

1 An improved rhythmicity analysis method using Gaussian  
2 Processes detects cell-density dependent circadian  
3 oscillations in stem cells

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## 6 Abstract

7 Detecting oscillations in time series remains a challenging problem even after decades of research.  
8 In chronobiology, rhythms in time series (for instance gene expression, eclosion, egg-laying and  
9 feeding) datasets tend to be low amplitude, display large variations amongst replicates, and  
10 often exhibit varying peak-to-peak distances (non-stationarity). Most currently available rhythm  
11 detection methods are not specifically designed to handle such datasets. Here we introduce a new  
12 method, ODeGP (**O**scillation **D**etection using **G**aussian **P**rocesses), which combines Gaussian  
13 Process (GP) regression with Bayesian inference to provide a flexible approach to the problem.  
14 Besides naturally incorporating measurement errors and non-uniformly sampled data, ODeGP  
15 uses a recently developed kernel to improve detection of non-stationary waveforms. An additional  
16 advantage is that by using Bayes factors instead of p-values, ODeGP models both the null (non-  
17 rhythmic) and the alternative (rhythmic) hypotheses. Using a variety of synthetic datasets we  
18 first demonstrate that ODeGP almost always outperforms eight commonly used methods in  
19 detecting stationary as well as non-stationary oscillations. Next, on analyzing existing qPCR  
20 datasets that exhibit low amplitude and noisy oscillations, we demonstrate that our method  
21 is more sensitive compared to the existing methods at detecting weak oscillations. Finally, we  
22 generate new qPCR time-series datasets on pluripotent mouse embryonic stem cells, which are  
23 expected to exhibit no oscillations of the core circadian clock genes. Surprisingly, we discover  
24 using ODeGP that increasing cell density can result in the rapid generation of oscillations in  
25 the *Bmal1* gene, thus highlighting our method's ability to discover unexpected patterns. In its  
26 current implementation, ODeGP (available as an R package) is meant only for analyzing single  
27 or a few time-trajectories, not genome-wide datasets.

## 28 Introduction

29 From the rapid ultradian oscillations of p53, NF- $\kappa$ B, Hes7, and the embryonic segmentation  
30 clock, to the slower seasonal flowering patterns in plants - oscillations in biological systems are  
31 ubiquitous across many length and time scales [1, 2]. These patterns, where repeatedly occurring  
32 peaks can be observed in the time series of interest, are often noisy, exhibit low amplitudes (peak-  
33 to-trough distance), and are non-stationary (peak-to-peak distance varies with time). Classic

34 examples of such noisy oscillations can be observed in circadian clock gene expression [3, 4],  
35 feeding, eclosion and egg-laying rhythms [5, 6, 7]. This makes it hard to distinguish rhythmic  
36 patterns from noise, necessitating the development of quantitative approaches to accurately in-  
37 fer the existence of oscillations, and subsequently extract parameters such as time period and  
38 amplitude [8]. Developing principled approaches to detecting oscillations is also important in  
39 differential rhythmicity analyses, where genes can lose or gain rhythmicity after perturbations  
40 [9, 10, 11]. Finally, biological oscillators are often coupled, and investigating the nature and con-  
41 sequences of coupling often depends on the ability to carefully measure and detect the individual  
42 oscillations in the first place [12, 13, 14].

43 Over the last few decades numerous methods have been developed to address the oscillation de-  
44 tection problem arising from experiments which generate, for example, qPCR, RNA-seq, feeding,  
45 eclosion and egg-laying time series datasets. Existing non-parametric methods used for oscilla-  
46 tion detection include JTK\_Cycle [15] and RAIN [16]. eJTK [17] is a more recently developed  
47 algorithm that improves on JTK\_Cycle by including non-sinusoidal reference waveforms. Meta-  
48 Cycle [18], an R package developed to identify oscillations, integrates the results of JTK\_Cycle,  
49 the Lomb Scargle Periodogram [19] (a parametric method) and ARSER [20] (a parametric  
50 method using autoregressive models, which cannot work on unevenly sampled data). Another  
51 existing R package for oscillation detection is DiscoRhythm [21], which builds on MetaCycle by  
52 additionally using the Cosinor [22] method (also parametric). Among the non-parametric meth-  
53 ods, RAIN requires that the period to test for (or a range of periods) be specified beforehand  
54 when trying to identify oscillations. Similarly, JTK\_Cycle and MetaCycle also require informa-  
55 tion about the expected oscillation time period to be provided as inputs to the algorithm. More  
56 recently, neural network based approaches have been used for classifying oscillatory versus non-  
57 oscillatory datasets [23], but these do not allow learning of the full waveform, besides requiring  
58 a lot of training data to perform well. Finally, wavelet-based techniques [24, 25] can extract  
59 time-dependent phase and periods from temporal datasets, but are not designed to specifically  
60 detect oscillations or handle replicate data. More details and comparisons of a variety of oscilla-  
61 tion detection algorithms can be found elsewhere - [8] provides a comprehensive review of earlier  
62 techniques while comparisons of more recent methods can be found in [26, 27, 28, 29, 30].

63 While these methods have improved over time and have become increasingly powerful, a number  
64 of challenges in oscillation detection are yet to be overcome [30] - (1) data is often available only  
65 at non-evenly spaced time points, which can make oscillation detection difficult, particularly with  
66 Fourier Transform based methods, (2) large error-bars at each time point from replicates are  
67 often not easy to incorporate into existing methods, (3) biological oscillations tend to be non-  
68 stationary (peak-to-peak distance varies over time), which parametric models cannot handle  
69 well due to difficulty in defining functional forms for such data, and finally (4) most current  
70 methods rely on calculating a p-value to classify a dataset as rhythmic versus non-rhythmic (for  
71 an exception, see [31]), but a major issue with p-value based approaches is that they model only  
72 the null, but not the alternative hypothesis [30, 32, 33, 34]. Existing methods often overcome  
73 one or few of these challenges, but no single method exists that addresses all these problems in  
74 a comprehensive manner.

75 To solve the above four challenges in a unified framework, here we develop ODeGP (**O**scillation  
76 **D**etection using **G**aussian **P**rocesses), a new approach to the oscillation detection problem com-  
77 bining Gaussian Process (GP) regression [35] with Bayesian model selection. Conceptually,  
78 the non-parametric nature of GPs allows ODeGP to flexibly model both stationary and non-  
79 stationary datasets, while the Bayesian model selection approach using Bayes factors allows us  
80 to model both the null as well as alternate hypotheses, unlike p-value based methods. In partic-  
81 ular, we use a recently developed non-stationary kernel that allows us to model non-stationary  
82 datasets [36, 37], improving the accuracy of oscillation detection over many of the popular ex-  
83 isting methods such as eJTK and MetaCycle. GPs also overcome issues related to unevenly  
84 spaced time-series data and can naturally incorporate error bars generated by replicates. Addi-  
85 tionally, ODeGP provides a simple Bonferroni-type multiple hypothesis correction [38], though  
86 this approach currently limits its use to settings where only one or a few trajectories are to be  
87 analyzed, not genome-wide datasets. Finally, a major additional advantage of GPs is that they  
88 provide the ability to quantify uncertainty predictions at any time point, including test points  
89 where no experimental data has been collected [35].

90 We extensively compare the performance of ODeGP with eight other existing methods on both  
91 simulated and experimental datasets and demonstrate that it is consistently better and more

92 sensitive at identifying oscillations. We also find that the Bayes factor usually has a large  
93 separation between oscillatory and non-oscillatory experimental datasets, suggesting that it is a  
94 good metric for the classification problem. Finally, to test the usefulness of ODeGP in learning  
95 patterns in new datasets, we generate time-series circadian clock gene expression profiles using  
96 mouse embryonic stem cells (mESCs). Intriguingly, we find that oscillations of *Bmal1* can be  
97 induced within a few days in mESCs by increasing cell density and that these oscillations get  
98 suppressed with the addition of MEK/ERK and GSK3b inhibitors. This interesting result adds  
99 to previous observations that while pluripotent mESCs exhibit no clock gene oscillations, retinoic  
100 acid mediated differentiation can induce oscillations after about two weeks [39, 40]. Our results  
101 indicate that increasing cell density might mimic the effects of directed differentiation, but with  
102 faster kinetics of emergence of circadian gene oscillations.

