ORIGINAL PAPER

Preferred hospitalization of COVID-19 patients using intuitionistic fuzzy set-based matching approach

Amalendu Si[1] · Sujit Das[2] (iD) · Samarjit Kar[3]

Abstract
Preferable hospitalization of COVID-19 patients has become an urgent and challenging task to save lives amidst the unexpected rising of the 3rd wave, where fuzzy set and matching techniques are considered due to their inherent capability to deal with uncertain suitable pair selection. The matching technique has been widely used to solve decision-making problems due to its capability to determine the suitable pair between the objects of two disjoint sets, whereas fuzzy set is well known to manage uncertain situations. This paper extends the matching technique using fuzzy set and proposes a novel fuzzy matching approach to solve uncertain decision-making problems. We also extend the fuzzy matching approach in the framework of an intuitionistic fuzzy set. A relation between the matching technique and fuzzy set theory is established by developing the preference sequence of the elements. The fuzzy entropy is used to measure the closeness among the elements between two distinct sets. Applicability of the proposed approach is measured by providing an illustrative case study concerned with the preferred hospitalization of the COVID-19 patients. Finally, a comparative study is given to analyze the effectiveness of the proposed approach, where the intuitionistic fuzzy set-based matching approach shows better performance compared to fuzzy and conventional matching based approach. For experimentation purpose, this study uses 9424 patients and 234 hospitals with a total available capacity of 18,024 beds.

Keywords Fuzzy matching · Intuitionistic fuzzy matching · Fuzzy entropy · COVID-19 · Preferred hospitalization

1 Introduction

Matching problems related to preference have been broadly studied in computer science, mathematics, marketing and economics through the publication of the seminal paper by Gale and Shapley in 1962 (Gale and Shapley 1962). They introduced a model to match two elements from two separate sets. The developed model provided the probable solution for the college admission problem. The authors defined a stable matching technique and used it to analyze the importance of two-side matching. The algorithm proposed in (Gale and Shapley 1962) followed the deferred acceptance (DA) mechanism. This proposed algorithm has been applied to solve various problems such as school selection problem, medical resident allocation, college admission, etc. Due to the special features of this procedure, it has been continually used by researchers to generate new conceptual options to solve real-life decision problems. Motivated by the wide applicability of the matching technique, many computer scientists, mathematicians, and many others domain experts extended the model to solve specifically computer science and mathematical problems according to the problem type and criteria. Liu and Ma (2015) proposed a decision model based on uncertain preference sequences, where a data processing method was used to generate uncertain preference sequences, which in turn was applied to compute the

✉ Sujit Das
  sujit.das@nitw.ac.in

  Amalendu Si
  amalendu.si@gmail.com

  Samarjit Kar
  samarjit.kar@maths.nitdgp.ac.in

[1] Department of Computer Science and Engineering, Haldia Institute of Technology, Haldia 721657, India

[2] Department of Computer Science and Engineering, NIT Warangal, Warangal 506004, India

[3] Department of Mathematics, NIT Durgapur, Durgapur 713209, India

preference distance of each matching pair. Similarly, Axtell and Kimbrough (2008) introduced unconventional and distributed matching techniques where the authors claimed to provide better functionality over the conventional deferred acceptance matching. The authors argued that this matching procedure was suitable for the problem, depending on the combination of several properties. A set of identical single-sided matching techniques-based algorithms were highlighted in (Lo and Wilson 2017), where the authors assigned a performance criterion associated with symmetry and analyzed the performance of queue-based algorithms which was found to be better than that of the stack-based algorithm.

Zadeh introduced the fuzzy sets (FS) theory in 1965 to overcome the limitation of the crisp sets, which is deduced as either one or zero to represent true or false, respectively (Zadeh 1965a, b). The FS provides a well-organized method for constructing uncertain decision-making systems with the help of uncertain information, where the uncertain information is used to represent experts' opinions. A number of researchers are contributing consistently towards the development of fuzzy set-based research works. Presently application of fuzzy set theory is found in many areas such as artificial intelligence (Hung and Yang 2004), decision-making (Xu and Yager 2008; De et al. 2019, 2020; Chellamani et al. 2021; Sun et al. 2021), optimization (Kumar and Kaur 2012), Pattern Recognition (Hung and Yang 2004; Gupta and Kumar 2021), weather forecasting (Tham et al. 2002), marketing (Patra 2022), medical science (Liu and Wang 2007; Sun et al. 2021; Verma and Rohtagi 2022), transportation (Kumar and Kaur 2012), social network analysis (Poulik and Ghorai 2021), etc. Fuzzy sets are considered as precise mathematical tools for processing data which is derived from vague information. The vagueness of language and its mathematical representation and processing is one of the major areas of study in fuzzy set theory. After the introduction of fuzzy sets in 1965, the researchers modified the fuzzy set in various ways to manage the unexpected situations, which are unable to handle by the fuzzy sets. Some of the extension of fuzzy sets are fuzzy soft sets (Cagman et al. 2011), intuitionistic fuzzy sets (Das et al. 2018), inter value fuzzy sets (Gorzałczany, 1987), multi-fuzzy sets (Das et al. 2013), fuzzy multi sets (Das et al. 2013), picture fuzzy sets (Si et al. 2019; Amanathulla et al. 2021), and adaptive neuro-fuzzy (Cabalar et al. 2012). The fuzzy set was generalized by Atanassov (1986) and converted into intuitionistic fuzzy sets (IFS). The IFS include hesitation margin as a degree of hesitation (hesitation margin is equal to complement of the sum of membership and non-membership degrees). Due to the inclusion of hesitation margin, IFS has become attractive and more useful to solve various real-life problems. The idea and semantic depiction of IFS

are more significant, imaginative and appropriate due to the introduction of the belongingness degree, non-belongingness degree, and hesitation margin. The authors in (Szmidt and Kacprzyk 2001) demonstrated that IFS is fruitful when explaining the problem by a linguistic variable which is given in terms of a membership function and the membership function alone is inadequate to represent the situation. Due to the flexibility of IFS in handling uncertainty, it has wide applicability under imperfectly defined facts and imprecise knowledge.

Entropy plays an important role in decision making application particularly in case of uncertainty when the uncertainty is expressed by fuzzy sets. The fuzzy set-based entropy illustrates the fuzziness scale of fuzzy sets (Zadeh 1965a, b). In 1965, Zadeh introduced the concept of fuzzy entropy (Zadeh 1965a, b), where the Shannon entropy of random variables (Vajda 1950) was modified and fitted in fuzzy environment. Luca and Termini (1972) introduced the axiom construction of entropy of fuzzy sets and referred to Shannon's probability entropy, and interpreted it as a measure of the amount of information. Kaufmann and Magens (1975) denoted that the entropy of a fuzzy set can be obtained by the distance from a fuzzy set to it is nearest non-fuzzy set. Similarly, Higashi and Klir (1982) followed the same idea to denote entropy using the distance between a fuzzy set and its complement. Trillas and Riera (1978) proposed the procedure to identify various type of entropies for finite fuzzy sets. Loo (1977) proposed a definition of entropy in respect of a fuzzy system by measuring the amount of information and distance between a fuzzy set and a nearest non-fuzzy set. Xuecheng (1992) provided the definitions of fuzzy similarity, distance and entropy. Fan and Ma (2002) determined the fuzzy entropy by measuring distance according to the definitions of fuzzy entropy and distance measurement procedure. Li and Liu (2006) announced a well-accepted definition of entropy in the context of fuzzy set-based on the axiomatic definition of fuzzy similarity, distance and entropy. Recently, Kadian and Kumar (2021) have introduced a novel fuzzy entropy measure for better discriminating the various fuzzy sets. Entropy helps us to measure the fuzziness degree of a collection of fuzzy membership values. Cross entropy is used to measure the relative closeness between two identical sets.

From late 2019, the world observed the outbreak of the novel coronavirus at Wuhan in the republic of China (Ren et al. 2020), and since then, it had a rapid spread all over the world from the beginning of the following year. This coronavirus hampered the normal activities of human civilization due to its uncontrolled and unpredicted behavior. People over the world became helpless, and their busy lifestyle came to an end. They were quarantined to isolate them from the coronavirus (Ren et al. 2020). Researchers

are continuously working to find the possible solutions. They are trying to dominate the virus in many ways like inventing effective treatment, destroying the virus, or protecting it. Mishra et al. (2020) studied the core area among the four mega-cities Delhi, Kolkata, Mumbai and Chennai in India and explored the mandatory pandemic precautions such as social distancing and lockdown. According to them, most of the underprivileged households in those four metro cities faced some difficulties to maintain physical distancing and isolation; they have the urgency to move out from the safe zone to collect water, food and to attend the nature's call. Si et al. (2021) proposed a decision-making method for selecting preferable medicine for the appropriate treatment of COVID-19 patients in a picture fuzzy environment through the hybrid approach of grey relational analysis and Dempster–Shafer theory. The government of every coronavirus-affected country has taken the necessary precautions, and several communities throughout the world are in cooperation with each other to jointly handle this pandemic situation. Despite all possible endeavor, there is no approved treatment available to serve the coronavirus-affected patients. The patients are also trying to get quality clinical service and top-class treatment based on their experience, financial status, and availability of resources. Patients and their family members are rushing towards the hospitals to get the necessary treatment. But the a few hospital managements were failed to provide the quality services to all of the patients and confronted problems regarding patients' selection because of their limited capacity.

This paper proposes a novel and unique matching technique in the domain of fuzzy set and intuitionistic fuzzy set to investigate the hospitalization of COVID-19 patients with difficult situation to mild symptoms. During the emergency crisis of COVID-19, the conventional procedure was found to be difficult to maintain the ranking of the patients and hospitals in a dynamic system to match each other or admit the patients within suitable hospitals. It was too complex for the hospitals to investigate the patient's condition precisely at the time of admission. Thereafter the admission procedure of the infected patients into the corresponding hospitals were not implemented properly. As a result, there was a possibility for the admission of highly infected patients to less equipped hospitals and less infected patients to highly equipped hospitals. Consequently, it was difficult to provide the quality treatment to severely infected patients as there was a chance that the bed with modern treatment equipment might have been booked by the mild symptom patients. Hence, there is need to represent the condition of the patients as well as the status of hospitals using fuzzy set or intuitionistic fuzzy sets to represent the condition of the patients as well as the status of hospitals. We estimate the fuzziness degree of the patients and hospitals using the entropy, and cross-entropy is applied to measure the relative closeness between patient and hospital. Fuzziness degree of the patients are analyzed by the health experts by observing the present symptoms and age of the patients along with past history of the patients regarding comorbidity. Similarly, the fuzziness degrees of the hospitals are assigned based on the treatment quality, ICU/HDU units, ventilator support and availability of experienced doctors and nurses. Fuzziness degree of patients and hospitals make the process smooth and assist the health department to execute a fair admission for providing better treatment. Initially, we consider the fuzzy set to represent the patient's condition and status of hospital. Then to improve the accuracy, we use intuitionistic fuzzy set to consider the level of infection of the patient and to represent strength and weak side of the hospital through the membership and non-membership degrees, respectively. Next entropy and cross-entropy are applied to measure the relative closeness between patient and hospital. The entropy values of patients and hospitals are considered as the threshold level, and the patients and hospitals are classified into two groups based on individual threshold values. The patients with conditional degree greater than the threshold and hospital with a status value greater than the threshold are provided more importance than the other patients and hospitals, respectively. All the hospitals have a finite capacity to admit the maximum number of patients. We use the distance measurement procedure to measure the difference between patient and hospital regarding patient's condition and hospital's status. If the difference is less than the predefined tolerance value and the hospital is not full, then the patient can take admission within the hospital. We use the proposed matching technique for hospitalization of coronavirus-affected patients for providing better and balance medical services among the various types of COVID-19 patients.

The main contributions of this paper can be summarized as follows:

- Extending the conventional matching technique in the context of fuzzy set, we propose fuzzy matching technique.
- We have also proposed intuitionistic fuzzy set-based matching technique by extending the proposed fuzzy matching technique.
- Fuzzy set-based entropy measure is used in the process to compute severity of patients and status of hospitals.
- Performance of the proposed matching techniques has been analyzed by performing comparative analysis with the conventional matching technique.

The paper is organized as follows: The basic concepts of fuzzy sets theory and entropy are reviewed in Sect. 2. The

basic concepts about matching are discussed in Sect. 3. The proposed fuzzy matching technique is described in detail in Sect. 4. Then the proposed fuzzy matching is extended using IFS in Sect. 5. Conventional matching is given in Sect. 6. In Sect. 7, one case study on the preferable hospitalization for COVID-19 patients is provided to show the effectiveness of the proposed method. Analysis and discussion are presented in Sect. 8. The paper summarizes the concluding remarks in Sect. 9.

2 Preliminaries

This section briefly discusses fuzzy sets, intuitionistic fuzzy sets and entropy.

