

The influence of a first-order antedependence model and hyperparameters in BayesC π for genomic prediction

Xiujin Li¹, Xiaohong Liu¹, and Yaosheng Chen^{1,*}

* Corresponding Author: Yaosheng Chen
Tel: +86-020-39332940, Fax: +86-02039332940,
E-mail: chyaosh@mail.sysu.edu.cn

¹ State Key Laboratory of Biocontrol, School of Life Sciences, Sun Yat-sen University, North Third Road, Guangzhou Higher Education Mega Center, Guangzhou, Guangdong 510006, China

ORCID

Xiujin Li
<https://orcid.org/0000-0002-8526-5547>
Xiaohong Liu
<https://orcid.org/0000-0001-8141-4835>
Yaosheng Chen
<https://orcid.org/0000-0002-3871-7651>

Submitted Jan 31, 2018; Revised Apr 19, 2018;
Accepted Jun 2, 2018

Objective: The Bayesian first-order antedependence models, which specified single nucleotide polymorphisms (SNP) effects as being spatially correlated in the conventional BayesA/B, had more accurate genomic prediction than their corresponding classical counterparts. Given advantages of BayesC π over BayesA/B, we have developed hyper-BayesC π , ante-BayesC π , and ante-hyper-BayesC π to evaluate influences of the antedependence model and hyperparameters for v_g and s_g^2 on BayesC π .

Methods: Three public data (two simulated data and one mouse data) were used to validate our proposed methods. Genomic prediction performance of proposed methods was compared to traditional BayesC π , ante-BayesA and ante-BayesB.

Results: Through both simulation and real data analyses, we found that hyper-BayesC π , ante-BayesC π and ante-hyper-BayesC π were comparable with BayesC π , ante-BayesB, and ante-BayesA regarding the prediction accuracy and bias, except the situation in which ante-BayesB performed significantly worse when using a few SNPs and $\pi = 0.95$.

Conclusion: Hyper-BayesC π is recommended because it avoids pre-estimated total genetic variance of a trait compared with BayesC π and shortens computing time compared with ante-BayesB. Although the antedependence model in BayesC π did not show the advantages in our study, larger real data with high density chip may be used to validate it again in the future.

Keywords: BayesC π ; Antedependence Model; Hyperparameter

INTRODUCTION

Methods for genomic prediction have been developed widely since Meuwissen et al [1] firstly proposed genomic selection methods. To date, there are mainly two classical types of methods applied to genomic prediction in livestock breeding. The first one is BLUP methods such as GBLUP [2], which assume that single nucleotide polymorphisms (SNP) effects independently and identically follow a normal distribution with an equal variance. The second one is Bayesian hierarchical models with different prior distributions on the variances of SNP effects. For example, BayesA assumes a scaled-inverted chi square distribution on SNP-specific variances [1].

Habier et al [3] firstly developed BayesC π for genomic prediction to address the drawbacks of BayesA and BayesB with respect to influences of prior hyperparameters and the prior probability π that a SNP has zero effect. BayesC π assumes that all SNPs have a common effect variance instead of locus-specific variances in BayesA or BayesB so that the influence of s_g^2 becomes smaller. $\pi = 0.95$ or 0.99 in BayesB is generally arbitrarily defined. It is improved in BayesC π where π is treated as an unknown being inferred from the data. Although the statistical drawbacks of BayesA and BayesB did not impair the prediction accuracy mainly

due to linkage disequilibrium (LD) information, accounting for reducing computing time and better fitting the genetic architecture of a trait, BayesC π can be deemed to have better merit for routine application compared with BayesA/B.

So far, most of Bayesian methods, such as BayesA, BayesB, and BayesC π , have a common feature that SNP effects are assumed to be independent. However, the existence of LD between SNPs makes the above assumption improper, and a close relationship should exist between SNPs. To account for LD between SNPs, Yang and Tempelman [4] brought first-order antedependence models which model SNP effects as correlated into the conventional BayesA and BayesB, and developed ante-BayesA and ante-BayesB. It is reported in their paper that the antedependence methods had significantly higher accuracies than the corresponding classical counterparts at higher LD levels ($r^2 > 0.24$) in the simulation study, and they were also more accurate in mice data and other benchmark data sets.

However, although there are advantages of BayesC π over BayesA and BayesB aforementioned, first-order antedependence models are still not brought to BayesC π . Thus, we will develop ante-BayesC π , and then investigate whether ante-BayesC π can improve the prediction accuracy and bias compared to BayesC π . Besides, it has not been widely appreciated that v_g and s_g^2 also can be estimated from the data [5]. This recognition is important as both hyperparameters can help define the genetic architecture of a trait of interest. The impact of adding hyperparameters to BayesC π for genomic prediction is little reported and is also an interesting topic for investigation.

Therefore, the objective of this study was to investigate the impact of adding a first-order antedependence model and hyperparameters into BayesC π for genomic prediction regarding the prediction accuracy and bias. We have demonstrated the predictive ability of BayesC π with first-order antedependence models and hyperparameters in simulation data from the study of Jiang et al [6], the 15th quantitative trait loci-marker assisted selection (QTL-MAS) workshop data set and the heterogeneous stock mice, compared to the classical BayesC π , ante-BayesA, and ante-BayesB.

