Heliyon 10 (2024) e34424

Contents lists available at ScienceDirect

Heliyon

journal homepage: www.cell.com/heliyon

Research article

New extended exponentially weighted moving average control chart for monitoring process mean

Hina Javed [a], Muhammad Ismail [b,*], Nadia Saeed [a]

[a] College of Statistical Sciences, University of the Punjab, Lahore Pakistan
[b] COMSATS University Islamabad Lahore, Campus Lahore Pakistan

ARTICLE INFO

Keywords:
Average run length
EWMA
EEWMA
NEEWMA
SPC
Normal distribution

ABSTRACT

In this article, we develop a new control chart based on the Exponentially Weighted Moving Average (EWMA) statistic, termed the New Extended Exponentially Weighted Moving Average (NEEWMA) statistic, designed to recognize slight changes in the process mean. We derive expressions for the mean and variance of the NEEWMA statistic, ensuring an unbiased estimation of the mean, with simulation results showing lower variance compared to traditional EWMA charts. Evaluating its performance using Average Run Length (ARL), our analysis reveals that the NEEWMA control chart outperforms EWMA and Extended EWMA (EEWMA) charts in swiftly recognizing shifts in the process mean. Illustrating its operational methodology through Monte Carlo simulations, an illustrative example using practical data is also provided to showcase its effectiveness.

1. Introduction

Statistical Process Control (SPC) plays a vital role in manufacturing by maintaining process reliability and reducing fluctuations. Variations stem from assignable causes or natural sources. A process under natural sources is deemed in-control (IC), while assignable causes indicate an out-of-control process. In SPC, charts are employed to distinguish these variations. There are two main types: memory-less and memory-type control charts.

Roberts [1] introduced the traditional EWMA control chart as a method to identify minor fluctuations in production, surpassing the capabilities of Shewhart charts. This statistical tool, utilizing both recent and historical data, proves invaluable in recognizing slight to moderate shifts in processes.

Utilizing similar statistics, numerous researchers introduced different statistics to detect the production process. When contrasted with the EWMA control chart, the newly developed EWMA control chart in Steiner [2] with time-varying lower and upper control limits rapidly identifies the change in process mean. Eyvazian et al. [3] found Exponential Weighted Moving Sample Variance (EWMSV) to observe process variability, resulting in smaller ARLs when contrasted with different strategies. Yang et al. [4] utilized a Nonparametric Exponentially Weighted Moving Average Sign (NEWMAS) statistic to observe the procedure mean that represents better execution in terms of more modest average run lengths. Abbas et al. [5] established the mixture of Exponentially Weighted Moving Average (EWMA) and cumulative sum (CUSUM) control charts and furnished that the proposed control chart yields superior results compared to individual EWMA and CUSUM control charts. Abbas et al. [6] introduced another EWMA statistic to increase the efficiency of a control chart by minimizing its variability, this EWMA statistic employs a regression estimator for solitary auxiliary variables. Abouelmagd et al. [7] reduced secular solutions around triangular equilibrium points to a periodic solution in the

* Corresponding author.
E-mail addresses: henajavd@gmail.com (H. Javed), muhammadismail@cuilahore.edu.pk (M. Ismail), nadia.stat@pu.edu.pk (N. Saeed).

https://doi.org/10.1016/j.heliyon.2024.e34424
Received 24 January 2024; Received in revised form 6 July 2024; Accepted 9 July 2024
Available online 9 July 2024

generalized restricted three-body problem.

Aslam et al. [8] utilized the EWMA control chart after normal approximation to an exponential distribution and worked on the outcome as far as more modest average run lengths. Hariba and Tukaram [9] established an economic model based on traditional EWMA statistic and represent the change in procedure mean expands and sample size expected to recognize the shift diminishes. Liu et al. [10] developed new stability criteria for Cohen–Grossberg neural networks, showcasing the potential impact on advancing stability analysis in neural network research. Du et al. [11] proved the global and existing exponential stability of periodic solutions in discrete time-neutral-type neural networks with time-varying delays, enhancing stability analysis. Naveed et al. [12] developed extended EWMA statistic to identify the modest shift more quickly as compared to the EWMA. Saeed and Abu-Shawiesh [13] introduced the Trimmed EWMA (T-EWMA) statistic, the Trimmed Weighted Standard Deviation Exponentially Weighted Moving Average (TWSD-EWMA) statistic and the Trimmed Weighted Variance Exponentially Weighted Moving Average (TWV-EWMA) statistic which showed that trimmed statistics performed better for non-normal processes in terms of out-of-control ARLs. For monitoring Zero-Inflated Poisson (ZIP) processes [14] an adaptive exponentially weighted moving average control chart, ZIP distribution was used to model count data with excessive zeros. The Markov Chain approach was utilized to estimate the performance of ARL and SDRL, where an adaptive exponentially weighted moving average control chart gives better results in terms of more modest ARL values.

Anastasopoulou and Rakitzis [15] studied lower and upper one-sided EWMA control charts with a finite range to monitor correlated counts, this model was preferable to the former when data has demonstrated extra-binomial variation and utilized to recognize downward or upward shifts in process mean level and provided better results. Khan et al. [16] introduced the fuzzy EWMA control chart, furnished that the fuzzy EWMA control chart gives better results as compared to traditional EWMA. Noor et al. [17] developed the hybrid EWMA control chart by using the Bayesian technique with two different loss functions asymmetric and symmetric loss functions which are known as squared error loss function and Linex loss function under non-informative prior (uniform and Jeffery prior) and informative (conjugate) prior, that represents better execution in terms of ARLs. Saeed et al. [18] modified the control limits for the EWMA control chart under the normal process by utilizing robust point M-scale estimators for observing the process mean. In SPC to recognize tiny shifts in process parameters the EWMA control chart is widely recognized. Five robust point M-scale estimators were compared. For comparing the estimator's performance, a simulation study was performed. Simulation results showed that all proposed control limits closely approximate the true limits of the process.

In light of existing literature, there's an opportunity to introduce a statistic that can detect shifts early using available data. While a traditional EWMA control chart is commonly used for shift recognition in continuous processes, it has limited capability in analyzing quantitative and qualitative effects. In our study, we introduce a generalized version of traditional EWMA statistics, where the conventional EWMA statistics become a specific instance of our proposed NEEWMA statistic. This new statistic incorporates both historical and current data from the study variable, along with weighted factors and prior data, to establish control limits and monitor the process mean. Our analysis demonstrates that the NEEWMA control chart outperforms other competing control charts, especially in detecting small shifts.

The rest of the paper is organized as follows: In Sections 2 and 3, the classical EWMA and extended EWMA charts are described while in Section 4, the details and necessary derivations about proposed chart are provided. The performance measures along with algorithms are given in Section 5. Section 6 is based on results and discussion. Section 7 comprised of comparative study while section 8 is based on illustrative examples consisting of simulated and practical data sets. Finally, the conclusion is provided in Section 9.

2. The traditional EWMA control chart

Suppose $T_1, T_2, T_3, \ldots, T_i, \ldots$ be a succession of IID random variables with mean μ and variance σ^2 drawn from a normal population. Therefore, the traditional EWMA statistics Z_i is:

$$Z_i = \alpha_1 T_i + (1-\alpha_1)Z_{i-1}, i = 1, 2, \ldots \tag{2.1}$$

where α_1 is smoothing constant with range $0 < \alpha_1 \leq 1$. A more modest worth of the smoothing constant represents that the present value of the variable under consideration gets less weight and the preceding value of the statistic gets more weight. If $\alpha_1 = 1$, the traditional EWMA statistic lessens to a statistic that utilizes only present data. The previous value of the statistic is represented by the quantity Z_{i-1} and the initial value Z_0 is taken as the target mean. For an IC process, mean and variance of the traditional EWMA statistic provided by Roberts (1959) are:

$$E(Z_i) = \mu, \mathrm{var}(Z_i) = \frac{\sigma^2 \alpha_1}{2-\alpha_1}\left(1-(1-\alpha_1)^{2i}\right), \tag{2.2}$$

Here μ and σ^2 show the target mean and variance of EWMA statistics Z_i respectively. We can estimate them from preliminary samples, in case if these target values are not known.

