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Research article

Innovative player evaluation: Dual-possibility Pythagorean fuzzy hypersoft sets for accurate international football rankings

Saraj Khan [a], Muhammad Imran Asjad [a], Muhammad Bilal Riaz [b,c], Abdul Wahab [a,*], Hira Ashaq [a], Taseer Muhammad [d]

[a] Department of Mathematics, University of Management and Technology, Lahore, Pakistan
[b] IT4Innovations, VSB - Technical University of Ostrava, Ostrava, Czech Republic
[c] Department of Computer Science and Mathematics, Lebanese American University, Byblos, Lebanon
[d] Department of Mathematics, College of Science, King Khalid University, P.O. Box: 9004, Abha 61413, Saudi Arabia

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ABSTRACT

This study introduces an advanced approach for ranking international football players, addressing the inherent uncertainties in performance evaluations. By integrating dual possibility theory and Pythagorean fuzzy sets, the model accommodates varying degrees of ambiguity and imprecision in player attributes. Additionally, the use of hypersoft set theory enriches the analysis by capturing the multifaceted nature of player evaluations. The proposed aggregation operators refine the synthesis of diverse information sources, leading to a comprehensive and nuanced assessment. This research significantly enhances player evaluation methodologies, providing a more adaptable framework for a fair assessment of international football talent. A practical example illustrates the application of dual-possibility Pythagorean fuzzy hypersoft sets (DP-PFHSS). A numerical technique is proposed for solving multi-criteria decision-making (MCDM) challenges with known dual possibility information using the proposed aggregation operators. This decision-making algorithm effectively determines a football player's worth, contributing to the overall ranking and evaluation process. The approach aids in scouting and recruitment by facilitating talent identification and informed player signings. Graphical analysis, comparing existing and proposed methods using average and geometric operators, demonstrates the superiority of the proposed approach in the players evaluation, indicating that $\mathcal{F}_1$ is in the top ranking.

1. Introduction

The most popular and extensively watched sport in the world is football, or soccer, as it is known in certain places. The history spanning several centuries, it has become a worldwide sensation, capturing the attention and emotions of billions of people [1]. People from all across the world are united by a common passion that transcends language, culture, and location because of the sport's global appeal. The accessibility and appeal of football across a variety of communities can be attributed to its simplicity; all it takes is a ball and a reasonably wide location. There are many different types of football contests across the world, from small-town leagues to major international events [2]. There are several top domestic football leagues, including the English Premier League, La Liga in Spain, serie A in Italy and the Bundesliga in Germany, that draw exceptional players from all over the world [3,4]. The top

* Corresponding author.
E-mail address: wahabgujjar175@gmail.com (A. Wahab).

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players have the opportunity to represent their countries on a large scale through international events like the FIFA World Cup and regional championships like the UEFA European Championship and Copa America [5,6].

The leagues use a sophisticated technique to evaluate international football players because the process is intricate and multi-dimensional. The dynamic character of the sport, where talents, tactics, and collaboration all come together, makes it difficult for traditional player assessment to fully capture the nuances of the game [7]. Innovative methods are crucial to providing complete understandings of a player's skills, performance, and worth to the team as the demand for accuracy in player assessments rises. A precise assessment of international football players requires taking into account a variety of data elements that represent individual and team performances [8]. Quantitative insights into a player's attacking potential and overall effect on the game may be obtained through statistical indicators like assists, passes completed and goals scored. Some defensive measures that reveal a player's potential to contribute defensively include tackles won, interceptions and aerial duels [9]. The contextual examination of a player's style of play, decision-making under duress and ability to adjust to various tactical tactics is far more important than simply looking at numbers. In order to get qualitative information about a player's mobility, vision and strategic game comprehension, video analysis and scouting reports are essential [10]. The method of evaluating players has been significantly enhanced with the introduction of modern analytics. Anticipated goals (xG), anticipated assists and player tracking data are examples of metrics that provide a more detailed picture of a player's impact on the field [11]. Physical characteristics, including sprint speed, distance traveled and work rate, are used to evaluate a player's endurance and fitness, which are important factors in the fast-paced, modern game. The total evaluation of a player considers both their on-field performance and their off-field attributes, such as leadership, teamwork, and adaptability to different playing environments [12]. The social and psychological aspects of a player, such as their marketability and media presence, also influence their worth within the football ecosystem.

For millennia, the sole means of characterizing and defining ambiguity has been through its incorporation into probability theory. Nowadays, being unclear is synonymous with being arbitrary. The emergence of theories other than probability theory by the year 1960 has altered this perspective, as it represents uncertainty in several dimensions. Because of the newly published theories, it is now recognized that vagueness is a multidimensional word and that randomness is merely one of its aspects [13]. It is now recognized that the concept of vagueness is based on the system's insufficient and inadequate information level. Uncertainty arises from a number of limitations, such as inadequate technology, time-varying systems and restrictions in the human sensory system.

A fuzzy set is a mathematical concept introduced by Lotfi Zadeh in 1965 as an extension of classical (crisp) set theory [14]. In classical set theory, an element either belongs to a set or does not, with no degrees of membership. Fuzzy set theory, on the other hand, allows for degrees of membership between 0 and 1, indicating the degree to which an element belongs to a set [15]. Formally, a fuzzy set A in a universe of discourse X is characterized by a membership function $\mu_{A(x)}$, which assigns a degree of membership to each element x in X. The membership function $\mu_A(x)$ yields values in the range $[0,1]$, where 0 indicates no membership, 1 indicates full membership and values in between represent degrees of partial membership. An intuitionistic fuzzy set (IFS) extends the traditional fuzzy set concept by incorporating a third parameter, hesitation, to capture the degree of uncertainty associated with an element's membership [16]. This concept was first introduced by Krassimir T. Atanassov in 1986, an IFS is defined by three functions: the membership function, which represents the degree of belonging; the non-membership function, indicating the degree of non-belonging; and the hesitation function, quantifying the degree of uncertainty or indecision [17]. This three-dimensional approach provides a more comprehensive representation of uncertainty, making intuitionistic fuzzy sets suitable for applications where imprecise information or ambiguity needs to be explicitly modeled, such as in decision-making and sentiment analysis [18].

A Pythagorean fuzzy set (PFS) is an extension of classical fuzzy set theory introduced by Wu et al. [19] in 2013, aiming to address limitations in representing uncertainty and ambiguity. Unlike traditional fuzzy sets, PFS involves two independent membership grades, namely the membership degree and the non-membership degree, both of which are real numbers between 0 and 1. Additionally, PFS adheres to the Pythagorean theorem, where the square of the membership degree plus the square of the non-membership degree equals one. This geometric interpretation introduces a more structured and rigorous framework for handling uncertainty, making Pythagorean fuzzy sets particularly useful in decision-making processes where clear distinctions between membership and non-membership are crucial, such as in risk assessment and expert systems. A soft set, introduced by Molodtsov in 1999, is a mathematical framework designed to handle uncertainty and vagueness in information [20].

In contrast to traditional sets with well-defined membership criteria, a soft set allows for the inclusion of elements based on certain parameters or characteristics rather than strict conditions. A soft set is defined by a pair of sets: the approximate set, which contains elements satisfying certain criteria and the boundary set, which represents elements that may or may not satisfy those criteria [21]. The flexibility of soft sets makes them applicable in various fields, such as decision-making, data analysis and information retrieval, where imprecise or incomplete information needs to be systematically managed. In a general sense, "possibility" refers to the condition or state of being possible, which means that something can happen, exist, or be true. Possibility is often associated with the likelihood or feasibility of a particular event or situation occurring [22]. In formal terms, the concept of possibility is frequently expressed in probability theory, where the likelihood of an event is quantified as a numerical value between 0 (impossible) and 1 (certain). In logic and philosophy, possibility is also explored through modal logic, which deals with modalities such as necessity, possibility, impossibility and contingency. Modal operators, like "necessarily" and "possibly," help express statements about the necessity or possibility of propositions. It's important to note that the definition and context of "possibility" can vary depending on the field of study or application, ranging from everyday language use to formal mathematical and philosophical frameworks [23].

The exploration of Pythagorean fuzzy sets and their diverse extensions has sparked a vibrant discourse within the realm of decision-making methodologies. Zhang et al. [24–27] seminal contribution in 2018 marked a pivotal moment with the introduction of dual-possibility Pythagorean fuzzy sets. This novel framework revolutionized decision-making by seamlessly integrating both membership and non-membership degrees, offering a nuanced perspective on how to handle uncertainty. Through a meticulous exploration of operational techniques and aggregation methodologies, their research not only expanded the theoretical boundaries of Pythagorean fuzzy sets but also laid a robust foundation for practical applications in decision support systems [28,29]. In tandem with this groundbreaking work, Xu's comprehensive review in 2019 provided a panoramic view of the evolving landscape of Pythagorean fuzzy sets. By synthesizing a vast body of literature, Xu offered valuable insights into the theoretical underpinnings and practical implications of Pythagorean fuzzy sets across diverse domains [30,31]. From fundamental operations to advanced aggregation strategies, Xu's review served as a cornerstone for researchers seeking to navigate the intricacies of Pythagorean fuzzy sets and harness their potential in decision-making processes [32].

Expanding beyond theoretical frameworks, in 2020 Lu et al. [33] ventured into the realm of practical applications, shedding light on the transformative impact of Pythagorean fuzzy sets in decision-making and fault diagnosis [34]. Their empirical studies underscored the efficacy of Pythagorean fuzzy sets in managing uncertainty and ambiguity, offering tangible solutions to real-world challenges. By presenting compelling case studies and demonstrating the superiority of Pythagorean fuzzy sets over traditional approaches, we paved the way for their widespread adoption in decision support systems and beyond [35]. Continuing this trajectory of innovation, Wang and Wenjun 2021 embarked on a quest to push the boundaries of Pythagorean fuzzy sets further with the introduction of Pythagorean fuzzy hypersoft sets. By integrating hypersoft information into the Pythagorean fuzzy framework, their research ushered in a new era of decision-making methodologies, capable of capturing and synthesizing complex, multidimensional data with unparalleled precision [36]. Through meticulous experimentation and theoretical analysis, Wang and Wenjun demonstrated the transformative potential of Pythagorean fuzzy hypersoft sets in addressing the multifaceted nature of decision-making problems across diverse domains. Collectively, these pioneering works represent a testament to the transformative power of Pythagorean fuzzy sets and their extensions in revolutionizing decision-making methodologies. From theoretical advancements to practical applications, these studies have not only expanded the horizons of knowledge but also paved the way for future innovations in the field of decision support systems. As researchers continue to push the boundaries of Pythagorean fuzzy sets, the journey towards harnessing uncertainty and complexity in decision-making processes will undoubtedly reach new heights of discovery and innovation.

A comprehensive literature review reveals a burgeoning interest in soft sets, originating from the seminal work of Molodtsov in 1999. Researchers have extensively explored the applications of soft sets in various domains, with a notable focus on decision-making processes. Soft sets offer a flexible framework for handling uncertainties and imprecise information inherent in decision-making scenarios [37]. Scholars such as Maji, Biswas and Roy have contributed significantly to this area, highlighting the effectiveness of soft sets in modeling and analyzing uncertain decision problems. Their research emphasizes the integration of soft sets with other mathematical theories, such as fuzzy logic and rough sets, to enhance the decision-making process. Moreover, studies by Hayat et al. [38,39] have demonstrated the utility of soft sets in multi-criteria decision analysis, where complex criteria with vague or incomplete information are involved. Furthermore, advancements in soft computing techniques have led to the development of novel decision-making algorithms based on soft sets. These algorithms, proposed by researchers like Dey and Roy, leverage the inherent flexibility of soft sets to provide robust and efficient solutions to decision problems. Additionally, the application of soft sets in group decision-making contexts has gained traction, with researchers like Liu and Qin exploring collaborative decision-making frameworks that integrate soft sets with other group decision models [40].

Research Gap

Despite advancements in player evaluation methodologies, there remains a notable gap in the development of a comprehensive approach that integrates dual-possibility theory, Pythagorean fuzzy sets and hypersoft sets for accurate international football rankings. Existing methods often rely on single-dimensional models or fail to adequately address the multifaceted nature of player assessments, leaving room for improvement in the accuracy and adaptability of player evaluation systems. **Novelty of Study:**

The novelty of the study is following:

- Introduces dual possibilities degrees with hypersoft sets and Pythagorean fuzzy sets theory for rating international football players.
- Utilizes a dual-possibility paradigm to consider both positive and negative factors, enhancing the complexity and precision of evaluations.
- Accounts for varying degrees of membership and non-membership, providing a detailed and accurate assessment of player attributes.
- Hypersoft sets improve the ability to manage uncertainty in the evaluation process.
- Employs advanced aggregation operators to effectively combine multiple criteria, addressing the complexities of player evaluations and advancing the mathematical modeling of football player rankings.

Motivation of Study:

The motivation of the study is following:

- **Addressing Uncertainties**: The study aims to address the inherent uncertainties and imprecision in traditional football player evaluations by incorporating advanced mathematical theories.
- **Comprehensive Assessment**: To develop a more comprehensive and nuanced method for evaluating international football players, considering multiple factors and criteria.
- **Enhanced Decision-Making**: To improve decision-making processes in scouting and recruitment by providing a robust framework that accounts for both positive and negative attributes of players.

- **Innovative Methodologies**: To introduce and validate novel approaches such as dual possibility theory, Pythagorean fuzzy sets, and hypersoft sets in the context of sports analytics.
- **Advanced Aggregation Techniques**: To leverage advanced aggregation operators to combine diverse information sources effectively, leading to more accurate and reliable player rankings.