MATERIALS AND METHODS

Statistical model

The general linear mixed model used for genomic prediction could be written as

$$y = X\beta + Zg + e \quad (1)$$

Here, y was a vector of phenotypes, β was a vector of fixed effects, g was a vector of random SNP effects, and e was a residual vector. X was a known incidence matrix linking β to y .

Z was a known matrix linking g to y in which SNP genotypes were coded as 0, 1, or 2 copies of one allele for each SNP (column) and animal (row). Furthermore, we specified $g \sim N(0, G)$, where $G = \text{diag}(\sigma_{g_i}^2)$ and $e \sim N(0, I\sigma_e^2)$.

BayesC π

Habier et al [3] firstly developed BayesC π , and the obvious difference between BayesC π and other methods such as GB-LUP, BayesA, and BayesB was the characterization of G . $\sigma_{g_i}^2$ in BayesC π had a two-component mixture with one component being 0 and the other component being one fixed value σ_g^2 , i.e.,

$$\sigma_{g_i}^2 | v_g, s_g^2, \pi \begin{cases} = 0 & \text{with probability } \pi \\ = \sigma_g^2 \sim \chi^{-2}(v_g, s_g^2) & \text{with probability } 1 - \pi \end{cases} \quad (2)$$

Here, π represented the proportion of SNPs having no genetics effects on the trait of interest. Based on Habier et al [3], the prior of π was treated as an unknown with uniform (0,1). The v_g was set to 4.2, and s_g^2 was set as follows:

$$s_g^2 = \frac{\tilde{\sigma}_g^2 (v_g - 2)}{v_g} \quad (3)$$

$$\tilde{\sigma}_g^2 = \frac{\tilde{\sigma}_a^2}{(1 - \pi) \sum_{k=1}^K 2p_k (1 - p_k)} \quad (4)$$

Where $\tilde{\sigma}_g^2$ was the additive genetic variance for a SNP, and $\tilde{\sigma}_a^2$ was the additive genetic variance explained by all SNPs.

Hyper-BayesC π

We labeled hyper-BayesC π when we sampled v_g using a Metropolis Hastings (MH) update and s_g^2 with a Gibbs update in BayesC π . Similar to BayesA and BayesB, we specified the following prior distributions on v_g and s_g^2 for hyper-BayesC π : $v_g \sim p(v_g) \propto (v_g + 1)^{-2}$, and $s_g^2 \sim \text{Gamma}(\alpha_s, \beta_s)$ with shape parameter $\alpha_s = 1.0$ and rate parameter $\beta_s = 1.0$. The full conditional density (FCD) of v_g did not have a recognizable form, and a random walk MH step other than a Gibbs step was used to sample v_g from this FCD. The detailed procedure can be seen Yang et al [7]. The FCD of s_g^2 had a recognizable form with a Gamma distribution, provided that a conditionally conjugate Gamma prior is used [8].

Ante-BayesC π

Ante-BayesC π was defined here as combining BayesC π and the first-order antedependence model proposed from the study of Yang and Tempelman [4]. Likewise, the following nonstationary first-order antedependence correlation structure for g based on the relative physical location of SNPs along the chromosome was used:

$$g_j = \begin{cases} \delta_1 & \text{if } j = 1 \\ t_{j,j-1}g_{j-1} + \delta_j & \text{if } 2 \leq j \leq m \end{cases} \quad (5)$$

Here, $t_{j,j-1}$ was the marker interval-specific antedependence parameter of g_j on g_{j-1} in the specified order. δ_j was the residual SNP effect following $N(0, \sigma_{\delta_j}^2)$. Different from ante-BayesA and ante-BayesB, $\sigma_{\delta_j}^2$ in ante-BayesC π had a two-component mixture prior with the following definition:

$$\sigma_{\delta_j}^2 | v_{\delta_j}, s_{\delta_j}^2, \pi \begin{cases} = 0 & \text{with probability } \pi \\ \sim \sigma_{\delta}^2 \sim x^{-2}(v_{\delta_j}, s_{\delta_j}^2) & \text{with probability } 1 - \pi \end{cases} \quad (6)$$

It should be noted that, here π represented the proportion of SNPs having no residual SNP effects, which was a little different from π of BayesC π and hyper-BayesC π .

Ante-hyper-BayesC π

We labeled the model as ante-hyper-BayesC π in which the hyperparameters for v_g and s_g^2 and a nonstationary first-order antedependence correlation structure for g were combined.

For ante-BayesC π and ante-hyper-BayesC π , we specified $t_{j,j-1} \sim N(u_t, \sigma_{t_j}^2)$ as a conjugate prior, and further $u_t \sim N(u_{t0}, \sigma_{t0}^2)$ and $\sigma_{t_j}^2 \sim x^{-2}(v_t, s_t^2)$ where u_{t0} , σ_{t0}^2 , v_t , and s_t^2 were specified to be known [4]. It should be noted that $t_{j,j-1}$ was set 0 between the last SNP of one linkage group or chromosome and the first SNP in the subsequent group. The remaining priors were also specified, such as $\pi \sim U(0,1)$, $v_{\delta} = v_g$ and $s_{\delta}^2 = s_g^2$ in ante-BayesC π , $s_{\delta}^2 \sim \text{Gamma}(\alpha_s, \beta_s)$ and $v_{\delta} \sim p(v_{\delta}) \propto (v_{\delta} + 1)^{-2}$ in ante-hyper-BayesC π . For all methods, the noninformative prior σ_e^2 is specified, i.e., $\sigma_e^2 \sim x^{-2}(-1,0)$, which is congruent with specifying uniform prior on σ_e^2 , and in line with recommendations for variance components by Gelman [9]. The full conditional densities of all unknown parameters and any necessary MH steps using Markov chain Monte Carlo are further provided in studies of Habier et al [3] and Yang and Tempelman [4].

