

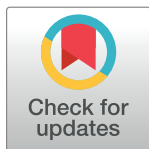
RESEARCH ARTICLE

Two-frequency chimera state in a ring of nonlocally coupled Brusselators

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Abstract

Chimera states, which consist of coexisting domains of spatially coherent and incoherent dynamics, have been intensively investigated in the past decade. In this work, we report a special chimera state, 2-frequency chimera state, in one-dimensional ring of nonlocally coupled Brusselators. In a 2-frequency chimera state, there exist two types of coherent domains and oscillators in different types of coherent domains have different mean phase velocities. We present the stability diagram of 2-frequency chimera state and study the transition between the 2-frequency chimera state and an ordinary 2-cluster chimera state.

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Citation: Dai Q, Liu D, Cheng H, Li H, Yang J (2017) Two-frequency chimera state in a ring of nonlocally coupled Brusselators. PLoS ONE 12 (10): e0187067. <https://doi.org/10.1371/journal.pone.0187067>

Editor: Changsong Zhou, Hong Kong Baptist University, HONG KONG

Received: July 27, 2017

Accepted: October 12, 2017

Published: October 27, 2017

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Data Availability Statement: All relevant data are within the paper.

Funding: This work was supported by National Natural Science Foundation of China (Grants Nos. 11575036 and 11505016), <http://www.nsf.gov.cn/>. The funders had no role in study design, data collection and analysis, decision to publish, or preparation of the manuscript.

Competing interests: The authors have declared that no competing interests exist.

Introduction

Chimera state refers to a type of fascinating hybrid dynamical states in which identically coupled units spontaneously develop into coexisting synchronous and asynchronous parts. Since its discovery in nonlocally coupled phase oscillators in 2002 [1], chimera state has become a very active research field [2, 3]. It has been extensively observed that chimera states can occur in globally coupled [4, 5] and locally coupled oscillators [6], periodic and chaotic maps [7], Stuart-Landau models [8, 9], Van der Pol oscillators [10], FitzHugh-Nagumo (FHN) oscillators [11], Hindmarsh-Rose models [12], Hodgkin-Huxley models [13] and Delayed-Feedback Systems [14]. Chimera states on random networks and on multiplex networks have been investigated [15–17]. Recently, chimera states were realized experimentally in chemical [18, 19], optical [20, 21], electrical [14], mechanical and electrochemical systems [22–25].

Different types of chimera states such as breathing chimeras [2], multi-cluster chimeras [26–28], and spiral chimeras [29, 30] have been discovered and investigated in details. However, in these chimera states, coherent oscillators always have the same mean phase velocity. In this work, we will report a new type of chimera state in which coherent oscillators may have different mean phase velocities.

Materials and methods

We consider a one-dimensional ring of N nonlocally coupled Brusselator [31] in which the individual unit is coupled to R neighbors on each side with coupling strength ϵ :

$$\begin{aligned} \dot{X}_k &= A - (B + 1)X_k + X_k^2 Y_k \\ &\quad + \frac{\epsilon}{2R} \sum_{j=k-R}^{k+R} D_{uu}(X_j - X_k) + D_{uv}(Y_j - Y_k), \\ \dot{Y}_k &= BX_k - X_k^2 Y_k \\ &\quad + \frac{\epsilon}{2R} \sum_{j=k-R}^{k+R} D_{vu}(X_j - X_k) + D_{vv}(Y_j - Y_k) \end{aligned} \tag{1}$$

The subscript k refers to the unit index, which has to be taken module N (or period boundary condition). Following Ref. [11], the coupling matrix is modelled as:

$$D = \begin{pmatrix} D_{uu} & D_{uv} \\ D_{vu} & D_{vv} \end{pmatrix} = \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix}. \tag{2}$$

Brusselator is a theoretical model for a type of autocatalytic reaction. Isolated Brusselator allows for an equilibrium at $X = A$ and $Y = B/A$. When $B > 1 + A^2$, the equilibrium becomes unstable and leads to a limit cycle.

To require that the Brusselator units work in the oscillatory regime, we set $A = 1, B = 2.1$. It is convenient to consider the ratio $r = R/N$, the coupling radius, which ranges from $1/N$ (nearest-neighbor coupling) to 0.5 (global coupling). In addition, we let $\phi = \pi/2 + \theta$. Throughout the paper, we numerically simulate Eq (1) by using the fourth-order Runge-Kutta method with a time step $\delta t = 0.01$. The total number of the Brusselator units is set to $N = 1000$.

