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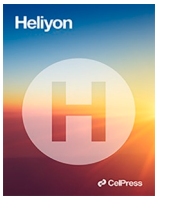

Review article

Review of modern models of porous media for numerical simulation of fluid flows

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ABSTRACT

The current models of porous media for numerical simulation of multiphase fluid flows are reviewed and classified herein. Methods for building these models and problems of creating the same are examined, and examples where the models were used are given. This review has been made as part of hydrocarbon deposit hydrodynamic simulation problems arising in the oil-and-gas producing industry. The approaches and possibilities for creating a digital twin of actual core have been evaluated. It is noted that the porous medium models used cannot presently meet the industry demands to the full and the reasons behind that have been analyzed. A conclusion was made that an in-depth fundamental study on processes and phenomena affecting the multiphase flow structure at the micro-scale is required to take them into account properly in the description of the flow at a deposit scale.

1. Introduction

Despite being widespread, multiphase fluid flows remain underexplored and difficult-to-model processes. The major reason for that is that they are governed by stochastic processes at the micro-level (capillary phenomena, wettability, adhesion and so on), which occur due to macro-scale factors. That said, continuum mechanics methods that imply averaging all the micro-phenomena at one point are employed to describe them at the micro-scale.

Addressing the problem of modeling multiphase flows is more relevant to the petroleum realm, as the demand for residual oil production and development of low-permeability reservoirs is increasing. This is manifested by an inconsistency between actual and estimated oil recovery indicators. As per Borisevich's estimates [1], the design or estimated hydrocarbon recovery factor is sometimes two or three times as high as the actual one, which cannot but affect the economic parameters of field development. The noted inconsistency evidences that the standards adopted in the industry and hydrodynamic simulation techniques do not allow the effective and rational use of natural reserves of raw hydrocarbons.

The most probable cause of the mismatching between the expected and actual hydrocarbon recoveries is related to insufficiently accurate hydrodynamic simulation that is performed using dedicated hydrodynamic simulators like ECLIPSE (Schlumberger) [2], T-Navigator (RFD company) [3], Tempest (Roxar) [4] and many others.

The simulation precision of a deposit is governed by multiple factors, for example, by complete information on beds and fluids

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thereof. But, in our view, one of the main problems lies in simulation techniques and acquisition methods of input parameters for that purpose.

It is common in the hydrodynamic simulation to describe the motion of fluids in line with the Darcy's law. Its classical variation is often used that describes the multiphase flow via the equation in the form (1):

$$\overline{w_i} = -k\frac{k_i}{\eta_i} grad\ P \tag{1}$$

where w_i is the filtration velocity of the i-th fluid phase, m/s; η_i is the viscosity of the i-th fluid phase, Pa•s; P is the formation pressure, Pa; k is the absolute permeability (AP) coefficient of the fluid or its i-th phase, which is estimated while the fluid is flowing in a porous medium in the absence of other fluids/phases, D; and k_i is the relative phase permeability (RPP) coefficient allowing for the flow of the i-th fluid in the presence of the other ones.

The porosity (m), permeability coefficient of porous medium, and fluid viscosity coefficient are measured experimentally by standard petroleum industry procedures and standardized equipment. The relative phase permeabilities for each phase are estimated as functions depending on phase concentrations in the total volume.

When Eq. (1) is used, it is common practice that the reservoir being a porous inhomogeneous body itself is replaced by a model continuum that is characterized by averaged fluid-flow parameters. Thus, the calculation region in which Eq. (1) is solved is a strong simplification of the geometry of the actual pore space of the reservoir.

On one hand, the use of the described approach towards hydrodynamic simulation is an *a-priori* rough generalization because the multiphase fluid flow is defined in reality by nonstationary, unstable, inert processes at the micro-level [5–7]. On the other hand, the averaged model was enough for stream engineering problems. This model enabled the results to be obtained at minimum requirements for source data amount and quality, and time-related and computing resources. This explains why the described approach is commonly spread in building a hydrodynamic model.

However, the experiments on measuring fluid-flow properties of the well core–a reservoir bed sample–especially in the case of multiphase fluid flow are expensive and labor-consuming even for advanced laboratories. This is explained by a number of reasons, as below [8,9].

- high-cost dedicated laboratory rooms and core warehouses, and setups and lab-scale equipment are required. Subsurface users have to erect entire factories for well-core research;
- one filtration experiment counts days; a considerable time is needed to solve variative problems, for example, to select an optimum method for enhanced oil recovery (EOR);
- following the filtration experiment, the well core cannot be used again, which also hinders research on EOR method selection;
- filtration experiments are sensitive to the procedure conduct; even the standardized procedures fail to achieve measurement repeatability.