Advantages of Study:

The advantages of the study are following:

- **Accuracy:** The method aims to provide accurate rankings of players by incorporating dual-possibility Pythagorean fuzzy hypersoft sets. This suggests a sophisticated approach that may capture nuances in player performance more effectively than traditional methods.
- **Innovation:** The method introduces a novel approach to player evaluation in football, which suggests a potential for fresh insights and perspectives in talent assessment and team selection.
- **International Scope:** By focusing on international football, the method addresses the need for evaluating players across diverse teams and competitions, offering a broader perspective on player performance.
- **Fuzzy Logic:** Fuzzy logic allows for the representation of uncertainty and ambiguity in player evaluation, which is a common feature in sports analytics where precise measurement is often challenging.

Lastly, the structure of the paper is as follows: Introduction and literature review are discussed in Section 1. Basic preliminaries are discussed in Section 2. The Section 3 explores a proposed model such as dual-possibility Pythagorean fuzzy hypersoft sets and aggregation operators, including arithmetic and geometric, and their basic operations. In Section 4, a systematic process for handling scenarios involving the solution of problems based on dual-possibility Pythagorean fuzzy hypersoft sets is provided. As an application, in Section 5 take into account a numerical method example for football players ranking based on the proposed model. Draw the graphs for both geometric and arithmetic data. In Section 6 of our paper, compare the new model with the existing model to demonstrate its superiority. Finally, a conclusion and future work are presented in Section 7.

2. Some basic concepts

This section provides a brief overview of the DP-PFHSS ideas and historical context. The basics are clarified through the defined definitions and dual possibility Pythagorean fuzzy hyper soft set are defined in this section.

Definition 1 *(Fuzzy set [41]).* A fuzzy set $\mathcal{F}$ in a universe of discourse X is defined as a set of ordered pairs $(x, \mu_{\mathcal{F}}(x))$, where $x \in X$ and $\mu_{\mathcal{F}} : X \to [0,1]$ is the membership function that assigns to each element x a degree of membership $\mu_{\mathcal{F}}(x)$ in the interval $[0,1]$. Formally, this can be expressed as:

$$\mathcal{F} = \left\{(x, \mu_{\mathcal{F}}(x)) \mid x \in X \text{ and } \mu_{\mathcal{F}}(x) \in [0,1]\right\}.$$

Properties of Fuzzy Sets: For any two fuzzy sets $\mathcal{F}$ and $\mathcal{G}$ and for all elements $x \in X$, the following properties hold:
(i) The union of $\mathcal{F}$ and $\mathcal{G}$ is given by $\mathcal{F} \cup \mathcal{G} = \{x, \max(\mu_{\mathcal{F}}(x), \mu_{\mathcal{G}}(x))\}$.
(ii) The intersection of $\mathcal{F}$ and $\mathcal{G}$ is expressed as $\mathcal{F} \cap \mathcal{G} = \{x, \min(\mu_{\mathcal{F}}(x), \mu_{\mathcal{G}}(x))\}$.
(iii) The complement of $\mathcal{F}$ is defined as $\mathcal{F}^c = \{(x, 1 - \mu_{\mathcal{F}}(x)) | x \in X\}$.

Fuzzy sets focus on membership degrees when dealing with uncertain situations. However, in various scenarios, it is essential to consider non-membership degrees to effectively apply fuzzy sets. To address this, Atanassov introduced intuitionistic fuzzy sets as an extension of fuzzy sets. Intuitionistic fuzzy sets provide a proper representation for both membership and non-membership degrees, offering an alternative perspective in situations where uncertainties exist [42].

Definition 2 *(Intuitionistic fuzzy set [43]).* An intuitionistic fuzzy set Λ in a universe of discourse X is defined as:

$$\Lambda = \left\{(\chi, \langle \alpha_\Lambda(\chi), \beta_\Lambda(\chi)\rangle) \mid \chi \in X\right\},$$

where, $\alpha_\Lambda : X \to [0,1]$ and $\beta_\Lambda : X \to [0,1]$ denote the degree of membership and degree of non-membership of χ in Λ, respectively. The conditions $0 \le \alpha_\Lambda(\chi) + \beta_\Lambda(\chi) \le 1$ ensure that the degrees of membership and non-membership are bounded and valid. The degree of hesitancy or uncertainty, denoted by $H_\Lambda(\chi)$, is defined as:

$$H_\Lambda(\chi) = 1 - \alpha_\Lambda(\chi) - \beta_\Lambda(\chi),$$

which captures the level of hesitation or uncertainty regarding the membership status of χ in the intuitionistic fuzzy set.

Definition 3 *(Pythagorean fuzzy set [44]).* A Pythagorean fuzzy set Λ in a universe of discourse X is defined as:

$$\Lambda = \left\{(\chi, \langle \alpha_\Lambda(\chi), \beta_\Lambda(\chi)\rangle) \mid \chi \in X\right\},$$

where, $\alpha_\Lambda : X \to [0,1]$ and $\beta_\Lambda : X \to [0,1]$ denote the degree of membership and degree of non-membership of χ in Λ, respectively. The condition

$$0 \leq (\alpha_\Lambda(\chi))^2 + (\beta_\Lambda(\chi))^2 \leq 1$$

ensures that the degrees of membership and non-membership are bounded and valid. The degree of hesitancy or uncertainty, denoted by $H_\Lambda(\chi)$, is defined as:

$$H_\Lambda(\chi) = 1 - \alpha_\Lambda(\chi) - \beta_\Lambda(\chi),$$

which captures the level of hesitation or uncertainty regarding the membership status of χ in the Pythagorean fuzzy set.

Soft sets, introduced by Molodtsov in 1999, provide a general mathematical framework for dealing with uncertainties that are inherent in various real-world problems. Unlike traditional sets, fuzzy sets, or rough sets, soft sets are more flexible and can handle indeterminate and vague information effectively.

Soft sets are used in decision-making processes to handle uncertainty and imprecise information effectively. They are applied in various fields such as data mining, pattern recognition and optimization, where they provide a flexible framework for modeling and analyzing complex systems. Additionally, soft sets are integrated with fuzzy and rough sets to enhance the analysis of ambiguous data in artificial intelligence and machine learning.

Definition 4 *(Soft set [45]).* A pair $(\mathfrak{S}, \Lambda)$ is referred to as a soft set Λ over a universal set $\mathcal{U}$. Here, $\mathfrak{S} : \Sigma \to \mathcal{P}(\mathcal{U})$ maps elements from a domain Σ to subsets of the universal set and Λ is a subset of the attribute set Σ.

Here is an example of soft set.

Let $X = \{a_1, a_2, a_3, a_4, a_5, a_6\}$, $U = \{u_1, u_2, \dots, u_9\}$ and $A = \{x_1, x_2, x_3, x_4\}$. The approximate elements of the soft set $\Lambda = (\Sigma, A)$ are defined as follows:

$\Sigma(u_1) = \{a_1, a_3, a_6\}$
$\Sigma(u_2) = \{a_2, a_3, a_5\}$
$\Sigma(u_3) = \{a_4, a_5, a_6\}$
$\Sigma(u_4) = \{a_1, a_2, a_5\}$

The soft set Λ can be expressed as

$$\Lambda = \{(u_1, \{a_1, a_3, a_6\}), (u_2, \{a_2, a_3, a_5\}), (u_3, \{a_4, a_5, a_6\}), (u_4, \{a_1, a_2, a_5\})\}.$$

Definition 5 *(Fuzzy soft set [46]).* A combination (Λ_Γ, Δ) is denoted as a fuzzy soft set over E, where $\Lambda_\Gamma : \Delta \to \mathcal{A}(\Gamma)$ and $\mathcal{A}(\Gamma)$ represents the collection of all fuzzy subsets over the universal set E. Here, Δ is a subset of the domain E.

Let's consider a concrete example.

Suppose we have a universal set $E = \{1, 2, 3, 4, 5\}$, a fuzzy set $\Gamma = \{(1, 0.8), (2, 0.6), (3, 0.4), (4, 0.7), (5, 0.2)\}$ and a subset $\Delta = \{2, 4, 5\}$.

A fuzzy soft set (Λ_Γ, Δ) is defined as follows:

$\Lambda_\Gamma(2) = \{(1, 0.2), (2, 0.6), (3, 0.4), (4, 0.7), (5, 0.2)\}$
$\Lambda_\Gamma(4) = \{(1, 0.3), (2, 0.6), (3, 0.4), (4, 0.7), (5, 0.2)\}$
$\Lambda_\Gamma(5) = \{(1, 0.1), (2, 0.6), (3, 0.4), (4, 0.7), (5, 0.2)\}$

So, the fuzzy soft set is expressed as $(\Lambda_\Gamma, \Delta) = \{(2, \{(1, 0.2), (2, 0.6), (3, 0.4), (4, 0.7), (5, 0.2)\}), (4, \{(1, 0.3), (2, 0.6), (3, 0.4), (4, 0.7), (5, 0.2)\}), (5, \{(1, 0.1), (2, 0.6), (3, 0.4), (4, 0.7), (5, 0.2)\})\}$.

Definition 6 *(Hypersoft set [47]).* Let $\Omega = \{x_1, x_2, ..., x_n\}$ be an initial universe and $\Sigma = \{\theta_1, \theta_2, ..., \theta_n\}$ a collection of criteria. Each element's corresponding attribute-valued non-overlapping sets of Σ are given by:

$A_1 = \{v_{11}, v_{12}, ..., v_{1n}\}$
$A_2 = \{v_{21}, v_{22}, ..., v_{2n}\}$
$A_3 = \{v_{31}, v_{32}, ..., v_{3n}\}$
$\vdots$
$A_n = \{v_{n1}, v_{n2}, ..., v_{nn}\}$

Let $A = A_1 \times A_2 \times A_3 \times \dots \times A_n = \{v_1, v_2, v_3, \dots, v_r\}$, here each $v_i (i = 1, 2, \dots, r)$ is an n-tuple element of A. Also, let $|A_i|$ denote the cardinality of set A_i, then $|A| = \prod_{i=1}^{n} |A_i|$.

A MAAF (Multiple Attribute Aggregation Function) is a mapping:

$$\phi_\Lambda : \Sigma \to \mathcal{P}(\Omega),$$

and is defined as:

$$\phi_\Lambda(\{v_1, v_2, ..., v_k\}) = \mathcal{P}(\{x_1, x_2, ..., x_n\}),$$

where $\mathcal{P}(\Omega)$ denotes the power set of Ω and $\Lambda \subset A$ with $k \leq r$. The pair (ϕ_Λ, Λ) is called hyper soft set.

Definition 7 *(Pythagorean fuzzy soft set [48]).* A pair $(\Gamma_{\Upsilon\Lambda}, \Delta)$ is termed a PFSS if a mapping

$$\Gamma_{\Upsilon\Lambda} : \Delta \to \mathcal{P}(\mathcal{P}\Upsilon\Lambda),$$

where $\mathcal{P}(\mathcal{P}\Upsilon\Lambda)$ denotes the power sets of pythagorean fuzzy set and is defined as

$$\Gamma_{\Upsilon\Lambda} = (A_\omega(\chi), B_\omega(\chi)),$$

referred to as Pythagorean fuzzy soft number (PFSN), with the condition

$$0 \le (A_\omega(\chi))^2 + (B_\omega(\chi))^2 \le 1,$$

for $\Gamma_{\Upsilon\Lambda} \in [0,1]$.

Definition 8 *(Pythagorean Fuzzy Hypersoft Set [49]).* Let Ω be a universe of discourse and $\mathcal{P}(\Omega)$ be the power set of Ω. Consider $X = \{x_1, x_2, x_3, ..., x_n\}$, where $n \ge 1$, as a set of attributes and let $R = \{r_1, r_2, r_3, ..., r_n\}$, where each r_i represents the collection of related sub-characteristics of r_i, such that $r_i \cap r_j = \emptyset$ for all $i, j \in 1, 2, 3, ..., n$ and $i \ne j$. Assume $\{r_1 \times r_2 \times r_3 \times ... \times r_n\} = \varrho = \{c'_1\alpha \times c'_2\beta \times ... \times c'_n\rho\}$, where $1 \le \alpha \le \beta$, $1 \le \gamma \le \delta$ and $1 \le \epsilon \le \rho$, with $\alpha, \beta, \gamma \in \mathbb{N}$. Let PFS^M be a collection of all Pythagorean fuzzy subsets over M.

Then, the pair $(\Theta_{P\Xi}, r_1 \times r_2 \times r_3 ... r_n = \varrho)$ is termed a PFHSS over M and the terminology of its mapping is:

$$\Theta_{P\Xi} : r_1 \times r_2 \times r_3 \times \ldots \times r_n = \varrho \to PFS^\Omega.$$

A Pythagorean fuzzy hyper soft number (PFHSN) within this context can be expressed as $\Theta_{P\Xi} = \{(A_u(\delta), B_u(\sigma))\}$, with the condition

$$0 \le \left(A_u(\delta)\right)^2 + \left(B_u(\sigma)\right)^2 \le 1.$$

3. Dual possibility-Pythagorean fuzzy hypersoft set (DP-PFHSS) aggregation operator

Within this section, we examine and proofs of arithmetic and geometric operators according to definition dual-possibility Pythagorean fuzzy hyper-soft set.

