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Research article

On soft parametric somewhat-open sets and applications via soft topologies

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A B S T R A C T

In this work, we adopt a new approach to study a new class of soft sets depending on the generalizations of open subsets in the parametric topological spaces. We first define the class of soft parametric somewhat-open sets and explore its basic features. We illustrate this class represents a proper extension of soft open and soft somewhat-open sets under a full soft topology. We derive the next formula

$$1+\prod_{\eta\in H}(|\Theta_\eta|-1)\leq|_F|\leq 1+(2^{|\mathfrak{U}|}-1)^{|H|},$$

which determines the lower and upper bounds of the cardinality number $_F$ of the family of soft parametric somewhat-open subsets of a soft topological space $(\mathfrak{U},\Theta,H)$, where Θ_η is a parametric topology inspired by Θ. Then, we introduce two novel kinds of soft compact and Lindelöf spaces inspired by the class of soft parametric somewhat-open sets and explain the relations between them with the aid of some counterexamples. We also examine the navigation of these spaces between soft and parametric (classical) structures and supply the necessary conditions that guarantee some directions. In the end, we introduce the concept of soft ps-connected spaces and give some of its equivalent descriptions. Furthermore, we prove the identity between this concept and soft hyperconnected spaces and show that the existence of a somewhat connected (parametric) space is used to confirm the possession of a soft ps-connected property.

1. Introduction

It was introduced the idea of soft sets in 1999 by Molodtsov [1] as a novel mathematical strategy for cope with uncertainties. This approach proved its useability and substance to address many real-life issues in different disciplines such as medical science (i.e. nutrition systems [2] and Covid-19 outbreak [3,4]), information system [5] and decision-making [6–8], etc. Over the past two decades, the essential principles of soft set theory have been established and studied by many authors [9,10]. During this period, it has been reviewed and adjusted the fundamentals of this theory in a way that is appropriate to use to deal with a large scope of theoretical and applied usage; see, [11,12].

Pakistani researchers Shabir-Naz [13], in 2011, set forth the structure so-called "soft topology" under the same corresponding terms of a classical topology, which are built under a constant set of parameters. In the same year, Turkish authors Çağman-Karataş-

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Enginoglu [14] presented soft topology in a different way than Shabir and Naz [13]; its terms are constructed under different subsets of parameters. It can be investigated the topological concepts via both structures by taking into account how these concepts behave under each one. In this work, we will follow the line of [13] and compare the obtained results and relationships with the published literature in this line.

Since the concept of soft topology was familiarized, many ideas of general topology have been explored and compared with their analogies in soft topologies. The basic concepts of soft operators and soft separation axioms were established by Shabir and Naz [13]. Min [15] proved that the relationships between soft T_3 and soft T_2 are similar to the systematic relation and he provided further properties of soft separation axioms. Losing of some properties by the previous types of soft separation axioms motivated us [16] to initiate new classifications inspired by new kinds of relations that describe the belonging and non-belonging of ordinary elements and soft sets. These relations were then exploited to initiate other soft sorts of separation axioms as introduced in the recent works by [17–19]. These concepts and others show the fruitful variety obtained from soft topological studies. Aygünoğlu and Aygün [20] displayed two of the main topological concepts "compactness and Lindelöfness" via soft topologies and scrutinized main features. They were popularized by Al-shami et al. [21]. By applying the relation (called recently total-belong), Hida [22] introduced and probed anther type of soft compact spaces. Compactness and Lindelöfness and their extensions (like almost and nearly compactness and Lindelöfness) have been recently defined with respect to some soft open sets generalizations such as soft regular closed [23], soft somewhere dense [24] and soft somewhat-open sets [25]. These contributions result in producing some pertinent examples with covering properties. As an important point, researchers and scholars should be noted that some classical properties of compactness and Lindelöfness are evaporated via soft topologies as elaborated by Al-shami [16]. Kočinac et al. [26] discussed selection principles via soft setting and introduced the concept of soft Menger spaces which were then generalized by him and Al-shami [27,28].

By analogue of classical connectedness, it was formulated the concept of soft connected spaces in [29,30]. Some contributions have been conducted to discover the main properties of this concept and generalized it. Among them, Asaad [31] defined extremally soft disconnected spaces and Ameen and Al-Ghour [32] presented the notion of a maximal soft connected topology. Also, Al-shami et al. described soft connectedness utilizing soft somewhat-open and soft somewhere dense sets which formulated in [25] and [24], respectively. One of the interesting findings that was proved is that the notions of soft sw-connectedness and soft hyperconnectedness are correspondent.

The definition of soft mappings was first given by Kharal and Ahmad [33] by combining two crisp mappings. This definition was updated by Al-shami [34] in order to decrease burden of calculation and difficulty that arises from the forgoing one. Soft continuity and soft homeomorphism were characterized by [30,35]. The concepts of soft expandable spaces and soft vietoris topology were introduced in [36] and [37]. The link between soft and fuzzy topologies was researched by Alcantud [38]. Production of soft topologies by operators was investigated in [39,40].

Generalizations of soft open sets and their applications have been studied by some authors. Chen [41] launched this line by presenting soft semi-open sets and studying basic features. Then, it was introduced the class of soft α-open sets by Akdag and Ozkan [42]. Al-shami [43] displayed a new class called somewhere dense sets and applied to investigate soft continuity and soft compactness. The notions of soft Q-sets [44] and minimal soft sets [45] were defined by Al-Ghour. Recently, Ameen et al. [46] have provided several soft functions motivated by the family of soft somewhat-open sets, which were exploited to initiate some classes of separation axioms by Al-shami [2]. We draw attention of the readers to that the existing generalizations of soft open sets are formulated following a similar technique of their counterparts in classical topologies; that is, it was utilizing soft interior and closure operators to define them. But, Al-shami with his co-authors [47–49] came up with another methodology to study generalizations of soft open sets inspired by classical topologies induced from a soft topology. This methodology is limited to *one* parametric topology. The approach adopted in this manuscript to create generalizations of soft open sets is also based on the parametric topologies, but it is different from the aforementioned approach by stipulating all *all* parametric topologies induced from a soft topology instead of *one* parametric topology.

As it is well-known that classical topologies are special frames obtained from the soft-topologies when the set of parameters is a singleton. Another method to produce classical topologies Θ_η from a soft topology Θ is given in [13] as follows: $\Theta_\eta = \{\mathcal{F}(\eta) : (\mathcal{F}, \mathrm{H}) \in \Theta\}$. The links and navigation of the concepts and their properties from a soft topology to its classical topologies and vise versa have been revealed and discussed in some interesting studies [44,25]. Al-shami and Kočinac [50] demonstrated the interchangeable characteristic for the operators of interior and closure between specific kind of soft topology namely "extended soft topology" and their classical topologies.

It is designed this article as follows. After this introduction, we recall the fundamentals that are prerequisites to being familiar with the manuscript topics in Section 2. The core idea of this work is the concept of soft parametric somewhat-open sets introduced and studied in Section 3. Then, in Section 4 we employ soft parametric somewhat-open sets to establish four types of covering properties namely soft ps-compact, almost soft ps-compact, soft ps-Lindelöf and almost soft ps-Lindelöf spaces. These spaces are characterized and some interesting examples that reveal the interrelations between them are furnished. Also, we investigate the condition under which some features navigate from soft topologies to their classical topologies. The last main part is Section 5 which is appointed to research a new sort of connectedness called "soft sp-connectedness". We give some descriptions for it and elaborate on the role of full soft topology to obtain some properties and relations. In the end, we write Section 6 to outline the major contributions and demonstrate their unique characteristics as well as plan for some future work.

2. Preliminaries

This section will mention the needful knowledge to be familiar with the results and relationships produced in this article.

2.1. Theory of soft set

Definition 2.1. [1] Take a nonempty set H to represent a set of parameters. Then, we call a pair $(\mathcal{F},\mathrm{H})$ a soft set over the universal set $\mathfrak{U}$ provided that $\mathcal{F}$ is an ordinary mapping from H to the power set $2^{\mathfrak{U}}$ of $\mathfrak{U}$. Symbolically, $(\mathcal{F},\mathrm{H})=\{(\eta,\mathcal{F}(\eta)):\eta\in\mathrm{H}\text{ and }\mathcal{F}(\eta)\in 2^{\mathfrak{U}}\}$.

The image of each parameter $\mathcal{F}(\eta)$ represents a component of $(\mathcal{F},\mathrm{H})$.

We adopt in this manuscript the symbols H and $\mathfrak{U}$ to denote respectively a set of parameters and the universal set.

Definition 2.2. [11] If $\mathcal{G}(\eta)=\mathfrak{U}-\mathcal{F}(\eta)$ for each $\eta\in\mathrm{H}$, then it is called $(\mathcal{G},\mathrm{H})$ a complement of $(\mathcal{F},\mathrm{H})$. For simplicity, it will be refereed for the complement of a soft set $(\mathcal{F},\mathrm{H})$ by $(\mathcal{F},\mathrm{H})^c=(\mathcal{F}^c,\mathrm{H})$.

Definition 2.3. [1,51] $(\mathcal{F},\mathrm{H})$ is said to be:

(i) absolute soft set, refereed by $\widetilde{\mathfrak{U}}$, if $\mathcal{F}(\eta)=\mathfrak{U}$ for all $\eta\in\mathrm{H}$;
(ii) null soft set, refereed by ϕ, if all components are empty. In other words, it is a complement of the absolute soft set.
(iii) a soft-point if there exists a $\upsilon\in\mathfrak{U}$ such that $\mathcal{F}(\eta)=\{\upsilon\}$ for a fixed parameter $\eta\in\mathrm{H}$ and the image of all $\rho\in\mathrm{H}-\{\eta\}$ are empty. Herein, we use the symbol υ_η to refer to a soft-point;
(iv) pseudo constant soft set if all images are $\mathfrak{U}$ or $\emptyset$. That is, for all $\eta\in\mathrm{H}$ we have $\mathcal{F}(\eta)=\mathfrak{U}$ or $\emptyset$ for each $\eta\in\mathrm{H}$;
(v) finite (resp., infinite) soft set if $\mathcal{F}(\eta)$ is finite (resp., infinite) for all (resp., some) $\eta\in\mathrm{H}$. Note that the complement of an infinite soft set need not be a finite soft set.

Definition 2.4. [9] $(\mathcal{F},\mathrm{H})$ is considered as a soft-subset of $(\mathcal{G},\mathrm{H})$ (or $(\mathcal{G},\mathrm{H})$ as a soft-superset of $(\mathcal{F},\mathrm{H})$), refereed by $(\mathcal{F},\mathrm{H})\widetilde{\subseteq}(\mathcal{G},\mathrm{H})$ if $\mathcal{F}(\eta)\subseteq\mathcal{G}(\eta)$ for all $\eta\in\mathrm{H}$.

Definition 2.5. We define the operators of union and intersection between soft sets $(\mathcal{F},\mathrm{H})$ and $(\mathcal{K},\mathrm{H})$ as follows.

(i) $(\mathcal{F},\mathrm{H})\widetilde{\bigcup}(\mathcal{K},\mathrm{H})=(\mathcal{G},\mathrm{H})$, in which for all $\eta\in\mathrm{H}$ $\mathcal{G}(\eta)=\mathcal{F}(\eta)\bigcup\mathcal{K}(\eta)$ [11].
(ii) $(\mathcal{F},\mathrm{H})\widetilde{\bigcap}(\mathcal{K},\mathrm{H})=(\mathcal{G},\mathrm{H})$, in which for all $\eta\in\mathrm{H}$ $\mathcal{G}(\eta)=\mathcal{F}(\eta)\bigcap\mathcal{K}(\eta)$ [10].

Definition 2.6. [16] The belonging relations of $\upsilon\in\mathfrak{U}$ with a soft set $(\mathcal{F},\mathrm{H})$ are defined as follows.

(i) $\upsilon\in(\mathcal{F},\mathrm{H})$ if $\upsilon\in\mathcal{F}(\eta)$ for every $\eta\in\mathrm{H}$.
(ii) $\upsilon\Subset(\mathcal{F},\mathrm{H})$ if $\upsilon\in\mathcal{F}(\eta)$ for some $\eta\in\mathrm{H}$.

