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Determining the Theoretical Effective Lens Position of Thick Intraocular Lenses for Machine Learning–Based IOL Power Calculation and Simulation

Damien Gatinel[1], Guillaume Debellemanière[1], Alain Saad[1], Mathieu Dubois[1], and Radhika Rampat[1]

[1] Department of Ophthalmology, Rothschild Foundation Hospital, Paris, France

Correspondence: Damien Gatinel, Department of Ophthalmology, Rothschild Foundation Hospital, 29 Rue Manin, Paris, France. e-mail: gatinel@gmail.com

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Purpose: To describe a formula to back-calculate the theoretical position of the principal object plane of an intraocular lens (IOL), as well as the theoretical anatomic position in a thick lens eye model. A study was conducted to ascertain the impact of variations in design and IOL power, on the refractive outcomes of cataract surgery.

Methods: A schematic eye model was designed and manipulated to reflect changes in the anterior and posterior radii of an IOL, while keeping the central thickness and paraxial powers static. Modifications of the shape factor (X) of the IOL affects the thick lens estimated effective lens position (ELP). Corresponding postoperative spherical equivalent (SE) were computed for different IOL powers (–5 diopters [D], 5 D, 15 D, 25 D, and 35 D) with X ranging from −1 to +1 by 0.1.

Results: The impact of the thick lens estimated effective lens position shift on postoperative refraction was highly dependent on the optical power of the IOL and its thickness. Design modifications could theoretically induce postoperative refraction variations between approximately 0.50 and 3.0 D, for implant powers ranging from 15 D to 35 D.

Conclusions: This work could be of interest for researchers involved in the design of IOL power calculation formulas. The importance of IOL geometry in refractive outcomes, especially for short eyes, should challenge the fact that these data are not usually published by IOL manufacturers.

Translational Relevance: The back-calculation of the estimated effective lens position is central to intraocular lens calculation formulas, especially for artificial intelligence–based optical formulas, where the algorithm can be trained to predict this value.

Introduction

Cataract surgery is one of the most frequently performed refractive surgical procedures globally.[1] The quality of the patient‘s postoperative vision largely depends on predictable selection of the intraocular lens (IOL)’s optical power, which influences the postoperative refraction. Precise biometric measurements and accurate IOL power calculation methods are required. The adequate collection and use of data in modern healthcare provides an opportunity for significant improvements within a wide range of sub-specialties.[2] Recently, IOL calculation approaches using artificial intelligence have shown improving performances, even though the superiority of artificial intelligence to previous methods still remains controversial.[3–10] Although the inner principles of the most recent IOL formulas have not been published, most of them are based on optical formulas, with only a few described as purely artificial intelligence based.[11] Regardless of the algorithms that an optical formula uses, the back-calculation of the estimated effective position of the lens (ELP) is necessary to train the algorithms to predict this value.

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The definition of the ELP varies according to the model eye, with refractive components which could equate to either thin or thick lenses. When the cornea and IOL are modeled as thick lenses, the ELP is congruent with the distance between the principal image plane of the cornea and the principal object plane of the IOL.[12] Supervised learning allows algorithms to be trained in predicting an outcome from labelled feature recognition, with increasing dataset leading to increased accuracy. Although it is possible to know the anatomic position of the IOL by imaging the anterior segment,[13–15] this information is not routinely available within large training datasets. These generally include information limited to the preoperative ocular biometry, and final refractive outcome achieved (spherical equivalent [SE]). The back-calculation of the "matching" estimated ELP can be performed systematically; this value integrates by definition within the same parameter, all the errors induced by the underlying assumptions made in the eye model.

Both the estimated ELP recalculated from the refractive outcome (target) as well as the biometric parameters obtained from the preoperative examination (predictors) can be used as values to aid prediction using the training dataset. To determine the estimated ELP in a double-refractive thick-lens schematic eye, the provision of the IOL radii, thickness and refractive index, is a prerequisite.

The objective of this article is two-fold. First, it aims to provide the equations necessary to estimate the effective lens implant position in a dual combination, thick-lens, paraxial, pseudophakic, schematic eye representing the cornea and the IOL as four refracting surfaces. It is possible to vary the radius of curvature of the anterior and posterior surfaces of the cornea and the implant, as well as the refractive indices of each segment (stroma, aqueous humor, IOL, and vitreous). The use of explicit equations, that account for the geometry of the intraocular implant, can be used to develop training datasets featuring the estimated ELP of the IOL modeled as a thick lens. Second, we use these equations to explore the impact of IOL design and power modifications on the refractive outcomes of cataract surgery.

Methods

Researchers involved in the development of biometric calculation formulas can access large databases collected in centers specializing in cataract surgery.[3,4] Datasets typically contain the preoperative ocular biometric parameters including anterior corneal curvature radius, corneal thickness, anterior chamber depth (measured from epithelium to lens), lens thickness, and corneal diameter, as well as the axial length. The postoperative refraction of each eye is provided as the value of the SE in the spectacle plane. Because the IOL surface curvature and thickness vary with power, the geometric characteristics of the inserted IOL such as anterior and posterior radii (R_{ai} and R_{pi}), thickness (d_i) and refractive index (n_i) should also ideally be available. The posterior corneal radius can be measured by many current biometers, but was previously usually inferred from the anterior corneal radius using the keratometric index.

Our main goal was to compute formulas to determine the estimated ELP of the thick IOL, for each eye in the training dataset, so that these data could be used for training purposes for a machine learning algorithm. The algorithm aimed to predict the ELP that matches the postoperative refractive error, from preoperative ocular biometry, allowing the suitable IOL power to be chosen for the desired refraction.

To compute and investigate the impact of the IOL power and design on the estimated ELP and postoperative refraction, we have designed a schematic eye model where the design of the IOL (its anterior and posterior radii) can vary for the same central thickness and paraxial power. In this context, the value of the ELP can be used to compute the anatomic lens position (ALP), which corresponds with the distance separating the anterior corneal and IOL vertices. The derived equations can then be used to study the theoretical influence of the power and the design of the IOL on the postoperative refraction for the same physical distance from the anterior corneal vertex.

Thick Lens Schematic Pseudophakic Eye Model

Paraxial optic formulas for calculating the respective optical power and principal planes' positions of the cornea and IOL, modeled as thick lenses, as well as the resultant power and principal planes' position of the eye are reviewed in Appendix A.

The cornea is comparable with a convex–concave lens whose refractive index is that of its main layer, the corneal stroma of index n_s. The total corneal power is denoted D_c. It can be obtained from the value of the anterior and posterior radii of curvature (R_{ca} and R_{cp}), and from the refractive indices of air, stroma and aqueous humor (n_a). The distances between the anterior and posterior vertices of the cornea with the main object planes and images of the cornea can be calculated from these values (see Appendix A1).

Positive power implants are usually biconvex lenses, the surfaces of which may have the same (symmetrical biconvex lens) or differing (asymmetric biconvex lens) paraxial curvatures. The optical power of a thick lens depends on the curvatures of its anterior and posterior surfaces, their separating distance (corresponding with the central IOL thickness, d_i), the index variations between that of the media in contact with these surfaces as well as that of the lens itself.