We used Fortran 90 to compile BayesC π , extent forms of BayesC π , ante-BayesA and ante-BayesB, and program codes can be seen in Supplementary Material (See e-version for Supplement). It should be pointed out that ante-BayesB with a pre-defined π in our paper was extended from BayesB of Meuwissen et al [1], while π was inferred from the data in ante-BayesB of Yang and Tempelman [4].

In order to evaluate the performance of our proposed methods, we used data sets from other reported studies, i.e., simulation data from the study of Jiang et al [6], the 15th QTL-MAS workshop data set and real data on the heterogeneous stock mice from studies of Yang and Tempelman [4] and Gao et al [10], to compare them with ante-BayesA and ante-BayesB with respect to the accuracy and bias of genomic prediction.

Simulation data

We downloaded simulation data from the study of Jiang et al [6], and the detailed simulation process could be seen in their study. Briefly, the default scenario was as follows: a total of 30 QTLs with a minor allele frequency >0.05 was randomly selected, and their effects on two traits were drawn from a standard bivariate normal distribution with correlation 0.5. Normal error deviates were added to achieve heritabilities of 0.5 for trait 1 and 0.1 for trait 2. The error covariance between two traits was set to 0.

Sampling every 10th and 25th SNPs from the full set of SNPs respectively was conducted to form two subsets of SNPs. Averaged over 30 replicates, the average number of SNPs were 4,119, 411, and 164 for the full set and two subsets, which resulted in three average LD levels ($r^2 = 0.33, 0.22, \text{ and } 0.14$) of adjacent SNPs corresponding to the full set of SNPs and other two subsets with SNP intervals of 10 and 25, respectively. In our study, we selected trait 1 with both phenotype and genotype for first 20 of 30 replicates with the full dataset and other two subsets of SNPs as test data to investigate the influence of LD between adjacent SNPs on the prediction accuracy and bias. Generation 5001 was considered as the reference population, and generation 5002 as the candidate population.

Analysis of the 15th QTL-MAS workshop data set

A base population was a collection of 20 sires and 200 dams. Each sire was mated to 10 dams, and each dam was mated to only one sire. Within each family, one dam gave birth to 15 offspring. The 10 progenies were randomly assigned to the reference population with trait phenotypes and marker genotypes information, and other 5 belonged to the candidate population, only recorded for genotype marker information. Thus, 2000 individuals formed the reference population, and the candidate population included 1,000 individuals. A genome consisting of 9,990 SNPs on five chromosomes with 1 Morgan each were simulated without any missing data and genotyping error. Eight QTLs (1 quadri-allelic, 2 linked in phase, 2 linked in repulsion, 1 imprinted and 2 epistatic) were simulated. Random error was added to create an heritability of 0.30 for the trait analyzed [11]. In the following analyses, we removed all SNPs with minor allele frequency = 0, leaving 7,121 SNPs on five chromosomes for comparing performances of different Bayesian methods aforementioned. These SNPs had been sorted by the physical position.

Application to heterogeneous stock mice data set

A population of heterogeneous stock mice was generated by the Wellcome Trust Centre for Human Genetics (WTCHG) (<http://gscan.well.ox.ac.uk/>). This population was formed by a crossing of eight inbred strains, followed by 50 generations of pseudorandom mating [12]. The extent of LD in this population is small, and average r^2 among adjacent SNPs is 0.62 [13]. It is well known for the family structure and history of

this population, and thus interpretation of results will be easy.

In order to compare performances of genomic prediction using different Bayesian methods, we selected four traits: body weight at 6 weeks (W6W), growth slope between 6 and 10 weeks of age (GSL), body mass index (BMI), and body length (BL) similar to the study of Legarra et al [13]. For computational simplicity, the pre-corrected phenotypes of these traits provided by Valdar et al [14] were used as pseudo phenotypes for the following analysis. Two sets of genotype information were used, which were directly from studies of Yang and Tempelman [4] and Gao et al [10]. Genotypic data processing of Yang and Tempelman [4] resulted in the data set involving records on 1,917 animals with 950 SNPs on the 19 chromosomes, and the average LD of r^2 was 0.35 between adjacent markers. In the study of Gao et al [10], there were 1,940 animals and 9,266 SNPs on the 19 chromosomes after their quality control steps, which resulted in the average LD of r^2 about 0.60.

Numbers of animals having both phenotypes and two sets of genotypes were 1,917, 1,901, 1,814, and 1,821 for W6W, GSL, BMI, and BL respectively. All animals for each trait were randomly divided into two nearly equal-sized partitions of reference and candidate data sets. This was replicated 20 times for comparison of genomic prediction among different Bayesian methods. SNPs were sorted based on their physical positions along the chromosome.