Definition 9 *(Dual-possibility Pythagorean fuzzy hypersoft set).*

The pair (Ξ_Λ, B) is identified as a dual-possibility Pythagorean fuzzy hypersoft set (DP-PFHS) over a hypersoft universe (Σ, B), if:

$$\Xi_\Lambda : B \to (I \times I)_\Sigma \times I_\Sigma$$

defined by

$$\Xi(\alpha) = (\Lambda(\alpha)(\upsilon), \Gamma(\alpha)(\upsilon))$$

with

$$\Xi(\alpha)(\upsilon) =< \chi_1(\upsilon), \chi_2(\upsilon) > \quad \forall \upsilon \in \Sigma$$

such that $0 \le \theta_1^2 + \theta_2^2 \le 1$, where

(i) $B = B_1 \times B_2 \times .... \times B_n$, B_i are disjoint attribute-valued sets corresponding to distinct attributes $b_i, i = 1, 2, ..., n$ respectively.
(ii) $\Xi : B \to (I \times I)_\Sigma$, $\Lambda : B \to I_\Sigma$, I_Σ and $(I \times I)_\Sigma$ represent all fuzzy and Pythagorean fuzzy subset collections of Σ correspondingly.
(iii) $[\Lambda(\eta)(\upsilon), \Gamma(\eta)(\upsilon)]$ is the degree of membership and non-membership of $\upsilon \in \Sigma$ in $\Xi(\alpha)$,
(iv) $[\Lambda(\alpha)(\upsilon), \Lambda(\beta)(\upsilon)]$ is the degree of dual possibility of membership and non-membership of $\upsilon \in \Sigma$ in $\Xi(\alpha)$.

Thus, $\Xi_\Lambda(\alpha_i)$ can be expressed as

$$\Xi(\alpha_i) = \left\{ \begin{array}{l} \left[\frac{\omega_1}{[\Lambda(\eta_i)(\omega_1), \Gamma(\eta_i)(\omega_1)]}, \left(\Lambda(\alpha_i)(\omega_1), \Lambda(\beta_i)(\omega_1)\right)\right], \left[\frac{\omega_2}{[\Lambda(\eta_i)(\omega_2), \Gamma(\eta_i)(\omega_2)]}, \left(\Lambda(\alpha_i)(\omega_2), \Lambda(\beta_i)(\omega_2)\right)\right] \\ \left[\frac{\omega_3}{[\Lambda(\eta_i)(\omega_3), \Gamma(\eta_i)(\omega_3)]}, \left(\Lambda(\alpha_i)(\omega_3), \Lambda(\beta_i)(\omega_3)\right)\right], \ldots \left[\frac{\omega_n}{[\Lambda(\eta_i)(\omega_n), \Gamma(\eta_i)(\omega_n)]}, \left(\Lambda(\alpha_i)(\omega_n), \Lambda(\beta_i)(\omega_n)\right)\right] \end{array} \right\}$$

where $i = 1, 2, 3, ..., n$.

The modefied version of [50] is represented by the score and accuracy functions below.

Definition 10 *(Score and accuracy function).* Let $\Gamma = (\Upsilon_\Gamma, \Phi_\Gamma)$ be a DP-PFHSS.

- A score function $\mathcal{S}$ of Γ is defined as

$$\mathcal{S}(\Gamma) = \Upsilon_\Gamma^2 - \Phi_\Gamma^2.$$

- An accuracy function $\mathcal{A}$ of Γ is defined as follows:

$$\mathcal{A}(\Gamma) = \Upsilon_\Gamma^2 + \Phi_\Gamma^2.$$

Definition 11 *(Averaging and geometric operator [50]).*
Suppose $\zeta_1, \ldots, \zeta_n$ exist a collection of DP-PFHSNs as:
The DP-PFHSAO is called

$$DP-PFHSA(\zeta_1,\ldots,\zeta_n) = \bigoplus_{i=1}^{n}\left(p_i'.p_i''(\zeta_i)\right) = \left[\left(1-\prod_{i=1}^{n}\left(1-(\aleph_{\zeta_i})^2\right)^{p_i'p_i''}\right)^{\frac{1}{2}}, \prod_{i=1}^{n}(\beth_{\zeta_i})^{p_i'p_i''}\right]. \tag{1}$$

The DP-PFHSGO is called

$$DP-PFHSG(\zeta_1,\ldots,\zeta_n) = \bigoplus_{i=1}^{n}(\zeta)^{p_i'.p_i''} = \left[\prod_{i=1}^{n}(\aleph_{\zeta_i})^{p_i'p_i''}, \left(1-\prod_{i=1}^{n}\left(1-(\beth_{\zeta_i})^2\right)^{p_i'p_i''}\right)^{\frac{1}{2}}\right]. \tag{2}$$

3.1. Dual possibility Pythagorean fuzzy hypersoft set averaging arithmetic aggregation operator

Definition 12. Consider a set of $DP-PFSNs$, denoted by $\Xi_j = \mathrm{W}\left((\alpha_j,\beta_j),(p_j',p_j'')\right)$ for $j=\{1,2,\ldots,k'\}$. Additionally, let $\vec{p'} = (\vec{p_1},\vec{p_2},\ldots,\vec{p_{k'}})^T$ and $\vec{p''} = (\vec{p_1},\vec{p_2},\ldots,\vec{p_{k'}})^T$ be dual possibility vectors corresponding to Ξj, with $\vec{pj} \geq 0$ for $j=1,\ldots,k'$. Furthermore, $\vec{p'}j, \in [0,1]$ and $\sum j = 1^{k'}\vec{p_j'} = 1$ and $\vec{p_j''} \in [0,1]$ with $\sum_{j=1}^{k'}\vec{p_j''} = 1$. The $DP-PFHS$ dual possibility averaging $(DP-PFA)$ operator, denoted as $DP-PFA : P^{k'} \to P$ (where P represents the collection of all $DP-PFSNs$), is defined based on these conditions.

Such that

$$DP-PFA\ \ Z(\Xi_1,\Xi_2,\ldots,\Xi_n) = \bigoplus_{j=1}^{k'}(p_j'p_j'')\Xi_j = p_1'p_1''\Xi_1 \oplus p_2'p_2''\Xi_2 \oplus \ldots \oplus p_k'p_k''\Xi_k.$$

The combined outcome of $DP-PFSNs$ through operational principles is presented.

Theorem 1. *Consider a set $\Xi_j = \mathrm{W}\left((\alpha_j,\beta_j),(p_j',p_j'')\right)$, where j ranges from 1 to k', representing a collection of $DP-PFSNs$. Additionally, examine the dual possibility vectors $\vec{p'} = (\vec{p_1},\vec{p_2},\ldots,\vec{p_{k'}})^T$ and $\vec{p''} = (\vec{p_1},\vec{p_2},\ldots,\vec{p_{k'}})^T$ associated with Ξj. Ensure that $\vec{pj} \geq 0$ for $j=1,\ldots,k'$, where $\vec{p_j'}, \in [0,1]$ and $\sum j = 1^{k'}\vec{p_j'} = 1$. Similarly, $\vec{p_j''} \in [0,1]$ with $\sum_{j=1}^{k'}\vec{p_j''} = 1$. This establishes the conditions for the dual possibility vectors and the corresponding $DP-PFSNs$ collection. Then*

$$DP-PFHSSG\ \ Z(\Xi_1,\Xi_2,\ldots,\Xi_k') = \bigcup_{\alpha_j\in\alpha_j,\beta_i\in\beta_j}\left\{\left(1-\prod_{i=1}^{n}\left(1-(\alpha_{\kappa_i})^2\right)^{p_i'p_i''}\right)^{\frac{1}{2}}, \prod_{i=1}^{n}(\beta_{\kappa_i})^{p_i'p_i''}\right\}. \tag{3}$$

Proof. As we know

$$\vec{p}_1'\vec{p}_1''\Xi_1 = \bigcup_{\alpha_1\in m_1,\beta_1\in n_1}\left\{\left\{\sqrt[2]{1-(1-\alpha_1^2)^{\vec{p}_1'\vec{p}_1''}}\right\},\left\{\beta_1^{\vec{p}_1'\vec{p}_1''}\right\}\right\},$$

and

$$\vec{p}_2'\vec{p}_2''\Xi_2 = \bigcup_{\alpha_2\in m_2,\beta_2\in n_2}\left\{\left\{\sqrt[2]{1-(1-\alpha_2^2)^{\vec{p}_2'\vec{p}_2''}}\right\},\left\{\beta_2^{\vec{p}_2'\vec{p}_2''}\right\}\right\}.$$

Firstly, we demonstrate the validity of Eq. (3) for $n=2$, we have

$$\vec{p}_1'\vec{p}_1''\Xi_1 \oplus \vec{p}_2'\vec{p}_2''\Xi_2 = \bigcup_{\alpha_1\in m_1,\beta_1\in n_1,\alpha_2\in m_2,\beta_2\in n_2}\left\{\left\{\sqrt[2]{\begin{matrix}1-(1-\alpha_1^2)^{\vec{p}_1'\vec{p}_1''}+1-(1-\alpha_2^2)^{\vec{p}_2'\vec{p}_2''}\\ -(1-(1-\alpha_1^2)^{\vec{p}_1'\vec{p}_1''})(1-(1-\alpha_2^2)^{\vec{p}_2'\vec{p}_2''})\end{matrix}}\right\},\left\{\beta_1^{\vec{p}_1'\vec{p}_1''}\beta_2^{\vec{p}_2'\vec{p}_2''}\right\}\right\}.$$

$$= \bigcup_{\alpha_1\in m_1,\beta_1\in n_1,\alpha_2\in m_2,\beta_2\in n_2}\left\{\left\{\sqrt[2]{1-(1-\alpha_1^2)^{\vec{p}_1'\vec{p}_1''}(1-\alpha_2^2)^{\vec{p}_2'\vec{p}_2''}}\right\},\left\{\beta_1^{\vec{p}_1'\vec{p}_1''}\beta_2^{\vec{p}_2'\vec{p}_2''}\right\}\right\}.$$

$$= \bigcup_{\alpha_j \in \alpha_j, \beta_j \in \beta_j} \left\{ \left\{ \sqrt[2]{1 - \Psi_{j=1}^{2} (1-\alpha_j^2)^{\vec{p}'_j \vec{p}''_j}} \right\}, \left\{ \Psi_{j=1}^{2} \beta_j^{\vec{p}'_j \vec{p}''_j} \right\} \right\}.$$

Assuming for now that Eq. (3) is valid for $n = k'$,

$$DP-PFHSSG\ A(\Xi_1, \Xi_2, ..., \Xi_{k'}) = \bigcup_{\alpha_j \in \alpha_j, \beta_j \in \beta_j} \left\{ \left\{ \sqrt[2]{1 - \Psi_{j=1} (1-\alpha_j^2)^{\vec{p}'_j \vec{p}''_j}} \right\}, \left\{ \Psi_{j=1} \beta_j^{\vec{p}'_j \vec{p}''_j} \right\} \right\}.$$

After increasing n by one unit, we have

$$DP-PFHSSG\ A(\Xi_1, \Xi_2, ..., \Xi'_k, \Xi_{k'+1}) = \vec{p}'_1 \vec{p}''_1 \Xi_1 \oplus \vec{p}'_2 \vec{p}''_2 \Xi_2 \oplus ... \oplus \vec{p}'_k \Xi'_k \oplus \vec{p}_{k'+1} \Xi_{k'+1} = (\vec{p}'_1 \vec{p}''_1 \Xi_1 \oplus \vec{p}'_2 \vec{p}''_2 \Xi_2 \oplus ... \oplus \vec{p}'_k \Xi'_k) \oplus \vec{p}_{k'+1} \Xi_{k'+1}$$

$$= \bigcup_{\alpha_j \in \alpha_j, \beta_j \in \beta_j} \left\{ \left\{ \sqrt[2]{1 - \Psi_{j=1} (1-\alpha_j^2)^{\vec{p}'_j \vec{p}''_j}} \right\}, \left\{ \Psi_{j=1} \beta_j^{\vec{p}'_j \vec{p}''_j} \right\} \right\} \oplus \bigcup_{\alpha_{k'+1} \in m_{k'+1}, \beta_{k+1} \in n_{k'+1}} \left\{ \left\{ \sqrt[2]{1 - (1-\alpha_{k'+1}^2)^{\vec{p}_{k'+1}}} \right\}, \left\{ \beta_{k'+1}^{\vec{p}_{k'+1}} \right\} \right\}.$$

$$= \bigcup_{\alpha_j \in \alpha_j, \beta_j \in \beta_j} \left\{ \left\{ \sqrt[2]{\begin{array}{l} 1 - \Psi_{j=1} (1-\alpha_j^2)^{\vec{p}'_j \vec{p}''_j} + (1 - (1-\alpha_{k'+1}^2)^{\vec{p}_{k'+1}}) \\ -(1 - \Psi_{j=1} (1-\alpha_j^2)^{\vec{p}'_j \vec{p}''_j})(1 - (1-\alpha_{k'+1}^2)^{\vec{p}_{k'+1}}) \end{array}} \right\}, \left\{ \Psi_{j=1} \beta_j^{\vec{p}'_j \vec{p}''_j} \lambda_{k'+1}^{\vec{p}_{k'+1}} \right\} \right\}.$$

$$= \bigcup_{\alpha_j \in \alpha_j, \beta_j \in \beta_j} \left\{ \left\{ \sqrt[2]{1 - \Psi_{j=1} (1-\alpha_j^2)^{\vec{p}'_j \vec{p}''_j} (1-\alpha_{k'+1}^2)^{\vec{p}_{k'+1}}} \right\}, \left\{ \Psi_{j=1} \beta_j^{\vec{p}'_j \vec{p}''_j} \lambda_{k'+1}^{\vec{p}_{k'+1}} \right\} \right\}.$$

$$= \bigcup_{\alpha_j \in \alpha_j, \beta_j \in \beta_j} \left\{ \left\{ \sqrt[2]{1 - \Psi_{j=1}^{k'+1} (1-\alpha_j^2)^{\vec{p}'_j \vec{p}''_j}} \right\}, \left\{ \Psi_{j=1}^{k'+1} \beta_j^{\vec{p}'_j \vec{p}''_j} \right\} \right\}.$$

