



# Testing sequential quantum measurements: how can maximal knowledge be extracted?

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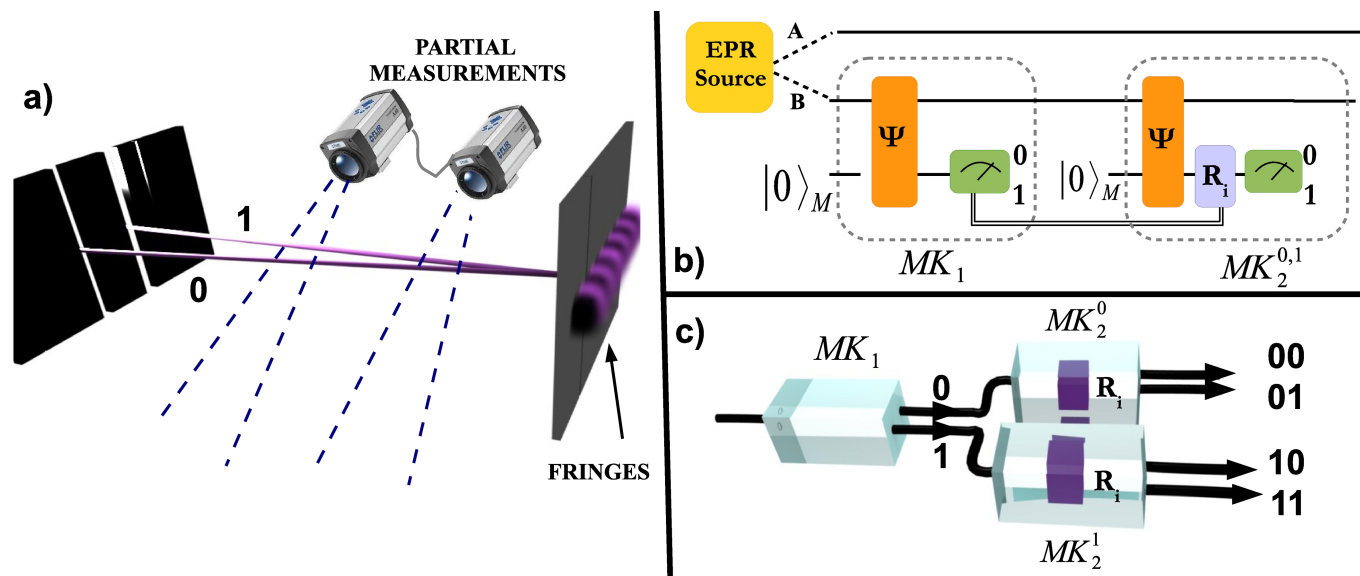
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**The extraction of information from a quantum system unavoidably implies a modification of the measured system itself. In this framework partial measurements can be carried out in order to extract only a portion of the information encoded in a quantum system, at the cost of inducing a limited amount of disturbance. Here we analyze experimentally the dynamics of sequential partial measurements carried out on a quantum system, focusing on the trade-off between the maximal information extractable and the disturbance. In particular we implement two sequential measurements observing that, by exploiting an adaptive strategy, it is possible to find an optimal trade-off between the two quantities.**