For each of the six Bayesian methods, ante-BayesA, ante-BayesB, BayesC π , hyper-BayesC π , ante-BayesC π , and ante-hyper-BayesC π , in both simulation data and real data on the heterogeneous stock mice, the Markov chains were run for 350,000 cycles of Gibbs sampling (for ante-BayesB, 100 additional cycles of MH sampling cycle), and the first 50,000 cycles were discarded as the burning period. After the burning period, every 10th cycles were subsequently saved for obtaining estimates of SNP effects. In ante-BayesB, we set $\pi = 0.95$ in both the simulation data of Jiang et al [10] and the heterogeneous stock mice application, and 0.99 in the 15th QTL-MAS workshop data set.

Direct genomic values (DGVs) for individuals with genotypes, but no phenotypes, were calculated as the sum of all SNP effects according to their SNP genotypes. The predic-

tion accuracy was calculated as Pearson's correlation between DGVs and true breeding values (TBVs) in simulation data (or pre-corrected phenotypes in mice data) for the candidate population, and the prediction bias was evaluated by the regression coefficient of DGVs on TBVs in simulation data (or pre-corrected phenotypes in mice data). For the simulation data of Jiang et al [10] and the heterogeneous stock mice, one-way analysis of variance was performed to determine the statistical significance of differences in the accuracy of genomic prediction among above six methods and in estimates of π among BayesC π , hyper-BayesC π , ante-BayesC π and ante-hyper-BayesC π . The stringent Bonferroni multiple test corrections were used.

RESULTS

Results from the simulations

The prediction accuracies and unbiased under three different LD levels can be seen in Table 1. For the prediction accuracy, there are not the statistical significance of differences in the accuracy of genomic prediction among six methods for all scenarios ($p > 0.05$). Hyper-BayesC π performed a little better or similar among all six methods in three scenarios of different LD levels. In the scenario of using all SNPs, ante-hyper-BayesC π performed as well as hyper-BayesC π , which was 0.6% higher than BayesC π and ante-BayesC π , 0.5% for ante-BayesB, and 0.8% for ante-BayesA, respectively. In other two scenarios, ante-hyper-BayesC π and ante-BayesC π performed a little worse than the counterparts. The prediction accuracy for ante-BayesB was much lower than other methods when using the SNP data set with the interval of 25 SNPs which was close to the level of significant difference ($p = 0.07$).

Regarding the prediction bias, the regression coefficients for all methods were close to 1.0, which indicated good unbiased prediction. Additionally, although there were similar prediction accuracies among BayesC π and extensions of BayesC π , the estimated π value from BayesC π were significantly lower than those from extensions of BayesC π in the scenario of using all SNPs, ante-(hyper-) BayesC π had significantly lower π value than (hyper-) BayesC π in the scenario

Table 1. Accuracies (mean \pm SE) and biases (mean \pm SE) of DGVs in the validation population of simulated data sets under different LD levels of adjacent markers over 20 replications

Method	All		Every 10th		Every 25th	
	Corr	Reg	Corr	Reg	Corr	Reg
Ante-BayesA	0.873 \pm 0.008	1.034 \pm 0.024	0.829 \pm 0.009	1.028 \pm 0.029	0.760 \pm 0.010	1.010 \pm 0.025
Ante-BayesB	0.876 \pm 0.008	1.037 \pm 0.025	0.824 \pm 0.010	1.065 \pm 0.036	0.719 \pm 0.014	1.069 \pm 0.039
BayesC π	0.875 \pm 0.008	1.044 \pm 0.025	0.832 \pm 0.008	1.056 \pm 0.031	0.760 \pm 0.010	1.023 \pm 0.025
Hyper-BayesC π	0.881 \pm 0.008	1.025 \pm 0.025	0.833 \pm 0.008	1.041 \pm 0.031	0.760 \pm 0.011	1.007 \pm 0.024
Ante-BayesC π	0.875 \pm 0.008	1.051 \pm 0.026	0.824 \pm 0.008	1.110 \pm 0.032	0.754 \pm 0.011	1.141 \pm 0.032
Ante-hyper-BayesC π	0.881 \pm 0.008	1.042 \pm 0.025	0.826 \pm 0.008	1.103 \pm 0.031	0.754 \pm 0.011	1.133 \pm 0.032

SE, standard error; DGVs, direct genomic values; LD, linkage disequilibrium; Corr, Pearson's correlation; Reg, regression coefficient.

Table 2. π values (mean \pm standard error) estimated from BayesC π and extensions of BayesC π using the reference population of simulated data sets under different LD levels of adjacent SNPs over 20 replications

Method	ALL	Every 10th	Every 25th
BayesC π ¹⁾	0.854 \pm 0.046 ^a	0.656 \pm 0.038 ^a	0.471 \pm 0.042
Hyper-BayesC π ¹⁾	0.969 \pm 0.022 ^b	0.767 \pm 0.028 ^b	0.542 \pm 0.041
Ante-BayesC π ²⁾	0.934 \pm 0.017 ^{ab}	0.551 \pm 0.028 ^a	0.425 \pm 0.022
Ante-hyper-BayesC π ²⁾	0.977 \pm 0.008 ^b	0.589 \pm 0.028 ^a	0.447 \pm 0.021

LD, linkage disequilibrium; SNPs, single nucleotide polymorphisms.

