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Research Report

Facilitating innovation diffusion in social networks using dynamic norms

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Abstract

Dynamic norms have recently emerged as a powerful method to encourage individuals to adopt an innovation by highlighting a growing trend in its uptake. However, there have been no concrete attempts to understand how this individual-level mechanism might shape the collective population behavior. Here, we develop a framework to examine this by encapsulating dynamic norms within a game-theoretic mathematical model for innovation diffusion. Specifically, we extend a network coordination game by incorporating a probabilistic mechanism where an individual adopts the action with growing popularity, instead of the standard best-response update rule; the probability of such an event captures the population's "sensitivity" to dynamic norms. Theoretical analysis reveals that sensitivity to dynamic norms is key to facilitating social diffusion. Small increases in sensitivity reduces the advantage of the innovation over status quo or the number of initial innovators required to unlock diffusion, while a sufficiently large sensitivity alone guarantees diffusion.

Keywords: innovation diffusion, dynamic norms, coordination games, network dynamics

Significance Statement:

Encouraging people to adopt important and helpful innovations, such as preventive vaccinations and sustainable practices, can be a difficult task. These innovations prevent potential future problems, but are lacking in two important aspects associated with innovation adoption: they offer a limited perceived advantage over the status quo, and few people are willing to be innovators by being the first to adopt. Dynamic norms refer to the sensitivity of people to behaviors that are not yet mainstream but growing in popularity. We use mathematical modeling to show that dynamic norms, an individual-level mechanism, can facilitate the population-level adoption of such innovations. These findings may help design strategies for promoting innovation diffusion by increasing the effects of dynamic norms.

Introduction

Innovation diffusion is indispensable in a modern society. It allows new ideas, behaviors, technologies, conventions, and practices to spread through a population, eventually replacing a status quo. Innovation diffusion has enabled the adoption of hybrid seed corn by farmers for improved crop yield (1), usage of new medicines by medical professionals (2), and the evolution of social norms and conventions (3–6).

Two notable factors that have been identified as key to allowing successful innovation diffusion are relative advantage and the presence of innovators (7). The relative advantage of the innovation over the status quo, such as hybrid seed corn offering improved crop yield over traditional corn, provides each individual with an incentive to select the innovation (7). Mathematical models of innovation diffusion have identified that diffusion is always guaranteed when the relative advantage is sufficiently large (3,8,9). The literature has also explored the importance of actors who promote the innovation, often referred to as innovators (7), opinion leaders (10,11), or committed minority (5), depending on the context. In particular, the number of innovators and their visibility within the population can often play an essential role in the adoption of the innovation, with some studies identifying a lower bound on the number of innovators that can guarantee that diffusion will occur (5,12–14).

These findings reflect the considerable challenges for the diffusion of many innovations, which are lacking in these two factors. For instance, the benefits for preventive innovations such as vaccination and family planning lie in stopping an undesirable event from occurring. Such innovations often struggle to become adopted as the relative advantage is not readily and immediately apparent (2,15). Morally motivated innovators, such as individuals who adopt meat-free diets to reduce environmental impact, can be ostracized from society and struggle to effectively promote the benefits of the innovation (16–18). In the battle against the climate crisis, the world now needs massive social change, including the widespread adoption of new sustainable technologies and prac-

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tices (19–22). Hence, we require now, more than ever, an understanding for facilitating diffusion of such innovations lacking in these two notable factors (23).

Dynamic norms may be one mechanism to facilitate behavior change—recent research has suggested that people are sensitive to observing or learning about the growth in popularity over time of a particular behavior, even if the behavior is currently in the minority, and be influenced to adopt it (24). Existing studies consider environmentally responsible behaviors around reducing meat consumption (24–27), usage of disposable coffee cups (28), and water usage (24,29), and other minority behaviors such as women's participation in STEM degrees and careers (30), or reducing electronic screen time before sleep (31). Online experiments have shown that exposure to dynamic norm information can increase a person's intention and attitude to adopt the minority behavior, in comparison to being exposed to static norm information or no information (24,29–31), while field intervention studies have found dynamic norm information can increase the number of individuals adopting the behavior that is increasing in popularity in various contexts (24,26,28,29). Several studies have also highlighted how the context and target audience can determine whether dynamic norms have a statistically significant effect in influencing behavior (24,26,29–31) or no significant effect (25–27).

While empirical approaches allow to examine the effects of dynamic norms at the individual level, including the contexts in which they are most powerful and the magnitude and significance of such effects, these approaches are limited in unveiling the downstream population-level consequences, i.e. how individual sensitivity to dynamic norms may collectively result in social change via the widespread adoption of the innovation. Mathematical modeling (perhaps informed and supported by empirical methods) has emerged as a powerful tool to explore the link between individual mechanisms and collective change (5,6,14,32). Indeed, a study on social convention formation and change has empirically identified the presence of an effect of dynamic norms at the individual level, and model simulations have suggested its presence helped to shape social diffusion patterns (14). However, a theoretical modeling framework for innovation diffusion that encapsulates dynamic norms and rigorous analysis is still missing.

