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Dynamic analysis and optimal control considering cross transmission and variation of information

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Cross-transmission of information has a profound influence on the progress of science and technology and the discipline integration in the field of education. In this work, knowledge gained from the viral recombination and variation in COVID-19 transmission is applied to information transmission. Virus recombination and virus variation are similar to the crossing and information fusion phenomena in information transmission. An *S2I4MR* model with information crossing and variation is constructed. Then, the local and global asymptotic stabilities of the information-free equilibrium and information-existence equilibrium are analyzed. Additionally, the basic reproduction number R_0 of the model is calculated. As such, an optimal control strategy is hereby proposed to promote the cross-transmission of information and generate variant information. The numerical simulations support the results of the theoretical analysis and the sensitivity of the system towards certain control parameters. In particular, the results show that strengthening information crossing promotes the generation of variant information. Furthermore, encouraging information exchange and enhancing education improve the generation of information crossing and information variation.

Information allows individuals to comprehend their surroundings and is critical in the development of human society. The concept of information is described in *Cybernetics* by Norbert Wiener, where he states, "Information is the content and name that human beings exchange with the external world in the process of adapting to and reacting to the external world." Furthermore, he adds, "Including social systems, it adjusts and determines its own movements according to certain changes in the surrounding environment"[1]. The entry of new information in the social system causes fluctuations. People need to estimate the impact of information on human society to formulate strategies for promoting the information transmission that is beneficial for social development[2,3], and at the same time suppress the information transmission that is harmful for social development[4,5].

The ownership and transmission mode of information are the main factors that affect its process of transmission. An open social system contains a plethora of homogeneous or heterogeneous information. In addition, different types of information diffuse together and generate new information. In principle, the transmission of information is very similar to the transmission of infectious diseases[6,7]. Several literature works have adopted the classical model of infectious diseases[8–10] to the research of rumor transmission[11] and information transmission[12,13]. Based on this, the process of studying information transmission in the present work is inspired by the spread of *SARS-COV-2*. It is found that the mutated virus, such as the Omicron variation (B.1.1.529)[14] has changed the transmissibility and pathogenicity of the original strain[15]. Moreover, it has also changed the way the virus impacts human society. This phenomenon of virus recombination and variation can also be applied to information cross-transmission and variation, as information also deviates from its original path during its transmission. Therefore, the model of virus recombination and variation can be employed to construct and describe the social phenomenon of information cross-transmission and variation.

The aforementioned works have presented extensive research on the cross-transmission of information. The results showed that the cross-transmission expands the scope of information transmission, and also enhances the intensity of information transmission. However, after the cross-transmission of information, new variants of information are formed, in a manner similar to a viral strain, and the mutated information generally changes the

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content of the original information. The natural variation of information has also been considered[27], but so far, a limited amount of works that consider the variations caused by the cross-transmission of such information exists.

Various works presented in literature focus on the cross-transmission of information. Zan[16] proposed *DSIR* and *C-DSIR* models and analyzed the interaction mechanism of dual rumor transmission in the BA network. The results showed that after an old rumor has been spreading for a certain period, releasing a new rumor is more favorable for the co-transmission of the two rumors. Yin et al.[17] showed that in real social networks, multiple pieces of information exist for collaborative transmission. Based on this, a *CT-FSI* model was constructed to analyze the cross-transmission behavior of multiple pieces of information on Chinese microblogs and the continuous attraction index was presented. The results showed that cross-transmission of multiple pieces of information continues to spread in a cyclic manner. Yin et al.[18] analyzed the hot topics of Chinese microblogs and observed that forwarding the information multiple times deepens the impression and spread of the topic. Therefore, the authors proposed the *MR-SFI* model which showed that the greater the number and intensity of re-forwarding, the wider the spread of hot topics is in microblogs. Huo et al.[19] constructed the $I_K I_U S_K S_U R$ model by considering two groups with and without scientific knowledge and analyzed the behavior of rumors spreading in each group. The results show that the group with scientific knowledge showed higher immunity to rumors. At the same time, the positive reinforcement of publicity can also resist the spread of rumors. Recently, the information transmission of *COVID-19* has attracted the focus of the research community. Yin et al.[20] discussed that part of epidemic information is unable to reach the public in an effective and timely manner. The authors constructed the $S_1S_2F_1F_2I_{1+}I_{1-}I_2$ model, which showed that cross-transmission of information makes the transmission scope wider. In fact, information cross-transmission is pivotal in spreading rumors in a multilingual environment. By analyzing the process of rumor spreading in multiple languages in the homogeneous, heterogeneous and scale-free networks, the authors constructed the *SIR*[21], *I2S2R*[22], *IE2S2R*[23], *2I2SR*[24], *IS2R2*[25], *ILSR*[26], and $S^{(1)}S^{(2)}IR$[12] models by considering two groups. The results obtained using these models showed a common feature, i.e., that rumors spread more widely due to the cross-transmission of multiple languages, while increasing the cross-contact rate and enhancing the intensity of rumor spread.

The aforementioned works have conducted extensive research on the cross transmission of information. The results show that the cross transmission expands the scope of information transmission, and also enhances the intensity of information transmission. However, after the cross transmission of information, new variants of information are formed similar to a viral strain, and the mutated information generally changes the content of the original information. Currently, some scholars have also considered the natural variation of information[27], but there are very few works that consider the variations caused due to the cross transmission of such information.

The modeling of cross-transmission and variation in information transmission is inspired by the phenomenon of virus recombination and variation in the COVID-19 transmission. However, information crossing and variation are desirable in certain situations. For instance, academics encourage the global integration of multidisciplinary information and employ the new cross-disciplines in various applications. Interdisciplinary fields, such as biomathematics, physical chemistry and biochemistry, are important in the development of a wide range of applications. Therefore, the aim of this work is to propose a model that considers the cross-transmission of multiple information and the resulting variant information to determine the impact of cross-transmission of information on the resulting variant information. Furthermore, the control strategies used to enhance the intensity of information crossing and the promotion of the generation of information variants are also discussed. A system of ordinary differential equations is created to describe the problem in question. The spreading scope of information crossing on the social system is obtained by calculating the basic regeneration number. The validity of the proposed model is obtained by analyzing the equilibrium point and the stability. Finally, the basic theorem of the model and the effectiveness of the control strategy are verified by selecting appropriate parameters as the control variables and numerical simulations. In contrast to previous literature, the existence of virus recombination and variation in COVID-19 transmission is hereby compared to information transmission. Meanwhile, an information transmission model is constructed considering cross-transmission and variation, which includes multi-information and multi-transmittable groups. In addition, the optimal control strategy of information transmission is quantified by scientific methods.

The rest of this paper is organized as follows. The *S2I4MR* model that considers the information cross-transmission on social media and the generated variations is presented in "The model " Section. The local and global stability of basic reproduction number R_0, information-free equilibrium, and information-existence equilibrium are presented in "Stabilityanalysisofthemodel" Section. The existence and strategy for controlling the information transmission and variation in an optimal manner are presented in "The optimalcontrolmodel" Section. The influence of parameter changes on information transmission and variation and the effect of optimal control strategy based on numerical simulations are illustrated in "Numericalsimulations " Section. The sensitivity analysis of control parameters in information transmission is presented in "Sensitivityanalysis" Section. Finally, "Conclusions" Section provides the conclusion.

The model

In this work, an open virtual community is considered. The population size is variable at any time t, and the total population is expressed as $N(t)$. The population can be divided into eight categories: (1) The easy adopters who are not exposed to information but easily adopt the information, denoted as $S(t)$; (2) People who are exposed to both kinds of information but choose to spread the first kind of information, denoted as $I_1(t)$; (3) The group exposed to both kinds of information that chooses to spread the second kind of information, denoted as $I_2(t)$; (4) The transmitters who are exposed to both kinds of information but spread the first kind of information and ultimately choose the variation group that believes in the first kind of information, denoted as $M_1(t)$; (5) The transmitters who are exposed to both kinds of information but spread the first kind of information and finally

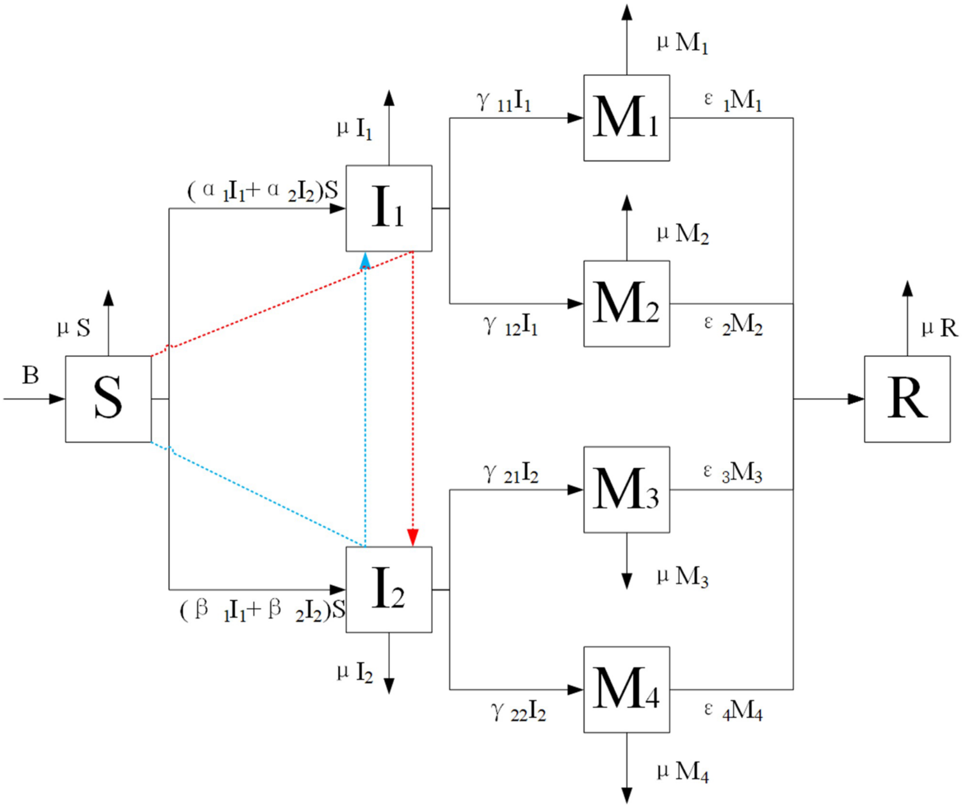

Figure 1. The flow diagram of the model.

choose the variation group that integrates the two kinds of information, denoted as $M_2(t)$; (6) The transmitters who are exposed to both kinds of information but spread the second kind of information and finally choose the variation group that integrates the two kinds of information, denoted as $M_3(t)$; (7) The transmitters who are exposed to both kinds of information but spread the second kind of information and ultimately choose the variation group who believes in the second kind of information, denoted as $M_4(t)$; and (8) The fleeing crowd that is not interested in any kind of information as well as the corresponding variation information, denoted as $R(t)$.

The model proposed in this work considers the common life phenomenon of "concept preconception". This means that when some easy-to-adopt populations have preferential access to any information, then in their minds, the information will be transmitted first. Even if they are exposed to another kind of information, they will not transmit it immediately. This is the main differentiation between populations I_1 and I_2. At the same time, when the transmitters that are already exposed to the first kind of information are exposed to the second kind of information, they fuse the two kinds of information after a period of analysis, thus forming the variation information group. However, since they prioritize the first type of information, they will use the second kind of information as a supplement to expand the content of the first kind of information. This is the main difference between populations M_2 and M_3. Infectious disease variants generally remain transmissible. However, the information variant does not spread easily due to the uncertainty in the content of the information. This is precise because humans have subjective judgments, whereas viruses are generally a result of natural selection.

