


Low Entropy Future Boundary Conditions

Lawrence S. Schulman 

Physics Department, Clarkson University, Potsdam, NY 13699, USA; schulman137@gmail.com

Abstract: A number of ways to detect future, low-entropy, boundary conditions are considered. The most important of these is the use of slowly-decaying isotopes and the observation (or prediction) of galactic dynamics. There is the expectation that future developments in experimental or observational technique will yield positive results.

Keywords: cosmology; nuclear physics; low entropy future boundary conditions

1. Introduction

This article is an attempt to see whether one can detect, observationally or experimentally, whether there are future boundary conditions. In particular we are concerned with future boundary conditions that involve low entropy. The conclusion is “probably not at this time”, but there is reason to try. The first reason is that one or two of the methods described here may soon succeed. Another is simple curiosity, but it is also true that there is a quantum measurement theory [1–4] which is difficult to justify in any other way.

It is clear that, if things have come to equilibrium between initial and final boundary conditions, nothing will be evident; see Figure 1. It is just as clear (from the figure) that a system that does not come to equilibrium is needed to see future boundary conditions (for a fuller explanation of the figure, see Appendix A).



Citation: Schulman, L.S. Low Entropy Future Boundary Conditions. *Entropy* **2022**, *24*, 976. <https://doi.org/10.3390/e24070976>

Academic Editor: Katarzyna Sznajd-Weron

Received: 14 June 2022

Accepted: 12 July 2022

Published: 14 July 2022

Publisher’s Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.



Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

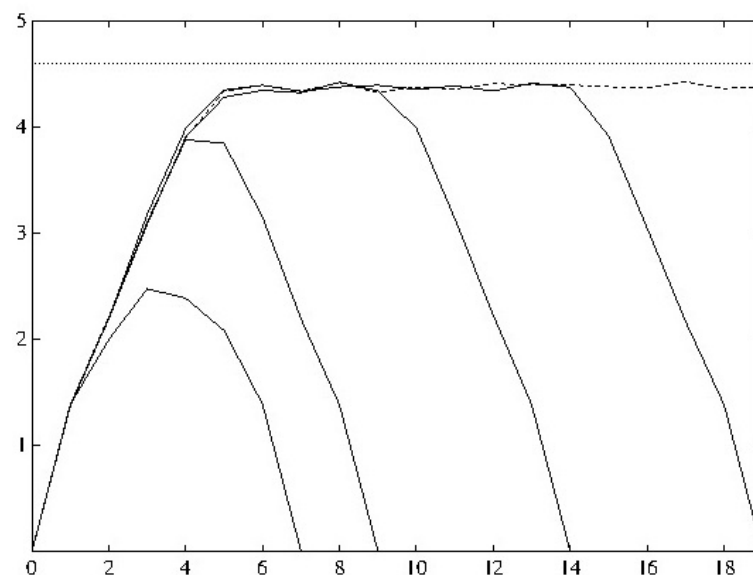


Figure 1. Entropy as a function of time for the “catmap”. (See Appendix A for further explanation). There are various future boundary conditions including (dashed line) none. Those boundary conditions where equilibrium can be established cannot be distinguished from unconstrained evolution. However, when equilibrium cannot be established—e.g., boundary conditions at time-7—differences are apparent.

Wheeler was aware of this problem, and proposed that long-lived nuclear isotopes might be an answer [5]. Unfortunately, he thought this could be done in the laboratory (and see a difference between an exponential and a hyperbolic cosine). At that time I wrote a paper [6] (see Appendix B) pointing out that you would require knowledge of the total abundance of the substance in the universe, presumably impossible to get.

All this would seem like idle speculation if, as now seems to be the case, the universe will go on expanding forever. However, doubts have been raised and there may be low entropy conditions in the distant future [7–9]. True, there is neither experimental nor observational evidence in favor of a “big bounce” or similar events, but one should bear in mind that this would not be the first time that there have been surprises in cosmology.

It is therefore relevant to ask: what are the slow relaxation times? We take up a number of slow processes.