Thus, for $n = k' + 1$, Eq. (3) holds true. Thus, the proof is complete for any value of n since Eq. (3) holds true. ■

The following introduces the intriguing properties of the suggested aggregation operators, which include idempotency, monotonicity, boundedness and symmetry:

Theorem 2 *(Idempotency).*

Consider a set $\Xi_j = \mathrm{W}\left((\alpha_j, \beta_j), (p'_j, p''_j)\right)$*, where* j *ranges from 1 to* k'*, representing a collection of* $DP-PFSNs$*. Additionally, examine the dual possibility vectors* $\vec{p'} = (\vec{p}_1, \vec{p}_2, ..., \vec{p}_{k'})^T$ *and* $\vec{p''} = (\vec{p}_1, \vec{p}_2, ..., \vec{p}_{k'})^T$ *associated with* Ξ_j*. Ensure that* $\vec{p}_j \geq 0$ *for* $j = 1, ..., k'$*, where* $\vec{p'_j}, \in [0,1]$ *and* $\sum j = 1^{k'} \vec{p'} j = 1$*. Similarly,* $\vec{p''_j} \in [0,1]$ *with* $\sum_{j=1}^{k'} \vec{p''_j} = 1$*. Then:*

$$DP-PFHS\ A(\Xi_1, \Xi_2, ..., \Xi'_k) = \Xi$$

Proof. From Theorem 1, we have

$$DP-qROFHS\ A(\Xi_1, \Xi_2, ..., \Xi'_k) = \bigcup_{\alpha_j \in \alpha_j, \beta_j \in \beta_j} \left\{ \left\{ \sqrt[2]{1 - \Psi_{j=1} (1-\alpha_j^2)^{\vec{p}'_j \vec{p}''_j}} \right\}, \left\{ \Psi_{j=1} \beta_j^{\vec{p}'_j \vec{p}''_j} \right\} \right\}$$

$$= \bigcup_{\alpha \in m, \beta \in n} \left\{ \left\{ \sqrt[2]{1 - \Psi_{j=1} (1-\alpha^2)^{\vec{p}'_j \vec{p}''_j}} \right\}, \left\{ \Psi_{j=1} \beta^{\vec{p}'_j \vec{p}''_j} \right\} \right\}$$

$$= \bigcup_{\alpha \in m, \beta \in n} \left\{ \left\{ \sqrt[2]{1 - (1-\alpha^2)^{\sum_{j=1}^{k'} \vec{p}'_j \vec{p}''_j}} \right\}, \left\{ \beta^{\sum_{j=1} \vec{p}'_j \vec{p}''_j} \right\} \right\}$$

$$= \bigcup_{\alpha \in m, \beta \in n} \left\{ \left\{ \sqrt[2]{1 - (1-\alpha^2)} \right\}, \{\beta\} \right\}$$

$$= \bigcup_{\alpha \in m, \beta \in n} \{ \{\alpha\}, \{\beta\} \}.$$

This implies that the $DP-PFHS$ aggregation operator $A(\Xi_1, \Xi_2, ..., \Xi'_k)$ results in Ξ, thereby providing the necessary proof for the theorem. Therefore, the conclusion is established. ■

Theorem 3 *(Boundedness).*

Consider a set Ξ_j comprising DP-PFSNs with parameters $\left((\alpha_j, \beta_j), (p'_j, p''_j)\right)$, where j ranges from 1 to k'. Additionally, let $\vec{p'} = (\vec{p}_1, \vec{p}_2, ..., \vec{p}_{k'})^T$ and $\vec{p''} = (\vec{p}_1, \vec{p}_2, ..., \vec{p}_{k'})^T$ be dual possibility vectors associated with Ξ_j. These vectors satisfy the conditions $\vec{p}_j \geq 0$ for $j = 1, ..., k'$, where $\vec{p'_j} \in [0,1]$ with $\sum j = 1^{k'} \vec{p'_j} = 1$ and $\vec{p''_j} \in [0,1]$ with $\sum_{j=1}^{k'} \vec{p''_j} = 1$,

$$\Xi^- \leq DP - PFHS \ A(\Xi_1, \Xi_2, ..., \Xi_n) = \Xi \leq \Xi^+,$$

where

$$\Xi^- = \Xi(m^-, n^+),\ \Xi^+ = \Xi(m^+, n^-),$$

such that

$$m^- = \bigcup_{\alpha_j \in \alpha_j} \min\{\alpha_j\}, \quad n^+ = \bigcup_{\beta_j \in \beta_j} \max\{\beta_j\},$$

$$m^+ = \bigcup_{\alpha_j \in \alpha_j} \max\{\alpha_j\}, \quad n^- = \bigcup_{\beta_j \in \beta_j} \min\{\beta_j\}.$$

Proof.

$$\bigcup_{\alpha_j \in \alpha_j} \min\{\alpha_j\} \leqslant \bigcup_{\alpha_i \in \alpha_j} \{\alpha_j\} \leqslant \bigcup_{\alpha_j \in \alpha_j} \max\{\alpha_j\},$$

and

$$\bigcup_{\beta_i \in \beta_j} \min\{\beta_j\} \leqslant \bigcup_{\beta_j \in \beta_j} \{\beta_j\} \leqslant \bigcup_{\beta_j \in \beta_j} \max\{\beta_j\}.$$

Now we have

$$\begin{aligned}
&\bigcup_{\alpha_j \in \alpha_j} \min\{\alpha_j\} \leqslant \bigcup_{\alpha_j \in \alpha_j} \{\alpha_j\} \leqslant \bigcup_{\alpha_j \in \alpha_j} \max\{\alpha_j\} \\
&\Leftrightarrow \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{\min\{(\alpha_j)^2\}} \leqslant \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{\{(\alpha_j)^2\}} \leqslant \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{\max\{(\alpha_j)^2\}} \\
&\Leftrightarrow \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{1 - \max\{(\alpha_j)^2\}} \leqslant \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{\{1 - (\alpha_j)^2\}} \leqslant \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{1 - \min\{(\alpha_j)^2\}} \\
&\Leftrightarrow \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{(1 - \max\{(\alpha_j)^2\})^{\vec{p}'_j \vec{p}''_j}} \leqslant \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{(1 - \{(\alpha_j)^2\})^{\vec{p}'_j \vec{p}''_j}} \leqslant \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{(1 - \min\{(\alpha_j)^2\})^{\vec{p}'_j \vec{p}''_j}} \\
&\Leftrightarrow \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{\underset{j=1}{\Psi} (1 - \max\{(\alpha_j)^2\})^{\vec{p}'_j \vec{p}''_j}} \leqslant \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{\underset{j=1}{\Psi} (1 - \{(\alpha_j)^2\})^{\vec{p}'_j \vec{p}''_j}} \leqslant \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{\underset{j=1}{\Psi} (1 - \min\{(\Im_j)^2\})^{\vec{p}'_j \vec{p}''_j}} \\
&\Leftrightarrow \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{(1 - \max\{(\alpha_j)^2\})^{\sum_{j=1} \vec{p}'_j \vec{p}''_j}} \leqslant \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{\underset{j=1}{\Psi} (1 - \{(\alpha_j)^2\})^{\vec{p}'_j \vec{p}''_j}} \leqslant \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{(1 - \min\{(\alpha_j)^2\})^{\sum_{j=1} \vec{p}'_j \vec{p}''_j}} \\
&\Leftrightarrow \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{1 - \max\{(\alpha_j)^2\}} \leqslant \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{\underset{j=1}{\Psi} (1 - \{(\alpha_j)^2\})^{\vec{p}'_j \vec{p}''_j}} \leqslant \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{1 - \min\{(\alpha_j)^2\}} \\
&\Leftrightarrow \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{-1 + \min\{(\alpha_j)^2\}} \leqslant \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{- \underset{j=1}{\Psi} (1 - \{(\alpha_j)^2\})^{\vec{p}'_j \vec{p}''_j}} \leqslant \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{-1 + \max\{(\alpha_j)^2\}} \\
&\Leftrightarrow \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{1 - 1 + \min\{(\alpha_j)^2\}} \leqslant \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{1 - \underset{j=1}{\Psi} (1 - \{(\alpha_j)^2\})^{\vec{p}'_j \vec{p}''_j}} \leqslant \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{1 - 1 + \max\{(\alpha_j)^2\}} \\
&\Leftrightarrow \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{\min\{(\alpha_j)^2\}} \leqslant \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{1 - \underset{j=1}{\alpha} (1 - \{(\alpha_j)^2\})^{\vec{p}'_j \vec{p}''_j}} \leqslant \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{\max\{(\alpha_j)^2\}} \\
&\Leftrightarrow \bigcup_{\alpha_j \in \alpha_j} \min\{(\alpha_j)\} \bigcup_{\alpha_j \in \alpha_j} \sqrt[2]{1 - \underset{j=1}{\Psi} (1 - \{(\alpha_j)^2\})^{\vec{p}'_j \vec{p}''_j}} \bigcup_{\alpha_j \in \alpha_j} \max\{(\alpha_j)\}
\end{aligned}$$

Now, we have

$$\Leftrightarrow \bigcup_{\beta_j\in\beta_j} \min\{\beta_j\} \leqslant \bigcup_{\beta_j\in\beta_j} \{\beta_j\} \leqslant \bigcup_{\beta_j\in\beta_j} \max\{\beta_j\}$$

$$\Leftrightarrow \bigcup_{\beta_j\in\beta_j} \min\{(\beta_j)^{\vec{p}'_j\vec{p}''_j}\} \leqslant \bigcup_{\beta_j\in\beta_j} \{(\beta_j)^{\vec{p}'_j\vec{p}''_j}\} \leqslant \bigcup_{\beta_j\in\beta_j} \max\{(\beta_j)^{\vec{p}'_j\vec{p}''_j}\}$$

$$\Leftrightarrow \bigcup_{\beta_j\in\beta_j} \Psi_{j=1} \min\{(\beta_j)^{\vec{p}'_j\vec{p}''_j}\} \leqslant \bigcup_{\beta_j\in\beta_j} \Psi_{j=1} \{(\beta_j)^{\vec{p}'_j\vec{p}''_j}\} \leqslant \bigcup_{\beta_j\in\beta_j} \Psi_{j=1} \max\{(\beta_j)^{\vec{p}'_j\vec{p}''_j}\}$$

$$\Leftrightarrow \bigcup_{\beta_j\in\beta_j} \min\{(\beta_j)^{\sum_{j=1}\vec{p}'_j\vec{p}''_j}\} \leqslant \bigcup_{\beta_j\in\beta_j} \Psi_{j=1} \{(\beta_j)^{\vec{p}'_j\vec{p}''_j}\} \leqslant \bigcup_{\beta_j\in\beta_j} \max\{(\beta_j)^{\sum_{j=1}\vec{p}'_j\vec{p}''_j}\}$$

$$\Leftrightarrow \bigcup_{\beta_j\in n_i} \min\{\beta_j\} \leqslant \bigcup_{\beta_j\in\beta_j} \Psi_{j=1} \{(\beta_j)^{\vec{p}'_j\vec{p}''_j}\} \leqslant \bigcup_{\beta_j\in\beta_j} \max\{\beta_j\}$$

$$\Leftrightarrow \bigcup_{\alpha_j\in\alpha_j} \min\{\alpha_j\} - \bigcup_{\beta_j\in\beta_j} \max\{\beta_j\} \leqslant \bigcup_{\alpha_j\in\alpha_j} \sqrt[2]{1 - \Psi_{j=1} (1-\{(\alpha_j)^2\})^{\vec{p}'_j\vec{p}''_j}} - \bigcup_{\beta_j\in\beta_j} \Psi_{j=1} \{(\beta_j)^{\vec{p}'_j\vec{p}''_j}\} \leqslant$$

$$\bigcup_{\alpha_j\in\alpha_j} \max\{(\alpha_j)\} - \bigcup_{\beta_j\in\beta_j} \max\{\beta_j\}$$

$$\Leftrightarrow \hat{S}(\Xi^-) \leqslant \hat{S}(\Xi) \leqslant \hat{S}(\Xi^+).$$

Therefore, from above equations and definition, we have

$$\Xi^- \le DP - PFHSS\ \ A(\Xi_1, \Xi_2, ..., \Xi_n) = \Xi \le \Xi^+ \quad \blacksquare$$

Theorem 4 *(Monotonicity).*

Consider the set Ξ_j*, composed of* $DP - PFSNs$ *with parameters* $\Big((\alpha_j, \beta_j), (p'_j, p''_j)\Big)$*, where* j *ranges from 1 to* k'*. Additionally, contemplate the dual possibility vectors* $\vec{p'} = (\vec{p}_1, \vec{p}_2, ..., \vec{p}_{k'})^T$ *and* $\vec{p''} = (\vec{p}_1, \vec{p}_2, ..., \vec{p}_{k'})^T$ *associated with* Ξ_j*, ensuring that* $\vec{p}_j \ge 0$ *for* $j = \{1, ..., k'\}$*. Here,* $\vec{p'_j} \in [0,1]$*, satisfying* $\sum j = 1^{k'} \vec{p'_j} = 1$ *and* $\vec{p''_j} \in [0,1]$*, with* $\sum_{j=1}^{k'} \vec{p''_j} = 1$*. Then:*

$$DP - PFMA(\daleth_1, \daleth_2, ..., \daleth_n) \ge DP - PFMA(L_1, L_2, ..., L_n)$$

Proof. From definition, it is clear that, if $\Xi_j \ge \mathcal{N}_j$, then $m_j \ge \varkappa_j$ and $\varpi_j \ge n_j$.