¹⁾ π represented the proportion of SNPs having no genetics effects on the trait.

²⁾ π represented the proportion of SNPs having no residual genetics effects.

^{a-b} The estimated π values within a column with no common superscript differ significantly ($p < 0.05$); no superscript within a column meant non-significant difference ($p > 0.05$).

of the interval of 10 SNPs, and the estimates of π were comparable in the scenario of the interval of 25 SNPs. We found that the π value became smaller as the density of SNPs decreased (Table 2).

Common data set of the 15th QTL-MAS workshop

Using the above six methods, we also analyzed the 15th QTL-MAS workshop data set. As shown in Table 3, ante-hyper-BayesC π and ante-BayesC π had the same prediction accuracy (0.942), and they were 0.1%, 0.3%, 0.6%, and 1.2% higher than hyper-BayesC π , BayesC π , ante-BayesB, and ante-BayesA, respectively. The regression coefficients ranged from 1.044 to 1.068 indicating the unbiased genomic prediction for all methods. The π values estimated from BayesC π , hyper-BayesC π , ante-BayesC π , and ante-hyper-BayesC π were very similar which were 0.996, 0.997, 0.996, and 0.997, respectively.

Real heterogeneous stock mice data set

As shown in Table 4, the genomic prediction for all six methods

Table 3. Accuracies and biases of DGVs in the validation population of the common data set from the fifteenth QTL-MAS workshop

Method	Pearson's correlation	Regression coefficient
Ante-BayesA	0.930	1.044
Ante-BayesB	0.936	1.056
BayesC π	0.939	1.062
Hyper-BayesC π	0.941	1.062
Ante-BayesC π	0.942	1.068
Ante-hyper-BayesC π	0.942	1.066

DGVs, direct genomic values; QTL-MAS, quantitative trait loci-marker assisted selection.

was comparable for all traits except for W6W in the scenario of low density SNP data set. The ante-BayesB using low density SNP data performed worse than other methods for all traits (statistical significance for W6W). The hyper-BayesC π and ante-hyper-BayesC π sometimes performed a little better than BayesC π and ante-BayesC π , such as for body weight trait. Using SNP data set from low density to high density did not reflect the advantage of ante-hyper-BayesC π (ante-BayesC π) over hyper-BayesC π (BayesC π) on the prediction accuracy for these four trait analyzed in mice data. Additionally, all the methods gave unbiased genomic prediction for each trait, which can be seen in Table 5.

As shown in Table 6, the estimated π were very different for different traits due to different genetic architectures of different traits. The estimated π values ranged from 0.327 \pm 0.010 to 0.949 \pm 0.009. The estimated π for W6W were higher than other three traits. The estimates of π from BayesC π , hyper-BayesC π , ante-BayesC π , and ante-hyper-BayesC π were significantly different ($p < 0.05$) for all scenarios except for BMI with high density SNP data.

Table 4. Prediction accuracies (mean \pm standard error) of DGVs in the validation population of mice data over 20 replications

Different SNPs ¹⁾	Method	W6W	GSL	BMI	BL
High density	Ante-BayesA	0.455 \pm 0.004	0.352 \pm 0.005	0.192 \pm 0.004	0.234 \pm 0.005
	Ante-BayesB	0.453 \pm 0.004	0.354 \pm 0.005	0.195 \pm 0.004	0.238 \pm 0.005
	BayesC π	0.449 \pm 0.004	0.352 \pm 0.005	0.193 \pm 0.004	0.236 \pm 0.005
	Hyper-BayesC π	0.453 \pm 0.004	0.353 \pm 0.005	0.192 \pm 0.004	0.237 \pm 0.005
	Ante-BayesC π	0.448 \pm 0.004	0.352 \pm 0.005	0.194 \pm 0.004	0.236 \pm 0.005
	Ante-hyper-BayesC π	0.452 \pm 0.004	0.353 \pm 0.005	0.193 \pm 0.004	0.237 \pm 0.005
Low density	Ante-BayesA	0.416 \pm 0.004 ^a	0.334 \pm 0.005	0.152 \pm 0.004	0.199 \pm 0.005
	Ante-BayesB	0.363 \pm 0.005 ^b	0.322 \pm 0.006	0.138 \pm 0.005	0.188 \pm 0.006
	BayesC π	0.408 \pm 0.004 ^a	0.334 \pm 0.005	0.151 \pm 0.004	0.203 \pm 0.004
	Hyper-BayesC π	0.416 \pm 0.004 ^a	0.334 \pm 0.005	0.153 \pm 0.004	0.203 \pm 0.005
	Ante-BayesC π	0.411 \pm 0.004 ^a	0.335 \pm 0.005	0.149 \pm 0.004	0.203 \pm 0.005
	Ante-hyper-BayesC π	0.415 \pm 0.004 ^a	0.335 \pm 0.005	0.153 \pm 0.004	0.203 \pm 0.005

DGVs, direct genomic values; SNPs, single nucleotide polymorphisms; W6W, body weight at 6 weeks; GSL, growth slope between 6 and 10 weeks of age; BMI, body mass index; BL, body length.