In this paper, we fill in this gap from a theoretical perspective, toward gaining insights into the crucial role of dynamic norms in facilitating innovation diffusion. We pursue such a goal through the development and analysis of a network model for innovation diffusion (3,8,9). In this game-theoretic model, each agent in a time-varying network can select between two actions, representing the status quo and the innovation. Agents revise their actions at discrete-time instants according to a best-response updating (33), selecting the action adopted by the majority of their neighbors on a time-varying network, with an adjustment made to account for relative advantage. To operationalize dynamic norms within the model (24,26,28,29,31), we expand the model by further proposing that at each time instant, an agent has a probability of replacing their best-response choice by selecting the action that is trending, i.e. the action whose adopters have increased in number since the last time instant.

Through a rigorous analysis of the proposed model, corroborated by numerical studies, we identify when innovation diffusion will occur and when the status quo will be maintained. This allows us to elucidate the downstream, population-level consequences of dynamic norms. Specifically, we can summarize our main findings with the observation that dynamic norms can lower both the relative advantage and the number of innovators necessary for unlocking innovation diffusion. Importantly, we identify that when the individuals' sensitivity to dynamic norms crosses a threshold value, even innovations without a clear advantage or that are disadvantageous will spread, irrespective of the number of innovators. Numerical simulations suggest this threshold behavior is a characteristic feature of the model, revealing that dynamic norms should indeed be pursued as a tool for achieving social change.

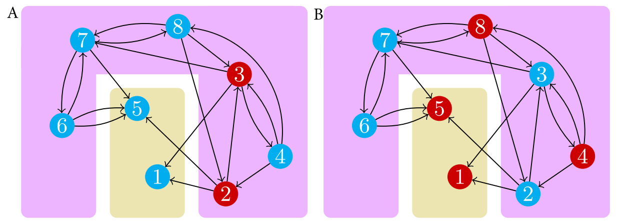

Fig. 1. One iteration of the model with $n = 8$ individuals; nodes in red (cyan) adopt the innovation (status quo). In panel (A), individuals that are following the coordination game (pink area) establish $k = 3$ interactions, while individuals in the yellow area follow dynamic norms. In panel (B), we illustrate the action update process with $\alpha = 0.5$ and $z(t) > z(t-1)$.

Modeling framework

A classical game-theoretic model will be the basis of our modeling framework for innovation diffusion over a network, with full details presented in the "Materials and methods" section. We consider a population of $n \in \mathbb{N}$ individuals, denoted by the set $\mathcal{V} = \{1, \ldots, n\}$, that interact on a time-varying network. At each discrete time step $t \in \mathbb{N}$, each individual $i \in \mathcal{V}$ can choose among two possible actions: the status quo ($x_i(t) = 0$) and the innovation ($x_i(t) = 1$). The action state vector $\mathbf{x}(t) = [x_1(t), \ldots, x_n(t)] \in \{0, 1\}^n$ gathers the actions of all individuals at time t. We let $\mathbf{1}$ and $\mathbf{0}$ be the n-dimensional vector of all 1s and all 0s, respectively.

To model innovation diffusion, we assume that initially all individuals are selecting the status quo, i.e. $\mathbf{x}(0) = \mathbf{0}$, and we let $\zeta > 0$ be the fraction of innovators in the population, who are the individuals acting as innovators and opinion leaders (7). Then, at time $t = 1$, a constant fraction $\zeta > 0$ of individuals adopt the innovation, $x_i(1) = 1$, while the rest of the population continues to adopt the status quo, $x_j(1) = 0$. From $t = 2$, every individual can revise their action, and diffusion is said to occur if $\mathbf{x}(t) = \mathbf{1}$ for some $t \geq 2$.

Coordination game

A popular paradigm used in game-theoretic diffusion models is that of network coordination games (3,4,8,34,35). In the simplest implementation, we suppose that at each time step t, each individual $i \in \mathcal{V}$ initiates k interactions among the members of the population, selected uniformly at random and independently of other selections, generating a time-varying network of social interactions (see Fig. 1). The selected individuals form the neighbor set of individual i at time t, denoted by $\mathcal{N}_i(t)$. Then, individual i engages in a symmetric two-player pure coordination games with each of their k interactions. The overall payoff that individual i would receive for selecting action 0 and 1 at time t, when the action state of the system is $\mathbf{x}(t)$ is equal to

$$\pi_i^{(0)}(\mathbf{x}(t)) := \sum_{j \in \mathcal{N}_i(t)} (1 - x_j(t)) \quad (1)$$

and

$$\pi_i^{(1)}(\mathbf{x}(t)) := \sum_{j \in \mathcal{N}_i(t)} (1+\alpha) x_j(t), \tag{2}$$

respectively, where the parameter $\alpha > -1$ represents the relative advantage (if positive) or disadvantage (if negative) of the innovation with respect to the status quo and captures possible differences in the intrinsic value of the two actions. For simplicity and consistency with the literature, we refer to α as the relative advantage even though α may be negative. From a game-theoretic perspective, the relative advantage captures risk dominance (4) (see Supplementary Material). In this setting, relative advantage also captures Pareto efficiency (4). Hence, innovations with $\alpha > 0$ can be interpreted as being beneficial for the population, while innovations with $\alpha < 0$ are inefficient.

