Heliyon 5 (2019) e01818

Contents lists available at ScienceDirect

Heliyon

www.elsevier.com/locate/heliyon

Light ray trajectories in an analog of conformal spacetimes

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ARTICLE INFO

Keywords:
Cosmology
Analogy
Conformal spacetimes
Standard cosmology

ABSTRACT

The kinematical implications of a dielectric analog of Robertson-Walker spacetime is discussed within a covariant postulation of transformation optics. It is found that the results thoroughly coincide with what we expect from standard model. Moreover, a moving cloaked region in the spacetime is also considered as a physically valid proposal.

1. Introduction

The connection between theory of transformation optics (TO) and engineered metamaterials [1, 2, 3] stems from the capability of these materials to be designed flexibly, to have variety of applications which are proposed by the theory. These may include cloaking devices [4], invisibility devices [5, 6] and perfect lenses [7]. It may also become important to develop a joint between general theory of relativity and man-made metamaterials. The importance of general relativistic modifications has been noted in engineering [8], for example in Global Positioning System (GPS). However, when fabrication of an optical device is desired which has to undergo general relativistic effects (for example, an orbiting telescope containing a superlens [9], or a satellite antenna based on TO [10]), these modifications have to be applied in interpreting the behavior of electromagnetic fields in materials, when for instance it is considered in curved spacetime [11]. Here it should be noted that despite the fact that general relativity and TO basically share a same mathematical language, it is a crucial task to fully understand an optical device in the context of general relativity, since TO is discussed on a fixed background, whereas general relativity is a dynamic theory of spacetime. For a recollection motivated by the recently celebrated centenary and for an overview of the challenges it currently faces, see [12, 13, 14].

This is the original idea of transformation optics. The underlying physics of TO, firstly proposed by Eddington [15], is that the trajectory of light in an arbitrary vacuum curved spacetime, could be regenerated in an appropriate dielectric media, residing in Minkowski spacetime. Later, Gordon [16] discussed the way of finding the analog curved vacuum spacetime, out of a definite dielectric, using an effective optical metric. However it was Plebanski [17] who found out that the constitutive equations for electromagnetic fields in an arbitrary vacuum curved spacetime, are equivalent to those in an appropriate dielectric media in Minkowski spacetime. Further, De Felice [18] and afterwards Reznik [19], used this equivalence in studying gravitational systems and De Felice generalized it to Friedmann-Robertson-walker spacetime in a universe with no spatial curvature. Note that, TO is based on transformation media performing coordinate transformations, which were initially considered to be purely spatial transformations [6, 20, 21, 22]. However since De Felice's approach is linked to differential geometry, one can perform both space and time transformations [9].

As stated above, the most important feature of the Plebanski's equivalence, is providing the possibility of studying an analog system in Minkowski spacetime that mimics some aspects of gravitational systems, like trajectories of light. Because of complexity of such systems and also lack of a unified theory of gravitation, some remarkably distinguishable phenomena are not still completely understood, even if they have been observationally approved. For example, there are some experimental evidences for stimulated emissions from an analog system [see for example [23]], that may provide some kind of Hawking-like radiation [24]. However since this appears to be a quantum phenomenon, we have yet not been able to properly explain it within available gravitational theories. Therefore it seems that if we could resemble spacetime properties using a definite dielectric with certain configuration, it might be possible to facilitate the investigation of gravitational systems, experiencing that spacetime.

In this paper as well, we consider such simplification to study the kinematical features of standard cosmology, by identifying Robertson-Walker (RW) geometry and a dielectric in Minkowski spacetime. However the method we exploit is not the Plebanski-De Felice approach,

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https://doi.org/10.1016/j.heliyon.2019.e01818
Received 25 October 2018; Received in revised form 9 December 2018; Accepted 22 May 2019

because of its limitations, specially lack of covariance which prompts us to take only a certain class of transformations.

Note that, the isotropy of RW geometry is in accordance with what Plebanski-De Felice TO relies on, and therefore the appearance of magnetoelectric coupling can be regarded as the velocity of an isotropic media at low speeds. However instead, we will apply a covariant theory of TO, introduced in [25], developed in [26] and studied in the context of vacuum solutions of general relativity in [27] and [11]. This method, because of its covariance, provides a greater freedom in considering diversified types of motions and transformations (even for time-varying dielectric [28]), while designing materials. Note that, the applications of this covariant theory in the FRW spacetime has been elaborated before. A thorough discussion of spherical cloaks has been given in [29], whereas the analog FRW model has been taken care, in [30]. Here, the approaches mentioned above, are discussed once more and the whole discussions, are brought in a single article.

The paper is organized as follows: in section 2, a general survey on classical electrodynamics is made. In section 3, we provide a background for Plebanski-based TO, followed by a constructive introduction to the mentioned covariant approach and the concept of the dielectric analog of a spacetime. In section 4, this method will be used to form a dielectric analog for RW spacetime, in order to reobtain the kinematical relations which are derived from standard cosmology. In section 5, we assume a peculiar transformation to a certain region in RW spacetime, which has been electromagnetically cloaked and the configuration of this functioning dielectric is calculated. The concluding remarks are given in section 6.

2. Background

2.1. Covariant electromagnetic theory

Here we bring a formulation of electrodynamics which manifests itself in terms of tensorial objects and spacetime metric, and therefore it is covariant. Our introduction is brief, containing only remarkable notions which we will use in this paper; further detailed notes on the covariant theory of electromagnetism could be found in classical text books like [31, 32, 33].

Taking the differential forms representation, we introduce the potential 1-form by the covector $\mathbf{A} = A_\mu$ [25] and the field strength 2-form by $\mathbf{F} = F_{\mu\nu}$, defined by the following exterior derivative:

$$\mathbf{F} = \mathrm{d}\mathbf{A}, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

For Minkowski spacetime (in $(-+++)$ sing convention with $c = 1$), the Cartesian components of $\mathbf{F}$ are

$$\begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{pmatrix},$$

in terms of the electric and magnetic vector field components E_i and B_i. These fields could also be included in the excitation 2-form $\mathbf{G}$, which in its component form for Cartesian coordinates in Minkowski spacetime is

$$\begin{pmatrix} 0 & H_x & H_y & H_z \\ -H_x & 0 & D_z & -D_y \\ -H_y & -D_z & 0 & D_x \\ -H_z & D_y & -D_x & 0 \end{pmatrix}. \tag{1}$$

Defining the Hodge dual operator as [26]

$$\star : \wedge^k T_p^*(M) \longrightarrow \wedge^{(m-k)} T_p^*(M),$$

one can provide a bijection between the spaces of k-forms and $(m-k)$-forms, on manifold M. In other words, a bijection between tensors of rank k and $(m-k)$, where m is the dimension of M. Also the component form of a Hodge dual, which is applied to 2-forms reads as

$$\star_{\alpha\beta}{}^{\mu\nu} = \frac{1}{2}\sqrt{|g|}\, \epsilon_{\alpha\beta\sigma\rho} g^{\sigma\mu} g^{\rho\nu}. \tag{2}$$

