to be factored in nursing needs' assessment because studies have shown that nurses' workload is correlated to patient outcomes such as hospital-related mortality and failure to rescue [25,3,15].

An example of total workload induced activity accounting can be found in Adenso-Díaz et al. [2] who proposed a method to determine the minimum number of staff needed for a predefined level of quality, given a particular mix of patients. They first calculated the theoretical number of staff needed in previous months by taking into account the

average number of activities per patient and the estimate of time needed to perform various activities. Then, they calculated the ratio of the actual to the theoretical staffing levels. Finally, based on the relationship between quality outcomes as a function of the real-to-theoretical ratio, they proposed a minimum staffing level for each mix of patients.

We turn next to the literature on nurse scheduling. The vast majority of such papers propose a mathematical program to find a minimum cost staff schedule that meets the estimated level of nursing needs. For example, Kao and Tung [18] used results from their ARIMA forecasting model (described in [17]) in a linear programming model to determine the number of permanent nurses required, the size of the pool of float nurses, and the need for overtime/temporary nurses by medical specialty at an aggregate (monthly) level. However, these authors do not report the overall performance of their schedule, after factoring in both forecast errors and inefficiencies induced by scheduling constraints. Some also use goal programming and artificial intelligence methodologies; see [10,8] for reviews of that body of literature.

Kao and Queyranne [16] formulated a single-period aggregate (over nursing skills) deterministic model and a multi-period disaggregate probabilistic model for budgeting nursing workforce requirements. They concluded that ignoring staffing-need uncertainty would lead to underestimates of nurse requirements and unnecessary staffing costs. Abernathy et al. [1] proposed that nurse staffing processes can be divided into three decision stages: (a) policy decisions such as operating procedures for service centers and for the staff control processes, (b) staff planning such as hiring, discharge, training and personnel reallocation decisions, and (c) short-term scheduling of available staff subject to the constraints determined by the two previous stages. They formulated the planning and scheduling stages as stochastic programming problems to account for staffing-need fluctuations.

Realizing that nurses' schedules developed several weeks in advance invariably require adjustments, several authors have focused on the problem of re-scheduling. Moz and Pato [20] proposed a multicommodity flow model that minimizes the difference between the original and the revised schedule. A unit cost is assessed for each change in a nurse's assignment (task/shift) if the change does not violate negotiated work rules or skill requirements. A much higher cost is assessed if any one of these constraints are violated. This approach minimizes the impact of re-scheduling on nursing staff by minimizing the dissimilarity between the current and the revised schedules.

Bard and Purnomo [5] use an integer programming model to generate a revised daily schedule based on an estimate of expected staffing need over the next 24 h. The goal of this model is to satisfy nursing requirements with minimum cost while honoring nurses' preferences for shift assignments. Its output identifies candidate nurse-shift combinations for overtime, floating to another unit, shift cancellations, and temporary staffing.

Schedule staggering shares a common goal with re-scheduling approaches. However, the former exploits a different type of scheduling flexibility available to the nurse manager—it delays the time when the hospital needs to fix

the schedule for a portion of its nursing staff. In contrast, rescheduling utilizes the ability to react to the differences between scheduled and required staffing levels. Clearly, schedule staggering can be used in combination with rescheduling approaches to help unit managers respond to uncertainty in staffing needs and staff availability. To the best of our knowledge, no previous study has either proposed or evaluated this staggering approach. Moreover, forecasting and scheduling are typically treated as two separate problems. No study has evaluated the overall performance of schedules when forecast errors as well as scheduling inefficiencies are simultaneously present. This paper contributes to the literature on health care operations management by performing forecasting and scheduling in a single study and demonstrating how this approach could be used to evaluate strategies for improving nurse schedules.

Some papers interpret staggering differently. In particular, the term staggering is also used to describe the practice of having different shift start times (during the day) of certain staff members. Sinreich and Jabali [23] proposed a linear optimization model and a simulation based algorithm to obtain a schedule that can better meet the staffing-need pattern within a day without adding staff members. Staggering start times of work shifts may increase the flexibility to respond to staffing-need fluctuations within a day, but it does not address the problem of developing medium-term staff schedules.

Finally, there are a variety of commercial software packages available for nurse scheduling. Burke et al. [8] provide a review of the suitability of various approaches/software packages for different scheduling environments. To the best of our knowledge, these packages do not come with a module for forecasting staffing needs. Moreover, they do not provide the flexibility to the user to evaluate the impact of different strategies for developing nurses' schedules.

The organization of the remainder of this article is as follows. We present our methodology in Section 2 and report the results of our experiments using real data from a large urban hospital in Section 3. Section 4 explains why staggering did not improve performance in our examples and lists other strategies for improving a nurse manager's ability to realize high performing schedules that could be evaluated with the help of the approach reported in this paper.

2. Materials and methods

A nursing unit manager needs two types of data to apply our approach for evaluating strategies for improving nurse scheduling. The first type of data consists of the number of beds in the test unit, the number of staff assigned to this unit and each staff member's weekday and weekend work patterns, work rules, and common management practices. The second type of data consists of patient movement time-stamps, which are used to calculate hourly unit-level bed occupancy and activity levels. Most hospitals have some type of bed management system in place that keeps track of patient movement time stamps. Still, some effort may be required in aggregating this information into hourly census and ADT counts.

We describe our methodology in two steps. First, we mention general features of our methodology that any nursing unit manager would need to know in order to implement our approach. Then, we describe how this approach was applied to the example unit for evaluating scheduling staggering approach. As explained in the previous section, our methodology has two parts—nursing needs' forecasting and nurse scheduling according to this forecast. Thus, these two steps are described for each part separately, beginning with forecasting.

The timing of nursing needs' forecasting decisions precludes the use of real-time data. Furthermore, as explained next, it may be difficult to include historical patient-specific data in forecasts of nursing needs. Typical nursing units have a variable staffing plan that converts each census level (by time of day) into a minimum number of registered nurses (RNs) needed to care for that many patients. The variable staffing plan is approximately based on nurse to patient ratios corresponding to an aggregate mix of patient acuity. We found that such practices are common at large hospitals, although not all hospitals use the terminology variable staffing plan. Our test unit did not have electronic records of patients' nursing care requirements. Patients' needs varied such that some required a dedicated nurse, whereas up to four patients could be assigned to a single nurse in other cases.