3. The EEWMA control chart

Suppose $T_1, T_2, T_3, \ldots, T_i, \ldots$ be a succession of IID random variables with mean μ and variance σ^2 drawn from a normal population. Therefore, the extended EWMA statistics with smoothing constants α_1 and α_2 is:

$$Z_i = \alpha_1 T_i - \alpha_2 T_{i-1} + (1-\alpha_1+\alpha_2) Z_{i-1}, i = 1, 2, \ldots \tag{3.3}$$

where $0 < \alpha_1 \leq 1, 0 \leq \alpha_2 < \alpha_1$, here Z_i shows a statistic named as EEWMA, and the range or scope of the smoothing constant α_1 and α_2

are $0 < \alpha_1 \le 1$ and $0 \le \alpha_2 < \alpha_1$ respectively. For an in-control process, The EEWMA statistics offered [12] have the following mean and variance:

$$E(Z_i) = \mu, \mathrm{var}(Z_i) = \sigma^2\left[(\alpha_1^2+\alpha_2^2)\left(\frac{1-(1-\alpha_1+\alpha_2)^{2i}}{2(\alpha_1-\alpha_2)-(\alpha_1-\alpha_2)^2}\right) - 2A\alpha_1\alpha_2\left(\frac{1-(1-\alpha_1+\alpha_2)^{2i-2}}{2(\alpha_1-\alpha_2)-(\alpha_1-\alpha_2)^2}\right)\right], \tag{3.4}$$

where the target mean is represented by μ and variance by σ^2 of T_i, we estimate these from initial samples, in case these values are unknown.

4. The proposed NEEWMA control chart

The Shewhart control chart utilizes only the details of the present sample, however, the traditional EWMA control chart is developed in such a manner that the latest subgroup is given more weight, and the other observations are given mathematically diminishing weights. In this paper, we present a new extended EWMA statistic that gives positive weight to the current sample and negative weight to the preceding samples, resulting in a smaller variance of the proposed statistic.

Suppose $T_1, T_2, T_3, \ldots., T_i, \ldots.$ be a succession of IID random variables with mean μ and variance σ^2 drawn from a normal population. Therefore, the NEEWMA statistics with smoothing constants $\alpha_1, \alpha_2,$ and α_3 is:

$$Z_i = \alpha_1 T_i - \alpha_2 T_{i-1} - \alpha_3 T_{i-2} + (1-\alpha_1+\alpha_2+\alpha_3)Z_{i-1}, i = 1, 2, \ldots.. \tag{4.5}$$

where $0 < \alpha_1 \le 1$, $0 \le \alpha_2 < \alpha_1$ and $0 \le \alpha_3 < \alpha_2$.

In this proposed statistic Z_i, the sum of weights is less than or equal to unity. The quantities T_{i-1}, and T_{i-2} denote the prior values of the variable and Z_{i-1} represents the prior value of the statistic Z_i. The values of Z_0 and T_0 are taken as the target mean.

4.1. The mean and variance of the proposed NEEWMA statistic

Here, the mean and variance of the proposed NEEWMA statistic are derived. Suppose $A = (1-\alpha_1+\alpha_2+\alpha_3)$ with $i = 1$, equation (4.5) becomes:

$$Z_1 = \alpha_1 T_1 - \alpha_2 T_0 - \alpha_3 T_{-1} + A\, Z_0. \tag{4.6}$$

$$\text{Similarly for } i = 2, Z_2 = \alpha_1 T_2 - \alpha_2 T_1 - \alpha_3 T_0 + A\, Z_1. \tag{4.7}$$

Substituting the value of Z_1, equation (4.7) becomes:

$$Z_2 = \alpha_1 T_2 - (\alpha_2 - A\alpha_1)T_1 - (\alpha_3 + A\,\alpha_2)T_0 - A\alpha_3 T_{-1} + A^2\, Z_0. \tag{4.8}$$

Suppose $B = (\alpha_2 - A\alpha_1)$ and $C = \alpha_3 + A\,\alpha_2$,

$$Z_2 = \alpha_1 T_2 - BT_1 - CT_0 - A\,\alpha_3 T_{-1} + A^2\, Z_0. \tag{4.9}$$

Similarly for $i = 3$,

$$Z_3 = \alpha_1 T_3 - (\alpha_2 - A\,\alpha_1)T_2 - \left(\alpha_3 + A\alpha_2 - A^2\alpha_1\right)T_1 - \left(A\alpha_3 + A^2\,\alpha_2\right)T_0 - A^2\alpha_3 T_{-1} + A^3\, Z_0. \tag{4.10}$$

Suppose $D = \alpha_3 + A\alpha_2 - A^2\,\alpha_1$, equation (4.10) can be rewritten as:

$$Z_3 = \alpha_1 T_3 - BT_2 - DT_1 - ACT_0 - A^2\alpha_3 T_{-1} + A^3\, Z_0. \tag{4.11}$$

In general, the expression of Z_i can be expressed as:

$$Z_i = \alpha_1 T_i - BT_{i-1} - DT_{i-2} - ADT_{i-3} - A^2DT_{i-4}\ldots\ldots\ldots - A^{i-3}DT_1 - A^{i-2}CT_0 - A^{i-1}\alpha_3 T_{-1} + A^i\, Z_0. \tag{4.12}$$

After applying expectation to both sides of equation (4.12), we get:

$$E(Z_i) = \alpha_1\mu - B\mu - D\mu - AD\mu - A^2D\mu - \ldots - A^{i-3}D\mu - A^{i-2}C\mu - A^{i-1}\,\alpha_3\mu + A^i\,\mu. \tag{4.13}$$

Substituting the values of B, C and D and after simplifying, equation (4.13) becomes:

$$E(Z_i) = \mu\left[\alpha_1\left(1+A+A^2+\ldots+A^{i-1}\right) - \alpha_2\left(1+A+A^2+\ldots+A^{i-1}\right) - \alpha_3\left(1+A+A^2+\ldots+A^{i-1}\right) + A^i\right]. \tag{4.14}$$

After using the formula for calculating the sum of a finite geometric series, equation (4.14) becomes:

$$E(Z_i) = \mu\left[(\alpha_1 - \alpha_2 - \alpha_3)\left(\frac{1-A^i}{1-A}\right) + A^i\right]. \tag{4.15}$$

$$E(Z_i) = \mu\left[(1-A)\left(\frac{1-A^i}{1-A}\right) + A^i\right]. \tag{4.16}$$

$$E(Z_i) = \mu \,. \tag{4.17}$$

For the expression of variance of NEEWMA statistic, applying variance to both sides of equation (4.12), we get:

$$\mathrm{var}(Z_i) = \alpha_1^2\sigma^2 + B^2\sigma^2 + D^2\sigma^2 + A^2D^2\sigma^2 + A^4D^2\sigma^2 \ldots\ldots\ldots + A^{2(i-3)}D^2\sigma^2 + A^{2(i-2)}C^2\sigma^2 + A^{2(i-1)}\alpha_3^2\sigma^2 + A^{2i}\sigma^2. \tag{4.18}$$

Substituting the values of B, C, and D, equation (4.18) becomes:

$$\mathrm{var}(Z_i) = \left[\alpha_1^2\left(1 + A^2 + A^4 + \ldots + A^{2(i-1)}\right) + \alpha_2^2\left(1 + A^2 + A^4 + \ldots + A^{2(i-1)}\right) + \alpha_3^2\left(1 + A^2 + A^4 + \ldots + A^{2(i-1)}\right) - 2A\alpha_1\alpha_2\left(1 + A^2 + A^4 + \ldots + A^{2(i-2)}\right) + 2A\alpha_2\alpha_3\left(1 + A^2 + A^4 + \ldots + A^{2(i-2)}\right) - 2A^2\alpha_1\alpha_3\left(1 + A^2 + A^4 + \ldots + A^{2(i-3)}\right)\right]\sigma^2. \tag{4.19}$$

Using the formula for the sum of the finite geometric series, we obtain.