The negation of these relations is defined as follows:

(i) $\upsilon\notin(\mathcal{F},\mathrm{H})$ if $\upsilon\notin\mathcal{F}(\eta)$ for some $\eta\in\mathrm{H}$.
(ii) $\upsilon\not\Subset(\mathcal{F},\mathrm{H})$ if $\upsilon\notin\mathcal{F}(\eta)$ for each $\eta\in\mathrm{H}$.

In connection with a soft-point υ_η, we write $\upsilon_\eta\in(\mathcal{F},\mathrm{H})$ if $\upsilon\in\mathcal{F}(\eta)$.

Definition 2.7. [34] Let $\mathsf{M}:\mathfrak{U}\to\mathfrak{V}$ and $\ell:\mathrm{H}\to\mathrm{E}$ be ordinary (classical) mappings. It is defined a soft mapping (or soft function) M_ℓ from the class of soft points defined over $\mathfrak{U}$ with H (as a domain) to the class of soft points defined over $\mathfrak{V}$ with E (as a codomain) is a relation associated every soft point in the domain with only one soft point in the codomain in which

$$\mathsf{M}_\ell(\upsilon_\eta)=\mathsf{M}(\upsilon)_{\ell(\eta)}\text{ for each }\upsilon\in\mathfrak{U}\text{ and, }\eta\in\mathrm{H}.$$

In addition,

$$\mathsf{M}_\ell^{-1}(\omega_\gamma)=\begin{cases}\widetilde{\bigcup}_{\substack{\upsilon\in\mathsf{M}^{-1}(\omega)\\ \eta\in\ell^{-1}(\gamma)}}\upsilon_\eta & : \ \ell^{-1}(\gamma)\text{ and }\mathsf{M}^{-1}(\omega)\text{ are nonempty}\\ \phi & : \ \ell^{-1}(\gamma)\text{ or }\mathsf{M}^{-1}(\omega)\text{ are empty}\end{cases}$$

2.2. Soft topology

Definition 2.8. [13] A soft topology Θ (in short, ST) on $\mathfrak{U}$ as a universal set and H as a set of parameters is a class of soft sets, defined over $\mathfrak{U}$ with H, that is closed under finite soft intersections and arbitrary soft unions as well as contains $\widetilde{\mathfrak{U}}$ and ϕ.

We name the elements of Θ soft open sets and the term of "soft closed set" is given for the complement of a soft open set. The triplet $(\mathfrak{U},\Theta,\mathrm{H})$ is called a soft topological space (in short, ST-space).

Definition 2.9. [13] A soft subset $(\mathcal{F}, \mathrm{H})$ of an ST-space $(\mathfrak{U}, \Theta, \mathrm{H})$ is called a soft neighbourhood of a soft-point υ_η if there exists a soft open set $(\mathcal{G}, \mathrm{H})$ such that $\upsilon_\eta \in (\mathcal{G}, \mathrm{H}) \widetilde{\subseteq} (\mathcal{F}, \mathrm{H})$.

Definition 2.10. [13] Respectively, it is defined the soft interior and closure of a soft subset $(\mathcal{F}, \mathrm{H})$ of an ST-space $(\mathfrak{U}, \Theta, \mathrm{H})$:

(i) the soft union of all soft open subsets of $(\mathcal{F}, \mathrm{H})$; referred by $inr(\mathcal{F}, \mathrm{H})$.
(ii) the soft intersection of all soft closed supersets of $(\mathcal{F}, \mathrm{H})$; referred by $clr(\mathcal{F}, \mathrm{H})$.

Definition 2.11. An ST-space $(\mathfrak{U}, \Theta, \mathrm{H})$ is said to be:

(i) soft compact [20] there is a finite subcover for every soft open cover of $\widetilde{\mathfrak{U}}$.
(ii) almost soft compact [21] if every soft open cover has a finite subcover such that the closures of whose members cover $\widetilde{\mathfrak{U}}$.

In the above definition, if we replace the word “finite” by “countable”, then we obtain the definitions of soft Lindelöf and almost soft Lindelöf spaces.

Definition 2.12. [21] Let $\{(\mathcal{F}_\kappa, \mathrm{H}) : \kappa \in I\}$ be a family of soft sets. If for any finite (resp., countable) subset δ of I, we have $\widetilde{\bigcap}_{\kappa\in\delta}(\mathcal{F}_\kappa, \mathrm{H}) \neq \phi$, then we say that this family has the FIP (resp., CIP).

Definition 2.13. [29] An ST-space $(\mathfrak{U}, \Theta, \mathrm{H})$ is said to be:

(i) soft connected if the only soft clopen subsets (i.e., soft open and soft closed) are absolute and null soft sets.
(ii) soft hyperconnected if Θ does not contain disjoint non-null soft open subsets.

Definition 2.14. [42,46,25,41] A soft subset $(\mathcal{F}, \mathrm{H})$ of $(\mathfrak{U}, \Theta, \mathrm{H})$ is said to be:

(i) soft somewhat-open (shortly, soft sw-open) providing that it is a null soft set or its soft interior is non-null.
(ii) soft α-open (resp. soft semi-open) if $(\mathcal{F}, \mathrm{H}) \widetilde{\subseteq} inr(clr(inr(\mathcal{F}, \mathrm{H})))$ (resp. $(\mathcal{F}, \mathrm{H}) \widetilde{\subseteq} clr(inr(\mathcal{F}, \mathrm{H}))$).
(iii) soft dense if $clr(\mathcal{F}, \mathrm{H}) = \widetilde{\mathfrak{U}}$.

The complements of soft sw-open, soft α-open and soft semi-open sets are respectively called soft somewhat closed (briefly, soft sw-closed), soft α-closed and soft semi-closed sets.

Definition 2.15. (see, [43]) An ST-space $(\mathfrak{U}, \Theta, \mathrm{H})$ is said to be soft α-compact (resp., soft semi-compact, soft pre-compact) if every soft α-open (resp., soft semi-open, soft pre-open) cover of $\widetilde{\mathfrak{U}}$ has a finite subcover.

The next result offers an interesting method to generate some classical topologies from an ST.

Proposition 2.16. *[13] Let $(\mathfrak{U}, \Theta, \mathrm{H})$ be an ST-space. Then $\Theta_\eta = \{\mathcal{F}(\eta) : (\mathcal{F}, \mathrm{H}) \in \Theta\}$ produces a topology (we name parametric topology) on $\mathfrak{U}$ for each $\eta \in \mathrm{H}$.*

Definition 2.17. [50] For a soft subset $(\mathcal{F}, \mathrm{H})$ of an ST-space $(\mathfrak{U}, \Theta, \mathrm{H})$, we respectively define $(inr(\mathcal{F}), \mathrm{H})$ and $(clr(\mathcal{F}), \mathrm{H})$ by $inr(\mathcal{F})(\eta) = inr(\mathcal{F}(\eta))$ and $clr(\mathcal{F})(\eta) = clr(\mathcal{F}(\eta))$, in which $inr(\mathcal{F}(\eta))$ and $clr(\mathcal{F}(\eta))$ represent the interior and closure of $\mathcal{F}(\eta)$ in $(\mathfrak{U}, \Theta_\eta)$, respectively.

Remark 2.18. When it is necessary, we will write int_η and cl_η instead of int and cl, respectively, to refer that the interior and closure operators that are calculated with respect to the parametric topological space $(\mathfrak{U}, \Theta_\eta)$.

Definition 2.19. An ST Θ is called:

(i) an enriched ST [20] if Θ contains all pseudo constant soft sets;
(ii) an extended ST [51] if $(\mathcal{F}, \mathrm{H}) \in \Theta$ iff $\mathcal{F}(\eta) \in \Theta_\eta$ for each $\eta \in \mathrm{H}$.
(iii) a full ST [25] if all $\mathcal{F}(\eta)$ are nonempty for every $(\mathcal{F}, \mathrm{H}) \in \Theta$.

In [50], it was shown the corresponding between extended and enriched soft topologies. For the sake of unite terminology, we name this kind of ST an extended ST and name $(\mathfrak{U}, \Theta, \mathrm{H})$ an extended ST-space. The below fact helps us to describe the characteristics of classical and soft topological concepts using their analogs in both settings.

Theorem 2.20. *[50] An ST-space $(\mathfrak{U}, \Theta, \mathrm{H})$ is extended iff $(inr(F), \mathrm{H}) = inr(\mathcal{F}, \mathrm{H})$ and $(clr(F), \mathrm{H}) = clr(\mathcal{F}, \mathrm{H})$ for every soft subset $(\mathcal{F}, \mathrm{H})$ of $(\mathfrak{U}, \Theta, \mathrm{H})$.*

Definition 2.21. [35] A soft mapping $M_{\ell} : (\mathfrak{U}, \Theta, H) \to (\mathfrak{V}, \Lambda, E)$ is said to be soft continuous if the inverse image of every soft open set in Λ is a soft open set in Θ.

Theorem 2.22. *[50] If $M_{\ell} : (\mathfrak{U}, \Theta, H) \to (\mathfrak{V}, \Lambda, E)$ is soft continuous, then $M : (\mathfrak{U}, \Theta_{\eta}) \to (\mathfrak{V}, \Lambda_{\ell(\eta)})$ is continuous for all $\eta \in H$.*

3. Soft parametric somewhat-open sets

In this section, we define the concept of soft parametric somewhat-open sets as a new class of soft subsets of an ST-space $(\mathfrak{U}, \Theta, H)$. This definition is based on the corresponding generalizations of open subsets of all parametric topological spaces. We elucidate the relationship between the new class and the forgoing ones and discuss the condition under which they are identical. We also explore some characterizations and properties and provide some counterexamples.

Henceforth, $\mathbb{N}$ will denote the set of natural numbers.

Definition 3.1. A soft subset $(\mathcal{F}, H)$ of an ST-space $(\mathfrak{U}, \Theta, H)$ is said to be a soft parametric somewhat-open set (briefly, soft ps-open set) if $(\mathcal{F}, H) = \phi$ or $inr(\mathcal{F}(\eta))$ is nonempty for all $\eta \in H$. The term of "soft ps-closed set" is given for the complement of soft ps-open set.

Recall that a subset of a topological space is called somewhat-open if it is empty or its interior points is nonempty. So we can say that $(\mathcal{F}, H)$ is a soft ps-open set if $(\mathcal{F}, H) = \phi$ or $\mathcal{F}(\eta)$ is nonempty somewhat-open for all $\eta \in H$.

We furnish the next example which we will need to show some results and relationships presented herein.

Example 3.2. Taking $\mathfrak{U} = \{\upsilon, \omega, \mu\}$ and $H = \{\eta, \rho\}$ as a universal set and a set of parameters, respectively. Let $(\mathcal{F}, H)$, $(\mathcal{G}, H)$ and $(\mathcal{K}, H)$ be soft sets over $\mathfrak{U}$ given as follows
$(\mathcal{F}, H) = \{(\eta, \{\upsilon\}), (\rho, \{\omega\})\}$;
$(\mathcal{G}, H) = \{(\eta, \{\upsilon\}), (\rho, \{\omega, \mu\})\}$ and
$(\mathcal{K}, H) = \{(\eta, \{\upsilon, \mu\}), (\rho, \{\omega, \mu\})\}$.

Then the family $\Theta = \{\phi, \widetilde{\mathfrak{U}}, (\mathcal{F}, H), (\mathcal{G}, H), (\mathcal{K}, H)\}$ forms an ST on $\mathfrak{U}$. Now, $\{(\eta, \{\upsilon, \mu\}), (\rho, \{\upsilon, \omega\})\}$ is a soft ps-open set because $int_{\eta}(\{\upsilon, \mu\})$ and $int_{\rho}(\{\upsilon, \omega\})$ are nonempty, whereas $\{(\eta, \{\omega\}), (\rho, \mathfrak{U})\}$ is not a soft ps-open set because $int_{\eta}(\{\omega\}) = \emptyset$.

The following proposition gives an equivalent condition for proper soft ps-closed subsets.

