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SPECIAL TOPIC

The Divine Proportion: Origins and Usage in Plastic Surgery

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Background: The aim of this study was to summarize the existing literature on the origin of the concept of the divine proportion and its usage in the plastic surgery field.
Methods: In PubMed, the search terms [golden ratio AND (plastic surgery OR aesthetic surgery OR face)] were used, resulting in 65 articles. Among them, 15 articles were excluded and 50 abstracts were reviewed, of which 45 were excluded. The remaining 5 full articles and 11 other mined articles were reviewed.
Results: No evidence was found that the golden ratio had been used in any architecture or paintings before Pacioli's *Divina proportione* (*Divine proportion*) (1509), after which painters begin to use this ratio in their art. Fechner (1876) found that the golden ratio (1:1.618) was more aesthetically pleasing than any other proportion of rectangles. Recently Marquardt invented a facial mask containing decagons and pentagons that embody φ in all dimensions, and claimed that this mask yielded the most beautiful shape of the human face. However, it did not fit the average facial features of northwestern Europeans or their perceptions of ideal femininity.
Conclusions: Mathematical regularity can be found in nature and in the human body. However, this does not necessarily mean that a "formula of beauty" exists in mathematics. From the contributions of modern aestheticians, we now know that the so-called "essence of beauty" cannot be derived from the mathematical "formula of beauty" in the object itself. Therefore, it is suggested that beauty is based on biology, rather than on mathematics. *(Plast Reconstr Surg Glob Open 2021;9:e3419; doi: 10.1097/GOX.0000000000003419; Published online 22 February 2021.)*

Without mathematics, there is no art.
— Fra. Luca Pacioli (1447–1517).

INTRODUCTION

In aesthetic surgery, new golden proportions are regularly presented and published. Some researchers believe that a "formula of beauty" really exists and can be found. However, others suggest that mathematical regularity can be found in nature and the human body.

We hypothesize that these controversies can be resolved through a careful, historically grounded, comparative evaluation of the corresponding philosophical ideas. The aim of this study was to summarize the existing literature on the origin of the concept of the divine proportion and its usage in the plastic surgery field.

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METHODS

In a search of PubMed, the search terms [golden ratio AND (plastic surgery OR aesthetic surgery OR face)] were used, resulting in 65 articles. Among them, 15 articles were excluded and 50 abstracts met our inclusion criteria ("golden ratio" and "plastic surgery or aesthetic surgery or face" appeared in the title). Studies that did not have sufficient content were excluded. Using these exclusion criteria, 45 abstracts were excluded. The remaining 5 full articles and 11 other mined articles were reviewed.[1–16]

No restrictions on language and publication form were imposed. All articles were read by 2 independent reviewers who extracted data from the articles.

RESULTS

What is the Divine Proportion?

In mathematics and the arts, 2 quantities are in the golden ratio if the ratio of the sum of the quantities to the larger quantity is equal to the ratio of the larger quantity to the smaller one: $a + b/a = a/b$ for $a > b$.

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Euclid

Euclid's *Elements* (13 Books, 300 B.C.) provided the first known written definition of the golden mean (Fig. 1). Book 2 contained a number of lemmas concerning the equality of rectangles and squares, sometimes referred to as "geometric algebra." This book constructed the golden ratio and a square equal in area to any rectilineal plane figure.