In order to reflect the phenomenon of cross transmission and information variation in information transmission, an *S2I4MR* model is constructed in this work. The model flow diagram is given in Fig. 1.

The parameters of *S2I4MR* model are interpreted as follows:

- In a social system, the number of individuals generally varies over time. Therefore, this work defines B as the number of immigrants in the social system. At the same time, it considers some individuals that may withdraw from the social system due to some force majeure factors. μ is defined as the emigration rate in this work;
- When information begins to spread in a social system, there is a certain progression rate that the easy adopters will contact the transmitters of the information. When the easy adopters are first exposed to the first kind of information and then to the second kind, they prioritize spreading the first kind of information. The contact rate of the first information is defined as α_1, and the contact rate of the second information as α_2. In order to express the phenomenon of "concept preconception", here, $\alpha_1 \geq \alpha_2$, and the group exposed to the first kind of information transmitters with the progression rate of α_1 must include the group exposed to the second kind of information transmitters with progression rate of α_2. In the same way, for the group that preferentially transmits the second kind of information, the contact rate of the first kind of information is

Parameter	Description
$S(t)$	The number of easy adopters at the time t
$I_1(t)$	The number of individuals choose to spread the first kind of information at the time t
$I_2(t)$	The number of individuals choose to spread the second kind of information at the time t
$M_1(t)$	The transmitters who ultimately choose the variation group that believes in the first kind of information
	at the time t
$M_2(t)$	The transmitters who finally choose the variation group that integrates the two kinds of information at the time t
$M_3(t)$	The transmitters who finally choose the variation group that integrates the two kinds of information at the time t
$M_4(t)$	The transmitters who ultimately choose the variation group that believes in the second kind of information
	at the time t
$R(t)$	The number of individuals that not interested in both kinds of information as well as the corresponding
	variation information at the time t
B	The number of immigrants in the social system per unit time
α	Progression rate from state S to I_1
β	Progression rate from state S to I_2
γ_{11}	Progression rate from state I_1 to M_1
γ_{12}	Progression rate from state I_1 to M_2
γ_{21}	Progression rate from state I_2 to M_3
γ_{22}	Progression rate from state I_2 to M_4
ε_1,ε_2,ε_3,ε_4	Progression rate from state M_1, M_2, M_3, M_4 to R
μ	Removal rate per unit time

Table 1. The parameters description of *S2I4MR* model.

β_1, and the contact rate of the second kind of information is β_2. In order to represent the phenomenon of "concept preconception", $\beta_1 \leq \beta_2$, and the group exposed to the second kind of information transmitters with the progression rate of β_2 must include the group exposed to the first transmitters with the progression rate of β_1;

- When the transmitters of two kinds of information fuse the information, the information variation is generated. When the transmitters of both types of information believe in the original information, they are mutated into M_1 and M_4 groups with the progression rate of γ_{11} and γ_{22}, respectively. When the transmitters of the two kinds of information choose to fuse the information, they mutate with the progression rate of γ_{12} into the M_2 group that regards the first kind of information as the main and the second kind of information as the auxiliary. The transmitters mutate with the progression rate of γ_{21} into the M_3 group that considers the second kind of information as the main and the first kind of information as the auxiliary;
- After the information exists the social system after a certain period of time, it is often eliminated by the society, or is no longer accepted by people. Therefore, the variation group chooses to escape with the progression rate of ε_1, ε_2, ε_3, and ε_4.

Based on the aforementioned analysis, we constructed the *S2I4MR* model by considering the cross transmission and variation of information. In order to facilitate the understanding and analysis, a special case is considered where the easy-adopter populations are exposed to both kinds of information with equal progression rate, i.e., $\alpha_1 = \alpha_2 = \alpha$ and $\beta_1 = \beta_2 = \beta$. Therefore, the fraction of the population as proceeding form from S to I_1 is α. In addition, the fraction of the population as proceeding form from S to I_2 is β.

The parameters of *S2I4MR* model are summarized in Table 1.

The system dynamics are mathematically expressed as follows:

$$\begin{cases} \frac{dS}{dt} &= B - \alpha(I_1 + I_2)S - \beta(I_1 + I_2)S - \mu S, \\ \frac{dI_1}{dt} &= \alpha(I_1 + I_2)S - \gamma_{11}I_1 - \gamma_{12}I_1 - \mu I_1, \\ \frac{dI_2}{dt} &= \beta(I_1 + I_2)S - \gamma_{22}I_2 - \gamma_{21}I_2 - \mu I_2, \\ \frac{dM_1}{dt} &= \gamma_{11}I_1 - \varepsilon_1 M_1 - \mu M_1, \\ \frac{dM_2}{dt} &= \gamma_{12}I_1 - \varepsilon_2 M_2 - \mu M_2, \\ \frac{dM_3}{dt} &= \gamma_{21}I_2 - \varepsilon_3 M_3 - \mu M_3, \\ \frac{dM_4}{dt} &= \gamma_{22}I_2 - \varepsilon_4 M_4 - \mu M_4, \\ \frac{dR}{dt} &= \varepsilon_1 M_1 + \varepsilon_2 M_2 + \varepsilon_3 M_3 + \varepsilon_4 M_4 - \mu R. \end{cases} \tag{1}$$

Where:

$$\begin{aligned} &B > 0, \mu > 0, \varepsilon_1 > 0, \varepsilon_2 > 0, \varepsilon_3 > 0, \varepsilon_4 > 0, \\ &\alpha \in (0,1], \beta \in (0,1], \gamma_{11} \in (0,1], \gamma_{12} \in (0,1], \lambda_{21} \in (0,1], \gamma_{22} \in (0,1], \end{aligned} \tag{2}$$

and

$$S(t) + I_1(t) + I_2(t) + M_1(t) + M_2(t) + M_3(t) + M_4(t) + R(t) = N(t). \tag{3}$$

It is easy to know that $\frac{dN(t)}{dt} = B - \mu N$, so $N(t) = \left(N_0 - \frac{B}{\mu}\right)e^{-\mu t} + \frac{B}{\mu}$, where $N_0 = N(0)$, and then $\lim_{t\to\infty} N(t) = \frac{B}{\mu}$. The positive invariant set of System (1) is $\Gamma = \left\{(S, I_1, I_2, M_1, M_2, M_3, M_4, R) \in R_8^+ : S + I_1 + I_2 + M_1 + M_2 + M_3 + M_4 + R \le \frac{B}{\mu}\right\}$.

Stability analysis of the model

Firstly, it is necessary to demonstrate the existence of equilibrium $E = (S, I_1, I_2, M_1, M_2, M_3, M_4, R)$ of the system dynamics Eq. (1). The information-free equilibrium point of System (1) can be easily obtained as $E^0 = (B/\mu, 0, 0, 0, 0, 0, 0, 0)$, which means the number of information disseminators tend to zero in System (1).

Then, the basic reproduction number R_0 of System (1) can be defined by the next generation matrix[28]. The basic reproduction number is important to intervene for a system, which represents the number of next generation from a single information disseminator produced.

Let $X = (I_1, I_2, S, M_1, M_2, M_3, M_4, R)^T$, then System (1) can be written as:

$$\frac{dX}{dt} = F(X) - V(X) \tag{4}$$

$$F(X) = \begin{pmatrix} \alpha(I_1 + I_2)S \\ \beta(I_1 + I_2)S \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, V(X) = \begin{pmatrix} \gamma_{11}I_1 + \gamma_{12}I_1 + \mu I_1 \\ \gamma_{22}I_2 + \gamma_{21}I_2 + \mu I_2 \\ -B + \alpha(I_1 + I_2)S + \beta(I_1 + I_2)S + \mu S \\ -\gamma_{11}I_1 + \varepsilon_1 M_1 + \mu M_1 \\ -\gamma_{12}I_1 + \varepsilon_2 M_2 + \mu M_2 \\ -\gamma_{21}I_2 + \varepsilon_3 M_3 + \mu M_3 \\ -\gamma_{22}I_2 + \varepsilon_4 M_4 + \mu M_4 \\ -\varepsilon_1 M_1 - \varepsilon_2 M_2 - \varepsilon_3 M_3 - \varepsilon_4 M_4 + \mu R \end{pmatrix}. \tag{5}$$

We can get:

$$F = \begin{pmatrix} \alpha S & \alpha S \\ \beta S & \beta S \end{pmatrix}, V = \begin{pmatrix} \gamma_{11} + \gamma_{12} + \mu & 0 \\ 0 & \gamma_{22} + \gamma_{21} + \mu \end{pmatrix}. \tag{6}$$

where F and V represent the infection and transition matrices respectively[29]. Hence, the basic reproduction number R_0 of System (1) is the spectral radius of the next generation matrix FV^{-1}. R_0 can be computed as:

$$R_0 = \rho(FV^{-1}) = \frac{B\alpha(\gamma_{22} + \gamma_{21} + \mu) + B\beta(\gamma_{11} + \gamma_{12} + \mu)}{\mu(\gamma_{11} + \gamma_{12} + \mu)(\gamma_{22} + \gamma_{21} + \mu)}. \tag{7}$$

While the information will be spread if $R_0 > 1$. The information-existence equilibrium point of System (1) can be expressed as $E^* = (S^*, I_1^*, I_2^*, M_1^*, M_2^*, M_3^*, M_4^*, R^*)$, which means the information will spread widely. The information-existence equilibrium E^* should satisfy:

$$\begin{cases} B - \alpha(I_1{}^* + I_2{}^*)S^* - \beta(I_1{}^* + I_2{}^*)S^* - \mu S^* & = 0, \\ \alpha(I_1{}^* + I_2{}^*)S^* - \gamma_{11}I_1{}^* - \gamma_{12}I_1{}^* - \mu I_1{}^* & = 0, \\ \beta(I_1{}^* + I_2{}^*)S^* - \gamma_{22}I_2{}^* - \gamma_{21}I_2{}^* - \mu I_2{}^* & = 0, \\ \gamma_{11}I_1{}^* - \varepsilon_1 M_1{}^* - \mu M_1{}^* & = 0, \\ \gamma_{12}I_1{}^* - \varepsilon_2 M_2{}^* - \mu M_2{}^* & = 0, \\ \gamma_{21}I_2{}^* - \varepsilon_3 M_3{}^* - \mu M_3{}^* & = 0, \\ \gamma_{22}I_2{}^* - \varepsilon_4 M_4{}^* - \mu M_4{}^* & = 0, \\ \varepsilon_1 M_1{}^* + \varepsilon_2 M_2{}^* + \varepsilon_3 M_3{}^* + \varepsilon_4 M_4{}^* - \mu R^* & = 0. \end{cases} \tag{8}$$

Let $\varphi = I_1^* + I_2^*$. The information-existence equilibrium E^* can be deduced as the following equations by solving Eqs. (8):

$$S^* = \frac{B}{\alpha\varphi + \beta\varphi + \mu}, I_1^* = \frac{\alpha\varphi S^*}{(\gamma_{11} + \gamma_{12} + \mu)}, I_2^* = \frac{\beta\varphi S^*}{(\gamma_{22} + \gamma_{21} + \mu)}. \tag{9}$$