## 103 **Methods**

### 104 **ODeGP - Gaussian Process regression for oscillation detection**

105 A brief mathematical introduction to the theory of Gaussian Processes (GPs) is provided in  
106 SI Section 3. Here we provide the basic outline of our oscillation detection algorithm using  
107 GPs. The time points at which gene expression data is collected using qPCR are specified as  
108 a list  $X$ . If the data for  $d$  distinct replicates is available for the times given in  $X$ , these  $d$  lists  
109  $Y_1, Y_2, \dots, Y_d$  are taken as input. Otherwise, if the averaged qPCR data  $Y$  is available along  
110 with the corresponding error bar values  $S$  for each time point,  $Y$  and  $S$  are taken as input. In  
111 either case, the qPCR data may be collected at irregular intervals or have missing points. In the  
112 former case,  $Y$  and  $S$  are calculated from  $Y_1, Y_2, \dots, Y_d$  before proceeding.  $Y$  is then detrended  
113 via linear regression to remove long-term trends, and zero-centred to simplify the computations  
114 involved in performing Gaussian process regression.

115 The entries of the covariance matrix  $\Sigma_{XX}$  are determined by a positive semi-definite kernel  
116 function  $K$ . ODeGP uses two different kernel functions,  $K_D$  and  $K_{NS}$ . The diagonal ker-  
117 nel  $K_D(x, x') = \epsilon^2 \cdot \delta_{xx'}$  is used to represent non-oscillatory functions, where  $\delta$  is the Kro-  
118 necker delta. This kernel encodes the prior belief that there is no correlation between the

119 values of the data at different time points, thus all the non-diagonal entries of the covari-  
120 ance matrix are 0. To represent oscillatory functions, the non-stationary kernel  $K_{NS}(x, x') =$   
121  $w(x)w(x')k_{\text{gibbs}}(x, x') \cos(2\pi(x\mu(x) - x'\mu(x')) + \epsilon^2 \cdot \delta_{xx'})$  is used, where  $k_{\text{gibbs}}$  is the Gibbs ker-  
122 nel. The hyperparameters  $w$  and  $\mu$ , along with  $l$  (which is part of the expression of  $k_{\text{gibbs}}$ ), are  
123 functions of  $x$ . Thus the covariance matrix will depend on  $x$  and is not solely a function of  
124  $|x - x'|$ .

125 The error bar values  $S$  are incorporated into the covariance matrices generated by these kernels  
126 as follows: for  $s_i \in S$ ,  $s_i^2$  is added to the  $i$ th diagonal entry of the matrix. These terms model  
127 local noise, which arise from technical variations amongst the replicates. Global noise, which  
128 includes other sources of variations beyond technical noise, is modeled by a hyperparameter  $\epsilon$   
129 such that  $\epsilon^2$  is added to all diagonal entries of the covariance matrix. This  $\epsilon^2$  term is included in  
130 the expressions of  $K_D$  and  $K_{NS}$ , allowing us to learn the best value of  $\epsilon$  along with the remaining  
131 hyperparameters.

132 Optimal hyperparameters for the non-stationary and diagonal kernels are obtained through  
133 maximization of the marginal log-likelihoods (MLL) (see Equation 17 in the SI), to obtain the  
134 optimal  $\text{MLL}_{NS}^*$  and  $\text{MLL}_D^*$  respectively. The kernel that represents a better prior belief for  
135 the given dataset is identified through Bayesian model selection. The Bayes factor is defined  
136 as the ratio of the marginal likelihood of two competing hypotheses. Here, it is calculated as  
137  $k = \exp(\text{MLL}_{NS}^* - \text{MLL}_D^*)$  and compared to a decided threshold  $T$ . If  $k \geq T$ , the dataset is  
138 declared to be oscillatory, otherwise not. A discussion on the choice of  $T$  is provided in the  
139 Discussion section.

140 Finally, while ODeGP has been designed primarily to detect rhythms in single time-series trajec-  
141 tories, it also provides a Bonferroni-type multiple-hypothesis correction term if multiple trajec-  
142 tories from the same dataset are intended to be analyzed simultaneously. A single multiplicative  
143 correction term to the Bayes Factors (which acts as the prior odds) is calculated based on the  
144 number of trajectories to be analyzed:  $\text{Prior Odds} = (1 - \Pi_0^{1/k})/(\Pi_0^{1/k})$ , where  $k$  is the number  
145 of trajectories and  $\Pi_0$  is by default taken to be 0.5. The prior odds, multiplied to each of the  
146 Bayes Factors of the different trajectories analyzed, provides the posterior odds that can be used  
147 to judge rhythmicity in the multiple hypothesis setting. More details are provided in Section 4

148 of the SI. The use of this Bonferroni-type correction factor currently limits ODeGP to the anal-  
149 ysis of one or few trajectories only, as opposed to genome-wide datasets, since the Bonferroni  
150 correction is very stringent and greatly increases false negatives when many comparisons are  
151 performed.

## 152 **Cell lines and culture conditions**

153 A passage 12 (p12) mESC line (E14TG2a.4) was expanded under conditions expected to maintain  
154 pluripotency, and all cells used for the experimental data reported here were within 10 additional  
155 passages. The p12 cells were thawed and expanded on 0.1% gelatin-coated, cell-culture treated 10  
156 cm plastic dishes. GTES ES cell media (GMEM, 15% FBS, Glutamax, Sodium Pyruvate, Non-  
157 Essential Amino Acids, BME, and LIF) was used to propagate the cells in pluripotent conditions.  
158 Cells were passaged at approximately 70% confluency to avoid crowding-induced differentiation.  
159 For all experiments that required thawing out new vials, the cells were always initially maintained  
160 in cell-culture treated plastic dishes coated with 0.1% gelatin, and then transferred to various  
161 other conditions, such as glass dishes with fibronectin or laminin coating.

## 162 **qPCR experiments - data collection protocol and error analysis**

163 For the qPCR experiments, cells were collected in Trizol, total RNA was extracted and converted  
164 to cDNA, and finally, qPCR was performed using the SYBR Green dye. Each time point  
165 (beginning from time 0, corresponding to immediately after Dex synchronization) corresponds  
166 to cells obtained from an independent well of a 24-well plate. To avoid any artifacts in cell  
167 synchronization due to differences in cell density at different times of cell collection, we devised  
168 a protocol to ensure an approximately equal number of cells collected for every time point:  
169 instead of synchronizing all the samples at one time point and collecting cells at different time  
170 points, we synchronized cells at different times and Trizol collected the cells at a single time.  
171 Details of the protocol, seeding densities, and various substrate conditions for the samples are  
172 provided in SI Section 1. The error analysis is explained in SI Section 2.

## 173 Results

### 174 **ODeGP - an oscillation detection algorithm based on Gaussian Pro-** 175 **cesses**

176 We developed a new method for detecting oscillations, ODeGP (**O**scillation **D**etection using  
177 **G**aussian **P**rocesses), combining Gaussian Process (GP) regression and Bayesian inference. A  
178 brief intuition of how GP regression works is provided in Figure 1A and an outline of the  
179 ODeGP pipeline is displayed in Figure 1B. In brief, GP regression is a non-parametric approach  
180 to learning non-linear trends in data, where instead of specifying a function and learning its  
181 optimal parameters (parametric regression), the functional form itself is learnt by specifying a  
182 prior over functions [35]. Placing a prior over functions is achieved by the use of a multivariate  
183 Gaussian (Figure 1A, left), whose covariance matrix is determined by a kernel that prioritizes  
184 certain classes of functions based on prior expectations of smoothness and characteristic length-  
185 scales associated with the problem of interest. After data is obtained, the instantiations of the  
186 prior that best describe the data are obtained via the posterior (Figure 1A, right), which can  
187 be described via the posterior mean and variance. More details can be found in Methods and in  
188 the SI Sections 3 and 4.

189 ODeGP detects oscillations in time-series data by initializing two GP models or kernels (one  
190 encoding a belief of the data containing oscillations, and the other encoding a belief of oscillations  
191 being absent), performing GP regression on the data with each kernel separately by optimizing  
192 their respective marginal log-likelihoods (MLLs), and finally comparing these likelihoods to  
193 determine the model that better describes the data. Optimization of the MLL automatically  
194 incorporates the trade-off between maximizing the fitting of the data while minimizing model  
195 complexity. This complexity is represented by the determinant of the covariance matrix in the  
196 expression for the MLL (Equation 17 in the SI). This term can penalize an increase in the  
197 number of kernel hyperparameters used to compute the covariance matrix, and thus prevent  
198 overfitting of the data. A more detailed analysis of this is presented in SI Section 6, Figure S1  
199 and Table S7. Finally, the possibility of performing corrections for multiple hypothesis testing  
200 is also provided - for details, see Methods and SI Section 4.