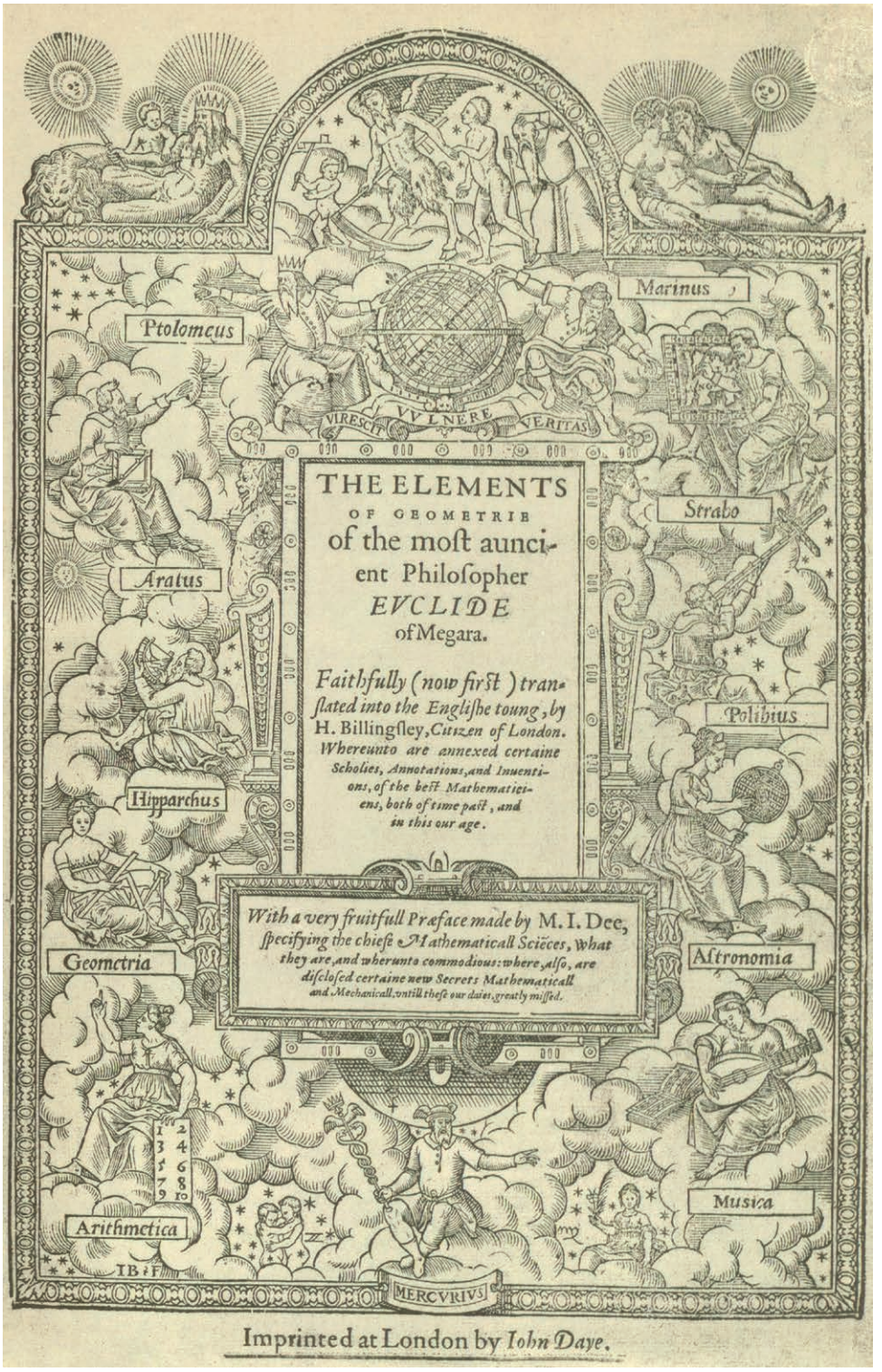

Fig. 1. Euclid's Elements (13 Books, 300 B.C.), which provided the first known written definition of the golden ratio. Available at https://commons.wikimedia.org/wiki/File:Title_page_of_Sir_Henry_Billingsley%27s_first_English_version_of_Euclid%27s_Elements,_1570_(560x900).jpg

Table 1. Fibonacci Connection

F_n		F_{n+1}/F_n	
F_0	1	F_1/F_0	1
F_1	1	F_2/F_1	2
F_2	2	F_3/F_2	1.5
F_3	3	F_4/F_3	1.6667
F_4	5	F_5/F_4	1.6
F_5	8	F_6/F_5	1.625
F_6	13	F_7/F_6	1.6153
F_7	21	F_8/F_7	1.6190
F_8	34	F_9/F_8	1.6176
F_9	55	F_{10}/F_9	1.618181
F_{10}	89	F_{11}/F_{10}	1.617977
F_{11}	144	F_{12}/F_{11}	1.618055
F_{12}	233	F_{13}/F_{12}	1.618025
F_{13}	377		

"A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the less." Euclid gives a construction for cutting a line "in extreme and mean ratio" (ie, the golden ratio). Several propositions and their proofs employ the golden ratio. Some of these propositions show that the golden ratio is an irrational number.