Proposition 3.3. *A proper soft subset $(\mathcal{F}, H)$ of an ST-space $(\mathfrak{U}, \Theta, H)$ is soft ps-closed iff $clr(\mathcal{F}(\eta)) \neq \mathfrak{U}$ for all $\eta \in H$.*

Proof. Let $(\mathcal{F}, H)$ be a proper soft ps-closed subset. Then, $int[(\mathcal{F}^c(\eta))] \neq \emptyset$ for all $\eta \in H$. So $clr(\mathcal{F}(\eta)) \neq \mathfrak{U}$ for all $\eta \in H$. Conversely, let $(\mathcal{F}, H)$ be a soft subset such that $clr(\mathcal{F}(\eta)) \neq \mathfrak{U}$ for all $\eta \in H$. Then, $inr(\mathcal{F}^c(\eta)) \neq \emptyset$ for all $\eta \in H$. Thus, $(\mathcal{F}^c, H)$ is soft ps-closed, as required. □

Proposition 3.4. *Every soft superset of a non-null soft ps-open set is also soft ps-open.*

Proof. Suppose that $(\mathcal{F}, H)$ is a non-null soft ps-open set and let $(\mathcal{G}, H)$ be a soft set with $(\mathcal{F}, H) \widetilde{\subseteq} (\mathcal{G}, H)$. By assumption, $inr(\mathcal{F}(\eta)) \neq \phi$ for all $\eta \in H$, we obtain $\emptyset \neq inr(\mathcal{F}(\eta)) \subseteq inr(\mathcal{G}(\eta))$ for all $\eta \in H$. Hence, $(\mathcal{G}, H)$ is also soft ps-open. □

The proofs of the next corollaries are obvious, so we omit them.

Corollary 3.5. *Every soft subset of a proper soft ps-closed is soft ps-closed.*

Corollary 3.6. *The arbitrary soft union of soft ps-open subsets of an ST-space $(\mathfrak{U}, \Theta, H)$ is soft ps-open.*

Corollary 3.7. *The arbitrary soft intersection of soft ps-closed subsets of an ST-space $(\mathfrak{U}, \Theta, H)$ is soft ps-closed.*

The finite soft intersection of soft ps-open subsets need not be soft ps-open and the finite union of soft ps-closed subsets need not be soft ps-closed as the next example shows.

Example 3.8. Let $\mathbb{N}$ be the set of natural numbers and take $H = \{\eta, \rho\}$ as a set of parameters. Then $\Theta = \{\phi, \widetilde{\mathbb{N}}, (\mathcal{F}, H) \widetilde{\subseteq} \widetilde{\mathbb{N}} : 1 \in \mathcal{F}(\eta)$ and $2 \notin \mathcal{F}(\rho)\}$ be an ST on $\mathbb{N}$. It is obvious that $(\mathcal{G}, H) = \{(\eta, \{1, 5\}), (\rho, \{1, 2\})\}$ and $(\mathcal{K}, H) = \{(\eta, \mathbb{N}), (\rho, \{2, 3\})\}$ are soft ps-open sets, and $(\mathcal{I}, H) = \{(\eta, \{6, 7\}), (\rho, \{1\})\}$ and $(\mathcal{J}, H) = \{(\eta, \{6, 8\}), (\rho, \mathbb{N} \setminus \{1\})\}$ are soft ps-closed sets. Now, neither $(\mathcal{G}, H) \widetilde{\cap} (\mathcal{K}, H)$ is soft ps-open nor $(\mathcal{I}, H) \widetilde{\cup} (\mathcal{J}, H)$ is soft ps-closed.

The relationship among the current class and some of the previous ones are discussed in the following.

Proposition 3.9.

(i) *Every soft sw-open subset of a full ST-space $(\mathfrak{U}, \Theta, \mathrm{H})$ is soft ps-open.*
(ii) *Every soft ps-open subset of an extended ST-space $(\mathfrak{U}, \Theta, \mathrm{H})$ is soft sw-open.*

Proof. (i): Let $(\mathcal{F}, \mathrm{H})$ be a soft sw-open set. Then there is a soft open set $(\mathcal{G}, \mathrm{H})$ such that

$$(\mathcal{G}, \mathrm{H}) \widetilde{\subseteq} (\mathcal{F}, \mathrm{H}). \tag{1}$$

Since $(\mathfrak{U}, \Theta, \mathrm{H})$ is full, $inr(\mathcal{G}(\eta)) \neq \emptyset$ for all $\eta \in \mathrm{H}$. According to relation (1), we obtain $\mathcal{G}(\eta) \subseteq \mathcal{F}(\eta)$ for all $\eta \in \mathrm{H}$. Thus, $inr(\mathcal{F}(\eta)) \neq \emptyset$ for all $\eta \in \mathrm{H}$. Hence, $(\mathcal{F}, \mathrm{H})$ is a soft ps-open set.

The proof of (ii) follows from the fact that the extended ST grantees the equality $(inr(F), \mathrm{H}) = inr(\mathcal{F}, \mathrm{H})$ as given in Theorem 2.20. □

Corollary 3.10. *Every soft open (soft α-open, soft semi-open) subset of a full ST-space $(\mathfrak{U}, \Theta, \mathrm{H})$ is soft ps-open.*

Proof. As it is well-known that every soft open (soft α-open, soft semi-open) is soft sw-open; so by (i) of Proposition 3.9, we get the desired result. □

It can be seen from Example 3.2 that the converses of (i) in the above proposition fail. The next example demonstrates that the converse of (ii) need not be true.

Example 3.11. $\Theta = \{\phi, \widetilde{\mathfrak{U}}, (\mathcal{F}_\kappa, \mathrm{H}) : \kappa = 1, 2, 3, 4\}$ be an extended ST over $\mathfrak{U} = \{\upsilon, \omega, \mu\}$ with $\mathrm{H} = \{\eta, \rho\}$, where
$(\mathcal{F}_1, \mathrm{H}) = \{(\eta, \mathfrak{U}), (\rho, \emptyset)\}$;
$(\mathcal{F}_2, \mathrm{H}) = \{(\eta, \emptyset), (\rho, \mathfrak{U})\}$;
$(\mathcal{F}_3, \mathrm{H}) = \{(\eta, \{\upsilon\}), (\rho, \emptyset)\}$;
$(\mathcal{F}_4, \mathrm{H}) = \{(\eta, \{\upsilon\}), (\rho, \mathfrak{U})\}$.
Then, $\{(\eta, \{\upsilon\}), (\rho, \{\mu\})\}$ is a soft sw-open set but it is not a soft ps-open set because $int_\rho(\{\mu\}) = \emptyset$.

Proposition 3.12. *The finite soft intersection of soft ps-open subsets of an ST-space $(\mathfrak{U}, \Theta, \mathrm{H})$ is soft ps-open provided that all parametric topological spaces $(\mathfrak{U}, \Theta_\eta)$ are hyperconnected.*

Proof. Let $(\mathcal{F}, \mathrm{H})$ and $(\mathcal{G}, \mathrm{H})$ be soft ps-open subsets of an ST-space $(\mathfrak{U}, \Theta, \mathrm{H})$. Putting $(\mathcal{K}, \mathrm{H}) = (\mathcal{F}, \mathrm{H}) \widetilde{\cap} (\mathcal{G}, \mathrm{H})$. Now, $inr(\mathcal{K}(\eta)) = inr(\mathcal{F}(\eta) \cap \mathcal{G}(\eta)) = inr(\mathcal{F}(\eta)) \cap inr(\mathcal{G}(\eta))$. Suppose that $inr(\mathcal{K}(\eta)) = \emptyset$ for some $\eta \in \mathrm{H}$. Then we get that $inr(\mathcal{F}(\eta))$ and $inr(\mathcal{G}(\eta))$ are disjoint nonempty open subsets for some $\eta \in \mathrm{H}$, which means that some parametric topological spaces are dishyperconnected. But this contradicts the given. So, it must $inr(\mathcal{K}(\eta)) \neq \emptyset$ for all $\eta \in \mathrm{H}$. Hence, $(\mathcal{F}, \mathrm{H}) \widetilde{\cap} (\mathcal{G}, \mathrm{H})$ is a soft ps-open set. □

Corollary 3.13. *If every topological space produced by an ST-space $(\mathfrak{U}, \Theta, \mathrm{H})$ is hyperconnected, then the class of soft ps-open subsets forms an ST.*

Proof. By Definition 3.1 the null and absolute soft sets belong to this class. The closedness of this class under arbitrary soft unions and finite soft intersections are respectively obtained from Corollary 3.6 and Proposition 3.12. □

Proposition 3.14. *A full ST-space $(\mathfrak{U}, \Theta, \mathrm{H})$ is soft hyperconnected iff all parametric topological spaces $(\mathfrak{U}, \Theta_\eta)$ are hyperconnected.*

Proof. *Necessity*: Let the given conditions be satisfied. Suppose that there is $\eta^* \in \mathrm{H}$ such that $(\mathfrak{U}, \Theta_{\eta^*})$ is dishyperconnected. Then Θ_{η^*} contains nonempty disjoint open subsets; say, V, W. This means there are non-null soft open subsets $(\mathcal{F}, \mathrm{H})$ and $(\mathcal{G}, \mathrm{H})$ such that $\mathcal{F}(\eta^*) = V$ and $\mathcal{G}(\eta^*) = W$. By a condition of full, it must $(\mathcal{F}, \mathrm{H})$ and $(\mathcal{G}, \mathrm{H})$ are disjoint, which contradicts the given. This means that $(\mathfrak{U}, \Theta_\eta)$ is hyperconnected for all $\eta \in \mathrm{H}$.
Sufficiency: Let every topological space $(\mathfrak{U}, \Theta_\eta)$ produced by a full ST-space $(\mathfrak{U}, \Theta, \mathrm{H})$ be hyperconnected. Suppose, to the contrary, that $(\mathfrak{U}, \Theta, \mathrm{H})$ is soft dishyperconnected. Then Θ contains non-null disjoint soft open subsets $(\mathcal{F}, \mathrm{H})$ and $(\mathcal{G}, \mathrm{H})$. By a condition of full, $\mathcal{F}(\eta)$ and $\mathcal{G}(\eta)$ are nonempty disjoint open subsets. So that, all $(\mathfrak{U}, \Theta_\eta)$ are dishyperconnected. But this contradicts the given. Hence, $(\mathfrak{U}, \Theta, \mathrm{H})$ is soft hyperconnected, as required. □

Corollary 3.15. *The class of soft ps-open subsets of a full soft hyperconnected TS forms an ST.*

Proof. Follows from Corollary 3.13 and Proposition 3.14. □

Two examples below confirm that the term "full ST" is necessary.

Example 3.16. Let $\Theta = \{\phi, \widetilde{\mathfrak{U}}, (\mathcal{F}, \mathrm{H}), (\mathcal{G}, \mathrm{H})\}$ be an ST over $\mathfrak{U} = \{\upsilon, \omega, \mu\}$ with $\mathrm{H} = \{\eta, \rho\}$, where
$(\mathcal{F}, \mathrm{H}) = \{(\eta, \mathfrak{U}), (\rho, \emptyset)\}$ and
$(\mathcal{G}, \mathrm{H}) = \{(\eta, \emptyset), (\rho, \mathfrak{U})\}$.

Then, it is obvious that $(\mathfrak{U}, \Theta, \mathrm{H})$ is soft dishyperconnected. In contrast, both parametric topological spaces are hyperconnected.

Example 3.17. Let $\mathfrak{U} = \{\upsilon, \omega, \mu\}$ be the universal set and $\mathbb{N}$ be a set of parameters. The class $\{\phi, \widetilde{\mathfrak{U}}, (\mathcal{F}_\kappa, \mathbb{N}) : \mathcal{F}_\kappa(n) \neq \mathfrak{U}$ for finite number $n \in \mathbb{N}$, where $\kappa \in \mathbb{N}\}$ structures an ST over $\mathfrak{U}$ with $\mathbb{N}$. It can be noted that this ST does not contain any disjoint proper soft open subsets, so $(\mathfrak{U}, \Theta, \mathbb{N})$ is soft hyperconnected. On the other hand, all parametric topologies are discrete; hence, $(\mathfrak{U}, \Theta_n)$ is dishyperconnected for all $n \in \mathbb{N}$.