2. Various Indicators

Observations and N-body results: Dynamics on a cosmological scale are certainly slow, a condition for seeing future boundary conditions (cf. Figure 1). One could make the case for low-entropy future boundary conditions if one could see that galaxies (as points) were not equilibrating. The points would still follow the equations of motion, and it would just appear to be unusual behavior. However, as far as I know there is no such indication. (They seem to virialize quickly [10], but presumably that is not related.) There are two possibilities. The first is *N*-body simulations. However, simulations are not sufficiently accurate to answer this question. After all, there are uncertainties connected with dark matter as well as other sources of possible error. Another problem is that the non-thermodynamic behavior might be so far into the future as to be impossible to see. This deals with an issue in any theory that purports to establish future boundary conditions: how far into the future are we talking about?

But there are other data. The range of fractality is subject to dispute as well as the definition of the range of inhomogeneity (and they are closely related [11]). At the time of recombination the range of inhomogeneity was small, near zero. It is now large, but is it infinite? This is where a dispute occurs [12], but the consensus is that it is finite [13]. How large depends on your definition and what era you are looking at.

Graphs of the range of inhomogeneity are given in [14–16] and in Figure 2 (see also [17–19]). There is little indication of a decline at $z = 0$, that is, at present. It could be that the first signs of this slowdown may be in the distant future—at most this can give a bound. It is possible, however, that with greater precision it would be possible to see this effect.

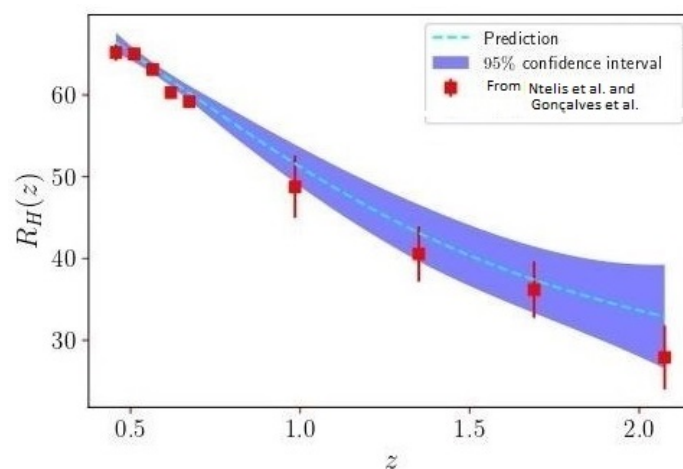


Figure 2. Reconstruction of the homogeneity scale function $R_H(z)$ (dashed line) and the R_H measurements (red squares) presented in [15,16]. There is some hint of a slowdown at “recent” times but it is not significant, nor are definitions definitive. Adapted from [14], Figure 2.

Dirac's theory: The classical theory of the electron's radiation involves future boundary conditions [20], and in fact Dirac calls this (on page 157 of the cited paper) "the most beautiful feature of the theory." Radiation is a damping process and Dirac ends up with odd-order derivatives having definite signs. In order to suppress runaway solutions he must invoke future boundary conditions. However, this has particular consequences: there is pre-acceleration. In other words, the electron "knows" it is about to be accelerated and undergoes some degree of acceleration *prior* to the forces that cause it to be accelerated.

There are two defects in looking for this effect. Firstly, the theory is classical. Secondly, the time of "pre-acceleration" is extremely short, on the order of m/e^2 , which for an electron corresponds to about 6×10^{-24} s (it is actually $\frac{2}{3} \frac{e^2}{mc^3}$ that is 6×10^{-24} s).

This is not an example of reduced entropy, but is included to disturb those who see no need for future boundary conditions.

Nuclei: There are a number of long-lived isotopes in the half life range of 10^{15} to 10^{24} years; see Table 1. Can these be used to indicate that there are future boundary conditions?

Table 1. Isotopes with half-lives in excess of 10^{15} years. For reference, the consensus on the time since the Big Bang is about 1.4×10^{10} years.