Since

$$\varphi = I_1 + I_2 = \frac{\alpha\varphi S^*}{(\gamma_{11} + \gamma_{12} + \mu)} + \frac{\beta\varphi S^*}{(\gamma_{22} + \gamma_{21} + \mu)}, \tag{10}$$

we can get

$$S^* = \frac{(\gamma_{11} + \gamma_{12} + \mu)(\gamma_{22} + \gamma_{21} + \mu)}{\alpha(\gamma_{22} + \gamma_{21} + \mu) + \beta(\gamma_{11} + \gamma_{12} + \mu)} = \frac{B}{\mu R_0}, \tag{11}$$

and

$$\varphi = \frac{\mu(R_0 - 1)}{\alpha + \beta}, \tag{12}$$

the other equilibrium points can be obtained as:

$$\begin{aligned} I_1^* &= \frac{B\alpha\varphi}{\mu(\gamma_{11}+\gamma_{12}+\mu)R_0}, I_2^* = \frac{B\beta\varphi}{\mu(\gamma_{22}+\gamma_{21}+\mu)R_0}, \\ M_1^* &= \frac{B\alpha\gamma_{11}\varphi}{\mu(\varepsilon_1+\mu)(\gamma_{11}+\gamma_{12}+\mu)R_0}, M_2^* = \frac{B\alpha\gamma_{12}\varphi}{\mu(\varepsilon_2+\mu)(\gamma_{11}+\gamma_{12}+\mu)R_0}, \\ M_3^* &= \frac{B\beta\gamma_{21}\varphi}{\mu(\varepsilon_3+\mu)(\gamma_{22}+\gamma_{21}+\mu)R_0}, M_4^* = \frac{B\beta\gamma_{22}\varphi}{\mu(\varepsilon_4+\mu)(\gamma_{22}+\gamma_{21}+\mu)R_0}. \end{aligned} \tag{13}$$

Theorem 1 *If $R_0 < 1$, $B(\alpha+\beta) < \mu(\gamma_{11}+\gamma_{12}+\mu)(\gamma_{22}+\gamma_{21}+\mu)$ and $(\gamma_{11}+\gamma_{12}+\mu) = (\gamma_{22}+\gamma_{21}+\mu)$, the information-free equilibrium point $E^0 = (B/\mu, 0, 0, 0, 0, 0, 0, 0)$ of System (1) is locally asymptotically stable.*

Proof 1 The Jacobin matrix of System (1) at information-free equilibrium point $E^0 = (B/\mu, 0, 0, 0, 0, 0, 0, 0)$ can be written as:

$$J(E^0) = \begin{bmatrix} -\mu & -\frac{B\alpha}{\mu} - \frac{B\beta}{\mu} & -\frac{B\alpha}{\mu} - \frac{B\beta}{\mu} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{B\alpha}{\mu} - (\gamma_{11}+\gamma_{12}+\mu) & \frac{B\alpha}{\mu} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{B\beta}{\mu} & \frac{B\beta}{\mu} - (\gamma_{22}+\gamma_{21}+\mu) & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma_{11} & 0 & -\varepsilon_1-\mu & 0 & 0 & 0 & 0 \\ 0 & \gamma_{12} & 0 & 0 & -\varepsilon_2-\mu & 0 & 0 & 0 \\ 0 & 0 & \gamma_{21} & 0 & 0 & -\varepsilon_3-\mu & 0 & 0 \\ 0 & 0 & \gamma_{22} & 0 & 0 & 0 & -\varepsilon_4-\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu \end{bmatrix}. \tag{14}$$

The negative eigenvalues of $J(E^0)$ can be easily obtained as $\Lambda_{01} = \Lambda_{02} = -\mu < 0$, $\Lambda_{03} = -\varepsilon_1 - \mu < 0$, $\Lambda_{04} = -\varepsilon_2 - \mu < 0$, $\Lambda_{05} = -\varepsilon_3 - \mu < 0$, $\Lambda_{06} = -\varepsilon_4 - \mu < 0$, and the other eigenvalues are the characteristic roots of $\left|hE - J(E^0)\right|$, where:

$$\left|hE - J(E^0)\right| = \begin{vmatrix} h - \frac{B\alpha}{\mu} + (\gamma_{11}+\gamma_{12}+\mu) & -\frac{B\alpha}{\mu} \\ -\frac{B\beta}{\mu} & h - \frac{B\beta}{\mu} + (\gamma_{22}+\gamma_{21}+\mu) \end{vmatrix}. \tag{15}$$

The eigenvalues of Eq. (15) can be obviously obtained as:

$$\Lambda_{07} = \frac{\left[\frac{B\alpha}{\mu} + \frac{B\beta}{\mu} - (\gamma_{11}+\gamma_{12}+\mu) - (\gamma_{22}+\gamma_{21}+\mu)\right]}{2} + \frac{\sqrt{\left[\frac{B\alpha}{\mu} + \frac{B\beta}{\mu} - (\gamma_{11}+\gamma_{12}+\mu) - (\gamma_{22}+\gamma_{21}+\mu)\right]^2 + 4(\gamma_{11}+\gamma_{12}+\mu)(\gamma_{22}+\gamma_{21}+\mu)(R_0-1)}}{2}, \tag{16}$$

and

$$\Lambda_{08} = \frac{\left[\frac{B\alpha}{\mu} + \frac{B\beta}{\mu} - (\gamma_{11}+\gamma_{12}+\mu) - (\gamma_{22}+\gamma_{21}+\mu)\right]}{2} - \frac{\sqrt{\left[\frac{B\alpha}{\mu} + \frac{B\beta}{\mu} - (\gamma_{11}+\gamma_{12}+\mu) - (\gamma_{22}+\gamma_{21}+\mu)\right]^2 + 4(\gamma_{11}+\gamma_{12}+\mu)(\gamma_{22}+\gamma_{21}+\mu)(R_0-1)}}{2}. \tag{17}$$

If $B(\alpha+\beta) < \mu(\gamma_{11}+\gamma_{12}+\mu)(\gamma_{22}+\gamma_{21}+\mu)$ and $(\gamma_{11}+\gamma_{12}+\mu) = (\gamma_{22}+\gamma_{21}+\mu)$, so $\Lambda_{07} < 0$ and $\Lambda_{08} < 0$. Hence, the information-free equilibrium point $E^0 = (B/\mu, 0, 0, 0, 0, 0, 0, 0)$ of System (1) is locally asymptotically stable based on the Routh–Hurwitz criterion. □

Theorem 2 *If $R_0 < 1$ and $B(\alpha+\beta) \le \mu^2$, the information-free equilibrium point $E^0 = (B/\mu, 0, 0, 0, 0, 0, 0, 0)$ of System (1) is globally asymptotically stable.*

Proof 2 It is easy to know that $S(t) + I_1(t) + I_2(t) + M_1(t) + M_2(t) + M_3(t) + M_4(t) + R(t) = N(t)$ and satisfy $\frac{dN(t)}{dt} \le B - \mu S(t)$. It illustrates that:

$$\lim_{t\to 0} \sup N(t) \le \frac{B}{\mu}. \tag{18}$$

For $t \ge 0$, the positive invariant set of System (1) can be written as:

$$\begin{aligned} T &= \{(S(t), I_1(t), I_2(t), M_1(t), M_2(t), M_3(t), M_4(t), R(t)) \in R_8^+ : \\ &\quad S(t) + I_1(t) + I_2(t) + M_1(t) + M_2(t) + M_3(t) + M_4(t) + R(t) \leq \tfrac{B}{\mu}\}. \end{aligned} \tag{19}$$

Then, the Lyapunov function $L(t) = I_1(t) + I_2(t) + M_1(t) + M_2(t) + M_3(t) + M_4(t) + R(t)$ can be constructed and $L'(t)$ can be computed as:

$$\begin{aligned} L'(t) &= \alpha(I_1 + I_2)S - \gamma_{11}I_1 - \gamma_{12}I_1 - \mu I_1 + \beta(I_1 + I_2)S - \gamma_{22}I_2 - \gamma_{21}I_2 \\ &\quad - \mu I_2 + \gamma_{11}I_1 - \varepsilon_1 M_1 - \mu M_1 + \gamma_{12}I_1 - \varepsilon_2 M_2 - \mu M_2 + \gamma_{21}I_2 - \varepsilon_3 M_3 \\ &\quad - \mu M_3 + \gamma_{22}I_1 - \varepsilon_4 M_4 - \mu M_4 + \varepsilon_1 M_1 + \varepsilon_2 M_2 + \varepsilon_3 M_3 + \varepsilon_4 M_4 - \mu R \\ &= (-\mu + \alpha S + \beta S)(I_1 + I_2) - \mu M_1 - \mu M_2 - \mu M_3 - \mu M_4 - \mu R \\ &\leq \left(-\mu + \frac{B\alpha}{\mu} + \frac{B\beta}{\mu}\right)(I_1 + I_2) - \mu(M_1 + M_2 + M_3 + M_4 + R), \end{aligned} \tag{20}$$

it is easy to know that $L'(t) \leq 0$ if $S \leq \frac{B}{\mu}$ and $B(\alpha + \beta) \leq \mu^2$.

In addition, $L'(t) = 0$ holds if and only if $S(t) = S^0, I_1 = I_2 = M_1 = M_2 = M_3 = M_4 = R = 0$. From System (1), it is known that E^0 is the only solution in T when $L'(t) = 0$. Therefore, based on the Lyapunov-LaSalle Invariance Principle[30], it is shown that every solution of System (1) approach E^0 for $t \to \infty$. Hence, the information-free equilibrium point $E^0 = (B/\mu, 0, 0, 0, 0, 0, 0, 0)$ of System (1) is globally asymptotically stable. □

Theorem 3 *If $R_0 > 1$ and $\mu^2 R_0(R_0 - 1) > B(\alpha + \beta)$, the information-existence equilibrium point $E^* = (S^*, I_1^*, I_2^*, M_1^*, M_2^*, M_3^*, M_4^*, R^*)$ of system (1) is locally asymptotically stable.*

Proof 3 The Jacobin matrix of System (1) at information-existence equilibrium point $E^* = (S^*, I_1^*, I_2^*, M_1^*, M_2^*, M_3^*, M_4^*, R^*)$ can be written as:

$$J(E^*) = \begin{bmatrix} -\alpha\varphi^* - \beta\varphi^* - \mu & -\alpha S^* - \beta S^* & -\alpha S^* - \beta S^* & 0 & 0 & 0 & 0 & 0 \\ \alpha\varphi^* & \alpha S^* - (\gamma_{11} + \gamma_{12} + \mu) & \alpha S^* & 0 & 0 & 0 & 0 & 0 \\ \beta\varphi^* & \beta S^* & \beta S^* - (\gamma_{22} + \gamma_{21} + \mu) & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma_{11} & 0 & -\varepsilon_1 - \mu & 0 & 0 & 0 & 0 \\ 0 & \gamma_{12} & 0 & 0 & -\varepsilon_2 - \mu & 0 & 0 & 0 \\ 0 & 0 & \gamma_{21} & 0 & 0 & -\varepsilon_3 - \mu & 0 & 0 \\ 0 & 0 & \gamma_{22} & 0 & 0 & 0 & -\varepsilon_4 - \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu \end{bmatrix}. \tag{21}$$

The negative eigenvalues of $J(E^*)$ can be easily obtained as $\Lambda_{11} = -\mu < 0, \Lambda_{12} = -\varepsilon_1 - \mu < 0, \Lambda_{13} = -\varepsilon_2 - \mu < 0, \Lambda_{14} = -\varepsilon_3 - \mu < 0, \Lambda_{15} = -\varepsilon_4 - \mu < 0$, and the other eigenvalues are the characteristic roots of $|hE - J(E^*)|$, where:

$$\left|hE - J(E^*)\right| = \begin{vmatrix} h + \alpha\varphi^* + \beta\varphi^* + \mu & \alpha S^* + \beta S^* & \alpha S^* + \beta S^* \\ -\alpha\varphi^* & h - \alpha S^* - (\gamma_{11} + \gamma_{12} + \mu) & -\alpha S^* \\ -\beta\varphi^* & -\beta S^* & h - \beta S^* - (\gamma_{22} + \gamma_{21} + \mu) \end{vmatrix}. \tag{22}$$