The measurement process represents one of the most distinctive aspects of quantum mechanics with respect to classical physics<sup>1,2</sup>. The main result of quantum measurement theory is the unavoidable disturbance of the quantum state by the measuring process, as epitomized by the early Heisenberg x-ray microscope thought experiment<sup>3</sup>. The duality between the information available on an unknown quantum system and the disturbance induced by a measurement process is of utmost relevance when investigating the quantum world<sup>4–6</sup> and lies at the basis of the security of quantum cryptographic protocols<sup>7</sup>. In this framework, a partial measurement approach can be adopted to extract only a limited amount of information from the quantum system at the cost of limited induced disturbance<sup>8–12</sup>. Such partial measurement technique allows to perform consecutive observations (i.e. sequential measurements) on the same quantum system in order to investigate its properties without destroying it<sup>13–21</sup>. In such context a question arises whether it is possible to extract an optimal amount of information from sequential measurements, compared to the degree of disturbance induced on the system. The aim of this paper is to investigate experimentally the trade-off between information gained and disturbance induced by partial sequential measurements on a quantum system<sup>22</sup>. Conceptually our experiment is similar to a double-slit experiment Fig. 1-a where the which-way information is acquired via successive measurements on the same particle. In particular we implement two sequential measurements performed on the same quantum system and observe that the optimal trade-off that characterizes the single measurement can be retrieved by adopting a proper adaptive strategy. Such result, observed for  $N = 2$  sequential measurements, can be extended for any value of  $N$ .

The experimental analysis is carried out through the interaction between a quantum state and an ancillary qubit (the ‘meter’) on which projective measurements have been performed<sup>11</sup>. Such quantum states have been encoded in two different degrees of freedom of a single photon. The performed analysis on the trade-off between knowledge and disturbance has also allowed us to observe a Zeno-like behavior<sup>23</sup> of the measurement dynamics as a function of the strength of the interaction between the system and the meter.

In order to quantify the parameters involved in the experiment here presented, we refer to two complementary figures of merit that lie at the basis of the quantum properties of a system. Let us refer to the schematic representation of the well-known double-slit experiment, shown in Fig. 1-a. According to quantum mechanics theory, the presence of a photon passing through the two slits is manifested by fringes with a defined visibility. Performing a partial measurement corresponds to an observer who tries to distinguish if the photon passes through path 0 or 1. Such which-path information quantifies the knowledge  $K$  extracted from the system, and affects the visibility of the fringes<sup>24</sup>. Indeed, once it is perfectly known where the photon passes, no fringes are observed. According to these observations, in this experiment we adopt the following figures of merit:



**Figure 1** | (a) Scheme of the two-slits experiment with the interference fringes (on the right), whose visibility depends on the information extracted on the path followed by the single photon. (b) Circuitual scheme of the sequential measurements scenario. The qubit B is coupled with the ancillary qubit  $|0\rangle_M$ . The measurement result affects the next sequential one by adapting the measurement basis through a rotation  $R_1$  in the next measurement process ( $MK_2$ ). (c) Scheme for the implementation of two sequential measurements strategy on single photon states. An individual photon passing through the whole measurement apparatus is detected only in one of the four output ports, which indicates which combination of results for  $MK_1$  and  $MK_2$  is obtained.

- i) The *knowledge*  $K \in [0,1]$  corresponds to the capability of correctly discriminating the quantum states belonging to the computational basis  $\{|0\rangle, |1\rangle\}$ , analogously to the capability of discriminating between path 0 or 1 in the double-slit experiment. Mathematically  $K$  can be defined as  $K = \left| \sum_i p(i,i) - \sum_{i \neq j} p(i,j) \right|$  where  $p(i, j)$  is the probability that the state is identified as  $|i\rangle$  by the measurement, when the input is  $|j\rangle$ .  $K = 1$  corresponds to maximal knowledge and is achieved by projective measurements  $\{|0\rangle\langle 0|, |1\rangle\langle 1|\}$ , while  $K < 1$  can be achieved by a measurement with reduced interaction strength to which we refer as partial measurement. Partial measurements are interactions between two systems, a meter and a target, that leave the meter in one of a set of non-orthogonal states, as opposed to strong measurements that leave the meter in one of a set of perfectly distinguishable orthogonal states. Indeed, the strength of a measurement can be related to the distinguishability between elements of the set of output states of the meter. When  $K \rightarrow 0$ , we perform a weak measurement<sup>25,26</sup>.
- ii) In order to quantify the overall disturbance related to the measurement process we need to define a figure of merit analogous to the fringes visibility in the double slit experiment. Here we choose to consider as quantum system a qubit belonging to bipartite entangled state: see Fig. 1-b. Indeed in this case any irreversible disturbance effect on a single subsystem  $B$  could be revealed by estimating the degree of correlation in the whole quantum state  $\rho_{AB}$ . The adoption of an entangled state allows to exploit quantum state tomography to estimate the disturbance of the channel via concurrence  $C \in [0,1]$ <sup>27,28</sup>. Such parameter is complementary to the knowledge, and it gets lower ( $C < 1$ ) as the information extracted from the system increases.