Individuals revise their actions to maximize their overall payoffs according to best-response dynamics, which is a standard protocol adopted in evolutionary game theory (33) with support from social psychology empirical studies (35). Specifically individual i revises their action to

$$x_i(t+1) = \begin{cases} 1 & \text{if } \pi_i^{(1)}(\mathbf{x}(t)) > \pi_i^{(0)}(\mathbf{x}(t))\,, \\ 0 & \text{if } \pi_i^{(1)}(\mathbf{x}(t)) \leq \pi_i^{(0)}(\mathbf{x}(t))\,, \end{cases} \tag{3}$$

where we have assumed that an individual i prefers the status quo if the two payoffs are equal, consistent with the social psychology and empirical literature on inertia and status-quo bias, which provides evidence that people are more likely to be consistent with their previous choices, at least in the absence of an advantage for changing (14,36).

For the sake of simplicity, we present the simplest incarnation of a coordination game on (time-varying) networks. In a more general formulation, detailed in the Supplementary Material and used for numerical simulations, further real-world features are incorporated, namely the presence of a backbone network that constrains the possible interactions between agents, and heterogeneity across the population in terms of the number of interactions and the relative advantage.

Our analysis finds that the innovation will diffuse to the entire population irrespective of the fraction of innovators in the population ζ if the relative advantage is greater than $k-2$, or if there are enough innovators and the relative advantage is not too negative (the formal result is presented in the sequel, after technical definitions are introduced). This result closely parallels the findings from other diffusion models employing coordination games on networks (8,9).

Dynamic norms

We operationalize dynamic norms through the following mechanism. An individual i under the influence of dynamic norms will choose to adopt that action whose number of adopters has increased in the previous time-step. Formally, we introduce the quantity $z(t) := \frac{1}{n}\sum_{i \in \mathcal{V}} x_i(t)$, which is the fraction of the population adopting the innovation at time $t \in \mathbb{N}$. Thus, dynamic norms will lead individual i to revise their action according to

$$x_i(t+1) = \begin{cases} 1 & \text{if } z(t) > z(t-1)\,, \\ 0 & \text{if } z(t) < z(t-1)\,, \\ x_i(t) & \text{if } z(t) = z(t-1)\,. \end{cases} \tag{4}$$

Eq. (4) captures individual i adopting the action that has increased in popularity over the previous time-step (is trending upward), even if it is the current minority action in the population. If the fraction of adopters of both actions is unchanged, then no dynamic norm is present and the individual does not change their action, consistent with the presence of inertia in social systems (14,36). Our model and findings can be straightforwardly extended to scenarios in which a trend is perceived only if a minimum number of individuals have changed action (see the Supplementary Material).

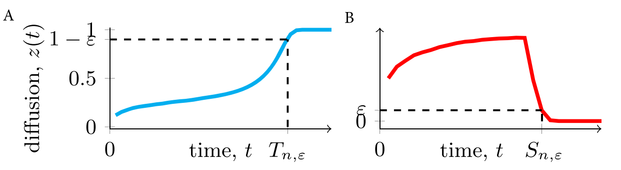

Fig. 2. Examples of (A) innovation diffusion and (B) status quo maintenance, with the random times highlighted in Eqs. (6,7).

Unveiling the role of dynamic norms

Our interest is to understand whether dynamic norms can unlock innovation diffusion. To investigate this, we suppose that individuals can make a decision on which action to adopt either through the classical coordination mechanism or through dynamic norms. Formally, we propose that at each time-step $t \in \mathbb{N}$, each individual $i \in \mathcal{V}$ is influenced by dynamic norms with probability $\gamma \in [0,1]$, independent of other individuals and past occurrences. The parameter γ captures the sensitivity of the population to dynamic norms. Hence, the action of individual i is revised as

$$x_i(t+1) = \begin{cases} \text{Eq. (3)} & \text{with probability } 1-\gamma\,, \\ \text{Eq. (4)} & \text{with probability } \gamma\,. \end{cases} \tag{5}$$

An example of this revision protocol in illustrated in Fig. 1.

A direct analysis of Eq. (5) (reported in the Supplementary Material) establishes that the action state will converge in finite time to a consensus state, in which either the entire population will adopt the innovation ($\mathbf{x} = \mathbf{1}$) or the diffusion fails and the status quo is maintained ($\mathbf{x} = \mathbf{0}$). The key question of our study can thus be formulated: *what characteristics of the population will ensure that diffusion is always achieved?*

Our diffusion model has four key parameters: the number of social contacts k, relative advantage α, sensitivity γ, and fraction of innovators in the population ζ. The key question can thus be answered by determining whether a population with given (k, α, γ, ζ) will approach $\mathbf{x} = \mathbf{1}$ in finite time. Since $\mathbf{x}(t)$ is a stochastic process, we will characterize its behavior using the notion of high probability, which occurs when an event is verified with probability converging (at least polynomially) to 1 as n grows large. To this aim, let us define for any positive constant $\varepsilon > 0$, the random times

$$T_{n,\varepsilon} := \inf\{t \in \mathbb{N}_+ : z(t) \geq (1-\varepsilon)n\}\,, \tag{6}$$

$$S_{n,\varepsilon} := \inf\{t \in \mathbb{N}_+ : z(t) \leq \varepsilon n\}\,, \tag{7}$$

which can be expressed as functions of the population size n and of the constant ε. In other words, $T_{n,\varepsilon}$ and $S_{n,\varepsilon}$ are the times at which the fraction of adopters of the innovation become greater than $1-\varepsilon$ and less than ε, respectively, as illustrated in Fig. 2. Using these random times, we can formalize our research question through the following two definitions.