Because of lack of reliable data, we did not include patients' acuity levels in our method for determining nursing needs. Until such data becomes available in electronic form, our approach is reasonable because of the following reasons. Each unit's aggregate patient composition is stable (so long as patient placement rules do not change) and the variable staffing plan takes into account aggregate patient acuity when deciding the appropriate nurse-to-patient ratios. It is for this reason that different units use different nurse-to-patient ratios. These ratios also vary by shift to account for shift-to-shift differences in workload due to doctors' orders, distribution of medications and meals, and various call-button requests. In addition, patient acuity is not constant over time and to our knowledge there is no database that updates patients' acuity information over time. Therefore, estimating patient acuity from available electronic records is not currently feasible. However, if such data were available, an acuity-based adjustment could be applied to our nursing needs' forecast in a manner similar to the ADT adjustment described below.

We asked several experienced charge nurses¹ to predict how they would staff the test unit, assuming a typical mix of patients in terms of their acuity, if they knew the census profile and the count of hourly ADT. We did this to determine what features of the census profile these experts considered important for the purpose of ascertaining nursing needs. Our experiment revealed that nurses used beginning-of-shift census to determine a base staffing level (from the variable staffing plan) and adjusted it upwards to account for the ADT activity and for additional nurses required for periods when census is higher than the

¹ A charge nurse is a nurse who is in charge of a unit (ward).

starting census. Based on this evidence, our nursing needs' forecast model predicts for each shift, by day of week, the beginning census of a shift and the additional workload due to the change in patient numbers and ADT. This information is then converted into nursing needs according to the variable staffing plan. The technical details of this model can be found in Appendix A. Next, we describe how we applied our method to the example unit.

We obtained time-stamp data for calendar years 2005, 2006, and the first four months of 2007. All data were de-identified by the collaborating hospital. We took a variety of steps to ensure data accuracy (e.g. comparing midnight bed census levels obtained from different databases) and to infer certain missing data based on rules provided by the unit manager (e.g. all patient movements from ED to the telemetry unit were treated as new admissions per hospital protocol). These steps are important but not described in

detail here in the interest of brevity. From the time-stamp data we obtained beginning census for each shift of each day of operation for the unit under investigation. We also aggregated this data into hourly, daily and weekly census levels to study seasonality by the time of the day, by the day of the week, and by the week of the year. The results are shown in Fig. 2.

Fig. 2 shows that there is no identifiable seasonal pattern by week; however, there are significant day of week and shift-based seasonal effects. Our approach compares the performance of a variety of forecasting techniques that utilize these seasonal patterns and picks the best among them. The goodness of each approach may be measured by the mean absolute deviation (MAD), i.e. the average of the absolute difference between the forecast and the actual staff requirements. It is also possible in our approach to weight shortages and overages differently when calculat-

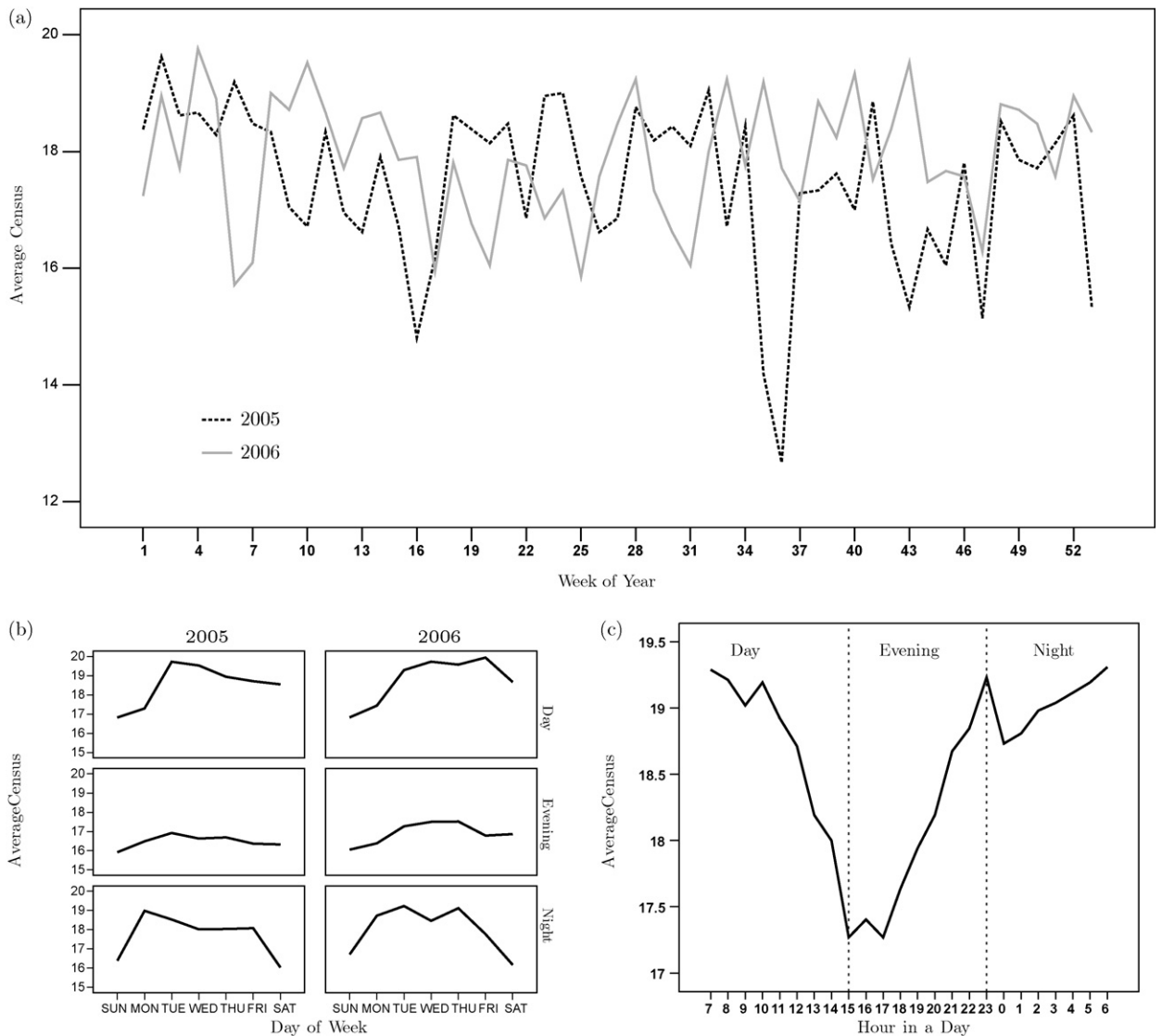


Fig. 2. Seasonal patterns of bed census. (a) Weekly average census. (b) Average start-of-shift census by day of week. (c) Average Tuesday hourly census (2006).

ing MAD or to use a different measure of the goodness of the forecast.