Results and discussion

We report a peculiar chimera state at $r = 0.35, \theta = -0.1$, and $\epsilon = 0.02$ in Fig 1. The snapshot of the variable X_k in (a) and the snapshot of the phase of oscillator Θ_k , defined as $e^{i\Theta_k} = (\dot{X}_k + i\dot{Y}_k)/|\dot{X}_k + i\dot{Y}_k|$, in (b) show the coexistence of spatially coherent domains, in which oscillators distribute their variables in space in a continuous way, and incoherent domains, in which the variables of oscillators are scattered. There exist two large and several small coherent domains. Oscillators in the same large coherent domain are nearly in phase while those in different large coherent domains have a phase difference between them at around π . In contrast, coherent oscillators in small coherent domains may disperse their variables over a large range such as the phase in the range of 2π . The snapshot of the oscillators in the (X, Y) plane in Fig 1(c) shows that oscillators do not fall onto the orbit of isolated oscillators.

Coherent and incoherent domains can be identified more clearly by the mean phase velocity of oscillators which is defined as $\omega_k = \lim_{t-t' \rightarrow \infty} [\Theta_k(t) - \Theta_k(t')]/(t - t')$ with t' the transient time. Oscillators in a same coherent domain share the same mean phase velocity while those in a same incoherent cluster have different mean phase velocities. As shown by the profile of ω_k in Fig 1(d), there exists two large coherent domains and six small coherent domains. In an ordinary view on chimera state containing multi-coherent-cluster, all coherent oscillators share the same mean phase velocity. However, Fig 1(d) shows an extraordinary feature: oscillators in the two large coherent domains share a same mean phase velocity Ω_1 while those in the other six small coherent domains share another mean phase velocity Ω_2 . $\Omega_1 \neq \Omega_2$ suggests that