$$\mathrm{var}(Z_i) = \left[\left(\alpha_1^2 + \alpha_2^2 + \alpha_3^2\right)\left\{\frac{1 - A^{2i}}{1 - A^2}\right\} - 2A(\alpha_1\alpha_2 - \alpha_2\alpha_3)\left\{\frac{1 - A^{2(i-1)}}{1 - A^2}\right\} - 2A^2\alpha_1\alpha_3\left\{\frac{1 - A^{2(i-2)}}{1 - A^2}\right\}\right]\sigma^2. \tag{4.20}$$

Finally,

$$\mathrm{var}(Z_i) = \frac{\sigma^2}{1 - A^2}\left[\left(\alpha_1^2 + \alpha_2^2 + \alpha_3^2\right)\left(1 - A^{2i}\right) - 2A(\alpha_1\alpha_2 - \alpha_2\alpha_3)\left(1 - A^{2(i-1)}\right) - 2A^2\alpha_1\alpha_3\left(1 - A^{2(i-2)}\right)\right]. \tag{4.21}$$

Under $\alpha_2 = \alpha_3 = 0$, reduced to traditional EWMA statistic, the NEEWMA statistic. For an IC process mean and variance of NEEWMA statistic are given lower than the EEWMA and EWMA statistics.

Finally, the mean and variance of NEEWMA are in Equations (4.17) and (4.21), which are the target mean and variance of T_i are μ and σ^2 respectively.

4.2. Control limits based on NEEWMA statistic

Based on the mean and variance of NEEWMA statistic, the upper and lower control limits (UCL and LCL) of the proposed NEEWMA statistic are developed as:

$$UCL = \mu_0 + K\sigma\sqrt{\mathrm{var}(Z_i)}. \tag{4.22}$$

$$LCL = \mu_0 - K\sigma\sqrt{\mathrm{var}(Z_i)}, \tag{4.23}$$

where constant K is the control coefficient and μ_0 represents the IC process as well as the central line (CL). The value of K is determined in such a way that the desired IC average run length of the NEEWMA control chart is obtained. The average run length is a typical measure utilized to rate the effectiveness of a control chart which follows a geometric distribution. Subsequently, in the derivation of average run lengths, the first point is noted which is to be out-of-control process.

5. Performance measures

A regular control chart usually contains three horizontal lines, named as, Upper Control Limit (UCL), Central Line (CL) and Lower Control Limit (LCL). The control chart process is said to be IC if the charting statistic values falls in between lower control and upper control limits while for an out-of-control situation, at least one point must be plotted outside these limits.

In the average related control chart, the LCL and UCL are setting at three times standard error from the central line. Hence the main focus of our research is to design the control chart for process average which can effectively detect an out-of-control signal. To evaluate the performance of any control chart an important yardstick is known as ARL. The ARL is a measure of how well a control chart detects the process shift.

When the process is IC, the ARL can be calculated as:

$$ARL_{0=}\frac{1}{\acute{\alpha}}, \tag{5.24}$$

where $\acute{\alpha}$ is the probability of type-I error.

When the process is out-of-control, the ARL can be calculated as:

$$ARL_{1=}\frac{1}{1 - \acute{\beta}}, \tag{5.25}$$

where $\acute{\beta}$ is the probability of type-II error.

The standard deviation of average run length is represented as SDRL which can also be used as a performance indicator depicting the consistency of Run Length (RL) values when the process is repeated a large number of times. Hence

$$SDRL = SD(RL). \tag{5.26}$$

5.1. Monte Carlo method

Monte Carlo methods encompass a wide range of computational algorithms that achieve numerical results through repeated random sampling. The core idea is to employ randomness to solve problems that are deterministic in nature. Crowder [19] explored the selection of the smoothing constant and the control limit coefficient using Monte Carlo simulations. In this context, the Monte Carlo method involved generating a large number of random samples to simulate the performance of different combinations of smoothing constants and control limit coefficients. By analyzing these simulations, Crowder was able to identify optimal values that improve the performance of control charts, providing insights into how these parameters affect the detection of process changes and variability.

5.2. Algorithms

To determine the ARLs and the control chart coefficient, the following algorithms are used. The algorithmic advances engaged with the Monte-Carlo simulation study. The coding is done in R-language which is given beneath.

Algorithm-1
Proposed NEEWMA for an in-control process

Step-1	Computation of the proposed NEEWMA statistic $\mathbf{Z_i}$.	
	Step-1.1	Specify the value of the in-control ARL, say b_0, and smoothing constants α_1, α_2, andα_3.
	Step-1.2	Create a random sample from standard normal distribution i.e., $T_i \sim N(0,1)$ that has specified parameters for the IC process; Create 2500 such subgroups.
	Step-1.3	Calculate equation 3, the NEEWMA statistic Z_i.
Step-2	Calculate the unestablished control limits (CLs).	
	Step-2.1	Choose the value of K in such a manner that the IC average run length of the NEEWMA control chart approaches to the target value b_0.
	Step-2.2	From 2500 subgroups, calculate $LCL_{(i)}$ and $UCL_{(i)}$.
	Step-2.3	If the statistic $LCL_{(i)} \le Zi \le UCL_{(i)}$, indicates that the process is to be IC, otherwise the process is declared to be OC.
	Step-2.4	If the process is stable and in control, continue repeating steps 1.1 through 2.3. If the process is declared OC, the number of subgroups should be counted as the run length.
Step-3	Calculate the average run length (ARL).	
	Step-3.1	To determine the in-control ARL, repeatedly do steps 1.1 through 2.4 thousand times (for example,10,000). Go to Algorithm 2 and halt the operation if the computed IC average run length is equal to the predefined b_0. Change the control chart coefficient value in a different way, then go over steps 1.1 to 3.1 again.

Algorithm-2
Proposed NEEWMA for an out-of-control process

Step-1	Computation of the proposed NEEWMA statistic Z_i.	
	Step-1.1	Specify the value of the in-control ARL, say b_0, and smoothing constants $\alpha_1, \alpha_{2,}$ and α_3, along with shift size d.
	Step-1.2	Create a random sample which has size one from an out-of-control normal process where the mean is shifted from μ_0 to μ_1; $\mu_1 = \mu_0 + d\,\sigma$ with $T \sim N(\mu_1, 1)$. Create 2500 such subgroups.
	Step-1.3	Calculate the NEEWMA statistic Z_i by using Equation (5).
Step-2	Calculate the unestablished control limits (CLs).	
	Step-2.1	Choose the control chart coefficient K value from the outcome of Algorithm 1.
	Step-2.2	From 2500 subgroups, drive $LCL_{(i)}$ and $UCL_{(i)}$
	Step-2.3	If the statistic $LCL_{(i)} \le Zi \le UCL_{(i)}$, the process is declared to be IC, otherwise the process is to be OC.
	Step-2.4	If the process is stable and in control, continue repeating steps 1.1 through 2.3. If the process is declared OC, the run length is equal to the number of subgroups.
Step-3	Calculate the average run length (ARL_1) and standard deviation of run length (SDRL) for the shifted process when the mean is shifted from μ_0 to μ_1 where $\mu_1 = \mu_0 + d\,\sigma$.	
	Step-3.1	To determine the ARL of the shifted process (ARL_1), repeat steps 1.1 through 2.4 thousand of times say 10,000 times.

6. Analysis and discussion

In this section, we discuss ARLs and SDRLs for EWMA, EEWMA, and proposed NEEWMA charts under specific values of smoothing constants (α_1,α_2,and α_3), control charts coefficient (K), shift size (d) and optimal in-control ARL values (b_0). Tables 1–6 show ARLs for various values of smoothing constants α_1,α_2,and α_3, when $b_0 = 300, 370,$ and 500. Crowder [19] explored the selection of smoothing constants, in-control average run length values and control limit coefficient. The values of α_1, α_2 and α_3 were set up in such a manner that $0 < \alpha_1 \leq 1$, $0 \leq \alpha_2 < \alpha_1$ and $0 \leq \alpha_3 < \alpha_2$. In practice, smoothing constants (α_1,α_2 and α_3) are often set within the interval [0.05, 0.30], with 0.10 and 0.20 being popular choices, also average run length b_0 most popular choices are 300, 370, and 500.