Isotope	Half-Life (years)	Isotope	Half-Life (years)
hafnium-174	2.002×10^{15}	zirconium-96	20×10^{18}
osmium-186	2.002×10^{15}	bismuth-209	20.1×10^{18}
neodymium-144	2.292×10^{15}	calcium-48	23.01×10^{18}
samarium-148	7.005×10^{15}	cadmium-116	31.02×10^{18}
cadmium-113	7.7×10^{15}	selenium-82	110×10^{18}
vanadium-50	140×10^{15}	barium-130	1.2×10^{21}
tungsten-180	1.801×10^{18}	germanium-76	1.8×10^{21}
europium-151	5.004×10^{18}	xenon-136	2.165×10^{21}
molybdenum-100	7.804×10^{18}	krypton-78	9.2×10^{21}
neodymium-150	7.905×10^{18}	xenon-124	18×10^{21}
tellurium-130	8.806×10^{18}	tellurium-128	2.2×10^{24}

As indicated, you would need to know global abundance of an isotope. All those in the table decay by rare modes, the double electron mode being the longest lived. Recently, the double electron mode of ^{124}Xe was observed [21,22], and is the longest lifetime actually measured. There is little information, however, on the overall abundance of some of these isotopes. Even what is available applies to the solar system or at most to our galaxy. However, this is an active area of research [23–29] and, for our galaxy, some of the isotopes in our table will have their abundance known. If we can assume that our galaxy is typical (or the relative production known) then the creation rate of those isotopes would also be known.

For example, Alibés et al. [23] discuss all elements up to the peak in iron (which includes V and Ca), in particular the abundance in the galaxy. The s-process deals with the heavier elements. There were indications in the work of Srinivasan et al. [30] (on the Murchison meteorite) that "stardust", could be an important factor. Bisterzo et al. [24] are concerned with the flux of neutrons and what effect that would have on isotopic composition. Another factor is the effect of cosmic rays, and that is studied in Cook et al. [25]—there is an increase in tungsten-180 that concerns them. More specific information is sought by Den Hartog et al. [26], in that the paper deals with the abundance of elements in the sun and metal poor stars. Peek [27] is concerned with europium and reaches conclusions about the overall frequency, although the vagaries of data availability induce a confinement to stars suspected of having exoplanets. Roederer et al. [28] have an interest in r-process tellurium; again they look at metal poor stars, but this gives an idea of galactic abundance.

Finally (in my list of citations), Travaglio et al. [29] see the galactic abundance of Sr, Y and Zr, so the question revolves (as it does for most contributors) around whether our galaxy is typical or at what stage it is. It should also be realized that my list of citations is not complete. Moreover, although this work is at most galactic in nature, the production rate for our galaxy is known for some elements. Moreover, the place of our galaxy among those that exist or existed may be guessed at with some degree of confidence.

A problem is that, even if long times are involved, it is difficult to distinguish exponential decay from the hyperbolic cosine (as predicted by Wheeler and verified *in some cases* by me [5,6,31]). The quantity $\exp(-x) - \cosh(X_{\max} - x) / \cosh(X_{\max})$ differs from zero by less than 10^{-2} for $X_{\max} = 5$ (and $x \geq 0$). Furthermore, this assumes there is a definite value for the decay rate. For suppose there was a hyperbolic cosine that dominated. Wouldn't that be interpreted as a slower decay?

On the other hand, with rare isotopes continuously being created, a simple hyperbolic cosine instead of an exponential would not be the dependence. However, given good (maybe unrealistic) control over global decay rates, there may be a way. As discussed in [6,31], in a small environment, for example a laboratory where the lifetime is measured, the fundamental lifetime would be observed. However, when looking at global events, the lifetime would be adjusted so that final conditions would be observed, those final conditions having none of the long lived isotope present, so the observed lifetime would be shorter than the laboratory value. There are two difficulties. First to make sure it is global, and not confined to our galaxy, and second to determine the lifetime of the remote or galactic sample.

Long lived substances between stars: Diamonds are not forever, but they and other solids can last a long time [32]. In fact, if time is symmetric on some scale, one would expect "relics" from the future. There would be two requirements: first this relic must escape from the fiery holocaust of a sun exploding. Second, it would be necessary to distinguish these relics from forward-time objects.