Hence, the components of $\star\mathbf{F}$ will be

$$(\star\mathbf{F})_{\alpha\beta} = \frac{1}{2}\sqrt{|g|}\, \epsilon_{\alpha\beta\sigma\rho} g^{\sigma\mu} g^{\rho\nu} F_{\gamma\delta} = \begin{pmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & E_z & -E_y \\ -B_y & -E_z & 0 & E_x \\ -B_z & E_y & -E_x & 0 \end{pmatrix}. \tag{3}$$

Regarding (1) and (3), it turns out that

$$\mathbf{G} = \chi(\star\mathbf{F}), \tag{4}$$

or in component form

$$G_{\mu\nu} = \chi_{\mu\nu}{}^{\alpha\beta} (\star\mathbf{F})_{\alpha\beta}$$

Where $\chi_{\mu\nu}{}^{\alpha\beta}$ is also independently antisymmetric with respect to exchanges in $\mu\nu$ and $\alpha\beta$. The parameter χ in (4), contains information about the material properties, whereas $\star$ exhibits spacetime geometry. Note that, the usual vacuum constitutive relation is $\mathbf{G} = \star\mathbf{F}$, so if we are concerning with a vacuum, then $\chi_{\text{vac}}(\star\mathbf{F}) = \star\mathbf{F}$, therefore χ_{vac} is independent of coordinate system (see relation (8)).

In terms of these parameters, the homogeneous and inhomogeneous Maxwell's equations are respectively

$$\mathrm{d}\mathbf{F} = 0,$$

$$\mathrm{d}\mathbf{G} = \mathbf{J},$$

in which the charge current 3-form $\mathbf{J}$, in its tensorial form would be

$$J_{\alpha\beta\gamma} = \sqrt{|g|}\, \epsilon_{\alpha\beta\gamma\rho}\, j^\mu,$$

where j^μ is the four-vector current. The general covariant constitutive equation in (4), provides 6 relations for the electric and magnetic responses in a material [25]:

$$\overrightarrow{H} = \check{\mu}^{-1}\,\overrightarrow{B} + \check{\gamma}_1{}^*\,\overrightarrow{E}, \quad \overrightarrow{D} = \check{\varepsilon}^*\,\overrightarrow{E} + \check{\gamma}_2{}^*\,\overrightarrow{B}, \tag{5}$$

or by rearranging

$$\overrightarrow{B} = \check{\mu}\,\overrightarrow{H} + \check{\gamma}_1\,\overrightarrow{E}, \quad \overrightarrow{D} = \check{\varepsilon}\,\overrightarrow{E} + \check{\gamma}_2\,\overrightarrow{H}. \tag{6}$$

The 3×3 matrices of permittivity $\check{\varepsilon}$ and permeability $\check{\mu}$, and the magnetoelectric couplings $\check{\gamma}_1$ and $\check{\gamma}_2$ in (6), are related to corresponding ones in (5) as follows:

$$\check{\mu} = \left(\check{\mu}^{-1}\right)^{-1}, \qquad \check{\varepsilon} = \check{\varepsilon}^* - \check{\gamma}_2{}^*\check{\mu}\,\check{\gamma}_1{}^*,$$

$$\check{\gamma}_1 = -\check{\mu}\check{\gamma}_1{}^*, \qquad \check{\gamma}_2 = \check{\gamma}_2{}^*\check{\mu}. \tag{7}$$

Note that in this work, same as what has been presented in [25, 26], $\check{\mu}^{-1}$, $\check{\varepsilon}^*$, $\check{\gamma}_1{}^*$ and $\check{\gamma}_2{}^*$, constitute the components of χ, and the main material parameters are obtained from (7).

3. Theory

3.1. Susceptibility reference frame

Here, we bring some relations which are in use in this paper. According to $\chi_{\text{vac}}(\star\mathbf{F}) = \star\mathbf{F}$, the vacuum susceptibility is derived as [25]

$$\chi_{\text{vac}} = (\chi_{\text{vac}})_{\mu\nu}{}^{\alpha\beta} = \frac{1}{2} \times \begin{pmatrix} \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} & \begin{pmatrix} 0&1&0&0\\-1&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} & \begin{pmatrix} 0&0&1&0\\0&0&0&0\\-1&0&0&0\\0&0&0&0 \end{pmatrix} & \begin{pmatrix} 0&0&0&1\\0&0&0&0\\0&0&0&0\\-1&0&0&0 \end{pmatrix} \\ \begin{pmatrix} 0&-1&0&0\\0&0&0&0\\0&0&0&0\\1&0&0&0 \end{pmatrix} & \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} & \begin{pmatrix} 0&0&0&0\\0&0&1&0\\0&-1&0&0\\0&0&0&0 \end{pmatrix} & \begin{pmatrix} 0&0&0&0\\0&0&0&1\\0&0&0&0\\0&-1&0&0 \end{pmatrix} \\ \begin{pmatrix} 0&0&-1&0\\0&0&0&0\\1&0&0&0\\0&0&0&0 \end{pmatrix} & \begin{pmatrix} 0&0&0&0\\0&0&-1&0\\0&1&0&0\\0&0&0&0 \end{pmatrix} & \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} & \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&1\\0&0&-1&0 \end{pmatrix} \\ \begin{pmatrix} 0&0&0&-1\\0&0&0&0\\0&0&0&0\\1&0&0&0 \end{pmatrix} & \begin{pmatrix} 0&0&0&0\\0&0&0&-1\\0&0&0&0\\0&1&0&0 \end{pmatrix} & \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&-1\\0&0&1&0 \end{pmatrix} & \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} \end{pmatrix}. \quad (8)$$

Susceptibility reference matrix in spherical coordinates is [26]:

$$\chi_{\mu\nu}{}^{\alpha\beta} = \frac{1}{2} \begin{pmatrix} \mathbf{0} & * & * & * \\ \mathcal{A} & \mathbf{0} & * & * \\ \mathcal{B} & \mathcal{C} & \mathbf{0} & * \\ \mathcal{D} & \mathcal{E} & \mathcal{F} & \mathbf{0} \end{pmatrix}, \quad (9)$$

where $*$ are obtained using the antisymmetry of χ, and

$$\mathcal{A} = \begin{pmatrix} 0 & -\mu_{rr}^{-1} & -\frac{\mu_{r\theta}^{-1}}{r} & -\frac{\mu_{r\phi}^{-1}}{r\sin\theta} \\ \mu_{rr}^{-1} & 0 & -\frac{\gamma_{1r\phi}^{*}}{r} & -\frac{\gamma_{1r\theta}^{*}}{r\sin\theta} \\ \frac{\mu_{r\theta}^{-1}}{r} & \frac{\gamma_{1r\phi}^{*}}{r} & 0 & -\frac{\gamma_{1rr}^{*}}{r^2\sin\theta} \\ \frac{\mu_{r\phi}^{-1}}{r\sin\theta} & \frac{\gamma_{1r\theta}^{*}}{r\sin\theta} & \frac{\gamma_{1rr}^{*}}{r^2\sin\theta} & 0 \end{pmatrix},$$