The power of the natural or artificial lens is denoted D_i. It can be calculated from the characteristics of the implant: curvatures of the anterior and posterior surfaces (R_{ia} and R_{ip}), central thickness d_i, refractive index and the refractive indices of aqueous humor and vitreous. The main object planes and images of the implant can also be calculated from these values (see Appendix A2).

Similarly, we can calculate the position of the principal planes H_e and H'_e of the entire eye (cornea + IOL) from the paraxial thick lens formulas (see Appendix A3).

In a thick lens model, depicting the distance between the refractive elements involves the position of the principal planes of these elements: the distance between the cornea and the implant is reduced to the distance between the position of the principal image plane of the cornea and the principal object plane of the IOL.

The dioptric power of the eye *De* is expressed by the Gullstrand formula (thick lenses):

$$D_e = D_C + D_i - \left(D_c\, D_i \frac{ELP_T}{n_a} \right) \tag{1}$$

Where $ELP_T = \overline{H'_c H_i}$ is referred to here as the ELP of a thick IOL, that is, the distance separating the principal image plane of the cornea (H'_c) from the principal object plane of the IOL (H_i). Note that all algebraic distances must be converted in meters for numerical applications using the formulas presented in this article.

Expression of the Anatomic and ELPs of the Thick IOL

The relationship between the thick lens position ELP_T and the ALP of the IOL defined as the anterior IOL vertex position (ALP = S1S3) is expressed as follows:

$$\overline{H'_c H_i} = ELP_T = \overline{H'_c S_1} + \overline{S_1 S_3} + \overline{S_3 H_i} \tag{2}$$

Which can be rewritten as:

$$ELP_T = ALP + \overline{H'_c S_1} + \overline{S_3 H_i} \tag{3}$$

This equation can be rearranged to express ALP as a function of ELP_T:

$$ALP = ELP_T - \overline{H'_c S_1} - \overline{S_3 H_i} \tag{3b}$$

Expression of the Anatomic and Optical Axial Lengths of the Paraxial Emmetropic Pseudophakic Eye

The anatomic axial length here corresponds to the value which connects the anterior corneal vertex to the photoreceptors' plane at the fovea. Optical biometers provide the distance between the anterior corneal vertex and the retinal pigment epithelium using the technique of partial coherence interferometry.

In the case of an emmetropic eye, it is equal to the distance $\overline{S_1 F'_e}$, where F'_e is the back focal point of the paraxial schematic pseudophakic eye.

We can split the anatomic axial length into two algebraic segments:

$$AL_A = \overline{S_1 H'_e} + \overline{H'_e F'_e} \tag{4}$$

This expression can be developed and rearranged (see Appendix B):

$$AL_A = \overline{S_1 H'_c} + ELP_T + \overline{H_i H'_i} + \frac{n_v - \frac{n_v}{n_a} D_c ELP_T}{D_e} \tag{5}$$

Let us define:

$$AL_T = AL_A - \overline{S_1 H'_c} - \overline{H_i H'_i} \tag{6}$$

AL_T is equal to the anatomic axial length of the emmetropic eye decreased by the distance between the principal planes of the implant ($\overline{H_i H'_i} > 0$) and the distance between the anterior surface of the cornea and the secondary principal plane of the cornea ($\overline{S_1 H'_c} < 0$).

Finally, we obtain:

$$AL_T = ELP_T + \frac{n_v - \frac{n_v}{n_a} D_c ELP_T}{D_e} \tag{7}$$

Which can be expanded using Equation 1:

$$AL_T = ELP_T + \frac{n_v - \frac{n_v}{n_a} D_c ELP_T}{D_c + D_i - \frac{D_c D_i ELP_T}{n_a}} \tag{7b}$$

This equation can be solved for ELP_T and Dc. Figure 1 provides a geometric representation of the relationships between ELP_T and ALP, and between AL_A and AL_T of this emmetropic pseudophakic eye.

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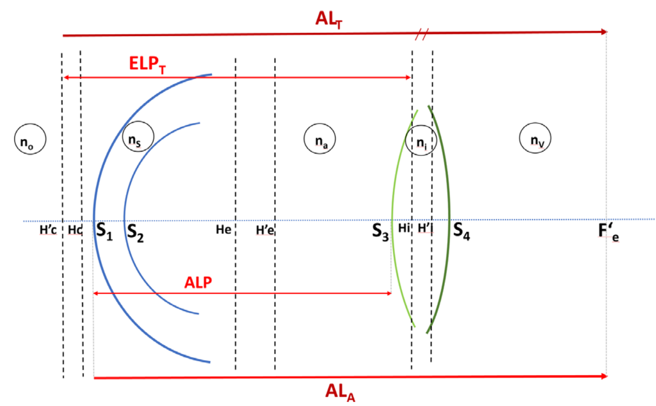


Figure 1. The AL_A is the anatomic axial length, from the anterior surface of the cornea to the photoreceptors' plane at the fovea F'_e: it is equal to the distance $S_1 F'_e$. The AL_T is the thick lens axial length, which connects the image principal plane of the cornea H'_c to the photoreceptor's plane and is reduced by the distance $\overline{H_i H'_i}$ separating the two principal planes of the IOL. ALP is the anterior lens position, connecting the corneal vertex S_1 to the IOL vertex S_3. ELP_T is the effective thick lens position, joining the image principal plane of the cornea to the object principal plane of the IOL.

Determination of the Effective Thick Lens Position ELP_T

The following two scenarios should be distinguished, depending on the value of the SE of the considered eye.

Emmetropic Pseudophakic Eye (Postoperative SE = 0)

Solving Equation 7b for ELP_T leads to the following explicit formula:

$$ELP_T = \frac{\pm\sqrt{\begin{array}{l}(D_c\,(n_a - n_v + AL_T D_i) + n_a D_i)^2 \\ \quad - 4 n_a D_c D_i\,(D_c AL_T + AL_T D_i - n_v)\end{array}} + D_c\,(n_a - n_v + AL_T D_i) + n_a D_i}{2 D_c D_i} \tag{8}$$

The ± sign must be replaced by – when $D_i > 0$, and by + when $D_i < 0$.

The same equation can be used to determine the effective thin lens position, denoted ELP_t, which would be obtained in a thin lens model where the cornea and IOL have null thickness. It is generally not identical to the position of the anterior vertex, posterior vertex, or the principal planes. In this scenario, $S_1S_2 = S_3S_4 = 0$, $AL_T = AL_A$.

Ametropic Pseudophakic Eye (postoperative SE ≠ 0)

The ELP_T value can be computed in an ametropic eye (SE ≠ 0) after replacing D_c by D_{ce} in Equation 8, where D_{ce} is the sum of Dc and the vergence in the corneal plane of a spectacle lens of power equal to SE, placed at distance d from the corneal vertex (ignoring the distance $\overline{S_1 H_c}$).

$$D_{ce} = D_c + \frac{SE}{(1 - dSE)} \tag{9}$$

Influence of the Design of the IOL on the ELP_T

The Coddington shape factor (X) is a formal measure of the bending of a lens. It is calculated using the usual sign convention for radii of curvatures of the anterior and posterior surfaces of a lens (see Appendix C). The bending influences the amount of spherical aberration and the location of the principal plane in relation to the lens surfaces. The Coddington shape factor is equal to 0 for a symmetrical biconvex or biconcave lens. It is equal to –1 and 1 for planoconvex lenses (for a flat lens surface located anteriorly and posteriorly, respectively). It is less than –1 or more than 1 for meniscus lenses, depending on the sign of the radii and their relative absolute value.

