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Optimal control analysis of *Taenia saginata* bovine cysticercosis and human taeniasis

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ABSTRACT

Bovine cysticercosis and human taeniasis are neglected food-borne diseases that pose challenge to food safety, human health and livelihood of rural livestock farmers. In this paper, we have formulated and analyzed a deterministic model for transmission dynamics and control of taeniasis and cysticercosis in humans and cattle respectively. The analysis shows that both the disease free equilibrium (DFE) and endemic equilibrium (EE) exist. To study the dynamics of the diseases, we derived the basic reproduction number R_0 by next generation matrix method which shows whether the diseases die or persist in humans and cattle. The diseases clear if $R_0 < 1$ and persist when $R_0 > 1$. The normalized forward sensitivity index is used to derive sensitive indices of model parameters. Sensitivity analysis results indicate that human's and cattle's recruitment rates, infection rate of cattle from contaminated environment, probability of humans to acquire taeniasis due to consumption of infected meat, defecation rate of humans with taeniasis and the consumption rate of raw or undercooked infected meat are the most positive sensitive parameters whereas the natural death rates for humans, cattle, *Taenia saginata* eggs and the proportion of unconsumed infected meat are the most negative sensitive parameters in diseases' transmission. These results suggest that control measures such as improving meat cooking, meat inspection and treatment of infected humans will be effective for controlling taeniasis and cysticercosis in humans and cattle respectively. The optimal control theory is applied by considering three time dependent controls which are improved meat cooking, vaccination of cattle, and treatment of humans with taeniasis when they are implemented in combination. The Pontryagin's maximum principle is adopted to find the necessary conditions for existence of the optimal controls. The Runge Kutta order four forward-backward sweep method is implemented in Matlab to solve the optimal control problem. The results indicate that a strategy which focuses on improving meat cooking and treatment of humans with taeniasis is the optimal strategy for diseases' control.

1. Introduction

Bovine cysticercosis is an infection of cattle caused by the larval stage of *Taenia saginata* tapeworm (Kumar and Tadesse, 2011).

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Human taeniasis is the dwelling of adult tapeworms in human's small intestine due to consumption of raw or undercooked meat of cattle infected by *Taenia saginata* tapeworm larval cysts (Symeonidou et al., 2018). When humans with taeniasis defecate in open spaces, they release tapeworm eggs in their feaces which contaminate the environment (Dermauw et al., 2018; Symeonidou et al., 2018). Cattle acquire cysticercosis through consumption of *Taenia saginata* eggs that were shed in human feaces during grazing on contaminated pastures, or when they ingest contaminated fodder or water (Symeonidou et al., 2018). Human taeniasis can be controlled through cooking meat of an infected cattle to a sufficient internal temperature (56–65 ˚C) to ensure that all cysts are killed (Grindle, 1978; Lesh and Brady, 2019).

Usually human taeniasis and bovine cysticercosis are common in rural areas where people keep cattle under free range system and humans defecate in the fields (Alemneh and Adem, 2017; Flisser et al., 2006; Trevisan et al., 2017). In such areas there is poor sanitation, low standard of slaughter facilities, inadequate or no meat inspection and the treatment of these diseases is not readily available (Mkupasi et al., 2011). Human taeniasis and bovine cysticercosis are globally distributed, affecting both developed and developing countries. In developed countries, the diseases' prevalence rate is very low and are reemerging in diseases free areas due to migration of infected humans and cattle exchange (Yimer and Gebrmedehan, 2019). The diseases' prevalence rates are higher in developing countries of Latin America, Africa and Asia (Braae et al., 2018). In Africa, *Taenia saginata* parasite is prevalent in almost all regions with higher prevalence rates in eastern and southern Africa including Ethiopia, Sudan, Kenya, South Africa, Tanzania, Botswana, Zambia and Zimbabwe (Dermauw et al., 2018). In such countries people keep cattle for their livelihoods, serving as a source of food, income, draft power and manure (Dermauw et al., 2018; Swanepoel et al., 2010). *Taenia saginata* parasite causes few symptoms in humans such as mild abdominal pain, anal pruritus and distress (Dermauw et al., 2018). In cattle, the infection is sub-clinical but may cause huge economic losses due to carcass condemnation or treatment upon detection of tapeworm larval cysts and related insurance costs (Dermauw et al., 2018). Economic losses due to *Taenia saginata* human taeniasis and bovine cysticercosis are measured in terms of grade of an infected cattle, potential market price of cattle, disease prevalence, medical costs for infected humans and the treatment cost for detained carcasses (Alemneh and Adem, 2017; Grindle, 1978). Human taeniasis can be treated through the use of prescribed medication of albendazole, praziquantel, niclosamide and tribendimidine (Okello and Thomas, 2017). Disease control measures in cattle involve the use of TSA–9 and TSA–18 vaccines (Kumar and Tadesse, 2011; Lightowlers et al., 1996), and treatment with drugs such as oxfendazole, fenbendazole, flubendazole, nitazoxanide and praziquantel (WHO, 2005; Winskill et al., 2017).

The theory of optimal control has become an important mathematical tool in making decisions that involve complex biological situations (Lenhart and Workman, 2007). It helps in making decision on intervention strategy that can be implemented to curtail the spread of infectious diseases (Hugo et al., 2017; Okosun et al., 2011). For instance, making decision on percentage of the population to be vaccinated over time so as to minimize the number of infected individuals and the cost of implementing the vaccination strategy. Usually, the underlying dynamical system is described by state variables with suitable optimal control time-dependent functions that are incorporated in an epidemic model comprising of ordinary differential equations, and thus affecting the dynamics of the model system (Lenhart and Workman, 2007). The objective is to adjust these optimal controls so as to optimize the given objective function. Over the past two decades, a number of mathematical models have been formulated and analyzed to determine the optimal control strategies for various infectious diseases (Asamoah et al., 2020; Khan et al., 2020; Nyerere et al., 2020; Okosun et al., 2016; Osman et al., 2020; Tilahun et al., 2017). In particular, there are only a few deterministic models that have been formulated and analyzed to study the transmission dynamics and control of taeniasis and cysticercosis in humans and pigs due to *Taenia solium* tapeworm parasite.