The eigenvalues of Eq. (22) can be obviously obtained as:

$$\begin{aligned} \left|hE - J(E^*)\right| &= h^3 + \Big[\mu + (\gamma_{11} + \gamma_{12} + \mu) + (\gamma_{22} + \gamma_{21} + \mu) + \alpha(\varphi^* - S^*) \\ &\quad + \beta(\varphi^* - S^*)\Big]h^2 + \Big\{\mu(\gamma_{11} + \gamma_{12} + \mu + \gamma_{22} + \gamma_{21} + \mu) \\ &\quad + (\gamma_{11} + \gamma_{12} + \mu)(\gamma_{22} + \gamma_{21} + \mu) + \alpha(\gamma_{22} + \gamma_{21} + \mu)(\varphi^* - S^*) \\ &\quad + \beta(\gamma_{11} + \gamma_{12} + \mu)(\varphi^* - S^*) + \alpha\Big[(\gamma_{11} + \gamma_{12} + \mu)\varphi^* - \mu S^*\Big] \\ &\quad + \beta\Big[(\gamma_{22} + \gamma_{21} + \mu)\varphi^* - \mu S^*\Big]\Big\}h + \Big\{\mu(\gamma_{11} + \gamma_{12} + \mu)(\gamma_{22} \\ &\quad + \gamma_{21} + \mu) + \alpha(\gamma_{22} + \gamma_{21} + \mu)\Big[(\gamma_{11} + \gamma_{12} + \mu)\varphi^* - \mu S^*\Big] \\ &\quad + \beta(\gamma_{11} + \gamma_{12} + \mu)\Big[(\gamma_{22} + \gamma_{21} + \mu)\varphi^* - \mu S^*\Big]\Big\}. \end{aligned} \tag{23}$$

Then we construct a cubic polynomial and replace the coefficient with a_3, a_2, a_1, a_0 to determine the other eigenvalues of System (21). Hence, Eq. (23) can be rewritten as:

$$a_3h^3 + a_2h^2 + a_1h + a_0 = 0, \tag{24}$$

where:

$$a_3 = 1, \tag{25}$$

$$a_2 = [\mu + (\gamma_{11} + \gamma_{12} + \mu) + (\gamma_{22} + \gamma_{21} + \mu) + \alpha(\varphi^* - S^*) + \beta(\varphi^* - S^*)], \tag{26}$$

$$\begin{aligned} a_1 = {} & \mu(\gamma_{11} + \gamma_{12} + \mu + \gamma_{22} + \gamma_{21} + \mu) + (\gamma_{11} + \gamma_{12} + \mu)(\gamma_{22} + \gamma_{21} + \mu) \\ & + \alpha(\gamma_{22} + \gamma_{21} + \mu)(\varphi^* - S^*) + \beta(\gamma_{11} + \gamma_{12} + \mu)(\varphi^* - S^*) \\ & + \alpha[(\gamma_{11} + \gamma_{12} + \mu)\varphi^* - \mu S^*] + \beta[(\gamma_{22} + \gamma_{21} + \mu)\varphi^* - \mu S^*], \end{aligned} \tag{27}$$

$$\begin{aligned} a_0 = {} & \mu(\gamma_{11} + \gamma_{12} + \mu)(\gamma_{22} + \gamma_{21} + \mu) + \alpha(\gamma_{22} + \gamma_{21} + \mu)[(\gamma_{11} + \gamma_{12} \\ & + \mu)\varphi^* - \mu S^*] + \beta(\gamma_{11} + \gamma_{12} + \mu)[(\gamma_{22} + \gamma_{21} + \mu)\varphi^* - \mu S^*], \end{aligned} \tag{28}$$

then, let

$$\begin{cases} \omega_1 & = \mu + (\gamma_{11} + \gamma_{12} + \mu) + (\gamma_{22} + \gamma_{21} + \mu), \\ \omega_2 & = \alpha(\varphi^* - S^*), \\ \omega_3 & = \beta(\varphi^* - S^*), \\ \omega_4 & = \mu(\gamma_{11} + \gamma_{12} + \mu + \gamma_{22} + \gamma_{21} + \mu) + (\gamma_{11} + \gamma_{12} + \mu)(\gamma_{22} + \gamma_{21} + \mu), \\ \omega_5 & = \alpha[(\gamma_{11} + \gamma_{12} + \mu)\varphi^* - \mu S^*], \\ \omega_6 & = \beta[(\gamma_{22} + \gamma_{21} + \mu)\varphi^* - \mu S^*], \end{cases} \tag{29}$$

and

$$\begin{aligned} a_2 a_1 - a_3 a_0 = {} & \omega_1\omega_4 + (\gamma_{22} + \gamma_{21} + \mu)\omega_1\omega_2 + (\gamma_{11} + \gamma_{12} + \mu)\omega_1\omega_3 \\ & + \omega_2\omega_4 + (\gamma_{22} + \gamma_{21} + \mu)\omega_2^2 + (\gamma_{11} + \gamma_{12} + \mu)\omega_2\omega_3 \\ & + \omega_3\omega_4 + (\gamma_{22} + \gamma_{21} + \mu)\omega_2\omega_3 + (\gamma_{11} + \gamma_{12} + \mu)\omega_3^2 \\ & - \mu(\gamma_{11} + \gamma_{12} + \mu)(\gamma_{22} + \gamma_{21} + \mu) + [\omega_1 + \omega_2 + \omega_3 \\ & - (\gamma_{22} + \gamma_{21} + \mu)]\omega_5 + [\omega_1 + \omega_2 + \omega_3 - (\gamma_{11} + \gamma_{12} + \mu)]\omega_6. \end{aligned} \tag{30}$$

The condition of information-existence equilibrium point $E^* = (S^*, I_1^*, I_2^*, M_1^*, M_2^*, M_3^*, M_4^*, R^*)$ is locally asymptotically stable and the conditions: (i) $a_3, a_2, a_1, a_0 > 0$ and (ii) $a_2a_1 - a_3a_0 > 0$ based on the Routh–Hurwitz criterion. It is easy to know that $a_3 > 0$.

If $\mu^2 R_0(R_0 - 1) > B(\alpha + \beta)$ and $R_0 > 1$, then $a_2, a_1, a_0 > 0$ and $a_2a_1 - a_3a_0 > 0$. In this case, the Routh–Hurwitz criterion are satisfied. Hence, the information-existence equilibrium point $E^* = (S^*, I_1^*, I_2^*, M_1^*, M_2^*, M_3^*, M_4^*, R^*)$ of System (1) is locally asymptotically stable. □

Theorem 4 *If $R_0 > 1$, the information-existence equilibrium point $E^* = (S^*, I_1^*, I_2^*, M_1^*, M_2^*, M_3^*, M_4^*, R^*)$ of System (1) is globally asymptotically stable.*

Proof 4 We construct the Lyapunov function as:

$$\begin{aligned} W(t) = {} & [(S(t) - S^*) + (I_1(t) - I_1^*) + (I_2(t) - I_2^*) + (M_1(t) - M_1^*) \\ & + (M_2(t) - M_2^*) + (M_3(t) - M_3^*) + (M_4(t) - M_4^*) + (R(t) - R^*)]^2, \end{aligned} \tag{31}$$

and

$$\begin{aligned} W'(t) = {} & 2[(S(t) - S^*) + (I_1(t) - I_1^*) + (I_2(t) - I_2^*) + (M_1(t) - M_1^*) \\ & + (M_2(t) - M_2^*) + (M_3(t) - M_3^*) + (M_4(t) - M_4^*) + (R(t) \\ & - R^*)][S'(t) + {I_1}'(t) + {I_2}'(t) + {M_1}'(t) + {M_2}'(t) + {M_3}'(t) + {M_4}'(t) + R(t)] \\ = {} & 2[(S(t) - S^*) + (I_1(t) - I_1^*) + (I_2(t) - I_2^*) + (M_1(t) - M_1^*) \\ & + (M_2(t) - M_2^*) + (M_3(t) - M_3^*) + (M_4(t) - M_4^*) + (R(t) \\ & - R^*)][B - \mu S - \mu I_1 - \mu I_2 - \mu M_1 - \mu M_2 - \mu M_3 - \mu M_4 - \mu R]. \end{aligned} \tag{32}$$

Because of the existence of $E^* = (S^*, I_1^*, I_2^*, M_1^*, M_2^*, M_3^*, M_4^*, R^*)$, we can know that $B - \mu S^* - \mu I_1^* - \mu I_2^* - \mu M_1^* - \mu M_2^* - \mu M_3^* - \mu M_4^* - \mu R^*$=0, i.e., B=$\mu S^* + \mu I_1^* + \mu I_2^* + \mu M_1^* + \mu M_2^* + \mu M_3^* + \mu M_4^* + \mu R^*$.

Then, Eq. (32) can be computed as:

$$\begin{aligned} W'(t) = {} & 2[(S(t) - S^*) + (I_1(t) - I_1^*) + (I_2(t) - I_2^*) + (M_1(t) - M_1^*) \\ & + (M_2(t) - M_2^*) + (M_3(t) - M_3^*) + (M_4(t) - M_4^*) + (R(t) \\ & - R^*)][\mu S^* + \mu I_1^* + \mu I_2^* + \mu M_1^* + \mu M_2^* + \mu M_3^* + \mu M_4^* + \mu R^* \\ & - \mu S - \mu I_1 - \mu I_2 - \mu M_1 - \mu M_2 - \mu M_3 - \mu M_4 - \mu R] \\ = {} & -2[(S - S^*) + (I_1 - I_1^*) + (I_2 - I_2^*) + (M_1 - M_1^*) \\ & + (M_2 - M_2^*) + (M_3 - M_3^*) + (M_4 - M_4^*) + (R - R^*)^2. \end{aligned} \tag{33}$$

Besides that, $W'(t) = 0$ holds if and only if $S(t) = S^*, I_1(t) = I_1^*, I_2(t) = I_2^*, M_1(t) = M_1^*, M_2(t) = M_2^*, M_3(t) = M_3^*, M_4(t) = M_4^*, R(t) = R^*$. Hence, the information-existence equilibrium point $E^* = (S^*, I_1^*, I_2^*, M_1^*, M_2^*, M_3^*, M_4^*, R^*)$ of System (1) is globally asymptotically stable based on Lyapunov-LaSalle Invariance Principle[30]. □

The optimal control model

Based on the information transmission model established above, we consider the fact that the educational fields all over the world encourage interdisciplinary application. This is consistent with the information crossing we have constructed and the new variant information that has been generated. Therefore, in order to promote large-scale information crossing and strengthen the generation of variant information after crossing, two control objectives are accordingly proposed. On one hand, there are more and more people who are exposed to cross information, and on the other hand, there are more and more variations that fuse the two kinds of information. For this reason, the four proportionality constants α, β, γ_{12}, and γ_{21} in the model are transformed into control variables $\alpha(t)$, $\beta(t)$, $\gamma_{12}(t)$, and $\gamma_{21}(t)$, respectively. The control variable $\alpha(t)$ is used to control the rate of exposure to the first kind of information and then to the second kind of information. Similarly, the control variable $\beta(t)$ is used to control the rate of exposure to the second kind of information and then to the first kind of information. Generally, the rate of exposure is improved by improving the flow of people or organizing the information exchange activities. The control variable $\gamma_{12}(t)$ is used to control the rate of variation that considers the first kind of information as the main and the second information as the auxiliary. Moreover, the control variable $\gamma_{21}(t)$ is used to control the rate of variation that considers the second kind of information as the main and the first kind of information as the auxiliary. Generally, the variation rate can be improved based on educational guidance or policy encouragement.