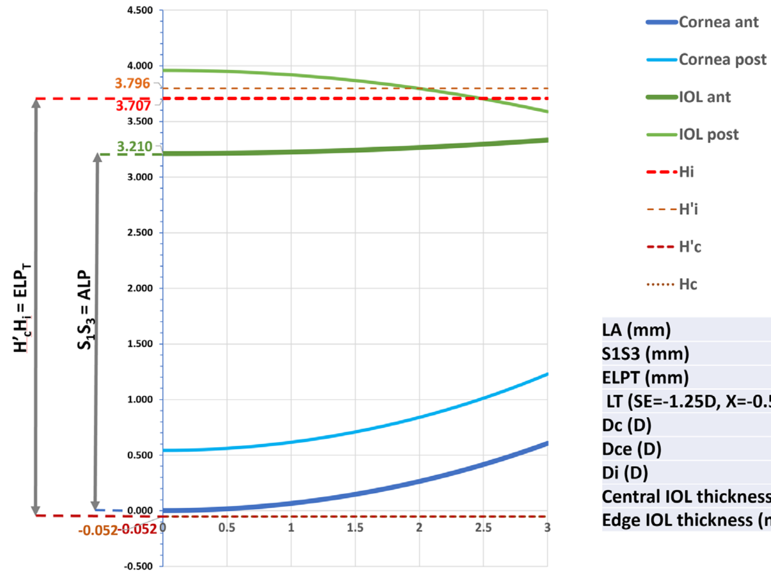
The optical power of a thick spherical lens and its Coddington shape factor are essential parameters that characterize its image quality; the amount of spherical aberration in a lens made from spherical surfaces depends on its shape. Biconvex and biconcave lenses have shape factors between –1 ($R_{ia} \to \infty$, planar posterior IOL) and 0 ($R_{ip} \to \infty$, planar anterior IOL).[12] For the same ALP, optical power, and central thickness, a variation in the shape factor results in an axial displacement of the principal planes of the IOL (see in Appendix A2 Equations A6 and A7, which depend on Rip and Ria, respectively). This variation also affects the length of the segment $\overline{S_3 H_i}$, which is involved in the calculation of ELP_T (Equation 8).

Influence of the ELP_T on the Postoperative Refraction

Solving Equation 8 for D_c provides the expected total corneal power D_{ce}, for which the considered eye is emmetropic when an IOL of power D_i is positioned so that its object principal plane H_i is located at distance ELP_T from the image principal plane of the cornea.

$$D_{ce} = \frac{n_a\,(n_v + ELP_T D_i - AL_T D_i)}{n_a AL_T - ELP_T\,(AL_T D_i - D_i ELP_T + n_a - n_v)} \tag{10}$$

The refraction in the spectacle plane of a given pseudophakic eye, is equal to the difference between D_c



LA (mm)	23.55
S1S3 (mm)	3.210
ELPT (mm)	3.759
LT (SE=-1.25D, X=-0.5) (mm)	23.513
Dc (D)	43.23
Dce (D)	42.00
Di (D)	20
Central IOL thickness (mm)	0.75
Edge IOL thickness (mm)	0.253

Figure 2. A cross-sectional diagram of the refracting components of the paraxial schematic eye. The principal object and image planes of the cornea and IOL are plotted with dashed lines.

and D_{ce} in the corneal plane:

$$SE = \frac{D_{ce} - D_c}{1 + (D_{ce} - D_c)\,d} \tag{11}$$

Results

Determination of ELP_T ($\overline{H'_c H_i}$) and ALP ($\overline{S_1 S_3}$) From a Known Dataset

Knowing the postoperative SE of the operated eye in the spectacle plane, the biometric variables necessary for calculations as well as the lens geometry, the first step is to determine the theoretical value of the corneal power D_{ce} needed to achieve emmetropia, using Equation 9. The AL_T is computed from Equation 6. Finally, we obtain the value of the ELP_T using Equation 8. Here we provide a numerical example, for which a comprehensive description of the corneal and IOL design parameters is available.

Preoperative biometry parameter values and postoperative refraction:

R_{ca} = 7.71 mm, R_{cp} = 6.91 mm ; t_c = 0.543 mm, n_s = 1.376, n_a = 1.337, n_v = 1.336, R_{ia} = 36.68 mm, R_{ip} = –12.23 mm, n_i = 1.52, t_i = 0.75 mm, Coddington shape factor X = –0.5, AL_A = 23.55 mm, SE = –1.25 D.

From these values and using appropriate equations, we obtain:

D_c = 43.23 D (Equation A1), D_i = 20 D (Equation A5), D_{ce} = 42 D (Equation 9), $\overline{S_1 H'_c}$ = 0.0522 mm (Equation A2), $\overline{H_i H'_i}$ = –0.0892 mm (Equation A8), AL_T = 23.513 mm (Equation 6)

Using Equation 8, we finally obtain: ELP_T= 3.758 mm. Equation 3b allows to get the physical position of the IOL, defined as the distance between the corneal and IOL vertices: ALP = $\overline{S_1 S_3}$ = 3.210 mm

These numerical data are used to depict schematically, the dual optic refracting system, and their respective principal planes and remarkable distances in Figure 2.

Determination of the Impact of Implant Design on Postoperative Refraction

In these simulations, the influence of parameters related to the optical design of a monofocal implant on the refraction in the spectacle plane is studied. This enables prediction of the potential refractive surprise that would be incurred from a deviation of the achieved ELP_T from its intended plane, caused by an IOL's design variation.

To assess the influence of the optical design of the implant on ocular refraction for the same anatomic position of the optic, simulations were carried out by modifying the Coddington shape factor for IOLs of selected powers. For each tested IOL power, varying the shape factor results in different pairs of R_{ia} and R_{ip} (see Appendix C), but the anterior vertex and posterior vertex were kept at the same distance from the corneal vertex ($\overline{S_1S_3}$ and IOL thickness were kept constant regardless of the value taken by the Coddington shape factor). The corneal parameters were identical for each of these theoretical examples. For each tested IOL configuration, the impact of a variation of the X shape factor on the ELP_T was computed using Equation A6 and Equation 3. For each of the selected IOL powers, the axial length was adjusted to induce emmetropia for a biconvex symmetrical IOL (X = 0) using Equation 7b. For positive powers, the central thickness of the IOL was determined so that the thickness of the 6 mm diameter optic at the haptic junction was 250 ± 5 microns.

The impact of the shape factor's induced variation of the ELP_T on the SE was computed for different IOL powers (–5 D, 5 D, 15 D, 25 D, and 35 D) with X varying from –1 to +1 by 0.1 steps using the method described in 1.5 of the Methods section. The theoretical locations of the ELP_T for X = –1, X = 0, X = +1, and of the ELP_t (corresponding with the position of an IOL of null thickness, making the considered eye emmetropic) were plotted for each of the computations (Figs. 3a through 3e).