We compared simple and weighted moving average and smoothing procedures to identify the best protocol for the test unit. For each method, the first twelve months' data were used to estimate any parameters needed to implement the method. The performance of the forecast was measured by calculating MAD for the second 12-month period. Upon comparing different forecasting methods, we found that in our example unit, the simple moving average procedure performed the best. Therefore, our forecasting model estimates the beginning census (by day of week) of each shift by the simple moving average of censuses of the most recent 52 weeks. The forecast of the workload adjustment due to ADT and increase in patient numbers

during a shift is calculated by the long-term (1 year) average of extra nursing hours required, also by day of week and shift. We did not use moving average for the second component because this number is relatively stable for the unit we investigated. For a nursing unit with more variable ADT workload throughout the year, it may be more appropriate to use a different method to calculate the workload due to ADT. The forecast of the number of RNs required to meet the staffing needs are then calculated by applying the unit's nurse-to-patient ratio to the estimates of the beginning census and the additional workload from activities.

We turn next to the second part of our approach that uses the forecast of staffing needs for each shift during the review period to develop a work schedule for

Table 1

An 1-week schedule (01/27/07–02/02/07) for the 45 RNs.

		Sat	Sun	Mon	Tue	Wed	Thu	Fri
RN	1							
RN	2				D(RT)	D(RT)	D(RT)	
RN	3			N(RT)				N(RT)
RN	4	E(RT)	E(RT)	E(RT)	E(RT)	E(RT)		
RN	5	D(RT)				D(RT)	D(RT)	
RN	6					D(RT)	D(RT)	E(RT)
RN	7	D(RT)	E(RT)	D(RT)	D(RT)	D(RT)		D(RT)
RN	8	N(RT)	N(RT)					
RN	9			D(RT)	E(RT)	E(RT)	E(RT)	D(RT)
RN	10			N ₁₂ (RT)				
RN	11			E(RT)	E(RT)	E(RT)	E(RT)	
RN	12						E(RT)	
RN	13							D ₁₂ (RT)
RN	14			N(RT)			N(RT)	
RN	15			D ₁₂ (RT)	D(RT)		D(RT)	
RN	16			N ₁₂ (RT)	N ₁₂ (RT)			N ₁₂ (ET)
RN	17				D(RT)	D(RT)		D(RT)
RN	18	N(RT)	N(RT)	N(RT)	N(RT)		N(RT)	
RN	19				D ₁₂ (RT)			E(RT)
RN	20				N(RT)			N(RT)
RN	21						E(RT)	
RN	22			E(RT)	E(RT)			E(RT)
RN	23						D(RT)	D(RT)
RN	24	E(RT)	E(RT)	E(RT)				
RN	25							
RN	26	E(RT)	D(RT)	D(RT)		E(RT)		
RN	27	D ₁₂ (RT)	D ₁₂ (RT)		E(RT)			
RN	28				D(RT)	D(RT)	D(RT)	D(RT)
RN	29	D(RT)	D(RT)	D(RT)	E(RT)	D(RT)		
RN	30	E(RT)		E(RT)				
RN	31	D(RT)	D(RT)		D(RT)		D(RT)	D(RT)
RN	32			E(RT)	E(RT)	E(RT)	E(RT)	E(RT)
RN	33							N ₁₂ (RT)
RN	34	N ₁₂ (RT)	N ₁₂ (RT)					
RN	35	N(RT)	N(RT)	N(RT)	N(RT)	N(RT)		
RN	36		D(RT)	D(RT)	D(RT)	D(RT)	E(RT)	
RN	37					N(RT)	N(RT)	N(RT)
RN	38					E(RT)		
RN	39							D ₁₂ (RT)
RN	40						E(RT)	E(RT)
RN	41		E(RT)			E(RT)	E(RT)	
RN	42				N(RT)	N(RT)	N(RT)	
RN	43	N(RT)	N(RT)		N(RT)	N(RT)	N(RT)	
RN	44						D(RT)	
RN	45					N(RT)	N(RT)	
Total	D	4 RT	4 RT	5 RT	7 RT	8 RT	8 RT	6 RT
	E	4 RT	4 RT	6 RT	7 RT	7 RT	8 RT	5 RT
	N	4 RT	4 RT	4 RT	5 RT	5 RT	6 RT	3 RT
	D ₁₂	1 RT	1 RT	2 RT	1 RT	0	0	2 RT
	N ₁₂	1 RT	1 RT	2 RT	1 RT	0	0	1 RT; 1 ET

nurses. Our scheduling model is an integer-programming model that minimizes the cost of meeting the staffing needs over the τ -week planning horizon, subject to constraints imposed by work rules (usually negotiated by the Nurses' Association (NA)), available RNs, their FTEs, work patterns in terms of allowable weekday and weekend duties for each nurse, and time-off requests that must be granted. [Recall that τ denotes the length of the review period.] Technical details of this model are presented in Appendix B.

In the example unit, there were 45 nurses. Most worked part time. Each nurse worked a particular pattern of Day–Evening, or Day–Night shifts and the majority worked alternate weekends. There were some exceptions who worked every third weekend or alternated between every other and every third weekend. Therefore the unit needed approximately twice as many nurses as there were weekend shifts. Our calculations revealed that 22 nurses were needed to staff each weekend's shifts at full capacity. This suggests that the unit employed a sufficient number of nurses (because 45 is approximately twice of 22) to meet weekend staffing need.