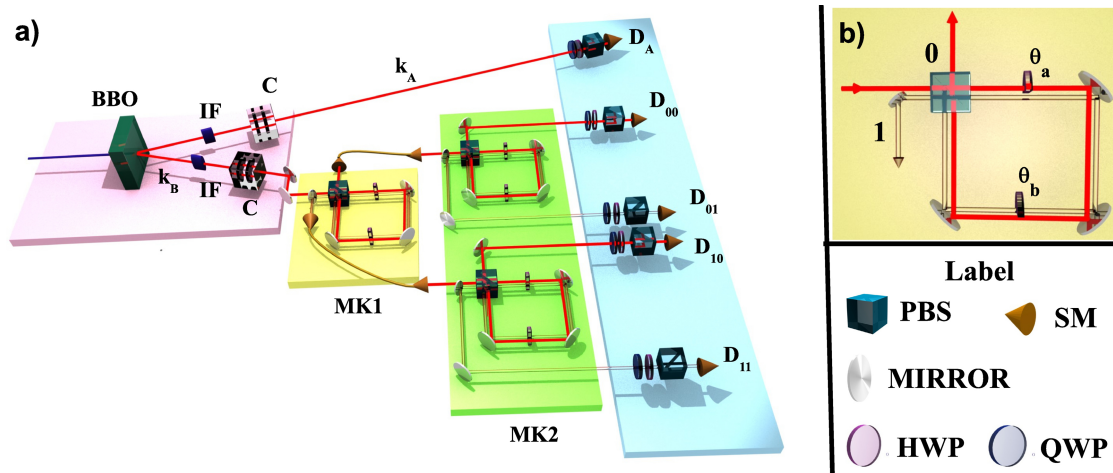
## Results

As first step, we analyze the trade-off in a single measurement strategy, represented by the quantum circuit in Fig. 1-b: box  $MK_1$ . We consider the singlet state  $|\psi^-\rangle_{AB} = \frac{1}{\sqrt{2}}(|10\rangle_{AB} - |01\rangle_{AB})$  shared by parties A and B. The measurement strategy gains information on qubit B by entangling it with the meter  $M$  through an interaction of

variable strength, parametrized by  $\psi$ , and then performing a projective measurement on  $M$ . Specifically, the qubit B interacts with  $M$ , initialized as  $|0\rangle_M$ , through the following unitary transformation:  $\hat{U}(\psi)|i\rangle_B|0\rangle_M = |i\rangle_B|\alpha_i\rangle_M$  where  $i = 0, 1$ , and  $|\alpha_i\rangle$  is a pure state. Both  $|\alpha_0\rangle$  and  $|\alpha_1\rangle$  can be expressed in terms of  $\psi$ :  $|\alpha_0\rangle_M = \cos \psi|0\rangle_M + \sin \psi|1\rangle_M$ ;  $|\alpha_1\rangle_M = \cos \psi|0\rangle_M - \sin \psi|1\rangle_M$ . In general the disturbance due to the measurement process can induce different quantum channels on the system, however, as predicted by theory<sup>22</sup>, the unbiased nondestructive measurement described above disturbs the state only by introducing a phase-damping channel. Hence the concurrence of the final state  $\rho_{AB}$  depends on the strength of the coupling with the meter, and reads  $C = |\langle \alpha_0 | \alpha_1 \rangle| = |\cos 2\psi|$ , equal to zero when  $|\alpha_0\rangle_M$  and  $|\alpha_1\rangle_M$  are orthogonal. The optimal value of extractable knowledge can be expressed in terms of  $\psi$ , and reads  $K = |\sin 2\psi|$ . This expression holds when we perform a projective measurement on the meter in the diagonal basis  $|\pm\rangle_M = \frac{|0\rangle_M \pm |1\rangle_M}{\sqrt{2}}$ , which is found out to be the one that maximizes the parameter  $K$ . Thus the optimal trade-off between knowledge and residual entanglement is represented by the curve  $C = \sqrt{1 - K^2}$ <sup>28</sup>, so that when all the information available is extracted ( $K = 1$ ), the initial entanglement is completely lost ( $C = 0$ ).