$$\mathcal{B} = \begin{pmatrix} 0 & -r\mu_{\theta r}^{-1} & -\mu_{\theta\theta}^{-1} & -\frac{\mu_{\theta\phi}^{-1}}{\sin\theta} \\ r\mu_{\theta r}^{-1} & 0 & -\gamma_{1\theta\phi}^{*} & \frac{\gamma_{1\theta\theta}^{*}}{\sin\theta} \\ \mu_{\theta\theta}^{-1} & \gamma_{1\theta\phi}^{*} & 0 & -\frac{\gamma_{1\theta r}^{*}}{r\sin\theta} \\ \frac{\mu_{\theta\phi}^{-1}}{\sin\theta} & -\frac{\gamma_{1\theta\theta}^{*}}{\sin\theta} & \frac{\gamma_{1\theta r}^{*}}{r\sin\theta} & 0 \end{pmatrix},$$

$$\mathcal{C} = \begin{pmatrix} 0 & -r\gamma_{2\phi r}^{*} & -\gamma_{2\phi\theta}^{*} & -\frac{\gamma_{2\phi\phi}^{*}}{\sin\theta} \\ r\gamma_{2\phi r}^{*} & 0 & -\varepsilon_{\phi\phi}^{*} & \frac{\varepsilon_{\phi\theta}^{*}}{\sin\theta} \\ \gamma_{2\phi\theta}^{*} & \varepsilon_{\phi\phi}^{*} & 0 & -\frac{\varepsilon_{\phi r}^{*}}{r\sin\theta} \\ \frac{\gamma_{2\phi\phi}^{*}}{\sin\theta} & -\frac{\varepsilon_{\phi\theta}^{*}}{\sin\theta} & \frac{\varepsilon_{\phi r}^{*}}{r\sin\theta} & 0 \end{pmatrix},$$

$$\mathcal{D} = \begin{pmatrix} 0 & -r\sin\theta\mu_{\phi r}^{-1} & -\sin\theta\mu_{\phi\theta}^{-1} & -\mu_{\phi\phi}^{-1} \\ r\sin\theta\mu_{\phi r}^{-1} & 0 & -\sin\theta\gamma_{1\phi\phi}^{*} & \gamma_{1\phi\theta}^{*} \\ \sin\theta\mu_{\phi\theta}^{-1} & \sin\theta\gamma_{1\phi\phi}^{*} & 0 & -\frac{\gamma_{1\phi r}^{*}}{r} \\ \mu_{\phi\phi}^{-1} & -\gamma_{1\phi\theta}^{*} & \frac{\gamma_{1\phi r}^{*}}{r} & 0 \end{pmatrix},$$

$$\mathcal{E} = \begin{pmatrix} 0 & r\sin\theta\gamma_{2\theta r}^{*} & \sin\theta\gamma_{2\theta\theta}^{*} & \gamma_{2\theta\phi}^{*} \\ -r\sin\theta\gamma_{2\theta r}^{*} & 0 & \sin\theta\varepsilon_{\theta\phi}^{*} & -\varepsilon_{\theta\theta}^{*} \\ -\sin\theta\gamma_{2\theta\theta}^{*} & -\sin\theta\varepsilon_{\theta\phi}^{*} & 0 & \frac{\varepsilon_{\theta r}^{*}}{r} \\ -\gamma_{2\theta\phi}^{*} & \varepsilon_{\theta\theta}^{*} & -\frac{\varepsilon_{\theta r}^{*}}{r} & 0 \end{pmatrix},$$

$$\mathcal{F} = \begin{pmatrix} 0 & -r^2\sin\theta\gamma_{2rr}^{*} & -r\sin\theta\gamma_{2r\theta}^{*} & -r\gamma_{2r\phi}^{*} \\ r^2\sin\theta\gamma_{2rr}^{*} & 0 & -r\sin\theta\varepsilon_{r\phi}^{*} & r\varepsilon_{r\theta}^{*} \\ r\sin\theta\gamma_{2r\theta}^{*} & r\sin\theta\varepsilon_{r\phi}^{*} & 0 & -\varepsilon_{rr}^{*} \\ r\gamma_{2r\phi}^{*} & -r\varepsilon_{r\theta}^{*} & \varepsilon_{rr}^{*} & 0 \end{pmatrix}.$$

Susceptibility reference matrix in Cartesian coordinates is [26]:

$$\chi_{\mu\nu}{}^{\alpha\beta} = \frac{1}{2} \begin{pmatrix} \mathbf{0} & * & * & * \\ \mathcal{M} & \mathbf{0} & * & * \\ \mathcal{N} & \mathcal{O} & \mathbf{0} & * \\ \mathcal{P} & \mathcal{Q} & \mathcal{R} & \mathbf{0} \end{pmatrix}, \quad (10)$$

where

$$\mathcal{M} = \begin{pmatrix} 0 & -\mu_{xx}^{-1} & -\mu_{xy}^{-1} & -\mu_{xz}^{-1} \\ \mu_{xx}^{-1} & 0 & -\gamma_{1xz}^{*} & \gamma_{1xy}^{*} \\ \mu_{xy}^{-1} & \gamma_{1xz}^{*} & 0 & -\gamma_{1xx}^{*} \\ \mu_{xz}^{-1} & -\gamma_{1xy}^{*} & \gamma_{1xx}^{*} & 0 \end{pmatrix},$$

$$\mathcal{N} = \begin{pmatrix} 0 & -\mu_{yx}^{-1} & -\mu_{yy}^{-1} & -\mu_{yz}^{-1} \\ \mu_{yx}^{-1} & 0 & -\gamma_{1yz}^{*} & \gamma_{1yy}^{*} \\ \mu_{yy}^{-1} & \gamma_{1yz}^{*} & 0 & -\gamma_{1yx}^{*} \\ \mu_{yz}^{-1} & -\gamma_{1yy}^{*} & \gamma_{1yx}^{*} & 0 \end{pmatrix},$$

$$\mathcal{O} = \begin{pmatrix} 0 & -\gamma_{2zx}^{*} & -\gamma_{2zy}^{*} & -\gamma_{2zz}^{*} \\ \gamma_{2zx}^{*} & 0 & -\varepsilon_{zz}^{*} & \varepsilon_{zy}^{*} \\ \gamma_{2zy}^{*} & \varepsilon_{zz}^{*} & 0 & -\varepsilon_{zx}^{*} \\ \gamma_{2zz}^{*} & -\varepsilon_{zy}^{*} & \varepsilon_{zx}^{*} & 0 \end{pmatrix},$$