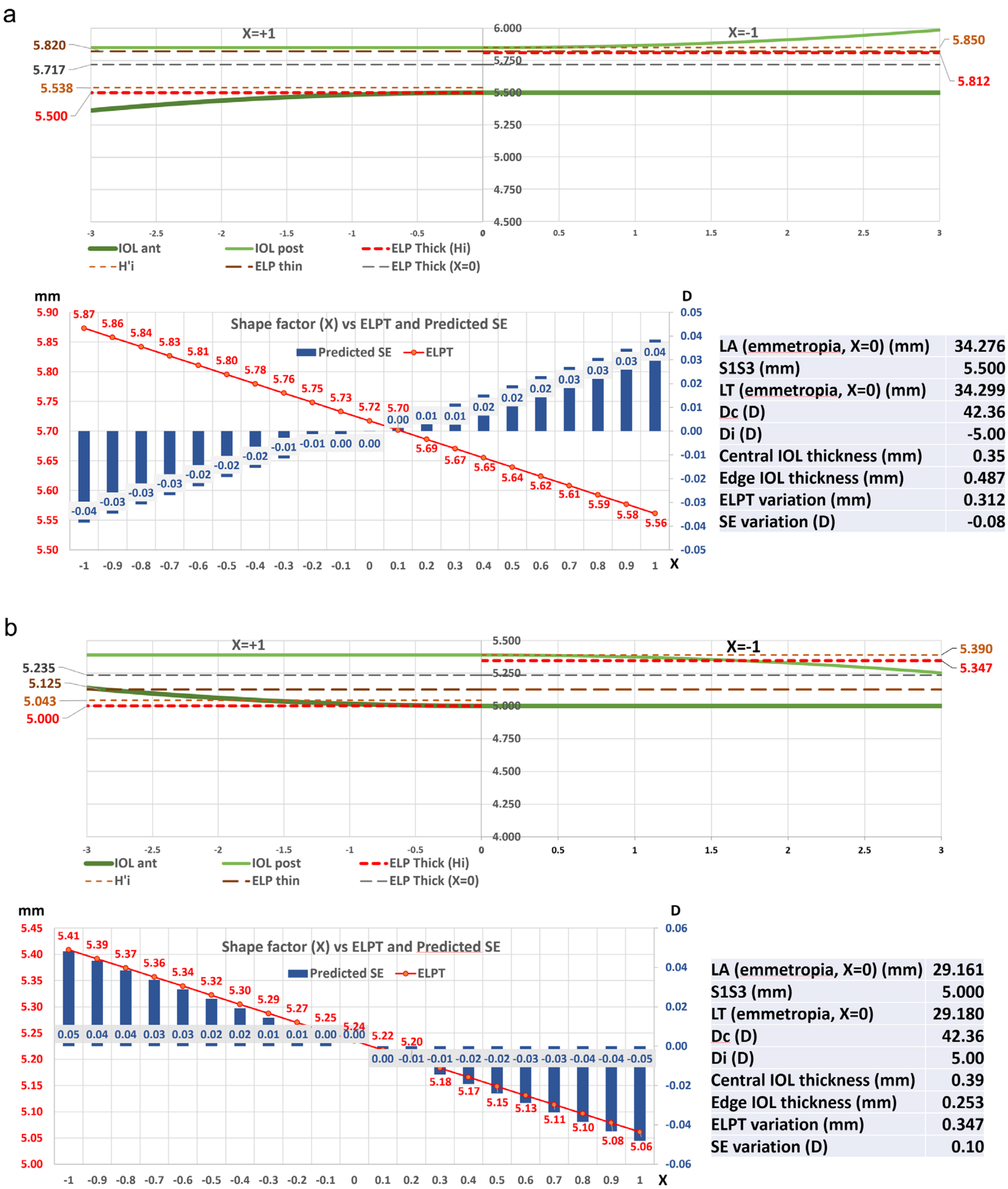
Discussion

An improvement in refractive outcome requires better methods for predicting the postoperative IOL position.[16,17,18] The estimation of postoperative IOL position is essential to IOL power calculations for cataract surgery and is also a critical variable in ray tracing. Similarly, the prediction of the ELP is an important issue to improve the refractive precision of the calculation of the IOL power with machine learning. Holladay et al.[19] were the first to publish an explicit formula for calculating the effective position of the implant in a thin lens model. Later, they discussed the relationship between the ELP of a thin versus thick lens of equivalent power.[20] At the time of this pioneering work, it was not routine practice to measure the posterior surface of the cornea, which was considered as a single dioptric surface. The net optical power of the cornea was obtained using a fictitious refractive index of the cornea equal to 4/3.

The effective power of an IOL depends on its geometrical characteristics and the exact intraocular locations of its refractive surfaces.[21] The determination of the ELP is formula dependent and does not need to reflect the real postoperative IOL position in the anatomic sense. However, knowing the geometry of the IOL makes it possible using a thick lens paraxial model to relate the optically estimated position of an IOL to its estimated anatomic position. Fernández et al.[22] studied the relationship between the measured ALP and the back-calculated ELP and demonstrated the differences between both values, induced by assumptions made in theoretical eye models.

In this article, we describe an explicit formula allowing back calculation of the theoretical position of the principal object plane of an IOL in a thick lens eye model, with four refractive surfaces and distinct refractive indices between the aqueous and vitreous humor. Our work could be useful for the development of methods based on machine learning to provide an estimate of the position of the implant that predicts the postoperative SE, from which it is possible to deduce the estimated anatomic position of the IOL if its geometry is known. Modern corneal topographers of the Scheimpflug or OCT type make it possible to measure the curvature of the anterior and posterior surfaces of the cornea, as well as its thickness. Conversely, these geometric data are generally not disclosed by manufacturers of IOLs. However, our results predict that, for the same anatomic position (distance between the corneal and IOL anterior vertices, defined as $\overline{S_1S_3}$ in our paraxial model) variations in the design of medium to high power IOLs can induce significant variations in postoperative refraction.

The definition of the position of the implant varies depending on the model used by the calculation formula: because the paraxial thin lens formulas assumes that the IOL has zero thickness, the computation of the ELP (labeled ELP_t in our model) does not provide any direct information about the position of the actual thick lens within the eye. Conversely, in a thick lens eye model, the ELP corresponds to the distance between the principal image plane of the cornea and the principal object plane of the IOL. In our model, this distance was labeled "ELP_T" and appears in the third term of the Gullstrand equation. For given corneal and IOL total powers, it suffices to know the ELP_T value to determine the paraxial refractive properties of that dual optics refracting component. Once these properties are known, it is necessary to know the axial length to predict the refraction of the eye in consideration. In our thick lens schematic eye model, the axial length considered for the calculation using Equation 7 is altered as it is augmented by the segment connecting the corneal vertex to the principal corneal image plane $\overline{S_1H'_c}$ and decreased by the interstice between the IOL's principal plane $\overline{H_iH'_i}$. This transformation is expected in our paraxial model, because it corresponds with the suppression of the interstice between the principal planes of



LA (emmetropia, X=0) (mm)	34.276
S1S3 (mm)	5.500
LT (emmetropia, X=0) (mm)	34.299
Dc (D)	42.36
Di (D)	-5.00
Central IOL thickness (mm)	0.35
Edge IOL thickness (mm)	0.487
ELPT variation (mm)	0.312
SE variation (D)	-0.08

LA (emmetropia, X=0) (mm)	29.161
S1S3 (mm)	5.000
LT (emmetropia, X=0)	29.180
Dc (D)	42.36
Di (D)	5.00
Central IOL thickness (mm)	0.39
Edge IOL thickness (mm)	0.253
ELPT variation (mm)	0.347
SE variation (D)	0.10