The second requirement may be satisfied by showing extreme age, for example in the absence of some slowly decaying isotope. A gold ring might have uranium as an impurity. Within 1.4×10^{10} years it will decay, so gold without uranium (and lead instead) is old. Of course, techniques of purification might eliminate the uranium (and hence the lead), but that too is reminiscent of the future. (To be precise, half the uranium will decay by that time, so for my conclusion a few half lives should have passed.)

Since I have no way to estimate this possibility, nor has any relic been found (although it may be sitting in a museum somewhere) it is speculative. However, should techniques of identification or estimation become available, this would represent another avenue.

3. Black Holes?

The formation of black holes would seem irreversible. Yes, there is black hole evaporation, but the time scale is long, orders of magnitude longer than, say, measured nuclear decays.

This means that if there are future boundary conditions and they force a reduction in entropy, than either they take place in the very distant future, or there is a flaw in the reasoning that leads one to believe in their effective irreversibility. The latter could easily occur, given the singularities that appear in those conclusions.

4. Doubts about Conventional Cosmology

The discovery of accelerated expansion [33] is a case in point. There have been doubts about this discovery [7,34–43]. These consist of under density of our local portion of the universe, leading to questions about the conclusion that there is expansion. Buchert [7,42,43] is particularly concerned with "backreaction", meaning the nonlinear effect of ignoring all but averages of density. The questioning and doubts are fairly common in cosmology and leads to surprises (which should not really be surprises).

5. Conclusions

There is insufficient observable evidence to establish a final restrictive boundary condition. There is theoretical reason to entertain this idea, but theory is not enough. In most cases, fortunately, there is hope that improved techniques will ultimately yield a positive result.

The techniques that seem hopeful are (1) slowdown in levels of homogeneity growth and (2) comparison of remote decay rates compared to local ones. These (and their flaws) are summarized as follows.

In the case of galactic dynamics, the range of the beginnings of homogeneity are available. If that level shows signs of slowing down it would indicate a future low entropy boundary condition. However, the range is subject to controversy and I would expect a slowdown would become controversial if only because of the definition of “homogeneity”. However, controversies are not new to cosmology.

Nuclear physics also has problems. The unaffected decay rate may be measured. The controversy would arise when galactic or extra-galactic rates are deduced. I would expect disagreement on galactic rates, even if naively they look slower.

As indicated I expect objections to be raised. However, this will not prevent measurements and observations. Moreover, if there is an indication of a slowdown, it will be noted.

Funding: This research received no external funding

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Explanation of Figure 1. The mapping (the catmap) is $x' = x + y$ and $y' = x + 2y$ (both mod 1, and x and y are taken in the unit square ($[0, 1) \times [0, 1)$). I imagine a gas of (say) 250 particles, all obeying the rule given above. To calculate entropy I divide the unit square into $M \times M$ sub-squares and count the number of points in each. This number divided by the total number of gas particles (250, in the example) is then used in the formula $S = -\sum p_\alpha \log p_\alpha$ with $p_\alpha = n_\alpha / N$ with α the sub-square in question, n_α the number of points in that sub-square, $N (=250)$ the total number of points and S the entropy.

To produce the figure I start with (slightly more than) $M^2 N$ points confined to a single sub-square (meaning a $\frac{1}{M}$ by $\frac{1}{M}$ square). The points are selected randomly. Then I take n steps, where $n = 7, 9, 14$ and 19 ; also I start with N points and do not give any future boundary condition. After n steps the points (typically) are everywhere in the unit square. I then select those points (N of them) that end within a given sub-square (it can be the same or different from the initial sub-square). Keeping only those points that satisfy being in a particular sub-square initially and finally guarantees that the entropy is zero at the beginning and at the end. Note that the dynamics is still only the catmap, since the particles do not interact with each other. This is the case for all but the unconditioned case. That case is different: I start with N random points in a sub-square and put no condition on their final location. The dotted line in the figure is the maximum entropy for this M value. Since $M < \infty$ this is not attained.