The proposed NEEWMA demonstrates superiority over EWMA and EEWMA, particularly noticeable for small sample sizes (d), where it detects process changes earlier, as evidenced by ARL values in Tables 1–6 However, as d increases, the performance gap

Table 1
ARLs(SDRLs) of proposed and existing control charts when $b_0 = 500$ and $K = 2.8485$.

Shift (**d**)	Proposed NEEWMA	EEWMA	EWMA
	$\boldsymbol{\alpha_1 = 0.1}$, $\boldsymbol{\alpha_2 = 0.03}$, $\boldsymbol{\alpha_3 = 0.01}$	$\boldsymbol{\alpha_1 = 0.1}$, $\boldsymbol{\alpha_2 = 0.03}$	$\boldsymbol{\alpha_1 = 0.1}$
0	500.89 (467.81)	502.67 (480.59)	502.26 (490.62)
0.05	433.08 (408.13)	429.51 (418.26)	450.92 (437.58)
0.07	372.37 (372.01)	388.85 (378.13)	402.03 (399.51)
0.10	298.03 (280.16)	306.48 (301.85)	331.49 (328.98)
0.12	252.42 (238.84)	264.33 (250.86)	288.13 (281.30)
0.15	196.88 (181.88)	210.01 (197.36)	227.09 (222.25)
0.17	165.58 (152.62)	183.58 (168.70)	194.12 (190.00)
0.20	132.19 (119.01)	142.31 (124.82)	155.28 (149.86)
0.22	112.12 (102.10)	124.86 (107.06)	131.27 (124.95)
0.25	91.59 (77.77)	98.42 (85.18)	108.13 (101.93)
0.27	81.47 (68.91)	85.68 (72.70)	94.03 (87.56)
0.30	68.32 (55.59)	72.04 (61.74)	78.06 (71.98)
0.35	50.54 (41.36)	53.32 (43.09)	58.54 (51.77)
0.40	40.02 (30.45)	42.04 (31.89)	45.48 (38.95)
0.45	31.81 (24.25)	33.91 (25.49)	36.28 (29.91)
0.50	23.77 (19.49)	27.46 (20.80)	29.36 (23.56)
0.60	16.25 (13.17)	19.73 (13.98)	20.91 (16.08)
0.70	13.94 (9.71)	15.02 (10.21)	15.56 (11.19)
0.80	10.67 (7.43)	11.95 (7.60)	12.36 (8.39)
0.90	9.48 (5.79)	9.96 (6.19)	10.03 (6.53)
1.00	8.18 (4.79)	8.20 (4.99)	8.78 (5.25)

Table 2
ARLs(SDRLs) of proposed and existing control charts when $b_0 = 500$ and $K = 3.001$.

Shift (**d**)	Proposed NEEWMA	EEWMA	EWMA
	$\boldsymbol{\alpha_1 = 0.2}$, $\boldsymbol{\alpha_2 = 0.06}$, $\boldsymbol{\alpha_3 = 0.02}$	$\boldsymbol{\alpha_1 = 0.2}$, $\boldsymbol{\alpha_2 = 0.06}$	$\boldsymbol{\alpha_1 = 0.2}$
0	498.87 (464.39)	500.65 (471.42)	504.00 (486.08)
0.05	461.19 (437.62)	477.12 (448.85)	495.39 (471.04)
0.07	422.81 (401.68)	437.37 (407.78)	460.27 (437.48)
0.10	362.89 (347.28)	374.99 (369.42)	409.59 (393.85)
0.12	315.78 (303.77)	334.15 (322.31)	364.34 (353.56)
0.15	258.52 (246.41)	267.47 (262.71)	301.54 (301.32)
0.17	222.34 (208.95)	232.81 (228.83)	264.84 (263.50)
0.20	182.48 (175.85)	190.45 (186.97)	224.48 (223.91)
0.22	160.64 (152.84)	167.34 (161.24)	195.50 (194.03)
0.25	130.14 (120.43)	139.56 (130.06)	159.62 (157.91)
0.27	117.28 (106.52)	129.94 (113.37)	144.40 (141.49)
0.30	97.03 (86.69)	110.72 (96.12)	119.60 (115.41)
0.35	73.22 (63.10)	78.05 (68.13)	88.82 (84.94)
0.40	55.53 (47.65)	59.80 (51.19)	68.17 (63.37)
0.45	44.16 (36.36)	47.07 (38.94)	53.12 (49.71)
0.50	34.48 (28.80)	39.69 (31.17)	42.25 (37.75)
0.60	23.08 (18.82)	25.59 (20.32)	28.94 (24.38)
0.70	18.21 (13.10)	19.57 (13.82)	20.88 (17.05)
0.80	12.60 (9.63)	14.39 (10.04)	15.68 (11.88)
0.90	11.38 (7.38)	11.44 (7.54)	12.11 (8.72)
1.00	9.41 (5.78)	9.64 (6.07)	9.97 (6.97)

Table 3
ARLs(SDRLs) of proposed and existing control charts when $b_0 = 370$ and $K = 2.7194$.

Shift (**d**)	Proposed NEEWMA	EEWMA	EWMA
	$\boldsymbol{\alpha_1 = 0.1}$, $\boldsymbol{\alpha_2 = 0.03}$, $\boldsymbol{\alpha_3 = 0.01}$	$\boldsymbol{\alpha_1 = 0.1}$, $\boldsymbol{\alpha_2 = 0.03}$	$\boldsymbol{\alpha_1 = 0.1}$
0	368.41 (359.45)	370.34 (358.65)	368.22 (367.61)
0.05	318.84 (309.30)	327.30 (323.89)	326.05 (331.58)
0.07	291.58 (281.29)	299.01 (289.79)	300.36 (297.11)
0.10	229.21 (221.33)	239.34 (234.29)	250.73 (250.69)
0.12	200.50 (188.89)	211.82 (198.92)	220.48 (218.70)
0.15	149.62 (143.93)	161.46 (154.66)	175.89 (173.52)
0.17	133.32 (123.00)	139.88 (127.82)	149.72 (143.86)
0.20	108.06 (94.88)	110.67 (103.92)	119.18 (115.91)
0.22	93.87 (83.85)	99.77 (87.46)	104.76 (100.39)
0.25	76.31 (67.27)	80.93 (71.65)	86.47 (81.37)
0.27	67.00 (56.93)	69.68 (62.01)	75.67 (70.73)
0.30	55.32 (47.93)	59.19 (50.30)	64.59 (59.08)
0.35	44.45 (35.34)	47.48 (37.56)	48.31 (42.57)
0.40	27.59 (27.22)	35.72 (29.07)	38.83 (33.74)
0.45	24.11 (21.29)	29.25 (22.72)	31.73 (26.69)
0.50	17.11 (17.30)	24.38 (18.62)	25.77 (20.56)
0.60	16.74 (12.20)	17.35 (12.45)	18.42 (13.91)
0.70	12.37 (8.82)	13.04 (7.27)	14.37 (10.38)
0.80	10.60 (6.88)	9.11 (5.79)	11.13 (7.65)
0.90	8.60 (5.50)	9.58 (4.66)	10.00 (5.95)
1.00	7.21 (4.65)	8.83 (5.58)	9.74 (4.98)

Table 4
ARLs(SDRLs) of proposed and existing control charts when $b_0 = 370$ and $K = 2.891$.