2.1 Fuzzy set

The fuzzy set theory describes and represents the uncertainty of real-life situations (Zadeh 1965a, b). The fuzzy set A in U is defined as a set of ordered pairs: $A = \{(u, \mu_A(u)|u \in U)\}$, where each element $u \in U$ is assigned to a real value, which is called the degree of membership and belongs to $[0, 1]$, and U be the universe of discourse. The membership function $\mu_A(u)$ denotes the characteristic of the object u (Si et al. 2019) or the degree of the belongingness of an object in a set. The fuzziness nature of the object is represented through the use of membership degree. The three basic operations of the fuzzy set are complement, intersection and union. These operations are usually referred to as standard fuzzy set operations and have a special significance in fuzzy set theory. The standard complement $\overline{A}$ of the fuzzy set A with respect to the universal set U is defined as $\overline{A}(u) = 1 - A(u)$ for all $u \in U$. The elements of U for which $\overline{A}(u) = A(u)$ are called equilibrium points of A. The cardinality ($|A|$) for a finite fuzzy set A is defined as $|A| = \sum_{u \in U} \mu_A(u)$, where the relative cardinality of fuzzy set A is evaluated by $\|A\| = |A|/|U|$. The fuzzy sets theory provides a mathematical framework that sharply manages the imprecision, uncertainty and vagueness of the data sets raised during the data collection due to lack of knowledge and incomplete information (Zimmermann 2010). One useful concept in the context of fuzzy set theory is $dist(\cdot, \cdot)$, which is defined as $dist(\mu_A(u), \mu_B(u)) = |\mu_A(u) - \mu_B(u)|$ and is used to measure the distance between two membership values, where $\mu_A(u)$ and $\mu_B(u)$ are two membership values. The $dist(\cdot, \cdot)$ function maintains the following properties:

1. Positiveness: $dist(\mu_A(u), \mu_B(u)) \geq 0$ and $dist(\mu_A(u), \mu_B(u)) = 0$ if and only if $\mu_A(u) = \mu_B(u)$.
2. Symmetry: $dist(\mu_A(u), \mu_B(u)) = dist(\mu_B(u), \mu_A(u))$.
3. Triangle inequality: $dist(\mu_A(u), \mu_B(u)) + dist(\mu_B(u), \mu_C(u)) \geq dist(\mu_A(u), \mu_C(u))$.

The standard intersection $A \cap B$, and standard union $A \cup B$ between two fuzzy sets A and B are, respectively, defined as $(A \cap B)(u) = \min[A(u), B(u)]$ and $(A \cup B)(u) = \max[\mu_A(u), \mu_B(u)]$ for all $u \in U$, where *min* and *max* denote the minimum operator and the maximum operator, respectively. Due to the associativity of *min* and *max*, these definitions can be extended to any finite number of fuzzy sets. The union (logical OR) and intersection (logical AND) of fuzzy sets are extended into different way based on some properties like t-conorm and t-norm. These types of extended operators are associative, monotonic and commutative. Some of the typical dual pair's operators as follows:

1a. drastic product:

$$t(\mu_A(u), \mu_B(u)) = \begin{cases} \dfrac{\min\{\mu_A(u), \mu_B(u)\} \ \ if \ \max\{\mu_A(u), \mu_B(u)\} = 1}{0 \ \ otherwise} \end{cases}.$$

1b. drastic sum

$$s(\mu_A(u), \mu_B(u)) = \begin{cases} \dfrac{\max\{\mu_A(u), \mu_B(u)\} \ \ if \ \min\{\mu_A(u), \mu_B(u)\} = 0}{1 \ \ otherwise} \end{cases}.$$

2a. bounded difference:

$$t_b(\mu_A(u), \mu_B(u)) = \max\{0, \ \mu_A(u) + \mu_B(u) - 1\}.$$

2b. bounded sum:

$$s_b(\mu_A(u), \mu_B(u)) = \min\{1, \ \mu_A(u) + \mu_B(u)\}.$$

3a. Hamacher product:

$$t_H(\mu_A(u), \mu_B(u)) = \frac{\mu_A(u).\mu_B(u)}{\mu_A(u) + \mu_B(u) - \mu_A(u).\mu_B(u)}.$$

3b. Hamacher sum:

$$s_H(\mu_A(u), \mu_B(u)) = \frac{\mu_A(u) + \mu_B(u) - 2.\mu_A(u).\mu_B(u)}{1 - \mu_A(u).\mu_B(u)}.$$

Atanassov (1999) proposed intuitionistic fuzzy set (IFS) which considered the non-membership degree to measure the dissatisfaction degrees with membership degree. IFS $\hat{A}$ in the universe of discourse U is defined as $\hat{A} = \{(u, \mu_{\hat{A}}(u), v_{\hat{A}}(u)|u \in U)\}$, where $\mu_{\hat{A}}(u)$ be the membership function and $v_{\hat{A}}(u)$ be the non-membership function, and $0 \leq \mu_{\hat{A}}(u) + v_{\hat{A}}(u) \leq 1$. Furthermore, we call $\pi_{\hat{A}}(u) = 1 - \mu_{\hat{A}}(u) - v_{\hat{A}}(u)$ be the IFS index or hesitancy degree of u. For a particular element u, $u \in U$, then an object $\{\mu_{\hat{A}}(u), v_{\hat{A}}(u)\}$ is usually called the intuitionistic fuzzy number (IFN). Consider two IFSs $\hat{A}$ and $\hat{B}$, where $\hat{A} = \{(u, \mu_{\hat{A}}(u), v_{\hat{A}}(u)|u \in U)\}$

$\hat{B} = \{(u, \mu_{\hat{B}}(u), v_{\hat{B}}(u)|u \in U)\}$. Some necessary relations (Li 2012) between then are narrated below.

a. $\hat{A} \subseteq \hat{B}$ if and only if $\mu_{\hat{A}}(u) \geq \mu_{\hat{B}}(u), v_{\hat{A}}(u) \leq v_{\hat{B}}(u), \forall u \in U$.

b. $\hat{A} = \hat{B}$ if and only if $\hat{A} \subseteq \hat{B}$ and $\hat{B} \subseteq \hat{A}$.

c. The complementary set of $\hat{A}$ denoted by $\overline{\hat{A}}$, is $\overline{\hat{A}} = \{(u, v_{\hat{A}}(u), \mu_{\hat{A}}(u)|u \in U)\}$.

d. $\hat{A} \leq \hat{B}$ called $\hat{A}$ less fuzzy than $\hat{B}$, i.e., for $\forall u \in U$,
   If $\mu_{\hat{B}}(u) \leq v_{\hat{B}}(u)$, then $\mu_{\hat{A}}(u) \leq \mu_{\hat{A}}(u), v_{\hat{A}}(u) \geq v_{\hat{B}}(u)$;
   If $\mu_{\hat{B}}(u) \geq v_{\hat{B}}(u)$, then $\mu_{\hat{A}}(u) \geq \mu_{\hat{A}}(u), v_{\hat{A}}(u) \leq v_{\hat{B}}(u)$.

e. The Hamming distance between $\hat{A}$ and $\hat{B}$ is denoted by $d(\hat{A}, \hat{B})$

$$d(\hat{A}, \hat{B}) = \frac{1}{2} \sum \left(|\mu_{\hat{A}}(u) - \mu_{\hat{B}}(u)| + |v_{\hat{A}}(u) - v_{\hat{B}}(u)| + |\pi_{\hat{A}}(u) - \pi_{\hat{B}}(u)| \right).$$

f. The Euclidean distance between $\hat{A}$ and $\hat{B}$ is denoted by $d(\hat{A}, \hat{B})$

$$d_E(\hat{A}, \hat{B}) = \frac{1}{2} \sum \left((\mu_{\hat{A}}(u) - \mu_{\hat{B}}(u))^2 + (v_{\hat{A}}(u) - v_{\hat{B}}(u))^2 + (\pi_{\hat{A}}(u) - \pi_{\hat{B}}(u))^2 \right)^{\frac{1}{2}}.$$

2.2 Fuzzy entropy

The entropy of a fuzzy set describes the fuzziness degree of a fuzzy set. However, according to the user point of view, the entropy of fuzzy variables satisfies the following three basic requirements: (a) entropy of a crisp number is minimum, i.e., 0, (b) entropy of an equipossible fuzzy membership degree is maximum, and (c) entropy is applicable not only to finite and infinite cases but also to discrete and continuous cases. The fuzziness characteristic of entropy was introduced in (Li and Liu 2006) due to the information deficiency in the fuzzy set. Since the fuzzy set considers a precise membership grade, often it becomes difficult to retrieve accurate information.

Let $x_i, i = 1, 2, ..., n$ be the elements of fuzzy set A and $\mu(x_i), 0 \leq \mu(x_i) \leq 1$ be the membership degree of the fuzzy set A. Then, the entropy of the fuzzy set A is defined as

$$H(A) = \frac{1}{n} \sum_{i=1}^{n} S(\mu(x_i)), \quad S(\mu(x_i))$$
$$= -\mu(x_i) \ln(\mu(x_i)) - (1 - \mu(x_i)) \ln(1 - \mu(x_i)). \quad (1)$$

It is simple to understand that $S(\mu(x_i))$ is systematic with respect to $\mu(x_i) = 0.5$, where the value of $S(\mu(x_i))$ increa-

ses during the range 0 to 0.5, obtains the peek value of $S(\mu(x_i))$ at $\mu(x_i) = 0.5$, and sharply decrease for the interval 0.5–1, which is shown in Fig. 1. The entropy value becomes zero when the fuzzy variable is degenerated into a crisp number. In this situation, the uncertainty is entirely removed. The entropy is used to partition the data set into a number segment using the membership function to establish fuzzy thresholds between classes of data. The minimum entropy value for the point x is considered as the threshold point.

Example 1 Consider a fuzzy set A with different membership values $\mu(x_i) = (0.7, 0.4, 0.3, 0.8, 0.7, 0.6, 0.4, 0.6)$. Then, the entropy $Ent(A)$ of the fuzzy set A is computed as

$$Ent(A) = -\frac{1}{8}[\{0.7\log(0.7) + 0.3\log(0.3)\}$$
$$+ \{0.4\log(0.4) + 0.6\log(0.6)\} + \{0.3\log(0.3)$$
$$+ 0.7\log(0.7)\} + \{0.8\log(0.8) + 0.2\log(0.2)\}$$
$$+ \{0.7\log(0.7) + 0.3\log(0.3)\} + \{0.6\log(0.6)$$
$$+ 0.4\log(0.4)\} + \{0.4\log(0.4) + 0.6\log(0.6)\}$$
$$+ \{0.6\log(0.6) + 0.4\log(0.4)\}] = -\frac{1}{8} \times 2.19 = 0.273.$$

Cross Entropy: Cross entropy is used to measure the discrimination information between two sets (Li and Liu 2006). Consider two independent discrete fuzzy variables as X and Y, where $0 \leq x_i \leq 1, x_i \in X$ and $0 \leq y_i \leq 1, y_i \in Y$, $i = 1, 2, 3, ..., n$. Then, the cross-entropy of X with respect to Y is denoted as

$$H(X, Y) = \frac{1}{n_1} \sum_{i=1}^{n_1} S(\mu(x_i), \mu(y_i)), \quad S(\mu(x_i), \mu(y_i))$$
$$= -\mu(x_i) \ln \frac{\mu(x_i)}{\mu(y_i)} - (1 - \mu(x_i)) \ln \frac{1 - \mu(x_i)}{1 - \mu(y_i)}. \quad (2)$$
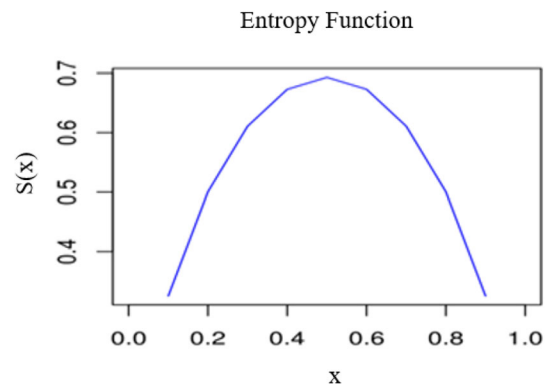

Fig. 1 Characterize of fuzzy entropy function

Example 2 Suppose the arrival delay of six consecutive landing flights in an airport in the winter season is represented by $X = (0.13, 0.33, 0.32, 0.13, 0.5, 0.03)$. Similarly, the arrival delay of those six flights in summer is represented by $Y = (0.12, 0.12, 0.12, 0.12, 0.12, 0.12)$. Here, all of the values given in X and Y are considered as fuzzy membership values. Then, the cross-entropy $CE(X, Y)$ between arrival delays of flights in two different seasons is estimated to determine the probable arrival delay of the flights at the airport.

$$
\begin{aligned}
CE(X, Y) = -\frac{1}{6} \Bigg[&\left\{ 0.13\log\left(\frac{0.13}{0.12}\right) + 0.87\log\left(\frac{0.87}{0.88}\right) \right\} \\
&+ \left\{ 0.33\log\left(\frac{0.33}{0.12}\right) + 0.67\log\left(\frac{0.67}{0.88}\right) \right\} \\
&+ \left\{ 0.32\log\left(\frac{0.32}{0.12}\right) + 0.68\log\left(\frac{0.68}{0.88}\right) \right\} \\
&+ \left\{ 0.13\log\left(\frac{0.13}{0.12}\right) + 0.87\log\left(\frac{0.87}{0.88}\right) \right\} \\
&+ \left\{ 0.5\log\left(\frac{0.5}{0.12}\right) + 0.5\log\left(\frac{0.5}{0.88}\right) \right\} \\
&+ \left\{ 0.03\log\left(\frac{0.03}{0.12}\right) + 0.97\log\left(\frac{0.97}{0.88}\right) \right\} \Bigg] \\
= -\frac{1}{6} &\times (-0.966) = 0.16.
\end{aligned}
$$

If the estimated value of cross-entropy $CE(X, Y)$ is near equal to the calculated value of entropy $Ent(X)$ or entropy $Ent(Y)$, then the probable approximation is considered to be good.