Gonzalez et al. (2002) formulated and analyzed a dynamic-stochastic model to assess the control of porcine cysticercosis. A number of control strategies with various combinations and duration of human and porcine treatment were implemented. Results showed that the treatment of both infected humans and pigs were more effective for diseases' control. Kyvsgaard et al. (2007) formulated and analyzed an SIR deterministic and stochastic version of Reed-Frost model for the dynamics and control of *Taenia solium* tapeworm parasite. Three interventions that were implemented in model simulation are cooking habits, meat inspection and the use of latrines; rapid detection, human treatment and pig vaccination; and treatment of either pig or human populations. The results showed that mass-treatment was effective in diseases' control. Braae et al. (2016) formulated and analyzed a cystiSim agent-based model to study the dynamics and control of *Taenia solium* parasite. Pig vaccination, pig treatment and human treatment were administered singularly or in combination to assess their impact in diseases' control. The results indicated that all controls targeting on pig population were effective provided that the coverage and efficacy was sufficiently high. Winskill et al. (2017) formulated and analyzed a deterministic model to study the impact of pig vaccination, pig treatment, improved animal husbandry, improved sanitation, improved meat inspection and treatment of humans with taeniasis on the control of *Taenia solium* taeniasis and cysticercosis in humans and pigs. The results showed that the treatment of infected humans or pigs was more effective in diseases' control when used singly, with annual treatment of pigs and humans. José et al. (2018) formulated and analyzed a deterministic model with some stochastic elements to study the dynamics of taeniasis and cysticercosis in humans and pigs basing on the life cycle of *Taenia solium* tapeworm through chemotherapy. The results indicated that chemotherapeutic interventions which focus on infected pigs or humans with taeniasis are effective in reducing the mean intensity of human taeniasis, porcine cysticercosis and human cysticercosis. Sánchez-Torres et al. (2019) developed and analyzed a deterministic model to assess the dynamics and control of taeniasis and cysticercosis in humans and pigs based on the life cycle of *Taenia solium* tapeworm. The developed model was an extension of the SI model in José et al. (2018). Pig vaccination and treatment of humans with taeniasis were considered in model simulation. The results showed that pig vaccination and human treatment has influence on the transmission dynamics among the vaccinated pigs and other hosts as well. Mwasunda et al. (2021) formulated and analyzed a deterministic model for transmission dynamics of cysticercosis and taeniasis in humans, pigs and cattle. Sensitivity analysis results indicated that recruitment rate of humans, probability of humans' infection with taeniasis and defecation rate by humans with taeniasis are the most positive sensitive parameters to diseases' transmission whereas the human

natural death rate is the most negative sensitive parameter. These results suggest that control measures such as treatment of humans with taeniasis, meat inspection and indoor keeping of cattle and pigs are essential for diseases' control. However, none of these studies have applied the theory of optimal control to study the dynamics and control of taeniasis and cysticercosis in humans and cattle. In this paper, we formulate the optimal control problem for determining the optimal control strategy for *Taenia saginata* bovine cysticercosis and human taeniasis.

The rest of this article is arranged as follows: In Section 2, we derive the deterministic model for dynamics and control of *Taenia saginata* taeniasis and cysticercosis in humans and cattle. The optimal control model is formulated and analyzed in Section 3 while Section 4 deals with numerical simulations of the optimal control model for assessing the optimal strategy for diseases' control. In Section 5 we present discussion on some previously established models in comparison with results of this study whereas conclusion and recommendations are provided in Section 6. Limitations of the study, strengths and future work are presented in Section 7.

2. Model formulation

A mathematical model for transmission dynamics and control of bovine cysticercosis and human taeniasis is formulated by considering the basic model for taeniasis and cysticercosis dynamics in humans, pigs and cattle in Mwasunda et al. (2021). The model divides humans into susceptible S_H and humans with taeniasis I_{HT}. Cattle are divided into susceptible, vaccinated, infected and recovered cattle denoted by S_C, V_C, I_C and R_C respectively. The classes B_I and E_T are meat of infected cattle and number of *Taenia saginata* eggs in the environment respectively.

Susceptibe humans are considered to be recruited through birth at a rate Λ_H and they move to infected class at a rate α_b due to consumption of raw or inadequately cooked meat of infected cattle. The parameter β_T is the probability of humans to acquire taeniasis due to consumption of meat of an infected cattle. Humans with taeniasis recover from cysticercosis at a rate χ and all humans suffer natural death a rate μ_h. The parameter ν is the rate at which humans with taeniasis release *Taenia saginata* eggs in the environment and μ_e is the natural death rate of *Taenia saginata* eggs. Susceptible cattle are assumed to be recruited through birth at a rate Λ_C and they are vaccinated at a rate ψ_s. Susceptible cattle move to infected class at a rate γ_b due to consumption of *Taenia saginata* eggs from the contaminated environment. The parameters ρ_b and λ are the vaccine efficacy for protecting vaccinated cattle from acquiring infection and the recovery rate of infected cattle from cysticercosis respectively. Susceptible, vaccinated, infected and recovered cattle are slaughtered for consumption at the rates σ_s, σ_v, η and σ_r respectively. All cattle are assumed to die naturally at a rate μ_b. The vaccine in cattle wane at the rate ψ_v and immunity wane in recovered cattle at the rate π. The parameter ε is the proportion of meat of infected cattle which is not consumed by susceptible humans.

To formulate a mathematical model for the dynamics and control of bovine cysticercosis and human taeniasis, we consider the free range farming system for cattle, and we do not consider migration. We assume that the number of *Taenia saginata* eggs consumed by cattle have negligible effect on the total number of eggs in the environment and that both infected humans and cattle cannot recover from infections without treatment. The contact rate of cattle with *Taenia saginata* eggs in the environment is assumed to be density dependent. We also assume that the cattle vaccines are not 100% effective and that they wane after sometime. We further assume that both cattle and humans do not suffer disease induced mortality, they become carriers for their life. Similarly, we assume that the rate at which susceptible humans consume raw or undercooked meat of infected cattle depends on the amount of meat of an infected cattle which is available. The compartmental model flow diagram for the dynamics of *Taenia saginata* human taeniasis and bovine cysticercosis with control measures is presented in Fig. 1. The state variables and model parameters are summarized in Tables 1 and 2

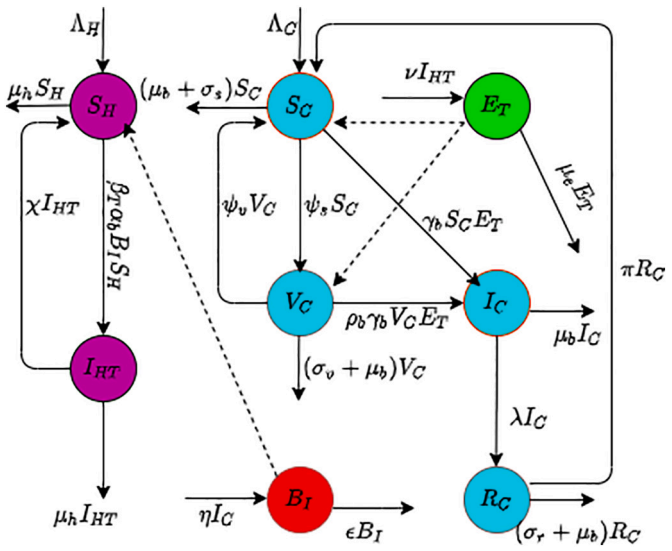

Fig. 1. The model flow diagram.

respectively.

Most parameter values are assumed since there is no study on mathematical modeling of *Taenia saginata* bovine cysticercosis and human taeniasis that has been carried out. Additionally, in most developing countries, there is no efforts to collect data related to neglected diseases which are common in rural areas. Also, in such areas there is inadequate or no meat inspection and treatment is not readily available (Mkupasi et al., 2011).