Definition 1 (Innovation diffusion)

A quadruple $(k, \alpha, \gamma, \zeta)$ is said to guarantee innovation diffusion if the family of events $E_{n,\varepsilon} := T_{n,\varepsilon} < S_{n,\varepsilon}$ holds with high probability, for any positive constant $0 < \varepsilon < \min\{\zeta, 1-\zeta\}$.

Definition 2 (Status quo maintenance)

A quadruple $(k, \alpha, \gamma, \zeta)$ is said to maintain the status quo if the family of events $E'_{n,\varepsilon} := T_{n,\varepsilon} > S_{n,\varepsilon}$ holds with high probability, for any positive constant $0 < \varepsilon < \min\{\zeta, 1-\zeta\}$.

Briefly, innovation diffusion is guaranteed if the system converges arbitrarily close to the innovation consensus state $\mathbf{x} = \mathbf{1}$ (with proximity ε) in finite time. In contrast, the status quo is maintained whenever the system converges arbitrarily close to the status-quo consensus state, $\mathbf{x} = \mathbf{0}$.

Results

Before presenting our main findings, we need to derive a closed-form expression for the action update rule of individual $i \in \mathcal{V}$. If i is influenced by dynamic norms at time t, then their next action is fully determined by Eq. (4). Otherwise, the probability that they update their state to $x_i(t+1) = 1$ depends on the state of other individuals through Eq. (3). Using these equations, we can derive an expression for the probability that a generic individual will adopt the innovation as a function that depends only on the individual's current action $x_i(t)$, and on the current and previous fraction of adopters, $z(t)$ and $z(t-1)$, respectively. The proof of this and of all the other results in this paper can be found in the Supplementary Material.

Proposition 1

Let us define

$$\Pi_{k,\alpha}(z) := \sum_{\ell=\lfloor k/(2+\alpha)\rfloor+1}^{k} \binom{k}{\ell} z^{\ell}(1-z)^{k-\ell}. \tag{8}$$

Then, Eq. (5) reduces *to*

$$\mathbb{P}[x_i(t+1)=1] = \begin{cases} \gamma + (1-\gamma)\Pi_{k,\alpha}(z(t)) & \text{if } z(t) > z(t-1) \\ & \text{or } z(t) = z(t-1),\ x_i(t) = 1, \\ (1-\gamma)\Pi_{k,\alpha}(z(t)) & \text{otherwise.} \end{cases} \tag{9}$$

For the purposes of offering a concise presentation of the results, we will focus on scenarios in which individuals establish multiple interactions, $k \geq 2$. When $k = 1$, the mechanism reduces to a simpler unbiased voter model (37) discussed in the Supplementary Material for completeness.

Advantage is key in the absence of dynamic norms

First and as anticipated below Eq. (3), we consider the scenario in which individuals are not sensitive to dynamic norms i.e. $\gamma = 0$. In this scenario, individuals make decisions solely on the basis of a coordination game, and we obtain the following.

Theorem 2

Let us assume that $\gamma = 0$ and $k \geq 2$. Then,

(1) If $\alpha > k - 2$, innovation diffusion occurs for any $\zeta > 0$.
(2) If $-1 + \frac{1}{k-1} < \alpha \leq k - 2$, innovation diffusion occurs if $\zeta > \zeta^*_{k,\alpha}$, while the status quo is maintained if $\zeta < \zeta^*_{k,\alpha}$, where $\zeta^*_{k,\alpha}$ is the unique solution of $\Pi_{k,\alpha}(z) = z$ *in* (0,1).
(3) If $\alpha \leq -1 + \frac{1}{k-1}$, status quo is maintained for any $\zeta \geq 0$.

Our findings in Theorem 2 are consistent with the diffusion literature (3,5,8,9,13,14). If the innovation is significantly better than the status quo ($\alpha > k - 2$), then diffusion occurs irrespective of the fraction of innovators, ζ. For instance, in a scenario in which people makes decisions on the basis of $k = 5$ social contacts —consistent with studies on preventive innovations (2), we require $\alpha > 3$, meaning the innovation must be four times better than the status quo to diffuse. When the relative advantage is not significantly better, i.e. $\alpha \in (-1 + (k-1)^{-1}, k-2]$, then diffusion can be driven by a sufficiently large fraction of innovators in the population, $\zeta > \zeta^*_{k,\alpha}$, as illustrated in Fig. 3. Notice that for innovations with a mild or negligible advantage ($\alpha \leq 1$), the required $\zeta^*_{k,\alpha}$ can increase rapidly to exceed what would be considered reasonable numbers of innovators in real-world situations (7). Of course, we have not accounted for other key ingredients of innovation diffusion, such as network structure effects (8), heterogeneity across the population (38,39), opinion leaders and committed minority (5,14), and social learning processes (3,40). Nonetheless, while these other ingredients may lower the relative advantage needed, it is still typical that a large advantage $\alpha > 0$ is required, as also suggested by the simulations reported in the Supplementary Material. Thus, for certain important innovations, including preventive innovations, that may have small relative advantage, the innovation may still struggle to spread even with the inclusion of the additional ingredients mentioned above (15).