Now if

$$\alpha_j \geqslant \varkappa_j \Rightarrow \bigcup_{\alpha_j\in\alpha_j} \{\alpha_j\} \geqslant \bigcup_{\breve{\vartheta}_j\in\varkappa_j} \{\breve{\vartheta}_j\} \Leftrightarrow \bigcup_{\alpha_j\in\alpha_j} \{(\alpha_j)^2\} \geqslant \bigcup_{\breve{\vartheta}_j\in\varkappa_j} \{(\breve{\vartheta}_j)^2\},$$

where $n \ge 1$

$$\Leftrightarrow \bigcup_{\alpha_j\in\alpha_j} \sqrt[2]{\{(\alpha_j)^2\}} \geqslant \bigcup_{\breve{\vartheta}_j\in\varkappa_j} \sqrt[2]{\{(\breve{\vartheta}_j)^2\}}$$

$$\Leftrightarrow \bigcup_{\breve{\vartheta}_j\in\varkappa_j} \sqrt[2]{1 - \{(\breve{\vartheta}_j)^2\}} \geqslant \bigcup_{\alpha_j\in\alpha_j} \sqrt[2]{1 - \{(\alpha_j)^2\}}$$

$$\Leftrightarrow \bigcup_{\breve{\vartheta}_j\in\varkappa_j} \sqrt[2]{(1 - \{(\breve{\vartheta}_j)^2\})^{\vec{p}'_j\vec{p}''_j}} \geqslant \bigcup_{\alpha_j\in\alpha_j} \sqrt[2]{(1 - \{(\alpha_j)^2\})^{\vec{p}'_j\vec{p}''_j}}$$

$$\Leftrightarrow \bigcup_{\breve{\vartheta}_j\in\varkappa_j} \sqrt[2]{\Psi_{j=1} (1 - \{(\breve{\vartheta}_j)^2\})^{\vec{p}'_j\vec{p}''_j}} \geqslant \bigcup_{\alpha_j\in\alpha_j} \sqrt[2]{\Psi_{j=1} (1 - \{(\alpha_j)^2\})^{\vec{p}'_j\vec{p}''_j}}$$

$$\Leftrightarrow \bigcup_{\alpha_j\in\alpha_j} \sqrt[2]{1 - \Psi_{j=1} (1 - \{(\alpha_j)^2\})^{\vec{p}'_j\vec{p}''_j}} \geqslant \bigcup_{\breve{\vartheta}_j\in\varkappa_j} \sqrt[2]{1 - \Psi_{j=1} (1 - \{(\breve{\vartheta}_j)^2\})^{\vec{p}'_j\vec{p}''_j}}. \tag{4}$$

Similarly,

$$\varpi_j \geqslant \beta_j, then \bigcup_{\breve{\eta}_j\in\varpi_j} \{\breve{\eta}_j\} \geqslant \bigcup_{\beta_i\in\beta_j} \{\beta_j\} \Leftrightarrow \bigcup_{\breve{\eta}_j\in\varpi_j} \{(\breve{\eta}_j)^{\vec{p}'_j\vec{p}''_j}\} \geqslant \bigcup_{\beta_j\in\beta_j} \{(\beta_j)^{\vec{p}'_j\vec{p}''_j}\}$$

$$\Leftrightarrow \bigcup_{\breve{\eta}_j\in\varpi_j} \{\Psi_{j=1} (\breve{\eta}_j)^{\vec{p}'_j\vec{p}''_j}\} \geqslant \bigcup_{\beta_j\in\beta_j} \{\Psi_{j=1} (\beta_j)^{\vec{p}'_j\vec{p}''_j}\} \tag{5}$$

Let $\Xi = DP - PFHSA(\Xi_1, \Xi_2, ..., \Xi_n)$ and $W = DP - PFMA(W_1, W_2, ..., W'_k)$. Then, from Eq. (4) and Eq. (5), we have $\check{S}(\Xi) \ge \check{S}(W)$

If $\check{W}(\Xi) > \check{S}(W)$, then $DP - PFMA(\Xi_1, \Xi_2, ..., \Xi'_k) > DP - PFMA(W_1, W_2, ..., W'_k)$. If $\check{S}(\Xi) = \check{S}(W)$, then $DP - PFMA(\Xi_1, \Xi_2, ..., \Xi'_k) = DP - PFMA(W_1, W_2, ..., W'_k)$,

$$\frac{1}{\#m}\sum_{\alpha\in m}\alpha^2-\frac{1}{\#n}\sum_{\beta\in n}\beta^2=\frac{1}{\#\varkappa}\sum_{\check{\vartheta}\in\varkappa}^{2}\check{\theta}^2-\frac{1}{\#\varpi}\sum_{\beta\in\varpi}^{2}\check{\eta}^2,$$

$$\frac{1}{\#m}\sum_{\alpha\in m}\alpha^2=\frac{1}{\#\varkappa}\sum_{\check{\vartheta}\in\varkappa}^{2}\check{\theta}^2 and \frac{1}{\#n}\sum_{\beta\in n}\beta^2=-\frac{1}{\#\varpi}\sum_{\beta\in\varpi}^{2}\check{\eta}^2.$$

Furthermore,

$$A(\Xi)=\frac{1}{\#m}\sum_{\alpha\in m}\alpha^2+\frac{1}{\#n}\sum_{\beta\in n}\beta^2=\frac{1}{\#\varkappa}\sum_{\check{\vartheta}\in\varkappa}^{2}\check{\theta}^2+\frac{1}{\#\varpi}\sum_{\beta\in\varpi}^{2}\check{\eta}^2 A(\Xi)=A(\mathrm{W}).$$

Therefore, $DP-PFMA(\Xi_1,\Xi_2,...,\Xi'_k)=DP-PFMA(\mathrm{W}_1,\mathrm{W}_2,...,\mathrm{W}'_k)$ ■

Theorem 5 *(Symmetry).*

Consider a set Ξ_j, denoted as $\mathrm{W}\Big((\alpha_j,\beta_j),(p'_j,p''_j)\Big)$, forming a collection of $DP-PFSNs$ with j ranging from 1 to k'. Additionally, examine the dual possibility vectors $\vec{p'}=(\vec{p}_1,\vec{p}_2,...,\vec{p}_{k'})^T$ and $\vec{p''}=(\vec{p}_1,\vec{p}_2,...,\vec{p}_{k'})^T$ associated with Ξ_j, ensuring that $\vec{p}_j\geq 0$ for $j=1,...,k'$. Furthermore, $\vec{p'_j}\in[0,1]$, satisfying $\sum j=1,\vec{p}'_j=1$ and $\vec{p''_j}\in[0,1]$, with $\sum_{j=1}\vec{p}''_j=1$. We have:

$$DP-PFHSSAW\Big((\alpha_j,\beta_j),(p_j,p_j)\Big)=DP-PFHSSAW\Big((m'_j,n'_j),(p'_j,p''_j)\Big). \tag{6}$$

Proof. The proof is Eq. (6) obvious. Hence, it is omitted. ■

3.2. Dual possibility Pythagorean fuzzy hypersoft set geometric aggregation operator

Definition 13. Let $\Xi_j=\mathrm{W}\Big((\alpha_j,\beta_j),(p'_j,p''_j)\Big),(j=1,2,3,...,k')$ represent a set of $DP-PFHNs$. Additionally, consider the dual possibility vectors $\vec{p'}=(\vec{p}_1,\vec{p}_2,...,\vec{p}_{k'})^T$ and $\vec{p''}=(\vec{p}_1,\vec{p}_2,...,\vec{p}_{k'})^T$ associated with Ξj, where $\vec{p}j\geq 0$ for $j=1,2,3,...,k'$. These vectors satisfy $\vec{p'_j},\in[0,1]$ with $\sum j=1\vec{p}'_j=1$ and $\vec{p''_j}\in[0,1]$ with $\sum_{j=1},\vec{p}''_j=1$. Subsequently, the function defining the dual possibility geometric ($DP-PFHG$) operator for $DP-PFHS$ is given by $DP-PFHG: P^{k'}\rightarrow P$, where P represents the collection of all $DP-PFHNs$.

$$DP-PFHSG\ \ Z(\Xi_1,\Xi_2,...,\Xi_n)=\bigoplus_{j=1}^{k'}\Xi_j^{\vec{p}'_j\vec{p}''_j}=\Xi_1^{\vec{p}'_1\vec{p}''_1}\oplus\Xi_2^{\vec{p}'_2\vec{p}''_2}\oplus...\oplus\Xi_{k'}^{\vec{p}'_k\vec{p}''_k}$$

Theorem 6. *Consider the set Ξ_j, defined as $\mathrm{W}\Big((\alpha_j,\beta_j),(p'_j,p''_j)\Big),(j=1,2,3,...,k')$, which comprises a collection of $DP-PFHNs$. Additionally, examine the dual possibility vectors $\vec{p'}=(\vec{p}_1,\vec{p}_2,...,\vec{p}_{k'})^T$ and $\vec{p''}=(\vec{p}_1,\vec{p}_2,...,\vec{p}_{k'})^T$ associated with Ξ_j, ensuring that $\vec{p}_j\geq 0$ for $j=\{1,2,3,...,k'\}$. Here, $\vec{p'_j},\in[0,1]$, satisfying $\sum j=1^{k'}\vec{p}'_j=1$ and $\vec{p''_j}\in[0,1]$, with $\sum_{j=1}\vec{p}''_j=1$. Then*

$$DP-PFHSG\ \ Z(\Xi_1,\Xi_2,...,\Xi'_k)=\bigcup_{\alpha_j\in\alpha_j,\beta_i\in\beta_j}\left\{\left\{\Psi_{j=1}\alpha_j^{\vec{p}'_j\vec{p}''_j}\right\}\right\},\left\{\sqrt[2]{1-\Psi_{j=1}^{k'}(1-\beta_j)^{\vec{p}'_j\vec{p}''_j}}\right\}.$$

Proof. Theorem 6 can be proved as Theorem 1. ■

Theorem 7 *(Idempotency).*

Let Ξ_j be a set defined as $\mathrm{W}\Big((\alpha_j,\beta_j),(p'_j,p''_j)\Big),(j=1,2,3,...,k')$, representing a collection of $DP-PFHNs$. Additionally, contemplate the dual possibility vectors $\vec{p'}=(\vec{p}_1,\vec{p}_2,...,\vec{p}_{k'})^T$ and $\vec{p''}=(\vec{p}_1,\vec{p}_2,...,\vec{p}_{k'})^T$ associated with Ξ_j, ensuring that $\vec{p}j\geq 0$ for $j=\{1,2,3,...,k'\}$. Here, $\vec{p'_j}\in[0,1]$, satisfying $\sum j=1,\vec{p}'_j=1$ and $\vec{p''_j}\in[0,1]$, with $\sum_{j=1}^{k'},\vec{p}''_j=1$. Then:

$$DP-PFHSSG\ \ A(\Xi_1,\Xi_2,...,\Xi'_k)=\Xi.$$

Proof. This theorem can be proved as Theorem 2. ■

Theorem 8 *(Boundedness).*

Let Ξ_j *represent a set defined as* $\mathrm{W}\Big((\alpha_j,\beta_j),(p'_j,p''_j)\Big),(j=1,2,3,...,k')$*, constituting a collection of* $DP-PFHNs$*. Additionally, examine the dual possibility vectors* $\vec{p'}=(\vec{p}_1,\vec{p}_2,...,\vec{p}_{k'})^T$ *and* $\vec{p''}=(\vec{p}_1,\vec{p}_2,...,\vec{p}_{k'})^T$ *associated with* Ξ_j*, ensuring that* $\vec{p}_j\geq 0$ *for* $j=\{1,2,3,...,k'\}$*. In this context,* $\vec{p'_j}\in[0,1]$*, satisfying* $\sum j=1\vec{p}'_j=1$ *and* $\vec{p''_j}\in[0,1]$*, with* $\sum_{j=1}^{k'}\vec{p}''_j=1$.