Figure 3. (*Top*) Schematic representation of the position of the ELP_T for specific geometries of the implant shown in section (left: X = 1, right: X = –1). The position of the ELP_T is shown for X = –1, X = 0, and X = +1. The position of the ELP_t (thin lens model) is also displayed. All distances (in mm) are computed from S_1. (*Bottom*) ELP_T shift and refraction variations predicted for an emmetropic eye with an IOL having zero form factor (X = 0, symmetrical biconvex IOL). (*Inset*) Summary of the results and main biometric variables used for the computations, including a fixed anatomic position ($\overline{S_1S_3}$), which value was chosen arbitrarily from commonly observed clinical cases. (a) D_i = –5 D, $\overline{S_1S_3}$ = 5.5 mm; (b) D_i = 5 D, $\overline{S_1S_3}$ = 5.0 mm; (c) D_i = 15 D, $\overline{S_1S_3}$ = 4.5 mm; (d) D_i = 25 D, $\overline{S_1S_3}$ = 4.0 mm; and (e) D_i =35 D, $\overline{S_1S_3}$ = 3.5 mm.

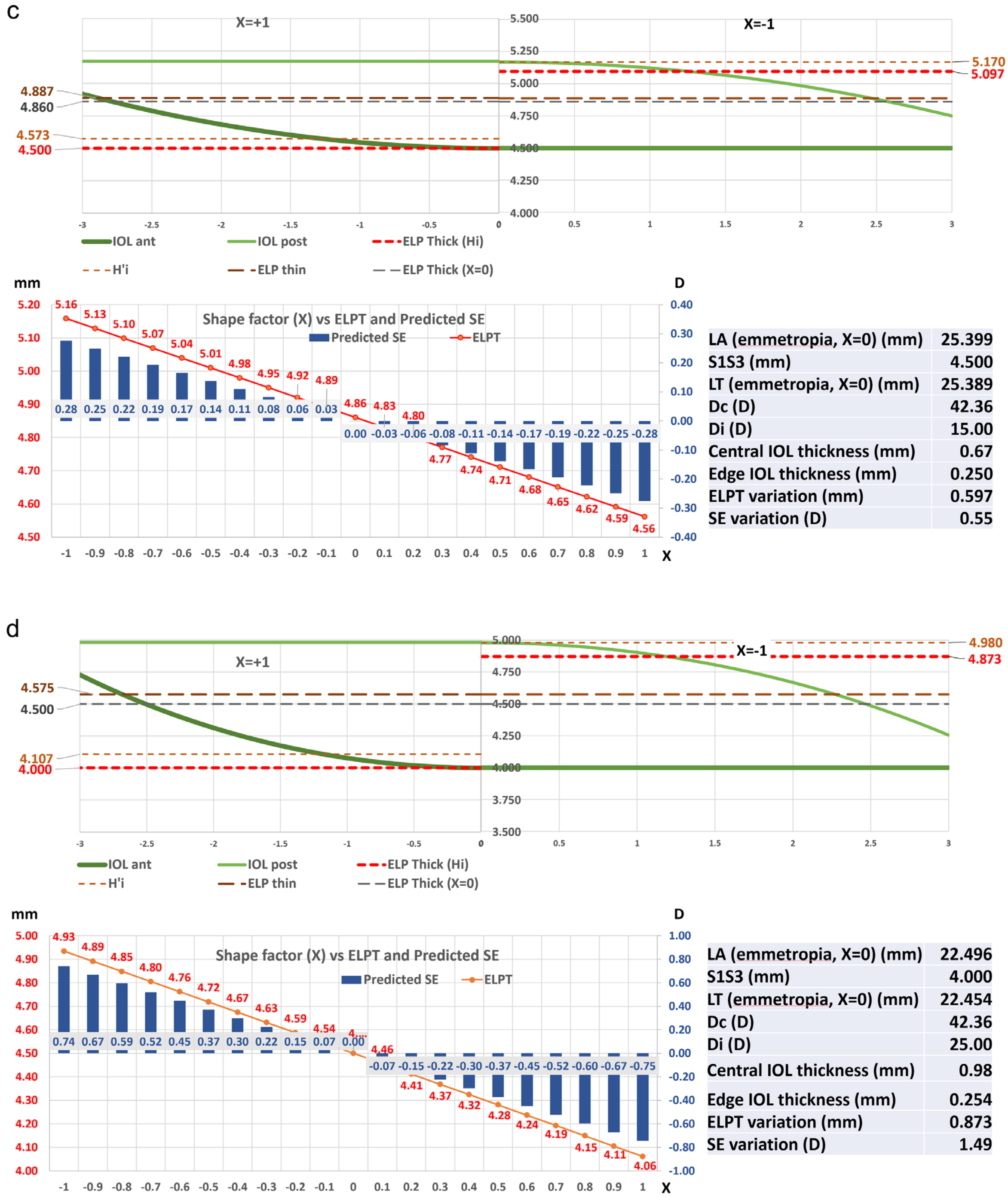


Figure 3. Continued

the system. Owing to the small dimensions of these algebraic segments and their opposite sign, this operation may have no clinically significant consequences on the results of numerical calculations involving the axial length. For a given pseudophakic eye, Equation 7 can be used to back-calculate an optimized axial length, which produces a refractive prediction error of zero, in the context of regression calculus such as that which has been used to improve the IOL power calculations in eyes with axial lengths of more than 25 mm.[23]

Our calculations allow estimation of the impact of an IOL of the same nominal power but differing designs, on the postoperative refraction. For bi or plano-convex implants (shape factors ranged from −1 to +1), the maximum amplitude of displacement of the principal object plane is equal to the central IOL thickness decreased by the distance between the