Shift (**d**)	Proposed NEEWMA	EEWMA	EWMA
	$\boldsymbol{\alpha_1 = 0.2}$, $\boldsymbol{\alpha_2 = 0.06}$, $\boldsymbol{\alpha_3 = 0.02}$	$\boldsymbol{\alpha_1 = 0.2}$, $\boldsymbol{\alpha_2 = 0.06}$	$\boldsymbol{\alpha_1 = 0.2}$
0	370.97 (367.77)	370.87 (359.56)	383.56 (374.89)
0.05	343.94 (335.54)	360.39 (343.75)	364.01 (355.80)
0.07	314.58 (310.10)	327.61 (319.41)	337.42 (334.37)
0.10	272.51 (267.67)	286.34 (273.95)	302.10 (297.40)
0.12	245.50 (236.65)	251.49 (246.01)	274.80 (273.26)
0.15	198.99 (189.90)	209.31 (208.98)	229.83 (228.82)
0.17	173.28 (164.01)	185.96 (176.35)	203.90 (203.72)
0.20	144.74 (133.89)	157.99 (143.14)	173.26 (170.23)
0.22	129.50 (122.94)	139.69 (123.94)	150.92 (150.01)
0.25	107.24 (99.25)	113.35 (103.12)	125.96 (123.90)
0.27	93.86 (86.04)	99.20 (91.04)	113.74 (111.52)
0.30	79.70 (70.38)	88.51 (77.02)	95.05 (93.23)
0.35	60.30 (54.10)	65.90 (54.79)	72.59 (68.31)
0.40	47.63 (41.15)	49.17 (42.36)	56.12 (52.88)
0.45	37.16 (31.16)	41.87 (33.54)	44.86 (40.58)
0.50	30.29 (24.98)	34.62 (26.59)	36.01 (31.99)
0.60	19.29 (16.54)	22.95 (17.57)	24.95 (21.18)
0.70	16.18 (11.63)	16.98 (12.37)	18.23 (14.67)
0.80	13.02 (8.56)	13.23 (9.30)	13.96 (10.64)
0.90	9.88 (6.78)	10.52 (6.96)	11.19 (8.12)
1.00	8.13 (5.39)	8.83 (5.58)	8.96 (6.24)

diminishes. In comparison to the EWMA and EEWMA control charts, it is found that the proposed control chart offered more modest ARLs for all shifts in the process mean. Additionally, the proposed control chart is more consistent in comparison to the EWMA and EEWMA control charts for having relatively lesser values of SDRL.

Besides the general findings, some specific results are also observed as follows.

i. When $d = 0$, the ARL is close to the target value b_0.
ii. For a specified value of shift size d, the values of ARL_1 increase as the smoothing constant α_1 increases.

Table 5
ARLs(SDRLs) of proposed and existing control charts when $b_0 = 300$ and $K = 2.631$.

Shift (**d**)	Proposed NEEWMA	EEWMA	EWMA
	$\alpha_1 = 0.1$, $\alpha_2 = 0.03$, $\alpha_3 = 0.01$	$\alpha_1 = 0.1$, $\alpha_2 = 0.03$	$\alpha_1 = 0.1$
0	300.00 (295.78)	300.09 (304.63)	300.17 (300.54)
0.05	252.80 (257.32)	265.23 (265.49)	265.39 (268.00)
0.07	225.23 (231.06)	238.49 (239.35)	244.30 (243.73)
0.10	188.65 (182.03)	193.93 (191.05)	206.32 (205.75)
0.12	164.10 (154.08)	176.61 (164.69)	180.31 (181.00)
0.15	132.88 (125.13)	138.73 (130.32)	145.81 (147.29)
0.17	114.67 (104.21)	120.40 (115.73)	129.29 (121.98)
0.20	94.25 (83.14)	100.34 (88.25)	107.09 (100.80)
0.22	80.02 (72.09)	86.37 (76.48)	90.69 (86.53)
0.25	62.28 (59.27)	68.34 (63.15)	74.31 (69.34)
0.27	60.39 (52.20)	62.39 (54.05)	67.49 (63.65)
0.30	49.53 (42.82)	53.75 (44.07)	59.27 (52.94)
0.35	37.65 (31.43)	40.60 (33.82)	43.87 (39.09)
0.40	30.90 (24.99)	32.09 (26.27)	34.74 (30.09)
0.45	25.31 (20.23)	26.50 (21.03)	28.36 (24.05)
0.50	21.76 (16.09)	22.18 (17.10)	23.79 (19.31)
0.60	14.02 (11.22)	16.37 (11.86)	17.20 (13.11)
0.70	11.63 (8.54)	12.79 (8.86)	13.20 (9.54)
0.80	10.28 (6.58)	10.34 (6.93)	10.45 (7.26)
0.90	8.11 (5.34)	8.45 (5.44)	8.65 (5.88)
1.00	7.11 (4.48)	7.17 (4.43)	7.26 (4.81)

Table 6
ARLs(SDRLs) of proposed and existing control charts when $b_0 = 300$ and $K = 2.8126$.

Shift (**d**)	Proposed NEEWMA	EEWMA	EWMA
	$\alpha_1 = 0.2$, $\alpha_2 = 0.06$, $\alpha_3 = 0.02$	$\alpha_1 = 0.2$, $\alpha_2 = 0.06$	$\alpha_1 = 0.2$
0	301.35 (300.36)	301.01 (302.50)	307.29 (311.56)
0.05	282.42 (277.36)	285.01 (281.09)	289.19 (295.44)
0.07	255.16 (249.66)	262.59 (254.92)	278.95 (280.96)
0.10	224.49 (212.99)	233.10 (230.09)	248.32 (248.11)
0.12	198.41 (192.16)	207.90 (203.41)	220.71 (220.44)
0.15	166.75 (160.27)	173.03 (167.08)	189.93 (187.16)
0.17	148.09 (138.14)	155.91 (148.64)	172.28 (170.70)
0.20	124.38 (116.90)	132.00 (122.06)	142.46 (139.20)
0.22	109.16 (101.49)	118.44 (105.41)	127.16 (126.04)
0.25	91.67 (82.85)	99.83 (88.92)	108.04 (104.72)
0.27	80.04 (73.26)	87.97 (77.76)	95.17 (92.56)
0.30	68.51 (64.21)	72.81 (65.50)	81.68 (78.87)
0.35	52.80 (46.50)	56.68 (49.58)	61.63 (58.85)
0.40	41.33 (36.17)	43.89 (38.15)	49.34 (45.49)
0.45	33.11 (28.28)	36.03 (30.23)	40.42 (36.13)
0.50	27.61 (22.84)	29.80 (23.97)	32.83 (28.82)
0.60	17.76 (15.24)	20.52 (16.09)	22.25 (18.78)
0.70	13.75 (10.80)	15.49 (11.43)	16.60 (13.56)
0.80	11.59 (8.26)	12.11 (8.44)	12.88 (9.78)
0.90	9.30 (6.43)	10.01 (6.61)	10.27 (7.32)
1.00	8.44 (5.20)	8.54 (5.46)	8.42 (5.85)

iii. For other specified values, the values of ARL_1 increase as the value of b_0 increases. For instance, when $b_0 = 370$, $\alpha_1 = 0.10, \alpha_2 = 0.03, \alpha_3 = 0.01$ and $d = 0.15$ the value of ARL_1 is 149.62 (Table 3), the value of ARL_1 is 196.88 when $b_0 = 500$ (Table 1). Hence it is noticed from Tables 1–6, that if the value of b_0 is high, the decreasing trend of ARL_1 is high.
iv. It is also observed that a large shift in the process is recognized more rapidly. For instance, when $b_0 = 300$, $\alpha_1 = 0.10, \alpha_2 = 0.03, \alpha_3 = 0.01$ and $d = 0.05$, the value of ARL_1 is 252.80 and for $d = 0.2$ it is just 94.25 (Table 5).