$$\mathcal{P} = \begin{pmatrix} 0 & -\mu_{zx}^{-1} & -\mu_{zy}^{-1} & -\mu_{zz}^{-1} \\ \mu_{zx}^{-1} & 0 & -\gamma_{1zz}^{*} & \gamma_{1zy}^{*} \\ \mu_{zy}^{-1} & \gamma_{1zz}^{*} & 0 & -\gamma_{1zx}^{*} \\ \mu_{zz}^{-1} & -\gamma_{1zy}^{*} & \gamma_{1zx}^{*} & 0 \end{pmatrix},$$

$$\mathcal{Q} = \begin{pmatrix} 0 & \gamma_{2yx}^{*} & \gamma_{2yy}^{*} & \gamma_{2yz}^{*} \\ -\gamma_{2yx}^{*} & 0 & \varepsilon_{yz}^{*} & -\varepsilon_{yy}^{*} \\ -\gamma_{2yy}^{*} & -\varepsilon_{yz}^{*} & 0 & \varepsilon_{yx}^{*} \\ -\gamma_{2yz}^{*} & \varepsilon_{yy}^{*} & -\varepsilon_{yx}^{*} & 0 \end{pmatrix},$$

$$\mathcal{R} = \begin{pmatrix} 0 & -\gamma_{2xx}^{*} & -\gamma_{2xy}^{*} & -\gamma_{2xz}^{*} \\ \gamma_{2xx}^{*} & 0 & -\varepsilon_{xz}^{*} & \varepsilon_{xy}^{*} \\ \gamma_{2xy}^{*} & \varepsilon_{xz}^{*} & 0 & -\varepsilon_{xx}^{*} \\ \gamma_{2xz}^{*} & -\varepsilon_{xy}^{*} & \varepsilon_{xx}^{*} & 0 \end{pmatrix}.$$

3.2. Transformation optics

Based on Plebanski's constitutive equations, it becomes possible to relate electromagnetic properties of a vacuum spacetime with arbitrary geometry, to those of a macroscopic media in Cartesian coordinates. Plebanski's equations for an impedance matched media ($\check{\mu} = \check{\varepsilon}$) are [17]

$$D_i = \varepsilon_0 \varepsilon^{ij} E_j + \epsilon_{ijk} v_j H_k$$
$$B_i = \mu_0 \mu^{ij} H_j - \epsilon_{ijk} v_j E_k \qquad i,j = 1,2,3 \quad (11)$$

where

$$\mu^{ij} = \varepsilon^{ij} = -\sqrt{\frac{|g|}{g_{00}}}\, g^{ij}$$

provides the permittivity and permeability of a dielectric media, and the bi-anisotropy vector [34]

$$v_j = \frac{g_{0j}}{g_{00}},$$

is pertinent to the velocity of the dielectric in a vacuum Minkowski spacetime. It turns out that this dielectric can regenerate same field equations as those in an arbitrary vacuum spacetime, described by the metric $g_{\mu\nu}$. Matching equations (6) and (11) for an impedance matched dielectric, one finds [25]

$$\check{\gamma}_1 = (\check{\gamma}_2)^T = -\epsilon_{ijk} v_j,$$

which are simply related to velocity. However, this simplified identification of magnetoelectric coupling and velocity does not always hold, unless the magnetoelectric dielectric is replaced by an isotropic moving media without any magnetoelectric coupling.

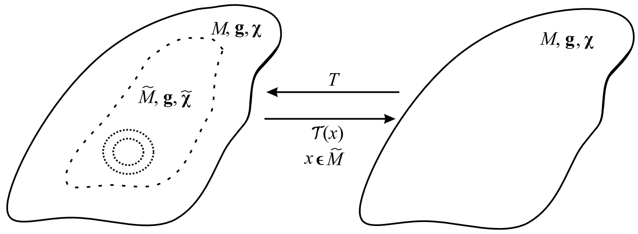

Fig. 1. Manifold M and its sub-manifold $\widetilde{M}$, which is obtained using a map like T from M to $\widetilde{M}$. $\widetilde{M}$ contains a material $\widetilde{\chi}$. The map could be defined, in a way to cerate a hole in $\widetilde{M}$ (cloaked region).

3.3. Covariant formulation of transformation optics

The Plebanski's theory of TO, derived from equation (11), suffers some shortcomings. Firstly, as it has been noted by Plebanski himself, these equations are nor covariant. Moreover, the indexing in both equations in (11) is not conserved. Therefore, it seems that we are in need of a more general approach. Based on covariantly developed classical electrodynamics, such method has been introduced in [27] and generalized for linear materials in [26]. Here we point out important considerations and results of the approach. For a detailed analysis, the reader is referred to these papers and specially to [35] for a very good review.

Firstly, a manifold M described by metric $\mathbf{g}$ is assumed, containing a material characterized by χ. On M it holds that $\mathrm{d}\mathbf{F}=0$, $\mathrm{d}\mathbf{G}=\mathbf{J}$ and $\mathbf{G}=\chi(\star\mathbf{F})$. Now consider a sub-manifold $\widetilde{M}\subseteq M$, onto which a map $T: M\longrightarrow\widetilde{M}$ could be defined. Since M does not physically change, $\widetilde{M}$ is also described by $\mathbf{g}$, however in order to get a correct physical configuration, $\widetilde{M}$ must contain a material characterized by $\widetilde{\chi}$, satisfying $\mathrm{d}\widetilde{\mathbf{F}}=0$, $\mathrm{d}\widetilde{\mathbf{G}}=\widetilde{\mathbf{J}}$ and $\widetilde{\mathbf{G}}=\widetilde{\chi}(\star\widetilde{\mathbf{F}})$ as depicted in Fig. 1. The map T, is a coordinate transformation from M to $\widetilde{M}$, which can shape the space contained in $\widetilde{M}$. For example one can produce a hole, in which no electromagnetic field exists. This procedure is pursued in cloaking process [6, 36].

Another crucial point here is that, although coordinates are transformed by T, the fields are transformed by $\mathcal{T}$, the inverse of T (note that, T may not have an inverse however, this method is essentially based on a well-defined $\mathcal{T}$ and T is ignored). Now the field strength and excitation tensors on $\widetilde{M}$ are derived as

$$\widetilde{\mathbf{G}}=\mathcal{T}^*(\mathbf{G})=\mathcal{T}^*\left(\chi(\star\mathbf{F})\right)=\widetilde{\chi}\left(\star\mathcal{T}^*(\mathbf{F})\right), \tag{12}$$

where $\mathcal{T}^*$ is the pullback of $\mathcal{T}$. Rearrangements lead to the following componentwise relation [26]:

$$\widetilde{\chi}_{\lambda\kappa}{}^{\tau\eta}(x)=-\Lambda^{\alpha}{}_{\lambda}\Lambda^{\beta}{}_{\kappa}\chi_{\alpha\beta}{}^{\mu\nu}|_{\mathcal{T}(x)}\star_{\mu\nu}{}^{\sigma\rho}|_{\mathcal{T}(x)}\left(\Lambda^{-1}\right)^{\pi}{}_{\sigma}\left(\Lambda^{-1}\right)^{\theta}{}_{\rho}\star_{\pi\theta}{}^{\tau\eta}|_{x}, \tag{13}$$

where $\mathbf{\Lambda}$ is the Jacobian matrix of $\mathcal{T}(x)$ which is calculated at $x\in\widetilde{M}$. Note that, there is no need to think of χ as a vacuum. Therefore equation (13) as well, holds for non-vacuum initial media [37]. Also in (13), the first Hodge dual is calculated at $\mathcal{T}(x)$, i.e. in the transformed coordinates. However, it is crucial to calculate both Hodge duals in same physical spacetimes.