3 Theory of matching

Matching is the process used to find the relevant pairs between two distinct sets of uncommon agents (person, organization, etc.) based on some suitable function for solving the real-life problem. An extensive study on how a particular matching process succeeds in finding the efficient matches, or failing to do so, has yielded some meaningful insights, mainly in data analytics. Let's consider a situation when a reputed IT industry wants to select experts in various domains based on the domain knowledge of the applicants. Then, the selection committee members apply the matching based on requirement criterion, qualifications and experience of the candidates through the various phases to select the experts according to the requirement (Axtell and Kimbrough 2008). Another well-known example of a matching problem is the marriage problem, where the common people choose each other

among the heterogeneous preference for matrimonial pairs. This type of problem involves two distinct or uncommon sets of participants known as men and women. Each member of both sets prepares a complete and strict preference list from the other set of participants. Then, the matching process yields the set of pairs (men, women) that are stable and perfect (Liu and Ma 2015). The matching technique in which one set actively participates and provides the preferences on the other set, while the other set has no choice, is called one-side matching. But the matching process when both sets actively anticipate and provide a preference list with each other and play the same importance during matching is called two-side matching. Combining both of the matching techniques into a single system is considered as hybrid matching.

A stable matching problem is the special type of matching which consists of a set of agents, where each agent has a preference list that maintains a subset of the other agents in order of preference (Knuth 1997). The pairing of two agents in stable matching is possible only when they prefer each other. The stability of the matching indicates that agents will be willing to participate in the matching in the sense that no one prefers to be unmatched. A matching is stable if it is not blocked by any pair.

Example 3 School student admission is a real-life stable matching problem. One set indicates qualified students and another set represents the selected schools with individual maximum student intake capacity. Each student provides a preference list of schools, where the students is willing to take the admission. An instance Z of the School students problem (ST) consists of a set of students $T = \{t_1, t_2, ..., t_{k_1}\}$ and a set of schools $S = \{s_1, s_2, ..., s_{k_2}\}$. Each student t_i has a preference list of schools S (normally subsets of S) in strict order of their choice (note that preference lists may be incomplete). Each school s_j has a preference list of students T in strict order of all those students that ranked s_j on their preference list. If t_i and s_j rank each other in their preference lists, then we say they find each other acceptable. Each school s_j is associated with a capacity c_j indicating the maximum number of students that may be admitted to the school s_j. A matching M of Z is a subset of $T \times S$, such that

(i) $(t_i, s_j) \in M$ implies that t_i and s_j find each other acceptable.

(ii) For each student $t_i \in T$, $\left| (t_i, s_j) \in M : s_j \in S \right| \leq 1$.

(iii) For each school $s_j \in S$, $\left| (t_i, s_j) \in M : t_i \in T \right| \leq c_j$.

If $(t_i, s_j) \in M$, then we say that t_i is matched with s_j, and s_j is matched with t_i. A student t_i may be either unmatched in M, or matched to some school denoted by $M(t_i)$. The set

of students matched with school s_j is denoted by $M(s_j)$. M is stable until and unless it admits a blocking pair $(t_i, s_j) \notin M$.

3.1 One-side matching

One-side matching process is one way, i.e., when any agent on a set can be matched with any agent of the other set (Gale and Shapley 1962). For example, when a person wants to reserve a room in a hotel as per his/her preferences and the hotel authority assigns the room as per the preferences of the visitor, then this type of matching is called as one-side matching since the hotel authority does not have any preferences on the visitors.

The well-known one-side matching problem is the room allotment (RA) problem which consists of two agents (Deng et al. 2003). The first agent is the visitor, which is represented by $B = \{b_1, b_2, b_3 ..., b_{k_1}\}$ and another agent is the room denoted by $R = \{r_1, r_2, r_3 ..., r_{k_2}\}$. Let E be a set of a suitable pair of visitor and room, i.e., $E \subseteq B \cup R$. The probable quantity of E is m and $m = |E|$. The finite list $L(b_i)$ is the preference list of room for the visitor $b_i \in B$, where $L(b_i) = \{r_j \in R : (b_i, r_j) \in E\}$. Similarly, every room has an allocated visitors list $L(r_l)$ where $L(r_l) = \{b_i \in B : (b_i, r_j) \in E\}$ which is prepared as per the preferences of the visitors.

The preference list of rooms for the visitor $b_i (b_i \in B)$ is denoted by $L(b_i)$ and followed by strict order of preference rooms. For the visitor $b_i (b_i \in B)$, initially, an instance of a preference list of rooms are $r_j (r_j \in R, j = 1, 2, ...k_2)$ (say). Now b_i prefer the room r_l over the room r_j if the $r_l, r_j \in L(b_i)$. Hence r_l will be executed before r_j according to preference list of rooms of the visitor b_i. The hostel authority does not have any preference list over the visitor; this is the property of one-side matching and it is different from two-side matching.

Example 4 In this example, we have formulated a problem named Visitors/Rooms (VR) problem, which is concerned with booking the hotel room by the visitors. Here, the set $V = \{vt_1, vt_2, vt_3, vt_4, vt_5, vt_6\}$ represents the list of

Table 1 List of visitors and their preference rooms

Visitors	Preference rooms
vt_1	rm_3, rm_2, rm_6
vt_2	rm_4, rm_2, rm_1
vt_3	rm_1, rm_5, rm_3, rm_4
vt_4	rm_2, rm_1, rm_4, rm_6
vt_5	rm_2, rm_5, rm_3
vt_6	rm_3, rm_6, rm_1, rm_4

six visitors, and set $R = \{rm_1, rm_2, rm_3, rm_4, rm_5, rm_6\}$ represents the list of six rooms in the hotel. The instance of the individual visitor's preference list of rooms is shown in Table 1, and the initial assignment of hotel rooms among the six visitors is represented by matching $M_0 = \{(vt_1, rm_6), (vt_2, rm_2), (vt_3, rm_5), (vt_4, rm_1), (vt_5, rm_3), (vt_6, rm_4)\}$. Next, the initial matching is upgraded into $M_1 = \{(vt_1, rm_2), (vt_2, rm_1), (vt_3, rm_5), (vt_4, rm_4), (vt_5, rm_3), (vt_6, rm_6)\}$ based on the preference list given in Table 1. Similarly, present matching further updated to $M_2 = \{(vt_1, rm_2), (vt_2, rm_4), (vt_3, rm_5), (vt_4, rm_1), (vt_5, rm_3), (vt_6, rm_4)\}$. At last, the final matching should be measured as $M_3 = \{(vt_1, rm_2), (vt_2, rm_4), (vt_3, rm_1), (vt_4, rm_6), (vt_5, rm_5), (vt_6, rm_3)\}$. Then the final matching M_3 strongly blocks the initial matching M_0 and intermediate matching M_1 and M_2 due to the visitors vt_2, vt_3 and vt_6 are reassigned rooms according to a better choice. M_3 is the most optimal and considered as a Pareto optimal.

3.2 Two-side matching

In the two-side matching, both of the agents of two distinct groups are actively involved in the process, such that no agent from one group is matched to more than one agent in the other group. The agent of one side is matched with the agent of another side and vice versa. Both of the agents have a preference list over each other's (Knuth 1997). Since both of the agents in two-sided matching maintain the preference lists over each other, an effective conclusion might be possible based on the properties of the matching technique. The Hospitals–Residents (HR) problem was considered as a two-side matching in (Gale and Shapley 1962). Afterwards, many researchers (Gusfield and Irving 1989; Sotomayor 1990; Irving and Manlove 2008) have contributed towards two-side matching for solving different types of problems.

Assume I be the instance of the HR problem, which involve two sets R and H, where the set $R = \{r_1, r_2, r_3 ..., r_{k_1}\}$ is represented as the set of residences and $H = \{h_1, h_2, h_3 ..., h_{k_2}\}$ is represented as the set of hospitals. The capacity (c_j) of the hospital h_j indicates the number of vacancies has for the residents. The acceptance pair of residences–hospitals is represented by the set E, $E \subseteq R \times H$ or $E \subseteq H \times R$. The magnitude 'm' of E indicates the acceptance pairs and $m = |E|$. For each resident $r_i \in R$ has an acceptable set of hospitals $L(r_i)$, where $L(r_i) = \{h_j \in H : (r_i, h_j) \in E\}$. Similarly, each hospital $h_j \in H$ has an acceptable set of residents $L(h_j)$, where $L(h_j) = \{r_i \in R : (h_j, r_i) \in E\}$.

We consider the agents a_k which belong to I are either residents or hospitals in $R \times H$. Each agent $a_k \in R \cup H$ has a preference list $A(a_k)$ in which the list follows a strict

sequence. For any resident $r_i, r_i \in R$, the preference hospitals list of the resident denoted as $A(r_i)$, then the resident r_i prefer the hospital h_j over h_l if $h_j, h_l \in A(r_i)$. In this case, the hospital h_j proceeds before the hospital h_l. Similarly, the preferences relation among the residents maintain for the hospitals.

Example 5 Consider the following Hospitals/Residents (HR) instances (as shown in Table 2) in which six residents represented by $RS = \{rs_1, rs_2, rs_3, rs_4, rs_5, rs_6\}$ are to be selected among the four hospitals $H = \{h_1, h_2, h_3, h_4\}$. For the hospital h_1 and h_4 have one vacancy for each, whereas h_2 and h_3 have two vacancies individually.

Here, the resident rs_1 prefers the hospital in the sequence h_1, h_2, h_3 and h_4 and capacity of hospital h_1 is one and preferable residents list of hospital h_1 is rs_1, rs_2, rs_3 and rs_4. So, resident rs_1 is assigned to hospital h_1 and the assignment is demoted as (rs_1, h_1). Similarly, the resident rs_2 is matched with hospital h_2 with assignment (rs_2, h_2). In the process, the final matching based on the preferences of residents and hospitals be $M = \{(rs_1, h_1), (rs_2, h_2), (rs_4, h_3), (rs_5, h_4)\}$, where the resident's rs_3 and rs_6 are not assigned to any hospital. Also, in this matching M, hospitals h_1 hand h_2 are full as per their capacity, whereas h_3 and h_4 are under assigned. The resident rs_6 is not assigned within any hospital because the preference hospitals list of resident rs_6 are h_3, h_2 and h_1 but the resident rs_6 is not included in the preference lists of those hospitals. On the other hand, for the resident rs_3, the preference hospitals are h_4, h_2 and h_1 and the resident rs_3 is existing in the preference list of the hospital h_1; however, the resident rs_3 is not assigned for the hospital h_1. Thereafter, matching M is not stable because (rs_3, h_1) is a blocking pair.

There may be some conflicting situation in the two-side matching, as given below in Example 6.

Example 6 Consider an instance of Hospitals/Residents (HR) problem where two residents rs_1 and rs_2 want to join the two hospitals h_1 and h_2, where the preferable hospitals of residents and preferable residents of the hospitals are shown in Table 3.

Table 3 Contradiction of residents and hospitals preference

Resident	Preference hospitals list	Hospital	Preference residents list
rs_1	h_1, h_2	h_1	rs_2, rs_1
rs_2	h_2, h_1	h_2	rs_1, rs_2

There are two stable matching $M_1 = \{(rs_1, h_1), (rs_2, h_2)\}$ and $M_2 = \{(rs_1, h_2), (rs_2, h_1)\}$ according to Table 3. Both of the matchings are the same status and one of them may be accepted. Once we apply any matching technique and choose the matching M_1, then the hospital h_1 announce that only resident rs_2 is acceptable, then matching M_1 became unstable. On the other hand, when applying another matching M_2, resident rs_2 declares that the hospital h_1 is not preferable. Henceforth, a conflicting situation is generated.

The pairing or assignment is done based on the accurate preference list in both side matching and single side matching. The actual preference lists of the elements are generated based on the agent's criteria and provided information that is often certain and have substantial evidence. In many cases, due to the crisis of information acquisitions and incomplete data sets or uncertain and vague information, the decision-maker may not provide strong evidence to generate accurate ranks. If one or more agents within a set can have the same preference degree, then the preference sequence cannot be isolated from each other. One common example of this situation is found in dynamic processes, when the elements are running, i.e., the entry and exit operation are executed simultaneously within the system. These types of situations often fail to generate an accurate rank according to the present situation. As found in the literature, the conventional matching technique is found to be difficult to manage the situation smoothly. This study considers such challenging contradictions and proposes a fuzzy-based matching technique under preference without strong evidence and analyses its importance. We have used fuzzy set theory to normalize and manage imprecise situations. The preferences of the

Table 2 List of residents, hospitals, and their preference list

Residents	Preference hospital list	Hospital [capacity]	Preference residents list
rs_1	h_1, h_2, h_3, h_4	$h_1[1]$	rs_1, rs_2, rs_3, rs_4
rs_2	h_2, h_1, h_3	$h_2[2]$	rs_5, rs_2, rs_1
rs_3	h_4, h_2, h_1	$h_3[2]$	rs_4, rs_1, rs_5
rs_4	h_3, h_2, h_1	$h_4[1]$	rs_6, rs_1, rs_2
rs_5	h_2, h_1, h_4		
rs_6	h_3, h_2, h_1		

agents are represented by the fuzzy membership degree instead of the preference sequence.