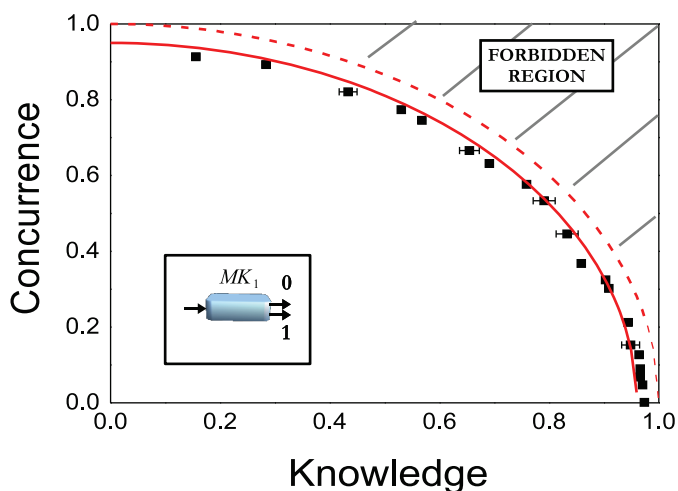
The former relations have been experimentally implemented encoding the initial singlet state in the polarization of two photons A and B,  $\frac{1}{\sqrt{2}}(|HV\rangle_{AB} - |VH\rangle_{AB})$ , where (H, V) are linear horizontal and vertical polarization, respectively. The experimental setup is reported in Fig. 2 and we provide all details in the Methods section. The meter qubit  $M$  has been encoded in a different degree of freedom of the photon B, the linear momentum. In this way the extracted information corresponds to the correlations between the polarization of the photon A and the path of the photon B. When all the information is extracted, and thus the strength of the coupling is maximum, the correlation is perfect.

In order to control the interaction between the qubit B and the meter, we exploit a Sagnac interferometer with a polarizing beam splitter (PBS), that interfaces the polarization to the transmitted and reflected spatial modes<sup>29–33</sup>; Fig. 2-b. We note that such interfacing between the two degrees of freedom could be achieved also in a



**Figure 2** | (a) Experimental setup adopted for the sequential measurement strategy for  $N = 2$  measurement processes. The entangled state has been generated via spontaneous parametric down conversion. (b) Scheme of the measurement kit  $MK_i$ ; the kit is based on a Sagnac interferometer which allows to separately manipulate the horizontal and vertical polarizations adopting two half-waveplates (HWP) oriented at angles  $\theta_a$  and  $\theta_b$ , related by a shift of  $\pi/4$  for the optimal configuration.

Mach-Zehnder<sup>34</sup>, however the Sagnac interferometer provides a higher stability and thus is more suitable for experimental purposes. The interferometer, here and after denoted as the *measurement kit MK*, has been aligned in a non-degenerate configuration, where the two internal modes are spatially separated and propagate clockwise (mode  $a$ ) and anticlockwise (mode  $b$ ). Such configuration allows an independent manipulation of the polarization on  $a$  and  $b$  by two half-waveplates rotated at angles  $\theta_a$  and  $\theta_b$ , respectively, providing a controlled modification of the coupling strength  $\psi$  associated to the information extraction. Indeed  $\psi$  is fixed by the waveplates in the interferometer by the relation  $\psi = \frac{\pi}{4} - 2\theta_b$ . The latter relations have been expressed in terms of the physical angle  $\theta_b$  as in order to perform a projective measurement on the meter qubit in the basis  $|\pm\rangle_M$ , corresponding to the output modes of the interferometer, the two waveplates of the Sagnac have a fixed shift relation, equal to  $\theta_b = \theta_a + \pi/4$ . The projective measurement on the meter qubit corresponds to the selection of one of the output modes 0 and 1 of the PBS. According to definition in (i), in the single measurement