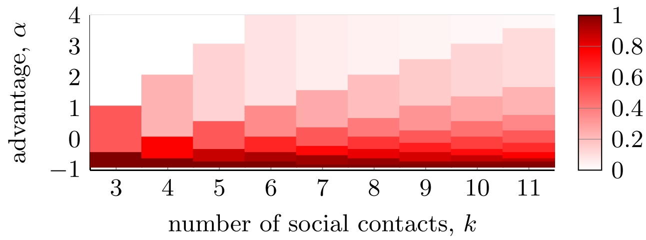

Fig. 3. Numerical computation of the fraction of innovators in the population $\zeta^*_{k,\alpha}$ needed to unlock diffusion, when dynamic norms are absent ($\gamma = 0$), for different values of k and α.

Dynamic norms can unlock diffusion

We now show how dynamic norms can facilitate innovation diffusion. With a sufficiently high sensitivity γ, innovation diffusion will occur in parameter regimes for α and ζ that would normally see the status quo maintained.

Theorem 3

Let us suppose that $k \geq 2$. Then:

(1) If $\alpha > k - 2$, innovation diffusion occurs for any $\zeta > 0$.
(2) If $-1 + \frac{1}{k-1} < \alpha \leq k - 2$ and
  (a) $\gamma < \gamma^*_{k,\alpha}$, innovation diffusion occurs for $\zeta > \zeta^*_{k,\alpha,\gamma}$, while status quo is maintained for $\zeta < \zeta^*_{k,\alpha,\gamma}$;
  (b) $\gamma > \gamma^*_{k,\alpha}$, innovation diffusion occurs for any $\zeta > 0$.
(3) If $\alpha \leq -1 + \frac{1}{k-1}$ and
  (a) $\gamma < \gamma^*_{k,\alpha}$, status quo is maintained for any $\zeta \geq 0$;
  (b) $\gamma > \gamma^*_{k,\alpha}$, innovation diffusion occurs for any $\zeta > 0$,

for the threshold values

$$\gamma^*_{k,\alpha} := \inf\{\gamma \in [0,1] : f_\gamma(z) > 0,\ \forall z \in (0,1)\}, \tag{10}$$

$$\zeta^*_{k,\alpha,\gamma} := \inf\{\zeta \in [0,1] : f_\gamma(z) > 0,\ \forall z \in (\zeta,1)\}, \tag{11}$$

where $f_\gamma(z) := (1-\gamma)\Pi_{k,\alpha}(z) - z + \gamma$.

Next, we examine some important aspects of our theoretical results, and discuss their real-world implications for innovation diffusion. We also illustrate how even small sensitivity γ, below the threshold value $\gamma^*_{k,\alpha}$, can still facilitate innovation diffusion.

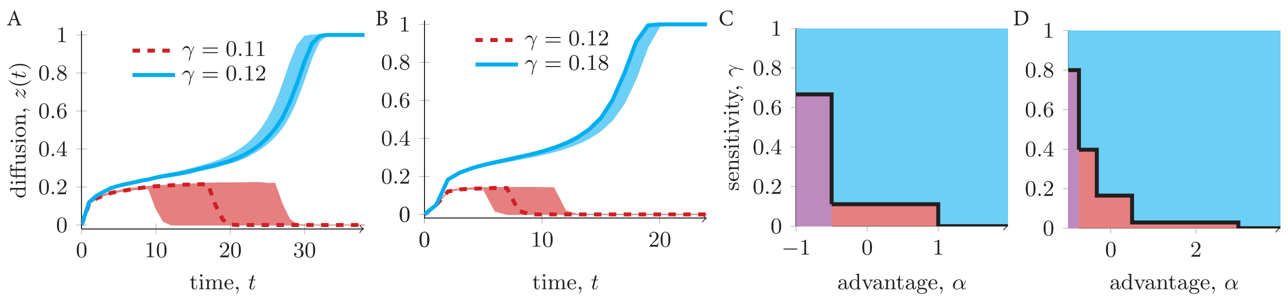

Fig. 4. In (A) and (B), we show simulated trajectories of the diffusion process with $n = 100,000$ individuals, $\alpha = -0.2$, $\zeta = 0.05$, and (A) $k = 3$ and (B) $k = 5$. Shaded areas are the envelopes of 100 independent simulations. In (C) and (D), we illustrate all the possible outcomes for (C) $k = 3$ and (D) $k = 5$, respectively, for different combinations of α and γ. In the cyan region, innovation diffusion is always guaranteed irrespective of the fraction of innovators in the population ζ; in the red region, innovation diffusion occurs only if ζ is sufficiently large, otherwise status quo is maintained; in the violet region, status quo is always maintained. The black curve is the threshold $\gamma^*_{k,\alpha}$.

Weak and lackluster innovations

A key consequence of Theorem 3 is that, for any k and α, there exists a threshold $\gamma^*_{k,\alpha}$ such that the innovation will successfully diffuse for any fraction of initial innovators, ζ, if the sensitivity $\gamma > \gamma^*_{k,\alpha}$. This has notable consequences for innovations with small relative advantage or even for inefficient innovations that are not beneficial to the population ($\alpha < 0$), such as some managerial fads and fashions (41). Ensuring the population is sufficiently sensitive to dynamic norms offers a pathway to innovation diffusion for weak and lackluster innovations.