The model for the transmission dynamics of *Taenia saginata* bovine cysticercosis and human taeniasis with control measures is described by the system of differential equations:

$$\begin{aligned}
\frac{dS_H}{dt} &= \Lambda_H + \chi I_{HT} - \beta_T \alpha_b B_I S_H - \mu_h S_H,\\
\frac{dI_{HT}}{dt} &= \beta_T \alpha_b B_I S_H - (\mu_h + \chi) I_{HT},\\
\frac{dS_C}{dt} &= \Lambda_C + \psi_v V_C + \pi R_C - \gamma_b S_C E_T - (\sigma_s + \psi_s + \mu_b) S_C,\\
\frac{dV_C}{dt} &= \psi_s S_C - \rho_b \gamma_b (1-\tau) V_C E_T - (\sigma_v + \psi_v + \mu_b) V_C,\\
\frac{dI_C}{dt} &= \gamma_b S_C E_T + \rho_b \gamma_b V_C E_T - (\eta + \lambda + \mu_b) I_C,\\
\frac{dR_C}{dt} &= \lambda I_C - (\pi + \sigma_r + \mu_b) R_C,\\
\frac{dB_I}{dt} &= \eta I_C - (\varepsilon + \alpha_b) B_I,\\
\frac{dE_T}{dt} &= \nu I_{HT} - \mu_e E_T,
\end{aligned} \tag{1}$$

with initial conditions:

$$S_H(0) > 0; \quad I_{HT}(0) \geq 0; \quad S_C(0) > 0; \quad V_C(0) > 0; \quad I_C(0) \geq 0; \quad R_C(0) \geq 0; \quad B_I(0) \geq 0 \quad \text{and} \quad E_T(0) \geq 0.$$

2.1. The basic model

When there are no interventions, the model system (1) reduces to the basic model given by:

$$\begin{aligned}
\frac{dS_H}{dt} &= \Lambda_H - \beta_T \alpha_b B_I S_H - \mu_h S_H,\\
\frac{dI_{HT}}{dt} &= \beta_T \alpha_b B_I S_H - \mu_h I_{HT},\\
\frac{dS_C}{dt} &= \Lambda_C - \gamma_b S_C E_T - (\sigma_s + \mu_b) S_C,\\
\frac{dI_C}{dt} &= \gamma_b S_C E_T - (\eta + \mu_b) I_C,\\
\frac{dB_I}{dt} &= \eta I_C - (\varepsilon + \alpha_b) B_I,\\
\frac{dE_T}{dt} &= \nu I_{HT} - \mu_e E_T,
\end{aligned} \tag{2}$$

with initial conditions:

$$S_H(0) > 0; \quad I_{HT}(0) \geq 0; \quad S_C(0) > 0; \quad I_C(0) \geq 0; \quad B_I(0) \geq 0 \quad \text{and} \quad E_T(0) \geq 0.$$

2.1.1. Disease free equilibrium and basic reproduction number R_0

When there are no infections in humans and cattle, we obtain the disease free equilibrium E^0 given by:

Table 1
Description of the state variables.

Variable	Description	Variable	Description
S_H	Susceptible humans	I_C	Infected cattle
I_{HT}	Infected humans	R_C	Recovered cattle
S_C	Susceptible cattle	B_I	Meat of infected cattle
V_C	Vaccinated cattle	E_V	*Taenia saginata* eggs

Table 2
Parameters' description and their values (unit: yr^{-1}).

Parameter	Description	Value	Source
Λ_H	Per capita recruitment rate of human population	2247	Wu et al. (2013)
μ_h	Per capita natural death rate of humans	0.0141	Wang et al. (2013)
α_b	Rate of eating raw or undercooked meat of infected cattle	0.023	Assumed
β_T	Probability of human infection with *Taenia saginata* tapeworm	0.093	Assumed
ν	Defecation rate by humans with taeniasis	0.150	Assumed
χ	Recovery rate of humans from taeniasis	0.225	Assumed
Λ_C	Per capita recruitment rate of cattle	750	Assumed
γ_b	*Taenia saginata* eggs to susceptible cattle transmission coefficient	0.00625	Assumed
ρ_b	Vaccine efficacy for protecting vaccinated cattle against infection	0.1968	Assumed
η	Slaughter rate of infected cattle	0.235	Assumed
μ_b	Per capita natural death rate of cattle	0.33	Wang et al. (2013)
ε	Proportion of unconsumed meat of infected cattle	0.225	Assumed
σ_s	Harvesting rate of susceptible cattle	0.213	Assumed
σ_v	Harvesting rate of vaccinated cattle	0.183	Assumed
σ_r	Harvesting rate of recovered cattle	0.153	Assumed
λ	Recovery rate of infected cattle from cysticercosis	0.125	Assumed
π	Cattle's immunity waning rate	0.213	Assumed
ψ_s	Vaccination rate of susceptible cattle	0.115	Assumed
ψ_v	Vaccine waning rate in cattle	0.248	Assumed
κ	Proportion of adequately cooked meat of infected cattle	0.350	Assumed
μ_e	Per capita death rate of *Taenia saginata* eggs	10.42	Assumed

$$E^0(S_H, I_{\mathrm{HT}}, S_C, I_C, B_I, E_T) = \left(\frac{\Lambda_H}{\mu_h}, 0, \frac{\Lambda_C}{\sigma_s + \mu_b}, 0, 0, 0\right). \tag{3}$$

The basic reproduction number R_0 is the expected number of secondary infections that may arise as a result of introducing one infected individual in a fully susceptible population (Diekmann et al., 1990). When $R_0 < 1$, the disease clears whereas when $R_0 > 1$, the disease persists within the population. In computing R_0, we adopt the next generation matrix method as used by Van den Driessche and Watmough (Van den Driessche and Watmough, 2002). Let $\mathcal{F}_i$ be the new infections in compartment i and $\mathcal{V}_i^+$ and $\mathcal{V}_i^-$ be the transfer terms in and out of the compartment i respectively, then infected classes in model system (2) can be written as:

$$\frac{\mathrm{dx}_i}{\mathrm{dt}} = \mathcal{F}_i(x) - \mathcal{V}_i^+(x) - \mathcal{V}_i^-(x).$$

Using the next generation matrix method, we define $\mathcal{F}_i$ and $\mathcal{V}_i$ by:

$$\mathcal{F}_i = \begin{pmatrix} \beta_T \alpha_b B_I S_H \\ \gamma_b S_C E_T \\ 0 \\ 0 \end{pmatrix}, \qquad \mathcal{V}_i = \begin{pmatrix} \mu_h I_{\mathrm{HT}} \\ (\eta + \mu_b) I_C \\ -\eta I_C + (\varepsilon + \alpha_b) B_I \\ -\nu I_{\mathrm{HT}} + \mu_e E_T \end{pmatrix}. \tag{4}$$

The Jacobian matrices $\boldsymbol{F}$ and V at the disease free equilibrium E^0 are given by:

$$F = \frac{\partial \mathcal{F}_i}{\partial x_j}\left(E^0\right), \qquad V = \frac{\partial \mathcal{V}_i}{\partial x_j}\left(E^0\right). \tag{5}$$

Thus the basic reproduction number R_0 is given by:

$$R_0 = \rho\left(\mathrm{FV}^{-1}\right). \tag{6}$$

From Eq. (5), F and V are:

$$F = \begin{pmatrix} 0 & 0 & \dfrac{\beta_T \alpha_b \Lambda_H}{\mu_h} & 0 \\ 0 & 0 & 0 & \dfrac{\gamma_b \Lambda_C}{(\sigma_s + \mu_b)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad V = \begin{pmatrix} \mu_h & 0 & 0 & 0 \\ 0 & \eta + \mu_b & 0 & 0 \\ 0 & -\eta & (\varepsilon + \alpha_b) & 0 \\ -\nu & 0 & 0 & \mu_e \end{pmatrix}.$$

Using definition (6), the basic reproduction number R_0 is:

$$R_0 = \sqrt{\frac{\beta_T \nu \alpha_b \gamma_b \eta \Lambda_H \Lambda_C}{\mu_h^2 \mu_e (\eta + \mu_b)(\alpha_b + \varepsilon)(\mu_b + \sigma_s)}}. \tag{7}$$