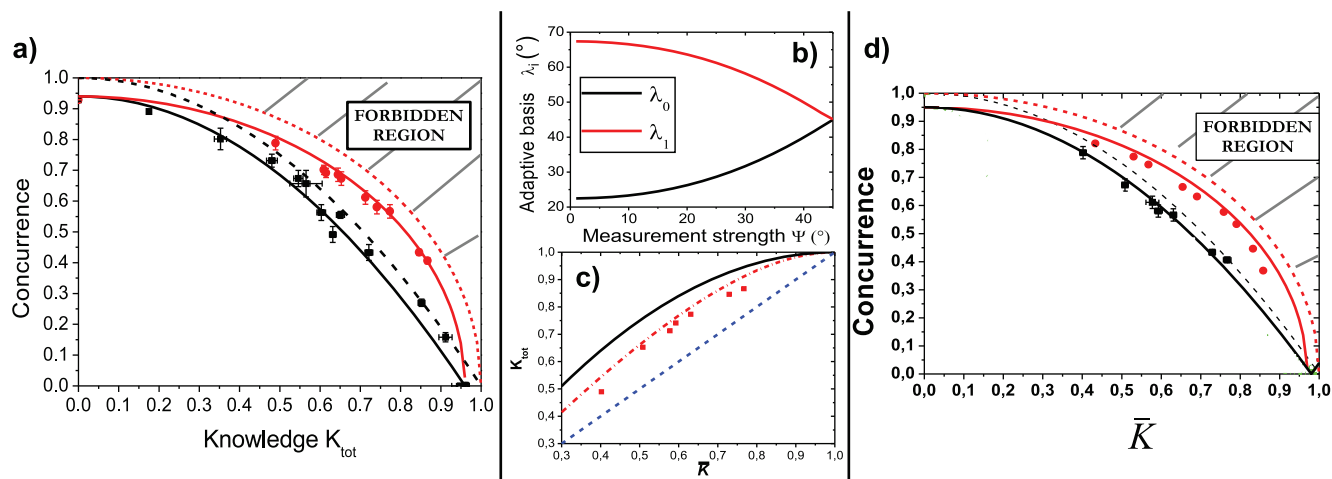
**Figure 3** | Theoretical expectations (red dashed line) compared to experimental data (black squares) for concurrence as function of the knowledge for the single measurement case. Continuous line reports theoretical expectations rescaled to experimental imperfections as the parameters of the PBS ( $t_H = r_V = 0.992$ ,  $r_H = t_V = 0.008$ ) and the concurrence of the initial state ( $C_{in} = (0.95 \pm 0.01)$ ).

case the knowledge  $K$  has been measured as  $K = \frac{p(0,H) + p(1,V) - p(1,H) - p(0,V)}{2}$ , where  $p(i, j)$  is the probability that an input photon with polarization  $j$  emerges on the mode corresponding to the outcome  $i$ . On the other hand the concurrence of the state  $\rho_{AB}$  after the measurement process acting on qubit  $B$  has been estimated from the density matrix reconstructed via quantum state tomography<sup>27</sup>. In order to estimate probabilities  $p(i, j)$  and the elements of the density matrix, we have recorded the coincidence counts between the single photon detector  $D_A$  and detectors  $D_0$  and  $D_1$  on the output modes of the first measurement kit, measuring around 600 events per second. Experimental results are reported in Fig. 3, and compared to theoretical expectations evaluated taking into account imperfections due to the PBS and to the source of entangled states. We observe that the single measurement process saturates the optimal trade-off between information extracted and disturbance, allowed by quantum mechanics<sup>22</sup>.