We illustrate this through some examples presented in Fig. 4. Trajectories of the diffusion process in Fig. 4A and B for an innovation with negative advantage ($\alpha = -0.2$) and different numbers of social contacts reveal that innovation diffusion is achieved if the sensitivity to dynamic norms is sufficiently high (cyan curves). The case of $k = 5$ is interesting as we now explain. After Theorem 2, we discussed that with $k = 5$ and no dynamic norms, a significant relative advantage would be necessary to unlock innovation diffusion. Fig. 4B illustrates that dynamic norms can instead enable innovation diffusion for $k = 5$, even in the presence of a (mild) relative disadvantage. We also note that, if we change the number of social contacts, the sensitivity to dynamic norms needed to unlock innovation diffusion may be affected. For instance, while $\gamma = 0.12$ is sufficient to unlock diffusion when $k = 3$ (cyan curve in Fig. 4A) as $\gamma^*_{3,-0.2} = 1/9$, it yields status quo maintenance for $k = 5$ (red curve in Fig. 4B). However, a threshold value always exists such that innovation diffusion is guaranteed if γ is greater than the threshold (in our example, we numerically estimate $\gamma^*_{5,-0.2} \approx 0.1652$). A better understanding of such a threshold behavior can be grasped by observing Fig. 4C and D, in which we depict the trade-off between the relative advantage α and the sensitivity γ needed to unlock innovation diffusion for $k = 3$ (computed analytically in the Supplementary Material) and $k = 5$ (computed numerically).

Reducing the number of required innovators

A key factor in driving diffusion, besides the size of the relative advantage, α, is the fraction of innovators in the population, ζ. Having established that dynamic norms can help weak and lackluster innovations overcome having a small or negative α, and successfully diffuse, we now show that dynamic norms can also reduce the fraction of innovators in the population ζ necessary for diffusion. From Eq. (11) and the expression of $f_\gamma(z)$ in Theorem 3, we can easily conclude the following corollary.

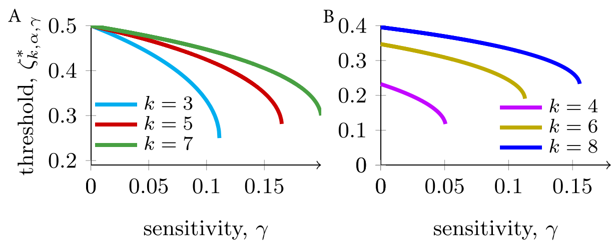

Fig. 5. Analytical ($k = 3$) and numerical ($k > 3$) computation of the threshold $\zeta^*_{k,\alpha,\gamma}$ as a function of γ for (A) $\alpha = 0$ and (B) $\alpha = 0.2$.

Corollary 4

The threshold $\zeta^*_{k,\alpha,\gamma}$ in Eq. (11) is monotonically decreasing with γ.

Corollary 4 suggests that dynamic norms can help facilitate innovation diffusion even when it is not sufficient to drive diffusion on its own (i.e. γ is below the threshold $\gamma^*_{k,\alpha}$). In fact, increasing γ reduces the fraction of innovators in the population, $\zeta^*_{k,\alpha,\gamma}$, needed to guarantee innovation diffusion. Fig. 5 elucidates this important role of dynamic norms by reporting the value of the threshold $\zeta^*_{k,\alpha,\gamma}$ for increasing values of sensitivity to dynamic norms $\gamma < \gamma^*_{k,\alpha}$, in the presence of no or small relative advantage.

This conclusion has particular relevance for innovations in the context of social conventions. The benefits of a convention, such as how we greet one another, are primarily tied to whether others also adopt it (coordination), and hence $\alpha = 0$ is expected. The spontaneous emergence of new social conventions and the replacement of status quo conventions with a new alternative are often studied by assuming the presence of a committed minority or initial set of seeding innovators (5,14). Making the population sensitive to dynamic norms can thus help facilitate a smaller, committed minority to still affect social convention change.

Robustness of the threshold behavior

Our theoretical analysis exhibited a threshold behavior with respect to the sensitivity to the dynamic norms parameter γ. If people are sufficiently sensitive, then diffusion of an innovation can be unlocked, even in the absence of a positive relative advantage or many initial supporters. We performed Monte Carlo numerical simulations to investigate the robustness of such a threshold behavior with respect to the addition of features in the mathematical model. Specifically, we introduced a backbone network structure that constrains who individuals can interact with, heterogeneity in the number of contacts established by each individual, and hetrogeneity in the relative advantage across the population. The details of our numerical analysis are presented in the Supplementary Material. The results reported therein suggest that the threshold behavior is a universal feature of the mathematical model, confirming the key role that dynamic norms can play in facilitating innovation diffusion. Interestingly, the critical value of sensitivity γ^* at which the phase transition occurs seems to be affected by the network structure and by the heterogeneity in the distribution of relative advantages across the population, paving the way for several avenues of future research.

Discussion

We have used mathematical modeling to operationalize dynamic norms, viz. an individual's sensitivity to growing trends, within the context of innovation diffusion. Our theoretical analysis of the model identified a threshold behavior with respect to the sensitivity. Above the sensitivity threshold, innovation diffusion occurred independently of the number of innovators in the population and social contacts. Below the threshold, sensitivity lowered the relative advantage and number of innovators necessary to unlock diffusion. Numerical simulations suggest this threshold behavior is a characteristic feature of the model, as it is robust to changes in network structure and population heterogeneity in the relative advantage and social contacts.