Hence, an objective function can be proposed as:

$$\begin{aligned} J(\alpha, \beta, \gamma_{12}, \gamma_{21}) &= \int_0^{t_f} [I_1(t) + I_2(t) + M_2(t) + M_3(t) - c_1/2\alpha^2(t) \\ &- c_2/2\beta^2(t) - c_3/2\gamma_{12}^2(t) - c_4/2\gamma_{21}^2(t)], \end{aligned} \tag{34}$$

and satisfy the follow state system

$$\begin{cases} \frac{dS}{dt} &= B - \alpha(t)(I_1 + I_2)S - \beta(t)(I_1 + I_2)S - \mu S, \\ \frac{dI_1}{dt} &= \alpha(t)(I_1 + I_2)S - \gamma_{11}I_1 - \gamma_{12}(t)I_1 - \mu I_1, \\ \frac{dI_2}{dt} &= \beta(t)(I_1 + I_2)S - \gamma_{22}I_2 - \gamma_{21}(t)I_2 - \mu I_2, \\ \frac{dM_1}{dt} &= \gamma_{11}I_1 - \varepsilon_1 M_1 - \mu M_1, \\ \frac{dM_2}{dt} &= \gamma_{12}(t)I_1 - \varepsilon_2 M_2 - \mu M_2, \\ \frac{dM_3}{dt} &= \gamma_{21}(t)I_2 - \varepsilon_3 M_3 - \mu M_3, \\ \frac{dM_4}{dt} &= \gamma_{22}I_2 - \varepsilon_4 M_4 - \mu M_4, \\ \frac{dR}{dt} &= \varepsilon_1 M_1 + \varepsilon_2 M_2 + \varepsilon_3 M_3 + \varepsilon_4 M_4 - \mu R. \end{cases} \tag{35}$$

The initial conditions for System (35) are satisfied:

$$\begin{aligned} &S(0) = S_0, I_1(0) = I_{1,0}, I_2(0) = I_{2,0}, M_1(0) = M_{1,0}, M_2(0) = M_{2,0}, \\ &M_3(0) = M_{3,0}, M_4(0) = M_{4,0}, R(0) = R_0, \end{aligned} \tag{36}$$

where:

$$\begin{aligned} \alpha(t), \beta(t), \gamma_{12}(t), \gamma_{21}(t) \ \in \ & U \triangleq \{(\alpha, \beta, \gamma_{12}, \gamma_{21}) | (\alpha(t), \beta(t), \gamma_{12}(t), \gamma_{21}(t)) \\ & measurable, 0 \leq \alpha(t), \beta(t), \gamma_{12}(t), \gamma_{21}(t) \leq 1, \\ & \forall t \in [0, t_f]\}, \end{aligned} \tag{37}$$

while U is the admissible control set. The time interval of control is between 0 and t_f. c_1, c_2, c_3, c_4 are positive weight coefficients shown the control strength and importance of four control measures.

Theorem 5 *An optimal control pair* $(\alpha^*, \beta^*, {\gamma_{12}}^*, {\gamma_{21}}^*) \in U$ *exists so that the function is established below*:

$$J(\alpha^*, \beta^*, \gamma_{12}^*, \gamma_{21}^*) = \max\{J(\alpha, \beta, \gamma_{12}, \gamma_{21}) : (\alpha, \beta, \gamma_{12}, \gamma_{21}) \in U\}. \tag{38}$$

Proof 5 Let $X(t) = (S(t), I_1(t), I_2(t), M_1(t), M_2(t), M_3(t), M_4(t), R(t))^T$ and

$$\begin{aligned} L(t; X(t), \alpha(t), \beta(t), \gamma_{12}(t), \gamma_{21}(t)) &= I_1(t) + I_2(t) + M_2(t) + M_3(t) - c_1/2\alpha^2(t) \\ &- c_2/2\beta^2(t) - c_3/2\gamma_{12}^2(t) - c_4/2\gamma_{21}^2(t). \end{aligned} \tag{39}$$

The existence of an optimal pair must satisfy: (i) the set of control variables and state variables is nonempty, (ii) the control set U is convex and closed, (iii) the right-hand side of the state system is bounded by a linear function in the state and control variables, (iv) the integrand of the objective functional is convex on U, (v) there exist constants $d_1, d_2 > 0$ and $\rho > 1$ such that the integrand of the objective functional satisfies:

$$-L(t; X(t), \alpha; \beta; \gamma_{12}; \gamma_{21}) \geq d_1(|\alpha|^2 + |\beta|^2 + |\gamma_{12}|^2 + |\gamma_{21}|^2)^{\rho/2} - d_2. \tag{40}$$

Conditions (i)-(iii) are clearly established, we just prove the condition (iv) and (v). One can easily obtain inequality:

$$\begin{aligned} &S' \leq B, I_1{}' \leq \alpha(t)(I_1 + I_2)S, I_2{}' \leq \beta(t)(I_1 + I_2)S, M_1{}' \leq \gamma_{11}I_1, M_2{}' \leq \gamma_{12}(t)I_1, \\ &M_3{}' \leq \gamma_{21}(t)I_2, M_4{}' \leq \gamma_{22}I_1, R' \leq \varepsilon_1 M_1 + \varepsilon_2 M_2 + \varepsilon_3 M_3 + \varepsilon_4 M_4. \end{aligned} \tag{41}$$

Hence, condition (iv) is established. Then, for any $t \geq 0$, there is a positive constant M which is satisfied $|X(t)| \leq M$, therefore

$$\begin{aligned} -L(t; X(t), \alpha; \beta; \gamma_{12}; \gamma_{21}) =& (c_1\alpha^2(t) + c_2\beta^2(t) + c_3\gamma_{12}^2(t) + c_4\gamma_{21}^2(t))/2 \\ &- I_1(t) - I_2(t) - M_2(t) - M_3(t) \\ &\geq d_1(|\alpha|^2 + |\beta|^2 + |\gamma_{12}|^2 + |\gamma_{21}|^2)^{\rho/2} - 2M. \end{aligned} \tag{42}$$

Let $d_1 = \min\left\{\frac{c_1}{2}, \frac{c_2}{2}, \frac{c_3}{2}, \frac{c_4}{2}\right\}, d_2 = 2M$ and $\rho = 2$, then condition (v) is established. Hence, the optimal control can be realized. □

Theorem 6 *For the optimal control pair* $(\alpha^*, \beta^*, \gamma_{12}^*, \gamma_{21}^*)$ *of state System* (35), *there exist adjoint variables* $\delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6$,

δ_7, δ_8 *that satisfy*:

$$\begin{cases} \frac{d\delta_1}{dt} &= (\delta_1 - \delta_2)\alpha(t)(I_1 + I_2) + (\delta_1 - \delta_3)\beta(t)(I_1 + I_2) + \delta_1\mu, \\ \frac{d\delta_2}{dt} &= 1 + (\delta_1 - \delta_2)\alpha(t)S + (\delta_1 - \delta_3)\beta(t)S + (\delta_2 - \delta_4)\gamma_{11} + (\delta_2 - \delta_5)\gamma_{12}(t) + \delta_2\mu, \\ \frac{d\delta_3}{dt} &= 1 + (\delta_1 - \delta_2)\alpha(t)S + (\delta_1 - \delta_3)\beta(t)S + (\delta_3 - \delta_7)\gamma_{22} + (\delta_3 - \delta_6)\gamma_{21}(t) + \delta_3\mu, \\ \frac{d\delta_4}{dt} &= (\delta_4 - \delta_8)\varepsilon_1 + \varepsilon_4\mu, \\ \frac{d\delta_5}{dt} &= 1 + (\delta_5 - \delta_8)\varepsilon_2 + \varepsilon_5\mu, \\ \frac{d\delta_6}{dt} &= 1 + (\delta_6 - \delta_8)\varepsilon_3 + \varepsilon_6\mu, \\ \frac{d\delta_7}{dt} &= (\delta_7 - \delta_8)\varepsilon_4 + \varepsilon_7\mu, \\ \frac{d\delta_8}{dt} &= \delta_8\mu. \end{cases} \tag{43}$$

With boundary conditions:

$$\delta_1(t_f) = \delta_2(t_f) = \delta_3(t_f) = \delta_4(t_f) = \delta_5(t_f) = \delta_6(t_f) = \delta_7(t_f) = \delta_8(t_f) = 0. \tag{44}$$

In addition, the optimal control pair $(\alpha^*, \beta^*, \gamma_{12}^*, \gamma_{21}^*)$ *of state System* (35) *can be given by*:

$$\alpha^*(t) = \min\left\{1, \max\left\{0, \frac{(\delta_1 - \delta_2)(I_1 + I_2)S}{c_1}\right\}\right\}, \tag{45}$$

$$\beta^*(t) = \min\left\{1, \max\left\{0, \frac{(\delta_1 - \delta_3)(I_1 + I_2)S}{c_2}\right\}\right\}, \tag{46}$$

$$\gamma_{12}^*(t) = \min\left\{1, \max\left\{0, \frac{(\delta_2 - \delta_5)I_1}{c_3}\right\}\right\}, \tag{47}$$

$$\gamma_{21}^*(t) = \min\left\{1, \max\left\{0, \frac{(\delta_3 - \delta_6)I_2}{c_4}\right\}\right\}. \tag{48}$$

Proof 6 Define a Hamiltonian function enlarged with penalty term to obtain the expression of optimal control system and optimal control pair. The Hamiltonian function enlarged can be written as:

$$\begin{aligned} H =& -I_1(t) - I_2(t) - M_2(t) - M_3(t) + c_1/2\alpha^2(t) + c_2/2\beta^2(t) + c_3/2\gamma_{12}^2(t) + c_4/2\gamma_{21}^2(t) \\ &+ \delta_1[B - \alpha(t)(I_1 + I_2)S - \beta(t)(I_1 + I_2)S - \mu S] + \delta_2[\alpha(t)(I_1 + I_2)S - \gamma_{11}I_1 \\ &- \gamma_{12}(t)I_1 - \mu I_1] + \delta_3[\beta(t)(I_1 + I_2)S - \gamma_{22}I_2 - \gamma_{21}(t)I_2 - \mu I_2] + \delta_4[\gamma_{11}I_1 - \varepsilon_1 M_1 \\ &- \mu M_1] + \delta_5[\gamma_{12}(t)I_1 - \varepsilon_2 M_2 - \mu M_2] + \delta_6[\gamma_{21}(t)I_2 - \varepsilon_3 M_3 - \mu M_3] + \delta_7[\gamma_{22}I_1 \\ &- \varepsilon_4 M_4 - \mu M_4] + \delta_8[\varepsilon_1 M_1 + \varepsilon_2 M_2 + \varepsilon_3 M_3 + \varepsilon_4 M_4 - \mu R] - \lambda_{11}\alpha(t) - \lambda_{12}(1 - \alpha(t)) \\ &- \lambda_{21}\beta(t) - \lambda_{22}(1 - \beta(t)) - \lambda_{31}\gamma_{12}(t) - \lambda_{32}(1 - \gamma_{12}(t)) - \lambda_{41}\gamma_{21}(t) - \lambda_{42}(1 - \gamma_{21}(t)), \end{aligned} \tag{49}$$

which the penalty term is $\lambda_{ij}(t) \geq 0$, and it is satisfied that $\lambda_{11}(t)\alpha(t) = \lambda_{12}(t)(1-\alpha(t)) = 0$ at optimal control α^*, $\lambda_{21}(t)\beta(t) = \lambda_{22}(t)(1-\beta(t)) = 0$ at optimal control β^*, $\lambda_{31}(t)\gamma_{12}(t) = \lambda_{32}(t)(1-\gamma_{12}(t)) = 0$ at optimal control γ_{12}^* and $\lambda_{41}(t)\gamma_{21}(t) = \lambda_{42}(t)(1-\gamma_{21}(t)) = 0$ at optimal control γ_{21}^*.