Fibonacci Sequence

From the ratio of F_{n+1}/F_n in the Fibonacci sequence (1,1,2,3,5,8,13,21,34,55,89,144,…), φ can be calculated ($L = 1 + 1/L$, $L = \varphi$) (Table 1).

Golden Rectangle, Golden Spiral, Golden Triangle, Golden Angle

A rectangle is called a golden rectangle if its sides are in the ratio of the golden mean. From a golden rectangle, a golden spiral can be constructed by dividing the space inside that rectangle into a square and a smaller golden rectangle. The golden spiral is a logarithmic spiral, where $\Theta = a\ln(b \cdot r)$, in polar coordinates. A golden triangle is an isosceles triangle where the ratio of the longer side to the base is φ. If the ratio of arcs $a/b = \varphi$, then angle subtended by the smaller arc is called the golden angle (approximately 137.51 degrees, about 2.399963 radians) (Fig. 2).

Before Luca Pacioli (before 1509)

No evidence was found that any architecture or painting used the golden ratio before Luca Pacioli, even Leonardo's paintings or the Parthenon.

Divina Proportione (Divine Proportion) (1509) by Luca Pacioli (1445–1517)

The divine proportion is a mathematical and artistic proportion that reflects the mathematics of the golden ratio and its applications in architecture (Fig. 3).[1] Leonardo da Vinci drew illustrations of regular solids in *Divina proportione* while living with and taking mathematics lessons from Pacioli.

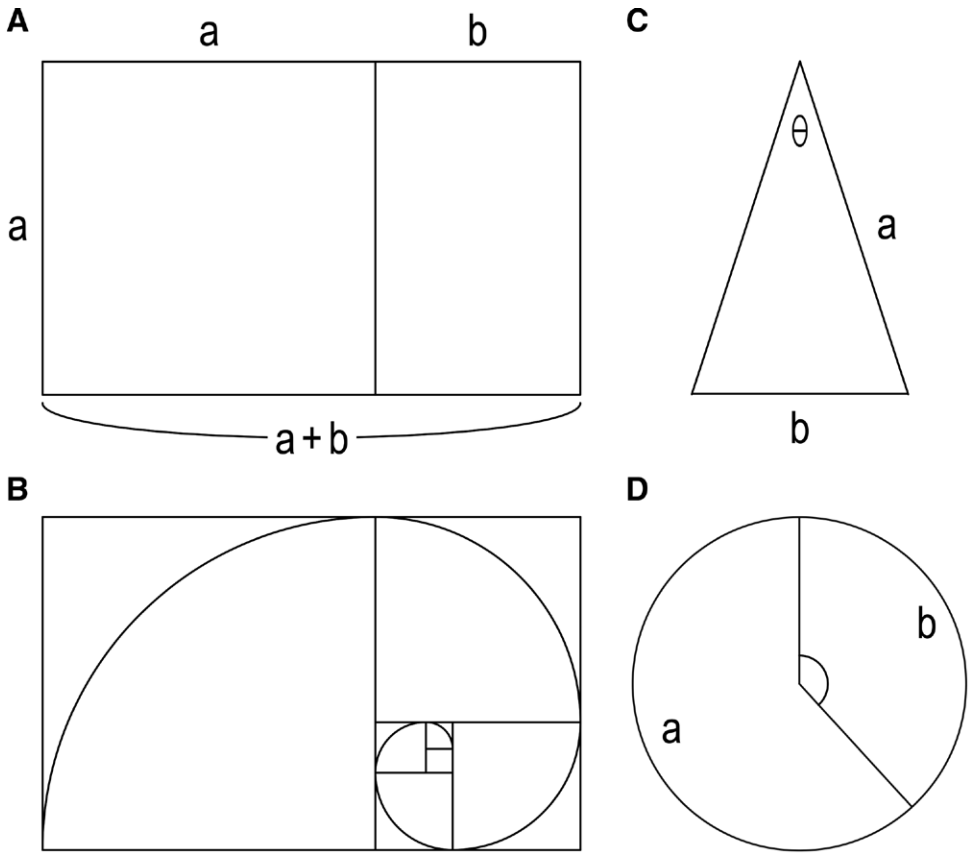

Fig. 2. Geometric figures with golden ratio. A: golden rectangle, B: golden spiral, C: golden triangle, D: golden angle.