To give the biological meaning of the basic reproduction number, we rewrite R_0 in the form:

$$R_0 = \sqrt{\beta_T \gamma_b \frac{\Lambda_H}{\mu_h} \frac{\nu}{\mu_e} \frac{1}{\mu_h} \frac{\Lambda_C}{(\mu_b + \sigma_s)} \frac{\eta}{(\eta + \mu_b)} \frac{\alpha_b}{(\alpha_b + \varepsilon)}}. \tag{8}$$

The terms in (8) can be interpreted as follows: ν/μ_e is the density of *Taenia saginata* eggs released by humans with taeniasis, $1/\mu_h$ is the human life expectancy and β_T is the probability of humans to acquire taeniasis due to consumption of raw or insufficiently cooked meat of cattle which is infected with tapeworm larval cysts. The terms Λ_H/μ_h and $\Lambda_C/(\mu_b + \sigma_s)$ are initial populations of susceptible humans and cattle respectively; $1/(\mu_b + \sigma_s)$ is the average time that cattle spend in susceptible class; $1/(\eta + \mu_b)$ is the infectious period of infected cattle; $1/(\alpha_b + \varepsilon)$ is the average infectious period for infected meat of cattle whereas $\eta/(\eta + \mu_b)$ is the proportion of infected cattle that are slaughtered for consumption. The term $\alpha_b/(\alpha_b + \varepsilon)$ is the proportion of raw or insufficiently cooked infected meat of cattle which is eaten by susceptible humans whereas γ_b is the rate at which *Taenia saginata* eggs are consumed by cattle. Generally, the basic reproduction number R_0 will increase in proportion to slaughter rate of infected cattle, the rate of eating raw or undercooked infected meat of cattle, probability of human to acquire taeniasis, human and cattle recruitment rates, defecation rate by humans with taeniasis and the rate at which *Taenia saginata* eggs are consumed by cattle whereas R_0 decreases when the slaughter rate of susceptible cattle, rate of unconsumed infected meat of cattle and the mortality rates of human, cattle and *Taenia saginata* eggs are increased.

2.1.2. Sensitivity analysis

The normalized forward sensitivity index method is used to determine the sensitivity indices of model parameters (Chitnis et al., 2008). If δ is a parameter in R_0 then its sensitivity index is given by:

$$\Gamma_{\delta}^{R_0} = \frac{\partial R_0}{\partial \delta} \times \frac{\delta}{R_0}. \tag{9}$$

Using Eq. (9) and parameter values in Table 2, the sensivity indices for each parameter are presented in Table 3. The positive sign of the sensitivity index shows that an increase in parameter value while keeping other parameters constant causes an increase in the magnitude of expected secondary infections while the negative sign indicates that an increase in parameter value leads to a decrease in expected secondary infections.

The most positive sensitive parameters are human's recruitment rate Λ_H, cattle's recruitment rate Λ_C, infection rate of cattle from contaminated environment γ_b, probability of humans to acquire taeniasis due to meat consumption β_T, defecation rate of humans with taeniasis ν and the consumption rate of raw or undercooked meat α_b are the most positive sensitive parameters whereas the most negative sensitive parameter are the natural human mortality rate μ_h, cattle's natural death rate μ_b, natural death rate of *Taenia saginata* eggs μ_e and the proportion of unconsumed infected meat ε. These results suggest that, more efforts should be directed to improved meat cooking, treatment of humans with taeniasis, meat inspection, and improved hygiene and sanitation to control the spread of these diseases.

2.2. The endemic equilibrium

When cysticercosis and taeniasis persist in cattle and humans respectively, we obtain the endemic equilibrium $E^* = (S_H^*, I_{HT}^*, S_B^*, I_B^*, B_I^*, E_T^*)$ for model system (2), where:

Table 3
Sensitivity indices.

Parameter	Sensitivity index	Parameter	Sensitivity index
Λ_H	+0.5000	Λ_C	+0.5000
β_T	+0.5000	μ_e	−0.5000
μ_h	−1.0000	η	+ 0.2920
ν	+ 0.5000	μ_b	−0.5959
α_b	+0.4536	ε	−0.4536
γ_b	+0.5000	σ_s	−0.1961

$$
\begin{aligned}
S_H^* &= \frac{\Lambda_H(\eta+\mu_b)(\varepsilon+\alpha_b)(\gamma_b E_T^*+\sigma_s+\mu_b)}{\beta_T\eta\alpha_b\gamma_b\Lambda_C E_T^*+\mu_h(\eta+\mu_b)(\varepsilon+\alpha_b)(\gamma_b E_T^*+\sigma_s+\mu_b)},\\
I_{\mathrm{HT}}^* &= \frac{\beta_T\eta\alpha_b\gamma_b\Lambda_C\Lambda_H E_T^*}{\mu_h\beta_T\eta\alpha_b\gamma_b\Lambda_C+\mu_h(\eta+\mu_b)(\varepsilon+\alpha_b)(\gamma_b E_T^*+\sigma_s+\mu_b)},\\
S_C^* &= \frac{\Lambda_C}{(\gamma_b E_T^*+\sigma_s+\mu_b)},\\
I_C^* &= \frac{\gamma_b\Lambda_B E_T^*}{(\eta+\mu_b)(\gamma_b E_T^*+\sigma_s+\mu_b)},\\
B_I^* &= \frac{\eta\gamma_b\Lambda_C E_T^*}{(\eta+\mu_b)(\varepsilon+\alpha_b)(\gamma_b E_T^*+\sigma_s+\mu_b)},\\
E_T^* &= \frac{\nu\Lambda_H(\sigma_s+\mu_b)(R_0+1)(R_0-1)}{R_0^2\mu_e\mu_h(\sigma_s+\mu_b)+\gamma_b\nu\Lambda_H}.
\end{aligned}
\tag{10}
$$

All variables are expressed in terms of E_T^* whereby E_T^* depends on the basic reproduction R_0. It can be easily observed from (10) that the model system (2) has a unique endemic equilibrium when the basic reproduction number $R_0 > 1$. This result is summarized in the following theorem:

Theorem 1. *The model system(2)has a unique endemic equilibrium when the basic reproduction number* $R_0 > 1$.

2.3. Model with interventions

In this subsection, we consider the model system (1) that involves interventions for controlling taeniasis and cysticercosis in humans and cattle respectively.