¹⁾ High density, 9266 SNPs; low density, 950 SNPs.

^{a-b} The prediction accuracies within each combination of traits and types of SNPs with no common superscript differ significantly ($p < 0.05$) among six methods; No superscript within each combination of traits and types of SNPs meant non-significant difference ($p > 0.05$).

Table 5. Prediction biases (mean±standard error) of DGVs in the validation population of mice data over 20 replications

Different SNPs ¹⁾	Method	W6W	GSL	BMI	BL
High density	Ante-BayesA	0.991 ± 0.021	0.982 ± 0.029	0.947 ± 0.039	0.919 ± 0.059
	Ante-BayesB	0.986 ± 0.021	0.998 ± 0.031	0.946 ± 0.044	0.927 ± 0.060
	BayesCπ	0.970 ± 0.018	1.018 ± 0.031	0.975 ± 0.041	0.998 ± 0.064
	Hyper-BayesCπ	1.000 ± 0.022	1.010 ± 0.031	0.992 ± 0.053	0.952 ± 0.061
	Ante-hyper-BayesCπ	0.970 ± 0.019	1.016 ± 0.031	0.974 ± 0.044	0.994 ± 0.062
	Ante-hyper-BayesCπ	1.000 ± 0.022	1.010 ± 0.031	0.992 ± 0.052	0.953 ± 0.061
Low density	Ante-BayesA	1.006 ± 0.025	1.009 ± 0.035	0.866 ± 0.044	0.923 ± 0.060
	Ante-BayesB	0.989 ± 0.026	1.046 ± 0.046	0.871 ± 0.061	1.032 ± 0.080
	BayesCπ	0.975 ± 0.024	1.024 ± 0.038	0.899 ± 0.057	1.014 ± 0.065
	Hyper-BayesCπ	1.015 ± 0.027	1.017 ± 0.039	0.924 ± 0.055	0.968 ± 0.070
	Ante-hyper-BayesCπ	1.022 ± 0.023	1.032 ± 0.038	0.912 ± 0.071	0.996 ± 0.065
	Ante-hyper-BayesCπ	1.055 ± 0.025	1.028 ± 0.038	0.912 ± 0.055	0.964 ± 0.066

DGVs, direct genomic values; SNPs, single nucleotide polymorphisms; W6W, body weight at 6 weeks; GSL, growth slop between 6 and 10 weeks of age; BMI, body mass index; BL, body length.

¹⁾ High density, 9266 SNPs; low density, 950 SNPs.

DISCUSSION

Our study is the first one to comprehensively investigate the influence of a first-order ante-dependence model and key hyperparameters on BayesCπ using simulation data sets and real mice data set. According to our results, the prediction accuracies for hyper-BayesCπ, ante-BayesCπ, and ante-hyper-BayesCπ were comparable with BayesCπ, ante-BayesB and ante-BayesA except the situation where ante-BayesB performed worse when using a few SNPs and π = 0.95. BayesCπ with an antedependence model and hyperparameters did not work better than BayesCπ regarding the prediction accuracy and bias. Meanwhile, ante-(hyper-) BayesCπ had a longer computing time than BayesCπ and hyper-BayesCπ, and there were similar computing times between BayesCπ and hyper-BayesCπ. When SNP density increased, computing time for ante-(hyper-) BayesCπ increased faster than (hyper-) BayesCπ.

BayesA, BayesB, BayesCπ and their extended forms assume

that the prior distribution of variances of SNP non-zero effects is a scaled inverse chi-square distribution with degree of freedom v_g and a scale parameter s_g^2 [15]. Preliminarily, v_g and $v_g s_g^2$ were fixed to 4.01 and 0.0020 in BayesA, and 4.23 and 0.0429 in BayesB in the first genomic prediction paper of Meuwissen et al [1]. However, it is more reasonable that these two fixed hyperparameters are assigned as different values in different genetic architectures of traits of interest. In the study of Habier et al [3], s_g^2 was calculated using formula (3) and (4) with different genetic variances for different traits in BayesA, BayesB, and BayesCπ. Many studies have also proposed some alternative priors to estimate these two hyperparameters. Habier et al [3] proposed BayesDπ treating s_g^2 as unknown with Gamma (1, 1). Yi and Xu [8] developed a model to assign a uniform density on $1/v_g$ for the range (0,1) and a uniform distribution on s_g^2 for the range (0, A) with A being a large number in the extent of BayesA. Yang and Tempelman [4] used a Gamma (0.1, 0.1) prior distribution

Table 6. π values (mean±standard error) estimated from BayesCπ and extensions of BayesCπ using the reference population of mice data over 20 replications