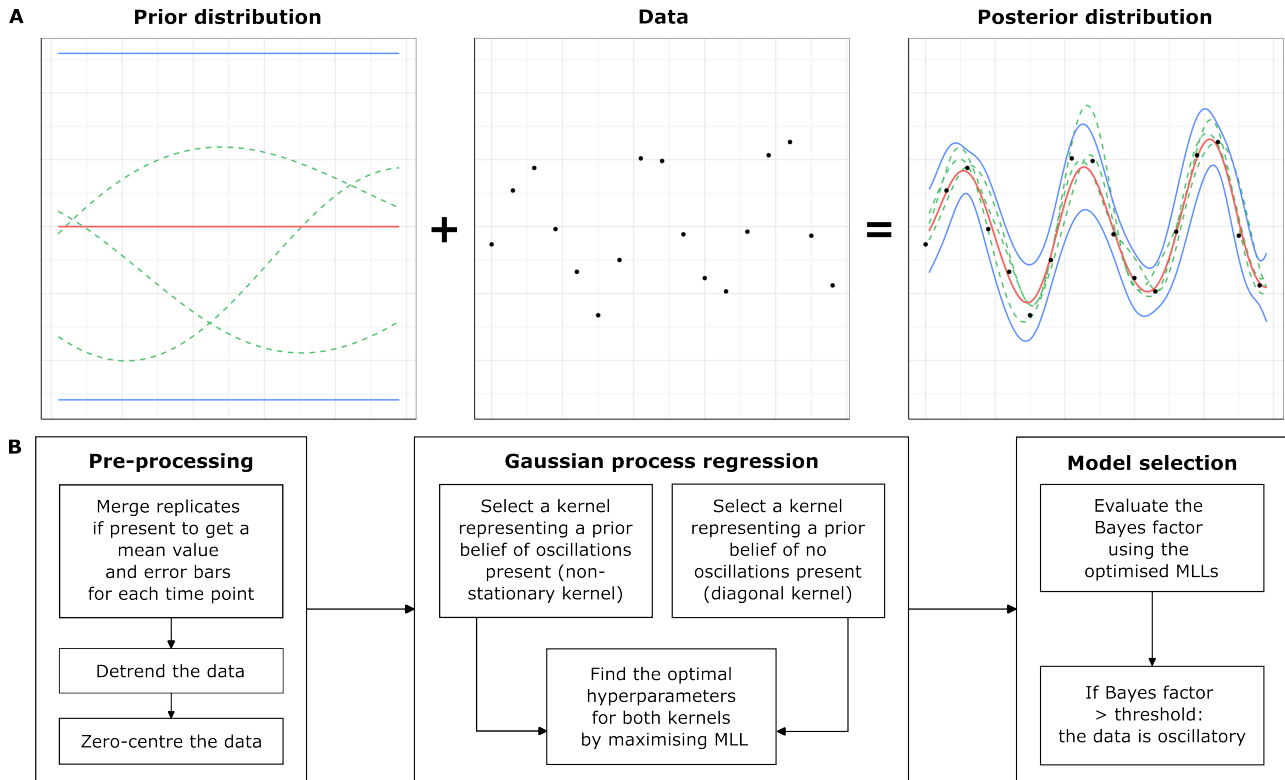


Figure 1: An overview of the workflow of ODeGP. (A) An intuitive schematic of Gaussian Process (GP) regression, where a GP is an indexed collection of random variables with a multivariate normal joint probability density. Left: A GP prior defining a distribution over functions (instantiations of the functions are shown in green lines; mean and confidence intervals are in red and blue respectively). Middle: Observed data. Combining the observed data and the GP prior allows the construction of the data likelihood. Right: Posterior distribution generated after Bayesian inference, which models the given data closely. Green lines are instantiations of the posterior, red and blue lines are the posterior mean and standard deviation respectively. (B) The workflow of ODeGP. The pre-processing stage involves formatting and normalizing the data. Next, two separate regressions are performed with the diagonal and non-stationary kernels respectively. The optimized marginal log likelihood (MLL) of the data is found for each case. These MLL values are then used to compute the Bayes factor for model selection. This Bayes factor is the final output metric used to determine whether the data is oscillatory or not.

All datasets: 3 replicates, 48 hr duration, 3 hr spacing, Noise level: low/high, Fraction of missing data: 0/0.5		
Simulated non-oscillatory datasets	Gaussian noise (Random sample from $\mathcal{N}(0, \sigma)$ at each timepoint)	
Simulated oscillatory datasets	Stationary symmetric data (combinations of sine waves)	Stationary asymmetric data (sawtooth waves)
	Non-stationary symmetric data (continuously decreasing or randomly varying time period)	Non-stationary asymmetric data (randomly varying time period)

Table 1: Categories of simulated data used for comparing the performance of existing oscillation-detection methods with ODeGP.

201 We generated a wide variety of simulated datasets where the ground truth (oscillatory or non-  
202 oscillatory) is known. These datasets are summarized in Table 1, and comparisons of various  
203 methods on these datasets are discussed in the next sections (see also SI Section 5). To mimic  
204 experimental qPCR data, all simulated waves were generated as sets of 3 replicates, with a  
205 total duration of 48 hours and an interval of 3 hours between consecutive observations. Two  
206 relative levels of noise (low and high), along with two different fractions of missing data (none  
207 and half), were used to create further variation among the datasets. Finally, three different ways  
208 of generating non-stationary data were tested (details in Section 5 of the SI).

## 209 **Detecting oscillations in simulated stationary datasets**

210 We first tested ODeGP on simulated stationary data. Datasets consisting of both non-oscillatory  
211 and stationary oscillatory waves were generated, and the ability of each method to distinguish  
212 the two was evaluated through the construction of receiver operating characteristic (ROC)  
213 curves.

214 Non-oscillatory waves were simulated by randomly sampling from a standard normal distribution  
215 with standard deviation  $\sigma$  at each timepoint in consideration:  $f(t) = \mathcal{N}(0, \sigma)$ , as shown in Figure  
216 2A. The three replicates are shown with red, blue and green lines. Symmetric stationary waves,  
217 as in Figure 2B, were generated by the addition of two sine waves:  $f(t) = A_1 \sin(2\pi t/\tau_1) +$   
218  $A_2 \sin(2\pi t/\tau_2) + \mathcal{N}(0, \sigma)$ .

219 The performance of various methods in terms of correctly identifying the presence of oscillations  
220 in symmetric stationary waves was evaluated by generating ROC curves on a collection of 500  
221 oscillatory and 500 non-oscillatory waves. These ROC curves for data corresponding to the  
222 waves in Figure 2A and 2B are shown in Figure 2C. ODeGP performs the best on this set of  
223 1000 waves, with an AUC value of 0.838. AUC values for all other datasets are reported in SI  
224 Section 5.

225 Similarly, performance on asymmetric stationary oscillatory waves was evaluated by generat-  
226 ing ROC curves on a collection of 500 non-oscillatory waves (as shown in figure 2D) and 500  
227 asymmetric stationary oscillatory waves. The latter were generated using the functional form  
228 of sawtooth waves:  $f(t) = A \cdot \{\frac{t}{\tau}\} + \mathcal{N}(0, \sigma)$ , where  $\{\}$  represents the fractional part function.