2.3.1. Disease free equilibrium and effective reproduction number R_e

When there is no human taeniasis and bovine cysticercosis, the disease free equilibrium for the model system (1) is given by:

$$
P^0(S_H, I_{\mathrm{HT}}, S_C, V_C, I_C, B_I, R_C, E_T) = \left(\frac{\Lambda_H}{\mu_h}, 0, \frac{c_0\Lambda_C}{K_0}, \frac{\psi_s\Lambda_C}{K_0}, 0, 0, 0, 0\right),
\tag{11}
$$

where

$$
c_0 = (\sigma_v+\psi_v+\mu_b) \quad \text{and} \quad K_0 = c_0(\sigma_s+\mu_b)+\psi_s(\sigma_v+\mu_b).
$$

The effective reproduction number R_e is the expected number of secondary infections that may occur as a result of introducing one infected individual in a susceptible population when interventions are implemented to control the spread of the disease (Diekmann et al., 1990). The control measures are ineffective when $R_e < 1$ whereas if $R_e > 1$ then the controls are effective. Using the next generation matrix approach Van den Driessche and Watmough (2002) as in Section 2.1.1, we obtain the effective reproduction number R_e for model system (1) given by:

$$
R_e = \sqrt{\frac{(\sigma_v+\psi_v+\mu_b+\rho_b\psi_s)\beta_T\nu\alpha_b\gamma_b\eta\Lambda_H\Lambda_C}{((\sigma_v+\psi_v+\mu_b)(\sigma_s+\mu_b)+\psi_s(\sigma_v+\mu_b))\mu_h\mu_e(\mu_h+\chi)(\eta+\lambda+\mu_b)(\alpha_b+\varepsilon)}}.
\tag{12}
$$

When there are no controls ($\psi_s = \psi_v = \lambda = \chi = c_h = 0$), the effective reproduction number R_e reduces to basic reproduction number R_0.

3. The optimal control model

Based on sensitivity analysis results, we focus on the time dependent control variable $u_1(t)$ which measures the effect of improved meat cooking for reducing the possibility of human infection with *Taenia saginata* tapeworm, $u_2(t)$ that measures the control efforts due to cattle vaccination and $u_3(t)$ that measures treatment efforts for humans with taeniasis. Thus, incoporating these control variables in the model system (1), we obtain:

$$\begin{aligned}
\frac{dS_H}{dt} &= \Lambda_H + u_3(t)I_{HT} - \beta_T\alpha_b(1-u_1(t))B_IS_H - \mu_hS_H,\\
\frac{dI_{HT}}{dt} &= \beta_T\alpha_b(1-u_1(t))B_IS_H - (\mu_h+u_3(t))I_{HT},\\
\frac{dS_C}{dt} &= \Lambda_C + \psi_vV_C + \pi R_C - \gamma_bS_CE_T - (\sigma_s+u_2(t)+\mu_b)S_C,\\
\frac{dV_C}{dt} &= u_2(t)S_C - \rho_b\gamma_bV_CE_T - (\sigma_v+\psi_v+\mu_b)V_C,\\
\frac{dI_C}{dt} &= \gamma_bS_CE_T + \rho_b\gamma_bV_CE_T - (\eta+\lambda+\mu_b)I_C,\\
\frac{dR_C}{dt} &= \lambda I_C - (\pi+\sigma_r+\mu_b)R_C,\\
\frac{dB_I}{dt} &= \eta I_C - (\varepsilon+\alpha_b)B_I,\\
\frac{dE_T}{dt} &= \nu I_{HT} - \mu_eE_T,
\end{aligned} \tag{13}$$

We aim at minimizing the number of infected humans, cattle and the cost associated with implementation of these interventions. The objective function that minimizes the cost for administering these interventions is given as:

$$J = \int_0^{T_f} (C_1I_{HT} + C_2I_C + C_3u_2S_C + \frac{1}{2}\sum_{i=1}^{i=3}A_iu_i^2)dt \tag{14}$$

subject to system of differential equations (13), where C_1 and C_2 are the constants for minimizing prevalence of humans with taeniasis and infected cattle respectively whereas the term u_2S_C aims at minimizing the number of vaccines used to vaccinated cattle with weight constants C_3 (Martcheva, 2015). The coefficients A_1, A_2 and A_3 are relative cost weights for each individual control measure that are used to transform the integral into cost expended over a period of T_f years which is the time period for applying the control strategy (Rong et al., 2021). The initial values are chosen to be 1800, 1500, 340, 130, 250, 90, 83 and 100 for S_H, I_{HT}, I_{HC}, S_C, V_C, I_C, R_C, B_I and E_T classes respectively.

Therefore, we seek to find the optimal controls u_1^*, u_2^* and u_3^* such that:

$$J(u_1^*, u_2^*, u_3^*) = \min_U J(u_1, u_2, u_3), \tag{15}$$

where $U = \{u : u$ is measurable and $0 \le u_i(t) \le 1$ for $t \in [0, T_f]\}$ is the control set.

3.1. Characterization of the optimal control problem

We apply the Pontryagin's maximum principle (Biswas et al., 2017; Pontryagin, 1962) which provides the necessary conditions that an optimal control problem must satisfy. This principle converts the system of differential equations (13) and equation (14) into minimization problem point-wise Hamiltonian ($\mathcal{H}$), with respect to control variables (u_1, u_2, u_3).

If we defined a Lagrangian $\mathcal{L}$ for the control problem by:

$$\mathcal{L} = C_1I_{HT} + C_2I_C + C_3u_2S_C + \frac{1}{2}\sum_{i=1}^{i=3}A_iu_i^2, \tag{16}$$

then the Hamiltonian function $\mathcal{H}$ for the control problem is given as:

$$\mathcal{H} = \mathcal{L} + \lambda_1\frac{\partial S_H}{\partial t} + \lambda_2\frac{\partial I_{HT}}{\partial t} + \lambda_3\frac{\partial S_C}{\partial t} + \lambda_4\frac{\partial V_C}{\partial t} + \lambda_5\frac{\partial I_C}{\partial t} + \lambda_6\frac{\partial R_C}{\partial t} + \lambda_7\frac{\partial B_I}{\partial t} + \lambda_8\frac{\partial E_T}{\partial t}, \tag{17}$$

where λ_i, $i = 1, 2, 3, \ldots, 8$ are the adjoint variables associated with the states S_H, I_{HT}, S_C, V_C, I_C, R_C, B_I and E_T.

If we let $k_1 = (\sigma_s + \mu_b + u_2)$ and $k_2 = (\eta + \lambda_b + \mu_b)$, the Hamiltonian function $\mathcal{H}$ becomes:

$$\begin{aligned}
\mathcal{H} = {} & C_1I_{HT} + C_2I_C + C_3u_2S_C + \frac{1}{2}\sum_{i=1}^{i=3}A_iu_i^2 + \lambda_1(\Lambda_H + u_3I_{HT} - \beta_T(1-u_1)\alpha_bB_IS_H - \mu_hS_H)\\
& + \lambda_2(\beta_T(1-u_1)\alpha_bB_IS_H - (u_3+\mu_h)I_{HT}) + \lambda_3(\Lambda_C + \pi_bR_C + \psi_vV_C - \gamma_bS_CE_T - k_1S_C)\\
& + \lambda_4(u_2S_C - \rho_b\gamma_bV_CE_T - (\sigma_v+\mu_b+\psi_v)V_C) + \lambda_5(\gamma_bS_CE_T + \rho_b\gamma_bV_CE_T - k_2I_C)\\
& + \lambda_6(\lambda_bI_C - (\sigma_b+\pi_b+\mu_b)R_C) + \lambda_7(\eta I_C - (\varepsilon+\alpha_b)B_I) + \lambda_8(\nu I_{HT} - \mu_eE_T).
\end{aligned} \tag{18}$$