$$\Xi^-\leq DP-PFHSG(\Xi_1,\Xi_2,...,\Xi_n)=\Xi\leq\Xi^+,$$

where

$$\Xi^-=\Xi(m^-,n^+),\ \Xi^+=\Xi(m^+,n^-),$$

such that

$$m^-=\underset{\alpha_j\in\alpha_j}{\cup}\min\{\alpha_j\},\quad n^+=\underset{\beta_j\in\beta_j}{\cup}\max\{\beta_j\},$$

$$m^+=\underset{\alpha_j\in\alpha_j}{\cup}\max\{\alpha_j\},$$

$$n^-=\underset{\beta_j\in\beta_j}{\cup}\min\{\beta_j\}.$$

Proof. The proof of this theorem can be proved as Theorem 3. ■

Theorem 9 *(Monotonicity).*

Consider a set Ξ_j*, defined as* $\mathrm{W}\Big((\alpha_j,\beta_j),(p'_j,p''_j)\Big),(j=1,2,3,...,k')$*, constituting a collection of* $DP-PFHNs$*. Additionally, examine the dual possibility vectors* $\vec{p'}=(\vec{p}_1,\vec{p}_2,...,\vec{p}_{k'})^T$ *and* $\vec{p''}=(\vec{p}_1,\vec{p}_2,...,\vec{p}_{k'})^T$ *associated with* Ξ_j*, ensuring that* $\vec{p}_j\geq 0$ *for* $j=\{1,2,3,...,k'\}$*. In this context,* $\vec{p'_j},\in[0,1]$*, satisfying* $\sum j=1\vec{p}'_j=1$ *and* $\vec{p''_j},\in[0,1]$*, with* $\sum_{j=1}^{k'}\vec{p}''_j=1$*. Then:*

$$DP-PFMG(\daleth_1,\daleth_2,...,\daleth_n)\geq DP-PFMG(L_1,L_2,...,L_n)$$

Proof. The proof of this theorem can be proved as Theorem 4. ■

Theorem 10 *(Symmetry).*

Let Ξ_j *denote a set, characterized as* $\mathrm{W}\Big((\alpha_j,\beta_j),(p'_j,p''_j)\Big),(j=1,2,3,...,k')$*, representing a collection of* $DP-PFHNs$*. Additionally, consider the dual possibility vectors* $\vec{p'}=(\vec{p}_1,\vec{p}_2,...,\vec{p}_{k'})^T$ *and* $\vec{p''}=(\vec{p}_1,\vec{p}_2,...,\vec{p}_{k'})^T$ *associated with* Ξ_j*, ensuring that* $\vec{p}_j\geq 0$ *for* $j=\{1,2,3,...,k'\}$*. In this context,* $\vec{p'_j}\in[0,1]$*, with* $\sum j=1,\vec{p}'_j=1$ *and* $\vec{p''_j},\in[0,1]$*, with* $\sum_{j=1}^{k'}\vec{p}''_j=1$*. Then we have:*

$$DP-PFHSSG\mathrm{W}\Big((\alpha_j,\beta_j),(p_j,p_j)\Big)=DP-PFHSSG\mathrm{W}\Big((m'_j,n'_j),(p'_j,p''_j)\Big).\tag{7}$$

Proof. Proof of this Eq. (7) is obvious. Hence, it is omitted. ■

4. Decision-making involving dual-possibility PFHSS operators

This section presents a method based on dual-possibility PFHSS operators for solving MCDM problems according the flow chart in Figure (1). Let $\mathrm{N}=\{n_1,n_2,n_3,...,n_n\}$ represent any finite assortment of n alternatives and let $\mathrm{I}=\{i_1,i_2,i_3,...,i_m\}$ be a finite set of m criteria. Where $\xi=\{(\alpha,\beta),(p',p'')\}$ gathered by using dual possibility Pythagorean fuzzy hyper soft set. The requirement for the quantitative part of ξ is $0\leq\alpha^2+\beta^2\leq 1$.

Step 1: **Data collection:**

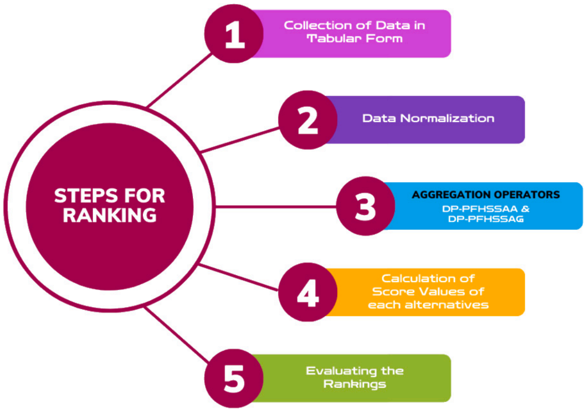

Fig. 1. Flow diagram for the suggested method.

Gather the evaluation data from the decision-makers and arrange it in a matrix $G=[N_{nm}]$ as follows:

$$M=\begin{pmatrix} N_{11} & N_{12} & \cdots & N_{1m} \\ N_{21} & N_{22} & \cdots & N_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ N_{n1} & N_{n2} & \cdots & N_{nm} \end{pmatrix}$$

Step 2: **Normalization:**

The matrix $G=[N_{ij}]$ decision is changed to $\bar{M}=[\bar{N_{ij}}]$, the normalized matrix at this point uses the following formula:

$$\bar{M}_{ij}=\begin{cases} N_{ij}, & \text{if it is from benefit parameter }, \\ (N_{ij})^c, & \text{if it is from cost parameter }, \end{cases} \tag{8}$$

where, N_{ij}^c is called the complement of N_{ij}.

Step 3: **Aggregation:**

Aggregate the DP-PFHSS N_{ij}(j=1,2,3,...,m) for all alternative $W_i(i=1,2,3,...,n)$ into the general preference value N by applying the proposed DP-PFHSSAA or DP-PFHSSGA operators.

Mathematically, it can be written as;

$$N_i = DP-PFHSSAA_{\delta'}(N_{i1},N_{i2},N_{i3},...,N_{im}),$$

$$N_i = DP-PFHSSGA_{\delta'}(N_{i1},N_{i2},N_{i3},...,N_{im}),$$

Step 4: **Identify the score values:**

Determine the score values $Sc(N_i)$ and $(i=1,2,3,...,m)$ of all DP-PFHSS N_i and $(i=1,2,3,...,m)$ in accordance with Definition 10.

Step 5: **Ranking:**

Arrange the choices in the best possible sequence. $Sc(N_i)$ score values are used to calculate $w_i,(i=1,2,3,..,m)$.

5. Numerical example

This section elaborates on the consequences and applicability of the proposed strategy using an illustrative example pertaining to the men's football players ranking process. One key indicator of a football player's success in the game is their ranking on the International scene. Federation International Football Association (FIFA), updates the rankings on a regular basis. Players ranking are essential for assessing the effectiveness and worth of goalkeepers, defenders, midfielders and forwards in the context of men's football. The ranking system helps in squad selection, performance assessment and strategic decision-making by offering insightful information about the relative status of players within each category. But in this instance, we are only concerned about forward rankings. Forwards are ranked based on a rigorous evaluation of multiple factors that gauge their effectiveness and contribution to the team. These parameters include some benefits parameters such as skill levels, physical attributes, scoring and assists, defensive abilities, tackling and intercepting, consistency, versatility, leadership and mentality and some cost parameters such as injury history, disciplinary record, age and longevity, contract and transfer costs, adaptability, market value, off-field behavior, contract length. The decision makers observe that the accompanying standards are applied to evaluate most of forward position players: Skill levels $(\mathcal{I}_1)$, Scoring and assists $(\mathcal{I}_2)$, Consistency $(\mathcal{I}_3)$, Injury history $(\mathcal{I}_4)$ and Disciplinary record $(\mathcal{I}_5)$. Then, at that point, how FIFA chooses the best player in which five top rated ranking of football players are following such as: $\{\mathcal{F}_1,\mathcal{F}_2,\mathcal{F}_3,\mathcal{F}_4,\mathcal{F}_5\}$. The interaction between football players is obviously an MCDM problem, with five alternatives $\{f_1,f_2,f_3,f_4,f_5\}$ and a specialist d. The optimal layout at

Table 1
Dual-possibility Pythagorean fuzzy hypersoft set decision matrix taken by d for q = 2.

	$\mathcal{I}_1$	$\mathcal{I}_2$	$\mathcal{I}_3$	$\mathcal{I}_4$	$\mathcal{I}_5$
$\mathcal{F}_1$	$\langle(0.3, 0.8), 0.1, 0.5\rangle$	$\langle(0.8, 0.5), 0.3, 0.5\rangle$	$\langle(0.5, 0.7), 0.2, 0.9\rangle$	$\langle(0.6, 0.8), 0.3, 0.4\rangle$	$\langle(0.5, 0.8), 0.1, 0.6\rangle$
$\mathcal{F}_2$	$\langle(0.7, 0.4), 0.3, 0.5\rangle$	$\langle(0.7, 0.8), 0.2, 0.3\rangle$	$\langle(0.6, 0.8), 0.1, 0.2\rangle$	$\langle(0.8, 0.7), 0.2, 0.6\rangle$	$\langle(0.5, 0.7), 0.2, 0.9\rangle$
$\mathcal{F}_3$	$\langle(0.2, 0.9), 0.2, 0.6\rangle$	$\langle(0.8, 0.4), 0.1, 0.3\rangle$	$\langle(0.2, 0.9), 0.2, 0.4\rangle$	$\langle(0.5, 0.7), 0.2, 0.3\rangle$	$\langle(0.8, 0.3), 0.3, 0.4\rangle$
$\mathcal{F}_4$	$\langle(0.6, 0.7), 0.3, 0.5\rangle$	$\langle(0.7, 0.8), 0.2, 0.4\rangle$	$\langle(0.6, 0.7), 0.2, 0.5\rangle$	$\langle(0.7, 0.7), 0.2, 0.4\rangle$	$\langle(0.6, 0.8), 0.1.0.2\rangle$
$\mathcal{F}_5$	$\langle(0.6, 0.7), 0.3, 0.6\rangle$	$\langle(0.5, 0.8), 0.2, 0.4\rangle$	$\langle(0.6, 0.7), 0.2.0.5\rangle$	$\langle(0.7, 0.4), 0.2, 0.5\rangle$	$\langle(0.6, 0.8), 0.1, 0.6\rangle$

that moment can then be found using the generated approach. It's vital to remember that the precise weight given to these cost and benefit factors may change depending on the ranking system employed by the body in charge of player rankings, like the Federation International Football Association (FIFA).

We can rank players like $\{\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4, \mathcal{F}_5\}$ by devising a scoring system that evaluates multiple performance metrics. These metrics might include goals scored, assists provided, successful passes, dribbles completed, defensive actions and overall impact on the team. Each criterion can be weighted according to its importance in determining a player's overall contribution. This approach allows for a nuanced understanding of the players' strengths and weaknesses across different aspects of the game. Additionally, fuzzy values $[0, 1]$, can be employed to represent these rankings, acknowledging the uncertainty and variability inherent in assessing player performance. This method's validity stems from its comprehensive consideration of various performance indicators and its ability to capture the complexity of player contributions in a flexible and adaptable manner.

Step 1: Collection of information in a matrix format (For q = 2) (see Table 1).

Step 2: Normalize the data using the suggested methodology according to Eq. (8).

	$\mathcal{I}_1$	$\mathcal{I}_2$	$\mathcal{I}_3$	$\mathcal{I}_4$	$\mathcal{I}_5$
$\mathcal{F}_1$	$\langle(0.3, 0.8), 0.1, 0.5\rangle$	$\langle(0.8, 0.5), 0.3, 0.5\rangle$	$\langle(0.5, 0.7), 0.2, 0.9\rangle$	$\langle(0.8, 0.6), 0.3, 0.4\rangle$	$\langle(0.8, 0.5), 0.1, 0.6\rangle$
$\mathcal{F}_2$	$\langle(0.7, 0.4), 0.3, 0.5\rangle$	$\langle(0.7, 0.8), 0.2, 0.3\rangle$	$\langle(0.6, 0.8), 0.1, 0.2\rangle$	$\langle(0.7, 0.8), 0.2, 0.6\rangle$	$\langle(0.7, 0.5), 0.2, 0.9\rangle$
$\mathcal{F}_3$	$\langle(0.2, 0.9), 0.2, 0.6\rangle$	$\langle(0.8, 0.4), 0.1, 0.3\rangle$	$\langle(0.2, 0.9), 0.2, 0.4\rangle$	$\langle(0.7, 0.5), 0.2, 0.3\rangle$	$\langle(0.3, 0.8), 0.3, 0.4\rangle$
$\mathcal{F}_4$	$\langle(0.6, 0.7), 0.3, 0.5\rangle$	$\langle(0.7, 0.8), 0.2, 0.4\rangle$	$\langle(0.6, 0.7), 0.2, 0.5\rangle$	$\langle(0.7, 0.7), 0.2, 0.4\rangle$	$\langle(0.8, 0.6), 0.1.0.2\rangle$
$\mathcal{F}_5$	$\langle(0.6, 0.7), 0.3, 0.6\rangle$	$\langle(0.5, 0.8), 0.2, 0.4\rangle$	$\langle(0.6, 0.7), 0.2.0.5\rangle$	$\langle(0.4, 0.7), 0.2, 0.5\rangle$	$\langle(0.8, 0.6), 0.1, 0.6\rangle$

Step 3: The aggregation operators (DP-PFHSSAA and DP-PFHSSGA) by using dual-possibilities by using step 2 according to Eq. (1) and (2).

The obtained results are as follows:

- DP-PFHSSAA:
  $\mathcal{F}_1 = \{0.5704, 0.7541\}$, $\mathcal{F}_2 = \{0.5449, 0.7358\}$, $\mathcal{F}_3 = \{0.2942, 0.8896\}$, $\mathcal{F}_4 = \{0.4617, 0.8644\}$ and $\mathcal{F}_5 = \{0.4503, 0.8319\}$.
- DP-PFHSSGA:
  $\mathcal{F}_1 = \{0.7721, 0.4989\}$, $\mathcal{F}_2 = \{0.8252, 0.4959\}$, $\mathcal{F}_3 = \{0.6099, 0.6160\}$, $\mathcal{F}_4 = \{0.8276, 0.5183\}$ and $\mathcal{F}_5 = \{0.7382, 0.5526\}$

Step 4: Alternative's score values of step 3 are as follows.