The basic features of the scheduling problems in our example unit are as follows. The hospital develops 4-week staffing plans for three standard 8-h shifts. Every staffing plan starts on a Saturday. Day shifts are from 7 AM to 3 PM; Evening shifts are from 3 PM to 11 PM; Night shifts are from 11 PM to 7 AM. A majority of the nurses work either 8-h shifts or 12-h shifts, although some do work a combination of 8- and 12-h shifts. A 12-h shift can be either from 7 AM to 7 PM or from 7 PM to 7 AM. In this section, we use $i = 1, \dots, 5$ to index Day, Evening, Night, 7 AM–7 PM, and 7 PM–7 AM shifts, respectively. These shifts are also denoted by acronyms D, E, N, D12, and N12, respectively.

We used the forecast requirements developed in the first part of our approach as input to the scheduling model and obtained the average weekly costs associated with imple-

menting either a single-cohort or a two-cohort strategy. The key schedule-performance metric of cost associated with single- and two-cohort strategies was obtained by adding the cost of the optimal schedule for that strategy (produced by our scheduling model with nursing needs' forecast as input) and the cost of staffing the realized uncovered shifts. The cost of staffing each uncovered shift is set equal to the overtime cost for that type of shift because last-minute staff additions were usually paid the overtime rate.

3. Results

Upon using the forecast of nursing needs along with current FTEs, shift times, and weekend on/off patterns to obtain the optimal assignment of nurses to shifts for each week of a 4-week period, we obtained a detailed work schedule for all nurses assigned to a unit. A partial solution to this problem is shown in Table 1. In this table, RT, ET and OT are used to indicate regular-time, extra-time and overtime shift assignments, respectively.

Because of work rules, nurses' weekday and weekend work patterns, and because of the fact that staffing levels (total FTE attached to a unit) often include time-off considerations, it is either not possible or not necessary to schedule all available nurse shifts. [Note that time-off requests were not known at the time of developing the schedule.] Unscheduled shifts for a 4-week period for each nurse are shown in Table 2. The shifts are calculated in terms of standard 8-h shifts. Thus, a 12-h shift counts as 1.5 shifts and a nurse working 0.4 FTE must work 8 shifts during a 4-week period. An unscheduled shift can be utilized in a different unit, or this time can be used for continuing education and training, if work rules and individual contract terms with a nurse permit such an assignment.

Realizing that RNs would decide which cohort they would like to join, we randomly assigned each RN to one of the two groups. In absence of data on nurse preferences,

Table 2
Number of scheduled and unscheduled shifts (1 shift = 8 h) during a 4-week period (01/20/07–02/16/07).

RN	FTE	Scheduled	Unscheduled	RN	FTE	Scheduled	Unscheduled
1	0.4	8	0	24	0.4	8	0
2	0.7	14	0	25	0.8	10	6
3	0.8	7.5	8.5	26	0.8	14	2
4	0.8	16	0	27	0.4	6.5	1.5
5	0.6	12	0	28	0.8	10	6
6	0.8	16	0	29	0.8	16	0
7	1	20	0	30	0.7	14	0
8	0.8	10	6	31	0.9	18	0
9	0.8	16	0	32	1	16	4
10	0.9	9	9	33	0.9	9	9
11	0.8	16	0	34	0.9	7.5	10.5
12	0.6	9	3	35	0.6	10	2
13	0.9	7.5	10.5	36	0.8	13	3
14	1	10	10	37	0.6	12	0
15	0.9	9	9	38	0.4	8	0
16	0.8	7.5	8.5	39	0.8	9	7
17	0.7	9	5	40	0.7	8	6
18	0.8	10	6	41	0.8	12	4
19	0.9	9	9	42	0.6	12	0
20	1	10	10	43	1	20	0
21	0.5	10	0	44	0.7	14	0
22	0.8	16	0	45	0.9	10	8
23	0.8	16	0				

Table 3

Comparison of forecast requirements with different strategies, number of regular-time shifts scheduled, and the actual requirements during a 1-week period.

		Single cohort		Two cohorts		Required
		Forecast	Scheduled	Forecast	Scheduled	
1/20/2007	D	5	5	5	5	6
	E	5	5	5	5	5
	N	5	5	5	5	6
1/21/2007	D	5	5	5	5	6
	E	5	5	5	5	6
	N	5	5	5	5	6
1/22/2007	D	7	7	7	7	9
	E	7	7	8	8	8
	N	6	6	6	6	6
1/23/2007	D	8	8	8	8	9
	E	8	8	8	8	8
	N	6	6	6	6	6
1/24/2007	D	8	8	8	8	9
	E	8	8	7	7	8
	N	5	5	5	5	6
1/25/2007	D	8	8	8	8	8
	E	8	8	8	8	8
	N	6	6	6	6	6
1/26/2007	D	8	8	8	8	8
	E	7	7	7	6.5	7
	N	5	5	5	4	5

random splitting (with equal probability of joining either cohort) serves to simulate a situation in which the RNs are allowed to choose a cohort. Results are reported in Tables 3 and 4. Table 3 shows the staffing needs' forecast and scheduled shifts (regular-time only) for each day of the week during the week of January 20 and January 26. We also show the actual number of shifts required. On January 26, there is a half shift in the evening because a 12-h day shift was scheduled in the morning. Although there was a matching 12-h shift in the night, that shift was scheduled as an overtime shift. Overtime shifts are not reported in this table because they are not scheduled unless needed after observing the realized staffing need in the previous shift.

Table 4 shows the costs of meeting staffing needs in each 2-week period as a percentage of minimum attainable cost ($P\%$). The latter is the cost incurred when all requirements are met with the regular-time hourly rate. The biweekly staffing need (in terms of standard 8-h shifts) as well as the number of shifts short and in excess are also shown.

Note, this table does not show unscheduled shifts and their cost is not included in calculating the performance of the schedule because it is possible in many cases to use the nurses' time for other purposes. That is, unscheduled shifts are not charged to direct patient care budget for the unit in question.

In some cases, the cost performance of the staggering strategy is slightly worse than the "single-cohort $\tau = 4$ " and no strategy dominates the other. A similar picture also emerges when we examine the number of shifts short (SH) and the number of shifts over (OV). The average shortage per 2-week period is 20.8 shifts and the average overage is 5.0 shifts if the forecast is updated every 4 weeks ($\tau = 4$) and staggering is not used. When we compare this to the staggering strategy, the performance with staggering turns out to be slightly worse. At a first glance, the above observations appear counterintuitive because more frequent information updating is not expected to lead to worse results. However, upon careful consideration these

Table 4

Biweekly performance of the staggering strategy with the current RN composition. SH = number of shifts short, OV = number of scheduled shifts that are not needed, P = relative cost compared to the attainable minimum cost, and AVG = average.