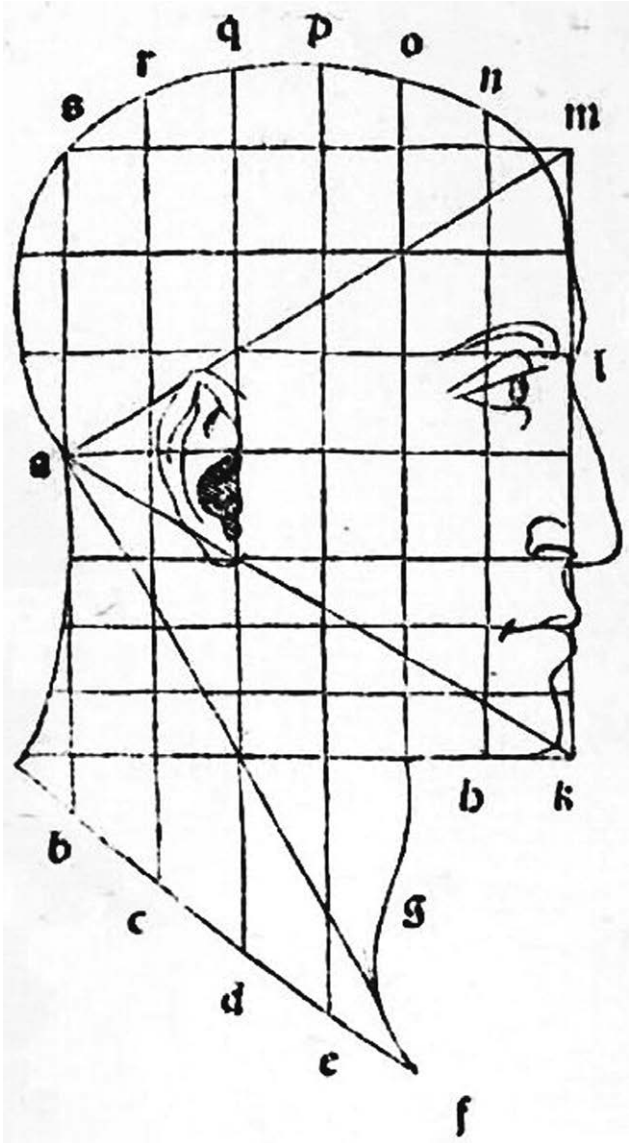

Fig. 3. Proportions of the human face from "Treatise on Architecture," the second part of *Divine Proportion* by Luca Pacioli. Available at https://commons.wikimedia.org/wiki/File:Pacioli_De_Divina_Proportione_Head_Equilateral_Triangle_1509.jpg

Usage of the Divine Proportion after Pacioli (after 1509)

The first painter known to use the golden ratio in painting was Paul Sérusier (1864–1927). Le Corbusier (1887–1965), an architect, found the golden ratio to be present at the neck, umbilicus, and knee. He insisted that this ratio can be applied at scales ranging from public squares to bookshelves.

Fechner's Rectangle Experiment and Contradiction thereof

Ten different sizes of rectangles were shown to participants and they were asked which one they preferred the most. Gustav Fechner (1801–1887) found that the average proportion of the most-preferred rectangles was always close to the golden ratio (1:1.618) (Fig. 4).[2] Through further study, he concluded that the golden ratio was more aesthetically pleasing than any other proportions of rectangles.[3]

Recently, an experimental study on preferences for proportions conducted using over 100 Korean traditional objects revealed that Korean participants had a significant preference for the root ratio (1:1.414), instead of the golden ratio (1:1.618). Based on that study, it is thought that golden ratio is not always the best proportion for a good design, although it may be a preferred proportion.[4]

Marquardt's Golden Mask and Refutation thereof

Marquardt argued that beauty is related to the golden ratio, insisting that the same relationship held for both genders and for all races and cultures, and claiming that his mask yielded the most beautiful shape of the human face. He used decagons and pentagons that embody φ in all their dimensions (Fig. 5).[5]