Theorem 3.18. *Let $\digamma$ be the family of soft ps-open subsets obtained from an ST-space $(\mathfrak{U}, \Theta, \mathrm{H})$. Then*

$$1 + \prod_{\eta \in \mathrm{H}} (|\Theta_\eta| - 1) \leq |\digamma| \leq 1 + (2^{|\mathfrak{U}|} - 1)^{|\mathrm{H}|}.$$

Proof. Let $\digamma$ be the family of soft ps-open subsets obtained from an ST-space $(\mathfrak{U}, \Theta, \mathrm{H})$. Since the interior of each component of any non-null soft ps-open subset is a nonempty open set, we select every η-component as a superset of a nonempty open subset of $(\mathfrak{U}, \Theta_\eta)$. This implies that each component is selected by $|\Theta_\eta| - 1$ distinct ways at least. By Definition 3.1 the null soft set is soft ps-open, so by the counting principle we find that $1 + \prod_{\eta \in \mathrm{H}} (|\Theta_\eta| - 1) \leq |\digamma|$. Moreover, if Θ_η is the discrete topology for all $\eta \in \mathrm{H}$, then the number of nonempty open subsets of $(\mathfrak{U}, \Theta_\eta)$ is $2^{|\mathfrak{U}|} - 1$. Again, by the counting principle we find that the family of soft ps-open subsets obtained from $(\mathfrak{U}, \Theta, \mathrm{H})$ is $(2^{|\mathfrak{U}|} - 1)^{|\mathrm{H}|}$. According to Definition 3.1 the null soft set is soft ps-open, so $|\digamma| \leq 1 + (2^{|\mathfrak{U}|} - 1)^{|\mathrm{H}|}$. Hence, we obtain the desired inequality. □

Proposition 3.19. *The surjective soft continuous pre-image of a soft ps-open set is also a soft ps-open set.*

Proof. Suppose that a soft mapping $\mathsf{M}_\ell : (\mathfrak{U}, \Theta, \mathrm{H}) \to (\mathfrak{B}, \Lambda, \mathrm{E})$ is soft continuous and let $(\mathcal{F}, \mathrm{E})$ be a soft ps-open subset of $(\mathfrak{B}, \Lambda, \mathrm{E})$. Now, for all $\varepsilon \in \mathrm{E}$ $inr(\mathcal{F}(\varepsilon)) \neq \emptyset$. Let $\ell(\eta) = \varepsilon$. It comes from Theorem 2.22 that the crisp mapping $\mathsf{M} : (\mathfrak{U}, \Theta_\eta) \to (\mathfrak{B}, \Lambda_{\ell(\eta)=\varepsilon})$ is continuous; therefore, $\mathsf{M}^{-1}[inr(\mathcal{F}(\varepsilon))] \subseteq int[\mathsf{M}^{-1}(\mathcal{F}(\varepsilon))]$. By surjectiveness of M, we obtain $int[\mathsf{M}^{-1}(\mathcal{F}(\varepsilon))]$ is a nonempty open subset of $\mathfrak{U}$. It follows from Definition 2.7 that $\mathsf{M}_\ell^{-1}(\mathcal{F}, \mathrm{E})$ is a soft ps-open subset of $(\mathfrak{U}, \Theta, \mathrm{H})$. Hence, the proof is complete. □

Corollary 3.20. *A soft ps-open set is a topological property.*

Proposition 3.21. *The class of soft ps-open subsets is closed under finite product.*

Proof. It is well-known that $\prod_{\kappa \in I} int_\kappa(A_\kappa) = inr(\prod_{\kappa \in I} A_\kappa)$, where int_κ is the interior operator in a parametric TS $(\mathfrak{U}_\kappa, \Theta_{\kappa\eta})$ and int is the interior operator in the product of classes of parametric TSs $\{(\mathfrak{U}_\kappa, \Theta_{\kappa\eta}) : \kappa \in I\}$, so the result is obtained. □

4. Some types of soft compact and Lindelöf space inspired by soft ps-open sets

In this part, we will investigate novel sorts of soft compactness and Lindelöfness inspired by the class of soft ps-open sets. We characterize them and establish main features. The relationships between them are pointed out with the assistance of interesting examples. Also, we discuss the necessary conditions to preserve these types of covering property between soft topologies and parametric topologies.

Henceforth, $\mathbb{R}$ will denote the set of real numbers.

4.1. Soft ps-compactness and soft ps-Lindelöfness

Definition 4.1. A class $\{(\mathcal{F}_\kappa, \mathrm{H}) : \kappa \in I\}$ of soft ps-open sets in $(\mathfrak{U}, \Theta, \mathrm{H})$ is called a soft parametric somewhat-open cover (shortly, soft ps-open cover) of $\widetilde{\mathfrak{U}}$ if $\widetilde{\mathfrak{U}} = \widetilde{\bigcup}_{\kappa \in I} (\mathcal{F}_\kappa, \mathrm{H})$.

Definition 4.2. An ST-space $(\mathfrak{U}, \Theta, \mathrm{H})$ is named:

- **(i)** soft parametric somewhat-open compact (briefly, soft ps-compact) providing that for any soft ps-open cover $\{(\mathcal{F}_\kappa, \mathrm{H}) : \kappa \in I\}$ of $(\mathfrak{U}, \Theta, \mathrm{H})$, there exists a finite set $\delta \subseteq I$ in which $\widetilde{\mathfrak{U}} = \widetilde{\bigcup}_{\kappa \in \delta} (\mathcal{F}_\kappa, \mathrm{H})$.
- **(ii)** soft parametric somewhat-open Lindelöf (briefly, soft ps-Lindelöf) providing that for any soft ps-open cover $\{(\mathcal{F}_\kappa, \mathrm{H}) : \kappa \in I\}$ of $(\mathfrak{U}, \Theta, \mathrm{H})$, there exists a countable set $\delta \subseteq I$ in which $\widetilde{\mathfrak{U}} = \widetilde{\bigcup}_{\kappa \in \delta} (\mathcal{F}_\kappa, \mathrm{H})$.

It is clear that any soft ps-compact space is soft ps-Lindelöf.

The following examples show the existence and uniqueness of soft ps-compactness and soft ps-Lindelöfness.

Example 4.3. Let $\Theta = \{\phi, (\mathcal{F}, \mathrm{H}) \widetilde{\subseteq} \widetilde{\mathbb{R}} : (\mathcal{F}, \mathrm{H})$ is finite$\}$ be an ST over $\mathbb{R}$, where a set of parameters H is finite. It is obvious that Θ is a full ST, so every soft open set is soft ps-open. Also, it can be noted that every soft ps-open set is soft open. Hence, $(\mathbb{R}, \Theta, \mathrm{H})$ is soft ps-compact.

Example 4.4. Let $\Theta = \{\widetilde{\mathbb{R}}, (\mathcal{F}, \mathrm{H}) \widetilde{\subseteq} \widetilde{\mathbb{R}} : 1 \notin (\mathcal{F}, \mathrm{H})\}$ be an ST over $\mathbb{R}$ with H as a set of parameters. Now, the class $\{(\mathcal{F}, \mathrm{H}) \widetilde{\subseteq} \widetilde{\mathbb{R}} : \mathcal{F}(\eta) = \{1, \upsilon\}$ for each $\eta \in \mathrm{H}$ and $\upsilon \in \mathbb{R} - \{1\}\}$ is a soft ps-open cover of $(\mathbb{R}, \Theta, \mathrm{H})$; hence, it is not soft ps-Lindelöf.

The ideas of soft compact (resp. soft Lindelöf) and soft ps-compact (resp. soft ps-Lindelöf) spaces are independent of each other. To point out this independency, see Example 4.4 which is soft compact but not soft ps-Lindelöf, and the next example which is soft ps-compact but not soft Lindelöf.

Example 4.5. Let $\Theta = \{\phi, \widetilde{\mathfrak{U}}, (\mathcal{F}, \mathbb{R}) \widetilde{\subseteq} \widetilde{\mathfrak{U}} : \mathcal{F}(\eta) = \emptyset$ or $\mathfrak{U}$ for each $\eta \in \mathbb{R}\}$ be an ST on $\mathfrak{U} = \{\upsilon, \omega\}$ with $\mathbb{R}$ as a set of parameters. It can be checked that $(\mathfrak{U}, \Theta, \mathbb{R})$ is soft ps-Lindelöf but not soft compact.

Proposition 4.6. *If $(\mathfrak{U}, \Theta, \mathrm{H})$ is a full soft ps-compact (resp., full soft ps-Lindelöf) space, then it is soft semi-compact (resp., soft semi-Lindelöf).*

Proof. Follows from Corollary 3.10. □

Example 4.5 confirms that a condition of a full ST in Proposition 4.6 is indispensable.

Proposition 4.7. *Let $(\mathfrak{U}, \Theta_1, \mathrm{H})$ and $(\mathfrak{U}, \Theta_2, \mathrm{H})$ be soft TSs such that $\Theta_1 \subseteq \Theta_2$. If $(\mathfrak{U}, \Theta_2, \mathrm{H})$ is soft ps-compact (resp., soft ps-Lindelöf), then $(\mathfrak{U}, \Theta_1, \mathrm{H})$ is soft ps-compact (resp., soft ps-Lindelöf).*

Proof. Since $\Theta_1 \subseteq \Theta_2$, we obtain $\Theta_{1\eta} \subseteq \Theta_{2\eta}$ for all $\eta \in \mathrm{H}$. So all soft ps-open subsets of Θ_1 are also soft ps-open subsets of Θ_2. Hence, the wanted result is proved. □

By taking a set of parameters $\mathrm{H} = \{\eta, \rho\}$ in Example 4.4, we note that any indiscrete ST-space defined over $\mathbb{R}$ with a set of parameters H is contained in the space of ST displayed in Example 4.4, which we illustrate it is not a soft ps-Lindelöf. This elaborates that the converse of Proposition 4.7 is generally false.

It is worthily noting that the fact obtained in Proposition 4.7 is not hold true for some types of extensions of soft open sets like soft pre-open sets. To elaborate this matter, take an ST-space displayed in Example 4.4. It is clear that any proper soft subset $(\mathcal{F}, \mathrm{H})$ with $1 \Subset (\mathcal{F}, \mathrm{H})$ is not soft pre-open, which means that this ST-space is soft pre-compact. In contrast, if we replace this ST by the soft indiscrete topology, then we obtain a non soft pre-Lindelöf space.

The following two facts can be proved following similar lines of the proof of their counterparts proved in previous studies.

Proposition 4.8. *All soft ps-closed subsets of a soft ps-compact TS $(\mathfrak{U}, \Theta, \mathrm{H})$ are soft ps-compact.*

Corollary 4.9. *The soft intersections of soft ps-compact and soft ps-closed sets are soft ps-compact.*

The above two results are still valid if we replace "soft ps-compact" by "soft ps-Lindelöf".

To give a complete description for the spaces of soft ps-compact and soft ps-Lindelöf, we furnish the next result.

Theorem 4.10. *An ST-space $(\mathfrak{U}, \Theta, \mathrm{H})$ is soft ps-compact (resp., soft ps-Lindelöf) iff $\widetilde{\bigcap}_{\kappa \in I} (\mathcal{F}_\kappa, \mathrm{H}) \neq \phi$ for every class $C = \{(\mathcal{F}_\kappa, \mathrm{H}) : \kappa \in I\}$ of soft ps-closed sets has a FIP (resp., a CIP).*

Proof. ⇒: For a class of soft ps-closed subsets $\{(\mathcal{F}_\kappa, \mathrm{H}) : \kappa \in I\}$ of a soft ps-compact space $(\mathfrak{U}, \Theta, \mathrm{H})$, let $\widetilde{\bigcap}_{\kappa \in I} (\mathcal{F}_\kappa, \mathrm{H}) = \phi$. Then, $\widetilde{\mathfrak{U}} = \widetilde{\bigcup}_{\kappa \in I} (\mathcal{F}^c_\kappa, \mathrm{H})$. By assumption, $\widetilde{\mathfrak{U}} = \widetilde{\bigcup}^n_{\kappa=1} (\mathcal{F}^c_\kappa, \mathrm{H})$. This means that $\phi = (\widetilde{\bigcup}^n_{\kappa=1} (\mathcal{F}^c_\kappa, \mathrm{H}))^c = \widetilde{\bigcap}^n_{\kappa=1} (\mathcal{F}_\kappa, \mathrm{H})$. Hence, this family has a FIP, as required.