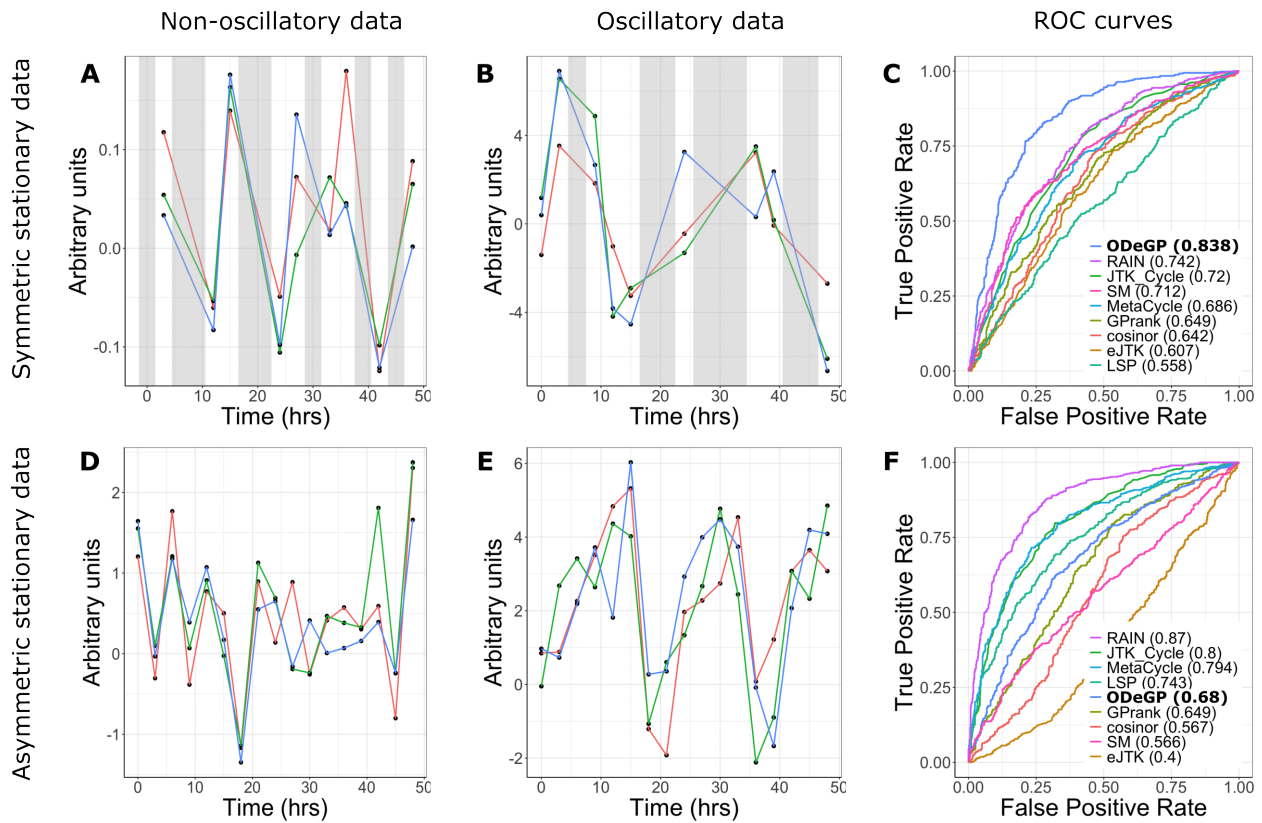


Figure 2: Detecting oscillations in simulated stationary datasets. All waves shown are sets of three replicates. (A) Simulated non-oscillatory data with  $\sigma = 0.1$  and half the points missing. (B) Simulated symmetric stationary data with  $A_1 = A_2 = 3, \tau_1 = 18, \tau_2 = 26, \sigma = 0.1$  and half the points missing. (C) ROC curves for all methods considered on the dataset consisting of waves generated like (A) and (B). Numbers in brackets denote AUC values. (D) Simulated non-oscillatory data with  $\sigma = 1$  and no points missing. (E) Simulated asymmetric stationary data with  $A = 5, \tau = 18$  and  $\sigma = 1$ . (F) ROC curves for all methods considered on the dataset consisting of waves generated like (D) and (E). Grey shaded areas represent regions with missing data. LSP - Lomb Scargle Periodogram; SM - spectral mixture kernel.

229 Figure 2E shows a wave generated as such. RAIN performs the best on this set of 1000 waves  
 230 as seen in Figure 2F with an AUC value of 0.87, while the non-stationary kernel has a relatively  
 231 poorer AUC value of 0.68.

232 The relative performance of the methods tested on all stationary datasets generated is sum-  
 233 marised in Table 2 (all AUC values are provided in SI Section 5). ODeGP is consistently among  
 234 the best 3 performing methods in all symmetric stationary datasets considered. Cosinor and  
 235 eJTK, which are among the best 3 methods the second-most times, also fall among the worst  
 236 3 methods a significant number of times. In asymmetric stationary datasets, RAIN is among  
 237 the best 3 methods the most consistently, whereas ODeGP here has a relatively average per-  
 238 formance (neither being many times among the best 3 nor many among the worst 3). RAIN's

239 better performance compared with our method on these datasets is expected because it sepa-  
 240 rately groups the rising and falling parts of the waves for comparison, boosting its ability to  
 241 identify asymmetric waveforms [16].

Stationary Datasets				
Method	Symmetric Datasets (Total: 9)		Asymmetric Datasets (Total: 8)	
	Times among best 3 methods	Times among worst 3 methods	Times among best 3 methods	Times among worst 3 methods
Cosinor	5	3	3	4
eJTK	5	4	4	4
GPrank	0	6	0	5
JTK_Cycle	1	0	3	0
LSP	0	6	1	4
MetaCycle	1	0	4	0
ODeGP	9	0	2	1
SM Kernel	2	5	0	6
RAIN	4	3	7	0

Table 2: Comparison of the number of times each method tested appears among the best and worst performing 3 methods, in all stationary datasets considered. LSP - Lomb Scargle Periodogram; SM - spectral mixture.

## 242 Detecting oscillations in simulated non-stationary datasets

243 Since experimental time-series qPCR data tends to be non-stationary (peak-to-peak distance  
 244 varies with time), we next tested the ability of each method to distinguish non-stationary oscil-  
 245 latory waves from non-oscillatory waves.

246 Symmetric non-stationary oscillatory waves with a monotonically decreasing time period, as  
 247 shown in Figure 3B, were generated using the functional form  $f(t) = A \cdot \sin(\frac{2\pi t}{1+|t-\tau|}) + \mathcal{N}(0, \sigma)$ ,  
 248 where  $\tau \geq 48$ . The ROC curves in Figure 3C indicate that ODeGP performs the best on a  
 249 dataset consisting of 500 non-oscillatory waves generated like in Figure 3A and 500 symmetric  
 250 non-stationary oscillatory waves generated like in Figure 3B, with an AUC value of 0.814.

251 Since there can be many ways of generating non-stationary data, an additional approach to  
 252 generating time-varying periodicities in a single wave was explored. Symmetric oscillatory waves  
 253 were generated with a randomly varying time period instead of a monotonically decreasing one,  
 254 as follows: at the start of each new oscillation, a time period  $\tau$  is sampled from  $\mathcal{N}(\mu, \sigma)$ . If the  
 255 total time elapsed up to the start of this new oscillation is  $\phi$ , then  $f(t) = A \cdot \sin(\frac{2\pi(t-\phi)}{\tau}) + \mathcal{N}(0, \sigma)$