7. Comparative study

Here, we examine the comparison between the NEEWMA chart, the EEWMA chart, and the traditional EWMA chart. In terms of the

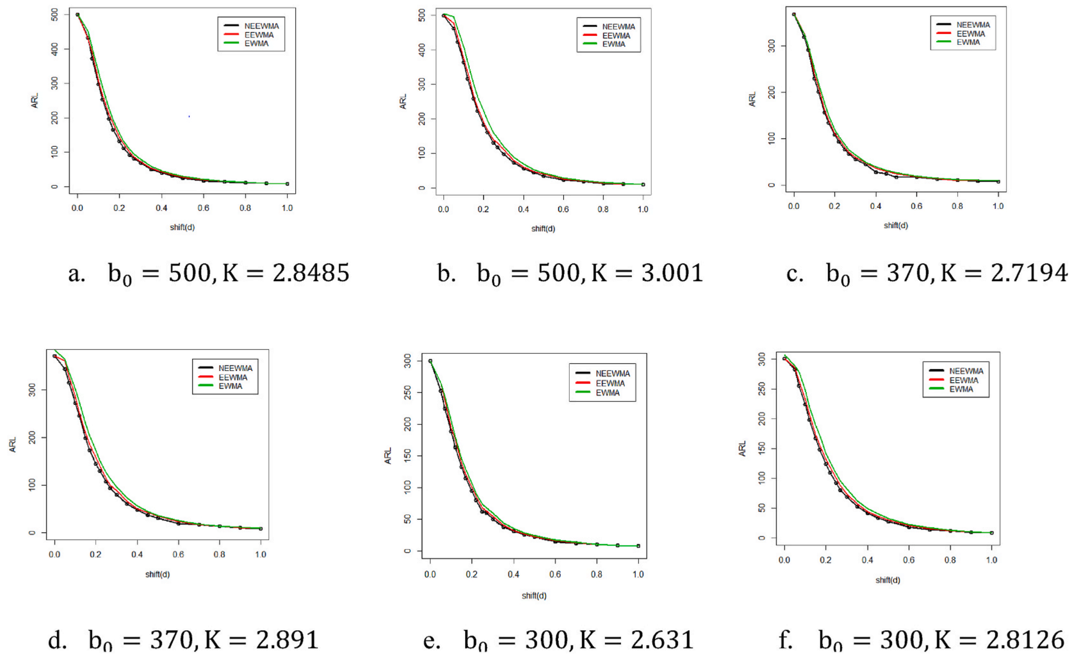

Fig. 1. Average Run Lengths (ARL) comparison of proposed and existing control charts.

ARL, the efficiency of the proposed control chart was examined in Tables 1–6 The graphical illustration of Fig. 1 (Panels a–f) shows that the proposed chart identifies the process shift quickly as compared to the other existing charts.

7.1. NEEWMA control chart versus EEWMA control chart

Here, the superiority of the NEEWMA control chart in comparison with the EEWMA control chart proposed by Naveed et al. [12] is discussed. Using target in-control ARL values as $b_0 = 500, 370, 300$, the out-of-control ARLs of the EEWMA and NEEWMA control charts are computed in Tables 1–6. It has been examined that the ARL_1 values for the NEEWMA control chart are less than the EEWMA chart for all values of shift parameter d. For instance, when $b_0 = 370$, $\alpha_1 = 0.10, \alpha_2 = 0.03, \alpha_3 = 0.01$ and $d = 0.15$, the ARL_1 value for the proposed control chart was 149.62, whereas it was 161.46 for EEWMA chart (Table 3), which represents that NEEWMA control chart is more sensitive as compared to EEWMA control chart, illustrates the proposed control chart has a superior ability to recognize slight changes in the process.

7.2. NEEWMA control chart versus EWMA control chart

The superiority and sensitivity of the NEEWMA control chart in comparison with the traditional EWMA control chart are discussed. For this purpose, Using target in-control ARL values $b_0 = 500, 370, 300$, we computed the out-of-control ARLs for the EEWMA and NEEWMA control charts in Tables 1–6. Our analysis revealed that ARL_1 values for the NEEWMA control chart are consistently lower than those for the EEWMA chart across all shift parameter values (d). For example, with $b_0 = 370$, $\alpha_1 = 0.10, \alpha_2 = 0.03, \alpha_3 = 0.01$ and $d = 0.15$, the ARL_1 value for the proposed control chart was 149.62, compared to 175.89 for the EWMA chart (Table 3). This indicates that the NEEWMA control chart is more sensitive than the EWMA control chart, highlighting its superior ability to detect even slight changes in the process.

We can also observe from the line graphs of ARL values for EWMA and NEEWMA control charts under specific values of parameters in the different panels of Fig. 1. It is examined that ARL_1 values for the NEEWMA control chart are smallest in every case and at every shift size d which represents the greater efficiency of the proposed control chart to identify small to large shifts in the process when contrasted with the EWMA control chart.

8. Illustrative examples

In this section, two datasets are taken into consideration to justify the practical use of the proposed NEEWMA control chart. In the first example simulated data set is generated while in the second example, the UTI data set taken from Ref. [20] is used.

Table 7
Simulated data set.