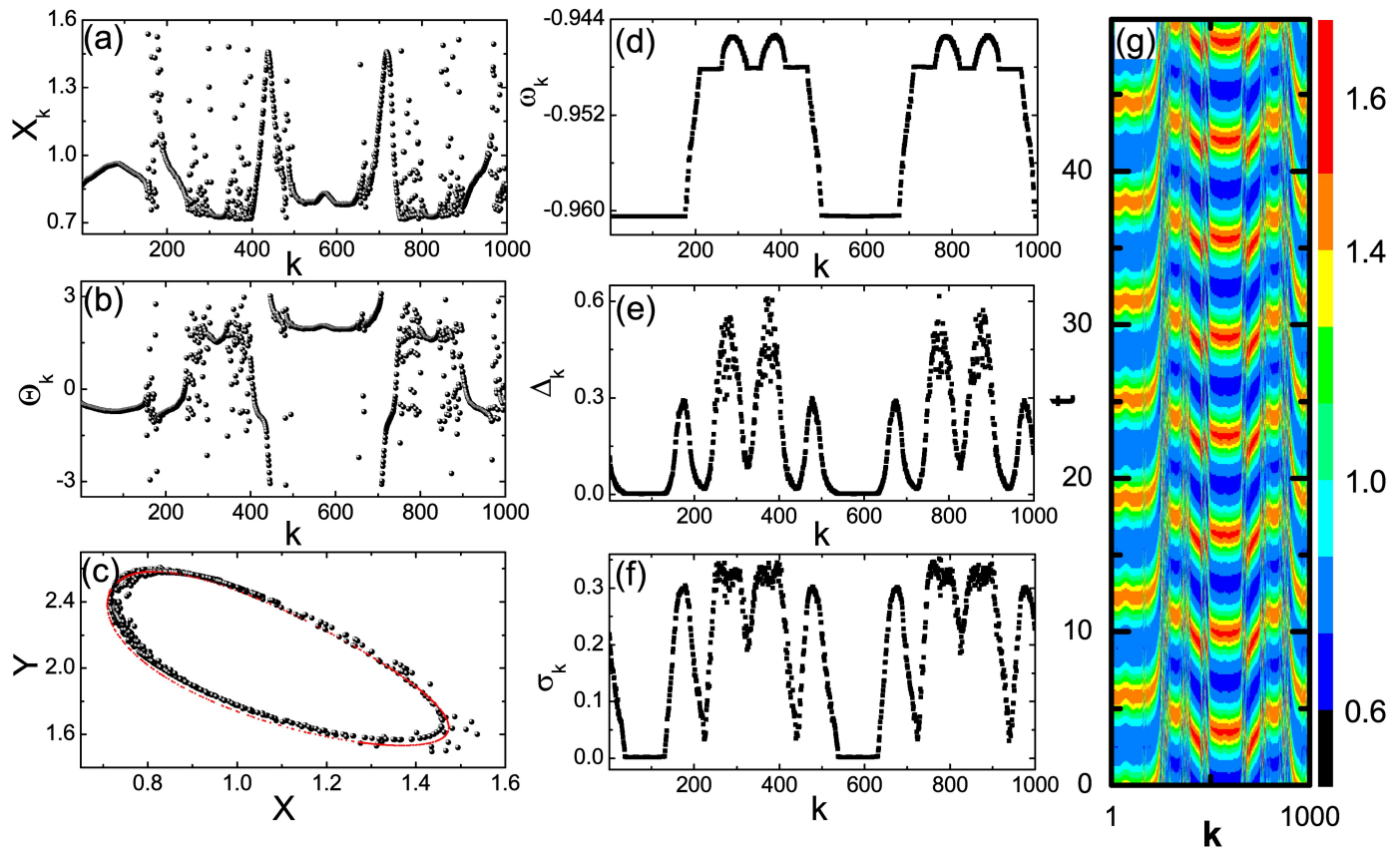


Fig 1. Two-frequency chimera state. (a) Snapshot of the variable X_k . (b) Snapshot of the phase Θ_k . (c) Snapshot of oscillators in the (X, Y) plane (in black). Red curve denotes the orbit of isolated Brusselator. (d) The profile of the mean phase velocity ω_k . (e) The profile of the difference between adjacent oscillators Δ_k . (f) The profile of the variance σ_k . (g) The spatiotemporal plot of the variable X_k after transient time. $A = 1$, $B = 2.1$, $r = 0.35$, $\theta = -0.1$, and $\epsilon = 0.02$.

<https://doi.org/10.1371/journal.pone.0187067.g001>

there are no fixed phase difference between oscillators in the large and the small coherent domains. From now on, we call the chimera state as 2-frequency chimera state. The profile of the mean phase velocity shows that the six small coherent domains are partitioned evenly into two groups and spatially separated by the large coherent domains. Regardless of the antiphase between the two large coherent domains, the 2-frequency chimera state is symmetric in space under the transformation, $k \rightarrow 2k_0 - k$ with k_0 the location of the center of the large coherent domain or the location of the center of the middle one among the three adjacent small coherent domains (For convenience, we call the middle one in the adjacent three small coherent domains as M-domain and others as S-domains). Thereby, the coherent domains are classified as the large domain, the M-domain, and the S-domain. Within the same type of coherent domain, different domains have the same domain size.

To further characterize the 2-frequency chimera state, we consider two other measures. One is the difference between adjacent oscillators, defined as $\Delta_k = \langle \Delta_k(t) \rangle_t = \langle \sqrt{(X_k - X_{k+1})^2 + (Y_k - Y_{k+1})^2} \rangle_t$ with $\langle \cdot \rangle_t$ the time average and the other is the variance σ_k of $\Delta_k(t)$. The profile of Δ_k in Fig 1(e) shows that Δ_k reaches its minima in coherent domains. Δ_k is nearly zero in the two large coherent domains, which confirms that coherent oscillators in the