Using the Pontryagin's maximum principle (Pontryagin et al., 1962; Pontryagin, 2018), there exist adjoint variables that satisfy:

$$\frac{d\lambda_i}{\mathrm{d\ t}} = -\frac{\partial \mathcal{H}}{\partial i} \tag{19}$$

with transversality conditions:

$$\lambda_i(T_f) = 0. \tag{20}$$

Therefore, the adjoint system is given as:

$$\begin{aligned}
\frac{d\lambda_1}{\mathrm{d\ t}} &= \beta_T(1-u_1)(\lambda_1-\lambda_2)\alpha_b B_I + \mu_h\lambda_1,\\
\frac{d\lambda_2}{\mathrm{d\ t}} &= (u_3+\mu_h)\lambda_2 - C_1 - u_3\lambda_1 - \nu\lambda_8,\\
\frac{d\lambda_3}{\mathrm{d\ t}} &= \gamma_b(\lambda_3-\lambda_5)E_T + (\sigma_s+\mu_b+u_2)\lambda_3 - u_2\lambda_4 - u_2C_3,\\
\frac{d\lambda_4}{\mathrm{d\ t}} &= \rho_b\gamma_b(\lambda_4-\lambda_5)E_T + (\sigma_v+\mu_b+\psi_v)\lambda_4 - \psi_v\lambda_3,\\
\frac{d\lambda_5}{\mathrm{d\ t}} &= (\eta+\lambda_b+\mu_b)\lambda_5 - \lambda_b\lambda_6 - \eta\lambda_7 - C_2,\\
\frac{d\lambda_6}{\mathrm{d\ t}} &= (\sigma_b+\pi_b+\mu_b)\lambda_6 - \pi_b\lambda_3,\\
\frac{d\lambda_7}{\mathrm{d\ t}} &= \beta_T(1-u_1)(\lambda_1-\lambda_2)\alpha_b S_H + (\varepsilon+\alpha_b)\lambda_7,\\
\frac{d\lambda_8}{\mathrm{d\ t}} &= \gamma_b(\lambda_3-\lambda_5)S_C + \rho_b\gamma_b(\lambda_4-\lambda_5)V_C + \mu_e\lambda_8.
\end{aligned} \tag{21}$$

To obtain the optimality conditions, we differentiate the Hamiltonian function (18) with respect to the control variables and solve it when derivative is zero, that is:

$$\begin{aligned}
\frac{\partial \mathcal{H}}{\partial u_1} &= A_1u_1 - (\lambda_2-\lambda_1)\beta_T\alpha_b B_I S_H = 0,\\
\frac{\partial \mathcal{H}}{\partial u_2} &= A_2u_2 - (\lambda_3-\lambda_4-C_3)S_C = 0,\\
\frac{\partial \mathcal{H}}{\partial u_3} &= A_3u_3 - (\lambda_2-\lambda_1)I_{\mathrm{HT}} = 0.
\end{aligned} \tag{22}$$

Since the characterization of the optimal control problem holds on the interior of the control set U, thus we have:

$$\begin{aligned}
u_1^* &= \max\{0, \min(1, \frac{(\lambda_2-\lambda_1)\beta_T\alpha_b B_I S_H}{A_1})\},\\
u_2^* &= \max\{0, \min(1, \frac{(\lambda_3-\lambda_4-C_3)S_C}{A_2})\},\\
u_3^* &= \max\{0, \min(1, \frac{(\lambda_2-\lambda_1)I_{\mathrm{HT}}}{A_3})\}.
\end{aligned} \tag{23}$$

where λ_i for $i = S_H, I_{HT}, S_C, V_C, I_C, R_C, B_I, E_T$ are solutions of the adjoint system (21).

4. Numerical simulations

In this section, numerical simulations of the optimal control model for the dynamics and control of taeniasis and cysticercosis in humans and cattle is carried out. To solve numerically the optimal control problem, we implement the forward-backward sweep method for the model system (13) and the adjoint system (21) in Matlab using parameter values in Table 2. The method begins by solving the model system (13) forward in time using Runge Kutta method of the fourth order relying on the supplied initial values of the controls. Then, the backward fourth order Runge Kutta method uses the obtained values of the state variables and initial values of controls to solve the adjoint equations (21) with given final condition (20). The control variables $u_1(t)$, $u_2(t)$, $u_3(t)$ are then updated and used to solve the state and adjoint systems. Since implementation of only one intervention may not be effective in disease control, a combination of various strategies will be assessed. Strategy 1: Combination of improving meat cooking rate $u_1(t)$, cattle vaccination $u_2(t)$ and treatment of humans with taeniasis $u_3(t)$, Strategy 2: Combination of cattle vaccination $u_2(t)$ and treatment of humans with taeniasis $u_3(t)$, Strategy 3: Combination of improving meat cooking rate $u_1(t)$ and cattle vaccination $u_2(t)$, Strategy 4: Combination of improving meat cooking rate $u_1(t)$ and treatment of humans with taeniasis $u_3(t)$.

4.1. When all controls are implemented

This strategy involves the combination of improved meat cooking, vaccination of cattle, and treatment of humans with taeniasis. The results in Fig. 2 shows a significant decrease of infected humans, infected cattle and *Taenia saginata* eggs in the environment when all time dependent controls are implemented. Humans with taeniasis reduces to zero in 2.5 years while infected cattle and *Taenia saginata* eggs reduce in 8 and 2,5 years respectively. The control profiles in Fig. 2(d) show that initially the control variables $u_2(t)$ is at its peak and then declines gradually to zero in the first 4.5 years. The control profiles for $u_1(t)$ and $u_3(t)$ are fully utilized in the first 3 and 9.5 years respectively and eventually decline to zero.

4.2. Vaccination of cattle and treatment of humans with taeniasis

In this strategy, the control variables $u_1(t)$ and $u_3(t)$ for cattle vaccination and treatment of humans with taeniasis respectively are used to optimize the objective function J while the control $u_1(t)$ on improving meat cooking rate is set to zero. The results in Fig. 3 show a reduction in number of cases for humans with taeniasis, infected cattle and *Taenia saginata* eggs in the environment. However, with this strategy it is not possible to control the disease prevalence in humans and cattle. The control profiles in Fig. 3(d) show that the control variables $u_2(t)$ and $u_3(t)$ are fully utilized in their first 8.5 and 9.8 years and then decline to zero in the final time.

4.3. Improved meat cooking and vaccination of cattle

In this strategy, we consider the combination of the control variables $u_1(t)$ for improved meat cooking and ($u_2(t)$) for vaccination of cattle. It can be observed in Fig. 4 that, although there is a decline in number of cases, however humans with taeniasis is maintained at 0.25×10^4 thoughout while infected cattle approaches zero after the first 1.5 years and a small increase in observed after the 9.5 year. On the other hand, *Taenia saginata* eggs decline in the first 0.5 years and stabilizes at 40. The control profiles in Fig. 4(d) show that the control variables $u_1(t)$ and $u_2(t)$ are at their peak in their first 8 and 9.9 years and finally decline to zero in the final time.