Sample Number	Simulated Data (**t**)	NEEWMA Control Chart			EEWMA Control Chart			EWMA Control Chart		
		$\alpha_1 = 0.3$	$\alpha_2 = 0.12$	$\alpha_3 = 0.04$	$\alpha_1 = 0.3$		$\alpha_2 = 0.12$	$\alpha_1 = 0.3$		
		$K = 2.952$			$K = 2.952$			$K = 2.952$		
		Z	LCL	UCL	Z	LCL	UCL	Z	LCL	UCL
1	−1.6175	−0.4853	−1.3286	1.3286	−0.4853	−0.9538	0.9538	−0.4853	−0.8856	0.8856
2	1.2523	0.1525	−1.2676	1.2676	0.1719	−1.0035	1.0035	0.0360	−1.0810	1.0810
3	0.5445	0.2089	−1.2206	1.2206	0.1540	−1.0356	1.0356	0.1886	−1.1649	1.1649
4	−0.6075	−0.1180	−1.1846	1.1846	−0.1213	−1.0566	1.0566	−0.0503	−1.2038	1.2038
5	1.7477	0.4739	−1.1573	1.1573	0.4978	−1.0705	1.0705	0.4891	−1.2224	1.2224
6	0.6117	0.4057	−1.1366	1.1366	0.3819	−1.0797	1.0797	0.5259	−1.2315	1.2315
7	−0.4785	0.0620	−1.1211	1.1211	0.0962	−1.0859	1.0859	0.2246	−1.2359	1.2359
8	0.4403	0.2184	−1.1095	1.1095	0.2684	−1.0900	1.0900	0.2893	−1.2380	1.2380
9	−2.0762	−0.4688	−1.1008	1.1008	−0.4556	−1.0928	1.0928	−0.4204	−1.2391	1.2391
10	0.1428	−0.1288	−1.0944	1.0944	−0.0816	−1.0946	1.0946	−0.2514	−1.2396	1.2396
11	−1.5573	−0.5120	−1.0896	1.0896	−0.5512	−1.0959	1.0959	−0.6432	−1.2398	1.2398
12	−0.4389	−0.3908	−1.0860	1.0860	−0.3968	−1.0967	1.0967	−0.5819	−1.2400	1.2400
13	1.873	0.3407	−1.0834	1.0834	0.2892	−1.0973	1.0973	0.1546	−1.2400	1.2400
14	−0.8742	−0.1764	−1.0814	1.0814	−0.2499	−1.0977	1.0977	−0.1541	−1.2401	1.2401
15	−0.0069	−0.1238	−1.0800	1.0800	−0.1021	−1.0979	1.0979	−0.1099	−1.2401	1.2401
16	−0.8811	−0.3350	−1.0789	1.0789	−0.3472	−1.0981	1.0981	−0.3413	−1.2401	1.2401
17	0.6933	0.0259	−1.0781	1.0781	0.0290	−1.0982	1.0982	−0.0309	−1.2401	1.2401
18	−0.7696	−0.2566	−1.0775	1.0775	−0.2903	−1.0983	1.0983	−0.2525	−1.2401	1.2401
19	1.2854	0.2296	−1.0771	1.0771	0.2399	−1.0983	1.0983	0.2089	−1.2401	1.2401
20	1.2179	0.4394	−1.0767	1.0767	0.4079	−1.0984	1.0984	0.5116	−1.2401	1.2401
21	0.0586	0.1979	−1.0765	1.0765	0.2059	−1.0984	1.0984	0.3757	−1.2401	1.2401
22	−0.4834	−0.0306	−1.0763	1.0763	0.0168	−1.0984	1.0984	0.1180	−1.2401	1.2401
23	0.2468	0.1034	−1.0762	1.0762	0.1458	−1.0984	1.0984	0.1566	−1.2401	1.2401
24	−0.9335	−0.2014	−1.0761	1.0761	−0.1901	−1.0984	1.0984	−0.1704	−1.2401	1.2401
25	1.4671	0.3691	−1.0760	1.0760	0.3963	−1.0984	1.0984	0.3208	−1.2401	1.2401
26	3.0964	1.1076	−1.0760	1.0760	1.0778	−1.0984	1.0984	1.1535	−1.2401	1.2401
27	−0.6644	0.3230	−1.0759	1.0759	0.3129	−1.0984	1.0984	0.6081	−1.2401	1.2401
28	1.0888	0.5603	−1.0759	1.0759	0.6629	−1.0984	1.0984	0.7523	−1.2401	1.2401
29	1.7993	0.9175	−1.0759	1.0759	0.9528	−1.0984	1.0984	1.0664	−1.2401	1.2401
30	−0.0737	0.5075	−1.0759	1.0759	0.5432	−1.0985	1.0985	0.7244	−1.2401	1.2401
31	−2.347	−0.3308	−1.0759	1.0759	−0.2498	−1.0985	1.0985	−0.1970	−1.2401	1.2401
32	−0.1644	−0.0492	−1.0759	1.0759	0.0275	−1.0985	1.0985	−0.1872	−1.2401	1.2401
33	−0.4831	−0.0736	−1.0758	1.0758	−0.1027	−1.0985	1.0985	−0.2760	−1.2401	1.2401
34	−1.305	−0.3903	−1.0758	1.0758	−0.4177	−1.0985	1.0985	−0.5847	−1.2401	1.2401
35	0.0218	−0.1532	−1.0758	1.0758	−0.1794	−1.0985	1.0985	−0.4027	−1.2401	1.2401
36	−0.2792	−0.1659	−1.0758	1.0758	−0.2335	−1.0985	1.0985	−0.3657	−1.2401	1.2401
37	0.2092	−0.0473	−1.0758	1.0758	−0.0952	−1.0985	1.0985	−0.1932	−1.2401	1.2401
38	1.5471	0.4095	−1.0758	1.0758	0.3610	−1.0985	1.0985	0.3289	−1.2401	1.2401
39	1.6299	0.6471	−1.0758	1.0758	0.5993	−1.0985	1.0985	0.7192	−1.2401	1.2401
40	−1.0066	−0.0029	−1.0758	1.0758	−0.0061	−1.0985	1.0985	0.2014	−1.2401	1.2401
41	1.1603	0.4012	−1.0758	1.0758	0.4639	−1.0985	1.0985	0.4891	−1.2401	1.2401
42	0.7083	0.4585	−1.0758	1.0758	0.4536	−1.0985	1.0985	0.5549	−1.2401	1.2401
43	−1.1312	−0.0764	−1.0758	1.0758	−0.0524	−1.0985	1.0985	0.0490	−1.2401	1.2401
44	0.588	0.2181	−1.0758	1.0758	0.2692	−1.0985	1.0985	0.2107	−1.2401	1.2401
45	2.0264	0.7702	−1.0758	1.0758	0.7581	−1.0985	1.0985	0.7554	−1.2401	1.2401
46	0.2618	0.4742	−1.0758	1.0758	0.4570	−1.0985	1.0985	0.6073	−1.2401	1.2401
47	−0.0456	0.2817	−1.0758	1.0758	0.3296	−1.0985	1.0985	0.4115	−1.2401	1.2401
48	0.6755	0.4399	−1.0758	1.0758	0.4784	−1.0985	1.0985	0.4907	−1.2401	1.2401
49	1.7025	0.8098	−1.0758	1.0758	0.8220	−1.0985	1.0985	0.8542	−1.2401	1.2401
50	0.0891	0.4918	−1.0758	1.0758	0.4965	−1.0985	1.0985	0.6247	−1.2401	1.2401

8.1. Simulated data example

To check the functioning strategy of the NEEWMA control chart, a simulation study is conducted. Fifty observations are randomly generated for this purpose. The first 25 observations are simulated from standard normal distribution i.e. $N(0,1)$ for an IC process while for contamination of shift in mean using the expression $\mu_1 = \mu_0 + d\sigma$, the next 25 observations are generated from normal distribution when $d = 0.20$ and standard deviation one i.e. $N(0.20,1)$. At fixed values of targeted ARL ($b_0 = 370$) and control charts co-efficient ($K = 2.952$), the estimated values of the proposed NEEWMA statistic are calculated at predetermined levels of charting parameters i.e. $\alpha_1 = 0.30, \alpha_2 = 0.12$ and $\alpha_3 = 0.04$. Correspondingly estimated values of the EEWMA statistic are calculated with control chart parameters $\alpha_1 = 0.30$ and $\alpha_2 = 0.12$. Likewise, estimated values of the traditional EWMA statistic are also calculated using $\alpha_1 = 0.30$. In Table 7, data and values of the NEEWMA, EEWMA, and EWMA statistics are presented. Furthermore, these values are plotted in Figs. 2–4. It can be observed in Fig. 2, the proposed NEEWMA control chart has recognized a shift in the 26th sample, on

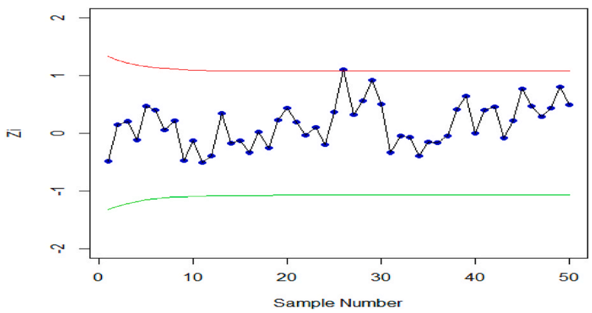

Fig. 2. NEEWMA control chart based on simulation data.

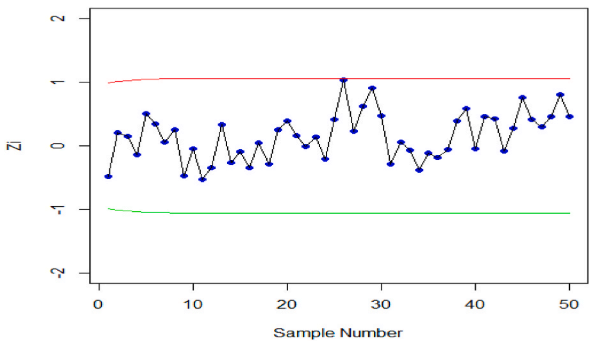

Fig. 3. EEWMA control chart based on simulation data.

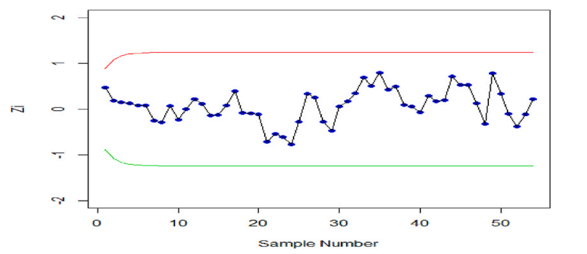

Fig. 4. EWMA control chart based on simulation data.

the other hand, EEWMA and EWMA control charts couldn't recognize that shift (Figs. 3–4). Subsequently, this indicates that the proposed NEEWMA control chart has a more prominent capacity to quickly identify the shifts when contrasted with the EWMA and EEWMA control charts.

8.2. Practical data example

Here, we made a comparison of the NEEWMA, EEWMA, and EWMA charts by utilizing information taken from Santiago and Smith (2013) for the purpose of application. The information is concerned about urinary tract infection (UTI) which was obtained from a clinic. The administration of the medical clinic needed to comprehend the quantity of patients (individuals) being released from the medical clinic who had obtained a UTI, as an approach to quickly distinguish the expansion in disease rate or, on the other hand, to see whether the cycle brought about a decrease of contamination. The information passages are recorded in Table 8, following an Exponential Distribution (ED) with a mean time of detection among male UTI individuals at 0.21 days, or around 5 h. Firstly, transformed information into Normal Distribution (ND) and then plotted for NEEWMA, EEWMA, and EWMA control charts, as illustrated in Figs. 5–7 and these figures show that the process is IC.