The time constant for relaxation in the figure is approximately 5, without units. The interpretation is related to the time constants of natural events. If they are short compared to “5” that is interpreted to mean they would be equilibrated. If longer than “5” that is interpreted to mean that they would not come to equilibrium. As a result, this graph applies to different times for equilibration.

This material also appears in [1], Chapters 2 and 4. This is an artificial model (in particular, because of the non-interaction assumption) but it serves to highlight the problem. The catmap is used in studies of relaxation and equilibrium. It is a mixing system.

Appendix B

Review of Relevant Results of Ref. [6]. In this appendix I will review the conclusions that one might reach based on “cosh” replacing “exp”. I will also discuss small samples.

The model studied is the Kac ring model (see [1] or [6]) and the demand is that $D(t) = \frac{1}{N}$ [number of white balls minus number of black balls at time t] satisfy $D(0) = \alpha$ and $D(T) = \beta$ for $T \ll N$. The answer (for $\alpha = \beta$) is

$$\text{Expectation of } D(t) = \frac{\alpha \cosh \gamma(\frac{T}{2} - t)}{\cosh \frac{\gamma T}{2}} \quad (\text{A1})$$

where $\gamma = -\log\langle \text{probability of being white minus probability of being black} \rangle$ (w.l.o.g. white is assumed to be more numerous). For t small, this replaces $\exp(-\gamma t)$, to which it is close.

However, for K balls, on the assumption that $1 \ll K \ll T \ll N$, you would have, for those K , the usual exponential, in other words, the color would be $K \exp(-\gamma(t - t_0))$ (where t_0 is the time of the initial experiment, in other words, when you had K of them white). This is because for $K \ll T \ll N$ the conditioning at t_0 does not affect (more precisely, has little effect) on overall abundance. You can think of $K = 10^{24}$ while $N = 10^{65}$ (see [6] for the whole story and also for conditioning the catmap).