Based on the Pontyragin maximum principle, the adjoint system can be written as:

$$\begin{aligned} \frac{d\delta_1}{dt} &= -\frac{\partial H}{\partial S}, \frac{d\delta_2}{dt} = -\frac{\partial H}{\partial I_1}, \frac{d\delta_3}{dt} = -\frac{\partial H}{\partial I_2}, \frac{d\delta_4}{dt} = -\frac{\partial H}{\partial M_1}, \\ \frac{d\delta_5}{dt} &= -\frac{\partial H}{\partial M_2}, \frac{d\delta_6}{dt} = -\frac{\partial H}{\partial M_3}, \frac{d\delta_7}{dt} = -\frac{\partial H}{\partial M_4}, \frac{d\delta_8}{dt} = -\frac{\partial H}{\partial R}, \end{aligned} \tag{50}$$

and the boundary conditions of adjoint system are

$$\delta_1(t_f) = \delta_2(t_f) = \delta_3(t_f) = \delta_4(t_f) = \delta_5(t_f) = \delta_6(t_f) = \delta_7(t_f) = \delta_8(t_f) = 0. \tag{51}$$

Let α^* as an example to give the optimality conditions. One have

$$\frac{\partial H}{\partial \alpha} = c_1\alpha(t) + \delta_1[-(I_1+I_2)S] + \delta_2[(I_1+I_2)S] - \lambda_{11} + \lambda_{12} = 0, \tag{52}$$

and the optimal control formulae can be written as:

$$\alpha^* = \frac{1}{c_1}(\delta_1 - \delta_2)(I_1+I_2)S + \lambda_{11} - \lambda_{12}. \tag{53}$$

To obtain the final optimal control formulae without λ_{11} and λ_{12} need to consider the following three situations.

The first situation is that $\lambda_{11}(t) = \lambda_{12}(t) = 0$ in set $\{t|0 < \alpha^*(t) < 1\}$, then the optimal control formulae can be written as:

$$\alpha^*(t) = \frac{1}{c_1}(\delta_1 - \delta_2)(I_1+I_2)S. \tag{54}$$

The second situation is that $\lambda_{11}(t) = 0$ in set $\{t|\alpha^*(t) = 1\}$, then the optimal control formulae can be written as:

$$1 = \alpha^*(t) = \frac{1}{c_1}[(\delta_1 - \delta_2)(I_1+I_2)S - \lambda_{12}]. \tag{55}$$

Due to $\lambda_{12}(t) \geq 0$, it is shown that $\frac{1}{c_1}(\delta_1 - \delta_2)(I_1+I_2)S \geq 1$.

The third situation is that $\lambda_{12}(t) = 0$ in set $\{t|\alpha^*(t) = 0\}$, then the optimal control formulae can be written as:

$$0 = \alpha^*(t) = \frac{1}{c_1}[(\delta_1 - \delta_2)(I_1+I_2)S + \lambda_{11}]. \tag{56}$$

Based on the above situation, the final optimal control formulae of $\alpha^*(t)$ can be written as

$\alpha^*(t) = \min\left\{1, \max\left\{0, \frac{(\delta_1-\delta_2)(I_1+I_2)S}{c_1}\right\}\right\}$. Similarly, the final optimal control formulae of $\beta^*(t)$ can be written as

$\beta^*(t) = \min\left\{1, \max\left\{0, \frac{(\delta_1-\delta_3)(I_1+I_2)S}{c_2}\right\}\right\}$, the final optimal control formulae of $\gamma_{12}^*(t)$ can be written as

$\gamma_{12}^*(t) = \min\left\{1, \max\left\{0, \frac{(\delta_2-\delta_5)I_1}{c_3}\right\}\right\}$, the final optimal control formulae of $\gamma_{21}^*(t)$ can be written as

$\gamma_{21}^*(t) = \min\left\{1, \max\left\{0, \frac{(\delta_3-\delta_6)I_2}{c_4}\right\}\right\}$.

So far, we get the optimal control system includes state System (35) with the initial conditions $S(0), I_1(0), I_2(0), M_1(0), M_2(0), M_3(0), M_4(0), R(0)$ and the adjoint System (43) with boundary conditions with the optimization conditions. The optimal control system can be written as:

$$\begin{cases} \frac{dS}{dt} &= B - \min\left\{1, \max\left\{0, \frac{(\delta_1-\delta_2)(I_1+I_2)S}{c_1}\right\}\right\}(t)(I_1+I_2)S \\ & - \min\left\{1, \max\left\{0, \frac{(\delta_1-\delta_3)(I_1+I_2)S}{c_2}\right\}\right\}(t)(I_1+I_2)S - \mu S, \\ \frac{dI_1}{dt} &= \min\left\{1, \max\left\{0, \frac{(\delta_1-\delta_2)(I_1+I_2)S}{c_1}\right\}\right\}(t)(I_1+I_2)S - \gamma_{11}I_1 \\ & - \min\left\{1, \max\left\{0, \frac{(\delta_2-\delta_5)I_1}{c_3}\right\}\right\}(t)I_1 - \mu I_1, \\ \frac{dI_2}{dt} &= \min\left\{1, \max\left\{0, \frac{(\delta_1-\delta_3)(I_1+I_2)S}{c_2}\right\}\right\}(t)(I_1+I_2)S - \gamma_{22}I_2 \\ & - \min\left\{1, \max\left\{0, \frac{(\delta_3-\delta_6)I_2}{c_4}\right\}\right\}(t)I_2 - \mu I_2, \\ \frac{dM_1}{dt} &= \gamma_{11}I_1 - \varepsilon_1 M_1 - \mu M_1, \\ \frac{dM_2}{dt} &= \min\left\{1, \max\left\{0, \frac{(\delta_2-\delta_5)I_1}{c_3}\right\}\right\}(t)I_1 - \varepsilon_2 M_2 - \mu M_2, \\ \frac{dM_3}{dt} &= \min\left\{1, \max\left\{0, \frac{(\delta_3-\delta_6)I_2}{c_4}\right\}\right\}(t)I_2 - \varepsilon_3 M_3 - \mu M_3, \\ \frac{dM_4}{dt} &= \gamma_{22}I_2 - \varepsilon_4 M_4 - \mu M_4, \\ \frac{dR}{dt} &= \varepsilon_1 M_1 + \varepsilon_2 M_2 + \varepsilon_3 M_3 + \varepsilon_4 M_4 - \mu R, \\ \frac{d\delta_1}{dt} &= (\delta_1-\delta_2)\min\left\{1, \max\left\{0, \frac{(\delta_1-\delta_2)(I_1+I_2)S}{c_1}\right\}\right\}(t)(I_1+I_2) \\ & + (\delta_1-\delta_3)\min\left\{1, \max\left\{0, \frac{(\delta_1-\delta_3)(I_1+I_2)S}{c_2}\right\}\right\}(t)(I_1+I_2) + \delta_1\mu, \\ \frac{d\delta_2}{dt} &= 1 + (\delta_1-\delta_2)\min\left\{1, \max\left\{0, \frac{(\delta_1-\delta_2)(I_1+I_2)S}{c_1}\right\}\right\}(t)S \\ & + (\delta_1-\delta_3)\min\left\{1, \max\left\{0, \frac{(\delta_1-\delta_3)(I_1+I_2)S}{c_2}\right\}\right\}(t)S \\ & + (\delta_2-\delta_4)\gamma_{11} + (\delta_2-\delta_5)\min\left\{1, \max\left\{0, \frac{(\delta_2-\delta_5)I_1}{c_3}\right\}\right\}(t) + \delta_2\mu, \\ \frac{d\delta_3}{dt} &= 1 + (\delta_1-\delta_2)\min\left\{1, \max\left\{0, \frac{(\delta_1-\delta_2)(I_1+I_2)S}{c_1}\right\}\right\}(t)S \\ & + (\delta_1-\delta_3)\min\left\{1, \max\left\{0, \frac{(\delta_1-\delta_3)(I_1+I_2)S}{c_2}\right\}\right\}(t)S \\ & + (\delta_3-\delta_7)\gamma_{22} + (\delta_3-\delta_6)\min\left\{1, \max\left\{0, \frac{(\delta_3-\delta_6)I_2}{c_4}\right\}\right\}(t) + \delta_3\mu, \\ \frac{d\delta_4}{dt} &= (\delta_4-\delta_8)\varepsilon_1 + \varepsilon_4\mu, \\ \frac{d\delta_5}{dt} &= 1 + (\delta_5-\delta_8)\varepsilon_2 + \varepsilon_5\mu, \\ \frac{d\delta_6}{dt} &= 1 + (\delta_6-\delta_8)\varepsilon_3 + \varepsilon_6\mu, \\ \frac{d\delta_7}{dt} &= (\delta_7-\delta_8)\varepsilon_4 + \varepsilon_7\mu, \\ \frac{d\delta_8}{dt} &= \delta_8\mu, \end{cases} \tag{57}$$

and

$$\begin{aligned} &S(0) = S_0, I_1(0) = I_{1,0}, I_2(0) = I_{2,0}, M_1(0) = M_{1,0}, M_2(0) = M_{2,0}, \\ &M_3(0) = M_{3,0}, M_4(0) = M_{4,0}, R(0) = R_0, \end{aligned} \tag{58}$$

$$\delta_1(t_f) = \delta_2(t_f) = \delta_3(t_f) = \delta_4(t_f) = \delta_5(t_f) = \delta_6(t_f) = \delta_7(t_f) = \delta_8(t_f) = 0. \tag{59}$$

□

Numerical simulations

In this section, the Rung-Kutta algorithm is adopted for performing numerical simulations to verify the rationality of the theoretical results by MATLAB R2017b. It is noteworthy that the value range of parameters is not clearly defined in previous studies. Therefore, in this work, we combine the value of the basic regeneration number R_0 and the stability condition for obtaining and presenting the parameter values of the model.

In order to verify the locally and globally asymptotically stability of information-free equilibrium in Theorem 1 and Theorem 2. Let $B = 1, \alpha = 0.01, \beta = 0.01, \mu = 0.1, \gamma_{11} = 0.2, \gamma_{12} = 0.7, \gamma_{22} = 0.2, \gamma_{21} = 0.7, \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 0.2$. It can be calculated that $R_0 = 0.2 < 1$. Figure 2 verifies the stability of the model and shows that variety groups eventually converge to 0 change over time.

In order to verify the locally and globally asymptotically stability of information-existence equilibrium in Theorem 3 and Theorem 4. Let $B = 3, \alpha = 0.7, \beta = 0.6, \mu = 0.1, \gamma_{11} = 0.2, \gamma_{12} = 0.5, \gamma_{22} = 0.2, \gamma_{21} = 0.5, \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 0.2$. It can be calculated that $R_0 = 48.75 > 1$. Figure 3 verifies the stability of the model and shows that variety groups eventually converge to E^* change over time.

