



# Possible observation of quantum spin-nematic phase in a frustrated magnet

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**Water freezes into ice in winter and evaporates into vapor in summer. Scientifically, the transformations between solid, liquid, and gas are called phase transitions and can be classified through the changes in symmetry which occur in each case. A fourth phase of matter was discovered late in the 19th century: the liquid crystal nematic, in which rod- or disk-shaped molecules align like the atoms in a solid, while continuing to flow like a liquid. Here we report thermodynamic evidence of a quantum analog of the classical nematic phase, the quantum spin nematic (SN). In an SN, the spins of a quantum magnet select a common axis, like a nematic liquid crystal, while escaping conventional magnetic order. Our state-of-the-art thermal measurements in high pulsed magnetic fields up to 33 T on the copper mineral volborthite with spin 1/2 on a frustrated lattice provide thermodynamic evidence for SN order, half a century after the theoretical proposal [Blume M, Hsieh YY (1969) *J Appl Phys* 40:1249; Andreev AF, Grishchuk IA (1984) *J Exp Theor Phys* 97:467–475].**

magnetism | high magnetic field | hidden order | spin nematic | calorimetry

The historical discovery of liquid crystals demonstrated the existence of a new state of matter beyond solid, liquid, and gas. Once a scientific curiosity, liquid crystals now form the basis for a wide range of technologies, including the majority of electronic displays. The identification of a spin nematic (SN) would be a significant achievement as it represents a new quantum phase of matter, with qualitatively different properties from any known phase in a magnet. Moreover, SNs also have considerable potential for application, notably in magnetic refrigeration and quantum computation. However, searching for SNs in experiments is very challenging because it is a form of “hidden order” (1), a term used to describe thermodynamic phase transitions where the changes in symmetry are unknown or very hard to detect.

There is good theoretical evidence that SN order should occur in a wide range of systems, including thin films of <sup>3</sup>He; cold atoms in an optical lattice; and a wide variety of quantum magnets with competing, or “frustrated,” interactions. The simplest candidate is the spin-1 system in which spin quadrupole moments of spin-1 form a quadrupolar order (2), although its observation is still elusive (3). Among other widely discussed examples are quantum spin chains (4–8) and square-lattice Heisenberg models (9–12) where ferromagnetic first-neighbor interactions ( $J_1$ ) compete with antiferromagnetic second-neighbor interactions ( $J_2$ ), in applied magnetic field. In these systems, the magnon excitations of the high-field saturated state form a bound pair on the bond of  $J_1$  (effective spin-1 state), which then undergo Bose–Einstein condensation (BEC), much like bound pairs of electrons in a superconductor (7, 13) (Fig. 1). Since each bound pair of magnons carries a quadrupole moment (Fig. 2B), the resulting BEC is an SN, characterized by an ordered arrangement of quadrupole moments on first-neighbor bonds, which mimic the rod- or disk-like molecules in the liquid crystal nematic (12) (Fig. 2C). The observation of an SN in experiment would be very significant since, while single-particle BECs have been observed in magnets

(14), the SN would represent the first example of a pair condensate. However, the quadrupole moments in SNs do not break time-reversal symmetry (2), and so they cannot be detected using the common probes of magnetic order (12), rendering the SN a hidden-order phase.

SN order has been discussed in a number of materials, with candidates including LiCuVO<sub>4</sub> and NaCuMoO<sub>4</sub>(OH), as realizations of a frustrated ferromagnetic spin chain (15–17), and Ba<sub>2</sub>CdVO(PO<sub>4</sub>)<sub>2</sub> as an example of a square-lattice frustrated ferromagnet (18). To date, however, conclusive experimental evidence for an SN remains elusive. At present, the best-documented system is LiCuVO<sub>4</sub>, where a recent low-temperature NMR study identified a narrow range of magnetic field, approaching saturation, for which the local magnetization remains homogeneous and uniform, while its value is field dependent (16). This behavior is consistent with a SN state, but evidence for a corresponding thermodynamic phase transition is lacking.

Volborthite [Cu<sub>3</sub>V<sub>2</sub>O<sub>7</sub>(OH)<sub>2</sub>·2H<sub>2</sub>O], a mineral named after the Russian paleontologist Alexander von Volborth, is one of the most exotic quantum magnets in nature. First suggested as an example of a kagome antiferromagnet (Fig. 2A), volborthite is now considered to be a square-lattice frustrated ferromagnet (19), in which groups of three Cu atoms (trimers) combine to form spin-1/2 moments on a distorted square lattice, coupled by competing ferromagnetic ( $J_1$ ) and antiferromagnetic ( $J_2$  and  $J_2'$ ) exchange interactions (cf. Fig. 2B). Theoretical estimates place volborthite in a parameter range for which two-magnon bound states have a tendency to condense (19), leading to SN order (9). On the experimental side, measurements of magnetization, shown in Fig.