Period	Staffing need	Single-cohort $\tau = 4$			Two-cohort $\tau = 4$		
		SH	OV	$P(\%)$	SH	OV	$P(\%)$
01/06–01/19	288	42	2	111.7	42	2	111.7
01/20–02/02	263	23	8	109.6	12.5	18	110.3
02/03–02/16	282	13	1	103.9	21.5	3	104.1
02/17–03/02	287	25	8	109.4	30	3	108.9
03/03–03/16	279	16	5	106.2	18.5	4.5	106.6
03/17–03/30	289	23	2	106.7	21	2	106.2
03/31–04/13	271	11	10	106.8	11	10	106.8
04/14–04/27	279	13	4	105.0	12	3	104.3
AVG		20.8	5.0	107.4	21.1	5.7	107.4

results can be explained by the additional scheduling limitations imposed by the staggering strategy. Details are presented in the next section.

4. Conclusions and discussion

Hospitals can better meet nursing needs, and enhance patient safety and staff morale in the process of doing so, by increasing flexibility to respond to the uncertainty in patient-care needs while respecting work rules. We presented a general-purpose methodology for evaluating such approaches and evaluated an example, which we call schedule staggering, in this paper. Although more frequent forecast updating generally improves the accuracy of the forecast (this may not hold for every 2-week period due to noise in the data), the benefits of schedule staggering are not realized because scheduling becomes more constrained under staggering. The work schedule of half of the nurses is fixed at the time forecast is updated. Even when the updated forecast is more accurate, the hospital may not be in a position to utilize the benefits of staggering because of the various scheduling constraints that must be met. Therefore, the success of the staggering strategy depends on whether a hospital is able to utilize the improvement in forecast accuracy given more restricted scheduling flexibility because a proportion of the nurses' schedules are already fixed.

There are several avenues for further research along the lines presented in this paper. The evaluation approach developed in this paper can be used to study the benefits of staggering in other hospitals for which scheduling constraints are less stringent, as well as the potential benefits of other strategies. Examples of other strategies include pooling nursing needs from several similar units (e.g. when a hospital has several telemetry or med/surge units) for the purpose of developing nurse schedules, using elective surgery booking information to improve nursing needs' forecasts, and choosing a hiring plan that strategically selects the weekday and weekend work patterns of additional hires. In each case, a schedule would be developed using the appropriate forecast as input and overall schedule performance would be ascertained in a manner similar to the example presented in Section 2. Finally, because most hospitals operate in non-stationary environments in terms of nursing needs, quantifying the benefits of learning approaches to improve nursing needs' forecasts is also a worthy topic for future research.

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Appendix A. The forecasting model

Hospitals usually determine staffing levels based on the factors such as census, ADT activities, and/or patient-specific medical diagnoses. In a focus group discussion with experienced charge nurses of the unit we studied, we discovered that these nurses used beginning-of-shift census to determine the base staffing level, and adjusted it upwards to accommodate ADT activities and higher census when they had information about the exact census and ADT activities during a shift with a typical mix of patients. Therefore, our forecasting method utilizes the beginning census, ADT activities, and the number of patient hours when census during a shift is higher than the beginning-of-shift census. We would like to point out that the census properties and activity information for determining staffing levels may differ by unit or hospital.

When the underlying census properties and therefore the nurse requirements are stable and contain only unassignable variation, some commonly used forecasting methods include mean, median, mode, or a particular percentile of the nurse requirement distribution. When the general pattern of nurse requirements evolves over time, time-series methods, such as moving average, that utilize more recent information may be appropriate. If there exist seasonal patterns in nursing needs, adjusting for seasonal changes can improve the forecast. Some common forecast performance measures include mean absolute deviation (MAD) and mean square error (MSE) that respectively calculate the average deviation and the average square distance of the forecast values from the corresponding realized values. Note that whether a forecasting model performs well also depends on the costs of understaffing and overstaffing. Therefore, one can incorporate different weights for under- and overstaffing into the performance measures.

After testing a variety of approaches, we report only the model that gave the best result in our test unit (smallest MAD and MSE), assuming the same penalty for understaffing and overstaffing. Hospitals may choose other forecasting approaches depending on their costs and ability to deal with understaffing or overstaffing, or use other performance measures.

Let $d_{i,j}$ denote the number of RNs required for shift i on the j -th day of the staffing plan, where $i = 1, 2, 3$ and $j = 1, \dots, 7\tau$ indexes days in the planning period. Note that RN requirements are always calculated for the three 8-h shifts, although these requirements can be met by combining nursing staff that work 8- and 12-h shifts. We assume that the forecast for RN requirements is developed as late as permitted under the work rules.

In the example reported in Section 2, this happens 6 weeks before the start of the nurses' schedule for which the forecast is needed. The forecast depends on two estimates: $S_{i,j}$, a forecast of beginning census, and $A_{i,j}$, a forecast of activity adjustment. We next describe how we obtain both these quantities from time-stamp data on patient movements.

The simple moving average procedure calculates the start-of-shift census levels of each shift (Day, Evening, or Night) as the average of the m most recent beginning census of the same shift on the same day of the week ($S_{i,j}$). In other

words, suppose $B_{i,j}$ is the actual census at the beginning of shift i on day j , then $S_{i,j} = (1/m) \sum_{n=1}^m B_{i,j-7n}$.

Holidays such as Thanksgiving, Christmas, and New Year's day generally have lower census even when they fall on weekdays. Therefore, we used the moving average of the beginning census of each shift that falls on the ten Federal holidays within the most recent 365 days to forecast census level for the corresponding holiday shift.

Next, we used the time-stamp data to estimate the number of ADT activities in each shift and hourly bed census in each shift. This estimate allowed us to forecast activity adjustment $A_{i,j}$ based on the average of extra hours due to ADT and hourly changes in bed occupancy levels by shift and by the day of week. Nursing unit managers informed us that each transfer (in or out) took about 20 min and each admission/discharge took about 1 h of a nurse's time. Different units may use different parameters based on ADT-related workload in their setting.