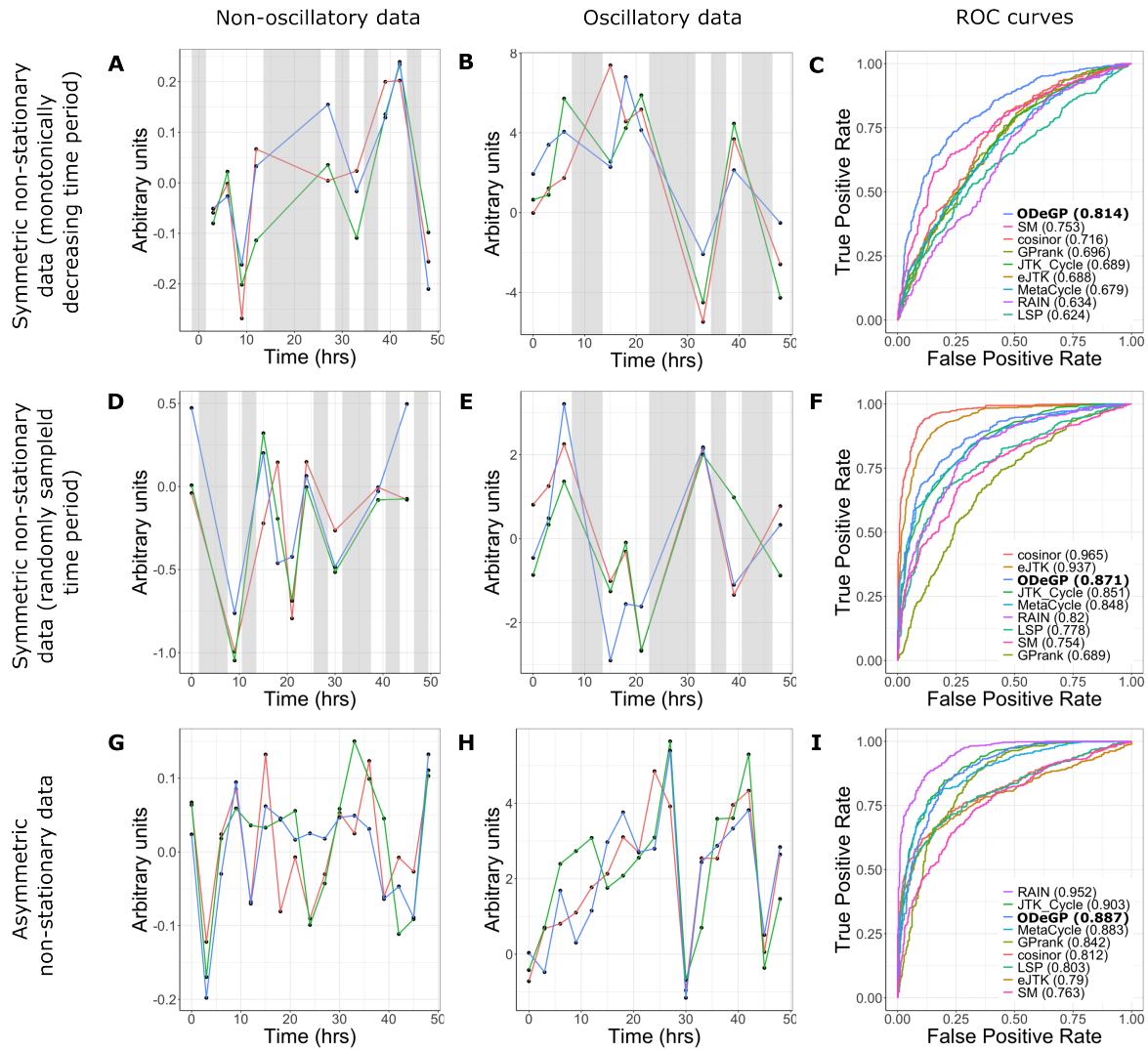


Figure 3: Detecting oscillations in simulated non-stationary datasets. All waves shown are sets of three replicates. (A) Simulated non-oscillatory data with  $\sigma = 0.1$  and half the points missing. (B) Simulated symmetric non-stationary data with a monotonically decreasing time period, with  $A = 5, \tau = 72, \sigma = 0.1$  and half the points missing. (C) ROC curves for all methods tested on the dataset consisting of waves generated like (A) and (B). (D) Simulated non-oscillatory data with  $\sigma = 0.5$  and half the points missing. (E) Simulated symmetric non-stationary data with a randomly varying time period, with  $A = 1.5, \mu = 24, \sigma = 1.33, \sigma = 0.5$ , and half the points missing. (F) ROC curves for all methods tested on the dataset consisting of waves generated like (D) and (E). (G) Simulated non-oscillatory data with  $\sigma = 0.1$  and no points missing. (H) Simulated asymmetric non-stationary data with  $A = 5, \tau_1 = 12, \tau_2 = 30$ , and  $\sigma = 0.1$ . (I) ROC curves for all methods tested on the dataset consisting of waves generated like (G) and (H). Grey shaded areas represent regions with missing data. Numbers in brackets in panels (C), (F) and (I) correspond to AUC values. LSP - Lomb Scargle Periodogram; SM - spectral mixture kernel.

256 for  $\phi \leq t < \phi + \tau$ . Figure 3E shows a wave generated in this way. Figure 3F demonstrates that  
 257 cosinor is the best performing method on the dataset consisting of 500 non-oscillatory waves  
 258 generated like in Figure 3D and 500 symmetric non-stationary oscillatory waves generated like  
 259 in Figure 3E. While ODeGP has a poorer AUC value of 0.871 compared to 0.965 for Cosinor

260 and 0.937 for eJTK, it still remains within the top three performing methods.

261 Asymmetric non-stationary oscillatory waves (like shown in Figure 3H) were generated using  
 262 a similar principle, but with a sawtooth waveform instead of a sine waveform. At the start of  
 263 each new oscillation, a time period  $\tau$  is sampled from  $U(\tau_1, \tau_2)$ . If the total time elapsed up to  
 264 the start of this new oscillation is  $\phi$ , then  $f(t) = A \cdot \{\frac{t-\phi}{\tau}\} + \mathcal{N}(0, \sigma)$  for  $\phi \leq t < \phi + \tau$ , where  
 265  $\{\}$  represents the fractional part function. RAIN can be seen as the best performing method in  
 266 Figure 3I for the dataset consisting of 500 non-oscillatory waves generated like in Figure 3G and  
 267 500 asymmetric non-stationary oscillatory waves generated like in Figure 3H. ODeGP however  
 268 remains among the top three methods here as well.

Non-stationary Datasets				
Method	Symmetric Datasets (Total: 15)		Asymmetric Datasets (Total: 12)	
	Times among best 3 methods	Times among worst 3 methods	Times among best 3 methods	Times among worst 3 methods
Cosinor	9	2	4	5
eJTK	7	4	4	6
GPrank	2	4	0	6
JTK_Cycle	4	2	8	0
LSP	0	14	0	8
MetaCycle	2	6	5	1
ODeGP	15	0	5	0
SM Kernel	4	7	0	10
RAIN	2	6	10	0

Table 3: Comparison of the number of times each method tested appears among the best and worst performing 3 methods, in all non-stationary datasets considered. LSP - Lomb Scargle Periodogram; SM - spectral mixture

269 A comparison of the performance of the methods tested across all simulated non-stationary  
 270 datasets is shown in Table 3 (AUC values from all datasets are provided in SI Section 5).  
 271 ODeGP is the best performer for symmetric non-stationary datasets, being among the top 3  
 272 methods (in terms of AUC values) for all 15 datasets tested. In the non-stationary case as  
 273 well, RAIN again emerges as the best method for asymmetric datasets. ODeGP shows a slight  
 274 improvement in its relative performance on non-stationary datasets compared with stationary  
 275 asymmetric datasets, being among the best 3 methods for a larger fraction of the datasets, and  
 276 also never falling among the worst 3 methods.



## 277 **ODeGP provides increased sensitivity at distinguishing oscillatory vs** 278 **non-oscillatory patterns in noisy qPCR datasets**

279 After analyzing simulated datasets, we next evaluated ODeGP's ability to distinguish oscillatory  
280 and non-oscillatory patterns in experimental datasets and benchmarked its performance against  
281 existing methods. We started with a published dataset on primary mouse marrow stromal  
282 cells, where the expression levels of a number of circadian clock genes were measured over  
283 48 hours using qPCR [4]. The cells were either treated with Dexamethasone (Dex) which is  
284 expected to synchronize or stimulate clock gene expression oscillations, or with vehicle (DMSO)  
285 where oscillations are not expected. Three independent experiments were done at each time  
286 point, thereby providing error bars for the expression levels as well. This dataset, therefore,  
287 represented a good test case for applying our rhythm detection method, since the ground truth  
288 is known.

289 The results from our analysis of the *Rev-ERB $\beta$*  and *Per1* genes are shown in Figure 4 (other  
290 genes are shown in Figure S2, SI Section 6). Raw data for *Rev-ERB $\beta$* , which exhibited large  
291 amplitude oscillations, are shown in Figure 4A and 4B along with the GP posteriors (mean and  
292 standard deviation) generated using the non-stationary kernel. Besides an AUC of its ROC curve  
293 being close to one, an additional characteristic of a good binary classifier is its ability to produce  
294 an output metric that is well-separated for the two classes in consideration - in our case, non-  
295 oscillatory and oscillatory. Applying ODeGP on the vehicle treated data (Figure 4A) produces  
296 a Bayes factor of 16.50, whereas it produces a Bayes factor of 50133.83 on the synchronized data  
297 in Figure 4B, a separation of more than three orders of magnitude. The significant separation  
298 between these two values shows that ODeGP is able to make a clear distinction between the  
299 non-oscillatory (or weakly oscillatory; see more in the Discussion section) and oscillatory qPCR  
300 data. The same trend is observed in the other genes we analyzed (Figure S2), where there is at  
301 least an order of magnitude increase in the Bayes Factor, usually even more, when the data is  
302 oscillatory.