Different SNPs ¹⁾	Method	W6W	GSL	BMI	BL
High density	BayesCπ ²⁾	0.941 ± 0.010 ^a	0.415 ± 0.014 ^a	0.606 ± 0.033	0.327 ± 0.010 ^a
	Hyper-BayesCπ ²⁾	0.661 ± 0.027 ^b	0.603 ± 0.020 ^b	0.589 ± 0.023	0.595 ± 0.021 ^b
	Ante-BayesCπ ³⁾	0.949 ± 0.009 ^a	0.485 ± 0.015 ^c	0.643 ± 0.030	0.333 ± 0.011 ^a
	Ante-hyper-BayesCπ ³⁾	0.694 ± 0.025 ^b	0.621 ± 0.016 ^b	0.613 ± 0.020	0.591 ± 0.024 ^a
Low density	BayesCπ ²⁾	0.626 ± 0.033 ^a	0.441 ± 0.020 ^a	0.641 ± 0.032 ^a	0.334 ± 0.012 ^a
	Hyper-BayesCπ ²⁾	0.345 ± 0.019 ^b	0.492 ± 0.024 ^a	0.524 ± 0.015 ^b	0.467 ± 0.014 ^b
	Ante-BayesCπ ³⁾	0.673 ± 0.017 ^a	0.589 ± 0.013 ^b	0.731 ± 0.027 ^c	0.427 ± 0.012 ^b
	Ante-hyper-BayesCπ ³⁾	0.525 ± 0.009 ^c	0.613 ± 0.014 ^b	0.590 ± 0.015 ^{ab}	0.572 ± 0.011 ^c

SNPs, single nucleotide polymorphisms; W6W, body weight at 6 weeks; GSL, growth slop between 6 and 10 weeks of age; BMI, body mass index; BL, body length.

¹⁾ High density, 9,266 SNPs; Low density, 950 SNPs.

²⁾ π represented the proportion of SNPs having no genetics effects on the trait.

³⁾ π represented the proportion of SNPs having no residual genetics effects.

^{a-c} The estimated π values within each combination of traits and types of SNPs with no common superscript differ significantly (p < 0.05) among six methods; No superscript within each combination of traits and types of SNPs meant non-significant difference (p > 0.05).

on s_g^2 and specified $v_g \sim p(v_g) \propto (v_g + 1)^{-2}$ for both BayesA and BayesB. In our study, we have applied $v_g \sim p(v_g) \propto (v_g + 1)^{-2}$ for v_g and a Gamma (1,1) prior distribution on s_g^2 in BayesC π , termed as hyper-BayesC π . The results from our analysis showed that hyper-BayesC π performed a little better than BayesC π regarding prediction accuracy and bias. Given needing a pre-estimated genetic variance to define s_g^2 in BayesC π , hyper-BayesC π is more feasible to be applied in livestock breeding compared with BayesC π .

Compared with hyper-BayesC π (BayesC π), ante-hyper-BayesC π (ante-BayesC π) did not show advantages in improving the prediction accuracy and bias for simulation data with LD levels r^2 ranging from 0.136 to 0.333 and mice data with two different LD levels. This phenomenon is very different from the performance of ante-BayesA (ante-BayesB) over BayesA (BayesB) in the study of Yang and Tempelman [4]. They reported that ante-BayesA and ante-BayesB had significantly higher accuracies than their corresponding classical counterparts at higher LD levels in simulation data and real small data. However, subsequent studies did not show that the antedependence model performed significantly better. Wang et al [16] evaluated the antedependence model performance in Danish pigs and found that ante-BayesA showed lower accuracy compared to other models. Jiang et al [6] also introduced the first-order antedependence model to multi-trait BayesA, and the analysis from simulation and mice data showed that multi-trait ante-BayesA had less than 1% higher accuracies than multi-trait BayesA. The results from these studies were similar to the performance of ante-hyper-BayesC π (ante-BayesC π) over hyper-BayesC π (BayesC π) from our study.

When using genotype data from low density to high density in mice data, the antedependence model was not more advantageous over the corresponding method for four traits with different heritabilities. This surprising phenomenon may result from the following reasons. On one hand, the antedependence model had higher number of effective parameters, which suggested that the accuracy estimating SNP effects may be poor in current data set (less than 1,000 individuals in reference population) due to model complexity [16]; On the other hand, the relationship for adjacent SNPs is more closed with increasing SNP density. The linear relationship may not be good to explain the relationship for adjacent SNPs, and non-linear relationship may be better. In the future, with sequence data widely being used, the performance of the antedependence model can be validated again in livestock with sequence data.

Habier et al [3] reported that estimates of π from BayesC π were sensitive to training data size and SNP density. From our results, estimates of π not only were sensitive to SNP density but also were sensitive to the genetic architecture of a quantitative trait. Compared with ante-BayesA and ante-BayesB, BayesC π and its extensions could provide more information about the genetic architecture of a trait of interest. Although

different estimates of π from BayesC π and its extensions led to little differences on the prediction accuracy, they may have different power for QTL mapping which is interesting to be further studied. Additionally, it should be noted that ante-BayesB in this study was extended from the classical BayesB proposed by Meuwissen et al [1]. Ante-BayesB performed significantly worse than other methods when using a few SNPs, which suggests that treating π as known with a high value may be a poor choice in some cases. This agrees with Daetwyler et al [17] who reported that GBLUP outperformed BayesC with a fixed π when the number of simulated QTL was large, which is also validated by Habier et al [3]. This may be the reason why the extent of classical BayesB in which π is estimated by using a prior distribution and data information was proposed, such as ante-BayesB developed by Yang and Tempelman [4].