Let us now address the following questions: how can knowledge be extracted from sequential measurements? Is it still possible to achieve the same optimal trade-off in such scenario? We consider two sequential measurement processes of equal strength. We note that if the first observer does not extract all the information on the state, the second one can still extract some information from the system. Such measurement process is represented by the whole scheme in Fig. 1-b,c. Each measurement process introduces a specific amount of decoherence, reflected by a lowering of the concurrence after each step. The degree of entanglement of the state after the two sequential measurements gives an indication of the total disturbance induced in the whole process. The analysis on sequential measurements has been carried out considering two separate cases: the first one concerning independent measurements, the second adaptive ones. In both cases in order to experimentally estimate the knowledge and the concurrence of the state, we recorded coincidence counts between detectors  $[D_A, D_{00}]$ ,  $[D_A, D_{01}]$ ,  $[D_A, D_{10}]$ , and  $[D_A, D_{11}]$ ; see Fig. 2-a.

Firstly we consider two independent sequential measurements, that is, the second projection on the state is performed independently of the outcome of the first one. In this case the concurrence shows a dependence from the whole amount of knowledge extracted  $K_{tot}$  as  $C^{ind} = 1 - (K_{tot}^{ind})^2$ , thus the maximum amount of information extractable from the system does not achieve the optimal trade-off with the decoherence induced. In Fig. 4-a we report the experimental behavior of the concurrence for this measurement strategy (black squares), where the total knowledge  $K_{tot}$  has been evaluated as in the single





**Figure 4** | (a) Concurrence  $C$  as function of the knowledge for  $N = 2$  sequential measurements adopting an independent strategy (black squares) and the adaptive one (red circles). Black and red lines represent theoretical expectations (dashed lines for the ideal case, continuous ones rescaled by experimental imperfections) for the two approaches. (b) Numerical determination of adaptive basis depending on the measurement carried out in the first kit, expressed by the parameter  $\psi$ . (c) Experimental knowledge after  $N = 2$  sequential adaptive measurements (red squares) compared to theoretical predictions for classical (black line), adaptive extraction (dashed-dot red line), and after  $N = 1$  measurement (dashed blue line). (d) Experimental and theoretical behavior of concurrence as function of  $\bar{K}$ . Black squares and line refer to experimental  $N = 2$  adaptive measurements and theoretical expectations, respectively. Analogously red dots and line refers to the experimental and theoretical results for the single quantum measurements.

measurement procedure, combining outcomes 00 with 01 and 10 with 11. This strategy is related to the scenario in which a series of independent observers estimate an unknown state of a quantum system by performing consecutive measurements over the very same system<sup>35</sup>.

As second benchmark we have considered whether it is possible to achieve the optimal trade-off exploiting sequential measurements. Thus we have analyzed the case of two sequential adaptive measurements, where a feed-forward between the two measurements allows to achieve the optimal trade-off  $C^{opt} = \sqrt{1 - K_{tot}^2}$ , analogously to what observed for the single measurement<sup>22</sup>. We note that such strategy is based on the one proposed for discrimination of multiple copies of quantum states<sup>24</sup>. As schematically shown in Fig. 1-b, the results from the first measurement kit determine an adaptation, i.e. a rotation in the meter basis, for the subsequent measurement process. Therefore, classical communication is required between the sequential measurements and they cannot be treated as independent anymore. Depending on the outcome 0 or 1 of  $MK_1$ , two different basis of analysis, generically indicated as  $\{|\beta_0\rangle, |\beta_0^\perp\rangle\}$  and  $\{|\beta_1\rangle, |\beta_1^\perp\rangle\}$ , are applied on the meter qubit in  $MK_2^0$  and  $MK_2^1$ , where  $|\beta_i\rangle = \cos \lambda_i |0\rangle + \sin \lambda_i |1\rangle$ . Both parameters  $\{\lambda_0, \lambda_1\}$  are determined in order to maximize the extracted knowledge and depend on the decoherence induced by the first measurement process. In Fig. 4-b we report the numerical determination of parameters  $\{\lambda_0, \lambda_1\}$  depending on the measurement strength  $\psi$  of the first kit.