303 The appropriateness of the non-stationary kernel for the classification problem is also highlighted  
304 by the narrower confidence intervals of the GP posterior in Figure 4B compared to those in Figure  
305 4A, which indicates that the non-stationary model is a less complex model for the oscillatory data

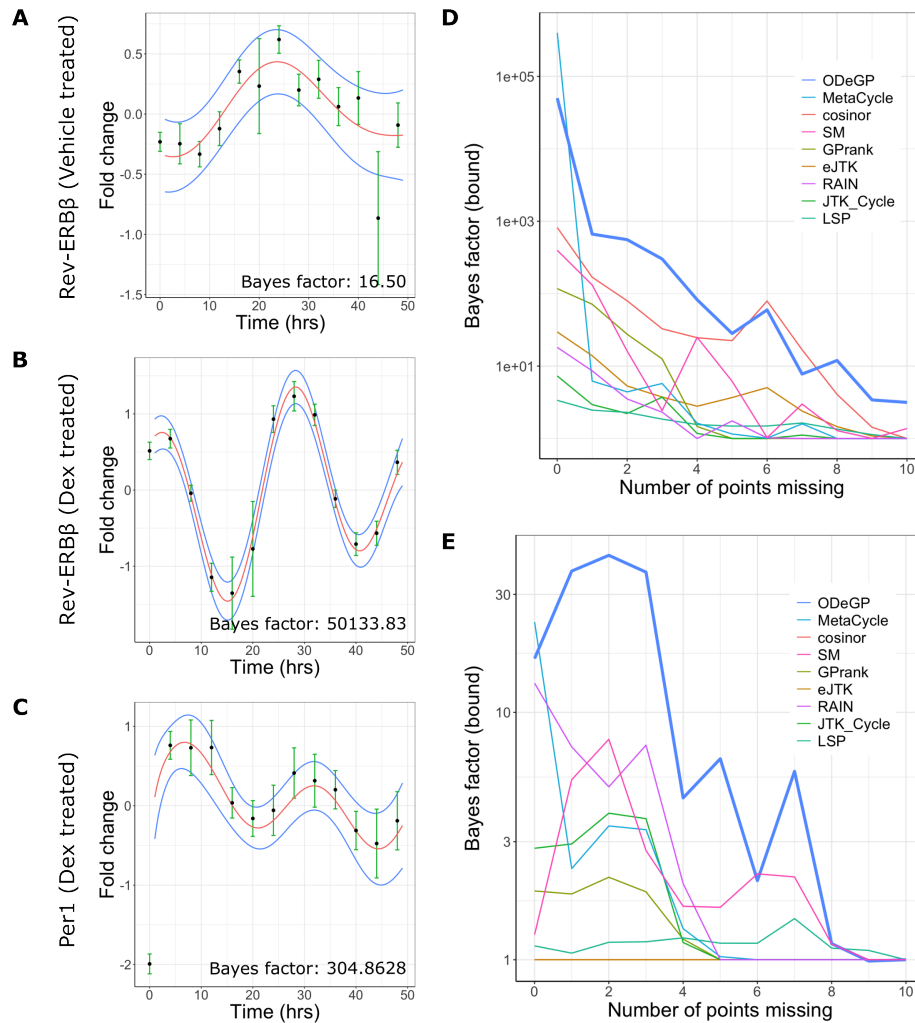


Figure 4: Application of ODeGP on *Rev-ERBβ* and *Per1* expression data [4] demonstrates the sensitivity of ODeGP at distinguishing oscillatory from non-oscillatory data. (A) The relative gene expression of *Rev-ERBβ* when treated with DMSO, measured at 4-hour intervals over a 48-hour duration, is shown by the black points with green error bars indicating an average taken over 3 biological replicates. Defining a GP prior using the non-stationary kernel and performing GP regression on this data produces the posterior distribution shown (mean in red, standard deviation in blue). (B) Relative gene expression of *Rev-ERBβ* when treated with Dex, representing an oscillatory dataset, is shown by the black points with green error bars. The GP posterior of the non-stationary kernel applied to this data is shown with the posterior mean and standard deviation in red, and blue respectively. (C) Relative gene expression of *Per1*, when treated with Dex, is shown by the black points with green error bars. This dataset represents ground-truth oscillatory data, but with smaller amplitude oscillations as compared to (B). The GP posterior of the non-stationary kernel applied to this data is shown with the posterior mean and standard deviation in red, and blue respectively. ODeGP classifies this as oscillatory with much more confidence than other existing methods (see main text and Table 4). (D) Points from the raw data in (B) were removed one by one in a random order, and all methods were applied to the resulting downsampled data at each step. The variation of the resulting Bayes factor (bounds) with increasing number of missing points is shown for each method. The thick blue line represents the ODeGP Bayes factor. (E) Analysis similar to panel (D) but for the raw data in panel (A).



306 than for the non-oscillatory data. Similar separations of Bayes factors and narrower confidence  
 307 intervals were observed when applying ODeGP on the corresponding data for the *Per2* and  
 308 *Npas2* genes as well (Fig. S2).

309 We next asked if ODeGP is more sensitive at detecting oscillations compared to other existing  
 310 methods. To evaluate this, we analyzed *Per1* data from the same paper [4], which visibly  
 311 exhibited lower amplitude oscillations with larger error bars (Figure 4C). To compare the p (or Q)  
 312 values generated by other methods against the Bayes factor produced by ODeGP, we converted  
 313 the p-values to Bayes factor bounds [33] (Table 4). The Bayes factor bound represents an upper  
 314 bound on Bayes factors corresponding to a given p-value, under very general assumptions on  
 315 the alternative hypothesis [33]. As is clear from Table 4, the only methods besides ODeGP that  
 316 correctly classified the data as rhythmic, were eJTK, MetaCycle and RAIN (p-values less than  
 317 0.05). However, the Bayes factor generated by ODeGP was much larger compared to the upper  
 318 bound values of eJTK, Metacycle or RAIN, demonstrating that ODeGP correctly classified the  
 319 oscillations with much more confidence. Indeed, if recent guidelines for rejecting the null based  
 320 on p-values less than 0.005 ([33]; see Discussion section) were to be used, ODeGP would be the  
 321 only method to correctly classify this dataset as oscillatory. The data's oscillatory trend was  
 322 also captured well in ODeGP's non-stationary kernel posterior shown in Figure 4C.

Method	Metric type	Metric value	Bayes factor (bound)
Cosinor	Q-value	0.08537	1.7512
eJTK	p-value	0.03577	3.0879
GPrank	log of Bayes factor	-6.7185e-05	0.9999
JTK_Cycle	p-value	0.12445	1.4185
LSP	p-value	0.43067	1.0140
MetaCycle	p-value	0.04496	2.6376
ODeGP	Bayes factor	304.8628	304.8628
SM Kernel	Bayes factor	1.2632	1.2632
RAIN	p-value	0.00617	11.7191

Table 4: Comparison of output metrics of all methods tested on the low-amplitude *Per1* oscillation data shown in Figure 4C. p-values/Q-values were converted to corresponding Bayes factor bound [33] values where applicable to allow comparison with the Bayes factor returned by ODeGP. While eJTK, MetaCycle and RAIN are the only other methods that correctly classify the dataset as oscillatory (based on a p-value cutoff of 0.05), the large difference between ODeGP's Bayes factor and the upper bounds generated by these methods suggest that ODeGP is more sensitive at detecting oscillations.

323 To more systematically test ODeGP's sensitivity of detecting oscillations against existing meth-

324 ods, we compared the Bayes factor generated by ODeGP with the Bayes factor bound values  
325 produced by the other methods across an increasingly down-sampled dataset. The raw data  
326 for Rev-ERB $\beta$  in Dex-treated cells (Figure 4B), which showed large amplitude oscillations, was  
327 considered as a starting point. Points were then removed from this dataset one by one in a  
328 random order, and at each step, all methods were applied to the sub-sampled data. Figure 4D  
329 demonstrates the rapid decrease in the Bayes factor and Bayes factor bounds with increasing  
330 missing points. At zero points missing all methods perform well (i.e. produce large Bayes factor  
331 bound values), though ODeGP and MetaCycle are distinctly better. As the number of miss-  
332 ing points increases, the ODeGP Bayes factor (blue thick line in Figure 4D) most consistently  
333 maintains a larger value compared to the Bayes factor bounds obtained from other methods.  
334 ODeGP thus performs better at identifying the data as oscillatory at an extent of missing points  
335 that causes other methods to fail. We also performed a similar downsampling analysis for the  
336 weakly oscillatory dataset in Figure 4A, the results of which are shown in Figure 4E. On com-  
337 paring Figures 4D and E, it is evident that for most points on the x-axis, the strong versus weak  
338 oscillation Bayes Factors are best separated for ODeGP.