4 Fuzzy matching

The Patients/Hospitals (*PH*) problem is defined as a practical problem, where a set of patients take admission among the set of quality hospitals to get better clinical service and treatments. There are different levels of hospitals such as higher level and lower level, where the higher-level hospitals can treat complex diseases and critical patients, whereas lower-level hospitals comparatively provide common services for the normal patients and common diseases. Now what type of mechanism is to be considered for the admission (matching) of the patients in the hospitals to provide better and quality medical service is an emerging issue. To manage this type of uncertain situation, we introduce the matching technique like fuzzy matching (*FM*) and intuitionistic fuzzy matching (*IFM*) to admit the patients to the hospital according to patient condition and hospital quality/status/level. In FM, each patient has a membership value indicate the condition of the patient and a preference list of hospitals. Similarly, each hospital has a finite number and a membership value to indicate the hospital's capacity and quality, respectively. For the admission of a patient in a hospital by the FM, the matching procedure is applied with the membership values and preference lists. The membership value of the patient and hospital represent the patient condition and hospital status, respectively. The preference lists of the patient and hospital show their choice sequence.

The fuzzy matching problem is formulated as follows. An instance of *I* of *PH* involves a set $P = \{p_1, p_2, p_3, ..., p_n\}$ to represent the patients and another set $H = \{h_1, h_2, h_3, ..., h_m\}$ to indicate set of the hospital. Each hospital $h_j \in H$ has a positive integer (c_j) to denote the capacity of the hospital h_j which implies that at most c_j number of patients can be admitted. Each hospital $h_j \in H$ has also a membership value (β_j) to indicate the status or quality of the hospitals. Similarly, each patient $p_i \in P$ has a membership value (μ_i) to represent the patient condition. Moreover, there is a set $E \in P \cup H$ to denote the acceptable patients-hospital pairs. The number of probable patients-hospital acceptable pairs is presented by m and m = |E|. Each patient $p_i \in P$ has an acceptable set of hospitals $L(p_i)$, where $L(p_i) = \{h_j \in H : (p_i, h_j) \in E\}$. Similarly, each hospital $h_j \in H$ has acceptable set of patients $L(h_j)$, where $L(h_j) = \{p_i \in P : \mu_i \bullet \beta_j, (p_i, h_j) \in E\}$. Here $\mu_i \bullet \beta_j$ represent the relation $\mu_i \geq \beta_j$, where μ_i and β_j are the fuzzy membership grade to represent the patient condition and hospital status, respectively. Consider

a_k is a set of agents and each agent $a_k \in P \cup H$ has a preference list. The finite list $A(a_k)$ represents the ranking list of preference which follows a strict order. Each patient $p_i \in P$ has a preference hospitals list $h_j \in H$, and the patient p_i prefer the hospital h_l over h_j if $h_l, h_j \in L(p_i)$ and h_l appears before h_j in the preference list. Each hospital follows the same procedure and relation as mentioned for patient during the patient admission.

An assignment *M* in *I* is a subset of *E*. If $(p_i, h_j) \in M$, p_i is assigned to h_j and h_j is assigned to p_i. For each agent $a_k : a_k \in P \cup H$, the set assignees of a_k in *M* is denoted by $M(a_k)$. If $p_i \in P$ and $M(p_i) = \emptyset$, then the patient p_i still waiting for taking admission otherwise p_i is admitted. Similarly, for a hospital $h_j \in H$ is under-load, full- or overload if the value of $|M(h_j)|$ less than, equal or greater than to c_j, respectively. A matching *M* in *I* is an assignment such that $|M(p_i)| \leq 1$ for each $p_i \in P$ and each hospital $h_j \in H$ should follow the relation $|M(h_j)| \leq c_j$. For a matching *M* and a patient $p_i \in P$ such that $M(p_i) \neq \emptyset, \forall i$, then there is no ambiguity in the matching, the notation $M(p_i)$ consider as a single member of the matching *M*.

An instance *I* of *PH* and a matching *M* of *I*, then $(p_i, h_j) \in E \backslash M$ is block by *M* if (i) patients p_i prefer hospital $h_j \in M(p_i)$ or p_i is unassigned and (ii) h_j prefer p_i over any other member of $M(h_j)$ or h_j is under-load. We consider the matching *M* as a stable matching if it does not have any blocking pair. If a patient–hospital pair (p_i, h_j) belongs into a stable matching in *I*, patient p_i is said to be a stable partner of hospital h_j and alternatively h_j is stable partner of p_i.

Example 7 Consider the following *PH* instances (as shown in Table 4) in which eighteen patients are to be assigned among the four hospitals. Each patient (p_i) has an illness condition and a list of preferable hospitals. Similarly, each hospital has a status and capacity. The capacity of the hospital h_1, h_2, h_3 and h_4 are 5, 3, 4 and 5, respectively. A patient can be admitted to a hospital if the hospital is present in the patients' preference list and the hospital is under-load. Simultaneously, the maximum difference between the patient's condition (represented using fuzzy membership grade) and the hospital status (represented using fuzzy membership grade) is maintained as 0.2 and hospital should be under-load.

Here, patient p_1 is with fuzzy membership grade 0.8 and preference hospitals list is h_4, h_1, h_3, h_2. Hospital h_4 has a capacity 4 and status 0.7, and $|0.8 - 0.7| \leq 0.2$. So, the hospital h_4 can take the admission of the patient p_1, where the matching is presented as (p_1, h_4). Similarly, patient p_2 has fuzzy membership grade 0.7 and its 1st preferable hospital h_1 has status 0.8 and $|0.7 - 0.8| \leq 0.2$. So,

Table 4 List of patients with status and their preference, list of hospital with status and capacity

Patients (condition)	Preference hospital list	Hospital (status)	Capacity
p_1 (0.8)	h_4, h_1, h_3, h_2	h_1 (0.8)	5
p_2 (0.7)	h_1, h_3, h_4	h_2 (0.6)	3
p_3 (0.4)	h_2, h_4, h_3	h_3 (0.4)	4
p_4 (0.6)	h_1, h_2, h_3	h_4 (0.7)	5
p_5 (0.7)	h_1, h_2, h_4		
p_6 (0.9)	h_4, h_2, h_1		
p_7 (0.2)	h_1, h_4		
p_8 (0.5)	h_2, h_3, h_4		
p_9 (0.6)	h_3, h_1, h_4		
p_{10} (0.6)	h_4, h_1, h_3		
p_{11} (0.8)	h_2, h_4, h_1		
p_{12} (0.7)	h_3, h_4, h_2		
p_{13} (0.5)	h_3, h_2, h_4		
p_{14} (0.3)	h_4, h_2, h_3		
p_{15} (0.7)	h_2, h_4, h_3		
p_{16} (0.3)	h_1, h_3, h_4		
p_{17} (0.9)	h_1, h_2, h_4		
p_{18} (0.4)	h_1, h_2, h_3, h_4		

Table 5 Taxonomy of symbols

Symbols	Discretion
H	The set $(h_1, h_2, h_3 ..., h_{k_2})$, List of hospital
H_c	The set $(c_1, c_2, c_3 ..., c_{k_2})$, Capacity of the hospitals
H_s	The set $(\beta_1, \beta_2, \beta_3 ..., \beta_{k_2})$, Status of hospitals
P	The set $(p_1, p_2, p_3 ..., p_{k_1})$, List of patients
P_c	The set $(\mu_1, \mu_2, \mu_3 ..., \mu_{k_1})$, Patients condition
H_i^p	The set $(ph_{i1}, ph_{i2}, ph_{i3} ..., ph_{ik_k})$, List of preference hospital of the ith patient
L	The set $(p_1, p_2, p_3 ..., p_{k_4})$, List of admitted patients
P_j^h	The set $(p_{j1}, p_{j2}, p_{j3} ..., p_{jk_{\#}})$, List of patients admitted within jth hospital
HPI	Hospital performance index
HI	Happiness index of patients
Tp	pivot values of patients
Th	Pivot values of hospitals

matching is possible with p_2 and h_1, and the matching is presented as (p_2, h_1). In the process, the probable matching is computed as $M = \{(p_1, h_4), (p_2, h_1), (p_3, h_2), (p_4, h_1), (p_5, h_1), (p_6, h_4), (p_8, h_2), (p_9, h_3), (p_{10}, h_4), (p_{11}, h_2), (p_{12}, h_4), (p_{13}, h_3), (p_{14}, h_3), (p_{15}, h_4), (p_{16}, h_1)\}$

In this matching M, each patient is assigned some hospital except p_7. Patient p_7 is not assigned to any hospitals because the set difference more than 0.2 for the preference hospitals. Furthermore, hospital h_2 and h_4 are full, whereas hospitals h_1 and h_3 are in under-load.

Result 1: Consider all the patients are same condition $(\mu_1 = \mu_2 = \mu_3 = ...\mu_k = \mu)$, so the entropy value of all the patients will be the same with respect to patients' condition. Similarly, when numbers of patients (k_1) of a particular condition (μ_*) and total capacity (k_2) of same status hospitals (β_*) are equal $(\mu_* = \beta_*, k_1 = k_2)$, then the cross-entropy of patient condition and hospital status is zero. After that, the patients are not required to follow the admission criteria, they only need to check the availability of the hospital according to the patients' preference list and process for matching.

Result 2: If the total capacity of the hospitals C $(C = \sum_j c_j)$ is greater than equal to the number of patients $|P|$, then the procedure generates the same result for any order of patients. Otherwise, the patient gets the flexibility according to their appearance in order.

Result 3: The fuzzy matching (M) generates the acceptable pairs between two distinct agents, which fulfills the essential condition of stable matching $(M \neq \emptyset)$. One agent is assigned to another agent based on the predefined criteria and preference list. When 1st choice fails, then only 2nd preference should be considered. So, it is observed that fuzzy matching is stable.

4.1 Proposed fuzzy matching approach

In this section, we propose the fuzzy matching approach. Consider that an instance of the simplified patient's hospitalization problem involves a set $P = \{p_1^{\mu_1}, p_2^{\mu_2}, p_3^{\mu_3}, ..., p_n^{\mu_n}\}$ of n patients, a set $H = \{h_1^{\beta_1}, h_2^{\beta_2}, h_3^{\beta_3}, ..., h_m^{\beta_m}\}$ of m hospitals. The variable c_j (non-negative finite integer) represents the capacity of hospital h_j. Two fractional parameters μ_i and β_j are the fuzzy membership values, those represent the condition of patient p_i and status of hospital h_j, respectively. The finite list $B_i = \left[h_k^{\beta_k}\right]$, $k \leq m_1$ indicates the set of preferable hospitals for the patient p_i. The capacity of hospital h_j is denoted by c_j, i.e., hospital h_j can admit utmost c_j number of patients. Using entropy, we measure the patient threshold value (Tp) to differentiate the criticalness of the patients. If the fuzzy membership degree

(μ_i) of the patient p_i is greater than the threshold value, then the patient is considered critical. Otherwise, the patient is considered as non-critical. Similarly, through the entropy, we measure the hospital threshold value (Th) and if the status grade (β_j) of the hospital h_j is greater than the threshold value (Th), then the hospital is treated as super specialty hospital. The critical condition patients are allowed to take admission within a super specialty hospital according to preference list otherwise, they can take admission into a common hospital from their preference hospital list and maintain the following criteria.

1. $dist(\mu_i, \beta_j) \leq T$: T is the predefined tolerance.
2. $h_j \in B_i$.
3. $\beta_j \prec Th, \mu_i \prec Tp$ or $\beta_j \succ Th, \mu_i \succ Tp$.

A list $A_j[k]$ indicates that currently, k number of patients are admitted in the hospital h_j. If the patient p_i is admitted in a hospital h_j then p_i is added within the list $A_j[k]$ and at the same time, patient p_i is removed from the patients list P. The admitted patients are stored within the list $L = \sum_{j=1}^{m_1} A_j[k]$. In the normal condition, the relation $|A_j[k]| \leq c_j$ is maintained. One can say that hospital h_j is under-load, full- and over-load according to the value of $|A_j[k]|$ is less than, equal to or greater than c_j, respectively. If $|L| \leq \sum_{j=1}^{m} c_j$ then the situation is under control; otherwise situation out of control. Happiness Index (HI) of the patients is considered as a fuzzy parameter which indicates the satisfied patient who has admitted according to their preferable hospital list. HI is obtained as given in (3).

$$HI = \frac{1}{|L|} \sum_{j=1}^{L} \beta^j. \tag{3}$$

The higher values of HI shows that a maximum number of patients are satisfied. Hospital Performance Index (HPI) indicates the number of critical patients a hospital (h_j) can handle.

$$HPI_j = \frac{1}{|A_j[.]|} \sum_{l=1}^{|A_j[.]|} \mu_l. \tag{4}$$

The proposed fuzzy matching algorithm is given below.