same large domain are almost in phase. On the other hand, Δ_k stays at nonzero values in both M-domains and S-domains, which is in agreement with the observation that oscillators in small coherent domains are off phase as shown in Fig 1(a) and 1(b). Actually, Δ_k fluctuates with the locations of oscillators in the incoherent domains and the strongest fluctuation appears at the center part of each incoherent domain. Accordingly, Fig 1(f) shows that the variance σ_k stays at its highest value at the center part of the incoherent domains. Furthermore, Fig 1(f) shows $\sigma_k = 0$ in the two large coherent domains while nonzero σ_k in the M- and S-domains. Fig 1(g) shows a typical spatiotemporal plot of the variable X_k for the 2-frequency chimera state. To be mentioned, synchronous state is stable at the parameters in Fig 1. That is, the 2-frequency chimera state coexists with the synchronous state. Moreover, the attraction basin of the synchronous state is overwhelmingly larger than that of the 2-frequency chimera state. Consequently, Eq (1) always builds up the synchronous state for arbitrary initial conditions and the establishment of the 2-frequency chimera state requires deliberately prepared initial conditions. However, at certain range of θ such as $\theta \in [0.6, 1.1]$, the synchronous state might be unstable in Eq (1) and chimera states can be easily built up for random initial conditions (The results on that are beyond the scope of this work and are not presented here.). Using these chimera states as initial conditions, we find that the 2-frequency chimera states are possible to be realized. For example, the 2-frequency chimera state in Fig 1 is generated using the chimera state at $\sigma = 0.09$ and $\theta = 0.6$.

To gain an overall view of 2-frequency chimera states, we explore the $\theta - \epsilon$ plane in the range $[-0.4, 0] \times [0.005, 0.05]$. We use the chimera state in Fig 1 as initial conditions and integrate Eq (1) for 10^5 time units. After this interval, if the final state possesses a profile of mean phase velocity with two different coherent frequencies, we classify the 2-frequency chimera state as stable. The stability diagram of the 2-frequency chimera states is presented in Fig 2. We observe a narrow stripe extending from $\theta \simeq -0.325$ and $\epsilon \simeq 0.04$ down to $\theta \simeq 0$ and $\epsilon \simeq 0.0125$. Ordinary chimera state with two coherent clusters are developed on the right side of the stable regime of the 2-frequency chimera state. In contrast, the coherent states including synchronous state and travelling wave states appear on the left side of the stability regime (To be noted, the synchronous state is always stable in Fig 2 if arbitrary initial conditions are adopted.)

Now we investigate the transition between the 2-frequency chimera state and the 2-cluster chimera state. The typical bifurcation scenario of the transition between them is presented in Fig 3, where we fix $\theta = -0.1$ and increase the coupling strength ϵ from 0.02 to 0.03. Each row in Fig 3 has been made for different coupling strengths, starting with the chimera state in Fig 1 as initial conditions. The column A presenting the snapshots of the phases of oscillators after transient time shows that increasing ϵ turns a 2-frequency chimera state to a 2-cluster one. During the process, the two large coherent domains remain while the other small coherent domains are eliminated. Oscillators in different coherent domains are in anti-phase for the 2-cluster chimera state, which provides an explanation for the anti-phase between two large coherent domains in a 2-frequency chimera state. The columns B and C, presenting the profiles of the mean phase velocity ω_k and the profiles of Δ_k , respectively, suggest that the transition is a continuous one. With the coupling strength ϵ increase, the sizes of the small coherent domains vanish gradually and, interestingly, the small coherent domains in the 2-frequency chimera state locate in the center part of the incoherent domains in the 2-cluster chimera state. The phenomenon that new coherent domains emerge out of incoherent domain with parameter change has been observed in Ref. [11]. Different from the 2-frequency chimera state, there the new coherent domains share the same mean phase velocity with previous ones. The continuous transition between the 2-frequency chimera state and the 2-cluster chimera state can be