## 339 **Cell-density dependent rapid emergence of oscillations in mouse em-** 340 **bryonic stem cells**

341 Finally, we tested ODeGP's ability to quantify oscillatory behaviour in new qPCR datasets, that  
342 could enable novel biological discoveries. For this purpose, we used early passage pluripotent  
343 mouse embryonic stem cells (mESCs). Previous work has demonstrated that pluripotent stem  
344 cells do not exhibit oscillations of the core circadian clock genes, even though the genes are  
345 expressed in these cells [39, 40]. We first confirmed these well-established results by culturing  
346 mESCs on a variety of substrates in the presence of LIF where pluripotency is expected to be  
347 maintained (gelatin on plastic, fibronectin on glass and laminin on glass; details in the SI Section  
348 1). We synchronized cells with Dex and collected the cells over a period of 24 hours at intervals of  
349 3 hours. Consistent with the previous literature [39, 40], application of our algorithm confirmed  
350 that there were no oscillations in any of the tested conditions, since the Bayes factors were in  
351 the range 2-8 (Figure 5A). As we had observed in the last section, in known cases where there

352 are no oscillations, the Bayes factor tends to be of the order of 10 while the presence of real  
 353 oscillations pushes up the Bayes factor by one or two orders of magnitude.

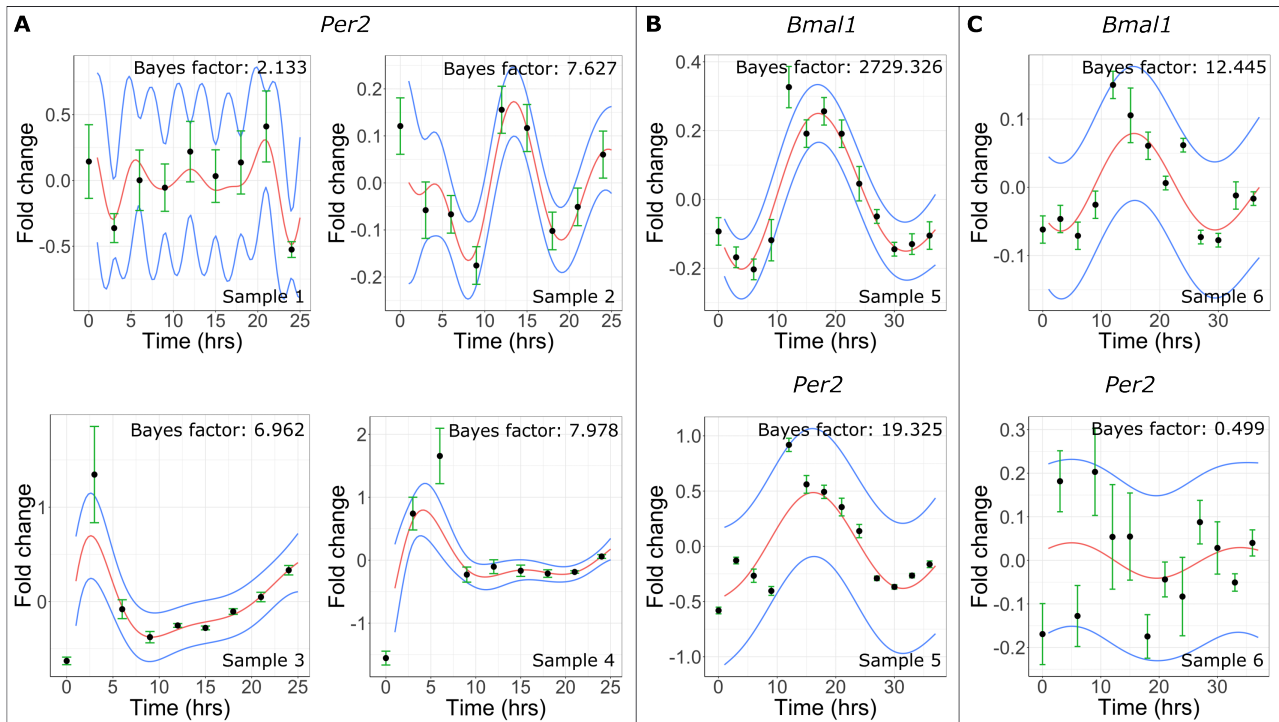


Figure 5: Cell-density dependent rapid emergence of oscillations in mESCs. GP posteriors of the non-stationary kernel (mean in red, standard deviation in blue) for (A) *Per2* gene expression from low-density samples 1,2,3 and 4 (see detailed descriptions in the SI), (B) *Bmal1* and *Per2* expression for high-density sample 5, and (C) *Bmal1* and *Per2* expression for high-density (but 2i treated) sample 6. In all plots, black circles represent the mean qPCR measurement from three technical replicates.

354 We next asked if increasing the cell density could lead to the generation of oscillations of circa-  
 355 dian clock gene expression. Previous work has demonstrated that about two weeks of Retinoic  
 356 Acid (RA) induced differentiation can induce oscillations in mESCs [39, 40], but to the best of  
 357 our knowledge, the kinetics of oscillation development upon cell density increase has not been  
 358 explored in this cell type. Since higher cell density can potentially cause some amount of differ-  
 359 entiation in stem cells [41], we explored the consequences of doubling the number of cells in our  
 360 culture dish before Trizol extraction and gene expression quantification. We also extended the  
 361 time over which cells were collected from 24 to 36 hours, to allow for better detection of potential  
 362 oscillations. Interestingly, though the higher cell density was maintained for only about 3-4 days  
 363 (see SI Section 1 for details), we saw clear signs of oscillation in *Bmal1* (Bayes factor of 2729)  
 364 and weak oscillations in *Per2* (Bayes factor of 19), as can be seen in Figure 5B. This was in  
 365 contrast to the earlier studies using RA, where oscillations emerged only after two weeks [39].

366 To check whether cell differentiation could potentially have played a role in generation of these  
367 oscillations, we added the dual inhibitors of MEK/ERK and GSK3b (commonly called 2i) to an  
368 otherwise identical experimental set up with the higher cell density. 2i has been shown to differ-  
369 entially kill cells with low *Nanog* expression levels, which are prone to undergoing differentiation  
370 [42]. This time we found much lower Bayes factors for both *Bmal1* as well as *Per2* - 12.4 and  
371 0.5 respectively (Figure 5C), clearly indicating that the oscillations are no longer present. These  
372 results suggest that increasing cell density might accelerate the development of oscillations of  
373 the core clock genes via some degree of differentiation, though the kinetics seem to be faster than  
374 that of RA induced differentiation. It is interesting to note that in some cases it is impossible  
375 to visually discern whether or not an oscillation is present (for example in Figure 5A). These  
376 examples serve to highlight the importance of a quantitative and methodical approach to the  
377 oscillation detection problem.

## 378 Discussion

379 Detecting biological oscillations from time-series datasets remains a challenging task even af-  
380 ter decades of research. Here we developed an oscillation detection method based on Gaussian  
381 Process (GP) regression for learning noisy patterns, combined with Bayesian model selection to  
382 distinguish between oscillatory and non-oscillatory datasets. Our method, ODeGP, is designed  
383 particularly to model non-stationarity in oscillatory data using the non-stationary kernels intro-  
384 duced recently [36, 37], thereby setting it apart from the few previous GP-based approaches to  
385 oscillation detection. Furthermore, the combination of GPs and Bayesian inference has a number  
386 of general advantages over existing methods: (1) the non-parametric nature of GPs allows for  
387 better learning of noisy, non-stationary and irregularly spaced patterns, (2) technical replicates  
388 can easily be incorporated via diagonal terms in the covariance matrix, (3) the learned function  
389 and error estimates are naturally generated via analytical expressions of the posterior mean and  
390 variance and (4) issues associated with p-values are circumvented by the Bayes factor, which ex-  
391 plicitly addresses the likelihood of the observed data under both (oscillatory and non-oscillatory)  
392 hypotheses, instead of just the null [32]. Additionally, we also provide a Bonferroni-like multiple  
393 hypothesis correction within the Bayesian setting that ODeGP uses [38].

394 We demonstrated the improved performance of ODeGP compared to eight existing methods  
395 using both artificial (simulated) as well as experimental datasets. On the 44 simulated datasets,  
396 we used ROC curves and the AUC metric to make the comparisons. Overall, ODeGP worked  
397 significantly better (most number of times amongst the top 3 performing methods) than all other  
398 methods tested, both for non-stationary as well as stationary symmetric datasets (Tables 2 and  
399 3). As expected however, RAIN consistently outperformed all other methods when the data was  
400 asymmetric. Importantly, even when ODeGP did not come out on top for particular datasets, it  
401 was still consistently good, as it almost never ranked amongst the 3 worst performing methods.  
402 This was in stark comparison to eJTK and Cosinor for example, which often performed very  
403 well, but also frequently performed very poorly (Tables 2 and 3). In the case of experimental  
404 data where the ground truth is known, we compared the performance of all the methods on  
405 low-amplitude oscillations of the *Per1* gene. While most methods incorrectly classified the data  
406 as arrhythmic, only ODeGP, eJTK and MetaCycle managed to detect the oscillations (Table  
407 4). However, on calculating the upper bounds of the Bayes factors corresponding to eJTK  
408 and MetaCycle's p-values, we found that these bounds were about two orders of magnitude  
409 less than the Bayes Factor generated by ODeGP, thus providing significantly less confidence  
410 in the rhythmicity classification. Furthermore, starting from a strongly oscillatory dataset and  
411 subsampling the data points, we demonstrated that ODeGP consistently produced higher Bayes  
412 factors than other methods. In summary, analysis of both simulated as well as experimental  
413 datasets suggests that ODeGP is a more sensitive and reliable oscillation detector in comparison  
414 to the existing methods tested here.