Algorithm 1: Fuzzy Matching

Input:
- H : list of hospitals, H_c : capacity of hospital, H_s : status of hospitals
- P : list of patients, P_c : condition of patients, H_i^p : list of preference hospitals of ith patient
- Th, Tp are threshold values

Output:
- P_j^h : list of admitted patients of jth hospital
- L : list of admitted patients with hospitals
- HI: happiness index of the patient, HPI: performance index of the hospitals

Begin

1 Tp =Ent (P)

2 Th = CE (H, P)

3 Initialized $hpi_j \leftarrow 0$ for all j

4 set initial happiness index, $h_i = 0$

5   For all item $\left(p_i \in P \right)$

6   For all item $\left(h_j \in H \right)$

7   If hospital h_j belong within H_i^p then

9   If hospital capacity $c_j \succ 0$ then

10   If patient's condition $\mu_i \geq Tp$ and hospital's status $\beta_j \geq Th$ then

11   If $dist\left(\mu_i, \beta_j \right) \leq 0.2$ then

12   patient $_{P_i}$ enlist within L

13   patient $_{P_i}$ remove from patient list P

14   capacity of hospital h_j reduce by 1: $c_j \leftarrow c_j - 1$

15   patient $_{P_i}$ enlist within P_j^h

16   performance index hpi_j update by adding μ_i

17   End_If

18   ElseIf patient's condition $\mu_i \leq Tp$ and hospital's status $\beta_j \leq Th$

19   If $dist\left(\mu_i, \beta_j \right) \leq 0.2$ then

20   patient $_{P_i}$ enlist within L

21   patient $_{P_i}$ remove from P

22   capacity of hospital h_j reduce by 1: $c_j \leftarrow c_j - 1$

23   patient $_{P_i}$ enlist with P_j^h

24   performance index hpi_j update by adding μ_i

25   End_If

26   End_If

27   End_If

28   End_If

29   h_i update by adding hospital status β_j

29   End_For

30 End_For

31 calculate happiness index of patient: $HI \leftarrow h_i / len\left(L \right)$

32 For all item $\left(h_j \in H \right)$

33   calculate performance index of each hospital $HPI_j \leftarrow hpi_j / len\left(hpi_j \right)$

34 End_For

Format of algorithm1 parameters
$H = \left\{ h_1, h_2, h_3 \dots, h_{k_2} \right\}$; list of hospitals
$H_c = \left\{ c_1, c_2, c_3 \dots, c_{k_2} \right\}$; capacity of hospitals
$H_s = \left\{ \beta_1, \beta_2, \beta_3 \dots, \beta_{k_2} \right\}$; status of hospitals
$P = \left\{ p_1, p_2, p_3 \dots, p_{k_1} \right\}$; list of patients
$P_c = \left\{ \mu_1, \mu_2, \mu_3 \dots, \mu_{k_1} \right\}$; condition of patients
$H_i^p = \left\{ phi_1, phi_2, phi_3 \dots, phi_{k_k} \right\}$; preference hospital of patients
$P_j^h = \left\{ p_{j1}, p_{j2}, p_{j3} \dots, p_{jk_\#} \right\}$; hospital wise admitted patients
$L = \left\{ p_1, p_2, p_3 \dots, p_{k_4} \right\}$; overall admitted patients

4.2 Summarization of the proposed algorithm

In this algorithm, we have considered six input data set denoted as H, H_c, H_s, P, P_c and H_i^p. The input parameters H and P represent the set of hospitals and set of patients, respectively. Individual hospital capacity and status are represented by the parameters H_c and H_s accordingly. The finite list P_c indicates the patient's condition. The preference hospital lists of the patients according to their choice sequence are represented by the parameter H_i^p. Executing the algorithm, two resultant parameters L and P_j^h are generated, which indicate the admitted patients list and hospitals wise admitted patients list, respectively. Initially, the threshold values Tp and Th of the respective patients and hospitals are calculated by the expressions (1) and (2), respectively, and given in line 1 and 2 of the proposed algorithm. After that, the initialization phase is started where the variable *hpi* is initialized to zero for the hospital performance index. Two successive for loops started at lines 5 and 6 to keep track of assigned the patient to the hospital according to the preference of the patient. Then, the belongingness of the hospital in the patient's preference list is checked in line 7. If the hospital is present within the list, then the vacancy of the hospital is checked in line 9. When the hospital capacity is not equal to zero, hospital status is compared with threshold value Th and patients' condition is compared with threshold value Tp in line 10. Next, we calculate the distance between hospitals status and patient's condition by the *dist* function and compare with predefined tolerance value in line 11 when both conditions satisfied in line 10. If the conditions are satisfied in line 10, then we enlist the patient in admitted patient list L, remove from patients list P, and reduce capacity of the hospital by one, the patients are enlisted with in the patient list of the desired hospital, and *hpi* variable is modified by adding assigned patient condition, respectively, in lines 12, 13, 14, 15 and 16. Again, we compare hospital status with threshold value Th and compare patients' condition with threshold value Tp in line 18 when both conditions remain unsatisfied in line 10. If the conditions are satisfied in line 18, we estimate the distance between the hospitals' status and the patients' condition by the *dist* function and compare it with the predefined tolerance value in line 19. After that, we enlist the patient in admitted patient list L, remove from patients list P, reduce the capacity of the hospital by

one, enlist the patients within the patient list of the hospital and *hpi* variable is modified by adding assigned patient condition, respectively, in lines 20, 21, 22, 23 and 24. The happiness index of the patients is calculated in line 31. The individual *HPI* is calculated by the loop as mentioned in line 33.

5 Intuitionistic fuzzy matching

The IFM is similar to FM, but in this type of matching technique, the patient condition and hospital status are represented by intuitionistic fuzzy number instead of fuzzy membership value in FM. The intuitionistic fuzzy numbers consist of positive membership degree as well as negative membership degree, and those are used to represent the satisfactory level and rejected tendency, respectively, of hospital or patient.

In the IFM technique, each patient has an intuitionistic fuzzy number to indicate the condition of the patient and a preference list of hospitals. Similarly, each hospital has a finite number and intuitionistic fuzzy number to indicate the capacity and level of the hospital, respectively. In this process, admission (matching) is made based on the intuitionistic fuzzy numbers and preference lists of patient and hospital.

The IFM problem can be formulated with the help of the previously defined *PH* problem. For an instance, I of *PH* problem involve a set $P = \{p_1, p_2, p_3, ..., p_n\}$ to represent the patients and another set $H = \{h_1, h_2, h_3, ..., h_m\}$ to indicate a set of hospitals. The capacity of the hospital $h_j \in H$ is represented by a finite positive integer c_j. The intuitionistic fuzzy number (μ_j, v_j) indicates the status of the hospital $h_j \in H$. Similarly, each patient $p_i \in P$ has an intuitionistic fuzzy number (μ_i, v_i) to represent the patient condition. The set E is used to represent the acceptable hospital–patient pairs, where $E \in P \cup H$. The variable m is represented the number of acceptable probable hospital–patient pair, m = |E|. Each patient $p_i \in P$ has an acceptable set of hospitals $L(p_i)$, where $L(p_i) = \{h_j \in H : (p_i, h_j) \in E\}$. Similarly, each hospital $h_j \in H$ has an acceptable set of patients $L(h_j)$, where $L(h_j) = \{p_i \in P : (\mu_i, v_i) \bullet (\mu_j, v_j), (p_i, h_j) \in E\}$, where $(\mu_i, v_i) \bullet (\mu_j, v_j)$ denotes Hamming or Euclidean distance and (μ_i, v_i), (μ_j, v_j) are two intuitionistic fuzzy numbers

that indicate the patient condition and hospital status, respectively. Consider a_k is a set of agents and each agent $a_k \in P \cup H$ has a preference list. The finite list $A(a_k)$ represents the ranking list of preference which follow a strict order. For any patient $p_i \in P$ has a preference hospitals list $h_j \in H$, and the patient p_i prefer the hospital h_l over h_j if $h_l, h_j \in L(p_i)$ and h_l appear before h_j in the preference list. Each hospital follows the same procedure and relation as mentioned for the patient during the patient admission.

An assignment M in I is a subset of E. If $(p_i, h_j) \in M$, p_i is called assigned to h_j and h_j admitted the p_i. For each agent a_k: $a_k \in P \cup H$, the set assigns of a_k in M is denoted by $M(a_k)$. If $p_i \in P$ and $M(p_i) = \emptyset$, then the patient p_i still waiting for taking admission otherwise, p_i is admitted. Similarly, for a hospital $h_j \in H$ is under-load, full- or over-load if the value of $|M(h_j)|$ less than, equal or greater than

to c_j, respectively. A matching M in I is an assignment such that $|M(p_i)| \leq 1$ for each $p_i \in P$ and for each hospital $h_j \in H$ should follow this relation $|M(h_j)| \leq c_j$. For a matching M and a patient $p_i \in P$ such that $M(p_i) \neq \emptyset$, $\forall i$, then there is no ambiguity in the matching M, the notation $M(p_i)$ is also used to represent a single member of the set M.

An instance I of PH and a matching M, then $(p_i, h_j) \in E \backslash M$ is block by M if (i) patient p_i has a preference hospital h_j over $M(p_i)$ or p_i is unallocated and (ii) h_j prefer p_i in respect of other member of $M(h_j)$ or h_j is under-loaded. We consider the matching M as a stable matching when there is no blocking pair. If a patient–hospital pair (p_i, h_j) belongs into a stable matching in I, the patient p_i is said to be a stable partner of hospital h_j and alternatively say that h_j is stable partner of p_i.

Algorithm 2: Intuitionistic Fuzzy Matching

For this algorithm, we provide six input variables and two lists, and two resultant values *HI* and *HPI* are generated after the execution of the algorithm. First three variables among the six input variables represent the list of hospitals (H), individual hospital capacity (H_c) and hospital status (H_s). Whereas, the remaining three represent the list of patients (P), patient's condition (P_c) and patient's preference list of hospitals (H_i^p). Two output lists P_j^h and L represent hospital wise admitted patients and the list of overall admitted patients. The two resultant variables *HI* and *HPI* respectively indicate the patient's happiness index and hospital performance index.

Input:
- H : list of hospitals, H_c : capacity of hospital, H_s : status of hospitals
- P : list of patients, P_c : condition of patients, H_i^p : list of preference hospitals of ith patient
- *Th, Tp are threshold values*

Output:
- P_j^h : list of admitted patients of jth hospital
- L : list of admitted patients with hospitals
- *HI: happiness index of the patient, HPI: performance index of the hospitals*

Begin

1 Initialized $hpi_j \leftarrow 0$ for all j

2 set happiness index, hi=0

3 For all item $\left(p_i \in P \right)$

4   For all item $\left(h_j \in H \right)$

5   If hospital h_j belong within H_i^p then

6   If capacity of hospital $c_j \succ 0$ then

7   If $d\left(P_i, H_j \right) \le 0.2$ then

8   patient p_i enlist within L

9   patient p_i remove from P

10   capacity of hospital h_j reduce by 1: $c_j \leftarrow c_j - 1$

11   patient p_i enlist within P_j^h

12   performance index hpi_j update by adding μ_i

13   End_If

14   End_If

15   End_If

16   End_For

17   hi update by adding β_j

18 End_For

19 calculate happiness index of patients $HI \leftarrow hi/len(L)$

20 For all item $\left(h_j \in H \right)$

21   calculate hospital performance index $HPI_j \leftarrow hpi_j/len(hpi_j)$

22 End_For

Format of algorithm2 parameters
$H = \{h_1, h_2, h_3 \ldots, h_{k_2}\}$; list of hospitals
$H_s = \{(\mu_1^h, v_1^h)(\mu_2^h, v_2^h)\ldots(\mu_{k_1}^h, v_{k_2}^h)\}$; status of hospitals
$H_c = \{c_1, c_2, c_3 \ldots c_{k_2}\}$; capacity of hospitals
$P = \{p_1, p_2, p_3 \ldots, p_{k_1}\}$; list of patients
$P_c = \{(\mu_1^P, v_1^P)(\mu_2^P, v_2^P)\ldots(\mu_{k_1}^P, v_{k_1}^P)\}$; condition of patients
$H_i^p = \{ph_{i_1}, ph_{i_2}, ph_{i_3} \ldots, ph_{ik_s}\}$; preference wise list hospitals
$P_j^h = \{p_{j_1}, p_{j_2}, p_{j_3} \ldots, p_{jk_3}\}$; hospital wise list of patients
$L = \{p_1, p_2, p_3 \ldots, p_{k_4}\}$; overall admitted patients.

Table 6 List of patients with their condition in the form of IFN and their preference hospital, list of hospital with status in the form of IFN and capacity

Patients (condition)	Preference hospital list	Hospital (status)	Capacity
p_1 (0.8, 0.2)	h_4, h_1, h_3, h_2	$h_1(0.8, 0.2)$	5
p_2 (0.7, 0.3)	h_1, h_3, h_4	$h_2(0.6, 0.4)$	3
p_3 (0.4, 0.6)	h_2, h_4, h_3	h_3 (0.4, 0.6)	4
p_4 (0.6, 0.4)	h_1, h_2, h_3	$h_4(0.7, 0.3)$	5
p_5 (0.7, 0.3)	h_1, h_2, h_4		
p_6 (0.9, 0.1)	h_4, h_2, h_1		
p_7 (0.2, 0.8)	h_1, h_4		
p_8 (0.5, 0.5)	h_2, h_3, h_4		
p_9 (0.6, 0.4)	h_3, h_1, h_4		
p_{10} (0.6, 0.4)	h_4, h_1, h_3		
p_{11} (0.8, 0.2)	h_2, h_4, h_1		
p_{12} (0.7, 0.3)	h_3, h_4, h_2		
p_{13} (0.5, 0.5)	h_3, h_2, h_4		
p_{14} (0.3, 0.7)	h_4, h_2, h_3		
p_{15} (0.7, 0.3)	h_2, h_4, h_3		
p_{16} (0.3, 0.7)	h_1, h_3, h_4		
p_{17} (0.9, 0.1)	h_1, h_2, h_4		
p_{18} (0.4, 0.6)	h_1, h_2, h_3, h_4		

Example 8 Consider the following *PH* instance (as Table 6 shows) in which eighteen patients are to be assigned among the four hospitals. Each patient (p_i) has an illness condition and a list of preferable hospitals. Similarly, each hospital has a status and capacity. The capacity of the hospital $h_1 h_2$, h_3 and h_4 are 5, 3, 4 and 5, respectively. A patient can be admitted to a hospital if the hospital belongs to the patients' preference hospital list, and the hospital should be under-load. Simultaneously, the maximum distance between the patient's condition (represented using an intuitionistic fuzzy number) and the hospital status (represented using an intuitionistic fuzzy number) is maintained as 0.2.