In order to analyze the effect of optimal control pair $(\alpha^*, \beta^*, \gamma_{12}^*, \gamma_{21}^*)$ on variety groups when adopt the optimal control strategy. One give the image of "optimal control $(\alpha = \alpha^*(t), \beta = \beta^*(t), \gamma_{12}^*(t), \gamma_{21}^*(t))$", "middle control measure", "single control measure" and "constant control measure" respectively.

First, different control strategies are adopted to increase the number of transmission groups I_1 and I_2. α^* and β^* are controlled, respectively. Then, let $\beta = 0.55, \mu = 0.07, \gamma_{11} = 0.2, \gamma_{12} = 0.6, \gamma_{22} = 0.2, \gamma_{21} = 0.7, \varepsilon_1 = 0.16, \varepsilon_2 = 0.22, \varepsilon_3 = 0.18, \varepsilon_4 = 0.23$ to control α^* and $\alpha = 0.65, \mu = 0.09, \gamma_{11} = 0.2, \gamma_{12} = 0.6, \gamma_{22} = 0.2, \gamma_{21} = 0.7, \varepsilon_1 = 0.21, \varepsilon_2 = 0.16, \varepsilon_3 = 0.23, \varepsilon_4 = 0.17$ to control β^*. Figure 4a,b show the variation trends in the density of $I_1(t)$ and $I_2(t)$ over time under different control strategies, respectively. As presented in Fig. 4, the populations of I_1 and I_2 reach the maximum when the optimal control strategy is adopted for the

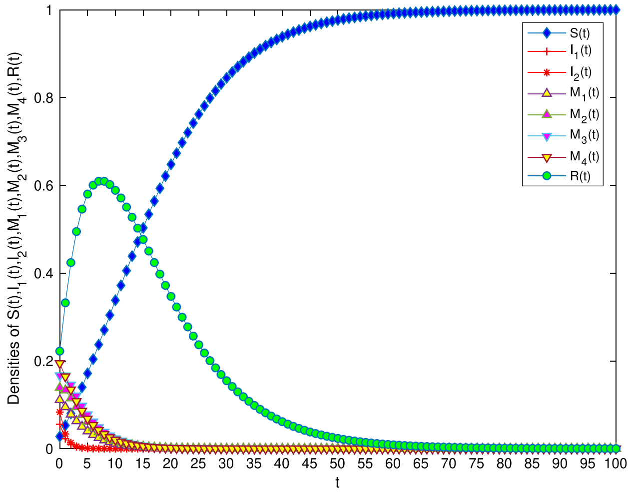

Figure 2. The stability of information-free equilibrium E^0 of system 1 with $R_0 < 1$.

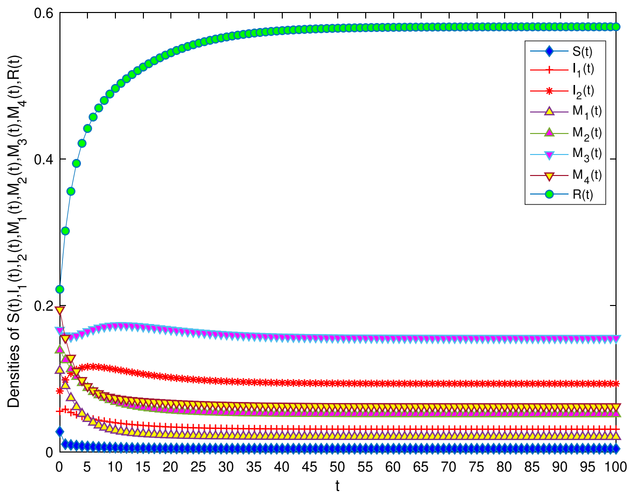

Figure 3. The stability of information-existence equilibrium E^* of system 1 with $R_0 > 1$.

control variables α^* and β^*. This shows that improving the mobility and contact rate of people enlarges the spreading scope of information.

Subsequently, different control strategies are adopted to increase the number of variation groups M_2 and M_3. The optimal control pairs $(\alpha^*, \gamma_{12}{}^*)$ and $(\beta^*, \gamma_{21}{}^*)$ are controlled, respectively. Then, let $\beta = 0.65, \mu = 0.05, \gamma_{11} = 0.2, \gamma_{22} = 0.2, \gamma_{21} = 0.7, \varepsilon_1 = 0.17, \varepsilon_2 = 0.22, \varepsilon_3 = 0.28, \varepsilon_4 = 0.21$ to control optimal control pair $(\alpha^*, \gamma_{12}{}^*)$ and $\alpha = 0.76, \mu = 0.09, \gamma_{11} = 0.2, \gamma_{12} = 0.75, \gamma_{22} = 0.2, \varepsilon_1 = 0.23, \varepsilon_2 = 0.19, \varepsilon_3 = 0.25, \varepsilon_4 = 0.18$ to control optimal control pair $(\beta^*, \gamma_{21}{}^*)$. Figure 5a,b show the variation trends in the densities of $M_2(t)$ and $M_3(t)$ over time under different control strategies, respectively. As presented in Fig. 5, $M_2(t)$ and $M_3(t)$ populations reach the maximum when the optimal control strategy is adopted for the optimal control pairs $(\alpha^*, \gamma_{12}{}^*)$ and $(\beta^*, \gamma_{21}{}^*)$. This shows that enhancing the education intensity and increasing the variation rate improves the variation of information.

Next, controlling the optimal control pairs (α^*, β^*) and $(\gamma 12^*, \gamma 21^*)$ is adopted to increase the number of transmission groups I_1 and I_2 and variation groups M_2 and M_3 simultaneously. Let $\mu = 0.09, \gamma_{11} = 0.2, \gamma_{22} = 0.2, \varepsilon_1 = 0.23, \varepsilon_2 = 0.19, \varepsilon_3 = 0.25, \varepsilon_4 = 0.18$. Figure 6a,b show the variation trends in the density of transmission groups $I_1(t)$ and $I_2(t)$ over time, respectively. When a single control α^* and β^* is adopted, the population of I_1 and I_2 reaches the maximum. Figure 6c,d show the variation trends in the density of variation groups $M_2(t)$ and $M_3(t)$ over time, respectively. When an optimal control strategy is adopted, the populations of M_2 and M_3 reach the maximum.

Lastly, $\alpha^*, \beta^*, \gamma_{12}{}^*$, and $\gamma_{21}{}^*$ are controlled to increase the number of transmission groups I_1 and I_2 as well as the variation groups M_2 and M_3, respectively. Let $\mu = 0.09, \gamma_{11} = 0.2, \gamma_{22} = 0.2, \varepsilon_1 = 0.23, \varepsilon_2 = 0.19, \varepsilon_3 = 0.25, \varepsilon_4 = 0.18$. Figure 7a,b show the variation trends in the density of transmission groups $I_1(t)$ and $I_2(t)$

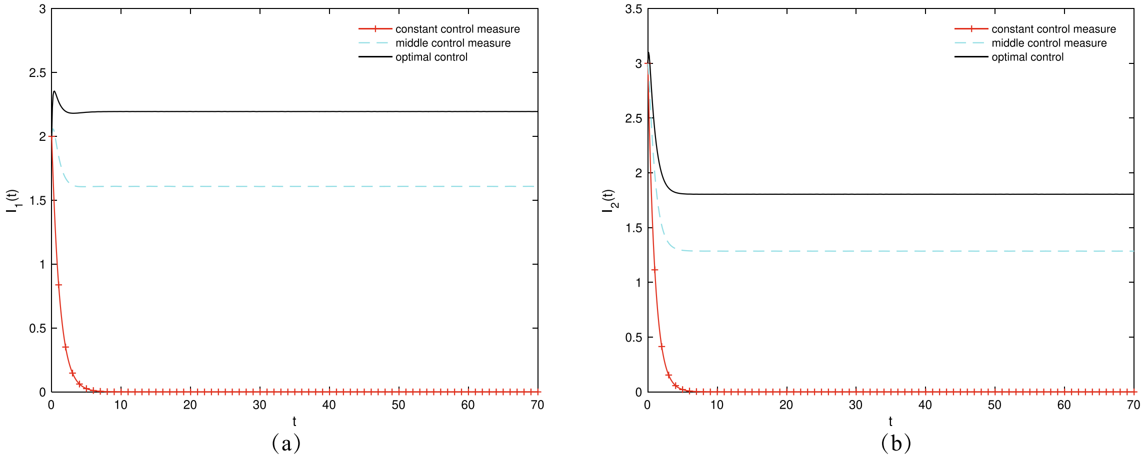

Figure 4. The densities of (**a**) $I_1(t)$, (**b**) $I_2(t)$ change over time under different control strategies.

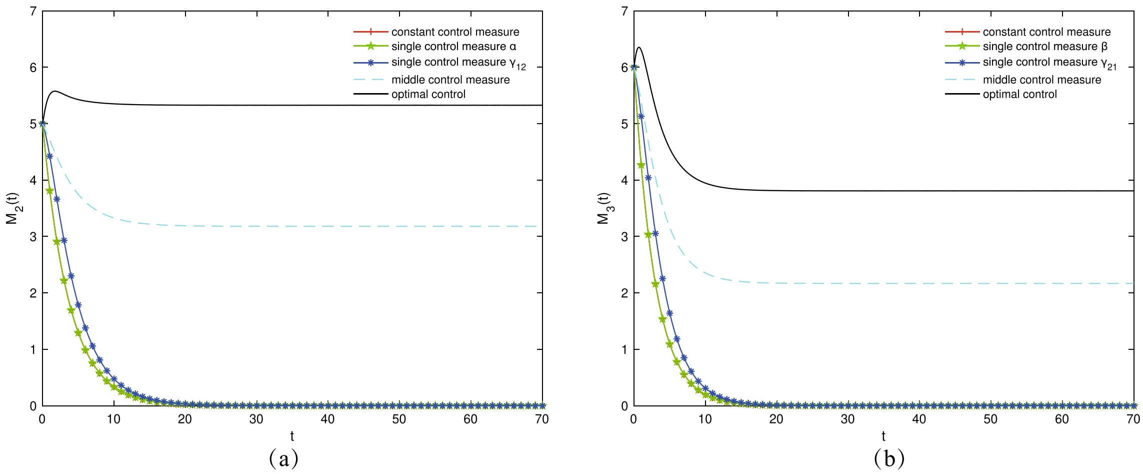

Figure 5. The densities of (**a**) $M_2(t)$, (**b**) $M_3(t)$ change over time under different control strategies.

over time, respectively. The I_1 population reaches its maximum when a single control α^* is used. The I_2 population reaches its maximum when a single control β^* is used. Figure 7c,d show the variation trends in the density of variation groups $M_2(t)$ and $M_3(t)$ over time, respectively. When the optimal control strategy is adopted, the populations of M_2 and M_3 reach the maximum.

Based on the aforementioned analysis, no matter what control method is chosen, the variation group reaches the maximum when an optimal control strategy is adopted. When the transmission group is in a single control, it reaches the maximum population size under the optimal control strategy. However, when the transmission group and variation group are controlled at the same time, the populations reach the maximum only under the single control of (α^*, β^*).

Finally, the choice of parameters values has no established principle in the illustrations of the numerical simulations. In relevant literature on information transmission, the choice of these parameters values does not have a fixed range. Most of them are limited to positive numbers and satisfy the stability condition. In the numerical simulation, the values in other relevant literature are mentioned and the requirements of stability conditions are combined to give the numerical values of the parameters in the model. As for practical problems, determination of the specific numerical parameters is proposed, referring to the relevant professional background knowledge and investigating the actual background with reference to relevant existing literature.