## Significance

**Spin-nematic state in frustrated ferromagnet is one of the exotic states of matter at high magnetic field that could arise as the Bose–Einstein condensation of the two-magnon bound state. Its experimental detection in a real material has long been hindered by the fact that the order parameter of the spin-nematic state, spin quadrupole moment, is hard to detect with conventional magnetic probes available in high magnetic field. Here we design a calorimeter for the pulsed magnetic field studies and perform state-of-the-art measurements of heat capacity and magnetocaloric effect up to 33 T. These results provide strong prima facie evidence for a spin-nematic state in the copper-based mineral volborthite.**

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fluctuation owing to MCE. We have developed a state-of-the-art technique to generate a modified pulsed magnetic field with its top truncated to become almost flat with a high field stability of  $\pm 0.01$  T, which allows us to measure heat capacity precisely at the flat-top field down to 0.8 K.

We first discuss the phase boundaries found using MCE measurements and then use corresponding result for heat capacity to characterize each of the phases found. Fig. 3A shows the evolution of curves of constant entropy,  $T(H)_S$ , taken from MCE measurements performed under quasi-adiabatic conditions (A-MCE), starting from the state found for  $H = 0$  at a given temperature. In all cases the adiabat  $T(H)_S$  initially decreases with increasing field, before passing through a broad minimum at  $H \sim 25$  T and then increasing, behavior consistent with a phase transition at low temperatures (22). Adiabats become tightly bunched at low temperatures, approaching the onset of the 1/3-magnetization plateau P at  $H_P \sim 27.5$  T, suggesting a dense set of low-energy excitations (22). A notable change occurs when the temperature falls below  $\sim 1.7$  K; the broad minimum in  $T(H)_S$  acquires small dips, indicative of phase transitions, at  $H_{s1} \sim 22.5$  T and  $H_{s2} \sim 25.5$  T and  $H_P \sim 27.5$  T. Each of these three critical fields is singled out by a corresponding anomaly in measurements of MCE under approximately isothermal conditions (I-MCE) of  $T = 1.25$ , 1.0, and 0.75 K (Fig. 3B) and the related changes in entropy (Fig. 3C). Together with measurements of heat

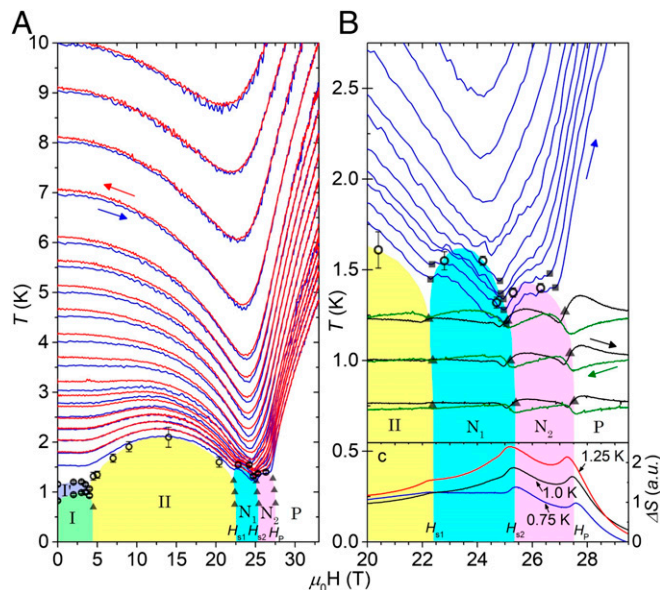
capacity, discussed below, these results clearly delineate two domes in the  $(H, T)$  plane, each corresponding to a different low-temperature phase. We label these phases  $N_1$  and  $N_2$  in Fig. 3;  $N_1$  corresponds to the phase N found in earlier NMR experiments (20, 21), and  $N_2$  fills the gap between N and P.