4.4. Improved meat cooking and treatment of humans with taeniasis

In this strategy, a combination of control variables ($u_1(t)$) for improved meat cooking and ($u_3(t)$) for treatment of humans with taeniasis are used to optimize the objective function J. The results for this strategy are similar to those for the first strategy which combines improved meat cooking, treatment of humans with taeniasis, and cattle vaccination. The control profile $u_1(t)$ in Fig. 5(d) is

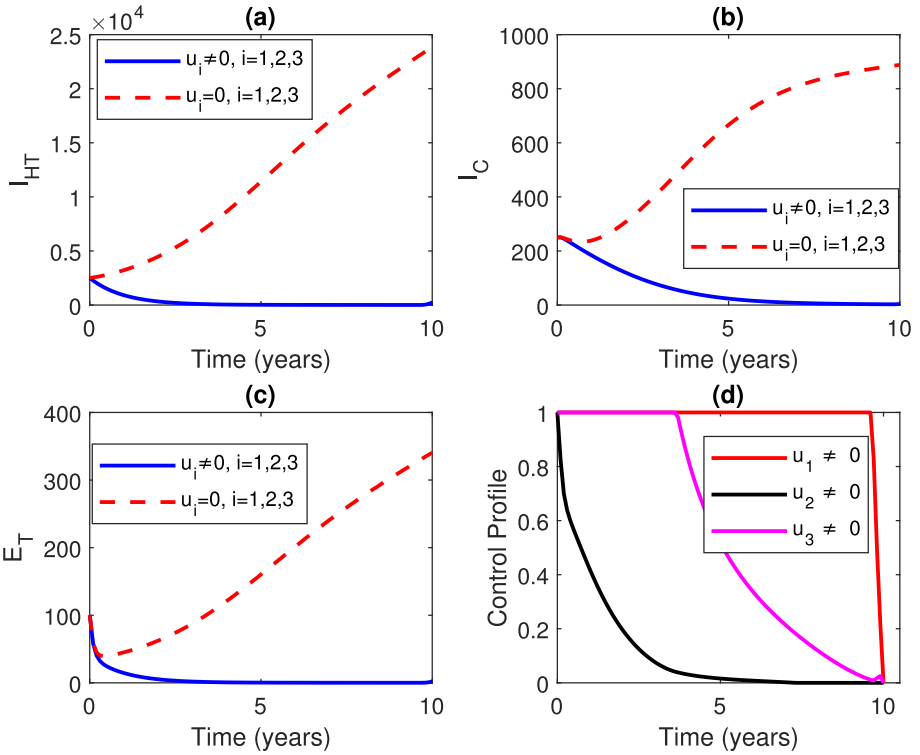

Fig. 2. Impact of applying all controls on infected humans, cattle and taenia eggs.

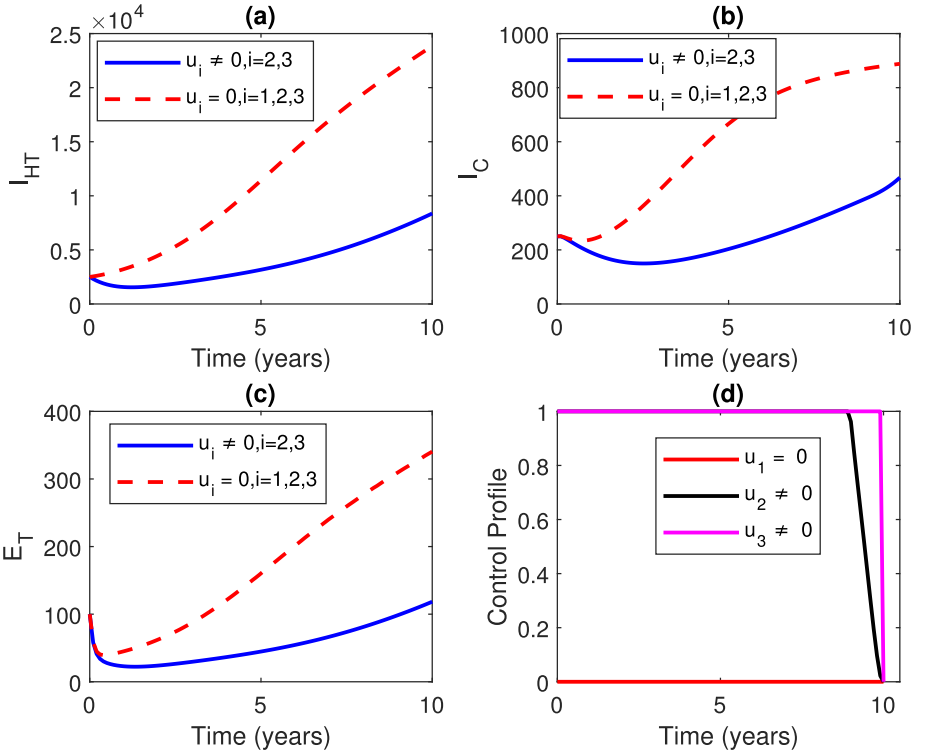

Fig. 3. Impact of vaccination of cattle and treatment of humans with taeniasis on infected humans, cattle and taenia eggs.

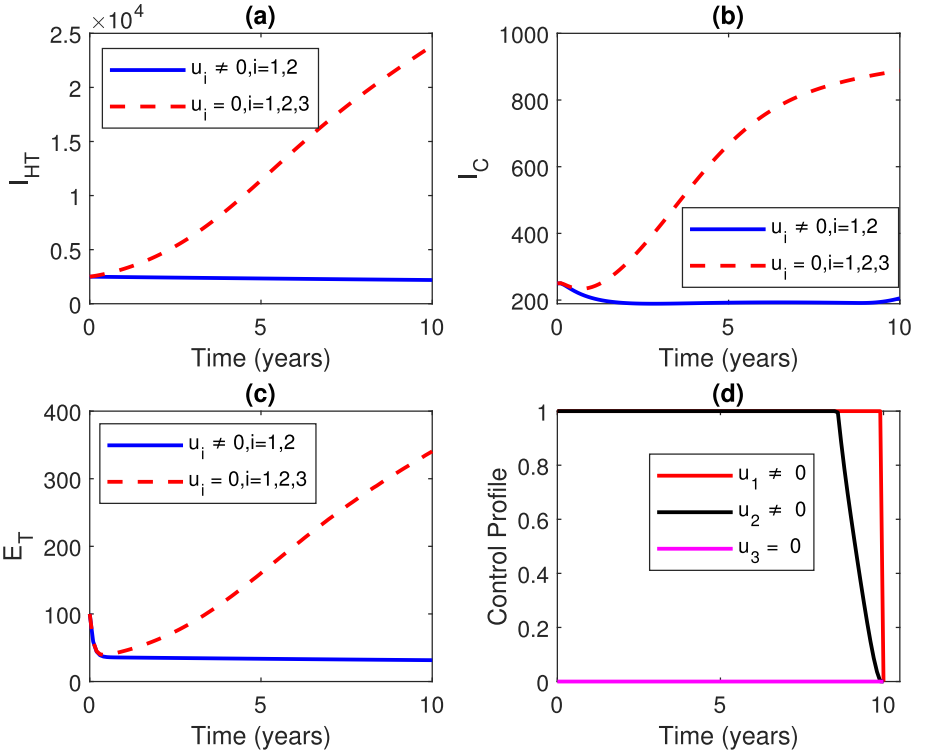

Fig. 4. Impact of improved beef cooking and vaccination of cattle on infected humans, cattle and taenia eggs.