Now taking the initial spacetime to be a vacuum, then $\chi=\chi_{\text{vac}}$, and since $\chi_{\text{vac}}\star=\star$, (13) results in

$$\widetilde{\chi}_{\lambda\kappa}{}^{\tau\eta}(x)=-\Lambda^{\alpha}{}_{\lambda}\Lambda^{\beta}{}_{\kappa}\star_{\alpha\beta}{}^{\sigma\rho}|_{\mathcal{T}(x)}\left(\Lambda^{-1}\right)^{\pi}{}_{\sigma}\left(\Lambda^{-1}\right)^{\theta}{}_{\rho}\star_{\pi\theta}{}^{\tau\eta}|_{x}, \tag{14}$$

which is the main result of the covariant formulation of TO. As it was mentioned above, the initial spacetime, described by $\mathbf{g}$, is arbitrary (may be Minkowski), and $\mathcal{T}$ changes the fields. Now what we are about to find out, is the configuration of the material, $\widetilde{\chi}$, which is supposed to perform such transformation. This can be figured out using (14).

3.4. Dielectric analog of a spacetime

Once again we mention a covariant formulation which has been derived and developed in [27]. The underlying physics is the same as that in the covariant formulation of TO, however the mathematical implementation somehow differs.

Recalling Plebanski's perspective of a transformation media, it has been stated that the behavior of electromagnetic fields in an arbitrary vacuum (may be curved) spacetime, could be identified to that in a dielectric, residing in Minkowski spacetime. Therefore, the question here is: how can we find the material properties of a dielectric, which is simulating a vacuum curved spacetime? Here this curved spacetime can be described by metric $g_{\mu\nu}$, a solution of Einstein equations.

Again we consider a manifold M, described by $\mathbf{g}$, however this time M is a flat Riemannian manifold, in which $g_{\mu\nu}=\eta_{\mu\nu}$ and $\eta_{\mu\nu}$ is the Minkowski metric. We also let M to contain a material χ, and corresponding $\mathbf{F}$ and $\mathbf{G}$. Now consider a map $\mathcal{T}(x)$, to transform every $x\in M$ to the vacuum curved manifold $\hat{M}$, described by $\hat{\mathbf{g}}$. $\hat{M}$ also contains a material $\hat{\chi}$, with $\hat{\chi}=\chi_{\text{vac}}$, and corresponding $\hat{\mathbf{F}}$ and $\hat{\mathbf{G}}$. The transformation

$$\mathcal{T}: x\longrightarrow\mathcal{T}(x)\quad\forall\ x\in M,$$

takes a tensor $\mathbf{G}_x$ to $\mathbf{G}_{\mathcal{T}(x)}$, however having its pullback $\mathcal{T}^*$, one can write equation (12) as

$$\mathbf{G}=\mathcal{T}^*(\hat{\mathbf{G}})=\mathcal{T}^*\left(\hat{\chi}(\hat{\star}\hat{\mathbf{F}})\right)=\chi\left(\star\mathcal{T}^*(\hat{\mathbf{F}})\right),$$

resulting in [27]

$$\chi_{\lambda\kappa}{}^{\tau\eta}(x)=-\Lambda^{\alpha}{}_{\lambda}\Lambda^{\beta}{}_{\kappa}\hat{\star}_{\alpha\beta}{}^{\sigma\rho}|_{\mathcal{T}(x)}\left(\Lambda^{-1}\right)^{\pi}{}_{\sigma}\left(\Lambda^{-1}\right)^{\theta}{}_{\rho}\star_{\pi\theta}{}^{\tau\eta}|_{x}, \tag{15}$$

where $\chi(x)$ in the rhs, provides the permittivity, permeability and magnetoelectric coupling of a spacetime analog dielectric, in Minkowski spacetime. In (15), once again $\mathbf{\Lambda}$ is the Jacobian matrix of $\mathcal{T}(x)$, calculated at $x\in M$. Also the first and second Hodge duals are respectively calculated in the vacuum curved, and Minkowski spacetimes. Therefore, one should be aware of the difference between the formulations (14) and (15). In TO theory formulated in (14), both manifolds were physically the same, whereas in (15), we are identifying two different manifolds. As a consequence, the coordinate transformation $\mathcal{T}(x)$, could be considered as the identity map $\mathcal{T}_0(x)=x$, for which (15) results in

$$\chi_{\lambda\kappa}{}^{\tau\eta}(x)=-\hat{\star}_{\lambda\kappa}{}^{\sigma\rho}|_{x}\star_{\sigma\rho}{}^{\tau\eta}|_{x}. \tag{16}$$

Having this, we just make a analogy between two spacetimes, without any change in coordinates. Such simplification could not be made in previously discussed method in subsection 3.3, since in that case we were dealing with a transformation media, and letting $\mathcal{T}(x)=\mathcal{T}_0(x)$, would result in the susceptibility in equation (8); an unnoticeable material.

4. Calculation

4.1. Dielectric analog of Robertson-Walker spacetime and ray tracing

The RW 2-form metric ($c=1$)

$$\mathbf{g}=-\mathrm{d}t\wedge\mathrm{d}t+\frac{a(t)^2}{1-Kr^2}\,\mathrm{d}\mathbf{r}\wedge\mathrm{d}\mathbf{r}+a(t)^2r^2\mathrm{d}\boldsymbol{\theta}\wedge\mathrm{d}\boldsymbol{\theta}+a(t)^2r^2\sin^2\theta\,\mathrm{d}\boldsymbol{\phi}\wedge\mathrm{d}\boldsymbol{\phi}, \tag{17}$$

with $a(t)$ as the scale factor and K as the spatial curvature, is the isotropic homogeneous interior solution to Einstein field equations. This metric is commonly supposed to describe the distribution of energy-momentum of a perfect fluid, however in this paper, we let describe a vacuum manifold, regardless of any energy constituents. This can be done, because the metric in (17), could be derived independently, for a homogeneous vacuum sphere (for a detailed analysis see [38]).