Sensitivity analysis

In order to analyze the effect of the above control variables α and β on the basic reproductive number R_0, one need to perform the sensitivity analysis of R_0. It has been figure out above $R_0 = \frac{B\alpha(\gamma_{22}+\gamma_{21}+\mu)+B\beta(\gamma_{11}+\gamma_{12}+\mu)}{\mu(\gamma_{11}+\gamma_{12}+\mu)(\gamma_{22}+\gamma_{21}+\mu)}$, thereby calculating:

$$\frac{\partial R_0}{\partial \alpha} = \frac{B(\gamma_{22} + \gamma_{21} + \mu)}{\mu(\gamma_{11} + \gamma_{12} + \mu)(\gamma_{22} + \gamma_{21} + \mu)} > 0, \tag{60}$$

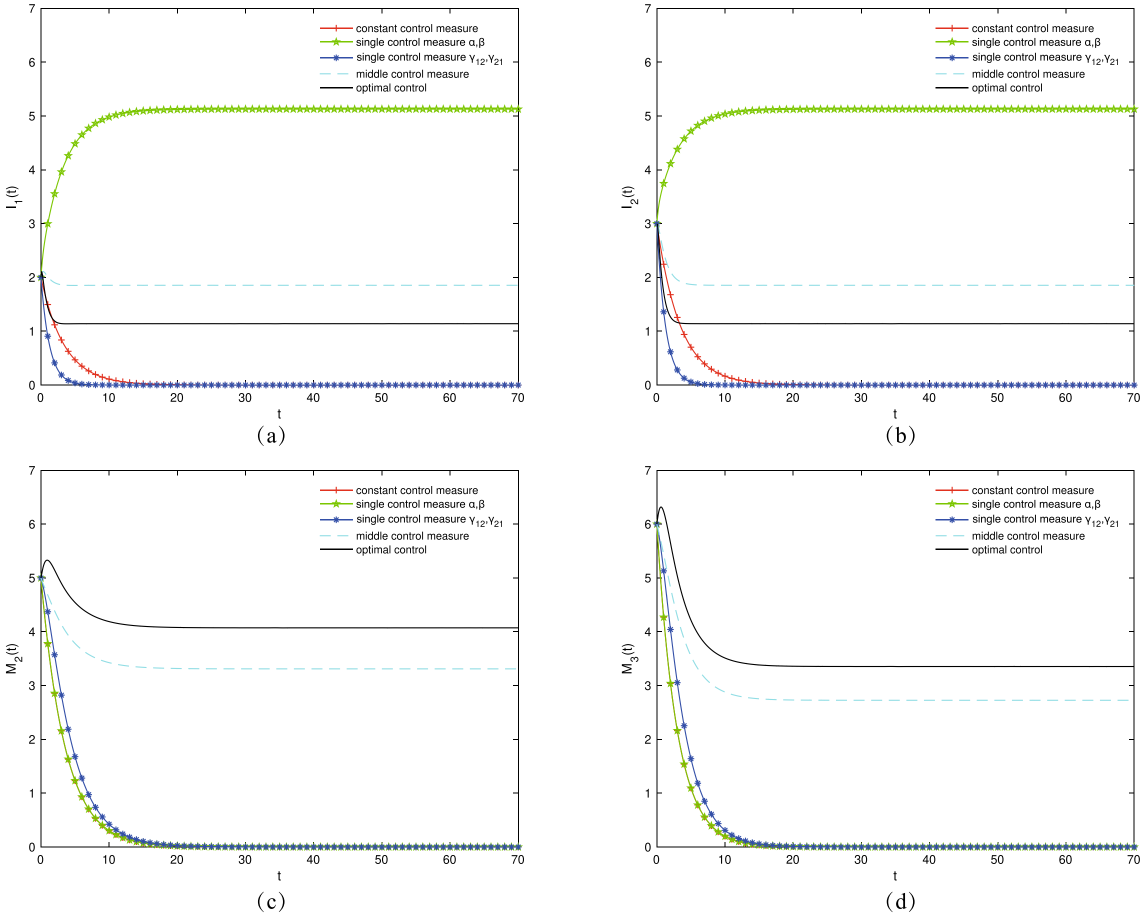

Figure 6. The densities of (**a**) $I_1(t)$, (**b**) $I_2(t)$, (**c**) $M_2(t)$, (**d**) $M_3(t)$ change over time under different control strategies.

$$\frac{\partial R_0}{\partial \beta} = \frac{B(\gamma_{11} + \gamma_{12} + \mu)}{\mu(\gamma_{11} + \gamma_{12} + \mu)(\gamma_{22} + \gamma_{21} + \mu)} > 0. \tag{61}$$

Thus it can be seen, R_0 is positively correlated with α and β. This indicates that improving the flow and contact between people can promote the transmission of information. In addition, the more information transmitters, the more information mutants.

The sensitivity analysis of R_0 is simulated with MATLAB R2017b in Fig. 8.

Conclusions

In this work, we consider the influence of information cross transmission and information variation on information transmission. We construct the *S2I4MR* model of information crossing and variation, calculated the basic regeneration number of the model, analyzed the equilibrium point and stability of the model, verified the existence of the optimal control of model, and proposed the optimal control strategy of the model. Based on numerical simulations, we verify the basic theorem of the model and the effectiveness of the optimal control strategy. Finally, we analyze the sensitivity of the optimal control parameters.

The main conclusions of this work are presented below.

- The phenomenon of cross-infection and variation has been a focus of research community due to Omicron, which is the variant of *SARS-CoV-2*. It can still be applied by analogy in information transmission. As compared to previous works, the presented optimal control strategy is based on the optimal value calculated by control variables;
- By promoting the flow of people or organizing information exchange activities, it effectively improves the exchange rate of information and promotes the large-scale integration of information. Therefore, improving the natural contact rate of the two kinds of information in the crowd effectively expands the communication and fusion of information;
- By strengthening the educational guidance or putting forward encouraging multi-information application policies, it effectively promotes the cross information to evolve into new usable information by combining common advantages and ensures that the people exposed to multiple information can integrate informa-

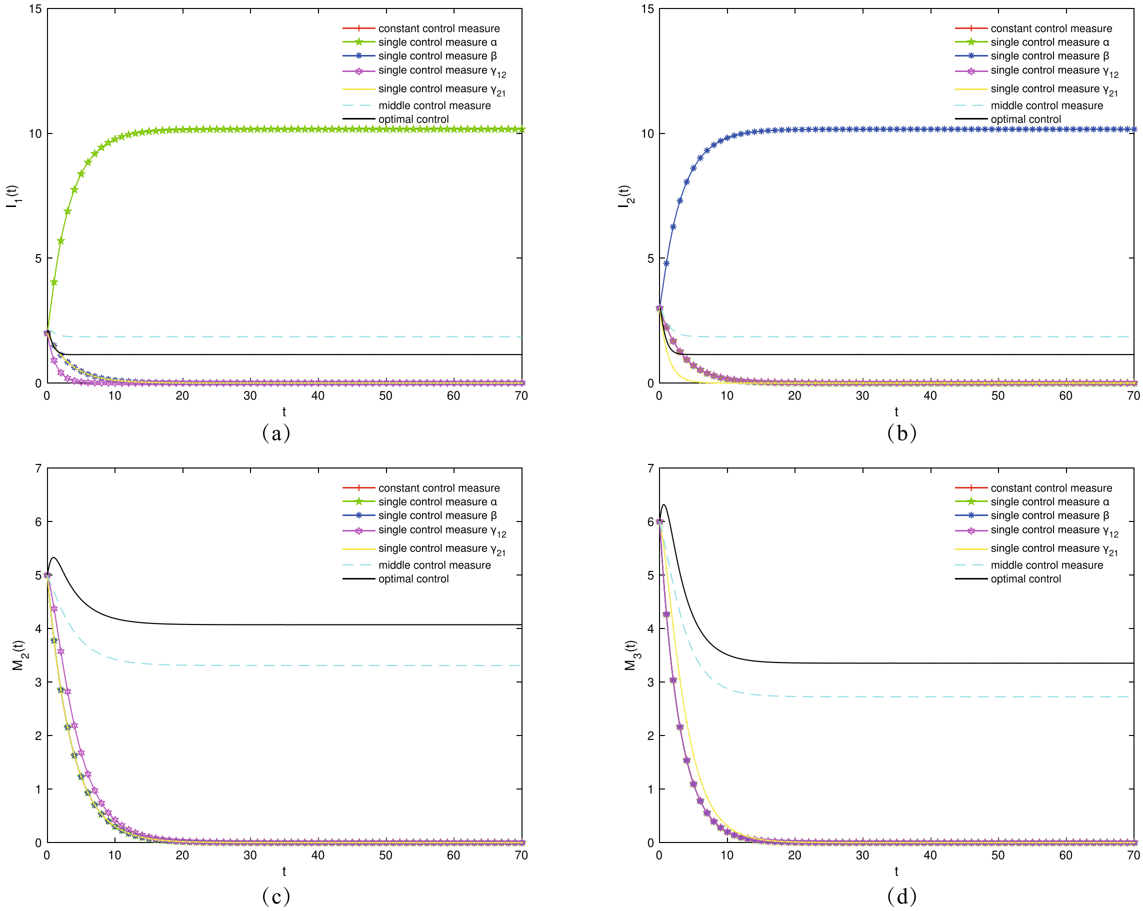

Figure 7. The densities of (**a**) $I_1(t)$, (**b**) $I_2(t)$, (**c**) $M_2(t)$, (**d**) $M_3(t)$ change over time under different control strategies.

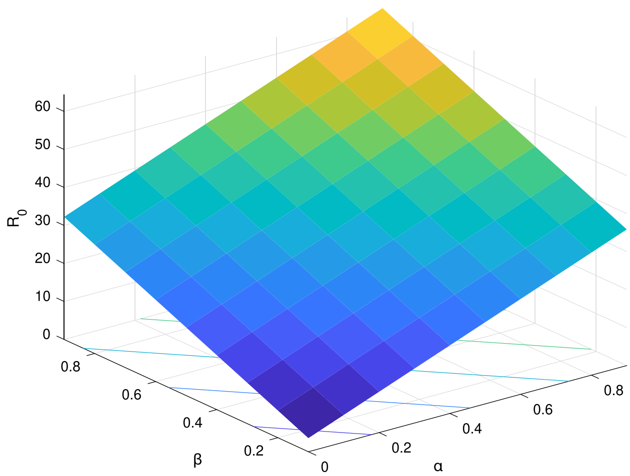

Figure 8. The sensitivity analysis of the basic reproduction number R_0.

tion from different fields into widely used new information. Therefore, increasing the variation rate in the population effectively enhances the generation of new information.

The phenomenon of information crossing and variation is universal in society. On one hand, in terms of positive information, we should strengthen the information crossing and variation, so that the information after such variation can be applied in additional fields. On the other hand, in terms of negative information, we should reduce the information crossing and variation, in order to reduce its adverse impacts on the society. In the future study, we will focus on the influence of random perturbation of parameters on information transmission. In addition, as the memory effect is the most important to control and disseminating information, we will construct an information transmission model considering memory effect in the subsequent research. And extend it to the fractional derivative with a non-local kernel in future research.

Data and code availability

All data analysed during this study are included in this published article.

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Author contributions

S.K. and X.H. conceptualization, S.K. and Y.H. methodology, S.K. and Y.H. software, S.K., X.H. and H.L. validation, S.K. and Y.H. formal analysis, S.K. and Y.H. investigation, S.K. and Y.H. data curation, S.K. writing—original draft preparation, X.H. and H.L. writing—review and editing. All authors reviewed the manuscript.

Competing interests

The authors declare no competing interests.

Additional information

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