Here, the patient p_1 has intuitionistic fuzzy number $(0.8, 0.2)$ and preference list of hospitals for patient p_1 is h_4, h_1, h_3, h_2. Hospital h_4 has a capacity 4 and status $(0.7, 0.3)$, then the hamming distance is $\frac{1}{2}(|0.8 - 0.7| + |0.3 - 0.2|) \leq 0.2$. So, the hospital h_4 can take the admission of patient p_1, where the matching is denoted as (p_1, h_4). Similarly, p_2 has the intuitionistic fuzzy number $(0.7, 0.3)$ and its 1st preferable hospital h_1 has status $(0.8, 0.2)$ and $\frac{1}{2}(|0.7 - 0.8| + |0.3 - 0.2|) \leq 0.2$. So matching of p_2 is possible with h_1 and, and the matching is presented as (p_2, h_1). In the process, the probable matching is computed as $M = \{(p_1, h_4), (p_2, h_1), (p_3, h_2), (p_4, h_1), (p_5, h_1), (p_6, h_4), (p_8, h_2), (p_9, h_3), (p_{10}, h_4), (p_{11}, h_2), (p_{12}, h_4), (p_{13}, h_3), (p_{14}, h_3), (p_{16}, h_1), (p_{17}, h_1)\}$

In this matching *M*, each patient is assigned some hospital except p_7, p_{17} and p_{18}. These three patients are not assigned to any hospital because the set difference more than 0.2 for their preference hospitals. Furthermore, hospital h_1 and h_2 are full, whereas hospitals h_3 and h_4 are in under-load.

6 Conventional matching

Conventional matching is straightforward, as there are no predefined conditions to be considered during matching (Sotomayor 1990; Knuth 1997; Liu and Ma 2015). One-side agent checks the availability of the most suitable option on the other side agents. If a suitable alternative is available, then they are matched; otherwise, a second suitable option is searched, and this process is continued until complete final matching. All the patients try to admit to a hospital, which is highly equipped with modern equipment, i.e., high-status hospital. Firstly, the patient selects the hospital with high status and checks the hospitals' availability from higher status to lower status and takes the admission based on availability.

Consider that an instance *I* of *PH* involve a set $P = \{p_1, p_2, p_3, ..., p_n\}$ to represent the patients and another set $H = \{h_1, h_2, h_3, ..., h_m\}$ to indicate a set of hospitals. Each hospital $h_j \in H$ has a positive integer (c_j) to denote the capacity of the hospital of h_j, i.e., at most c_j number of patients may be admitted and a membership value (β_j)

indicates the status or quality of the hospitals. Similarly, each patient $p_i \in P$ has a membership value (μ_i) to represent the patient condition. The set of hospital-patients pairs is denoted by E, where $E \in P \cup H$. The magnitude of E is m which indicates the number of probably acceptable pairs, where m = |E|. Each patient $p_i \in P$ has an acceptable set of hospitals $L(p_i)$, where $L(p_i) = \{h_j \in H : (p_i, h_j) \in E, \beta_j \succ \beta_{j+1} \forall j\}$. Similarly, each hospital $h_j \in H$ has an acceptable set of patients $L(h_j)$, where $L(h_j) = \{p_i \in P : (p_i, h_j) \in E\}$. Consider a_k is a set of agents and each agent $a_k \in P$ has a preference hospital list. The finite list $A(a_k)$ represents preference hospital which is ranked according to their status. When every patient $p_i \in P$ has a preference hospitals list $h_j \in H$, then the patient p_i prefers the hospital h_l over h_j if $h_l, h_j \in L(p_i)$ and $\beta_l \succ \beta_j$.

An assignment M in I is a subset of E. If $(p_i, h_j) \in M$, p_i is called assigned to h_j and h_j has admitted the p_i. For each agent a_k: $a_k \in P$, the set assignees of a_k in M is denoted by $M(a_k)$. If $p_i \in P$ and $M(p_i) = \emptyset$, then the patient p_i still waits for taking admission otherwise p_i is admitted. Similarly, a hospital $h_j \in H$ is under-load, full- or over-load if the value of $|M(h_j)|$ less than, equal or greater than to c_j, respectively. A matching M in I is an assignment such that $|M(p_i)| \leq 1$ for each $p_i \in P$ and for each hospital $h_j \in H$ should follow this relation $|M(h_j)| \leq c_j$. For a matching M and a patient $p_i \in P$, if $M(p_i) \neq \emptyset$, $\forall i$, then there is no ambiguity in the matching M, and the instance $M(p_i)$ is considered as a single member of the set M.

A matching M and an instance I of PH, a pair $(p_i, h_j) \in E \setminus M$ is blocked by M if only if $|M(h_j)| = c_j, \forall j$. We consider the matching M as a stable matching because there is no option of blocking pair. If a patient–hospital pair (p_i, h_j) belongs into a stable matching in I, the patient p_i is said to be a stable partner of hospital h_j and alternatively h_j is stable partner of p_i.

Algorithm 3: Conventional matching

In this algorithm, we consider five input variables and two lists, and two resultant values *HI* and *HPI* are generated in the process. First three variables among the five input variables represent the list of hospitals (*H*), individual hospital capacity (H_c) and hospital status (H_s). Whereas, the remaining two input represent the list of patients (*P*) and the patient's condition (P_c). Two output lists P_j^h and *L* represent hospital wise admitted patients and overall admitted patients. The two resultant variables *HI* and *HPI* respectively indicate the patient's happiness index and hospital performance index.

Input:
- *H* : list of hospitals, H_c : capacity of hospital, H_s : status of hospitals
- *P* : list of patients, P_c : condition of patients
- *Th, Tp are threshold values*

Output:
- P_j^h : list of admitted patients of jth hospital
- *L* : list of admitted patients with hospitals
- *HI: happiness index of the patient, HPI: performance index of the hospitals*

Begin

1 Initialized $hpi_j \leftarrow 0$ for all j

2 set initial value hi=0

3 For all item $\left(p_i \in P \right)$

4   set initial value k=1

5   For all item $\left(h_j \in H \right)$

6   If hospital capacity $c_j \succ 0$ then

7   If hospital status $\beta_j == k$ then

8   patient p_i enlist within *L*

9   patient p_i remove from patient list *P*

10   capacity of jth hospital reduce by 1: $c_j \leftarrow c_j - 1$

11   patient p_i enlist within P_j^h

12   performance index hpi_j update by adding μ_i

13   End_If

14   End_If

15   reduce the k value by single step (0.1): k=k-0.1

16   End_For

17   update hi value by adding β_j

18 End_For

19 calculate happiness index of patient $HI \leftarrow hi / len \left(L \right)$

20 For all item $\left(h_j \in H \right)$

21     $HPI_j \leftarrow hpi_j / len \left(hpi_j \right)$

22 End_For

Format of algorithm3 parameters
$H = \left\{ h_1, h_2, h_3 ..., h_{k_2} \right\}$; set of hospitals
$H_c = \left\{ c_1, c_2, c_3 ..., c_{k_2} \right\}$; capacity of hospitals
$H_s = \left\{ \beta_1, \beta_2, \beta_3 ..., \beta_{k_2} \right\}$; status of hospitals
$P = \left\{ p_1, p_2, p_3 ..., p_{k_1} \right\}$; list of patients
$P_j^h = \left\{ p_{j1}, p_{j2}, p_{j3} ..., p_{jk_\#} \right\}$; hospital wise admitted list of patients
$L = \left\{ p_1, p_2, p_3 ..., p_{k_4} \right\}$; overall admitted list of patients

Example 9 Consider the following *PH* instances (as shown in Table 7) in which eighteen patients are to be assigned among the four hospitals and each patient has a condition. Each hospital has a status and capacity. The capacity of the hospital h_1, h_2, h_3 and h_4 are 5, 3, 4 and 5, respectively. A patient tries to select the under-load hospital with the highest status.

The top five patients p_1 to p_5 consider the hospital h_1, which is the highest status hospital and takes the admission. Then the hospital h_1 became full. Next, five patients p_6 to p_{10} select the second-highest status hospital h_4, which has a capacity 5 and can accommodate those five patients. The next three patients from p_{11} to p_{13} are admitted to hospital h_2, and the remaining four patients are in the hospital h_3. Then the final matching M is observed as $M = \{(p_1, h_1), (p_2, h_1), (p_3, h_1), (p_4, h_1), (p_5, h_1), (p_9, h_3), (p_{10}, h_4), (p_{11}, h_2), (p_{12}, h_4), (p_{13}, h_3), (p_{14}, h_3), (p_{15}, h_4), (p_{16}, h_1)\}$

M is a matching in which each patient is admitted except p_{18}, the patient p_{18} is not assigned to any hospital due to overload situation. All the hospitals are full, but the allotted patients within the hospital are not accurate.

7 Case study: preferred hospitalization of COVID-19 patients

The COVID-19 pandemic created an unprecedented challenge to the healthcare system due to the exponential rise in the number of active cases especially when the infection rate was high during the peak. The infected patients with comorbidities like diabetes, obesity, and hypertension were observed to be critical and needed proper treatment to save their lives. The healthcare systems found it challenging to extend their service in such a crucial situation, where the influx of patients with COVID-19 was continued to the dedicated hospitals. The severity of the patients and the treatment qualities of the hospitals are found to be very important factors to be considered during the patient admission within the hospitals, which are represented with help of the fuzzy membership degree in the study.

7.1 Analysis of COVID-19 patients and hospitals using fuzzy membership degree

In this article, three types of parameters such as current symptoms, associated comorbidities and age of the patients are considered to assign the fuzzy membership degree for
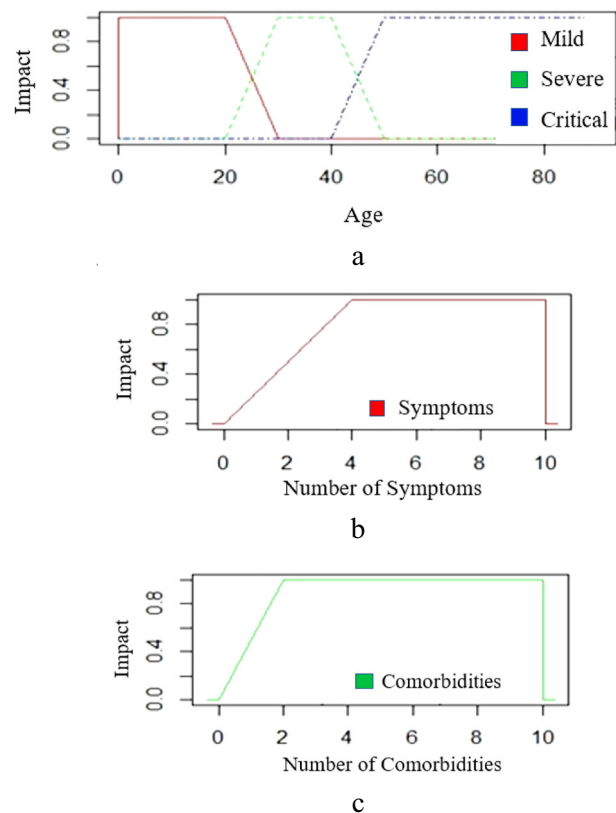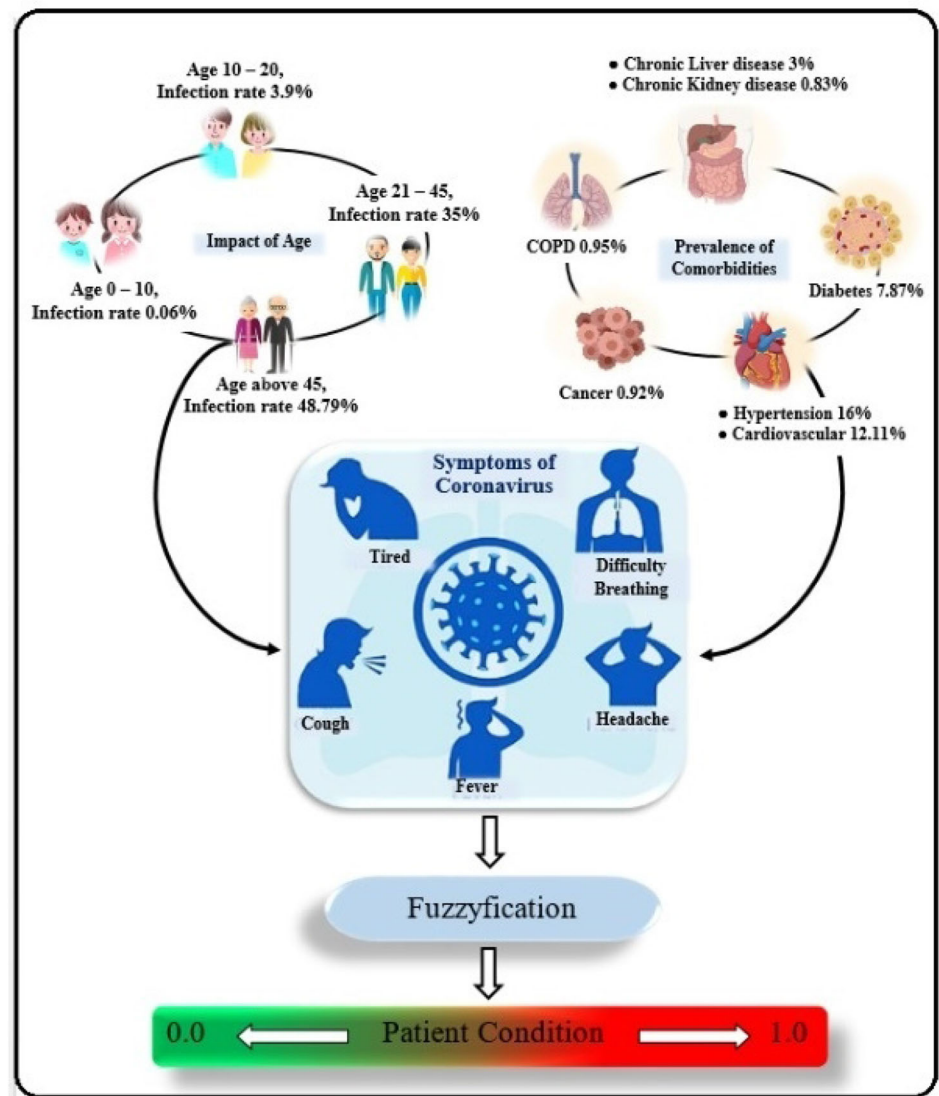
Table 7 List of patients with status and list hospitals with status and capacity