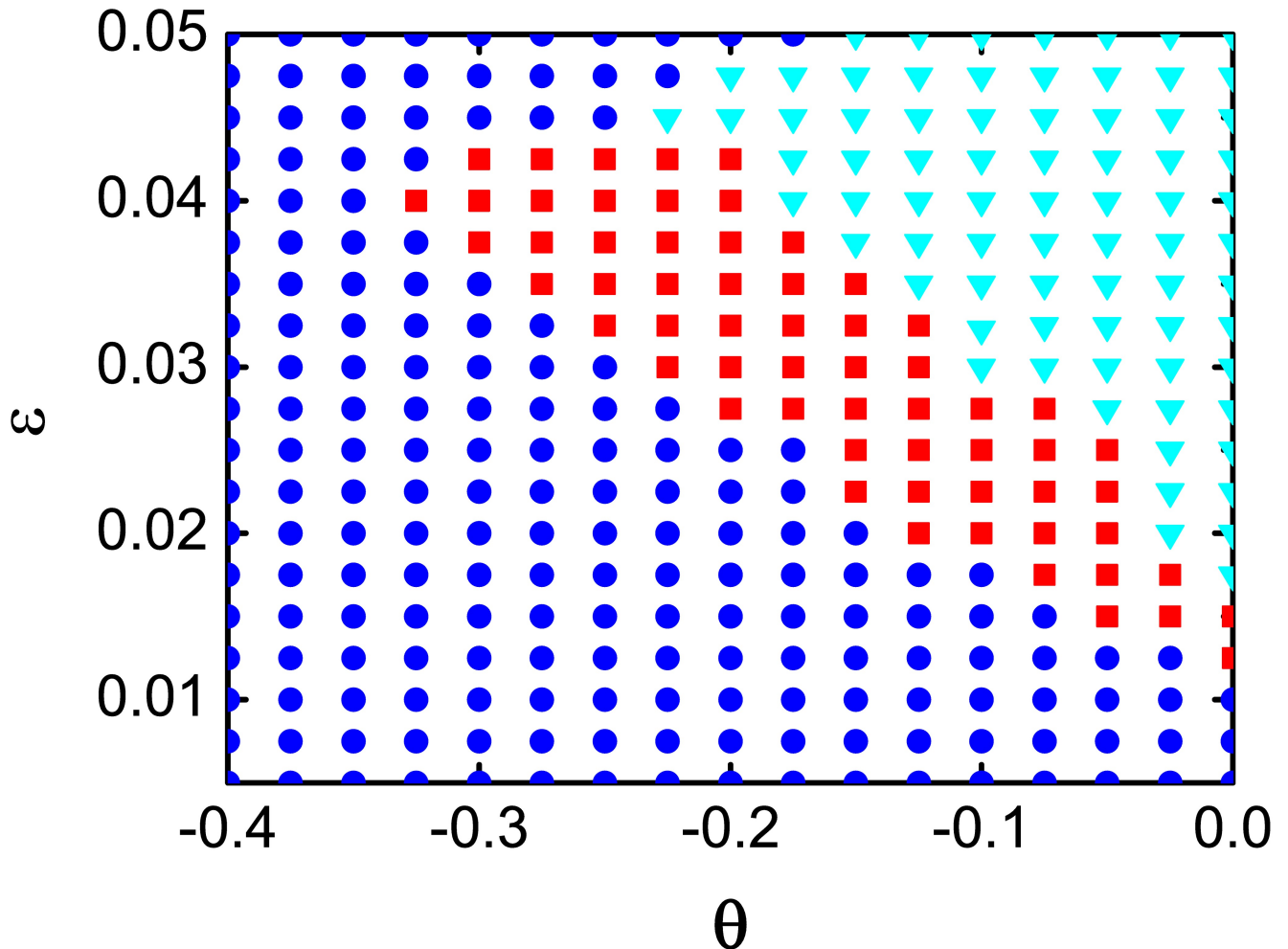


Fig 2. Stability diagram. The stability diagram of the 2-frequency chimera state in the $\theta - \epsilon$ plane. The 2-frequency chimera state, the 2-cluster chimera state, and the synchronous state are distinguished by the red squares, the cyan triangles, and blue circles, respectively. Other parameters are same as in Fig 1.

<https://doi.org/10.1371/journal.pone.0187067.g002>

supported by using the 2-cluster chimera states at $\epsilon = 0.03$ as initial conditions. With the coupling strength ϵ decrease, Fig 3 can be reproduced.

Conclusion

In conclusion, we have investigated nonlocally coupled Brusselators in a ring. We reported a new type of chimera states, 2-frequency chimera state. In a 2-frequency chimera state, there exist two types of coherent domains and oscillators in different types of coherent domains have different mean phase velocities. We explored the stability diagram of the 2-frequency chimera state in the parameter $\theta - \epsilon$ plane. We studied the transition between the 2-frequency chimera state and 2-cluster chimera state and found that the transition is a continuous one. The discovery of the 2-frequency chimera state may shed light on the future studies on chimera states.