⇐: If $\{(\mathcal{F}_\kappa, \mathrm{H}) : \kappa \in I\}$ is a soft ps-open cover of $(\mathfrak{U}, \Theta, \mathrm{H})$, then $\phi = \widetilde{\bigcap}_{\kappa \in I} (\mathcal{F}^c_\kappa, \mathrm{H})$. We get $\phi = \widetilde{\bigcap}^n_{\kappa=1} (\mathcal{F}^c_\kappa, \mathrm{H})$ by the FIP of this cover. Thus, $\widetilde{\mathfrak{U}} = \widetilde{\bigcup}^n_{\kappa=1} (\mathcal{F}_\kappa, \mathrm{H})$, which verifies that the soft ps-compactness of $(\mathfrak{U}, \Theta, \mathrm{H})$.

The case given between the parentheses can be proved in a similar way. □

Proposition 4.11. *The soft ps-compact (resp., soft ps-Lindelöf) sets are preserved under surjective soft continuous.*

Proof. Let a soft mapping $\mathsf{M}_\ell : (\mathfrak{U}, \Theta, \mathrm{H}) \to (\mathfrak{B}, \Lambda, \mathrm{E})$ be soft continuous and suppose that $(\mathcal{K}, \mathrm{H})$ is a soft ps-Lindelöf subset of $(\mathfrak{U}, \Theta, \mathrm{H})$. Consider $\{(\mathcal{F}_\kappa, \mathrm{E}) : \kappa \in I\}$ as a soft ps-cover of $\mathsf{M}_\ell(\mathcal{K}, \mathrm{H})$. So we obtain $(\mathcal{K}, \mathrm{H}) \widetilde{\subseteq} \widetilde{\bigcup}_{\kappa \in I} \mathsf{M}^{-1}_\ell (\mathcal{F}_\kappa, \mathrm{E})$. According to Proposition 3.19, $\mathsf{M}^{-1}_\ell (\mathcal{F}_\kappa, \mathrm{H})$ is soft ps-open for all $\kappa \in I$. By assumption of soft ps-Lindelöfness of $(\mathcal{K}, \mathrm{H})$, there is a countable set δ such that

$(\mathcal{K},\mathrm{H})\widetilde{\subseteq}\widetilde{\bigcup}_{\kappa\in\delta}\mathsf{M}_{\ell}^{-1}(\mathcal{F}_\kappa,\mathrm{E})$. Now, we obtain $\mathsf{M}_{\ell}(\mathcal{K},\mathrm{H})\widetilde{\subseteq}\widetilde{\bigcup}_{\kappa\in\delta}\mathsf{M}_{\ell}(\mathsf{M}_{\ell}^{-1}(\mathcal{F}_\kappa,\mathrm{E}))\widetilde{\subseteq}\widetilde{\bigcup}_{\kappa\in\delta}(\mathcal{F}_\kappa,\mathrm{E})$, which means that $\mathsf{M}_{\ell}(\mathcal{K},\mathrm{H})$ is soft ps-Lindelöf. It can be proved the case of soft ps-compact in a similar way. □

In what follows, we discuss the navigation of these types of covering properties between soft topologies and classical (parametric) topologies. First of all, we need to specify the analogous notions of soft ps-compact and ps-Lindelöf spaces via the classical topological spaces. It is convenient to adopt the concepts of compactness and Lindelöfness inspired by somewhat-open sets as the appropriate counterparts for soft ps-compactness and ps-Lindelöfness. According to this viewpoint, we begin studying the navigation by showing that the properties of soft ps-compactness and ps-Lindelöfness are transmitted to their parametric topologies without any imposed terms, which represents an unparalleled behaviour compared to compactness and Lindelöfness defined by several generalizations of soft compactness and Lindelöfness such as soft α-compact, soft semi-compact, soft pre-compact and soft b-compact spaces and their corresponding Lindelöfness spaces.

Recall that a topological space $(\mathfrak{U},\tau)$ is said to be somewhat compact (resp. somewhat Lindelöf) if every cover of somewhat-open subsets of $(\mathfrak{U},\tau)$ has a finite (resp. countable) subcover.

Theorem 4.12. *A topological space $(\mathfrak{U},\Theta_\eta)$ produced by a soft ps-compact (resp., soft ps-Lindelöf) space $(\mathfrak{U},\Theta,\mathrm{H})$ is somewhat compact (resp., somewhat Lindelöf) for each $\eta\in\mathrm{H}$.*

Proof. Take $\{\mathcal{F}_\kappa : \kappa\in I\}$ as a somewhat-open cover of a parametric topological space $(\mathfrak{U},\Theta_\eta)$. Then, for every $\kappa\in I$ there is a nonempty open subset $\mathcal{K}_\kappa$ of Θ_η in which $\mathcal{K}_\kappa\subseteq\mathcal{F}_\kappa$. So that, there is a soft open subset (V_κ,H) of Θ such that $V_\kappa=\mathcal{K}_\kappa$ for every $\kappa\in I$. Now, we build a class of soft ps-open sets (W_κ,H) as following $W_\kappa(\eta)=\mathcal{F}_\kappa$ and $W_\kappa(\eta')=\mathfrak{U}$ for $\eta'\neq\eta$. Then, $\{(W_\kappa,\mathrm{H}) : \kappa\in I\}$ represents a soft ps-open cover of $(\mathfrak{U},\Theta,\mathrm{H})$. By the soft ps-compactness of $(\mathfrak{U},\Theta,\mathrm{H})$, we obtain $\widetilde{\mathfrak{U}}=\bigcup_{\kappa=1}^{n}(W_\kappa,\mathrm{H})$. This directly leads to that

$$\mathfrak{U}=\bigcup_{\kappa=1}^{n}W_\kappa(\eta)=\bigcup_{\kappa=1}^{n}\mathcal{F}_\kappa.$$

Hence, $(\mathfrak{U},\Theta_\eta)$ is somewhat compact, as required. Following similar argument, one proves the case mentioned between parentheses. □

The target of the following example is to show that the converse side of Theorem 4.12 need not be true.

Example 4.13. Let $\Theta=\{\phi,(\mathcal{F},\mathbb{R})\widetilde{\subseteq}\widetilde{\mathbb{R}} : (\mathcal{F},\mathbb{R})$ is finite$\}$ be an ST over $\mathbb{R}$, where $\mathbb{R}$ is also the set of parameters. Now, all topologies produced by $(\mathbb{R},\Theta,\mathbb{R})$ are the co-finite topology, so they are ps-compact, but $(\mathbb{R},\Theta,\mathbb{R})$ is not soft ps-Lindelöf.

In the other forgoing kinds of Lindelöfness and compactness initiated by generalizations of soft open sets, Theorem 4.12 need not be true as elucidated by the next example.

Example 4.14. We construct an ST Θ over $\mathbb{R}$ and $\mathrm{H}=\{\eta,\rho\}$ which respectively represent the set of real numbers and parameters set such that the members of Θ are the absolute soft set $\widetilde{\mathbb{R}}$ and every soft set $(\mathcal{F},\mathrm{H})$ satisfying that $1\notin\mathcal{F}(\eta)$. One can note that the families of soft pre-open and soft open subset of this ST-space $(\mathbb{R},\Theta,\mathrm{H})$ are identical. Hence, $(\mathbb{R},\Theta,\mathrm{H})$ is soft pre-compact. Whereas, $(\mathbb{R},\Theta_\rho)$ is not pre-Lindelöf because all parametric topologies Θ_ρ are the discrete topology.

Next finding investigates the necessary terms to make the converse of Theorem 4.12 is correct.

Theorem 4.15. *If a set of parameters H is finite (resp., countable), then, all parametric topological spaces $(\mathfrak{U},\Theta_\eta)$ inspired by $(\mathfrak{U},\Theta,\mathrm{H})$ is somewhat compact (resp., somewhat Lindelöf) iff $(\mathfrak{U},\Theta,\mathrm{H})$ is soft ps-compact (resp., soft ps-Lindelöf).*

Proof. A proof is presented for the case of compactness.

⇒: Assume that $\{(\mathcal{F}_\kappa,\mathrm{H}) : \kappa\in I\}$ is a soft ps-open cover of $(\mathfrak{U},\Theta,\mathrm{H})$ and let $|\mathrm{H}|=m$. Then $\mathfrak{U}=\bigcup_{\kappa\in I}\mathcal{F}_\kappa(\eta)$ for each $\eta\in\mathrm{H}$. Then $\mathcal{F}_\kappa(\eta)$ is a somewhat open set for each $\eta\in\mathrm{H}$. By hypothesis, $(\mathfrak{U},\Theta_\eta)$ is somewhat compact for each $\eta\in\mathrm{H}$, so we obtain $\mathfrak{U}=\bigcup_{\kappa=1}^{n_1}\mathcal{F}_\kappa(\eta_1)$, $\mathfrak{U}=\bigcup_{\kappa=n_1+1}^{n_2}\mathcal{F}_\kappa(\eta_2),\ldots,\mathfrak{U}=\bigcup_{\kappa=n_{m-1}+1}^{n_m}\mathcal{F}_\kappa(\eta_m)$. This implies that $\widetilde{\mathfrak{U}}=\widetilde{\bigcup}_{\kappa=1}^{n_m}(\mathcal{F}_\kappa,\mathrm{H})$. Hence, $(\mathfrak{U},\Theta,\mathrm{H})$ is soft ps-compact.

⇐: Follows from Theorem 4.12. □

In classical topology, the property says that a topological space defined over a finite (countable) set is somewhat compact is not valid for soft ps-compact spaces.

Example 4.16. Let $(\mathfrak{U}, \Theta, \mathbb{R})$ be an ST-space, where $\mathfrak{U} = \{\upsilon, \omega\}$ is the universal set, Θ is the discrete ST and a set of parameters $\mathbb{R}$ is the set of real numbers. Clearly, all parametric topological spaces $(\mathfrak{U}, \Theta_r)$ produced by $(\mathfrak{U}, \Theta, \mathbb{R})$ is somewhat compact. In contrast, an ST-space $(\mathfrak{U}, \Theta, \mathbb{R})$ is not soft ps-Lindelöf.

In what follows, we present some characteristics of compactness, α-compactness and semi-compactness and their Lindelöfness counterparts that are evaporated for the spaces of soft ps-compact and ps-Lindelöf spaces; see Example 4.5.

Let $(\mathfrak{U}, \Theta, \mathrm{H})$ be an extended ST-space. Then,

- if all topological spaces $(\mathfrak{U}, \Theta_a)$ produced by $(\mathfrak{U}, \Theta, \mathrm{H})$ are compact (resp., σ-compact), then $(\mathfrak{U}, \Theta, \mathrm{H})$ is soft compact (resp., soft σ-compact), where $\sigma \in \{\alpha, pre, semi, b\}$.
- if all topological spaces $(\mathfrak{U}, \Theta_a)$ produced by $(\mathfrak{U}, \Theta, \mathrm{H})$ are Lindelöf (resp., σ-Lindelöf), then $(\mathfrak{U}, \Theta, \mathrm{H})$ is soft Lindelöf (resp., soft σ-Lindelöf), where $\sigma \in \{\alpha, pre, semi, b\}$.

The property says that "An extended ST-space $(\mathfrak{U}, \Theta, \mathrm{H})$ is soft j-compact (resp., soft j-Lindelöf) iff $\mathfrak{U}$ and H are finite (resp., countable)" is true for all celebrated generalizations of soft compact and Lindelöf spaces. But it does not hold true for the current types of covering properties as the next example demonstrates.

Example 4.17. Let $(\mathbb{R}, \Theta, \mathbb{R})$ be an ST-space, where the set of real numbers $\mathbb{R}$ is the universal and parameters sets and Θ consists of all pseudo constant soft sets. It is obvious that the only soft ps-open subsets are the absolute and null soft sets, so $(\mathbb{R}, \Theta, \mathbb{R})$ is soft ps-compact. But an ST-space $(\mathbb{R}, \Theta, \mathbb{R})$ is not soft Lindelöf, which means that it is not soft b-Lindelöf and soft sw-Lindelöf. So it is not soft α-Lindelöf, soft pre-Lindelöf, soft semi-Lindelöf.