Beginning with (17), we let $K = 1$ to get a closed geometry. Now the properties of the dielectric analog of RW geometry is derived from (16). To calculate this, the first Hodge dual must be calculated in RW spacetime and the second one, in Minkowski. No coordinate transformation is considered, so the identity map is performed

$$\mathcal{T}_0(t', r', \theta', \phi') = (t, r, \theta, \phi),$$

in which the primed coordinates are those in vacuum RW spacetime and the unprimed ones are in Minkowski. Using (2) in (16) and comparing with (9), the dielectric components in (6) are derived using relation (7).

$$\check{\mu} = \check{\varepsilon} = \begin{pmatrix} \sqrt{1-r^2}\, a(t) & 0 & 0 \\ 0 & \frac{a(t)}{\sqrt{1-r^2}} & 0 \\ 0 & 0 & \frac{a(t)}{\sqrt{1-r^2}} \end{pmatrix},$$

$$\check{\gamma}_1 = \check{\gamma}_2 = \mathbf{0}.$$

One can see that the scale factor now appears in the permittivity and the permeability of the dielectric analog. This means that the geometry has been merged into the properties of media and the spacetime expansion is vanished. Now the question here is: how we can study the propagation of light in this dielectric analog?

To find the answer, we deal with the geometric optics approximation, characterized by a solution like

$$A_\mu = \hat{A}_\mu(x^\rho)\, \mathrm{e}^{\mathrm{i}S(x^\rho)},$$

in which, $A_\mu(x^\rho)$ varies slowly with respect to S, and $S(x^\rho)$ has small deviations from linearity. Therefore one can write [35]

$$X_\kappa{}^\eta \hat{A}_\eta = 0,$$

in which the 4×4 matrix $X_\kappa{}^\eta$ is defined as

$$X_\kappa{}^\eta = g^{\mu\lambda} \chi_{\lambda\kappa}{}^{\tau\eta} k_\mu k_\tau, \tag{18}$$

and $k_\mu = \partial_\mu S$ could be regarded as the wave vector $k_\mu = (k_0, k_r, k_\theta, k_\phi)$. Applying (18) for the dielectric analog of RW spacetime, obtained from (16), it is found that $\det\left(X_\kappa{}^\eta\right) = 0$. Any nontrivial solution for k_μ requires this determinant to be vanished, however since this automatically is the case, instead of it we introduce a Hamiltonian

$$\mathcal{H} = \det\left(X j^m\right), \tag{19}$$

in which

$$X_j{}^m =$$
$$\begin{pmatrix} -\frac{-k_2{}^2+k_0{}^2r^2a^2-k_3{}^2\csc^2\theta}{2r^2\sqrt{1-r^2}a^3} & -\frac{k_1k_2}{2r^2\sqrt{1-r^2}a^3} & -\frac{k_1k_3\csc^2\theta}{2r^2\sqrt{1-r^2}a^3} \\ -\frac{k_1k_2(r-1)^2(r+1)^2}{2\sqrt{1-r^2}a^3} & \frac{(r-1)(r+1)\left(k_1{}^2r^4+k_0{}^2a^2r^2-k_1{}^2r^2-k_3{}^2\csc^2\theta\right)}{2r^2\sqrt{1-r^2}a^3} & -\frac{k_2k_3\sqrt{1-r^2}\csc^2\theta}{2r^2a^3} \\ -\frac{k_1k_3(r-1)^2(r+1)^2}{2\sqrt{1-r^2}a^3} & -\frac{k_2k_3\sqrt{1-r^2}}{2r^2a^3} & \frac{(r-1)(r+1)\left(k_1{}^2r^4-k_1{}^2r^2+k_0{}^2a^2r^2-k_2{}^2\right)}{2r^2\sqrt{1-r^2}a^3} \end{pmatrix}$$

is the spatial part of $X_\kappa{}^\eta$. Considering only radial propagation of light on the equatorial plane of dielectric, we let $k_\theta = k_\phi = 0$, and from (19) we have

$$\mathcal{H} = \frac{k_0{}^2(r-1)(r+1)\left[k_0{}^2a(t)^2 + k_r{}^2\left(r^2-1\right)\right]^2}{8\sqrt{1-r^2}\,a(t)^7}. \tag{20}$$

Accordingly, Hamilton equations are

$$\dot{x}^\alpha = \frac{\partial\mathcal{H}}{\partial k_\alpha}, \tag{21}$$

$$\dot{k}_\beta = -\frac{\partial\mathcal{H}}{\partial x^\beta}, \tag{22}$$

where dot stands for differentiation with respect to ray parametrization. Now for $\alpha = \beta = 0$, (21) and (22) together with (20) result in

$$\frac{\mathrm{d}k_0}{\mathrm{d}t} = \frac{k_0a'(t)\left[3k_0{}^2a(t)^2 + 7k_r{}^2\left(r^2-1\right)\right]}{2a(t)\left[3k_0{}^2a(t)^2 + k_r{}^2\left(r^2-1\right)\right]}. \tag{23}$$

To have a justification, we need to obtain an expression for k_r. This could be done using the Hamilton-Jacobi equation

$$\mathcal{H} = 0,$$

giving

$$k_r = -\frac{k_0\, a(t)}{\sqrt{1-r^2}}. \tag{24}$$

Considering a monochromatic light ray of frequency $k_0 = \omega$, equations (23) and (24) give

$$\omega(t) = \omega(0)\,\frac{1}{a(t)}, \tag{25}$$

indicating a redshift, imposed from the material properties, on the frequency of light. Based on the Plebanski based TO, such redshift has been also derived in [39] by implementing an optical analog of a spatially flat ($K = 0$) RW spacetime.

Now let us consider the radial evolution of the dielectric analog. From the Hamilton equations (21) and (22) for $a = 1$ and $\beta = 0$, and considering (24), one gets

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\sqrt{1-r^2}}{a(t)}. \tag{26}$$

If t_B is the moment in which the light ray commences propagating in the dielectric, then (26) can be rearranged to

$$\int_{t_B}^{t} \frac{\mathrm{d}t}{a(t)} = \int_0^{r_p} \frac{\mathrm{d}r}{\sqrt{1-r^2}}, \tag{27}$$

where r_p is the longest distance the ray has travelled, since it started propagation. Equations (25) and (27) are precisely the standard cosmological definitions for the cosmological redshift and particle horizon [40], which here, have been derived from a dielectric analog of RW spacetime. Note that the horizon has been claimed to be proportional to the Casimir energy, obtained in a dielectric analog of de Sitter spacetime [41].

5. Model

5.1. *Spherical cloaked region in Robertson-Walker spacetime*

Let us study a different situation; a transformation media, moving in RW geometry. To deal with this case, we exploit the covariant theory of TO, discussed in subsection 3.3. The material is supposed to be receding from an observer in RW spacetime, and it is desired to obtain its properties, however these results are experienced by another observer, located in the materials frame (a comoving observer). Therefore using appropriate tetrad, one should transform the results which are derived for RW spacetime, to a local Minkowski frame, in Cartesian coordinates.