An initial estimate of nurses required is obtained from the beginning-of-shift census forecast and the applicable nurse-to-patient ratio. The activity adjustment inflates this estimate to account for additional workload that is not captured in the initial estimate. Nurse-to-patient ratios, $1 : \gamma_{i,j}$ s, specify the maximum number of patients that can be assigned to a RN for each shift in the staffing plan. Usually, the same ratio is used for all Day/Evening shifts during weekdays, which is different from the ratio for the Night shifts or shifts during weekends. Acceptable ratios are negotiated between the hospital and the nurses' union.

In the example hospital, staff planning was done based on negotiated nurse-to-patient ratios and the actual number of nurses employed in a shift were adjusted depending on the realized nursing needs, actual staffing levels (which are affected by absenteeism and nurses reporting sick at the start of each shift) and the realized acuity of the patient mix. The hospital did not have a reliable record of patient acuity levels. Therefore, to make clean comparisons, we did not include patient acuity-based adjustments when calculating both the realized requirements and the forecast requirements. That is, both were based only on bed census and activity adjustment. However, when acuity indices are available, it would be possible to develop a forecast of acuity level by shift and day of week and to subsequently apply an acuity adjustment to the census estimates. In what follows, the parameter $q_{i,j}$ denotes the number of patients that are cared for by the charge nurse on the i th shift of the j th day of the schedule. This gives

$$d_{i,j} = \frac{1}{\gamma_{i,j}} \left[S_{i,j} + \frac{A_{i,j}}{8} - q_{i,j} \right] + 1.$$

For the unit we investigated, the hospital uses $\gamma_{i,j} = 3$ for all Day and Evening shifts during weekdays (i.e. for $i \in \{1, 2\}; j \notin \{\text{weekends}\}$) and $\gamma_{i,j} = 4$ for all other shifts. Similarly,

$$q_{i,j} = \begin{cases} 0 & \text{if } i \in \{1, 2\}; j \notin \{\text{weekends}\} \\ 3 & \text{otherwise} \end{cases}$$

because the charge nurse provides direct patient care only during night shifts and weekend shifts. After computing each $d_{i,j}$ as shown above, it is rounded up to an integer

number (or the next half or quarter nurse equivalent, as required) and used as input in the nurse scheduling module described in the next section. In different applications of our approach, forecasting will be based on similar steps. However, the estimates of parameters may be different.

Appendix B. The scheduling model

The majority of nurse scheduling models in the literature formulate the problem as an integer-program (IP). These models may contain different types of constraints for different hospital settings. Our formulation is also IP based and is designed specifically for the unit we studied. This formulation is an example of how to automate nurse scheduling with a variety of constraints. Hospitals may modify the parameters and/or constraints to suit their specific needs.

Our formulation includes the actual constraints faced by a nurse manager for generating an optimal 4-week schedule for each nurse in the unit. For the examples reported in Section 2, the problem contained 19,080 decision variables and 6749 constraints. Using OPL Studio 3.6.1 as the modeling environment and CPLEX 8.1 as the solver on a PC with Intel 2.0 GHz processor, the computation times ranged from a few seconds to 7 min depending on the values of the input parameters. The surprisingly modest computing requirements are in part due to the fact that contractual arrangements between the hospital and the nurses often result in limited degrees of freedom in shift assignments. In what follows, we describe the key blocks of our model formulation.

B.1. The decision variables

We use binary variables $r_{i,j,k}$, $e_{i,j,k}$, and $o_{i,j,k}$ to denote nurse k 's ($k \in \{1, \dots, K\}$) working status on shift i ($i \in \{1, \dots, 5\}$) of the j -th ($j \in \{1, \dots, 7\tau\}$) day of the plan. In particular, $r_{i,j,k}$, $e_{i,j,k}$, and $o_{i,j,k}$ represent regular-time shifts, extra-time shifts, and overtime shifts respectively. If $r_{i,j,k} = 1$ ($e_{i,j,k} = 1$; $o_{i,j,k} = 1$), then nurse k is scheduled to work on shift i in regular-time (extra-time; overtime) mode on the j -th day of the plan.

Nurses' regular hourly wages depend on their specialization, seniority and the time-of-day when their shift falls. In this study, nurses' specialization and seniority levels are not decision variables. Also, nurses can be assigned to different shifts within the flexibility allowed by work rules. Therefore, we used average hourly wage rate, by time of day, to determine the cost of staffing each shift. In our example, a day shift costs less than an evening shift and an evening shift costs less than a night shift.

An extra-time shift occurs when a nurse works more than his/her FTE hours during a 2-week period, but less than 80 h (full-time workload). Shifts that are in excess of 80 h per 2-week period are considered overtime shifts. For example, a 0.6 FTE nurse would be scheduled to work 48 h in each 2-week period. If (s)he works up to 4 extra 8-h shifts, then this would be counted as extra-time work. Overtime shifts would be those that exceed 80-h workload in a 2-week period. Overtime shifts cost more. In particular, for a given shift type (Day, Evening or Night), the overtime cost

was twice as much as the corresponding regular-time cost in our example. Extra-time shifts do not cost more than the corresponding regular-time shifts if a nurse volunteers to work more than his/her usual workload without exceeding full-time workload. In order to encourage this behavior, hospitals often provide an incentive (bonus payment) to part-time nurses to work extra-time shifts.

Nurses cannot be forced to work extra-time or overtime shifts. If they decline either extra- or overtime assignments that are recommended by the solution to our model, then this results in a manpower shortage. Because the extra-time shifts cost less than the cost of using either overtime or temporary staff to cover those shifts and all extra-time assignments may not materialize, the cost estimates generated by our formulation are a lower bound on the actual cost that may be experienced by a unit. Unfortunately, it is not possible to know which extra-time shifts will be picked up by part-time staff at the time of generating the schedule.

To complete the formulation, we also need non-negative integer decision variables $\delta_{i,j}$ that account for the total number of shifts of type i on day j that cannot be scheduled with regular-time, extra-time or overtime shifts. Variables $\delta_{i,j}$ can be interpreted as the number of temporary nurses needed to satisfy the anticipated RN requirements of the shift. We do not place a constraint on the size of $\delta_{i,j}$. This ensures that our model has a feasible solution in all cases. A unit manager may view the sum of uncovered shifts, scheduled extra-time and overtime shifts as the total shortfall.