415 Finally, we tested ODeGP's ability to provide new biological insights in qPCR data on circadian  
416 clock genes from pluripotent mESCs. Embryonic stem cells from both mouse [39] and human  
417 [43] are known to be deficient in the circadian clock oscillations, even though the genes are  
418 expressed in these cells. On directed differentiation using Retinoic Acid, oscillations have been  
419 shown to develop on a time scale of about two weeks in these cell types, raising intriguing  
420 questions on the role of gradual development of the oscillations [40]. Here we demonstrated that  
421 clear oscillations in one core circadian clock gene *Bmal1*, and to a lesser extent in *Per2*, can be  
422 induced within 3-4 days by increasing cell density (Figure 5B). Interestingly, we found that these

423 oscillations were prevented from developing in the presence of the inhibitors commonly known  
424 as 2i (Figure 5C), thereby suggesting a potential role of density-dependent mESC differentiation  
425 in the establishment of the oscillations. Recent experiments have uncovered cell-density effects  
426 on strengthening of the clock oscillations [44, 45], potentially via inter-cellular TGF $\beta$  signalling  
427 [45]. Our results suggest the intriguing possibility that the kinetics of oscillation development  
428 in stem cells could depend on cell-cell signalling during differentiation, and remains an exciting  
429 avenue to be further studied in the future.

430 The oscillatory (alternate hypothesis) versus non-oscillatory (null) classification problem will  
431 necessarily involve defining somewhat arbitrary cutoffs. However, based on recent discussions on  
432 p-values and interpretation of Bayes Factors as odds ratios, it has been proposed that a p-value of  
433 0.005 is a more sensible cutoff compared to the widely used 0.05 value, corresponding to a Bayes  
434 Factor Bound of  $\sim 14$  [33]. We empirically notice that this guideline seems to be approximately  
435 consistent with our own experimental observations. In our case, the real “gold standard” non-  
436 oscillatory datasets are the pluripotent mESC datasets (Fig. 5A), where we expect no oscillations  
437 to be present even at the single cell level. In these datasets, we consistently find that the Bayes  
438 Factors produced by ODeGP are below 14. The interpretation of the unsynchronized (vehicle  
439 treated), non-mESC datasets such as in Figure 4A is more challenging. While these datasets  
440 are supposed to be non-oscillatory, we observe Bayes Factors that are somewhat higher (16.5  
441 in Fig. 4A and as high as 274 in Fig. S2B). This suggests the presence of weak oscillations,  
442 which might be arising from plating of cells and/or addition of fresh media, which are both  
443 known to induce a small degree of synchronization between the single cell oscillators. After Dex  
444 synchronization however, the oscillations are expected to be stronger, which is correctly being  
445 reflected in each case by the much higher Bayes Factors (Fig. 4B,C). Overall, consistent with  
446 the recent recommendations in the statistics community, our results suggest that a Bayes Factor  
447 cutoff of 14 might be a good choice for classifying oscillatory versus non-oscillatory datasets. In  
448 addition, Bayes factors close to 14 could be classified as weak oscillations.

449 Though not explored extensively, there are a few prior examples of the use of GPs in modeling  
450 biological data, for example in identifying differentially expressed genes [36, 46], detecting os-  
451 cillations [47, 48] and the discovery of spatial patterns in gene expression [49]. The method in



452 [48] uses a principle similar to ours with the comparison of a non-oscillatory GP model to an  
453 oscillatory GP model. However, this method does not explicitly incorporate a non-stationary  
454 functional form to encode correlations between the data at different time points in its oscillatory  
455 model, and also compares its performance only with that of the LS Periodogram (which never  
456 falls among the best 3 performing methods in our analysis). GPrank [46] on the other hand uses  
457 GPs to model genome-wide time series data. Though it was not created for the specific purpose  
458 of detecting oscillations, the workflow followed by this method is similar to ours in terms of the  
459 combination of two GP models with Bayesian model selection. The significant difference between  
460 GPrank and ODeGP is in the choice of kernel used to define the alternate hypothesis (the RBF  
461 kernel is used in GPrank). Our results show that ODeGP almost always outperforms GPrank  
462 (Tables 2 and 3), demonstrating the importance of careful choice of the kernel for the specific  
463 problem at hand. This point also highlights the flexibility of GPs in modeling various kinds of  
464 datasets simply by appropriate choice of kernel functions. Similar to GPrank, the output metric  
465 used by ODeGP to make a decision on the presence of oscillations is the Bayes factor, which is a  
466 ratio of the marginal likelihoods of the competing models. The marginal likelihood  $\mathcal{L}$  computed  
467 during GP regression automatically incorporates a measure of the complexity of the model being  
468 considered through the  $\log|K|$  term. A larger  $\log|K|$  for a given model generally results from  
469 higher covariance values between the data's values at different time points, which produce wider  
470 confidence bounds in the GP posterior. Wider bounds allow a greater flexibility of functions in  
471 the posterior distribution, indicating that the model is more complex. For this reason, we did  
472 not additionally penalise the number of hyperparameters of both models when comparing the  
473 two (something that is generally done in regression methods which use the BIC/AIC for model  
474 selection). Importantly, we found that the Bayes factor was distinctly different between datasets  
475 that were known to be oscillatory versus non-oscillatory, thus highlighting the usefulness of this  
476 metric in the classification problem.

477 While our oscillation detection method seems to significantly improve upon currently used meth-  
478 ods, there are a number of limitations and potential areas of improvement. As can be seen from  
479 Tables 2 and 3, ODeGP is clearly inferior to RAIN in detecting oscillations in asymmetric wave-  
480 forms. Providing ODeGP an enhanced ability to model asymmetric oscillations represents a

481 clear avenue for improvement, which may be done by identifying new suitable functional forms  
482 for the hyperparameters of the non-stationary kernel. The runtime of our method is also sig-  
483 nificantly higher than that of most of the existing methods we tested (though it is comparable  
484 to that of RAIN), due to the hyperparameter optimization routine we used. Furthermore, the  
485 multiple hypothesis correction provided with ODeGP is similar to the Bonferroni correction in  
486 the frequentist setting [38]. This correction results in many false negatives when there are many  
487 hypotheses to be tested, and therefore ODeGP can currently be used only with smaller datasets  
488 such as those generated in qPCR, eclosion, egg-laying or feeding experiments. Detecting rhyth-  
489 mic or differentially rhythmic genes from genome-wide datasets is not possible with the current  
490 implementation of our method, and remain avenues for future improvement.

## 491 **Conclusions**

492 While much work has been done in developing algorithms to detect oscillations in time series  
493 datasets, there is clearly room for significant improvement. Our method combining Gaussian  
494 processes and Bayesian model selection demonstrates the flexibility of GPs, providing a highly  
495 sensitive approach for classifying oscillatory versus non-oscillatory datasets without using p-  
496 values. We hope that our results along with the user-friendly ODeGP R package, will spur  
497 more careful exploration of these approaches in the future. Applied to new experimental data,  
498 ODeGP provides initial evidence for rapid development of circadian clock oscillations upon  
499 increasing density of mESCs, and it would be exciting to see in future if these results have  
500 broader implications in the context of development and inter-cellular signalling.

## 501 **Software Availability**

502 ODeGP is provided as an easy to use R package with this manuscript, and can be downloaded  
503 from either <https://github.com/shabnamsahay/ODeGP> or <https://github.com/Shaoonlab/ODeGP>



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## 509 Author contributions

510 S.C. conceptualized and designed the study, S.S. developed and implemented the ODeGP algo-  
511 rithm, S.C. and S.A. performed the experiments, S.A. developed the error analysis method for  
512 qPCR datasets, S.S. and S.C. wrote the paper with inputs from S.A. and S.H., S.C. and S.H.  
513 supervised the work.

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