Ante-BayesA, ante-BayesB, and ante-(hyper-)BayesC π had a similar genomic prediction accuracy, and no one outperformed the other methods across all traits, which is consistent with the study of Habier et al [3] who reported the similar prediction accuracy between BayesA, BayesB, and BayesC π . However, computing time is very different among these three ante-Bayesian methods. From our study, ante-BayesA is the fastest, ante-(hyper-) BayesC π is next, and ante-BayesB is the slowest. Ante-BayesB had the longest computing time because the 100 cycles of MH step for sampling the locus-specific variances in the implementation of ante-BayesB is repeated in each iteration. Ante-BayesA and ante-(hyper-) BayesC π had the advantage on computing time, which becomes more and more important as SNP density increases. Compared with ante-BayesA, ante-(hyper-) BayesC π can shrink SNP effects and is more sensitive to the genetic architecture of a trait of interest, which results in gaining higher accuracies for traits with some large QTL effects, such as fat yield in Holstein populations [18,19].

CONCLUSION

In conclusion, BayesC π with prior distributions on v_g and s_g^2 (assigned as hyper-BayesC π) is recommended because it has the comparable prediction accuracy and bias and avoids pre-estimated total genetic variance of a trait compared with BayesC π . The performance of the first-order antedependence model in BayesC π did not show an advantage in improving the prediction accuracy and bias.

AUTHOR CONTRIBUTIONS

LXJ performed the analysis and wrote the manuscript. LXH and CYS conceived the study, gave help in the analysis and contributed to the manuscript. All authors have read and approved the final manuscript.

CONFLICT OF INTEREST

We certify that there is no conflict of interest with any financial organization regarding the material discussed in the manuscript.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (31601009), China Postdoctoral Science Foundation (2016M600699), and China Agriculture Research System (CARS-36).

REFERENCES

1. Meuwissen THE, Hayes BJ, Goddard ME. Prediction of total genetic value using genome-wide dense marker maps. *Genetics* 2001;157:1819-29.
2. VanRaden PM. Efficient methods to compute genomic predictions. *J Dairy Sci* 2008;91:4414-23.
3. Habier D, Fernando RL, Kizilkaya K, Garrick DJ. Extension of the bayesian alphabet for genomic selection. *BMC Bioinformatics* 2011;12:186.
4. Yang W, Tempelman RJ. A Bayesian antedependence model for whole genome prediction. *Genetics* 2012;190:1491-501.
5. Zhu B, Zhu M, Jiang J, et al. The impact of variable degrees of freedom and scale parameters in Bayesian methods for genomic prediction in Chinese Simmental beef cattle. *PLoS One* 2016;11: e0154118.
6. Jiang J, Zhang Q, Ma L, et al. Joint prediction of multiple quantitative traits using a Bayesian multivariate antedependence model. *Heredity (Edinb)* 2015;115:29-36.
7. Yang W, Chen C, Tempelman RJ. Improving the computational efficiency of fully Bayes inference and assessing the effect of misspecification of hyperparameters in whole-genome prediction models. *Genet Sel Evol* 2015;47:13.
8. Yi N, Xu S. Bayesian LASSO for quantitative trait loci mapping. *Genetics* 2008;179:1045-55.
9. Gelman A. Prior distributions for variance parameters in hierarchical models. *Bayesian Anal* 2006;1:515-34.
10. Gao N, Martini JWR, Zhang Z, et al. Incorporating gene annotation into genomic prediction of complex phenotypes. *Genetics* 2017;207:489-501.
11. Elsen J-M, Tesseydre S, Filangi O, Le Roy P, Demeure O. XVth QTLMAS: simulated dataset. *BMC Proc* 2012;6(Suppl 2):S1.
12. Valdar W, Solberg LC, Gauguier D, et al. Genome-wide genetic association of complex traits in heterogeneous stock mice. *Nat Genet* 2006;38:879-87.
13. Legarra A, Robert-Granié C, Manfredi E, Elsen JM. Performance of genomic selection in mice. *Genetics* 2008;180:611-8.
14. Valdar W, Solberg LC, Gauguier D, et al. Genetic and environmental effects on complex traits in mice. *Genetics* 2006;174:959-84.
15. Gianola D, De Los Campos G, Hill WG, Manfredi E, Fernando R. Additive genetic variability and the Bayesian alphabet. *Genetics* 2009;183:347-63.
16. Wang L, Edwards D, Janss L. Evaluation of antedependence model performance and genomic prediction for growth in Danish pigs. 10th World Congress on Genetics Applied to Livestock Production (WCGALP), Vancouver, Canada; 2014. p. 1-3.
17. Daetwyler HD, Pong-Wong R, Villanueva B, Woolliams JA. The impact of genetic architecture on genome-wide evaluation methods. *Genetics* 2010;185:1021-31.
18. Li X, Lund MS, Janss L, et al. The patterns of genomic variances and covariances across genome for milk production traits between Chinese and Nordic Holstein populations. *BMC Genet* 2017;18:26.
19. Ding X, Zhang Z, Li X, et al. Accuracy of genomic prediction for milk production traits in the Chinese Holstein population using a reference population consisting of cows. *J Dairy Sci* 2013;96:5315-23.