4.2. Almost soft ps-compactness and almost soft ps-Lindelöfness

We first introduce the next definition which will be the basis to define the main concepts of this subsection.

Definition 4.18. The soft ps-closure of a soft subset $(\mathcal{F}, \mathrm{H})$ of an ST-space $(\mathfrak{U}, \Theta, \mathrm{H})$, denoted by $spclr(\mathcal{F}, \mathrm{H})$, is the intersections of all soft ps-closed sets containing $(\mathcal{F}, \mathrm{H})$.

Definition 4.19. An ST-space $(\mathfrak{U}, \Theta, \mathrm{H})$ is named:

(i) almost soft parametric somewhat-open compact (briefly, almost soft ps-compact) if any soft ps-open cover has a finite subfamily in which the soft ps-closure of whose elements is a cover of $\widetilde{\mathfrak{U}}$.

(ii) almost soft parametric somewhat-open Lindelöf (briefly, almost soft ps-Lindelöf) if any soft ps-open cover has a countable subfamily in which the soft ps-closure of whose elements is a cover of $\widetilde{\mathfrak{U}}$.

It can be easily remarked that all almost soft ps-compact spaces are almost soft ps-Lindelöf.

The examples below show the existence and uniqueness of almost soft ps-compactness and almost soft ps-Lindelöfness.

Example 4.20. Let $\Theta = \{\phi, \widetilde{\mathbb{R}}, (\mathcal{F}, \mathrm{H}) \widetilde{\subseteq} \widetilde{\mathfrak{U}} : 1 \in \mathcal{F}(\eta)\}$ be an ST on $\mathbb{R}$ where $\mathrm{H} = \{\eta, \rho\}$. Similar to particular point topology, we get that $(\mathbb{R}, \Theta, \mathrm{H})$ is almost soft ps-compact.

Example 4.21. An ST-space $(\mathbb{R}, \Theta, \mathrm{H})$ mentioned in Example 4.4 is not almost soft ps-Lindelöf.

Proposition 4.22. *Any soft ps-compact (resp., soft ps-Lindelöf) space is almost soft ps-compact (resp., almost soft ps-Lindelöf).*

It can be seen from Example 4.20 that the converse of Proposition 4.22 need not be true in general.

The following two facts can be proved following similar lines of the proof of their counterparts given in the foregoing studies.

Proposition 4.23. *All soft ps-clopen subsets of an almost soft ps-compact space $(\mathfrak{U}, \Theta, \mathrm{H})$ are almost soft ps-compact.*

Corollary 4.24. *The intersection of almost soft ps-compact and soft ps-clopen subsetsis almost soft ps-compact.*

The above two results are still valid if we replace the word "compact" by "Lindelöf".

Definition 4.25. A class of soft sets $\{(\mathcal{F}_\kappa, \mathrm{H}) : \kappa \in I\}$ is said to have the 1st ps-CIP (resp., 1st ps-FIP) provided that $\widetilde{\bigcap}_{\kappa \in \delta} spinr(\mathcal{F}_\kappa, \mathrm{H}) \neq \phi$ for any countable (resp., finite) set $\delta \subseteq I$.

The next theorem characterizes the spaces of almost soft ps-Lindelöf and almost soft ps-compact.

Theorem 4.26. *An ST-space $(\mathfrak{U},\Theta,\mathrm{H})$ is an almost soft ps-compact (resp., almost soft ps-Lindelöf) iff every family $\{(\mathcal{F}_\kappa,\mathrm{H}):\kappa\in I\}$ of soft ps-closed subsets with a non-null soft intersection has the 1st ps-FIP (resp., 1st ps-CIP).*

Proof. A proof is provided for compactness.

$\Rightarrow$: Take $\{(\mathcal{F}_\kappa,\mathrm{H}):\kappa\in I\}$ as an arbitrary class of soft ps-closed subsets of an almost soft ps-compact $(\mathfrak{U},\Theta,\mathrm{H})$. If $\widetilde{\bigcap}_{\kappa\in I}(\mathcal{F}_\kappa,\mathrm{H})=\phi$, then $\widetilde{\mathfrak{U}}=\widetilde{\bigcup}_{\kappa\in I}(\mathcal{F}^c_\kappa,\mathrm{H})$. So, $\widetilde{\mathfrak{U}}=\widetilde{\bigcup}^n_{\kappa=1}spclr(\mathcal{F}^c_\kappa,\mathrm{H})$. Thus, $\phi=(\widetilde{\bigcup}^n_{\kappa=1}spclr(\mathcal{F}^c_\kappa,\mathrm{H}))^c=\widetilde{\bigcap}^n_{\kappa=1}spinr(\mathcal{F}_\kappa,\mathrm{H})$, as required.
$\Leftarrow$: Let $\{(\mathcal{F}_\kappa,\mathrm{H}):\kappa\in I\}$ be a soft ps-open cover of $(\mathfrak{U},\Theta,\mathrm{H})$. Then $\phi=\widetilde{\bigcap}_{\kappa\in I}(\mathcal{F}^c_\kappa,\mathrm{H})$. According to the 1st ps-FIP, we get $\phi=\widetilde{\bigcap}^n_{\kappa=1}spinr(\mathcal{F}^c_\kappa,\mathrm{H})$, which equivalently means that $\widetilde{\mathfrak{U}}=\widetilde{\bigcup}^n_{\kappa=1}spclr(\mathcal{F}_\kappa,\mathrm{H})$. Hence, $(\mathfrak{U},\Theta,\mathrm{H})$ is almost soft ps-compact. □

Theorem 4.27. *The surjective soft continuous image of an almost soft ps-compact (resp., almost soft ps-Lindelöf) set is almost soft compact (resp., almost soft Lindelöf) provided that the ST in the domain is full.*

Proof. Let a soft mapping $\mathsf{M}_\ell:(\mathfrak{U},\Theta,\mathrm{H})\to(\mathfrak{B},\Lambda,\mathrm{E})$ be soft continuous and suppose that $(\mathcal{K},\mathrm{H})$ is an almost soft ps-Lindelöf subset of $(\mathfrak{U},\Theta,\mathrm{H})$. Consider $\{(\mathcal{F}_\kappa,\mathrm{E}):\kappa\in I\}$ as a soft ps-open cover of $\mathsf{M}_\ell(\mathcal{K},\mathrm{H})$. So we obtain $(\mathcal{K},\mathrm{H})\widetilde{\subseteq}\widetilde{\bigcup}_{\kappa\in I}\mathsf{M}^{-1}_\ell(\mathcal{F}_\kappa,\mathrm{E})$. According to Proposition 3.19, $\mathsf{M}^{-1}_\ell(\mathcal{F}_\kappa,\mathrm{H})$ is soft ps-open for all $\kappa\in I$. By assumption of almost soft ps-Lindelöfness of $(\mathcal{K},\mathrm{H})$, there is a countable set δ such that $(\mathcal{K},\mathrm{H})\widetilde{\subseteq}\widetilde{\bigcup}_{\kappa\in\delta}spclr(\mathsf{M}^{-1}_\ell(\mathcal{F}_\kappa,\mathrm{E}))$. Since Θ is full, it follows from Corollary 3.10 that $\widetilde{\bigcup}_{\kappa\in\delta}spclr(\mathsf{M}^{-1}_\ell(\mathcal{F}_\kappa,\mathrm{E}))\widetilde{\subseteq}\widetilde{\bigcup}_{\kappa\in\delta}sclr(\mathsf{M}^{-1}_\ell(\mathcal{F}_\kappa,\mathrm{E}))$. Now, we obtain $\mathsf{M}_\ell(\mathcal{K},\mathrm{H})\widetilde{\subseteq}\widetilde{\bigcup}_{\kappa\in\delta}\mathsf{M}_\ell(sclr(\mathsf{M}^{-1}_\ell(\mathcal{F}_\kappa,\mathrm{E})))$. By soft continuity, $\widetilde{\bigcup}_{\kappa\in\delta}\mathsf{M}_\ell(sclr(\mathsf{M}^{-1}_\ell(\mathcal{F}_\kappa,\mathrm{E})))\widetilde{\subseteq}\widetilde{\bigcup}_{\kappa\in\delta}sclr(\mathsf{M}_\ell(\mathsf{M}^{-1}_\ell(\mathcal{F}_\kappa,\mathrm{E})))$ $\widetilde{\subseteq}\widetilde{\bigcup}_{\kappa\in\delta}sclr(\mathcal{F}_\kappa,\mathrm{E})$, which means that $\mathsf{M}_\ell(\mathcal{K},\mathrm{H})$ is almost soft Lindelöf. It can be proved the case of soft ps-compact in a similar way. □

We complete this subsection by studying the transmission of the introduced covering properties between soft topologies and classical topologies. First, we recall that a topological space $(\mathfrak{U},\tau)$ is said to be almost somewhat compact (resp. almost somewhat Lindelöf) if every cover of somewhat-open subsets of $(\mathfrak{U},\tau)$ has a finite (resp. countable) subcover such that the somewhat closure of whose members covers $\mathfrak{U}$.

Lemma 4.28. $(spclr(\mathcal{F}),\mathrm{H})\widetilde{\subseteq}spclr(\mathcal{F},\mathrm{H})$ *for any soft set $(\mathcal{F},\mathrm{H})$ in a full ST-space $(\mathfrak{U},\Theta,\mathrm{H})$.*

Proof. Let $\upsilon_\eta\notin spclr(\mathcal{F},\mathrm{H})$. Then, it can be found a soft ps-open set $(\mathcal{G},\mathrm{H})$ satisfying $\upsilon_\eta\in(\mathcal{G},\mathrm{H})$ and $(\mathcal{G},\mathrm{H})\widetilde{\bigcap}(\mathcal{F},\mathrm{H})=\phi$. Automatically, it follows that $\mathcal{G}(\eta)\cap\mathcal{F}(\eta)=\emptyset$ for each $\eta\in\mathrm{H}$. The characteristic of full of Θ implies that $\mathcal{G}(\eta)$ is a nonempty ps-open subset of Θ_η. Obviously, we obtain $\mathcal{G}(\eta)\cap spclr(\mathcal{F}(\eta))=\emptyset$. Therefore, $\upsilon_\eta\notin(spclr(\mathcal{F}),\mathrm{H})$. Hence, $(spclr(\mathcal{F}),\mathrm{H})\widetilde{\subseteq}spclr(\mathcal{F},\mathrm{H})$, as required. □

By taking a soft subset $(\mathcal{F},\mathrm{H})=\{(\eta,\{\upsilon\}),(\rho,\{\upsilon\})\}$ of full ST-space displayed in Example 3.2, we find that $spclr(\mathcal{F},\mathrm{H})=\widetilde{\mathfrak{U}}$ whereas $(spclr(H),\mathrm{H})=\{(\eta,\mathfrak{U}),(\rho,\{\upsilon\})\}$. This confirms that the converse of lemma mentioned above is wrong in general.

Theorem 4.29. *For a full ST-space $(\mathfrak{U},\Theta,\mathrm{H})$ with a finite (resp., countable) parameter set, every topological space $(\mathfrak{U},\Theta_\eta)$ produced by almost soft ps-compact (resp., almost soft ps-Lindelöf) $(\mathfrak{U},\Theta,\mathrm{H})$ is almost somewhat compact (resp., almost somewhat Lindelöf).*

Proof. This proof investigates the case of compactness.