- DP-PFHSSAA:
  $Sc(\mathcal{F}_1) = -0.2432$, $Sc(\mathcal{F}_2) = -0.2444$, $Sc(\mathcal{F}_3) = -0.7049$, $Sc(\mathcal{F}_4) = -0.5339$ and $Sc(\mathcal{F}_5) = -0.4892$.
- DP-PFHSSGA:
  $Sc(\mathcal{F}_1) = 0.3472$, $Sc(\mathcal{F}_2) = 0.3345$, $Sc(\mathcal{F}_3) = 0.1817$, $Sc(\mathcal{F}_4) = 0.2462$ and $Sc(\mathcal{F}_5) = 0.2396$.

Step 5: Ranking of score values obtained in step 4.

- DP-PFHSSAA:
  After conducting calculations, Fig. 2 illustrates the rankings of International football players using the DP-PFHSSAA method. Notably, $\mathcal{F}_1$ emerges with the highest rating when compared to other players.

  $$\mathcal{F}_1 > \mathcal{F}_2 > \mathcal{F}_5 > \mathcal{F}_4 > \mathcal{F}_3.$$

- DP-PFHSSGA:
  On the another hand, the DP-PFHSSAG method also highlights $\mathcal{F}_1$ as holding the top rating among players as shown in Fig. 3. The average outcomes from both methods give highly similar rankings, indicating that the results obtained by both approaches are nearly consistent.

  $$\mathcal{F}_1 > \mathcal{F}_2 > \mathcal{F}_4 > \mathcal{F}_5 > \mathcal{F}_3.$$

  The list provides a ranking of player skills among nations participating in International football. However, without specific knowledge about the nations behind these labels, the conclusions drawn remain speculative. Player rankings in International football

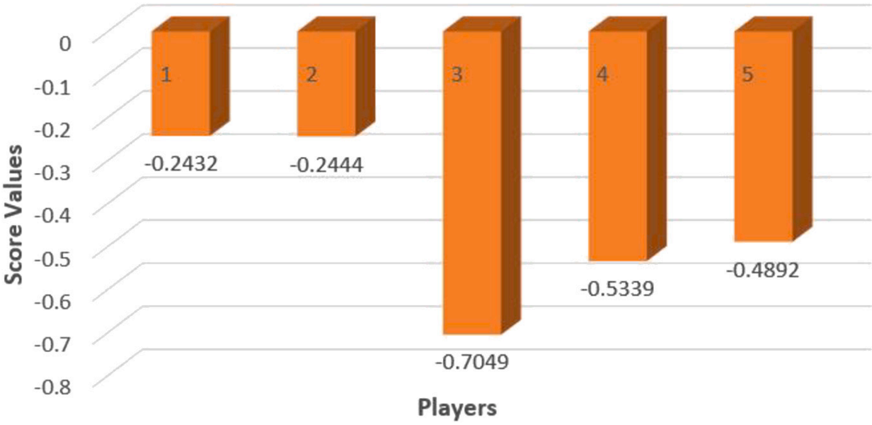

Fig. 2. Graphically representation of players by using DP-PHSSAA.

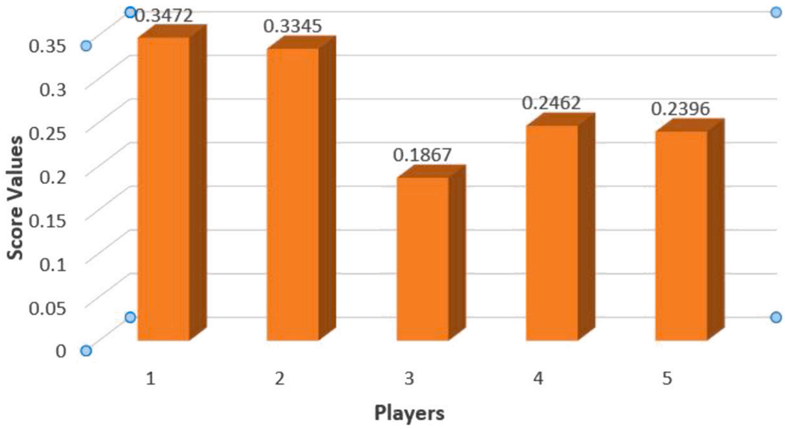

Fig. 3. Graphically representation of players by using DP-PHSSGA.

can fluctuate, so it's advisable to consult reliable sources for the latest and accurate information. For football enthusiasts, FIFA rankings serve as comprehensive and reliable assessments of player performances, ensuring accurate and consistent information, whether using average or geometric operators.

6. Comparison analysis

The process of examining two or more related items to determine their similarities and differences is known as comparative analysis. Its application across a wide range of contexts and sectors might help individuals comprehend the parallels and discrepancies between various objects. It can help businesses make informed decisions on crucial subjects. It might be beneficial when used to scientific data. Scientific data is information that has been gathered via scientific investigation and will be used for a certain objective. When compared to scientific data, it demonstrates how reliable and consistent the data. It also helps scientists make sure their data is accurate and legitimate. Comparative analyses play a vital role in providing answers to important issues and a deeper understanding of a subject. These are the main goals that businesses aim to achieve via comparative analysis. It encourages a full understanding of the opportunities associated with certain processes, departments, or business units. Furthermore, this study ensures that the real reasons of performance gaps are being addressed. It is widely used since it helps to comprehend the challenges that a firm has faced in the past as well as in the present. This method offers accurate, objective information on performance together with recommendations for improving it.

To further demonstrate the merits and benefits of the suggested methods, we lead the subsequent comparison likeness. We spearhead the ensuing comparative resemblance in order to further highlight the advantages and virtues of the recommended techniques.

6.1. *Comparison between the provided method with the one presented out by [50]*

To address the overhead issue, we utilize the framework proposed by Wahab et al. [50], shown in Table 2. The scores were calculated using the Pq-ROFHSSAG approach with a single possibility for ranking via q-ROFHSSAA operators. Our study introduces a dual-possibility method for enhanced sensitivity analysis.

Table 2
Comparison of suggested operators with existing operators.

Different Operators :	Score function values	Ranking
Pq-ROFHSSAA [50]	$Sc(\mathcal{F}_1)=0.1712$, $Sc(\mathcal{F}_2)=0.1639$, $Sc(\mathcal{F}_3)=0.0903$, $Sc(\mathcal{F}_4)=0.1294$, $Sc(\mathcal{F}_5)=0.0594$	$\mathcal{F}_1>\mathcal{F}_2>\mathcal{F}_4>\mathcal{F}_3>\mathcal{F}_5$
Pq-ROFHSSAG [50]	$Sc(\mathcal{F}_1)=0.1184$, $Sc(\mathcal{F}_2)=0.1354$, $Sc(\mathcal{F}_3)=0.0678$, $Sc(\mathcal{F}_4)=0.0868$, $Sc(\mathcal{F}_5)=0.0594$	$\mathcal{F}_2>\mathcal{F}_1>\mathcal{F}_4>\mathcal{F}_3>\mathcal{F}_5$
DP-PFHSSAA (Proposed Operator)	$Sc(\mathcal{F}_1)=-0.2432$, $Sc(\mathcal{F}_2)=-0.2444$, $Sc(\mathcal{F}_3)=-0.7049$, $Sc(\mathcal{F}_4)=-0.5339$, $Sc(\mathcal{F}_5)=-0.4892$	$\mathcal{F}_1>\mathcal{F}_2>\mathcal{F}_5>\mathcal{F}_4>\mathcal{F}_3$
DP-PFHSSGA (Proposed Operator)	$Sc(\mathcal{F}_1)=0.3472$, $Sc(\mathcal{F}_2)=0.3345$, $Sc(\mathcal{F}_3)=o.1817$, $Sc(\mathcal{F}_4)=0.2462$, $Sc(\mathcal{F}_5)=0.2396$	$\mathcal{F}_1>\mathcal{F}_2>\mathcal{F}_4>\mathcal{F}_5>\mathcal{F}_3$

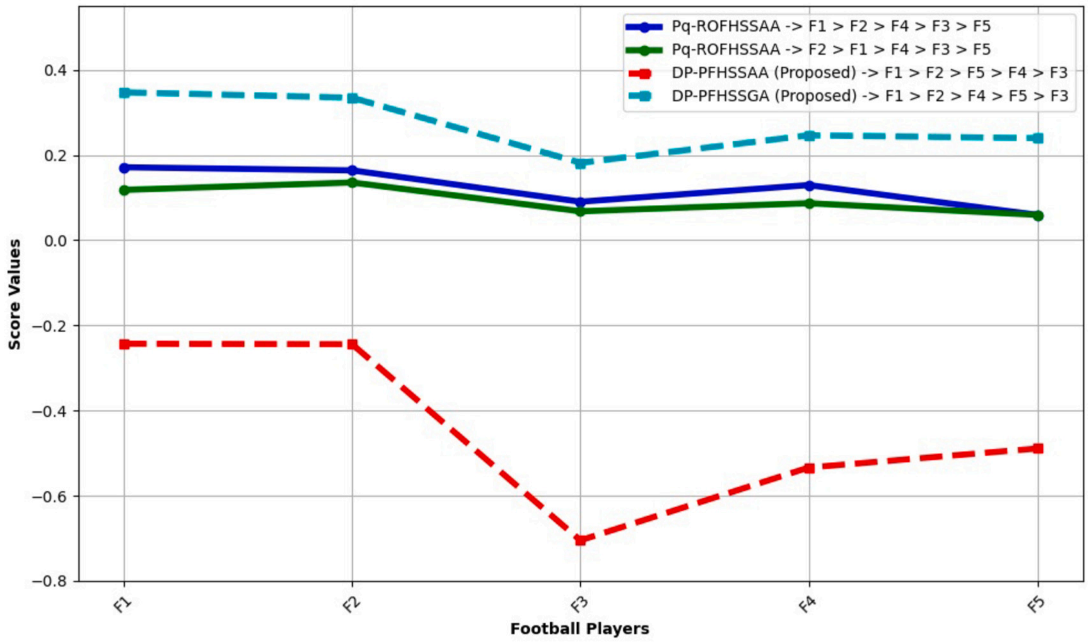

Fig. 4. Graphical representation comparison between existing and proposed methods.

We compare both methods and find that while the relative weights of options remain unchanged, the best solution is consistent across both approaches, as depicted in Fig. 4. This consistency demonstrates the robustness of our dual-possibility method. Additionally, our approach offers improved flexibility by using known possibilities attributes instead of fixed weights. This facilitates a more accurate and nuanced calculation of alternative rankings, effectively addressing uncertainties and imprecision inherent in decision-making processes. Furthermore, the dual-possibility approach can better capture complex scenarios where multiple potential outcomes are possible, providing a deeper insight into the decision-making landscape. By accommodating varying levels of uncertainty, our method enhances the overall reliability and precision of the rankings generated.

6.2. Comparison between the provided method with the one presented out by [51]

We also conducted a comparison between our recommended approach and the techniques proposed by Khan et al. [51] to validate the effectiveness and suitability of our technique. As detailed in Table 3, we applied the IVPFHSIWAO and IVPFHSIWGO methods, which utilize interval-valued data for ranking within the specified application. This comparison aims to evaluate how our approach stands against these established techniques. The results indicate that the rankings produced by our method are quite comparable to those generated by Khan et al.'s techniques, as illustrated in Fig. 5. This suggests that our approach performs on par with, if not better than, existing methods in terms of ranking accuracy. Additionally, we observed that the score values obtained using our method exhibit a mix of both negative and positive values, unlike Khan et al.'s methods, which tend to produce scores that are either uniformly negative or positive.

This characteristic highlights a key advantage of our approach: it provides a more balanced and nuanced perspective on the ranking outcomes. The ability to capture both negative and positive scores enhances the clarity and depth of the analysis, making our method more effective in revealing subtle differences between options. Overall, this comparison underscores the superior performance and greater insight offered by our approach, confirming its robustness and illuminating nature in addressing complex decision-making scenarios.

Table 3
Comparison of suggested operators with existing operators.