We now turn to measurements of heat capacity, which provide more insight into the nature of the phases  $N_1$  and  $N_2$ . Fig. 4 shows the temperature dependence of heat capacity for magnetic field ranging from 0 to 30.2 T. In the absence of magnetic field,  $C(T)$  exhibits small peaks at 0.8 and 1.2 K, as reported previously (24) (Fig. 4A). The onset of SDW order (II) at intermediate values of magnetic field is revealed in broad shoulders in  $C(T)$ ; these occur at  $T \sim 2$  K for  $H = 14$  T and  $T \sim 1.5$  K for  $H = 20.4$  T (Fig. 4A), in agreement with phase boundaries found in NMR (24). Very different behavior is observed for  $H > H_{s1}$ ; the transition into the phase  $N_1$  is marked by a substantial  $\lambda$ -shaped anomaly, occurring at  $T \sim 1.54$  K for  $H = 22.8$  T, in agreement with the phase boundary identified in MCE (Fig. 3). This feature becomes still more pronounced with increasing field (Fig. 4A). Meanwhile, the onset of the phase  $N_2$  is accompanied by an extremely sharp peak, consistent with a logarithmic divergence in  $C(T)$  (Fig. 4B). The crossover into the saturated state, for  $H > H_P$ , is not accompanied by any anomaly in  $C(T)$  (Fig. 4B). These results provide unequivocal evidence that  $N_1$  and  $N_2$  are bulk, thermodynamic phases; we find corresponding changes in entropy of  $\Delta S = 0.12R \ln 2$  entering phase  $N_1$  at  $H = 24.2$  T and  $\Delta S = 0.09R \ln 2$  entering phase  $N_2$  at  $H = 26.3$  T.

Spontaneous symmetry breaking can produce a gapless Goldstone excitation that gives power-law behavior in the low-temperature heat capacity. For most ordered states including SDW and SN states, one would expect linearly dispersing Goldstone modes (8, 12, 25). These usually contribute to the low-temperature heat capacity as  $C(T) \propto T^d$ , where  $d$  is the spatial dimension of the system. As shown in the log-log plots of Fig. 4C and D, the low-temperature heat capacities for phases I and II tend to be proportional to  $T^3$ , while for phases  $N_1$  and  $N_2$ ,  $C(T) \propto T^2$ . This implies that the character of magnetic excitations changes from 3D to 2D, although it is preferable to increase the temperature range of the fit (SI Appendix). Probably, for phases I and II, 3D antiferromagnetic magnons or phasons occur (8) owing to non-negligible interlayer couplings, while the dimensionality of magnetic excitations for phases  $N_1$  and  $N_2$  is qualitatively reduced in the measurement temperature range. This type of dimensional reduction is not surprising in a quasi-2D magnet with SN order, since the opening of a gap to transverse spin excitations will lead to a suppression of interplane exchange.

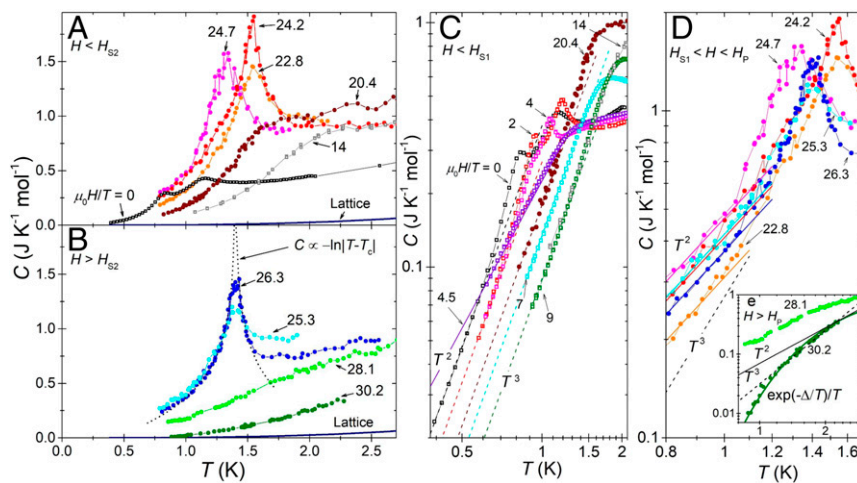
In the state P, for  $H > H_P$ , there is no phase transition above 0.8 K, and the low-temperature heat capacity shows exponentially activated behavior (Fig. 4E): the 30.2 T data are well described by the equation,  $C = \exp[-\Delta/k_B T]/T$  (26), with an activation energy  $\Delta = 6.7$  K. We interpret this as a gap to two-magnon excitations, as illustrated in Fig. 1B (19). Assuming that this gap opens at  $H_P = 27.5$  T and modeling it as  $\Delta = g\mu_B(H - H_P)$ , we arrive at an effective Landé  $g$  factor of  $g = 3.7$ . It should be noted that the critical field deduced from the phase boundary,  $H_P^{PB}$ , lies in the range 27.5–28.0 T, which yields  $g = 3.7$ –4.5 (SI Appendix). These values are nearly twice as large as the single-electron  $g$  factor found in electron spin resonance measurements at lower values of field [ $g_c = 2.04$  below 12 T (27)], consistent with a two-magnon excitation. The  $g$  factor estimated from NMR measurements on volborthite also shows an enhancement to  $g \sim 4.6$ –5.9 (24), providing further evidence for two-magnon bound states.