Let $\{(\mathcal{F}_\kappa,\mathrm{H}):\kappa\in I\}$ be a soft ps-open cover of $(\mathfrak{U},\Theta,\mathrm{H})$ such that $|\,\mathrm{H}\,|=m$. Then $\mathfrak{U}=\bigcup\limits_{\kappa\in I}\mathcal{F}_\kappa(\eta)$ for each $\eta\in\mathrm{H}$. Now, $\mathcal{F}_\kappa(\eta)$ is a nonempty somewhat-open subset for each $\eta\in\mathrm{H}$. By hypothesis, $(\mathfrak{U},\Theta_\eta)$ is almost somewhat compact for each $\eta\in\mathrm{H}$, we obtain $\mathfrak{U}=\bigcup\limits_{\kappa=1}^{n_1}spclr(\mathcal{F}_\kappa(\eta_1))$, $\mathfrak{U}=\bigcup\limits_{\kappa=n_1+1}^{n_2}spclr(\mathcal{F}_\kappa(\eta_2)),\ldots,\ \mathfrak{U}=\bigcup\limits_{\kappa=n_{m-1}+1}^{n_m}spclr(\mathcal{F}_\kappa(\eta_m))$. This implies that $\widetilde{\mathfrak{U}}=\widetilde{\bigcup}^{n_m}_{\kappa=1}(spclr(\mathcal{F}_\kappa),\mathrm{H})$. According to Lemma 4.28, we obtain $(spclr(H),\mathrm{H})\widetilde{\subseteq}spclr(\mathcal{F},\mathrm{H})$, so $\widetilde{\mathfrak{U}}=\widetilde{\bigcup}^{n_m}_{\kappa=1}spclr(\mathcal{F}_\kappa,\mathrm{H})$. Hence, $(\mathfrak{U},\Theta,\mathrm{H})$ is almost soft ps-compact. □

The above theorem collapses if the terms of finite or countable of a set of parameters are absent.

Example 4.30. Let $(\mathfrak{U},\Theta,\mathbb{R})$ be an ST-space given in Example 4.16. Then, all parametric topological space $(\mathfrak{U},\Theta_r)$ inspired by $(\mathfrak{U},\Theta,\mathbb{R})$ are almost somewhat compact. But an ST-space $(\mathfrak{U},\Theta,\mathbb{R})$ is not almost soft ps-Lindelöf.

5. Soft ps-connected spaces

We allocated this section to study a novel kind of soft connectedness inspired by soft ps-open sets. We explore its master characterizations and elucidate that every soft ps-connected is almost soft ps-compact. Moreover, we proved two interesting properties under a full ST, first, soft ps-connectedness is a proper generalization of soft connectedness. Second, the correspondence between soft ps-connected and soft hyperconnected spaces. Ultimately, we describe how this type of soft connectedness behaves between soft and classical topologies.

Definition 5.1. The soft subsets $(\mathcal{F},\mathrm{H})$ and $(\mathcal{K},\mathrm{H})$ of an ST-space $(\mathfrak{U},\Theta,\mathrm{H})$ are named ps-separated if $(\mathcal{F},\mathrm{H})\widetilde{\bigcap}spclr(\mathcal{K},\mathrm{H})=\phi$ and $spclr(\mathcal{F},\mathrm{H})\widetilde{\bigcap}(\mathcal{K},\mathrm{H})=\phi$.

It is easily to note that the stipulation of disjoint is proper weaker than stipulation of ps-separated.

Definition 5.2. If an ST-space $(\mathfrak{U},\Theta,\mathrm{H})$ contains non-null ps-separated soft subsets $(\mathcal{F},\mathrm{H})$ and $(\mathcal{K},\mathrm{H})$ such that their union is $\widetilde{\mathfrak{U}}$, then we call $(\mathfrak{U},\Theta,\mathrm{H})$ soft ps-disconnected and call $(\mathcal{F},\mathrm{H})$ and $(\mathcal{K},\mathrm{H})$ a soft ps-disconnection of $\widetilde{\mathfrak{U}}$. Otherwise, $(\mathfrak{U},\Theta,\mathrm{H})$ is named a soft ps-connected space.

By the next examples, it is shown there is no relationship between soft disconnected and soft ps-disconnected spaces.

Example 5.3. An ST-space $(\mathfrak{U},\Theta,\mathrm{H})$ presented in Example 4.4 is soft ps-disconnected space which is soft connected.

Example 5.4. An ST-space $(\mathfrak{U},\Theta,\mathrm{H})$ presented in Example 4.5 is soft disconnected which is soft ps-connected.

Proposition 5.5. *A full soft disconnected space $(\mathfrak{U},\Theta,\mathrm{H})$ is soft ps-disconnected.*

Proof. According to Corollary 3.10, we find that $spclr(\mathcal{F},\mathrm{H})\widetilde{\subseteq}clr(\mathcal{F},\mathrm{H})$ for each $(\mathcal{F},\mathrm{H})\widetilde{\subseteq}\widetilde{\mathfrak{U}}$. □

The above proposition is not conversely in general; the example below elaborates this point.

Example 5.6. Let $\Theta=\{\phi,\widetilde{\mathfrak{U}},(\mathcal{F},\mathrm{H}),(\mathcal{G},\mathrm{H}),(\mathcal{K},\mathrm{H})\}$ be an ST over $\mathfrak{U}=\{\upsilon,\omega,\mu\}$ with $\mathrm{H}=\{\eta,\rho\}$, where
$(\mathcal{F},\mathrm{H})=\{(\eta,\{\upsilon\}),(\rho,\{\upsilon\})\}$;
$(\mathcal{G},\mathrm{H})=\{(\eta,\{\omega\}),(\rho,\{\omega\})\}$ and
$(\mathcal{K},\mathrm{H})=\{(\eta,\{\upsilon,\omega\}),(\rho,\{\upsilon,\omega\})\}$.

Now, it can be checked that $(\mathfrak{U},\Theta,\mathrm{H})$ is full soft connected. On the other hand, $\{(\eta,\{\upsilon,\mu\}),(\rho,\{\omega\})\}$ and $\{(\eta,\{\omega\}),(\rho,\{\upsilon,\mu\})\}$ are non-null ps-separated soft sets with soft union equals the absolute soft set; so it is soft ps-disconnected. In contrast, $(\mathfrak{U},\Theta,\mathrm{H})$ is soft connected.

We furnish some descriptions for soft ps-connected spaces in the next finding.

Proposition 5.7. *The three characterizations below are corresponding.*

(i) $(\mathfrak{U},\Theta,\mathrm{H})$ *is soft ps-connected.*
(ii) *if $(\mathcal{F},\mathrm{H})$ and $(\mathcal{K},\mathrm{H})$ are disjoint soft ps-closed (or soft ps-open) subsets with soft union equals $\widetilde{\mathfrak{U}}$, then $(\mathcal{F},\mathrm{H})=\widetilde{\mathfrak{U}}$ or $(\mathcal{K},\mathrm{H})=\widetilde{\mathfrak{U}}$.*
(iii) *The null and absolute soft sets are the only soft ps-open and soft ps-closed.*

Proof. Following similar arguments given for proof of its counterpart result in general topology, the proof follows. □

The theorem below replaces soft ps-open sets with soft open sets to describe soft ps-disconnectedness when the ST is full.

Theorem 5.8. *A full ST-space $(\mathfrak{U},\Theta,\mathrm{H})$ is soft ps-disconnected iff it is soft dishyperconnected.*

Proof. ⇒: Let $(\mathfrak{U},\Theta,\mathrm{H})$ be full soft ps-disconnected. Then Θ contains proper non-null disjoint soft ps-open subsets; say, $(\mathcal{F},\mathrm{H})$ and $(\mathcal{G},\mathrm{H})$. By taking specific parameter; say $\eta^\star$, we obtain $inr(\mathcal{F}(\eta^\star))\neq\emptyset$ and $inr(\mathcal{G}(\eta^\star))\neq\emptyset$. This implies that there are soft open subsets $(\mathcal{K},\mathrm{H})$ and $(\mathcal{I},\mathrm{H})$ such that $\mathcal{K}(\eta^\star)=inr(\mathcal{F}(\eta^\star))$ and $\mathcal{I}(\eta^\star)=inr(\mathcal{G}(\eta^\star))$. Now, $(\mathcal{K},\mathrm{H})\widetilde{\bigcap}(\mathcal{I},\mathrm{H})$ is a member of Θ with an empty component; so it follows from the condition of full that $(\mathcal{K},\mathrm{H})$ and $(\mathcal{I},\mathrm{H})$ are disjoint. Hence, $(\mathfrak{U},\Theta,\mathrm{H})$ is soft dishyperconnected.
⇐: Follows from Corollary 3.10. □

Corollary 5.9. *The next properties are equivalent provided that $(\mathfrak{U},\Theta,\mathrm{H})$ is full.*

(i) $(\mathfrak{U},\Theta,\mathrm{H})$ *is soft ps-connected.*
(ii) *if $(\mathcal{F},\mathrm{H})$ and $(\mathcal{K},\mathrm{H})$ are non-null soft ps-open subsets, then $(\mathcal{F},\mathrm{H})\widetilde{\cap}(\mathcal{K},\mathrm{H})\neq\phi$.*
(iii) *if $(\mathcal{F},\mathrm{H})$ and $(\mathcal{K},\mathrm{H})$ are proper soft ps-closed subsets, then $(\mathcal{F},\mathrm{H})\widetilde{\cup}(\mathcal{K},\mathrm{H})\neq\widetilde{\mathfrak{U}}$.*
(iv) *all non-null soft open sets are soft dense.*
(v) *if $(\mathcal{F},\mathrm{H})$ is a proper soft closed subset, then $inr(\mathcal{F},\mathrm{H})=\phi$.*
(vi) *$clr(\mathcal{F},\mathrm{H})=\widetilde{\mathfrak{U}}$ or $inr(clr(\mathcal{F},\mathrm{H}))=\phi$ for any soft subset $(\mathcal{F},\mathrm{H})$.*
(vii) *No disjoint soft neighbourhoods separate two soft-points.*

Definition 5.10. A soft subset $(\mathcal{K},\mathrm{H})$ of $(\mathfrak{U},\Theta,\mathrm{H})$ which cannot be represented as a soft union of two non-null ps-separated soft sets $(\mathcal{G},\mathrm{H})$ and $(\mathcal{F},\mathrm{H})$ is named a soft ps-connected set. Otherwise we call $(\mathcal{K},\mathrm{H})$ a soft ps-disconnected set.

Lemma 5.11. *Let soft sets $(\mathcal{F},\mathrm{H})$ and $(\mathcal{G},\mathrm{H})$ be soft ps-disconnection of $(\mathfrak{U},\Theta,\mathrm{H})$ with the property that $(\mathcal{K},\mathrm{H})$ is soft ps-connected. Then $(\mathcal{K},\mathrm{H})\widetilde{\subseteq}(\mathcal{F},\mathrm{H})$ or $(\mathcal{K},\mathrm{H})\widetilde{\subseteq}(\mathcal{G},\mathrm{H})$.*

Proof. Since $(\mathcal{F},\mathrm{H})$ and $(\mathcal{G},\mathrm{H})$ are soft ps-disconnection sets of $(\mathfrak{U},\Theta,\mathrm{H})$, we obtain $(\mathcal{F},\mathrm{H})\widetilde{\bigcup}(\mathcal{G},\mathrm{H})=\widetilde{\mathfrak{U}}$ and $[(\mathcal{F},\mathrm{H})\widetilde{\bigcap}spclr(\mathcal{G},\mathrm{H})]\widetilde{\bigcup}[spclr(\mathcal{F},\mathrm{H})\widetilde{\bigcap}(\mathcal{G},\mathrm{H})]=\phi$. Now, $(\mathcal{K},\mathrm{H})=[(\mathcal{K},\mathrm{H})\widetilde{\bigcap}(\mathcal{F},\mathrm{H})]\widetilde{\bigcup}[(\mathcal{K},\mathrm{H})\widetilde{\bigcap}(\mathcal{G},\mathrm{H})]$. It is clear that

$$[((\mathcal{K},\mathrm{H})\widetilde{\bigcap}(\mathcal{F},\mathrm{H}))\widetilde{\bigcap}spclr((\mathcal{K},\mathrm{H})\widetilde{\bigcap}(\mathcal{F},\mathrm{H}))]\widetilde{\bigcup}[((\mathcal{K},\mathrm{H})\widetilde{\bigcap}(\mathcal{G},\mathrm{H}))\widetilde{\bigcap}spclr((\mathcal{K},\mathrm{H})\widetilde{\bigcap}(\mathcal{G},\mathrm{H}))]$$
$$\widetilde{\subseteq}[(\mathcal{K},\mathrm{H})\widetilde{\bigcap}spclr(\mathcal{F},\mathrm{H})]\widetilde{\bigcup}[(\mathcal{K},\mathrm{H})\widetilde{\bigcap}spclr(\mathcal{G},\mathrm{H})]=\phi.$$