To do this, firstly we consider the relative motion of our media, to be only in r direction. Therefore the four-velocity vector in spacetime coordinates could be written as

$$u^\mu = \left(\frac{\mathrm{d}t}{\mathrm{d}\tau}, \frac{\mathrm{d}r}{\mathrm{d}\tau}, 0, 0\right),$$

according to the trajectory parametrization τ. The time-like behavior of $g_{\mu\nu}u^\mu u^\nu = -1$, where ($\theta = \frac{\pi}{2}$)

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{a^2}{1-r^2} & 0 & 0 \\ 0 & 0 & r^2a^2 & 0 \\ 0 & 0 & 0 & r^2a^2 \end{pmatrix}, \tag{28}$$

together with the 0-component of geodesic equations, leads to

$$u^\mu = e_0^\mu = \frac{1}{\sqrt{\left(4-\frac{4}{r^2}\right)\dot{a}^2+1}}\left(1, -\frac{2\left(1-r^2\right)\dot{a}}{r\, a}, 0, 0\right). \tag{29}$$

Here and form now on, dot stands for $\frac{\mathrm{d}}{\mathrm{d}t}$. The tetrad should obey $g_{\mu\nu}\, e_A^\mu\, e_B^\nu = \eta_{AB}$, and for simplicity (as it is common in the literature), x coordinate is assumed to be along r. Therefore x is confined to time and the vector in (29). Consequently the non-zero components of e_1^μ should be only the first and second ones. The orthogonality condition with e_0^μ gives

$$e_1^\mu = \frac{2}{\sqrt{\frac{r^2}{(1-r^2)\dot{a}^2}-4}}\left(1, -\frac{r}{2\, a\, \dot{a}}, 0, 0\right). \tag{30}$$

Now since we let y and z to be respectively along θ and ϕ, according to (28), a suitable choice for e_2^μ and e_3^μ could be

$$e_2^\mu = \left(0, 0, \frac{1}{r\, a}, 0\right),$$
$$e_3^\mu = \left(0, 0, 0, \frac{1}{r\, a}\right).$$

These tetrad are able to transform geometric objects from RW spacetime, to local Minkowskian frame. To transform a 1-form n_μ in RW to a 1-form n_A in Minkowski, we use a transformation matrix ${S_A}^\mu$, consisting of the tetrad in (29) to (30) to form its rows. We have

$${S_A}^\mu = \begin{pmatrix} \frac{1}{\sqrt{\left(4-\frac{4}{r^2}\right)\dot{a}^2+1}} & -\frac{2(1-r^2)\dot{a}}{r\, a\sqrt{\left(4-\frac{4}{r^2}\right)\dot{a}^2+1}} & 0 & 0 \\ \frac{2}{\sqrt{\frac{r^2}{(1-r^2)\dot{a}^2}-4}} & -\frac{r}{a\, \dot{a}\sqrt{\frac{r^2}{(1-r^2)\dot{a}^2}-4}} & 0 & 0 \\ 0 & 0 & \frac{1}{r\, a} & 0 \\ 0 & 0 & 0 & \frac{1}{r\, a}. \end{pmatrix}. \tag{31}$$

One can easily check that the condition

$$g_{\mu\nu}\, {S_A}^\mu {S_B}^\mu = \eta_{AB} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

holds for the matrix in (31). Also note that to transform a vector, we should use ${S^A}_\mu$, the transpose of the inverse of ${S_A}^\mu$ [11]. Having these transformation matrices, the material properties, ${\widetilde{\chi}_{\lambda\kappa}}^{\tau\eta}(x)$ in equation (14), which here are derived for RW spacetime, can be transformed to local Cartesian frame as follows

$${\hat{\chi}_{AB}}^{CD} = {S_A}^\lambda {S_B}^\kappa {S^C}_\tau {S^D}_\eta\, {\widetilde{\chi}_{\lambda\kappa}}^{\tau\eta}. \tag{32}$$

5.2. *Spherical cloak*

Now let us think of our dielectric media, to perform a peculiar coordinate transformation. This transformation is defined, in a way to create a spherical cloak.

A spherical cloak consists of a spherical shell, with b_1 and b_2, respectively as its interior and exterior radii. The center of these concentric spheres "c", is located at distance r', from an observer "o". A light ray from a luminous object, enters the shell at $r' = r'_1$ and comes out at $r' = r'_2$, where

$${r'}_1 = r' + b_2,$$
$${r'}_2 = r' - b_2.$$

The cloak perform the following transformation from M (the vacuum manifold) to $\widetilde{M}$ (manifold containing the material):

$$T(t', r', \theta', \phi') = (t, r, \theta, \phi) = \left(t', f(r'), \theta', \phi'\right),$$

where [22]

$$f(r') = \begin{cases} b_1 + \frac{b_2-b_1}{b_2}\, r' & {r'}_2 \le r' \le {r'}_1 \\ r' & \text{elsewhere} \end{cases},$$

which is applied to the region ${r'}_2 \le r' \le {r'}_1$ and maps to the interval $b_1 \le r \le b_{21}$, in order to create a cavity. The maximum amount of ψ occurs when the light ray is incoming from $\phi' = 0$. Whilst ϕ' increases, ψ decreases until at $\phi' = \psi_{b_2}$ it vanishes; no deflection. In other words

$$\psi_{\max} = \psi|_{\phi'=0},$$
$$\psi_{\min} = \psi|_{\phi'=\psi_{b_2}} = \tan^{-1}\left(\frac{b_2}{r'}\right).$$

However it should be noted that the angular size ψ of the shell varies. This means that because of cloak's radial motion, at later times we have more or fewer rays passing through the cloak. Also note that we are mostly dealing with the only objects which are located on the radial axis (at $\phi' = 0$) and the cloak is moving towards or apart from them.

As it was stated in subsection 3.3, it is $\mathcal{T}$, the inverse of T, which is applicable in this approach. Therefore we need

$$\mathcal{T}(t, r, \theta, \phi) = (t', r', \theta', \phi') = (t, q(r), \theta, \phi), \tag{33}$$

with

$$q(r) = \begin{cases} \frac{(r-b_1)\, b_2}{b_2-b_1} & b_1 \le r \le b_2 \\ r & \text{elsewhere} \end{cases}.$$

The Jacobian of the transformation (33) would be

$${\Lambda^\mu}_\nu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \partial_r q & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{34}$$

Now using (14), (32), (31) and (34), and comparing with (10), the material properties of the spherical shell in local Cartesian frame can be derived. The non-zero components are

$$\check{\mu} = \check{\varepsilon}\ :$$

$$\mu_{xx} = \frac{q^2}{r^2 q'}\sqrt{\frac{1-q^2}{1-r^2}},$$

$$\mu_{yy} = \mu_{zz} = \sqrt{(1-q^2)(1-r^2)}\ \frac{q'\left(r^2 - 4\left(1-r^2\right)\dot{a}^2\right)}{r^2\left(1-q^2\right) - 4\dot{a}^2 q'^2\left(1-r^2\right)^2}, \tag{35}$$

$$\check{\gamma}_1 = (\check{\gamma}_2)^T\ :$$

$$\check{\gamma}_{1zy} = -\check{\gamma}_{1yz}$$
$$= \sqrt{\frac{\left(r^2 - 4\left(1-r^2\right)\dot{a}^2\right)\left(r^2\left(1-r^2\right)\dot{a}^2\right)}{r^2 + 4\left(1-r^2\right)\dot{a}^2}}\ \frac{1-q^2-q'^2(1-r^2)}{r^2\left(1-q^2\right) + 4\dot{a}^2 q'^2\left(1-r^2\right)^2}. \tag{36}$$

These are the properties of a spherical cloak; no field can enter nor can protrude. Therefore it becomes completely invisible to the observer located at o, and every luminous objects on the radial direction, are completely detected.