9. Conclusion

In this article, we introduced the NEEWMA control chart assuming a ND for the quality characteristic studied. By varying

Table 8
Urinary Tract Infections (UTI) data set among patients.

Detection time of UTI patients (in days)					
0.57014	0.03819	0.12014	0.01389	0.27083	0.24653
0.07431	0.24653	0.11458	0.03819	0.04514	0.04514
0.15278	0.29514	0.00347	0.46806	0.13542	0.01736
0.14583	0.11944	0.12014	0.22222	0.08681	1.08889
0.13889	0.05208	0.04861	0.29514	0.40347	0.05208
0.14931	0.12500	0.02778	0.53472	0.12639	0.02778
0.03333	0.25000	0.32639	0.15139	0.18403	0.03472
0.08681	0.40069	0.64931	0.52569	0.70833	0.23611
0.33681	0.02500	0.14931	0.07986	0.15625	0.35972

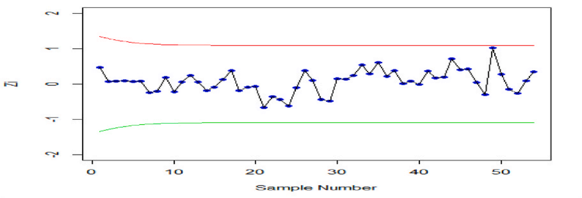

Fig. 5. NEEWMA control chart based on UTI data.

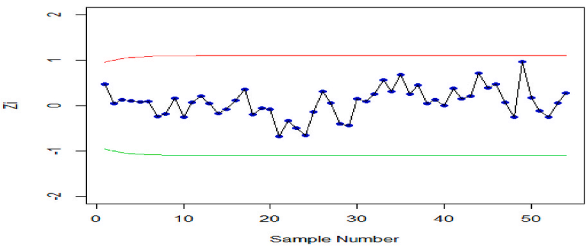

Fig. 6. EEWMA control chart based on UTI data.

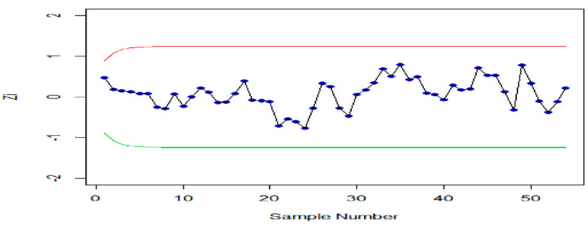

Fig. 7. EWMA control chart based on UTI data.

smoothing constants and shift levels, we computed Average Run Length (ARL) values to compare its performance with EWMA statistics and EEWMA control charts. The results clearly demonstrated the NEEWMA control chart's superior efficiency in detecting process shifts compared to EEWMA and EWMA control charts. Illustrative examples using both simulated and practical datasets further validated these findings, suggesting the potential utilization of NEEWMA in the industrial sector for monitoring manufacturing processes. However, it's important to note that this study's applicability is limited to situations where data follow a normal distribution or have been transformed to fit such a distribution. Future research avenues could explore integrating the proposed statistic into the development of hybrid EWMA statistics, as suggested [21,22] for more comprehensive analysis and applications.

Data availability statement

Data included in article. The data utilized in this analysis, as presented in Tables 8 and is sourced from a published article [20]. We have employed this dataset for our application, as it is readily available in their study.

CRediT authorship contribution statement

Hina Javed: Writing – review & editing, Visualization, Validation, Software, Methodology, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Muhammad Ismail:** Writing – original draft, Supervision, Methodology, Investigation, Conceptualization. **Nadia Saeed:** Writing – review & editing, Validation, Software, Methodology, Investigation, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] S.W. Roberts, Control chart tests based on geometric moving averages, Technometrics 1 (3) (1959) 239–250.
[2] S.H. Steiner, EWMA control charts with time-varying control limits and fast initial response, J. Qual. Technol. 31 (1) (1999) 75–86.
[3] M. Eyvazian, Naini S.G. Jalali, Vaghefi, A., Monitoring process variability using exponentially weighted moving sample variance control charts, Int. J. Adv. Manuf. Technol. 39 (2008) 261–270.
[4] S.F. Yang, J.S. Lin, S.W. Cheng, A new nonparametric EWMA sign control chart, Expert Syst. Appl. 38 (5) (2011) 6239–6243.
[5] N. Abbas, M. Riaz, R.J. Does, Mixed exponentially weighted moving average–cumulative sum charts for process monitoring, Qual. Reliab. Eng. Int. 29 (3) (2013) 345–356.
[6] N. Abbas, M. Riaz, R.J. Does, An EWMA-type control chart for monitoring the process mean using auxiliary information. Commun. Stat, Theory Methods 43 (16) (2014) 3485–3498.
[7] E.I. Abouelmagd, M.E. Awad, E.M.A. Elzayat, I.A. Abbas, Reduction the secular solution to periodic solution in the generalized restricted three-body problem, Astrophys. Space Sci. 350 (2014) 495–505.
[8] M. Aslam, M. Azam, C.H. Jun, A new control chart for exponential distributed life using EWMA, Trans. Inst. Meas. Control 37 (2) (2015) 205–210.
[9] P.S. Hariba, S.D. Tukaram, Economic design of a nonparametric EWMA control chart for location, Production 26 (2016) 698–706.
[10] Y. Liu, W. Liu, M.A. Obaid, I.A. Abbas, Exponential stability of Markovian jumping Cohen–Grossberg neural networks with mixed mode-dependent time-delays, Neurocomputing 177 (2016) 409–415.
[11] B. Du, Y. Liu, I.A. Abbas, Existence and asymptotic behavior results of periodic solution for discrete-time neutral-type neural networks, J. Franklin Inst. 353 (2) (2016) 448–461.
[12] M. Naveed, M. Azam, N. Khan, M. Aslam, Design of a control chart using extended EWMA statistic, Technologies 6 (4) (2018) 108.
[13] N. Saeed, M.O. Abu-Shawiesh, A trimmed EWMA control charts for non-normal processes. Prod, Eng. 2021 15 (3–4) (2021) 545–561.
[14] A.A. Aly, N.A. Saleh, M.A. Mahmoud, An adaptive EWMA control chart for monitoring zero-inflated Poisson processes. Commun, Stat. Simul. 51 (4) (2022) 1564–1577.
[15] M. Anastasopoulou, A.C. Rakitzis, EWMA control charts for monitoring correlated counts with finite range, J. Appl. Stat. 49 (3) (2022) 553–573.
[16] M.Z. Khan, M.F. Khan, M. Aslam, A.R. Mughal, A study on average run length of fuzzy EWMA control chart, Soft Comput. 26 (18) (2022) 9117–9124.
[17] S. Noor, M. Noor-ul-Amin, M. Mohsin, A. Ahmed, Hybrid exponentially weighted moving average control chart using Bayesian approach, Commun. Stat. Theor. Methods 51 (12) (2022) 3960–3984.
[18] N. Saeed, A.F. Obeidat, S. Kamal, M.O. Abu-Shawiesh, Robust point M-scale estimators for monitoring the mean of EWMA control chart under the normal process, Ain Shams Eng 13 (6) (2022) 101796.
[19] S.V. Crowder, A simple method for studying run–length distributions of exponentially weighted moving average charts, Technometrics 29 (4) (1987) 401–407.
[20] E. Santiago, J. Smith, Control charts based on the exponential distribution: adapting runs rules for the t chart, Qual. Eng. 25 (2) (2013) 85–96.
[21] A. Haq, A new hybrid exponentially weighted moving average control chart for monitoring process mean, Qual. Reliab. Eng. Int. 29 (7) (2013) 1015–1025.
[22] A. Haq, A new hybrid exponentially weighted moving average control chart for monitoring process mean: discussion, Qual. Reliab. Eng. Int. 33 (7) (2017) 1629–1631.