To recap, it has been suggested (19) that a narrow range of  $H$  just below  $H_P$ , bound pairs of magnons undergo a low-temperature BEC, leading to an antiferroquadrupolar SN order, as illustrated in Fig. 2C. At present the case for this SN order rests on a combination of theoretical analysis of the crystal structure and associated magnetic model (19) and NMR experiments which,



**Fig. 3.** Magnetocaloric effect and magnetic phase diagram of volborthite. (A) Experimental measurements of the MCE in volborthite, showing the evolution of temperature with changing magnetic field under (quasi-)adiabatic conditions (A-MCE). Results are shown for both rising (blue curves) and falling (red curves) magnetic field and reflect contours of constant entropy. The onset of the hidden-order phases,  $N_1$  and  $N_2$ , is associated with tight bunching curves and a dip in the entropy contour, corresponding to a change in sign of the MCE. Black triangles show phase boundaries at low temperature, extracted from complimentary measurements of MCE under isothermal conditions (I-MCE). The phase boundaries extracted from measurements of heat capacity,  $C(T)$ , are shown with open circles. (B) Detail of phases  $N_1$  and  $N_2$ , showing results for A-MCE measured in rising field (blue curves), I-MCE in rising field (black curves), and I-MCE in falling field (green curves). The black squares show the evolution of a corresponding feature in the A-MCE. Two distinct domes can be resolved at finite temperature, bounded by the critical fields  $H_{s1} = 22.5$  T,  $H_{s2} = 25.5$  T, and  $H_P = 27.5$  T. (C) Changes of entropy extracted from measurements of I-MCE at low temperature (Methods). The field boundaries of the SN phase,  $N_2$ , and presumed supersolid phase,  $N_1$ , are sharply distinguished by local anomalies in entropy.





**Fig. 4.** Heat capacity and thermodynamic evidence for low-energy excitations in volborthite. (A) Experimental results for heat capacity  $C(T)$ , for magnetic fields spanning the low-field phase I, SDW phase II, and phase  $N_1$ . The onset of  $N_1$  is distinguished by a sharp anomaly in  $C(T)$  for  $T \sim 1.5$  K. (B) Heat capacity within the hidden-order phase  $N_2$  and magnetization plateau P. The onset of  $N_2$  is distinguished by a sharp peak in  $C(T)$  for  $T \sim 1.4$  K. The form of this peak is consistent with the logarithmic divergence expected for an Ising phase transition in 2D. (C) Heat capacity for phases I and II, plotted on a log–log scale. Both phases exhibit a  $T^3$  scaling of heat capacity at low temperatures, consistent with a linearly dispersing excitation in 3D. (D) Heat capacity of phases  $N_1$  and  $N_2$ , plotted on a log–log scale. Both phases exhibit a  $T^2$  scaling of heat capacity at low temperatures, consistent with a linearly dispersing excitation in 2D. This form of dimensional reduction is expected for weakly coupled 2D SNs. (E) Heat capacity of magnetization plateau P, showing activated behavior at low temperatures, consistent with the opening of a gap  $\Delta = 6.7$  K at 30.2 K.

for the phase  $N_2$ , are incompatible with any conventional form of magnetic order (20, 21). The thermodynamic measurements reported in this manuscript supply the missing pieces of this puzzle.

The first crucial piece of evidence comes from the field dependence of the excitation gap with the 1/3-magnetization plateau, P. From measurements of heat capacity, we can identify the lowest-lying excitation within P as a two-magnon bound state, with gap which tends to zero approaching the transition into the phase  $N_2$  (Fig. 4E). This result explicitly links the mechanism identified in theory—the condensation of two-magnon bound states (9, 19)—to the phase transition observed in experiment, at low temperatures, as a function of magnetic field.