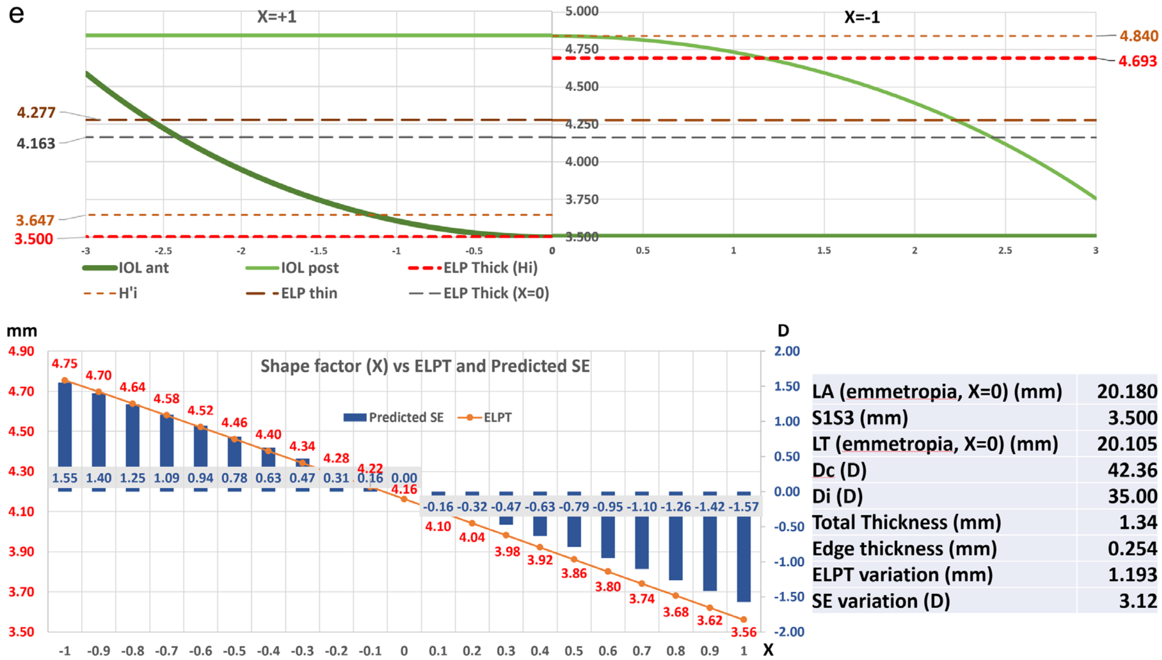


Figure 3. Continued

principal planes $\overline{H_i H'_i}$. The impact on postoperative refraction of the ELP_T shift is highly dependent on the optical power of the IOL as suggested by the Gullstrand equation. As the thickness of the implants increases with their power, this tendency is increased, and we calculated that these design variations could induce postoperative refraction variations between approximately 0.50 and 3.0 D, for implant powers ranging from 15 D to 35 D. Therefore, the use of high index, thinner implants reduce the influence of variations in optical design. We did not explore the influence of the corneal geometry and power in the ELP_T prediction. Using paraxial matrix optical calculations, Schröder and Langenbucher[24] have studied the relationship between the thin lens predicted ELP and axial position of a thick IOL achieving the same refraction (respectively referred to as ELP_t and ALP in the present work). They found that the corneal power had less influence than the lens power and design, on the difference between the ELP and the ALP. In all scenarios, the ALP was shorter than the ELP, which was confirmed in the present work where we also found that the ALP of a thick lens must be placed in front of the thin and thick ELP locations for achieving the same refraction. We limited our analysis to biconvex lenses, although these authors included concave–convex minus powered IOLs. Although the latter did increase the discrepancy between ALP and ELP, the influence of variations in the ELP_T distance on postoperative refraction appears to be relatively weak and not clinically significant for low power implants, even for negative implants, whose convex concave design could induce a more marked variation between the position of their vertices and the plane of the ELP_T. The improvement of the calculation formulas for long eyes requires other adjustments than those related to the actual position of the implant, which rather concern the measurement of the posterior segment of the eye and the historical assumptions used by current biometers to infer the axial length from the measured optical path length.[25,26] This finding is reflected in a recent study[27] in which there was no significant difference in the accuracy of thick lens IOL power formula based on calculated versus manufacturer's IOL data for eyes with ALs of 22 mm and more. Fernández et al.[22] suggested modifying the refractive index of the cornea to correct errors beyond the ELP prediction, including assumptions from the biometers.

The design characteristics of the IOLs are not usually known when using a dataset for machine learning training. As expected, the thin lens ELP is different from the symmetrical biconvex thick lens ELP ($X = 0$), although the difference is small and not clinically significant. A system trained to predict the ELP according to a thin lens approximation may be relatively efficient for other implant models if their design is similar and homogeneous throughout their power range, but performance would be degraded for medium to high power lenses for which different designs are employed at specific power intervals.

There are various other methods to predict the ELP with machine learning based techniques. These methods are usually based on the parameters of optical coherence tomography, biometry, and anterior segment optical coherence tomography acquired before and after cataract surgery.[14,28–30] In a work evaluating the prediction accuracy of ELP after cataract surgery using a multiobjective evolutionary algorithm,[31] the ELP was defined as the distance from the cornea to the anterior surface of the IOL at 3 months after surgery plus the distance to the presumed principal object point of the IOLs. The predicted ELP was obtained by a multiobjective evolutionary algorithm and multiple regression analysis, including a dozen parameters, and the difference between the predicted value and measured value (prediction accuracy) was compared between the two methods. The study demonstrated that ELP prediction by a multiobjective evolutionary algorithm was more accurate and was a method with less fluctuation than that of stepwise multiple regression and conventional formulas such as SRK/T and Haigis formulas.

The paraxial nature of our model does not consider corneal and IOL aberrations, which limits its accuracy, and several tools can be used to provide a better optical description of the eye which incorporate cornea aberrations and polychromatic estimations.[32,33] This might be even more important in eyes that have undergone corneal refractive surgery.[34]

In conclusion, we have described a set of equations to back-calculate both the optical and anatomic positions of an artificial lens implant in an eye model with thick lenses formulas.[35] This makes it possible to study the theoretical influence of factors linked to the design of the implant, on its effective position and the potential refractive variations that may result from it. This article aimed to provide specialists interested in the field of implant calculation and related problems, with a basis of explicit equations intended to solve numerically, problems related to dual thick lens optics with four refracting surfaces. It also underlines the critical importance of IOL geometry in IOL formulas training and calculation processes, especially for short eyes. This data is not usually released by the manufacturers; in an era where most of the optical parameters of the eye can be accurately measured or predicted, this consensus should be challenged.

Future studies could be of interest to compare the theoretical computations with the achieved position of the implant, using imaging techniques such as optical coherence tomography after cataract surgery,[36] to improve the accuracy of models useful for theoretical formulas and machine learning algorithms for IOL power calculation.

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References

1. Wang W, Yan W, Fotis K, et al. Cataract surgical rate and socioeconomics: a global study. *Invest Ophthalmol Vis Sci*. 2016;57:5872–5881.
2. Jiang F, Jiang Y, Zhi H, et al. Artificial intelligence in healthcare: past, present and future. *Stroke Vasc Neurol*. 2017;2:230–243.
3. Melles RB, Kane JX, Olsen T, Chang WJ. Update on intraocular lens calculation formulas. *Ophthalmology*. 2019;126:1334–1335.
4. Darcy K, Gunn D, Tavassoli S, Sparrow J, Kane JX. Assessment of the accuracy of new and updated intraocular lens power calculation formulas in 10 930 eyes from the UK National Health Service. *J Cataract Refract Surg*. 2020;46:2–7.
5. Savini G, Hoffer KJ, Balducci N, Barboni P, Schiano-Lomoriello D. Comparison of formula accuracy for intraocular lens power calculation based on measurements by a swept-source optical coherence tomography optical biometer. *J Cataract Refract Surg*. 2020;46:27–33.
6. Sramka M, Slovak M, Tuckova J, Stodulka P. Improving clinical refractive results of cataract surgery by machine learning. *PeerJ*. 2019;7:e7202.
7. Nemeth G, Modis L, Jr. Accuracy of the Hill-radial basis function method and the Barrett Universal II formula. *Eur J Ophthalmol*. 2020:1120672120902952.
8. Shajari M, Kolb CM, Petermann K, et al. Comparison of 9 modern intraocular lens power calculation formulas for a quadrifocal intraocular lens. *J Cataract Refract Surg*. 2018;44:942–948.
9. Szalai E, Toth N, Kolkedi Z, Varga C, Csutak A. Comparison of various intraocular lens formulas using a new high-resolution swept-source optical coherence tomographer. *J Cataract Refract Surg*. 2020;46:1138–1141.
10. Carmona-González D, Castillo-Gómez A, Palomino-Bautista C, Romero-Domínguez M, Gutiérrez-Moreno MÁ. Comparison of the