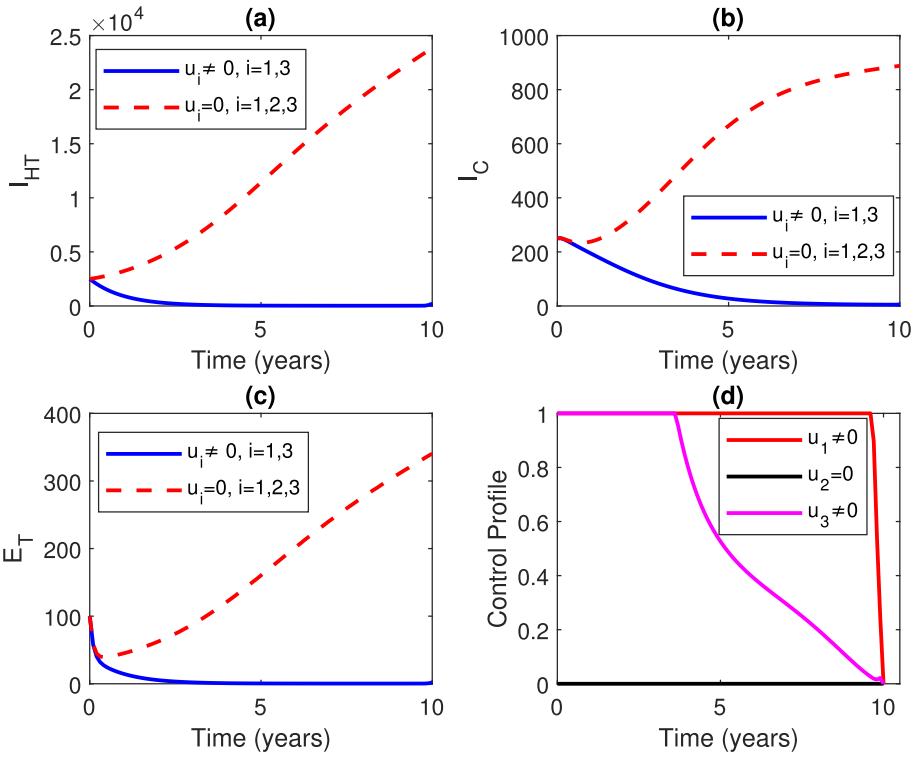

Fig. 5. Impact of improved meat cooking and treatment of humans with taeniasis on infected humans, cattle and taenia eggs.

fully utilized for the first 9.5 years and quickly drops to zero at the final time while $u_3(t)$ is at its peak for the first 3 years and gradually declines to zero in the final time.

Generally, it can be observed that a strategy which focus on improving meat cooking and treatment of humans with taeniasis is the most optimal strategy in controlling the transmission of taeniasis and cysticercosis in humans and cattle respectively.

5. Discussion

Bovine cysticercosis is a threat to rural livestock farmers who depend on cattle to earn their incomes. The disease affects market value of cattle by making cattle's meat unsafe for consumption. Although only few mathematical models have been formulated and analyzed to study the transmission dynamics and control of *Taenia solium* tapeworm parasite in humans and pigs, however no study has been carried out to study the dynamics and optimal control of *Taenia saginata* tapeworm parasite which is responsible to cause bovine cysticercosis and human taeniasis. Studies by Gonzalez et al. (2002), Kyvsgaard et al. (2007), Winskill et al. (2017) and José et al. (2018) have shown that treatment of infected humans and pigs have significant impact in controlling the transmission of *Taenia solium* parasite in humans and pigs whereas the study by Sánchez-Torres et al. (2019) has shown that pig vaccination and treatment of humans with taeniasis have influence on the dynamics of *Taenia solium* parasite. These results are in correspondence with our results presented in this paper which show that the treatment of human with taeniasis plays a significant role for controlling cysticercosis in cattle and taeniasis in humans. Our results indicate further that the treatment of infected humans is more effective when combined with adequate cooking of infected meat of cattle. Our previous study in Mwasunda et al. (2021) on the transmission dynamics of taeniasis and cysticercosis in humans, pigs and cattle has shown that among other sensitive parameters, recruitment rate of humans, probability of humans to acquire taeniasis and the defecation rate by humans with taeniasis are the most sensitive parameters to diseases' transmission whereas human natural mortality rate is the most negative sensitive parameter. These results agree with sensitivity indices of model parameters presented in this paper.

6. Conclusion and recommendations

In this paper, we have formulated and analyzed a deterministic model for transmission dynamics of *Taenia saginata* bovine cysticercosis and human taeniasis. The analysis shows that both the disease free and endemic equilibria exist. The basic reproduction number R_0 which determines whether bovine cysticercosis and human taeniasis persist or die in cattle and human populations has been computed by applying the next generation method approach. The diseases die in humans and cattle when $R_0 < 1$ and persist when $R_0 > 1$. The normalized forward sensitivity index approach has been employed to determine sensitivity indices of parameters in the

basic reproduction number. Results show that recruitment rates for human and cattle populations, infection rate of cattle from contaminated environment, probability of humans to acquire taeniasis due to consumption of raw or undercooked meat of cattle which is infected with tapeworm larval cysts, defecation rate of humans with taeniasis and the consumption rate of raw or undercooked meat are the most positive sensitive parameters whereas the natural death rates for humans, cattle, *Taenia saginata* eggs and the proportion of unconsumed infected meat are most negative sensitive parameters in diseases' transmission. These results suggest that more efforts should focus on improving meat cooking, meat inspection and treatment of infected humans so as to control the spread of cysticercosis and taeniasis in cattle and humans respectively. The optimal control model has been presented and analyzed to study the impact of various strategies on the control of human taeniasis and bovine cysticercosis. The Pontryagin's maximum principle has been adopted to find necessary conditions for existence of the optimal time dependent controls. The time dependent controls that have been considered in the model are improved meat cooking, vaccination of cattle and treatment of infected humans. Results indicate that a strategy which focuses on improving meat cooking and treatment of humans with taeniasis is the most optimal control strategy in diseases' control. Therefore, to control *Taenia saginata* bovine cysticercosis and human taeniasis, we suggest that more efforts should be directed to treat humans with taeniasis and improve meat cooking. The research work presented in this work differs from our previous study in Mwasunda et al. (2021) in the sense that this study investigates the optimal controls for cysticercosis and taeniasis in cattle and humans only while the former study focused on analyzing the dynamics of taeniasis and cysticercosis in humans, pigs and cattle without any control measure. However, sensitivity analysis results in the two papers agree that humans recruitment and the defecation rate by humans with taeniasis are the most sensitive parameters in diseases' transmission whereas human natural mortality rate is the most negative sensitive parameter.

7. Limitations, strength and future work

This study has given insight to the dynamics of *Taenia saginata* bovine cysticercosis and human taeniasis through determining parameters that drive the diseases. Results obtained from this research work are significant in the sense that they suggest appropriate measures to control the spread of the diseases. However, the study has got some limitations particularly due to lack of data that could have been fitted in the model to obtain actual parameter values. This is due to the fact that bovine cysticercosis and human taeniasis are neglected diseases that affect developing countries where low priority is given on data collection related to such diseases. In future, this study can be extended to study the impact of combining reinforcement learning and optimal control analysis with cost-effectiveness analysis so as to determine the most cost-effective strategy in diseases' control.

Availability of data and material

Most of data used in this paper were found from different literature and some were assumed.

Authors' contribution

J.A. Mwasunda: Conceptualization, Model formulation, model analysis and drafting of the manuscript; J.I. Irunde: Model formulation and supervision; D. Kajunguri: Supervision; and D. Kuznetsov: Supervision.

Declaration of Competing Interest

The authors declare that they have no competing interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

Table A.1
Summary of intervention strategies

Strategy	Description
1.	Improved meat cooking, vaccination of cattle and human treatment
2.	Vaccination of cattle and human treatment
3.	Improved meat cooking and vaccination of cattle
4.	Improved meat cooking and human treatment

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