The second crucial result is clear evidence for a thermodynamic phase transition into the phase  $N_2$ , at finite temperature and fixed magnetic field (Fig. 4B). This phase transition, which separates the high-temperature paramagnet from a low-temperature, broken-symmetry state, appears to be continuous, with a vanishingly small critical exponent, as found for, e.g., 3D-XY or 2D-Ising universality classes. [We note that an Ising transition is expected for an SN in the presence of Dzyaloshinskii–Moriya interactions (8).]

The third distinct piece of evidence comes from measurements of heat capacity at low temperatures for fixed magnetic field, within the phase  $N_2$ . The power-law behavior of heat capacity at low temperature provides clear evidence for the emergence of a Goldstone mode, associated with the breaking of spin rotation symmetry within phase  $N_2$  (Fig. 4D). Under other circumstances, such a Goldstone mode could be explained by conventional magnetic order. However, this is ruled out by published NMR results (20, 24), which fail to find the internal field associated with conventional magnetic order within the phase  $N_2$ . Taken together, these results make a strong case for SN order, which possesses a linearly dispersing Goldstone mode but does not break time reversal symmetry and so produces no internal field.

Our thermodynamic measurements also clearly identify a second new phase  $N_1$ , which is distinct in its properties from  $N_2$ . The nature of this phase remains an open question, and at this point it is only possible to speculate as to its origin. The phase  $N_1$  shares a number of features with  $N_2$ , including a sharp singularity in heat capacity at the ordering temperature and a  $T^2$  behavior at low  $T$ . However, the NMR spectra of phase  $N_1$  contain multiple peaks,

which is different from the single-peak feature in  $N_2$ , indicating the presence of an internal field in  $N_1$  (20, 24). One possible explanation for this would be a heterogeneity in the crystal (24), but this seems hard to reconcile with the sharpness of the phase transition or the single peak found in phase  $N_2$ . An intriguing alternative is the microscopic coexistence of a dipolar SDW and a quadrupolar SN order. It is already known that SDW and SN correlations coexist in the high-field states of frustrated ferromagnetic spin chains (5, 6). Any ordered phase in which both were present would break translational symmetry, through  $S^z$  (SDW), making it a solid, and spin-rotation symmetry, through the transverse quadrupole moments  $Q^{xy}$  and  $Q^{x^2-y^2}$  (SN), making it a superfluid (9, 19). In this sense the phase  $N_1$  could be a supersolid, a state long sought in  $^4\text{He}$  (28) and also widely discussed in the context of frustrated magnets. The data available for volborthite seem consistent with this interpretation of the phase  $N_1$ , but further experimental and theoretical work will be needed to establish its validity.

In summary, in this article we have presented a detailed study of the thermodynamic properties of the frustrated magnetic insulator volborthite, in high magnetic field. These results are summarized in the phase diagram, Fig. 3. We find evidence for the existence of two magnetic phases, labeled  $N_1$  and  $N_2$ , at fields approaching the known 1/3-magnetization plateau, P. The thermodynamic properties of the phase  $N_2$ , and its previously reported NMR line shape (21), appear to be consistent with the published prediction of SN order (19).

We note that while this article was under review, two studies have also reported the possibility of SN order in the square-lattice frustrated ferromagnet  $\text{BaCdVO}(\text{PO}_4)_2$  (29, 30).

## Methods

**Materials.** Single crystals of volborthite  $\text{Cu}_3\text{V}_2\text{O}_7(\text{OH})_2 \cdot 2\text{H}_2\text{O}$  were grown by the hydrothermal method (31).  $\text{CuO}$  (99.99%; 6.0 g) and  $\text{V}_2\text{O}_5$  (99.99%; 3.0 g) powders were mixed in the stoichiometric ratio and were put into a Teflon container together with 1.1 wt % nitric acid (400 mL). Then, the container was placed in a stainless steel vessel and heated to 443 K for 30 d and furnace-cooled to room temperature. Single crystals of volborthite thus obtained are transparent yellow–green, arrowhead shaped, and typically  $1 \times 0.5 \times 0.05$  mm in size. They crystallize in the monoclinic  $C2/c$  structure at room temperature. Upon cooling, two structural transitions take place to the  $I2/m$  structure at 290 K and to the  $P2_1/a$  structure at 155 K (31). Thus, the

crystal structure of volborthite at low temperatures concerned in the present study is the  $P2_1/a$  structure (supplemental material in ref. 20). In this structure, there are two kinds of crystallographically distinguished kagome layers instead of one kind in the  $I2/m$  structure. However, the two kagome layers have an identical arrangement of spin-carrying Cu  $3d_{x^2-y^2}$  orbitals and nearly equal bond lengths and angles, suggesting that the magnetic interactions there may be almost the same. In fact, the DFT + U calculations suggest nearly equal magnetic interactions for the two layers (19).