Different operators :	Score function values	Ranking
IVPFHSIWAO [51]	$Sc(\mathcal{F}_1)=0.8387$, $Sc(\mathcal{F}_2)=0.7656$, $Sc(\mathcal{F}_3)=0.2979$, $Sc(\mathcal{F}_4)=0.4526$, $Sc(\mathcal{F}_5)=0.3597$	$\mathcal{F}_1>\mathcal{F}_2>\mathcal{F}_4>\mathcal{F}_5>\mathcal{F}_3$
IVPFHSIWGO [51]	$Sc(\mathcal{F}_1)=0.2586$, $Sc(\mathcal{F}_2)=0.1329$, $Sc(\mathcal{F}_3)=0.0097$, $Sc(\mathcal{F}_4)=0.0489$, $Sc(\mathcal{F}_5)=0.0396$	$\mathcal{F}_1>\mathcal{F}_2>\mathcal{F}_4>\mathcal{F}_5>\mathcal{F}_3$
DP-PFHSSAA (Proposed Operator)	$Sc(\mathcal{F}_1)=-0.2432$, $Sc(\mathcal{F}_2)=-0.2444$, $Sc(\mathcal{F}_3)=-0.7049$, $Sc(\mathcal{F}_4)=-0.5339$, $Sc(\mathcal{F}_5)=-0.4892$	$\mathcal{F}_1>\mathcal{F}_2>\mathcal{F}_5>\mathcal{F}_4>\mathcal{F}_3$
DP-PFHSSGA (Proposed Operator)	$Sc(\mathcal{F}_1)=0.3472$, $Sc(\mathcal{F}_2)=0.3345$, $Sc(\mathcal{F}_3)=0.1817$, $Sc(\mathcal{F}_4)=0.2462$, $Sc(\mathcal{F}_5)=0.2396$	$\mathcal{F}_1>\mathcal{F}_2>\mathcal{F}_4>\mathcal{F}_5>\mathcal{F}_3$

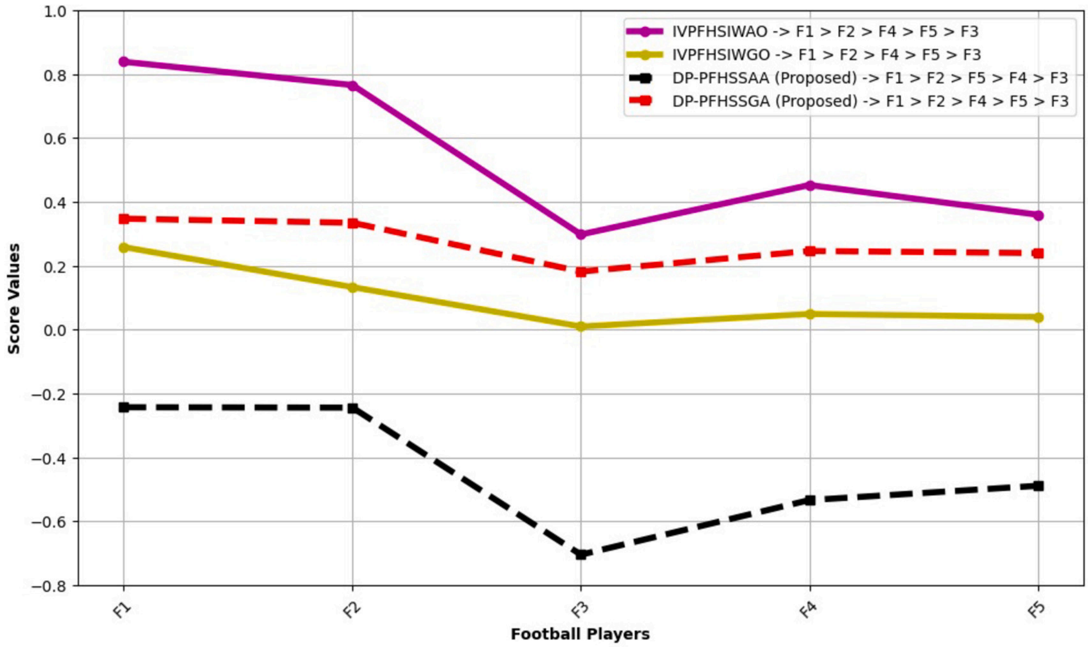

Fig. 5. Graphical representation comparison between existing and proposed methods.

Table 4
Comparison of suggested operators with existing operators.

Different operators :	Score function values	Ranking
GGIVq-ROFSWA [52]	$Sc(\mathcal{F}_1)=0.9087$, $Sc(\mathcal{F}_2)=0.8059$, $Sc(\mathcal{F}_3)=0.2033$, $Sc(\mathcal{F}_4)=0.5026$, $Sc(\mathcal{F}_5)=0.3997$	$\mathcal{F}_1>\mathcal{F}_2>\mathcal{F}_4>\mathcal{F}_5>\mathcal{F}_3$
GGIVq-ROFSWG [52]	$Sc(\mathcal{F}_1)=0.3042$, $Sc(\mathcal{F}_2)=0.1929$, $Sc(\mathcal{F}_3)=0.0297$, $Sc(\mathcal{F}_4)=0.1089$, $Sc(\mathcal{F}_5)=0.0376$	$\mathcal{F}_1>\mathcal{F}_2>\mathcal{F}_4>\mathcal{F}_5>\mathcal{F}_3$
DP-PFHSSAA (Proposed Operator)	$Sc(\mathcal{F}_1)=-0.2432$, $Sc(\mathcal{F}_2)=-0.2444$, $Sc(\mathcal{F}_3)=-0.7049$, $Sc(\mathcal{F}_4)=-0.5339$, $Sc(\mathcal{F}_5)=-0.4892$	$\mathcal{F}_1>\mathcal{F}_2>\mathcal{F}_5>\mathcal{F}_4>\mathcal{F}_3$
DP-PFHSSGA (Proposed Operator)	$Sc(\mathcal{F}_1)=0.3472$, $Sc(\mathcal{F}_2)=0.3345$, $Sc(\mathcal{F}_3)=0.1817$, $Sc(\mathcal{F}_4)=0.2462$, $Sc(\mathcal{F}_5)=0.2396$	$\mathcal{F}_1>\mathcal{F}_2>\mathcal{F}_4>\mathcal{F}_5>\mathcal{F}_3$

6.3. Comparison between the provided method with the one presented out by [52]

We conducted a thorough comparison between our recommended approach and the technique proposed by Hayat et al. [52] to establish its validity and suitability. Utilizing interval-valued data for ranking, we employed GGIVq-ROFSWA and GGIVq-ROFSWG to address the case described. The results are detailed in Table 4 and illustrated in Fig. 6.

Our analysis revealed that while the ranks of the options were comparable between the two methods, there were notable differences in the results. Specifically, our method produced score values that spanned both negative and positive ranges, offering a more nuanced and comprehensive evaluation. This variability in score ranges highlights the depth and flexibility of our approach in capturing subtle differences between options. In contrast, the method by Hayat et al. showed a more restricted range of scores. This indicates that our approach is not only superior in its ability to handle diverse data ranges but also provides a more illuminating perspective on the decision-making process. By incorporating both negative and positive values, our method enhances the overall analysis, offering deeper insights and a more robust evaluation framework. The study highlighted in the preceding discussion underscores the effectiveness of our recommended techniques in addressing decision-making (DM) problems. Our approaches surpass other methods in terms of flexibility and rationality for tackling multiple attribute decision making (MADM) issues. These approaches offer distinct advantages. Firstly, most discussed options hinge on DP-PFHSS, a unique method that allows the sum and square of positive participation degree, impartial participation degree and negative enrollment degree to exceed one. This characteristic minimizes data loss during MADM, empowering decision-makers with more freedom to articulate their ideas.

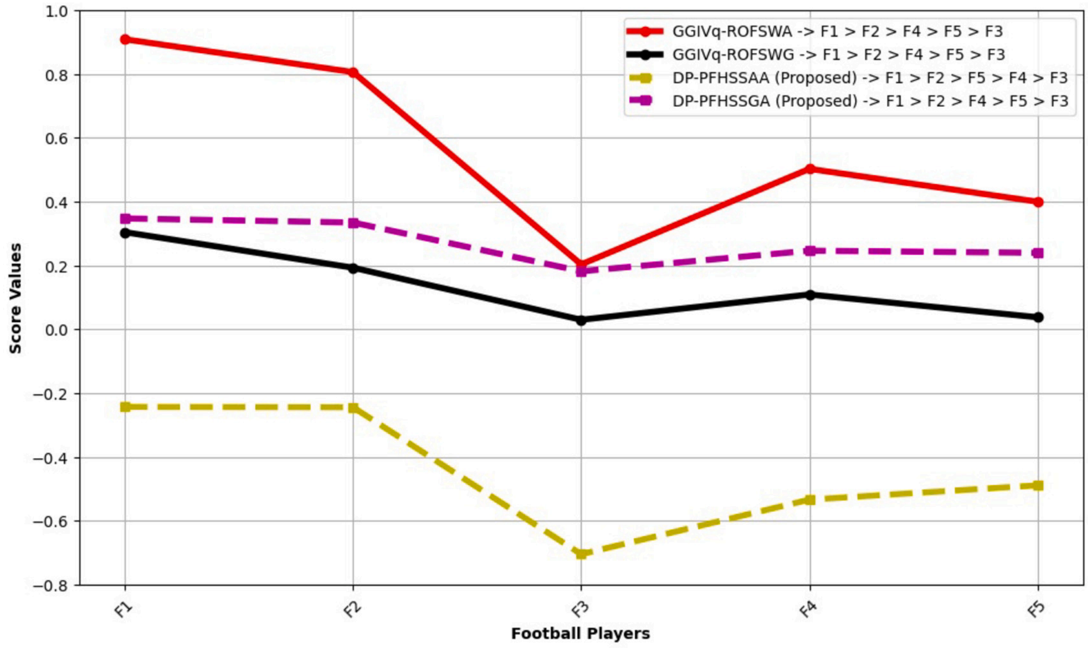

Fig. 6. Graphical representation comparison between existing and proposed methods.

Table 5
Comparison of Proposed Method with Existing Methods.

Method	Score Values	Ranking
Wahab et al. [50] (Pq-ROFHSSAA)	Sc(F1) = 0.1712, Sc(F2) = 0.1639, Sc(F3) = 0.0903, Sc(F4) = 0.1294, Sc(F5) = 0.0594	F1 > F2 > F4 > F3 > F5
Wahab et al. [50] (Pq-ROFHSSAG)	Sc(F1) = 0.1184, Sc(F2) = 0.1354, Sc(F3) = 0.0678, Sc(F4) = 0.0868, Sc(F5) = 0.0594	F2 > F1 > F4 > F3 > F5
Proposed (DP-PFHSSAA)	Sc(F1) = -0.2432, Sc(F2) = -0.2444, Sc(F3) = -0.7049, Sc(F4) = -0.5339, Sc(F5) = -0.4892	F1 > F2 > F5 > F4 > F3
Proposed (DP-PFHSSGA)	Sc(F1) = 0.3472, Sc(F2) = 0.3345, Sc(F3) = 0.1817, Sc(F4) = 0.2462, Sc(F5) = 0.2396	F1 > F2 > F4 > F5 > F3
Khan et al. [51] (IVPFHSIWAO)	Sc(F1) = 0.8387, Sc(F2) = 0.7656, Sc(F3) = 0.2979, Sc(F4) = 0.4526, Sc(F5) = 0.3597	F1 > F2 > F4 > F5 > F3
Khan et al. [51] (IVPFHSIWGO)	Sc(F1) = 0.2586, Sc(F2) = 0.1329, Sc(F3) = 0.0097, Sc(F4) = 0.0489, Sc(F5) = 0.0396	F1 > F2 > F4 > F5 > F3
Hayat et al. [52] (GGIVq-ROFSWA)	Sc(F1) = 0.9087, Sc(F2) = 0.8059, Sc(F3) = 0.2033, Sc(F4) = 0.5026, Sc(F5) = 0.3997	F1 > F2 > F4 > F5 > F3
Hayat et al. [52] (GGIVq-ROFSWG)	Sc(F1) = 0.3042, Sc(F2) = 0.1929, Sc(F3) = 0.0297, Sc(F4) = 0.1089, Sc(F5) = 0.0376	F1 > F2 > F4 > F5 > F3

Secondly, in considering the quantitative assumptions made by managers alongside their subjective judgments, DP-PFHSS proves to be not only sufficient but also valuable in illustrating assessments of various options by decision-makers. This feature enhances the applicability of our recommended techniques in real-world decision-making scenarios.

Thirdly, recognizing that credits are often intertwined with challenges in decision-making, our MADM method, based on operators DP-PFHSSGA or DP-PFHSSAA, takes contention relationships into account. This inclusion makes our approach adept at handling actual MADM problems, where the interaction of different elements plays a crucial role. The comparison of proposed work with existing work is given in Table 5.

7. Conclusions and future work

In conclusion, this technique contributes the hypersoft sets, Pythagorean fuzzy sets and dual possibility to rate football players, enhancing evaluation complexity and addressing difficulties in player ranking through advanced mathematical modeling. It significantly advances player evaluation methodologies, offering a holistic and adaptable framework for assessing International football talent. The dual possibility Pythagorean fuzzy hypersoft sets (DP-PFHSS) example illustrates the model's efficacy in multi-criteria decision-making based on aggregation operators. The decision-making algorithm successfully determines player worth, contributing to an overall ranking process. A practical example showcases the approach's applicability for scouting and recruitment. Graphical analysis with average and geometric operators highlights $\mathcal{F}_1$ top ranking, demonstrating the approach's superiority. In essence, this research provides a comprehensive and effective methodology for football player evaluations, with broader implications for talent assessment across various domains. One potential limitation is the focus on introducing a novel approach to player evaluation without delving into extensive empirical validation. This suggests an opportunity for future research to empirically test and refine the proposed methodology, thereby contributing to the ongoing advancement of player evaluation methodologies in international football.

The future directions for a comprehensive approach to players ranking using:

- Probabilistic q-rung orthopair linguistic interval-valued neutrosophic fuzzy soft set.
- Dual-Probabilistic (m,n) bi-polar neutrosophic fuzzy soft set.
- Dual-Probabilistic q-rung orthopair m-polar neutrosophic fuzzy soft set.
- Probabilistic q-rung orthopair hesitant neutrosophic fuzzy soft set.
- Generalized interval-valued (m,n) fuzzy soft aggregation operators etc.

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Ethical approval

This study does not involve the use of human subjects or animals, therefore ethical approval is not required. The research relies exclusively on publicly available data and no personally identifiable information is being collected or analyzed. All procedures and methodologies strictly adhere to established ethical guidelines and regulations.

CRediT authorship contribution statement

Saraj Khan: Writing – original draft, Validation, Conceptualization. **Muhammad Imran Asjad:** Formal analysis, Methodology, Supervision, Project administration. **Muhammad Bilal Riaz:** Writing – review & editing, Software, Methodology, Validation, Resources, Funding acquisition, Formal analysis. **Abdul Wahab:** Writing – original draft, Validation, Methodology, Investigation. **Hira Ashaq:** Writing – original draft, Software. **Taseer Muhammad:** Resources, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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