Finally, we use two additional decision variables— $u_{n,k}$ and $v_{n,k}$ —to keep track of the total number of unused 8- and 12-h shifts for nurse k during weeks $(2n - 1)$ and $2n$, where $n = 1, \dots, \tau/2$. Note that τ , the number of weeks in the review period, is a multiple of pay periods (2-weeks) and therefore always an even number. These decision variables help ensure that the unutilized time for each RN is divisible into an integer number of 8- and/or 12-h shifts.

B.2. The problem parameters

The average RN pay rates for shift type i are denoted by c_{r_i} and c_{o_i} in regular-time and overtime modes, respectively. Because nurses often received a bonus to work extra-time shifts that approximately equaled half the difference between c_{o_i} and c_{r_i} , we used the average cost of a regular-time shift and an overtime shift as the cost rate for an extra-time shift. This is denoted by c_{e_i} . The cost of a manpower shortage was the same as the cost of staffing that shift in overtime. However, NA rules required that all shortages be offered first to regular staff as a possible overtime assignment. Therefore, we set the cost of an uncovered

Table 5

Weekend patterns in a repeating 6-week plan. Each nurse is assigned to one weekend work pattern in which (s)he does not work on the weekends marked with OFF.

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6
Pattern 1	OFF		OFF		OFF	
Pattern 2		OFF		OFF		OFF
Pattern 3		OFF	OFF		OFF	
Pattern 4	OFF		OFF	OFF		OFF
Pattern 5	OFF	OFF		OFF	OFF	

shift, c_{δ_i} , slightly higher than the overtime cost. This prevented the occurrence of uncovered shifts in the optimal schedule when it was possible to schedule them as overtime shifts. When reporting the optimal cost, it would be necessary to adjust this cost such that uncovered shifts would be priced at the same level as overtime shifts. In all problem instances that we solved using the actual data, the optimal schedule did not have any uncovered shifts. Therefore, the last step was not necessary. However, it is important to set c_{δ_i} higher than c_{o_i} for developing an implementable schedule.

The number of RNs required for shift i on the j -th day of the schedule, $d_{i,j}$, is obtained from the forecasting model. The number of paid 8-h shifts (resp. 12-h shifts) during which nurse k is not available to care for patients during weeks $(2n - 1)$ and $2n$ are denoted by $\alpha_{n,k}$ (resp. $\beta_{n,k}$), where $n = 1, 2, \dots, \tau/2$. Paid shifts during which nurses are unavailable may result from education leaves, vacations, and additional training assignments.

We use ϑ_k to denote each nurse's FTE. For example, if a nurse's FTE equals 0.8, (s)he would expect to work 64 (0.8×80) hours in a 2-week period at the regular-time rate. Depending on seniority and the pre-existing arrangements between a nurse and the hospital, a nurse would either work every other weekend or every third weekend, and the shift type could be 8-h shifts or 12-h shifts. Because the smallest multiple of two and three is six, the weekend working pattern for the unit will repeat every 6 weeks. Specifically, each nurse would work one of the five repeating weekend work patterns shown in Table 5. Based on the NA rules, each nurse can have no more than two different regular shift-time assignments. This results in 14 possible shift combinations in our example unit, as shown in Table 6.

B.3. The objective function

The goal of our formulation is to minimize the total cost of meeting the forecast RN requirements via regular-time, extra-time, overtime, and uncovered shifts. That is,

Table 6

Shift types worked by nurses. D = Day shifts, E = Evening shifts, N = Night shifts, D_{12} = 7 AM–7 PM shifts, and N_{12} = 7 PM–7AM shifts.

Pattern #	1	2	3	4	5	6	7
Shift type	D	E	N	D + E	D + N	D_{12}	N_{12}
Pattern #	8	9	10	11	12	13	14
Shift type	$D_{12} + N_{12}$	$D + D_{12}$	$E + D_{12}$	$E + N_{12}$	$N + N_{12}$	$D + E + D_{12}$	$D + N + D_{12} + N_{12}$

our objective function is as follows.

$$\begin{aligned} \min c = & \sum_{i=1}^5 c_{r,i} \sum_{j=1}^{7\tau} \sum_{k=1}^K r_{i,j,k} + \sum_{i=1}^5 c_{e,i} \sum_{j=1}^{7\tau} \sum_{k=1}^K e_{i,j,k} \\ & + \sum_{i=1}^5 c_{o,i} \sum_{j=1}^{7\tau} \sum_{k=1}^K o_{i,j,k} + \sum_{i=1}^5 \sum_{j=1}^{7\tau} c_{\delta_i} \delta_{i,j}. \end{aligned}$$

Although for the hospital, the nurses' regular-time wages are sunk, in this formulation we only count the cost of scheduled shifts because the hospital applies only the cost of scheduled shifts to each unit's budget and unit managers are evaluated on the individual productivity of each unit. This encourages unit managers to make unused regular hours available for possible use by other units, or for scheduling vacation time for some of the nurses.

B.4. The constraints

The above cost function is minimized subject to a set of constraints. In what follows, we have organized these constraints into five major categories.

(a) **Taboo assignments.** For each nurse's non-working shift times, we set the corresponding $r_{i,j,k}$'s and $e_{i,j,k}$'s to zero. For example, if nurse k is a Day-shift only nurse, then $\sum_{i=2}^5 \sum_{j=1}^{7\tau} r_{i,j,k} = 0$. If the nurse takes times off, which includes vacation or education leave, then in addition to

$r_{i,j,k}$'s and $e_{i,j,k}$'s, we also set $o_{i,j,k}$'s to zero for those days.

(b) **Compulsory assignments.** If a nurse must work on a particular shift, the corresponding $r_{i,j,k}$ is set equal to 1. This situation usually occurs as a result of nurses requesting to work on specific shifts, or when nurses must attend a training session at a particular time.

(c) **NA rules.** There are a variety of rules negotiated with the NA that affect how nurses may be assigned to shifts. For example, each RN cannot have more than two different shift start times. A RN's weekend duty pattern can be one of two choices—either every other weekend off or every third weekend off. We describe key NA constraints below.