The adaptive strategy has been implemented experimentally by rotating the waveplates in the Sagnac of  $MK_2$ , thus modifying the basis of the meter qubit depending on the outcome of the measurement carried out by  $MK_1$ . We note that an intrinsic feed-forward takes place in the adaptation process since two different rotations of the basis are performed in the second measurement process, depending on which output arm of the first interferometer the photon gets out, as shown in Fig. 1-b and Fig. 2-a. Different values of parameters  $\{\lambda_0, \lambda_1\}$  leads to different values of the physical angles  $\{\theta_a, \theta_b\}$  and hence the basis for the meter projection is not in general the diagonal one, so that the relation  $\theta_b = \theta_a + \pi/4$  does not hold anymore.

In Fig. 4-a we report the theoretical behavior (red line) of concurrence as function of the global knowledge  $K_{tot}$  and the experimental results (red dots) for the adaptive measurement strategy, where the value of  $K_{tot}$  has been estimated with the same relation adopted for

the single measurement process, where outcomes  $i$  refer only to the second kit. We find a good agreement with theoretical predictions rescaled to experimental imperfections, thus achieving the predicted optimal trade-off. We conclude that even performing several weak transitions from quantum to classical world through the information extraction, is possible to keep the same degree of disturbance of a single transition.

Let us now analyze the previous results from the point of view of information extraction within the dynamics of sequential measurements, observing how knowledge accumulates after  $N = 2$  adaptive measurements as a function of  $\bar{K}$ , the knowledge that would be extracted from each measurement if it were the only one performed: see Fig. 4-c, red squares and dashed-dot line. We stress that we consider two sequential measurements of equal strength. As a general observation, we expect that, as the knowledge extracted from the first measurement gets closer to 1, a lowering of the extractable knowledge from the second process takes place. Here we experimentally verified that, according to theoretical predictions in<sup>22</sup>, the total knowledge extracted after two equal adaptive sequential measurements does not additively accumulate as  $K_{tot} = 2\bar{K}$ . Indeed the adaptation between the two processes introduces additional quadratic terms in the accumulation law, so that the total knowledge is equal to  $K_{tot} = \sqrt{2\bar{K}^2 - \bar{K}^4}$ . Such behavior can be generalized to  $N$  sequential adaptive measurements<sup>22</sup>.

Finally, Fig. 4-d experimentally demonstrates that the adaptive concurrence  $C_{tot}^{adapt}$  after two identical measurements is a concave function of  $\bar{K}$ , as in the single measurement case. This is at variance with the scenario in which the information is extracted with a classical strategy and where the concurrence is a convex function of  $\bar{K}$ , leading to an exponential decay. The qualitative behavior of the function curvature that characterizes strategies allowing an optimal trade-off between knowledge and decoherence, is responsible for a Zeno-like effect<sup>22</sup>. Indeed the quantum Zeno-effect refers to a situation in which a quantum system, if observed frequently by projective measurements, varies slower than the exponential decay law. Here we verified that for the case  $N = 2$  the concurrence scales with  $\bar{K}$  as  $C_{tot}^{adapt}(\bar{K}) \approx 1 - N \frac{\bar{K}^2}{2}$ . Moreover the amount of entanglement is weakly affected when a small amount of information is acquired in each single measurement.