accuracy of 11 intraocular lens power calculation formulas. *Eur J Ophthalmol*. 2020: 1120672120962030.
11. Savini G, Taroni L, Hoffer KJ. Recent developments in intraocular lens power calculation methods-update 2020. *Ann Transl Med*. 2020;8:1553.
12. Holladay JT. Standardizing constants for ultrasonic biometry, keratometry, and intraocular lens power calculations. *J Cataract Refract Surg*. 1997;23:1356–1370.
13. Satou T, Shimizu K, Tsunehiro S, et al. Relationship between crystalline lens thickness and shape and the identification of anterior ocular segment parameters for predicting the intraocular lens position after cataract surgery. *Biomed Res Int*. 2019;2019:3458548.
14. Goto S, Maeda N, Koh S, et al. Prediction of postoperative intraocular lns position with angle-to-angle depth using anterior segment optical coherence tomography. *Ophthalmology*. 2016;123:2474–2480.
15. Yoo Y-S, Whang W-J, Kim H-S, Joo C-K, Yoon G. New IOL formula using anterior segment three-dimensional optical coherence tomography. *PLoS One*. 2020;15:e0236137.
16. Atchison DA. Refractive errors induced by displacement of intraocular lenses within the pseudophakic eye. *Optom Vis Sci*. 1989;66:146–152.
17. Norrby S. Sources of error in intraocular lens power calculation. *J Cataract Refract Surg*. 2008;34:368–376.
18. Olsen T. Sources of error in intraocular lens power calculation. *J Cataract Refract Surg*. 1992;18:125–129.
19. Holladay JT, Prager TC, Chandler TY, Musgrove KH, Lewis JW, Ruiz RS. A three-part system for refining intraocular lens power calculations. *J Cataract Refract Surg*. 1988;14:17–24.
20. Holladay JT, Maverick KJ. Relationship of the actual thick intraocular lens optic to the thin lens equivalent. *Am J Ophthalmol*. 1998;126:339–347.
21. Naeser K. Intraocular lens power formula based on vergence calculation and lens design. *J Cataract Refract Surg*. 1997;23:1200–1207.
22. Fernández J, Rodríguez-Vallejo M, Martínez J, Tauste A, Piñero DP. New approach for the calculation of the intraocular lens power based on the fictitious corneal refractive index estimation. *J Ophthalmol*. 2019;2019:1–9.
23. Wang L, Shirayama M, Ma XJ, Kohnen T, Koch DD. Optimizing intraocular lens power calculations in eyes with axial lengths above 25.0 mm. *J Cataract Refract Surg*. 2011;37:2018–2027.
24. Schröder S, Langenbucher A. Relationship between effective lens position and axial position of a thick intraocular lens. *PLoS ONE*. 2018;13:e0198824.
25. Abulafia A, Barrett GD, Rotenberg M, et al. Intraocular lens power calculation for eyes with an axial length greater than 26.0 mm: comparison of formulas and methods. *J Cataract Refract Surg*. 2015;41:548–556.
26. Wang L, Cao D, Weikert MP, Koch DD. Calculation of axial length using a single group refractive index versus using different refractive indices for each ocular segment: theoretical study and refractive outcomes. *Ophthalmology*. 2019;126:663–670.
27. Næser K, Savini G. Accuracy of thick-lens intraocular lens power calculation based on cutting-card or calculated data for lens architecture. *J Cataract Refract Surg*. 2019;45:1422–1429.
28. Taroni L, Hoffer KJ, Barboni P, Schiano-Lomoriello D, Savini G. Outcomes of IOL power calculation using measurements by a rotating Scheimpflug camera combined with partial coherence interferometry. *J Cataract Refract Surg*. 2020;46:1618–1623.
29. Satou T, Shimizu K, Tsunehiro S, et al. Development of a new intraocular lens power calculation method based on lens position estimated with optical coherence tomography. *Sci Rep*. 2020;10: 6501.
30. Martinez-Enriquez E, Pérez-Merino P, Durán-Poveda S, Jiménez-Alfaro I, Marcos S. Estimation of intraocular lens position from full crystalline lens geometry: towards a new generation of intraocular lens power calculation formulas. *Sci Rep*. 2018;8:9829.
31. Tamaoki A, Kojima T, Tanaka Y, et al. Prediction of effective lens position using multiobjective evolutionary algorithm. *Transl Vis Sci Technol*. 2019;8:64.
32. Rosales P, Marcos S. Customized computer models of eyes with intraocular lenses. *Opt Express*. 2007;15:2204–2218.
33. Canovas C, Artal P. Customized eye models for determining optimized intraocular lenses power. *Biomed Opt Express*. 2011;2:1649–1662.
34. Canovas C, van der Mooren M, Rosén R, et al. Effect of the equivalent refractive index on intraocular lens power prediction with ray tracing after myopic laser in situ keratomileusis. *J Cataract Refract Surg*. 2015;41:1030–1037.
35. Barrett GD. Intraocular lens calculation formulas for new intraocular lens implants. *J Cataract Refract Surg*. 1987;13:389–396.

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36. Ding X, Wang Q, Xiang L, Chang P, Huang S, Zhao Y-E. Three-dimensional assessments of intraocular lens stability with high-speed swept-source optical coherence tomography. *J Refract Surg*. 2020;36:388–394.

Appendices

(All distances must be expressed in meters for numerical applications.)

The paraxial schematic refracting components of the cornea, IOL and pseudophakic eye are represented in Figure A1.

A) Determination of the Power and Position of the Principal Planes of the Cornea, IOL, and Total Pseudophakic Eye

The considered paraxial model is represented in Figure A1. The cornea and the implant are assimilated to thick lenses whose anterior and posterior curvature have specific radii of curvature.

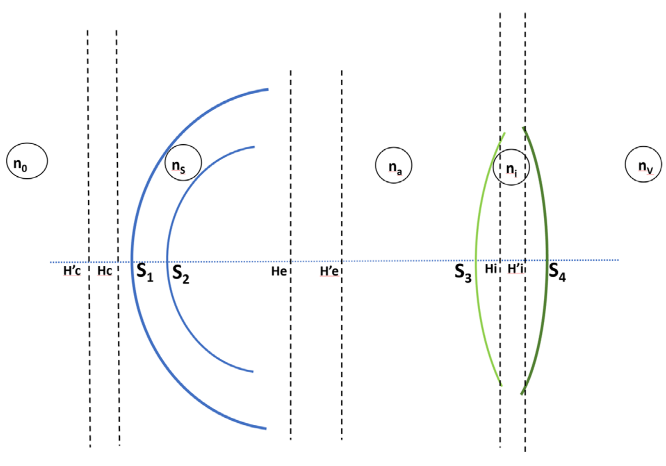

Figure A1. Representation of the refractive components of the paraxial schematic pseudophakic eye. n_a is the refractive index of aqueous humor, n_v the refractive index of the vitreous index. n_i is the refractive index of the IOL. The index of refraction of the air $n_o = 1$. S_1 and S_2 are the anterior and posterior vertices of the cornea, respectively. S_3 and S_4 are the anterior and posterior vertices of the IOL, respectively. H_c, H'_c, H_i and H'_i are the intercepts of the principal object and image planes of the cornea and the IOL with the optical axis. H_e and H'_e are the intercepts of the principal object and image planes of the eye considered as the centered optical dual system composed of the cornea and the IOL.