A RN who works only 8-h shifts cannot be scheduled in the regular or extra-time mode for more than 80 h in a 2-week period.

$$\begin{aligned} 8 \sum_{i=1}^3 \sum_{j=14(n-1)+1}^{14n} (r_{i,j,k} + e_{i,j,k}) \leq 80 \quad \forall k \in \{8\text{-h RNs}\}; \\ n = 1, \dots, \frac{\tau}{2}. \end{aligned}$$

A 12-h only or (8 + 12)-h RN cannot be scheduled for regular/extra time for more than 40 h per week.

$$\begin{aligned} 8 \sum_{i=1}^3 \sum_{j=7(n-1)+1}^{7n} (r_{i,j,k} + e_{i,j,k}) \\ + 12 \sum_{i=4}^5 \sum_{j=7(n-1)+1}^{7n} (r_{i,j,k} + e_{i,j,k}) \leq 40 \\ \forall k \in \{12 \text{ or } (8 + 12)\text{-h RNs}\}; n = 1, \dots, \tau. \end{aligned}$$

RNs may not work more than 120 h in a 2-week period, or work more than one 8- or 12-h shift each day when being paid regular or extra-time wages.

$$\begin{aligned} 8 \sum_{i=1}^3 \sum_{j=14(n-1)+1}^{14n} (r_{i,j,k} + e_{i,j,k} + o_{i,j,k}) \\ + 12 \sum_{i=4}^5 \sum_{j=14(n-1)+1}^{14n} (r_{i,j,k} + e_{i,j,k} + o_{i,j,k}) \leq 120 \\ \forall k = 1, \dots, K; n = 1, \dots, \frac{\tau}{2}. \end{aligned}$$

$$\sum_{i=1}^5 (r_{i,j,k} + e_{i,j,k}) \leq 1 \quad \forall j = 1, \dots, 7\tau; k = 1, \dots, K.$$

The following constraints ensure that a RN will not be assigned to more than three 12-h shifts on consecutive days.

$$\begin{aligned} \sum_{i \in \{4,5\}} \sum_{j=j'}^{j'+3} (r_{i,j,k} + e_{i,j,k} + o_{i,j,k}) \leq 3 \\ \forall j' = 1, \dots, (7\tau - 3); k = 1, \dots, K. \end{aligned}$$

The schedule starts with a Saturday. Because there might be some nurses who worked on the Friday before the first day of the schedule, in order to prevent violation of the “no more than three 12-h shifts on consecutive days” rule, these special cases need to be considered. For the nurses who might be scheduled to work on day 1, we impose the constraints below to ensure that the union rule is satisfied in the transition from the previously determined schedule.

$$\begin{aligned} \sum_{i \in \{4,5\}} \sum_{j=1}^3 (r_{i,j,k} + e_{i,j,k} + o_{i,j,k}) \leq 2 \\ \forall k \in \{\text{RNs who is on duty on day 1}\} \end{aligned}$$

(d) **RN requirements.** The schedule must satisfy RN requirements. The forecasting model gives RN requirements ($d_{1,j}$) by the three standard 8-h shifts ($i = 1, 2, 3$). Because a Day shift (7 AM–3 PM, $i = 1$) can be covered by a 12-h 7 AM–7 PM shift ($i = 4$), the 7 AM–7 PM shifts can also be used to meet the day-shift staffing need forecast $d_{1,j}$. This gives

$$\begin{aligned} \sum_{i \in \{1,4\}} \sum_{k=1}^K (r_{i,j,k} + e_{i,j,k} + o_{i,j,k}) + \delta_{1,j} + \delta_{4,j} \geq d_{1,j} \\ \forall j = 1, \dots, 7\tau. \end{aligned}$$

Similarly, a 7 AM–7 PM shift ($i = 4$) can cover the first 4 h of an Evening shift (3 PM–11 PM, $i = 2$) and a 7 PM–7 AM shift ($i = 5$) can cover the last 4 h of an Evening shift. The constraints below guarantee that the evening-shift

staffing need forecast $d_{2,j}$ is satisfied.

$$\sum_{k=1}^K (r_{2,j,k} + e_{2,j,k} + o_{2,j,k}) + \delta_{2,j} + 0.5 \left\{ \sum_{i \in \{4,5\}} \sum_{k=1}^K (r_{i,j,k} + e_{i,j,k} + o_{i,j,k}) + \sum_{i \in \{4,5\}} \delta_{i,j} \right\} \geq d_{2,j}$$

$$\forall j = 1, \dots, 7\tau$$

A Night shift (11 PM–7 AM, $i = 3$) can be covered by a 12-h 7 PM–7 AM shift ($i = 5$). Therefore, the 7 PM–7 AM shifts can also be used to meet the night-shift staffing need forecast $d_{3,j}$.

$$\sum_{i \in \{3,5\}} \sum_{k=1}^K (r_{i,j,k} + e_{i,j,k} + o_{i,j,k}) + \delta_{3,j} + \delta_{5,j} \geq d_{3,j}$$

$$\forall j = 1, \dots, 7\tau$$

(e) **Other constraints.** These constraints ensure a reasonable schedule. A 7 AM–7 PM shift covers the first 4 h of a Evening shift, whereas a 7 PM–7 AM shift covers the last 4 h of a Evening shift. In order to make sure that a 8-h Evening shift is covered uniformly, if a 7 AM–7 PM shift is scheduled, then there must be a matching 7 PM–7 AM shift and vice versa.

$$\sum_{k=1}^K (r_{4,j,k} + e_{4,j,k} + o_{4,j,k}) = \sum_{k=1}^K (r_{5,j,k} + e_{5,j,k} + o_{5,j,k})$$

$$\forall j = 1, \dots, 7\tau$$

The following constraints prevent each RN from being assigned to two (or more) shifts that have time conflicts.

$$\sum_i (r_{i,j,k} + e_{i,j,k} + o_{i,j,k}) \leq 1$$

$$\forall i \in \{1, 4\}, \{2, 4, 5\}, \{3, 5\}; j = 1, \dots, 7\tau; k = 1, \dots, K$$

Fractional shifts are not desirable because they cannot be utilized in other units. The constraints below exclude the assignments of fractional unused shifts and allow the RNs to work on regular-time shifts up to their FTE regular hours.

$$8 \left(\sum_{i=1}^3 \sum_{j=14(n-1)+1}^{14n} r_{i,j,k} + \alpha_{n,k} + u_{n,k} \right) + 12 \left(\sum_{i=4}^5 \sum_{j=14(n-1)+1}^{14n} r_{i,j,k} + \beta_{n,k} + v_{n,k} \right) = 80\vartheta_k$$

$$\forall k = 1, \dots, K; n = 1, \dots, \frac{\tau}{2}$$

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