Patients (condition)	Hospital (status)	Capacity
p_1 (0.8)	h_1 (0.8)	5
p_2 (0.7)	h_2 (0.6)	3
p_3 (0.4)	h_3 (0.4)	4
p_4 (0.6)	h_4 (0.7)	5
p_5 (0.7)		
p_6 (0.9)		
p_7 (0.2)		
p_8 (0.5)		
p_9 (0.6)		
p_{10} (0.6)		
p_{11} (0.8)		
p_{12} (0.7)		
p_{13} (0.5)		
p_{14} (0.3)		
p_{15} (0.7)		
p_{16} (0.3)		
p_{17} (0.9)		
p_{18} (0.4)		

Fig. 2 **a** Impact of age. **b** Impact of symptoms. **c** Impact of comorbidities

Fig. 3 Patient evaluation method

representing the condition of the patients. The COVID-19 affected persons initially may suffer from fever or/and dry cough, then various symptoms like fatigue, anorexia, myalgia and dyspnea may appear, and in this stage, patients' condition may be changed from mild to difficult and critical situation. Existence of the comorbidities such as cardiovascular disease, diabetes mellitus, hypertension, chronic lung disease, cancer and chronic kidney disease associated with the patients often cause more severe illness and increase the casualty. The age of a person is found to be a significant factor in COVID-19 infection and impacts. As reported so far, up to the second wave of COVID-19, the kids and the younger generation are less infected or infected with mild symptoms. The middle age group have moderate risk, whereas the COVID-19 virus is very dangerous for elderly persons and cause maximum loss. Figure 2a, b, c shows the fuzziness behavior of these three types of parameters age, symptoms and comorbidities,

respectively. The health experts consider those three decision variables to estimate the fuzziness degree of the patient and identify the patients who need treatment. The method of evaluation regarding the patient's condition based on three parameters is logically illustrated in Fig. 3.

The COVID-19-infected patients and -suspected persons require the necessary treatment to recover from it. The suspected persons are normally suggested to stay in home quarantine for more than two weeks to isolate them from others. The mild symptoms patients are kept under observation at a safe home. The severe affected and critical patients need proper treatments to recover from the unsafe situation. The dedicated COVID-19 hospitals have ultra-modern treatment policy, instrumental support and well experienced health experts for providing the best quality treatment to the patients. The fuzzy membership degree is used to consider the available facilities and intuitionistic fuzzy sets are used to estimate the available as well as lack

Table 8 Classification and status of hospital based on available functionalities

Hospital status	Classification of COVID-19 hospital	Available services						
		S1	S2	S3	S4	S5	S6	S7
0.1	Home quarantine							
0.2	Safe Home							
0.3	Temporary hospital facilities	Y						
0.4	COVID care center	Y						
0.5	Primary Health center with triage and temporary isolation rooms	Y	Y					
0.6	Hospital with triage and COVID-19 dedicated ward	Y	Y	Y				
0.7	Dedicated COVID health center	Y	Y	Y	Y			
0.8	Dedicated COVID hospital	Y	Y	Y	Y	Y	Y	
0.9	Super Speciality Hospital	Y	Y	Y	Y	Y	Y	
1	Mass critical care Hospital	Y	Y	Y	Y	Y	Y	Y

S1 Experience Health Experts, *S2* Oxygen Supply, *S3* Quality Clinical Lab, *S4* X-ray, *S5* CT scan, *S6* ICU/HDU, *S7* Ventilators

of facilities for the dedicated COVID-19 hospitals to represent the status. The classification and designation of the hospitals according to the available facilities are shown in Table 8.

7.2 Experimental results

The World Health Organization (WHO) declared COVID-19 a global pandemic on 11th March 2020 (Ren et al. 2020). The malicious coronavirus has been continuously changing its behavior according to country contexts and has been exploring unexpectedly. Most of the affected countries have already applied tough decision like lockdown to reduce the virus transmission. The health care system of the affected countries was over helmed due to the treatment of severe patients of this disease. A number of severe patients required the hospital, bed and supportive modern equipment during the treatment (Kapoor et al. 2020). In India, the number of affected persons increased rapidly, which led to a large influx of critically ill patients and subsequently the demand of ICU care and mechanical ventilation were increased.

The incubation period for COVID-19 is thought to be within 14 days following exposure, with most cases occurring approximately four to five days after exposure (Trillas and Riera 1978; Li and Liu 2006; Ren et al. 2020). The spectrum of symptomatic infection ranges from mild to critical; most infections are not severe. Comorbidities that have been associated with severe illness and mortality include; cardiovascular disease, diabetes mellitus, hypertension, chronic lung disease, cancer and chronic kidney disease (Atanassov 1986; Kapoor et al. 2020; Kaufmann and Magens 1975; Loo 1977; Molodtsov 1999; Irving and Manlove 2008). The possibility of COVID-19 should be

considered primarily in patients with fever and/or respiratory tract symptoms who reside in or have traveled to areas with community transmission or who have had recent close contact with a confirmed or suspected case of COVID-19. There are no specific clinical features that can yet reliably distinguish COVID-19 from other viral respiratory infections. The most common clinical features of the onset of COVID-19 disease are fever, fatigue, dry cough, anorexia, myalgia and dyspnea (Xu and Yager 2008).

According to the report published on 1st June 2021 in (Govt of WB, 1st June, 2021), newly infected persons and active cases were, respectively, 9424 and 78,613, in West Bengal, India, where 193 government and 43 private hospitals were declared as the dedicated COVID-19 hospital for treatment of COVID-19-infected patients. The total capacity of those hospitals was approximately 24,695, out of which 1298 beds were provided with ventilator supports and 3522 beds were equipped with ICU or HDU systems. At that moment, 6671 beds were occupied by the active COVID-19 patients and 18,024 beds were available.

In this study, we consider the said report of West Bengal regarding the newly infected 9424 COVID-19 patients with their probable diseases conditions which is represented using fuzzy membership degree or intuitionistic fuzzy number according to clinical measure done by the number of health experts based on mentioned fuzzification method given in previous section (Sect. 7.1). Each patient provided their preference by mentioning at least one hospital and at most eleven hospitals for taking admission through the proposed method. There were 234 hospitals with 18,024 total available beds supported with ICCU/HDU or ventilator. The status of each of the hospitals was assigned based on membership degree or intuitionistic fuzzy number according to treatment facilities as shown in Table 9 (lower

Table 9 Comparison among the three matching techniques

Applied procedure	No of patients	Details of hospitals		Non-allotted patients	HI	Average HPI
		Numbers	Total capacity			
Conventional matching	9424	234	18,024	0	0.51	0.56
Fuzzy matching	9424	234	18,024	0	0.77	0.71
Intuitionistic fuzzy matching	9424	234	18,024	0	0.68	0.80

value indicates the quarantine center and higher value denotes the hospitals which provide quality treatments) for treatment functionality. We apply our proposed approach for the patient's admission procedure in the hospitals. All of the 9424 COVID-19 infected patients as of 1st June, 2021 were admitted to the hospitals. The number of infected patients in a day was much less than the total number of available beds, hence the situation was under control. The patients were admitted within the hospitals based on their preferable choice. All the hospitals were under-load after completing the admission procedure, where the minimum and maximum availability of beds were 3 and 108, respectively. The performance report indicates the happiness index of the patients and hospital wise performance index of the methods namely convention matching, fuzzy matching and intuitionistic fuzzy matching which are statistically represented by Fig. 4a–c, respectively. In these figures, the status of the hospitals is denoted by red bubble whereas the hospital performance index is symbolized by black bubble. In Fig. 4a, the position of black bubbles is located below the red bubbles, which indicates that the resources utilized in hospitals are less than that of their expected level and very poor in conventional matching. Whereas, as per Fig. 4c, the resources utilized in hospitals are much better and above the normal limit in intuitionistic fuzzy matching. The box plot within the figures indicate statistical report of patients' happiness. According to Fig. 4c, both of the median and upper quartile of patients' happiness index (HI) are same which signifies the maximized happiness index and the magnitude is 0.71 in intuitionistic fuzzy matching. When the happiness index for the patients is 0.71, 71% of patients can avail their first-choice hospital and the remaining patients can avail of second choice or further choice hospitals for admission. The statistical report and mapping between the status of hospitals and hospital performance index within the figures represent that the performance of intuitionistic fuzzy matching is much better with respect of fuzzy matching and conventional matching.

8 Results and discussion

This study considers the COVID-19 health bulletin published in (Govt of WB, 1st June, 2021) on 1st June 2021, where 9424 infected patients are reported and 234 hospitals are granted for the COVID-19 treatment with a total available capacity of 18,024 beds. The respective threshold values estimated for the fuzzy matching method of the patient and hospital using the entropy method are 0.73 and 0.82. According to the final outcome of fuzzy matching, all the patients were admitted, and necessary treatment and medical service were started from their preferable hospital according to the approved conditions. The overall performances of the three approaches are presented in Table 9. The sample data set consists of 9424 patients and 234 hospitals with a total available capacity of 18,024 beds. The comparative performances among the conventional, fuzzy-based and intuitionistic fuzzy-based matching techniques are given in Table 9, where the patient's happiness index (HI) indicate the satisfaction of patient and the respective estimated values are 0.51, 0.77 and 0.68. Similarly, the performance of hospital is represented by hospital performance index (HPI) and the respective evaluated values are 0.56, 0.71 and 0.80.

In the conventional matching procedure, the patients are just assigned within the hospitals for admission based on patient's preference hospitals. This system initially checks the availability of the hospitals, then assigns patients if any vacancies are found, otherwise second choice hospitals of the patients are processed. This process is followed until the patients are assigned within their preferable list of hospitals. The performance of this type of conventional matching is not significant because the patients' who are at the beginning of the patients list get the opportunities to be assigned by some better service providing hospitals, although they may be less infected patients. Hence the patient admission procedure in this type of conventional matching is not fair and super specialty or mass critical care unit hospitals cannot utilize their ultra-modern medical units appropriately. Normally, the severe case patients are not admitted within the desired hospital for getting accurate treatment. After completing the admission
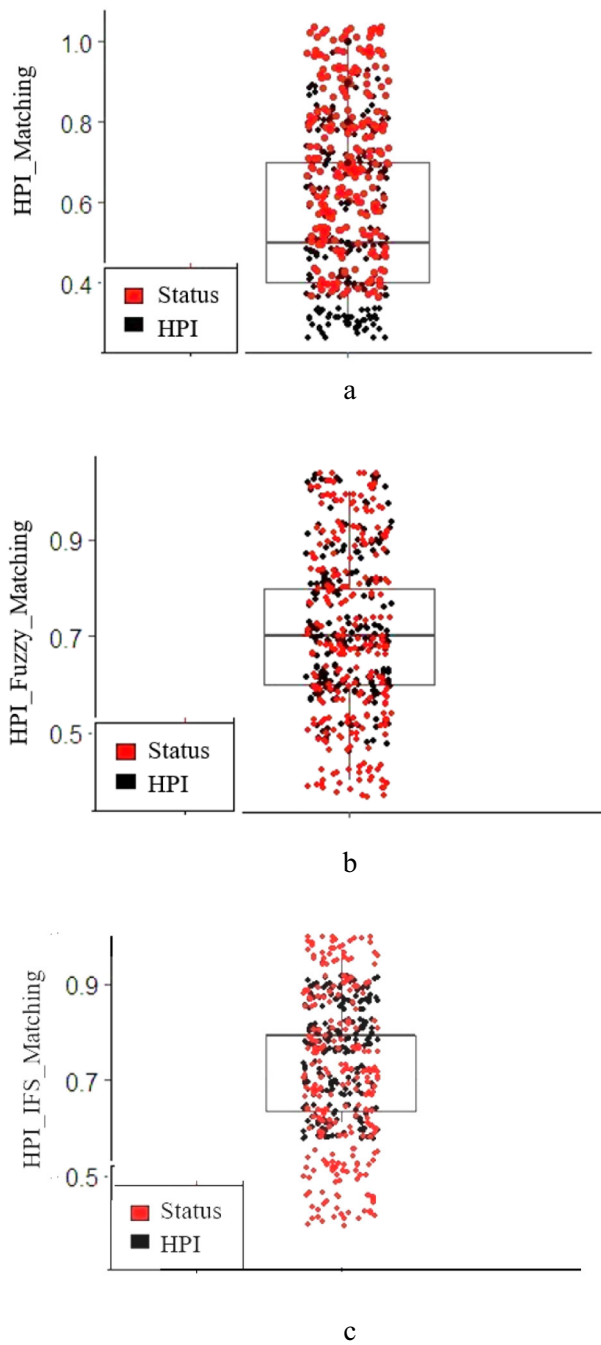
Fig. 4 a Patients allocation status of conventional matching. **b** Patients allocation status of fuzzy matching. **c** Patients allocation status of intuitionistic fuzzy matching