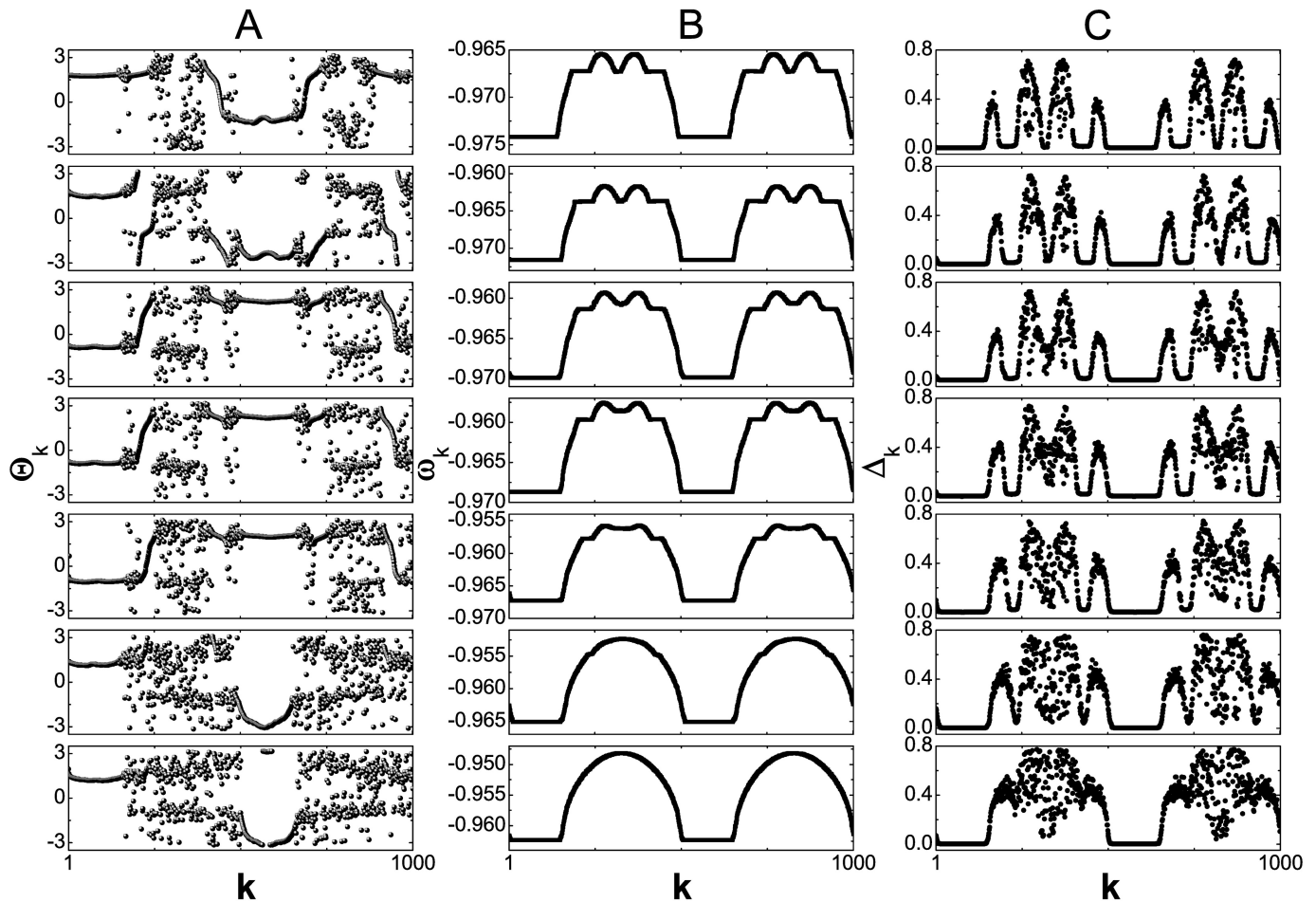


Fig 3. Bifurcation scenario. Typical bifurcation scenario for the 2-frequency chimera state with $\theta = -0.1$. For each value of the coupling strength ϵ (increasing from the top to the bottom, $\epsilon = 0.02, 0.0225, 0.024, 0.025, 0.026, 0.0275,$ and 0.03 , respectively) the snapshots of Θ_k (column A), the profile of the mean phase velocity ω_k (column B), and the profile of Δ_k (column C) are shown. Other parameters are same as in Fig 1.