So, we infer that $(\mathcal{K},\mathrm{H})$ has the following soft ps-disconnection sets $(\mathcal{K},\mathrm{H})\widetilde{\bigcap}(\mathcal{F},\mathrm{H})$ and $(\mathcal{K},\mathrm{H})\widetilde{\bigcap}(\mathcal{G},\mathrm{H})$, which contradicts that $(\mathcal{K},\mathrm{H})$ is soft ps-connected. Hence, $(\mathcal{K},\mathrm{H})\widetilde{\bigcap}(\mathcal{F},\mathrm{H})=\phi$ or $(\mathcal{K},\mathrm{H})\widetilde{\bigcap}(\mathcal{G},\mathrm{H})=\phi$, which means that $(\mathcal{K},\mathrm{H})\widetilde{\subseteq}(\mathcal{F},\mathrm{H})$ or $(\mathcal{K},\mathrm{H})\widetilde{\subseteq}(\mathcal{G},\mathrm{H})$. □

Theorem 5.12. *Let $(\mathcal{K},\mathrm{H})$ be a soft subset of $(\mathfrak{U},\Theta,\mathrm{H})$ such that for each $v_\eta,\omega_\rho\in(\mathcal{K},\mathrm{H})$ there is a soft ps-connected subset $(\mathcal{I},\mathrm{H})$ of $(\mathcal{K},\mathrm{H})$ containing v_η,ω_ρ. Then $(\mathcal{K},\mathrm{H})$ is soft ps-connected.*

Proof. Take $(\mathcal{K},\mathrm{H})$ as a soft ps-disconnected set. This means that $(\mathcal{K},\mathrm{H})$ has soft ps-disconnection sets, say, $(\mathcal{F},\mathrm{H})$ and $(\mathcal{G},\mathrm{H})$. Directly, we find soft-points v_η,ω_ρ in which $v_\eta\in(\mathcal{F},\mathrm{H})$ and $\omega_\rho\in(\mathcal{G},\mathrm{H})$. According to the given, we find a soft ps-connected set $(\mathcal{I},\mathrm{H})$ containing v_η,ω_ρ and $(\mathcal{I},\mathrm{H})\widetilde{\subseteq}(\mathcal{K},\mathrm{H})=(\mathcal{F},\mathrm{H})\widetilde{\bigcup}(\mathcal{G},\mathrm{H})$. By Lemma 5.11, we get $(\mathcal{I},\mathrm{H})\widetilde{\subseteq}(\mathcal{F},\mathrm{H})$ or $(\mathcal{I},\mathrm{H})\widetilde{\subseteq}(\mathcal{G},\mathrm{H})$. Consequentially, $(\mathcal{F},\mathrm{H})\widetilde{\bigcap}(\mathcal{G},\mathrm{H})\neq\phi$, which contradicts that $(\mathcal{F},\mathrm{H})$ and $(\mathcal{G},\mathrm{H})$ are soft ps-disconnection of $(\mathcal{K},\mathrm{H})$. This means that $(\mathcal{K},\mathrm{H})$ is soft ps-connected. □

Corollary 5.13. *$(\mathcal{K},\mathrm{H})$ is a soft ps-connected set provided that $(\mathcal{K},\mathrm{H})$ is a soft union of soft ps-connected sets $(\mathcal{F}_\kappa,\mathrm{H})$ which their soft intersections are non-null.*

Proposition 5.14. *Let M_ℓ be a soft continuous mapping of a soft ps-connected space $(\mathfrak{U},\Theta,\mathrm{H})$ onto an ST-space $(\mathfrak{B},\Lambda,\mathrm{E})$. Then $\mathsf{M}_\ell(\widetilde{\mathfrak{U}})$ soft ps-connected.*

Proof. Take $\mathsf{M}_\ell(\widetilde{\mathfrak{U}})=\widetilde{\mathfrak{B}}$ as a soft ps-disconnected set. Theorem 5.7 tells us that there are non-null disjoint soft ps-open subsets $(\mathcal{I},\mathrm{H})$ and $(\mathcal{F},\mathrm{H})$. By Proposition 3.19 we obtain $\mathsf{M}_\ell^{-1}(\mathcal{I},\mathrm{H})$ and $\mathsf{M}_\ell^{-1}(\mathcal{F},\mathrm{H})$ are disjoint soft ps-open sets in Θ. Surjectiveness of M_ℓ implies that these soft ps-open subsets are non-null and their soft union is the absolute soft set $\widetilde{\mathfrak{U}}$. We obtain $(\mathfrak{U},\Theta,\mathrm{H})$ is soft ps-disconnected which is a contradiction. This proves that $(\mathfrak{B},\Lambda,\mathrm{E})$ is soft ps-connected. □

Proposition 5.15. *Every soft ps-connected is almost soft ps-compact.*

With respect to the converse side of the above proposition, note that Example 5.6 provides a soft ps-disconnected space which is also almost soft ps-compact.

We close this part of work by discussing this type of connectedness between soft topologies and the general topologies inspired by it.

As we previously mentioned that the classical counterparts of the concepts introduced herein are those defined by somewhat-open sets. So we recall the definition of somewhat connectedness in the following

Definition 5.16. A topological space $(\mathfrak{U},\Theta)$ is called somewhat disconnected if there are two nonempty somewhat-open sets which their union is $\mathfrak{U}$. Otherwise, we call $(\mathfrak{U},\Theta)$ a somewhat connected.

Theorem 5.17. *An ST-space $(\mathfrak{U},\Theta,\mathrm{H})$ is soft ps-connected iff $(\mathfrak{U},\Theta_\eta)$ is somewhat connected for* some $\eta\in\mathrm{H}$.

Proof. ⇒: Let $(\mathfrak{U},\Theta,\mathrm{H})$ be soft ps-connected. Suppose, to the contrary, all parametric topological spaces $(\mathfrak{U},\Theta_\eta)$ are somewhat disconnected. Then there exist disjoint nonempty sw-open subsets V_η,W_η of $(\mathfrak{U},\Theta_\eta)$ with union equals $\mathfrak{U}$ for each $\eta\in\mathrm{H}$. This implies that $(\mathcal{F},\mathrm{H})=\{(\eta,\mathcal{F}(\eta)):\mathcal{F}(\eta)=V_\eta\}$ and $(\mathcal{G},\mathrm{H})=\{(\eta,\mathcal{G}(\eta)):\mathcal{G}(\eta)=W_\eta\}$ are disjoint non-null soft ps-open subsets with soft union equals $\widetilde{\mathfrak{U}}$, which means that $(\mathfrak{U},\Theta,\mathrm{H})$ is soft ps-disconnected. But this contradicts the given. Hence, $(\mathfrak{U},\Theta_\eta)$ is somewhat connected for some $\eta\in\mathrm{H}$.
⇐: Suppose, to the contrary, $(\mathfrak{U},\Theta,\mathrm{H})$ is soft ps-disconnected. Then there exist disjoint non-null soft ps-open subsets $(\mathcal{F},\mathrm{H})$ and $(\mathcal{G},\mathrm{H})$ with soft union equals $\widetilde{\mathfrak{U}}$. By the definition of a soft ps-open subset, we find that $\mathcal{F}(\eta)$ and $\mathcal{G}(\eta)$ are disjoint non-null sw-open subsets of $(\mathfrak{U},\Theta_\eta)$ with union equals $\mathfrak{U}$ for each $\eta\in\mathrm{H}$. Therefore, all $(\mathfrak{U},\Theta_\eta)$ are somewhat disconnected. But this contradicts the given. Hence, $(\mathfrak{U},\Theta,\mathrm{H})$ is soft ps-connected. □

Corollary 5.18. *If all parametric topological spaces* $(\mathfrak{U}, \Theta_\eta)$ *produced by an* ST*-space* $(\mathfrak{U}, \Theta, \mathrm{H})$ *are somewhat connected, then* $(\mathfrak{U}, \Theta, \mathrm{H})$ *is soft* ps*-connected.*

An ST-space $(\mathfrak{U}, \Theta, \mathrm{H})$ introduced in Example 3.8 is soft ps-connected; however, its parametric topological space $(\mathfrak{U}, \Theta_\rho)$ is somewhat disconnected. This confirms that the converse of Corollary 5.18 fails.

Theorem 5.19. *A full* ST*-space* $(\mathfrak{U}, \Theta, \mathrm{H})$ *is soft* ps*-connected iff* $(\mathfrak{U}, \Theta_\eta)$ *is somewhat connected for* all $\eta \in \mathrm{H}$.

Proof. $\Rightarrow$: Let $(\mathfrak{U}, \Theta, \mathrm{H})$ be soft ps-connected. Suppose, to the contrary, there is a parametric topological space $(\mathfrak{U}, \Theta_\eta)$ is somewhat disconnected. Then there exist disjoint nonempty sw-open subsets V_η, W_η of $(\mathfrak{U}, \Theta_\eta)$ with union equals $\mathfrak{U}$. It is obvious that $inr(V_\eta)$ and $inr(W_\eta)$ are nonempty open subsets of $(\mathfrak{U}, \Theta_\eta)$. This implies there exist non-null soft open subsets $(\mathcal{F}, \mathrm{H})$ and $(\mathcal{G}, \mathrm{H})$ such that $\mathcal{F}(\eta) = inr(V_\eta)$ and $\mathcal{G}(\eta) = inr(W_\eta)$. Now, $(\mathcal{F}, \mathrm{H})\widetilde{\cap}(\mathcal{G}, \mathrm{H})$ is soft open with an empty component. By a condition of full, $(\mathcal{F}, \mathrm{H})\widetilde{\cap}(\mathcal{G}, \mathrm{H}) = \phi$. Thus, $(\mathfrak{U}, \Theta, \mathrm{H})$ is soft hyperconnected. It follows from Theorem 5.8 that $(\mathfrak{U}, \Theta, \mathrm{H})$ is soft ps-disconnected, which contradicts the given. This contradiction finishes the proof that $(\mathfrak{U}, \Theta, \mathrm{H})$ is soft ps-connected.
$\Leftarrow$: It is proved in Theorem 5.17. □

6. Conclusion remarks and future works

Since it was introduced the frame of ST, many authors endeavored to convey the classical topological principles to this soft frame. As it clearly appeared from the published literature, ST provides a fruitful environment to expand soft topological concepts; for example, each T_i-space corresponds to four types of soft T_i-spaces. Also, it was successfully applied some abstract soft topological concepts to handle some practical problems as introduced in [5,8,2].

In this article, we have provided a new technique to initiate the well-known generalizations of soft open sets. This technique is based on these corresponding generalizations that are obtained from the classical topologies produced by an ST. We first have presented the notion of "soft ps-open sets" and demonstrated this class of soft ps-open subsets of full soft hyperconnected space constructs an ST. Then, we have displayed the concepts of soft ps-compact, soft ps-Lindelöf, almost soft ps-compact, almost soft ps-Lindelöf, and soft ps-connected spaces. The basic properties of these spaces have been established and some interesting examples have been provided to show the relationships between them. We have investigated their main characterizations and demonstrated their unique properties such as 1) transmission of soft ps-compactness and soft ps-Lindelöfness to all classical topologies without any imposed condition; 2) The equivalent between soft ps-connected and soft hyperconnected spaces. We also have obtained and illustrated with some counterexamples some exciting results which describe the behaviours of these spaces via an ST and its parametric topologies.

In closing, we give four possible directions for future work.

- **(i)** It would be interesting to replace the concept of somewhat-open sets restriction on parametric topologies by considering the other celebrated extensions of open sets like α-open and β-open subsets.
- **(ii)** One can redefine the previous notions of soft continuity and soft separation axioms by making use of the class of soft ps-open sets
- **(iii)** Also, it can be investigated the application given in [2] by replacing the concept of soft somewhat-open sets with soft ps-open sets.
- **(iv)** It would be of interest to research the concepts introduced herein in the frames of supra-soft and infra-soft topologies.

CRediT authorship contribution statement

Tareq M. Al-shami: Writing – review & editing, Writing – original draft, Validation, Supervision, Investigation, Formal analysis, Data curation, Conceptualization. **Abdelwaheb Mhemdi:** Writing – review & editing, Funding acquisition, Formal analysis, Data curation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

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