References

- Schulman, L.S. *Time's Arrows and Quantum Measurement*; Cambridge University Press: New York, NY, USA, 1997.
- Schulman, L.S. Special states demand a force for the observer. *Found. Phys.* **2016**, *46*, 1471–1494. [[CrossRef](#)]
- Schulman, L.S. Program for the special state theory of quantum measurement. *Entropy* **2017**, *19*, 343. [[CrossRef](#)]
- Schulman, L.S.; da Luz, M.G.E. Looking for the source of change. *Found. Phys.* **2016**, *46*, 1495–1501. [[CrossRef](#)]
- Wheeler, J.A. At some point in the 1970s this was mentioned by Wheeler. I do not have a reference and I asked Wheeler, but he did not recall. I am nevertheless sure that this should be attributed to him. The closest I could come was J. A. Wheeler. In *General Relativity and Gravitation (GR7)*; Shaviv, G., Rosen, J., Eds.; Wiley: New York, NY, USA; Israel University Press: Israel, 1975.
- Schulman, L.S. Illustration of reversed causality with remarks on experiment. *J. Stat. Phys.* **1977**, *16*, 217–231. [[CrossRef](#)]
- Buchert, T. Toward physical cosmology: Focus on inhomogeneous geometry and its non-perturbative effects. *Class. Quant. Grav.* **2011**, *28*, 164007. [[CrossRef](#)]
- Khoury, J.; Steinhardt, P.J.; Turok, N. Designing cyclic universe models. *Phys. Rev. Lett.* **2004**, *92*, 31302. [[CrossRef](#)]
- Steinhardt, P.J.; Turok, N. Cosmic evolution in a cyclic universe. *Phys. Rev. D* **2002**, *65*, 126003. [[CrossRef](#)]
- Lynden-Bell, D. Statistical mechanics of violent relaxation in stellar systems. *Mon. Not. R. Astronom. Soc.* **1977**, *136*, 101–121. [[CrossRef](#)]
- Yadav, J.K.; Bagla, J.S.; Khandai, N. Fractal dimension as a measure of the scale of homogeneity. *Mon. Not. R. Astronom. Soc.* **2010**, *405*, 2009–2015. [[CrossRef](#)]
- Cosmai, L.; Fanizza, G.; Labini, F.S.; Pietronero, L.; Tedesco, L. Fractal universe and cosmic acceleration in a Lemaitre–Tolman–Bondi scenario. *Class. Quant. Grav.* **2019**, *36*, 45007.
- Hogg, D.W.; Eisenstein, D.J.; Blanton, M.R.; Bahcall, N.A.; Brinkmann, J.; Gunn, J.E.; Schneider, D.P. Cosmic homogeneity demonstrated with luminous red galaxies. *Astrophys. J.* **2005**, *624*, 54–58. [[CrossRef](#)]
- Avila, F.; Bernui, A.; Nunes, R.C.; de Carvalho, E.; Novaes, C.P. The homogeneity scale and the growth rate of cosmic structures. *Mon. Not. R. Astron. Soc.* **2022**, *509*, 2994–3003. [[CrossRef](#)]
- Gonçalves, R.S.; Carvalho, G.C.; Andrade, U.; Bengaly, C.A.P.; Carvalho, J.C.; Alcaniz, J. Measuring the cosmic homogeneity scale with SDSS-IV DR16 quasars. *J. Cosmo. Astropart. Phys.* **2021**, *3*, 29. [[CrossRef](#)]
- Ntelis, P.; Hamilton, J.; Goff, J.L.; Burtin, E.; Laurent, P.; Rich, J.; Busca, N.G.; Tinker, J.; Aubourg, E.; de Bourboux, H.M.; et al. Exploring cosmic homogeneity with the BOSS DR12 galaxy sample. *J. Cosmo. Astropart. Phys.* **2017**, *6*, 19. [[CrossRef](#)]
- Gaite, J. The fractal geometry of the cosmic web and its formation. *Adv. Astron.* **2019**, *25*, 6587138. [[CrossRef](#)]
- Heinesen, A. Cosmological homogeneity scale estimates are dressed. *J. Cosmol. Astropart. Phys.* **2020**, *2020*, 52. [[CrossRef](#)]
- Li, M.; Lin, H. Testing the homogeneity of the universe using gamma-ray bursts. *Astronom. Astrophys.* **2015**, *582*, A111. [[CrossRef](#)]
- Dirac, P.A.M. Classical theory of radiating electrons. *Proc. R. Soc. Lond.* **1938**, *167*, 148–169.
- Volta, G. This was reported in a talk by G. Volta at Univ. of Zurich. In Proceedings of the Swiss Physics Society Annual Meeting, Zurich, Switzerland, 26–30 August 2019.
- XENON Collaboration. Observation of two-neutrino double electron capture in ^{124}Xe with XENON1T. *Nature* **2019**, *568*, 532–535. [[CrossRef](#)]
- Alibés, A.; Labay, J.; Canal, R. Galactic chemical abundance evolution in the solar neighborhood up to the iron peak. *Astronom. Astrophys.* **2001**, *370*, 1103–1121. [[CrossRef](#)]