A1) Cornea

The total power of the cornea is computed with the following formula:

$$D_c = \frac{n_s - 1}{R_{ca}} + \frac{n_a - n_s}{R_{cp}} + \frac{(n_s - 1)(n_a - n_s)\,d_c}{n_s R_{ca} R_{cp}} \quad \text{(A1)}$$

Where:

R_{ca} is the radius of curvature of the anterior cornea, R_{cp} is the radius of curvature of the posterior cornea, n_s is the refractive index of stroma, n_a is the refractive index of aqueous, and d_c is the corneal thickness.

Example: $n_s = 1.376$, $n_a = 1.337$, $d_c = 0.545$ mm (0.00545 m) $R_a = 7.75$ mm (0.00775 m), $R_p = 6.45$ mm (0.00645 m) → $D_c = 42.59$ D

The distance $\overline{S_1H_c}$ is given by:

$$\overline{S_1H_c} = h_{c1} = \frac{(n_a - n_s)\,dc}{n_s R_{cp} D_c} \quad \text{(A2)}$$

$\overline{S_1H_c} = –0.056$ mm

The distance $\overline{S_2H'_c}$is given by

$$\overline{S_2H'_c} = h_{c2} = -\frac{n_a (n_s - 1)\,dc}{n_s R_{ca} D_c} \quad \text{(A3)}$$

$\overline{S_2H'_c} = –0.60$ mm

The segment $\overline{S_1H'_c}$ is given by:

$$\overline{S_1H'_c} = \overline{S_1S_2} + \overline{S_2H'_c} = d_c + h_{c2} \quad \text{(A4)}$$

$\overline{S_1H'_c} = –0.058$ mm

A2) Intraocular Lens

The power of an IOL implant immersed in saline solution of index n_{ss} is given by:

$$D_i = \frac{n_i - n_{ss}}{R_{ia}} + \frac{n_{ss} - n_i}{R_{ip}} + \frac{(n_i - n_{ss})(n_{ss} - n_i)\,d_i}{n_i R_{ia} R_{ip}} \quad \text{(A5)}$$

Where D_i is the labeled IOL power, n_i is the refractive index of the IOL, d_i is the central thickness of the IOL, R_{ia} is the radius of curvature of the anterior surface, R_{ip} is the radius of curvature of the posterior surface, n_i is the refractive index of the IOL, n_a is the refractive index of aqueous, and d_i is the IOL thickness.

Example: For $n_i = 1.52$, $n_{ss} = 1.335$, $d_i = 0.7$ mm, $R_{ia} = 23.97$ mm, $R_{ip} = –12.91$ mm, we get $D_i = 22$ D

Once implanted in the eye, D_i may change because of the changes in refractive index. Using the (previous) equation with $n_a = 1.337$ and $n_v = 1.336$ appropriately substituted to n_{ss} we get $D_i = 21.84$ D.

In most of this article, we considered that the power of the IOL corresponded to that of a lens immersed in a solution of index n = 1.336. Our equations, however, allow us to assign different values and distinguish the refractive index of the aqueous humor from that of the vitreous when necessary.

The distance $\overline{S_3H_i}$ is given by:

$$\overline{S_3H_i} = h_{i1} = \frac{n_a\,(n_v - n_i)\,d_i}{n_i R_{ip} D_i} \tag{A6}$$

$\overline{S_3H_i}$=0.40 mm

The distance $\overline{S_4H'_i}$ is given by:

$$\overline{S_4H'_i} = h_{i2} = -\frac{n_v\,(n_i - n_a)\,d_i}{n_i R_{ia} D_i} \tag{A7}$$

$\overline{S_4H'_i}$ = –0.22 mm

The segment $\overline{H_iH'_i}$ is given by:

$$\overline{H_iH'_i} = \overline{S_3S_4} - \overline{S_3H_i} + \overline{S_4H'_i} = d_i - h_{i1} + h_{i2} \tag{A8}$$

$\overline{H_iH'_i}$= 0.083 mm

A3) Model Eye

The determination of the principal planes and focal lengths of the eye constituted of the cornea and IOL, is determined in the same manner. The eye is compared with a system of double lenses separated by aqueous humor and in contact with air on one side, and vitreous on the other.

The distance between the cornea and the crystalline lens is taken to exist between the principal image plane of the cornea H'$_c$ and the principal object plane of the IOL H$_i$. This distance corresponds with the ELP in a thick lens paraxial pseudophakic eye model, denoted ELP$_T$ in the manuscript. Hence, the total power of the eye is given by:

$$D_e = D_c + D_i - \frac{\overline{H'_cH_i}\,D_cD_i}{n_a} \tag{A9}$$

The position of the principal planes is given by:

Principal object plane:

$$\overline{H_cH_e} = \frac{D_i\,\overline{H'_cH_i}}{n_aD_e} \tag{A10}$$

Principal image plane:

$$\overline{H'_iH'_e} = -\frac{n_vD_c\,\overline{H'_cH_i}}{n_aD_e} \tag{A11}$$

B) Decomposition of the Axial Length

We can split the anatomic axial length into two algebraic segments:

$$AL_A = \overline{S_1H'_e} + \overline{H'_eF'_e} \tag{B1}$$

The $\overline{H'_eF'_e}$ segment is the image focal length of the entire eye and is given by:

$$\overline{H'_eF'_e} = \frac{n_v}{D_e} \tag{B2}$$

$\overline{S_1H'_e}$separates the anterior surface of the cornea from the main image plane of the eye and can be decomposed as:

$$\overline{S_1H'_e} = \overline{S_1H'_c} + \overline{H'_cH_i} + \overline{H_iH'_i} + \overline{H'_iH'_e} \tag{B3}$$

Because ELP$_T$ = $\overline{H'_cH_i}$ and $\overline{H'_iH'_e} = -\overline{H'_cH_i}\frac{n_vD_c}{n_aD_e}$

We get:

$$\overline{S_1H'_e} = \overline{S_1H'_c} + \overline{H'_cH_i} + \overline{H_iH'_i} - \overline{H'_cH_i}\frac{n_vD_c}{n_aD_e} \tag{B4}$$

Finally, summing Equation B2 and Equation B4, and rearranging the terms, we obtain:

$$AL_A = \overline{S_1H'_c} + ELP_T + \overline{H_iH'_i} + \frac{n_v - \frac{n_v}{n_a}D_cELP_T}{D_e} \tag{B5}$$

C) Coddington Shape Factor

The Coddington shape factor is given by:

$$X = \frac{\left(R_{ip} - R_{ia}\right)}{\left(R_{ip} + R_{ia}\right)} \tag{C1}$$

From this we get:

$$R_{ip} = \frac{(X\,R_{ia} + R_{ia})}{(1 - X)} \tag{C2}$$

Replacing R$_{ip}$ by this expression in Equation A5 produces an expression which allows computation of the value of R$_{ia}$ for a specific IOL power D$_i$ and Coddington shape factor X. The corresponding value of R$_{ip}$ is obtained using Equation C2.

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