<https://doi.org/10.1371/journal.pone.0187067.g003>

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Formal analysis: Qionglin Dai, Danna Liu, Hongyan Cheng, Junzhong Yang.

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Visualization: Qionglin Dai, Danna Liu.

Writing – original draft: Junzhong Yang.

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References

1. Kuramoto Y, Battogtokh D. Coexistence of coherence and incoherence in nonlocally coupled phase oscillators. *Nonlinear Phenomena in Complex Systems*. 2002; 5(4): 380–5.
2. Abrams DM, Strogatz SH. Chimera states for coupled oscillators. *Phys. Rev. Lett.* 2004 Oct; 93(17): 174102. <https://doi.org/10.1103/PhysRevLett.93.174102> PMID: 15525081
3. Panaggio MJ, Abrams DM. Chimera states: Coexistence of coherence and incoherence in networks of coupled oscillators. *Nonlinearity*. 2015 Feb; 28(3): R67. <https://doi.org/10.1088/0951-7715/28/3/R67>
4. Sethia GC, Sen A. Chimera states: The existence criteria revisited. *Phys. Rev. Lett.* 2014 Apr; 112(14): 144101. <https://doi.org/10.1103/PhysRevLett.112.144101> PMID: 24765967
5. Schmidt L, Krischer K. Clustering as a prerequisite for chimera states in globally coupled systems. *Phys. Rev. Lett.* 2015 Jan; 114(3): 034101. <https://doi.org/10.1103/PhysRevLett.114.034101> PMID: 25658999
6. Laing CR. Chimeras in networks with purely local coupling. *Phys. Rev. E*. 2015 Nov; 92(5): 050904. <https://doi.org/10.1103/PhysRevE.92.050904>
7. Omelchenko I, Maistrenko Y, Hövel P, Schöll E. Loss of coherence in dynamical networks: Spatial chaos and chimera states. *Phys. Rev. Lett.* 2011 Jun; 106(23): 234102. <https://doi.org/10.1103/PhysRevLett.106.234102> PMID: 21770506
8. Laing CR. Chimeras in networks of planar oscillators. *Phys. Rev. E*. 2010 Jun; 81(6): 066221. <https://doi.org/10.1103/PhysRevE.81.066221>
9. Zakharova A, Kapeller M, Schöll E. Chimera death: Symmetry breaking in dynamical networks. *Phys. Rev. Lett.* 2014 Apr; 112(15): 154101. <https://doi.org/10.1103/PhysRevLett.112.154101> PMID: 24785041
10. Omelchenko I, Zakharova A, Hövel P, Siebert J, Schöll E. Nonlinearity of local dynamics promotes multi-chimeras. *Chaos*. 2015 Aug; 25(8): 083104. <https://doi.org/10.1063/1.4927829> PMID: 26328555
11. Omelchenko I, Omelchenko OE, Hövel P, Schöll E. When nonlocal coupling between oscillators becomes stronger: Patched synchrony or multichimera states. *Phys. Rev. Lett.* 2013 May; 110(22): 224101. <https://doi.org/10.1103/PhysRevLett.110.224101> PMID: 23767727
12. Hizanidis J, Kanas V, Bezerianos A, Bountis T. Chimera states in networks of nonlocally coupled Hindmarsh-Rose neuron models. *Int. J. Bifurcat. Chaos*. 2014 Mar; 24(3): 1450030. <https://doi.org/10.1142/S0218127414500308>
13. Sakaguchi H. Instability of synchronized motion in nonlocally coupled neural oscillators. *Phys. Rev. E*. 2006 Mar; 73(3): 031907. <https://doi.org/10.1103/PhysRevE.73.031907>
14. Larger L, Penkovsky B, Maistrenko Y. Virtual chimera states for delayed-feedback systems. *Phys. Rev. Lett.* 2013 Aug; 111(5): 054103. <https://doi.org/10.1103/PhysRevLett.111.054103> PMID: 23952404
15. Zhu Y, Zheng Z, Yang J. Chimera states on complex networks. *Phys. Rev. E*. 2014 Feb; 89(2): 022914. <https://doi.org/10.1103/PhysRevE.89.022914>
16. Ghosh S, Kumar A, Zakharova A, Jalan S. Birth and death of chimera: Interplay of delay and multiplexing. *EPL*. 2016 Nov; 115(6): 60005. <https://doi.org/10.1209/0295-5075/115/60005>
17. Maksimenko VA, Makarov VV, Bera BK, Ghosh D, Dana SK, Goremyko MV, et al. Excitation and suppression of chimera states by multiplexing. *Phys. Rev. E*. 2016 Nov; 94(5): 052205. <https://doi.org/10.1103/PhysRevE.94.052205> PMID: 27967153
18. Tinsley MR, Nkomo S, Showalter K. Chimera and phasecluster states in populations of coupled chemical oscillators. *Nat. Phys.* 2012 Sep; 8(9): 662. <https://doi.org/10.1038/nphys2371>
19. Schmidt L, Schönleber K, Krischer K, García-Morales V. Coexistence of synchrony and incoherence in oscillatory media under nonlinear global coupling. *Chaos*. 2014 Mar; 24(1): 013102. <https://doi.org/10.1063/1.4858996> PMID: 24697364
20. Hagerstrom AM, Murphy TE, Roy R, Hövel P, Omelchenko I, Schöll E. Experimental observation of chimeras in coupled-map lattices. *Nat. Phys.* 2012 Sep; 8(9): 658. <https://doi.org/10.1038/nphys2372>
21. Viktorov EA, Habruseva T, Hegarty SP, Huyet G, Kelleher B. Coherence and incoherence in an optical comb. *Phys. Rev. Lett.* 2014 Jun; 112(12): 224101. <https://doi.org/10.1103/PhysRevLett.112.224101> PMID: 24949771
22. Martens EA, Thutupalli S, Fourrière A, Hallatschek O. Chimera states in mechanical oscillator networks. *Proc. Natl. Acad. Sci. U S A*. 2013 Jun; 110(26): 10563–7 <https://doi.org/10.1073/pnas.1302880110> PMID: 23759743
23. Kapitaniak T, Kuzma P, Wojewoda J, Czolczynski K, Maistrenko Y. Imperfect chimera states for coupled pendula. *Sci. Rep.* 2014 Sep; 4: 6379. <https://doi.org/10.1038/srep06379> PMID: 25223296