**MCE.** To determine the  $H$ - $T$  phase diagram, we performed MCE experiments in a pulsed magnetic field up to 33 T in two different conditions: one is an adiabatic condition (A-MCE), and the other is a nearly isothermal condition (I-MCE).

The field derivative of the isentropic curve  $T(H)_S$  obtained by the A-MCE measurement is given by

$$\left(\frac{\partial T}{\partial H}\right)_S = -\frac{T}{C} \left(\frac{\partial M}{\partial T}\right)_H,$$

where  $S$ ,  $M$ , and  $C$  are entropy, magnetization, and heat capacity, respectively (22). Thus, the  $T(H)_S$  curve is sensitive to any magnetic phase transition. This has been demonstrated for  $\text{Sr}_3\text{Ru}_2\text{O}_7$  (32), where multiple kinks in the  $T(H)_S$  curves are observed and successfully attributed to a series of field-induced phase transitions.

To decide a phase boundary at lower temperatures, we carried out I-MCE measurements in the nearly isothermal condition. Compared with A-MCE, I-MCE is less sensitive to phase boundaries but can be performed at lower temperatures. Moreover, it yields the field dependence of entropy at an almost constant temperature (32). The I-MCE data are converted to entropy change  $\Delta S(H)$  by the equation  $\Delta S = \int \kappa(T - T_b)/T dt$ , where  $\kappa$  is the thermal conductance between the sample and the surrounding thermal bath and  $T_b$  is the temperature of the thermal bath. Assuming a constant value of  $\kappa$ , the field dependence of  $\Delta S$  is estimated in the arbitrary unit and plotted in Fig. 3C.

**Heat Capacity Measurements at Flat-Top Pulsed Magnetic Fields.** Heat capacity measurements under high magnetic fields above 20 T are not possible in commercial equipment. Either static fields using a big facility or pulsed fields with short time duration are necessary to generate larger magnetic fields. We have employed the latter, but measuring heat capacity in a quickly varying field is challenging. We recently succeeded in developing an experimental

technique that allows us to generate a flat-top pulsed magnetic field up to 60 T with the high field stability of  $\pm 0.005$ – $0.01$  T.

Our strategy for generating a stabilized pulsed magnetic field is depicted in *SI Appendix, Fig. S1*. To generate a time-dependent field of a round arch profile with the top over 40 T, a large current pulse ( $\sim 10$  kA) from the 210-MJ flywheel motor generator is supplied to the main magnet. Slightly before the magnetic field reaches the maximum value, an additional small current (several hundred amperes) from the 50-V lead-acid battery is supplied to a minimagnet set inside the main magnet. When the magnetic field reaches a target value, the field-programmable gate array device starts a PID control of magnetic field for every 2.5  $\mu\text{s}$ . The current flow through the minimagnet (i.e., the magnetic field generated by the minimagnet) is regulated by the resistance of an insulated gate bipolar transistor as commonly used nowadays for motor control in electric cars, trams, and trains. During the PID feedback control, the total field strength was kept at a constant value within  $\pm 0.01$  T (*SI Appendix, Fig. S2*), and the temperature fluctuation was suppressed at least within  $\pm 0.01$  K. Within this temperature fluctuation, we could observe a clear temperature increase ( $\Delta T$ ) by the application of a heat ( $\Delta Q$ ) and obtained heat capacity ( $C = \Delta Q/\Delta T$ ) as seen in *SI Appendix, Figs. S3 and S4*. By measuring a Ge polycrystalline sample, we estimated the sensitivity of the present setup in high magnetic field of 43.5 T (*SI Appendix, Fig. S5*).

A series of heat capacity measurements were performed by the quasi-adiabatic method at each flat-top field with time duration of  $\sim 0.2$  s. A temperature increase  $\Delta T$  from a starting temperature  $T_0$  after a heat pulse with  $\Delta Q$  was measured, which yields  $C$  as  $\Delta Q/\Delta T$ . In one shot, more than 10 points of data with varying  $T_0$  were measured, and three to eight pulses were used to get one set of temperature-dependent  $C(T)$  data shown in Fig. 4 at each field. We have applied this state-of-the-art technique to study the SN phase of volborthite and obtained a complete  $H$ - $T$  phase diagram. Detailed explanations on the heat capacity measurements can be found in *SI Appendix*.

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