procedure in the conventional matching, the performance measuring parameters *HI* and *HPI* are obtained as 0.51 and 0.56, respectively. The score of *HPI* is closer to the lower quartile of the box plot, which is shown in Fig. 4a and according to this figure, the performance of lower-level hospitals are below the hospital status. In fuzzy matching, initially the system checks the patient's condition which is evaluated by health experts, and the status of the hospitals

which is assigned according to the availability of the service. Thereafter the admission process will be continued to admit the patients within the appropriate hospitals. The accuracy of the admission process for this matching technique is better with respect of conventional matching. The priority of the enlisted patients depends on the illness level of the patient and based on the illness level, the patients are admitted within appropriate hospitals. The resultant score of *HI* of the patients and *HPI* are, respectively, obtained as 0.77 and 0.71. As mentioned on the box plot of Fig. 4b, the *HPI* of the fuzzy matching lies nearby the median level, and the performance and status of hospitals are balanced with each other, which is shown in Fig. 4b. For the intuitionistic fuzzy matching, the admission procedure is executed through the bidirectional checking between condition of the patient and status of hospitals. In this matching technique, both of the appearing and non-appearing symptoms of the patients as well as available and non-available services of the hospitals are considered. This matching (intuitionistic fuzzy matching) is more accurate and provides the option for admission of the real patients who are critically ill to the quality service providing hospitals. Here, the obtained score value of the *HPI* is near equal to the upper quartile mentioned in Fig. 4c. The *HPI* score of this matching is obtained as 0.8, which is better than the other two matching techniques. However, the score of *HI* of the patients is 0.68, which is lower than that of fuzzy matching due to incorporating more options of the patient during this matching process.

9 Conclusion

Matching is the process to pair between two disjoint elements which belong to two distinct sets. In the one-side matching, only one agent provides their preference sequence and matching is done accordingly. Whereas in two sides matching, both side agents provide their preference and matching is done after fulfilling both predefined side conditions. In this paper, initially we have proposed fuzzy set-based matching by extending the conventional matching technique in the environment of uncertainty. We have also proposed intuitionistic fuzzy set-based matching technique by extending the proposed fuzzy matching technique. We have proposed both of the matching techniques to capture the uncertain information, where the actual preference sequence is difficult to construct due to imprecise and inadequate data. So, we have used the membership value in fuzzy matching, and membership as well as non-membership values in intuitionistic fuzzy matching. For the fuzzy matching, the threshold values of both side agents are estimated through the entropy measurements for comparing the two-side agents with some

predefined conditions. In the intuitionistic fuzzy matching, the comparison between two-side agents is performed based on hamming distance. In this study, we have applied the proposed method for the hospitalization of COVID-19 patients among the dedicated COVID-19 hospitals. The patients have a fuzzy membership value which indicates the patient condition and a preference list of hospitals. Similarly, a set of hospitals has a fuzzy membership value that represents the status of the hospital and a finite capacity. Using the entropy measure, the threshold values of the patients as well as hospitals are obtained. Then, the patient condition is checked with respect to the patient threshold value. According to the patient preference sequence, the suitable hospital is chosen with respect to a hospital threshold value for taking admission. Similarly, in the intuitionistic fuzzy matching, patients' condition and status of hospital are represented by the intuitionistic fuzzy number, and the hamming distance is measured between them. Patients are admitted to a hospital based on the hamming distance and some predefined conditions. Two parameters are used to measure the performance of the proposed method, which are *HI* and *HPI*. The happiness index (*HI*) indicates how many patients are admitted in their first preferences hospital, whereas hospital performance index (*HPI*), represent how much the individual hospital handles critical patients. The proposed fuzzy matching and intuitionistic fuzzy matching have been applied to the sample data set with conventional matching. In conventional matching, the patients are admitted according to their sequence within the patient list and most of the patients are not admitted according to their best choice. Whereas according to the fuzzy matching and intuitionistic fuzzy matching, initially, the list of unadmitted patients and vacancy in the available hospitals are checked, and then compared with each other for proper matching before admission. The obtained results of the fuzzy matching and intuitionistic fuzzy matching are observed as much better than the conventional matching. The performance of fuzzy matching and intuitionistic fuzzy matching are in conflict with each other due to the outcomes of *HI* and *HPI* values. In fuzzy matching, the value of *HI* is better than the intuitionistic fuzzy matching, alternatively, the *HPI* value of intuitionistic fuzzy matching is better. In future, the researchers can improve the performance of proposed matching by normalizing the fuzzy membership degree with a linguistic hedge for better accuracy and may use different statically analysis for finding the more accurate pairs.

Data availability statement All data generated or analyzed during this study are included in this published article.

References

Amanathulla S, Bera B, Pal M (2021) Balanced picture fuzzy graph with application. Artif Intell Rev 54(7):5255–5281

Atanassov KT (1986) Intuitionistic fuzzy sets. Fuzzy Sets Syst 20:87–96

Atanassov KT (1999) Intuitionistic fuzzy sets. Theory Appl Physica-Verlag Heidelberg 35:1–137. https://doi.org/10.1007/978-3-7908-1870-3_1

Axtell RL, Kimbrough SO (2008) The high cost of stability in two-sided matching: How much social welfare should be sacrificed in the pursuit of stability. In: Proceedings of the 2008 world congress on social simulation (WCSS-08)

Cabalar AF, Cevik A, Gokceoglu C (2012) Some applications of adaptive neuro-fuzzy inference system (ANFIS) in geotechnical engineering. Comput Geotech 40:14–33

Cagman N, Enginoglu S, Citak F (2011) Fuzzy soft set theory and its applications. Iran J Fuzzy Syst 8(3):137–147

Chellamani P, Ajay D, Broumi S, Ligori T (2021) An approach to decision-making via picture fuzzy soft graphs. Granul Comput 7:527–548

Das S, Malakar D, Kar S, Pal T (2018) A brief review and future outline on decision making using fuzzy soft set. Int J Fuzzy Syst Appl (IJFSA) 7(2):1–43

Das S, Kar MB, Kar S (2013) Group multi-criteria decision making using intuitionistic multi-fuzzy sets. J Uncertain Anal Appl 1(1):1–16

De A, Kundu P, Das S, Kar S (2020) A ranking method based on interval type-2 fuzzy sets for multiple attribute group decision making. Soft Comput 24(1):131–154

De A, Das S, Kar S (2019) Multiple attribute decision making based on probabilistic interval-valued intuitionistic hesitant fuzzy set and extended TOPSIS method. J Intell Fuzzy Syst 37(4):5229–5248

Deng X, Papadimitriou C, Safra S (2003) On the complexity of price equilibria. J Comput Syst Sci 67(2):311–324

Fan JL, Ma YL (2002) Some new fuzzy entropy formulas. Fuzzy Sets Syst 128(2):277–284

Gale D, Shapley LS (1962) College admissions and the stability of marriage. Am Math Mon 69(1):9–15

Gorzałczany MB (1987) A method of inference in approximate reasoning based on interval-valued fuzzy sets. Fuzzy Sets Syst 21(1):1–17

Govt of WB (1st June, 2021) COVID-19 Health Bulletin. https://www.wbhealth.gov.in/uploaded_files/corona/WB_DHFW_Bulletin_1st_JUNE_REPORT_FINAL2.pdf. Accessed 12 June 2022

Guidance on developing a national deployment and vaccination plan for COVID-19 vaccines (1st June, 2021). https://www.who.int/publications/i/item/WHO-2019-nCoV-Vaccine-deployment-2021.1-eng. Accessed 11 June 2022

Gupta R, Kumar S (2021) A new similarity measure between picture fuzzy sets with applications to pattern recognition and clustering problems. Granul Comput 7:561–576

Gusfield D, Irving RW (1989) The stable marriage problem: structure and algorithms. Cambridge MIT press, Cambridge

Higashi M, Klir GJ (1982) On measures of fuzziness and fuzzy complements. Int J Gen Syst 3(8):169–180

Hung WL, Yang MS (2004) Similarity measures of intuitionistic fuzzy sets based on Hausdorff distance. Pattern Recogn Lett 25(14):1603–1611

Irving RW, Manlove DF (2008) Approximation algorithms for hard variants of the stable marriage and hospitals/residents problems. J Comb Optim 16(3):279–292

Kadian R, Kumar S (2021) New fuzzy mean codeword length and similarity measure. Granul Comput 7:61–478

Kapoor G, Hauck S, Sriram A, Joshi J, Schueller E, Frost I, Balasubramanian R, Laxminarayan R, Nandi A (2020) State-wise estimates of current hospital beds, intensive care unit (ICU) beds and ventilators in India: Are we prepared for a surge in COVID-19 hospitalizations? https://doi.org/10.1101/2020.06.16.20132787

Kaufmann A, Magens N (1975) Introduction to the theory of fuzzy subsets: fundamental theoretical elements. Academic press, Cambridge

Knuth DE (1997) Stable marriage and its relation to other combinatorial problems: an introduction to the mathematical analysis of algorithms. Am Math Soc

Kumar A, Kaur A (2012) Optimization for different types of transportation problems with fuzzy coefficients in the objective function. J Intell Fuzzy Syst 23(5):237–248

Li DF (2012) Multiattribute decision making models and methods using intuitionistic fuzzy sets. J Comput Syst Sci 70:73–85

Li X, Liu B (2006) Cross-entropy and generalized entropy for fuzzy variables. Techn Rep

Liu HW, Wang GJ (2007) Multi-criteria decision-making methods based on intuitionistic fuzzy sets. Eur J Oper Res 179(1):220–233

Liu X, Ma H (2015) A two-sided matching decision model based on uncertain preference sequences. Math Probl Eng. https://doi.org/10.1155/2015/241379

Lo J, Wilson MC (2017) New algorithms for matching problems. https://doi.org/10.48550/arXiv.1703.04225

Loo SG (1977) Measure of fuzziness. Cursos Congr Univ Santiago De Compostela 20:201–210

De Luca A, Termini S (1972) A definition of a nonprobabilistic entropy in the setting of fuzzy sets theory. Inf Control 20(4):301–312

Mishra SV, Gayen A, Haque SM (2020) COVID-19 and urban vulnerability in India. Habitat Int 103:102230

Molodtsov D (1999) Soft set theory—first results. Comput Math Appl 37(4–5):19–31

Patra K (2022) Fuzzy risk analysis using a new technique of ranking of generalized trapezoidal fuzzy numbers. Granul Comput 7(1):127–140

Poulik S, Ghorai G (2021) New concepts of inverse fuzzy mixed graphs and its application. Granul Comput 7:549–559

Ren Z, Liao H, Liu Y (2020) Generalized Z-numbers with hesitant fuzzy linguistic information and its application to medicine selection for the patients with mild symptoms of the COVID-19. Comput Ind Eng 145:106517

Si A, Das S, Kar S (2019) An approach to rank picture fuzzy numbers for decision making problems. Decis Mak Appl Manag Eng 2(2):54–64

Si A, Das S, Kar S (2021) Picture fuzzy set-based decision-making approach using Dempster-Shafer theory of evidence and grey relation analysis and its application in COVID-19 medicine selection. Soft Comput. https://doi.org/10.1007/s00500-021-05909-9

Sotomayor MAO (1990) Two-sided matching: study in game-theoretic modeling and analysis. Cambridge University Press, Cambridge

Sun B, Tong S, Ma W, Wang T, Jiang C (2021) An approach to MCGDM based on multi-granulation Pythagorean fuzzy rough set over two universes and its application to medical decision problem. Artif Intell Rev 55:1887–1913

Szmidt E, Kacprzyk J (2001) Intuitionistic fuzzy sets in some medical applications. In: International Conference on Computational Intelligence, pp 148–151, https://doi.org/10.1007/3-540-45493-4_19

Tham CW, Tian SH, Ding L (2002) Weather forecasting system based on satellite imageries using neuro-fuzzy techniques. In: AFSS International Conference on Fuzzy Systems, pp 267–273, https://doi.org/10.1007/3-540-45631-7_36

Trillas E, Riera T (1978) Entropies in finite fuzzy sets. Inf Sci 15(2):159–168

Vajda S (1950) The mathematical theory of communication. Math Gaz 34(310):312–313

Verma R, Rohtagi B (2022) Novel similarity measures between picture fuzzy sets and their applications to pattern recognition and medical diagnosis. Granul Comput. https://doi.org/10.1007/s41066-021-00294-y

Xu Z, Yager RR (2008) Dynamic intuitionistic fuzzy multi-attribute decision making. Int J Approx Reason 48(1):246–262

Xuecheng L (1992) Entropy, distance measure and similarity measure of fuzzy sets and their relations. Fuzzy Sets Syst 52(3):305–318

Zadeh LA (1965a) Information and control. Fuzzy Sets 8(3):338–353

Zadeh LA (1965b) Fuzzy sets and systems. Proceedings of the Symposium on Systems Theory Polytechni(NY) 5:29–37

Zimmermann H (2010) Fuzzy set theory. Wiley interdisciplinary reviews: computational statistics, 2(3):317–332