24. Olmi S, Martens EA, Thutupalli S, Torcini A. Intermittent chaotic chimeras for coupled rotators. *Phys. Rev. E*. 2015 Sep; 92(3):030901. <https://doi.org/10.1103/PhysRevE.92.030901>
25. Wickramasinghe M, Kiss IZ. Spatially organized dynamical states in chemical oscillator networks: synchronization, dynamical differentiation, and chimera patterns. *PLoS ONE*. 2013 Nov; 8(11): e80586. <https://doi.org/10.1371/journal.pone.0080586> PMID: 24260429
26. Omelchenko I, Provata A, Hizanidis J, Schöll E, Hövel P. Robustness of chimera states for coupled Fitz-Hugh-Nagumo oscillators. *Phys. Rev. E*. 2015 Feb; 91(2): 022917. <https://doi.org/10.1103/PhysRevE.91.022917>
27. Maistrenko YL, Vasylenko A, Sudakov O, Levchenko R, Maistrenko VL. Cascades of multiheaded chimera states for coupled phase oscillators. *Int. J. Bifurcat. Chaos*. 2014 Aug; 24(8): 1440014. <https://doi.org/10.1142/S0218127414400148>
28. Zhu Y, Li Y, Zhang M, Yang J. The oscillating two-cluster chimera state in non-locally coupled phase oscillators. *EPL*. 2012 Jan; 97(1): 10009. <https://doi.org/10.1209/0295-5075/97/10009>
29. Martens EA, Laing CR, Strogatz SH. Solvable model of spiral wave chimeras. *Phys. Rev. Lett.* 2010 Jan; 104(4): 044101. <https://doi.org/10.1103/PhysRevLett.104.044101> PMID: 20366714
30. Gu C, St-Yves G, Davidsen J. Spiral wave chimeras in complex oscillatory and chaotic systems. *Phys. Rev. Lett.* 2013 Sep; 111(13): 134101 <https://doi.org/10.1103/PhysRevLett.111.134101> PMID: 24116782
31. Glansdorff P, Prigogine I. *Thermodynamic theory of structure, stability and fluctuations*. Chichester: Wiley Interscience; 1971.