## Discussion

In summary, we have reported the experimental analysis of the trade-off between acquired knowledge from a quantum state and the detrimental effect on the system itself, by performing sequential measurements. We have experimentally investigated how knowledge can be accumulated from two sequential measurements, verifying that an optimal trade-off can be achieved when an adaptive strategy is adopted. Finally we have observed that in sequential adaptive measurements, the knowledge accumulation rule can lead to a Zeno-like behavior of the entanglement dynamic of the system here considered. Future steps might be the study of the extension for multiple measurements and the application to continuous measurements for controlled dynamics and others quantum information protocols<sup>36,37</sup>. Here we considered sequential measurements of compatible observables, future work will focus on different scenarios of complementary measurements<sup>38</sup> and non-compatible observables.

## Methods

**Entangled state source.** The initial singlet state encoded in the polarization of single photons has been implemented through spontaneous parametric down conversion in a 1.5 mm thick  $\beta$ -borate of barium crystal (BBO) cut for type-II phase matching, pumped by 700 mW of the second harmonic of a Ti:Sa mode-locked laser beam with repetition rate equal to 76 MHz. The photons are generated with wavelength  $\lambda = 795$  nm and spectral bandwidth  $\Delta\lambda = 3$  nm, as determined by two interference filters (IF). The spatial and temporal walk-off is compensated by inserting a half-waveplate and a 0.75 mm thick BBO crystal (C) on each output mode  $k_A$  and  $k_B$ <sup>39</sup>, shown in Fig. 2-a. The detected photon-pair generation rate of the source is 8 kHz.

The photon on mode  $k_A$  is sent to a standard polarization analysis setup, composed by a half-waveplate (HWP), a quarter waveplate (QWP) and a polarizing beam splitter (PBS), and then coupled to a single mode fiber (SM) connected to a single photon counter module  $D_A$ . The photon on mode  $k_B$  is sent through a single mode fiber to the single or sequential measurement kit  $MK_i$ .

**Sequential adaptive quantum measurements.** The adaptive measurement can be described by the following operators, where the subscripts 1,2 refer to projective measurements on the first and the second meter qubit, respectively:

$$\begin{aligned} \Pi_{00} &= |+\rangle\langle+| \otimes |\beta_0\rangle\langle\beta_0| & ; & \quad \Pi_{01} = |+\rangle\langle+| \otimes |\beta_0^\perp\rangle\langle\beta_0^\perp| \\ \Pi_{10} &= |-\rangle\langle-| \otimes |\beta_1\rangle\langle\beta_1| & ; & \quad \Pi_{11} = |-\rangle\langle-| \otimes |\beta_1^\perp\rangle\langle\beta_1^\perp| \end{aligned} \quad (1)$$

The states  $|\beta_{0,1}\rangle$  are defined as follows:

$$|\beta_i\rangle = \cos \lambda_i |0\rangle + \sin \lambda_i |1\rangle$$

where  $|\pm\rangle$  is the projection basis adjusted to gain maximal knowledge from first meter qubit. The elements  $\Pi_{ij}$  satisfy the condition  $\sum_{ij} \Pi_{ij} = 1$ . The overall prediction on the computational basis states  $|0\rangle$  and  $|1\rangle$  of qubit B is performed only respectively to the last index of  $\Pi_{ij}$ . The optimal parameters  $\lambda_{0,1}$  are chosen to maximize the overall knowledge  $K_{tot}$  on the computation basis. By exploiting the adaptive measurement, maximal knowledge is reached with optimal collective measurement on both the meters. The minimal disturbance effect from this unbiased nondemolition measurement is pure phase damping and this optimal synthesis of the sequential measurements leads to a single optimal measurement with minimal phase decoherence<sup>22</sup>.

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## Author contributions

R.F., S.F., E.N., F.S. Conceived the theory and designed the experiments. V.D., P.D., S.F., E.N., F.S. performed the experiments and analyzed the data. All authors discussed the results and wrote the manuscript.

## Additional information

**Competing financial interests:** The authors declare no competing financial interests.

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