A study provided support for the use of this mathematical model to produce an objective system for measuring facial attractiveness. Applying the mask to 72 European White individuals and judging their appearance in surveys, Bashour found that the mask model correlated strongly with attractiveness.[6]

A plastic surgeon applied the facial golden mask to 40 preoperative and postoperative photographs and scored each photograph on a 1–5 scale and found that score was lower when the facial deformity was severe, whereas it was higher when the face was attractive. Thereafter he suggested that the facial golden mask may be used as an analytical tool.[7] However, a thin-plate spline analysis of the average facial features of northwestern Europeans revealed that Marquardt's mask best described the facial proportions of masculinized White women, as seen in fashion models, but did not seem to describe the "ideal" face shape even for White women, because its proportions

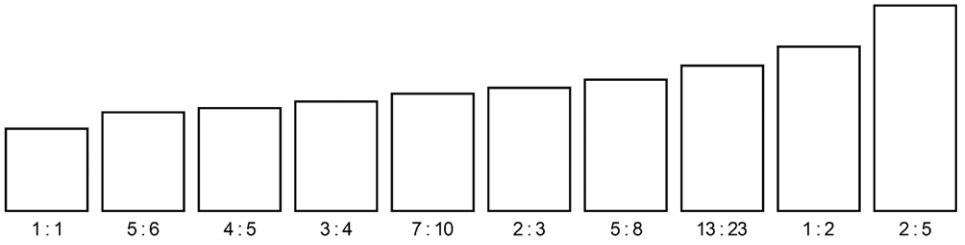

Fig. 4. Fechner's rectangle experiment. The average proportion of the most-preferred rectangles was always close to the golden ratio (1:1.618).

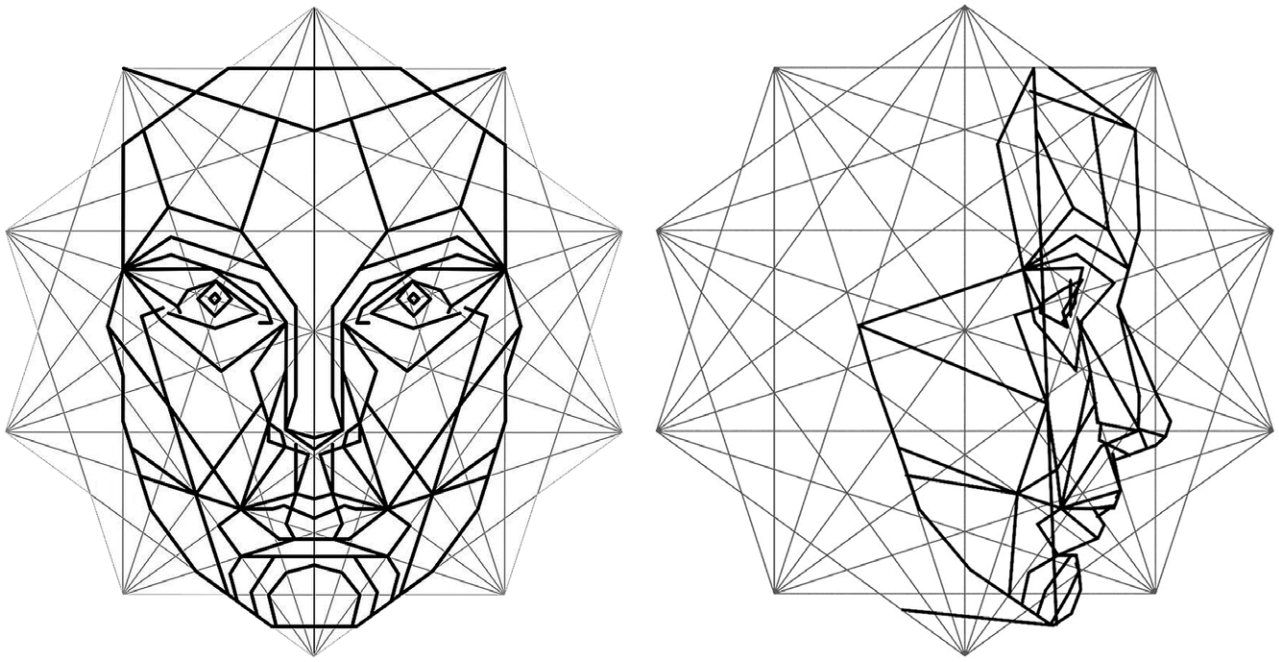

Fig. 5. Marquardt's golden mask, in which decagons and pentagons were used that embody φ in all their dimensions. Available at https://vali.de/wp-content/uploads/2011/06/FaceResearchMaskIn3D-SideFrontview-1024x576.png

were inconsistent with the optimal preferences of most people, especially with regard to femininity.[8]