To date, empirical methods such as experiments and field interventions have established that, dependent on a variety of factors, dynamic norms can have a significant effect or limited to no effect—this is consistent with the broader literature on social norm interventions (42,43). Ongoing efforts aim to characterize the effect of interventions involving exposure to dynamic norm information for different innovations (technology or sustainable practice), contexts (online or face-to-face, and target population characteristics), and the content and framing of the message (27). Our work can thus be viewed as complementary to these empirical approaches. The latter can help establish the amount of individual-level sensitivity of dynamic norms for the scenario and context of interest, while our model can help explore their downstream population-level consequences and inform policymakers and practitioners of the level of intervention necessary to facilitate diffusion.

Since we have demonstrated that dynamic norms can facilitate diffusion, scientists and practitioners may find particular relevance and interest in considering dynamic norm effects in the context of preventive and sustainable innovations (15,19), new social conventions (3,14), and fads and fashions (41,44). A new social convention can emerge if the committed minority promoting it reaches a critical mass (5,14), with literature reporting values up to 40% of the total population. This is a significant proportion of the population and may not be reached in many scenarios. Moreover, committed minority can often feel outcast and ostracized, including proponents of sustainable practices such as reducing meat consumption or improving recycling and reusing (16–18). Relying on a critical mass of committed minority for such innovations may thus yield limited results, whereas developing strategies centered around enhancing people's awareness of changing trends may be more effective.

A key challenge that inhibits the diffusion of beneficial preventive innovations (7), such as family planning and contraceptives, wearing seat-belts, and immunization is that the relative advantage could be small, since there are often no immediate and clear benefits for adopting the innovation. Our findings suggest that public authorities may consider exploiting dynamic norms, e.g. through specially crafted advertisement and messaging campaigns, to overcome the inherent challenges associated with preventive innovation diffusion. On the other hand, our findings also suggest that a population's sensitivity to dynamic norms may also favor the widespread adoption of disadvantageous or inefficient innovations, under some circumstances. Industry organizations have been observed to quickly adopt a given managerial fad or fashion, and then equally quickly adopt the next given fad or fashion to replace it, even if such fads and fashions are inefficient or counter-productive (41). Our analysis reveals that a sequence of successful innovation diffusion outcomes can occur with a high sensitivity to dynamic norms as might be the case if such organizations aim to present an image of being "innovators and industry leaders," or are exposed to "bandwagon pressures" (41).

Despite its generality, there are limitations of the model that we discuss in the following and which suggest further directions of research. First, our framework is build upon the assumption that individuals who revise their action according to a coordination mechanisms follow a myopic best-response. Such a mechanism, however, does not allow to separate the two important game-theoretic concepts of risk dominance and Pareto efficiency of an action (4). Therefore, the proposed formalism cannot be directly employed to analyze situations in which the risk-dominant action and the Pareto efficient one are different; the former is preferred by the individual, while the latter is beneficial for the population. Such a scenario may describe several important real-world social problems, such as in weakest-link games and coordination failures, where it has been empirically observed that risk dominance can shape dynamics, leading toward Pareto-inferior actions (45–47). Policymakers may wish to promote an innovation that is Pareto efficient but not risk dominant, as may be the case for certain sustainability practices or innovations (48). One could explore further theoretical tools to incorporate different revision protocols within our mathematical framework such as those based on imitation (49), where risk dominance and Pareto efficiency can be decoupled by adjusting the payoff function for the coordination game to a more general formulation, and possible tensions between them can be investigated.

Moreover, we have assumed that all the individuals have the same relative advantage α. In this scenario, there is a perfect alignment between an individual's payoff and the aggregate population payoff. Intuitively, such an action is simultaneously preferred by each individual and beneficial for the entire population if $\alpha > 0$. In the last section, we showed that the threshold behavior with respect to the sensitivity to dynamic norms γ is robust with respect to the presence of heterogeneous α_i. However, such a scenario implies that individuals may disagree on the action they prefer to adopt. While providing an univocal definition of what is beneficial for the population in such a scenario is nontrivial, a possible option within the modeling framework is to define the action that benefits the population as the one that maximizes the aggregate payoffs. However, this would only hold under the assumption that the society can redistribute the payoff ex-post to compensate for those individuals whose payoff has instead decreased.

Dynamic norms as examined through empirical studies and implemented in our model are a form of descriptive norm, i.e. an individual determines what is the normative behavior based on information about the actions of others (24). Our model assumes that individuals have access to complete, ongoing, and unbiased information regarding the trend, which reflects many real-life scenarios in which the number of innovation adopters or emerging trends are publicly available. However, in other cases, individuals may instead have a perceived dynamic norm based on partial information. As with other descriptive norms, such an estimation process can introduce bias and distortion, whereby the perceived norms may differ from the true norm. For instance, individuals may underestimate or overestimate trends if information is obtained only from their social circle, especially in populations where social interactions are highly fragmented or clustered. Moreover, the presence of heterogeneity in an individual's visibility and ability to influence others may lead to further biases, whereby changes in the behavior of a few opinion leaders or highly visible minorities may be perceived as larger trends, and thus such individuals may have a strong role in shaping the diffusion process. Studies on the social-psychological mechanisms that govern how an individual perceives dynamic norms and incorporating such findings into the modeling framework are desirable. This knowledge can aid in designing intervention policies that reduce potential biases that could limit the efficiency of leveraging dynamic norms for innovation diffusion.