5.3. *A note on magnetoelectric coupling*

It is known that moving materials are magnetoelectric and magnetoelectric coupling of a transformation media is sometimes interpreted as velocity [9]. However it has been shown in [25] that for an anisotropic magnetoelectric medium, it is impossible to reobtain the magnetoelectric couplings, solely from motion conditions. Here, we are about to ask whether the magnetoelectric couplings are consequences of the cloak's radial motion, or general relativistic contributions. To do so, it is more convenient to reconsider our case in a spatially flat RW metric, i.e.

$$\mathbf{g} = -\mathbf{dt} \wedge \mathbf{dt} + a(t)^2 \left(\mathbf{dx} \wedge \mathbf{dx} + \mathbf{dy} \wedge \mathbf{dy} + \mathbf{dz} \wedge \mathbf{dz} \right), \tag{37}$$

which is more similar to the Minkowski spacetime in Cartesian coordinates. Therefore, the transformation matrix to a comoving frame in this case becomes

$$S_A{}^{\mu} = \frac{1}{\sqrt{1-4a^2}} \begin{pmatrix} 1 & -2 & 0 & 0 \\ 2a & -\frac{1}{a} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{1-4a^2}}{a} & 0 \\ 0 & 0 & 0 & \frac{\sqrt{1-4a^2}}{a} \end{pmatrix}, \tag{38}$$

according to motion along x-direction. In this approach and since we chose a frame where the spherical r-direction is supposed to be along the Cartesian x-direction, the cloak transformation (33) could be rewritten as

$$\mathcal{T}(t,x,y,z) = (t',x',y',z') = (t, q(x), y, z), \tag{39}$$

which together with (14), (37), (38) and (32), we get [25]

$$\check{\mu} = \check{\varepsilon} = \begin{pmatrix} \frac{1}{q'} & 0 & 0 \\ 0 & \frac{(1-4a^2)q'}{1-4a^2q'^2} & 0 \\ 0 & 0 & \frac{(1-4a^2)q'}{1-4a^2q'^2} \end{pmatrix}, \tag{40}$$

$$\check{\gamma}_1 = \check{\gamma}_2{}^T = \frac{2a(1-q'^2)}{1-4a^2q'^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}. \tag{41}$$

To see whether these results can be recovered for a moving media in Minkowski spacetime or not, let us provide a Lorentz boost for an equivalent Cartesian transformation in Minkowski spacetime. Indeed we consider that the transformation (39) is boosted along x-direction with some speed β. Such a boost is defined in the following matrix representation:

$$L^{\mu}{}_{a} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{42}$$

with $\gamma = \frac{1}{\sqrt{1-\beta^2}}$. This frame transformation for 1-forms and its counterpart for vectors $L^{a}{}_{\mu}$ (obtained by letting $\beta \to -\beta$ in $L^{\mu}{}_{a}$) can be exploited in the same way as it is in (32) to obtain the material properties of a moving material in Minkowski spacetime [26].

$$\chi'_{ab}{}^{cd} = L^{\mu}{}_{a} L^{\nu}{}_{b} L^{c}{}_{\rho} L^{d}{}_{\lambda}\, \widetilde{\chi}_{\mu\nu}{}^{\rho\lambda}. \tag{43}$$

Using (42) and (43), together with (14) and (39) we obtain

$$\check{\mu} = \check{\varepsilon} = \begin{pmatrix} \frac{1}{q'} & 0 & 0 \\ 0 & \frac{(1-\beta^2)q'}{1-\beta^2q'^2} & 0 \\ 0 & 0 & \frac{(1-\beta^2)q'}{1-\beta^2q'^2} \end{pmatrix}, \tag{44}$$

$$\check{\gamma}_1 = \check{\gamma}_2{}^T = \frac{\beta(1-q'^2)}{1-\beta^2q'^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}. \tag{45}$$

Comparing the results in (40) and (41), with those in (44) and (45), one observes that the relativistic results in (40) and (41) which have been obtained for a spatially flat RW metric, can be recovered by setting $\beta = 2a$. This implies that the magnetoelectric couplings for a moving anisotropic transformation media in a spatially flat RW spacetime, are directly consequences of its motion. This may become more apparent when the slow speed limit is considered. In this limit, $\beta \to 0$ (or equivalently $a \to 0$ for which every spatial separation in RW metric and the coordinate velocity will vanish), the material properties become

$$\check{\mu} = \check{\varepsilon} = \begin{pmatrix} \frac{1}{q'} & 0 & 0 \\ 0 & q' & 0 \\ 0 & 0 & q' \end{pmatrix},$$

$\check{\gamma}_1$ and $\check{\gamma}_2{}^T = \mathbf{0}$.

Therefore we infer that it is velocity which induces magnetoelectric coupling in the anisotropic media in (40), when it is moving in a spatially flat RW universe and therefore they can be regenerated only from the medium's velocity. However this is not the case when spatial curvature is taken into account, since no expression for β in (44) and (45), can be found in order to recover the fully relativistic results in (35) and (36). This shows that beside impacts of motion, these results are highly adherent to spatial curvature contributions.

6. Conclusion

The behavior of dielectric media in a vacuum Robertson-Walker spacetime was studied, by applying a covariant method in Transformation Optics. The main idea is that the behavior of electromagnetic fields in arbitrary curved spacetime could be regenerated in an appropriate dielectric media. Since the complexity of gravitational systems may make studying cosmological phenomena somehow complicated, in this paper we made a simplification, by creating a dielectric analog of RW spacetime and as a consequence, the fundamental results of standard cosmology were regained. Moreover, a cloaked region with RW spacetime was also considered and corresponding configuration of the functioning material were derived. Such cloaked regions in space, if they are actually exist in nature, will be totally invisible although the object within may be massive and luminous. However we can not deal with such cloaks readily from approaches like the one pursued in section 5 of this paper, since the formulation of TO is on a fixed background and understanding the back-reaction of a cloak requires a dynamical theory, including full general relativity. Therefore this needs a completely clarified relationship between TO and general relativity. Consequently it is hard at this stage to investigate possible astrophysical cloaks. However it may make sense to consider transformation media in diversified cosmological models and look for possible evidences.

Declarations

Author contribution statement

Farrin Payandeh: Conceived and designed the analysis; Analyzed and interpreted the data; Contributed analysis tools or data; Wrote the paper.

Funding statement

This paper has been derived from a research project approved to be supported financially by Payame Noor University.

Competing interest statement

The author declares no conflict of interest.

Additional information

No additional information is available for this paper.

Acknowledgements

The author is grateful for the comments and suggestions of the reviewers in improving the manuscript.

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