Application of Divine Proportion in Mammaplasty and Abdominoplasty

Divine proportion between the perimeter of the waist and the hips was aimed for abdominoplasty. After fusiform skin resection, the rectus muscles aponeuroses were plicated through a medial tunnel. The umbilicus aponeurotic implantation was detached and reattached in a maximum proportion of 1/1.6 between the infra- and supraumbilical skin segment in 502 patients.[9]

For the standardization of navel surface anatomy and of proper placement in cosmetic abdominoplasty, Visconti analyzed navel shape and position on 81 high-quality pictures that had been chosen as top 2013 bikini models by editors of mass media. An on-line survey on navel shape and position has been made via facebook.com, involving 1682 people. The majority of participants chose the one with the navel relocated according to the golden ratio (1.618). Thereafter, the authors concluded that the most attractive navel position is located at the xiphoid–umbilicus:umbilicus-abdominal crease golden ratio.[10]

Recently, divine proportion was applied to plan mastopexy and breast reduction in 530 breasts. The constancy of the submammary fold and the orientation of the vertex of a V-shaped triangle opening at approximately 60 degrees at the umbilicus were maintained with each branch opening in the direction of the acromioclavicular joint. This strategy reduces the laxity in the axillary region, promotes bulk in the upper pole, and eases nipple–areola complex elevation.[11]

Phi Point and Cheek Shaping

To create aesthetic cheeks using hyaluronic acid fillers, Swift suggested that the apex of the beautiful cheek lies in a fixed proportion relative to Phi measurements on the face.[12] The Phi point is at the apex of the cheek mound and targeted for filler injections. Among the 113 patients who received fat injection on the Phi point during lower blepharoplasty, 102 patients got significant improvement in lower lid appearance, contour, transition to the cheek, and cheek projection.[13]

Golden Ratios in Facial Proportions in the 21st Century

Recent study reviewed photographs of beauty contest winners from 2001 to 2015. Twenty-six dominant points were identified. Comparisons of the facial proportion among Miss Universe Thailand, Miss Universe, neoclassical canons, and facial golden ratios were made. As a result, facial golden ratios were statistically significantly invalid in modern facial proportions of beauty.[14]

DISCUSSION

As stated in results, golden ratio is still being utilized in mammaplasty, abdominoplasty, and cheek shaping in plastic surgery.[9–14] It is also being used in analysis of beauty.

The golden ratio is attractively simple and beguiling as a theory; however, it is ultimately incomplete. Despite the golden ratio, we are yet far from establishing what constitutes beauty in quantitative terms, such that beauty is truly in the "eyes of the beholder."[15] The quest for a unifying theory continues, but there is evidence to support universally enduring standards of beauty, as defined by similar mathematical principles.[16]

Although its use as a universal criterion for beauty has been refuted several times, the golden ratio is still beautiful because it reflects the universal form of life and nature.

The golden ratio is a mirage created by yearning for absolute beauty, which transcends time. Longing for the rules of truth led to the discovery of phi (φ) in objects. In other words, the golden ratio found in the proportions of the human body is a philosophical construct derived from mysticism about numbers—that is, it solely reflects human beings' desire for mathematical proportions.

Mathematical regularity can be found in nature and in the human body. However, this does not necessarily mean that a "formula of beauty" exists in mathematics.

CONCLUSIONS

From the contributions of modern aestheticians, we now know that the "essence of beauty" cannot be derived from a mathematical "formula of beauty" in the object itself. Therefore, the longstanding efforts of plastic surgeons to identify a beauty formula have failed. Instead, it is suggested that beauty is based on biology, rather than on mathematics.

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