Several further research directions emerge from our theoretical results. First, our preliminary numerical simulations revealed that the threshold behavior is robust, but can be shaped by the backbone network structure and population heterogeneity. Further detailed study is desirable to identify for instance, the network structures that may accelerate or stop innovation diffusion (8,9,13,50). Previous work in (14) provided empirical evidence of dynamic norms in the diffusion of new social conventions and used numerical simulations to study an agent-based model parametrized from the data (14). It is desirable to continue such efforts to integrate empirical and modeling methods for the various innovations and contexts discussed above and for studying the potential interplay between dynamic norms and other factors that could be present in human decision making such as personal preferences and beliefs (4). Last, dynamic norms offer a new dimension in developing strategies for promoting innovation diffusion: besides identifying the size of the critical mass needed (4,5,14) and the optimal location in the network to place innovators (51), we may also consider optimizing the timing of the introduction of innovators and exposing people to information on dynamic norms (27) as a key method to generate a trend, make people sensitive to it, and, ultimately, unlock innovation diffusion and social change.

Materials and methods

Action and innovation

The term "action," which is taken from the game-theoretic literature, may represent a wide range of possible binary and mutually exclusive choices, depending on the particular scenario under consideration. In the diffusion of new hybrid seed corn (52) the status quo may represent regular seed corn, while the innovation represents hybrid seed corn with improved crop yield. In the context of social conventions such as greetings, the status quo may represent handshaking, while the innovation captures elbow-bumping that has recently appeared due to the COVID-19 pandemic. In relation to behaviors, the status quo and innovation may represent not fastening and fastening seat belts while in a car, respectively (15).

The term "innovation" does not necessarily imply that action 1 is better than action 0. Indeed, the relative advantage α may be negative, which would suggest that action 1 could provide a lower payoff than action 0. Nor does the term imply that the action 1 is "objectively" new. Rather, and consistent with the definition by Rogers (7), the term "innovation" simply refers to the fact that action 1 is perceived as newly introduced to the population currently adopting the status quo action 0.

Coordination games

Coordination games on networks have been widely used to study innovation diffusion (3,4,8,9,34). We briefly review how Eq. (3) is derived. At each time step t, individual $i \in \mathcal{V}$ initiates k interactions among their neighbors, and engages in a symmetric 2-player pure coordination game with each one of them (34). In the coordination game, a player i that interacts with a player j will receive a payoff characterized by the payoff matrix:

$$\begin{array}{c} \\ x_j = 1 \\ x_j = 0 \end{array} \begin{array}{c} x_i = 1 \quad x_i = 0 \\ \begin{bmatrix} 1+\alpha & 0 \\ 0 & 1 \end{bmatrix} \end{array}. \tag{12}$$

By summing up the payoffs from all k games, we can write the payoff for individual i playing action $s \in \{0, 1\}$ given action state $\mathbf{x}(t)$ to be

$$\pi(s, \mathbf{x}(t)) = \sum_{j \in \mathcal{N}_i(t)} [s \quad 1-s] \begin{bmatrix} 1+\alpha & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_j(t) \\ 1-x_j(t) \end{bmatrix}, \tag{13}$$

which is then easily seen to decompose into Eqs. (1) and (2).

We remark that if only the coordination mechanism is present, i.e. if the sensitivity $\gamma = 0$, our proposed model reduces to a (possibly biased) k-majority dynamics on a complete network. See (53,54) for more details.

Time-varying network

The coordination game mechanism induces a directed time-varying (multi-)graph $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$, where $(i, j) \in \mathcal{E}(t) \iff j \in \mathcal{N}_i(t)$. Since each interaction is sampled independently of other interactions in $\mathcal{V}$, for any individual i, the same individual can appear multiple times in $\mathcal{N}_i(t)$ and, consequently, multiple occurrences of the link $(i, j) \in \mathcal{E}(t)$ could be present. A realization of such a process is illustrated in Fig. 1. Note that the graph formation process is similar to the one of directed discrete-time activity-driven networks (55,56).

The parameter k reflects the number of social interactions established by each individual, whenever they are revising their action through the coordination mechanism in Eq. (3) at time t. We limit each individual to k interactions, to capture the fact that individuals generally make use of only a limited amount of information during their decision-making processes (2,7). The value of k can depend on the particular diffusion scenario being considered; different innovations may be more or less observable by others in the populapation. For instance, in the context of rioting (57), we would expect an individual to identify a large number of contacts who are rioting (innovation) and who are not (status quo). On the other hand, in the context of family planning (2), an individual would only have knowledge of the family planning status for a limited number of individuals, e.g. very close friends and family. Thus, the value of k in the former example would be larger than in the latter.

Supplementary Material

Supplementary material is available at *PNAS Nexus* online.

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Authors' Contributions

L.Z., M.Y., and M.C. designed research; L.Z. and M.Y performed research; L.Z. contributed new analytic tools; L.Z., M.Y., and M.C. analyzed data; and L.Z. and M.Y. wrote the paper with inputs from M.C.

Previous Presentation

Some preliminary results were presented at the 2021 IEEE Conference on Decision and Control, December 13−17, 2021 (58).

Data Availability

The code used and the data underlying this article are available at Zenodo [doi.org/10.5281/zenodo.6793397].

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