24. Bisterzo, S.; Gallino, C.T.a.R.; Wiescher, M.; Käppeler, F. Galactic chemical evolution and solar s-process abundances: Dependence on the ^{13}C -pocket structure. *Astrophys. J.* **2014**, *787*, 10–14. [[CrossRef](#)]
25. Cook, D.L.; Smith, T.; Leya, I.; Hilton, C.D.; Walker, R.J.; Schönabächler, M. Excess ^{180}W in IIAB iron meteorites: Identification of cosmogenic, radiogenic, and nucleosynthetic components. *Earth Planet. Sci. Lett.* **2018**, *503*, 29–36. [[CrossRef](#)]
26. Den Hartog, E.A.; Lawler, J.E.; Sneden, C.; Cowan, J.J. Improved laboratory transition probabilities for Nd II and application to the neodymium abundances of the sun and three metal-poor stars. *Astrophys. J. Suppl.* **2003**, *148*, 543–566. [[CrossRef](#)]
27. Peek, K.M.G. A new technique for determining europium abundances in solar-metallicity stars. *Pub. Astron. Soc. Pacific* **2009**, *121*, 755–766. [[CrossRef](#)]
28. Roederer, I.U.; Lawler, J.E.; Cowan, J.J.; Beers, T.C.; Frebel, A.; Ivans, I.I.; Schatz, H.; Sobeck, J.S.; Sneden, C. Detection of the second r-process peak element tellurium in metal-poor stars. *Astrophys. J. Lett.* **2012**, *747*, 5. [[CrossRef](#)]
29. Travaglio, C.; Gallino, R.; Arnone, E.; Cowan, J.; Jordan, F.; Sneden, C. Galactic evolution of Sr, Y, and Zr: A multiplicity of nucleosynthetic processes. *Astrophys. J.* **2004**, *601*, 864–884. [[CrossRef](#)]
30. Srinivasan, B.; Anders, E. Noble gases in the Murchison meteorite: Possible relics of s-process nucleosynthesis. *Science* **1978**, *201*, 51–60. [[CrossRef](#)]
31. Schulman, L.S. Normal and reversed causality in a model system. *Phys. Lett. A* **1976**, *57*, 305–306. [[CrossRef](#)]
32. Lazicki, A.; McGonegle, D.; Rygg, J.R.; Braun, D.G.; Swift, D.C.; Gorman, M.G.; Smith, R.F.; Heighway, P.G.; Higginbotham, A.; Suggit, M.J.; et al. Metastability of diamond ramp-compressed to 2 terapascals. *Nature* **2021**, *589*, 532–535; Erratum in *Nature* **2022**, *605*, 1. [[CrossRef](#)]
33. Wikipedia. Accelerated Expansion. Available online: https://en.wikipedia.org/wiki/Accelerating_expansion_of_the_universe (accessed on 30 May 2022).
34. Blanchard, A.; Douspis, M.; Rowan-Robinson, M.; Sarkar, S. An alternative to the cosmological “concordance model”. *Astronom. Astrophys.* **2003**, *412*, 35–44. [[CrossRef](#)]
35. Bolejko, K.; Krasiński, A.; Hellaby, C.; Calerier, M. *Structures in the Universe by Exact Methods*; Cambridge University Press: Cambridge, UK, 2009.
36. Clarkson, C.; Cortés, M.; Bassett, B. Dynamical dark energy or simply cosmic curvature? *J. Cosmol. Astropart. Phys.* **2007**, *8*, 11. [[CrossRef](#)]
37. Ellis, G. Patchy solutions. *Nature* **2008**, *452*, 158–161. [[CrossRef](#)] [[PubMed](#)]
38. Leith, B.M.; Ng, S.C.C.; Wiltshire, D.L. Gravitational energy as dark energy: Concordance of cosmological tests. *Astrophys. J.* **2008**, *672*, L91–L94. [[CrossRef](#)]
39. Maartens, R. Is the universe homogeneous? *Phil. Trans. Roy. Soc. A* **2011**, *389*, 1957. [[CrossRef](#)]
40. Mertens, J.B.; Giblin, J.T., Jr.; Starkman, G.D. Integration of inhomogeneous cosmological spacetimes in the BSSN formalism. *Phys. Rev. D* **2016**, *93*, 124059. [[CrossRef](#)]
41. Rácz, G.; Dobos, L.; Beck, R.; Szapudi, I.; Csabai, I. Concordance cosmology without dark energy. *Mont. Not. R. Astronom. Soc.* **2017**, *469*, L1–L5. [[CrossRef](#)]
42. Boudewijn, F.; Roukema, P.M.; Buchert, T.; Ostrowski, J.J. The background Friedmannian Hubble constant in relativistic inhomogeneous cosmology and the age of the universe. *Astronom. Astrophys.* **2017**, *591*, A111.
43. Vigneron, Q.; Buchert, T. Dark matter from backreaction? Collapse models on galaxy cluster scales. *Class. Quant. Grav.* **2019**, *36*, 175006. [[CrossRef](#)]