Such problematics gave birth to the idea of Digital Core that holds that a real filtration experiment can be carried over to a virtual environment through up-to-date software programs, numerical methods and imaging tools. The numerical experiment can be replicated repeatedly, even under conditions infeasible for laboratories; therefore, such an approach would enable an inexpensive and qualitative estimation of permeability with a possible parametric analysis in the future.

The above-described approach has been reported in recent Russian [8–11] and international studies [10,12–19].As any other problem of creating a digital model, the digital core constitutes a solution to issues of fluid flows in some computational region [20]. The finite-element method is most often used whereby a spatial grid on which flow equations are solved is created across the calculation region. The flow equations alone are standard in general, and, for example, the problem of flow in a cylindrical channel differs from that of fluid flow in the reservoir particularly by the choice of the calculation region, i.e. the geometry of the space region in which the flow is modeled.

For flows in porous media, the calculation region is defined by the actual pore space geometry that has a complex irregular structure that often has different-scale voids or local inhomogeneities. As a result, it is the choice of the porous medium geometric model that governs the complexity in simulating the fluid flows.

The present review focuses on the analysis of the ways used to represent porous bodies in simulating fluid flows, as well as on how the choice of any particular model would influence the quality of simulation results. Understanding that the porous medium also exhibits the properties of minerals it is made of, the present paper examines only those medium models that reflect the medium geometry. It is explainable by the fact that the features of the contact between the solid skeleton and fluids are factored in by using boundary conditions of the problem, whereas the porous medium geometry can even affect the choice of a flow simulation method, as will be shown hereinafter.

The present review aimed to choose directions for further research and identify a fluid-flow numerical simulation method in order to enhance the quality of hydrodynamic simulation. Herein, we also looked for answers to the following questions: (i) which models among the existing ones are optimum for the digital well-core concept; (ii) can one or a few models out of the existing ones be sufficient; and (iii) will the digital well-core simulation on any porous medium model help improve hydrodynamic simulation in perspective and enhance the prediction accuracy of hydrocarbon extraction?

Here, we looked into the modern and traditional simulation methods for the geometry of porous media; their application experience, conditions and limitations when used; as well as how the use of a chosen model could influence the problem of simulation of

flows in the porous medium on the whole.

In the present review, we called attention to how the models of porous media in question correlate to each other. Realizing that it is impossible to examine all the models and all modifications thereof within a single review paper, we combined them into groups by the key features and proposed their classification. Moreover, we arranged the model grades themselves in a hierarchy according to characteristics important to digital well-core: to what extent the porous medium model is close to the actual medium, and how big is the volume of computational resources required for the simulation of flow in such a geometry.

In the present review, each group of models was examined with reference to the classification, therefore Section 2 gives a brief description of the classification. The further sections consider each group of models from the classification in detail, describe key principles by which the models were classified into groups, make examples of using those methods with reference to the other studies, and describe how the choice of the model influenced the entire problem of fluid-flow simulation and how much the choice improved or worsened the problem.

The brief description of the Discussion section and major conclusions: following the interdisciplinarity trend, we looked into the history of solving this problem in the field of aerodynamic simulation in the Discussion section. This allowed us to analyze the simulation features of multiphase flows in porous media from the standpoint of industry and formulate conclusions about the adequacy of porous medium models considered herein. The drawn analogy also allowed us to delineate the vector for further work over the digital well-core concept on the whole.

2. Classification of porous medium models

It is customary to use core digitalization results or static data on core as the baseline information for constructing a porous medium model. In the former case, a sample is analyzed by instruments and the porous space is reconstructed as a 3D or 2D model; in the latter case, the fractional or mineral well-core composition, and shape, size and packing of grains forming the rock skeleton are analyzed.

The described methods for acquiring information on the porous medium have led to a classification of its models into two groups which are tabulated in Fig. 1 on account of accuracy and computational complexity.

The simulation of a core digital twin is considered to be the most precise method for reproducing the porous medium [8]. In the other studies, the fluid-flow problems actualized on such models were assigned to a stand-alone group–dynamic numerical simulation (DNS) [14,21,22].

The concept of the method holds that a 3D model that reproduces the actual geometry of the well-core porous space as much as possible serves as the computation region. It is achieved by instruments, for example, by electron microscopy-assisted (SEM) digitalization of thin sections of a rock.

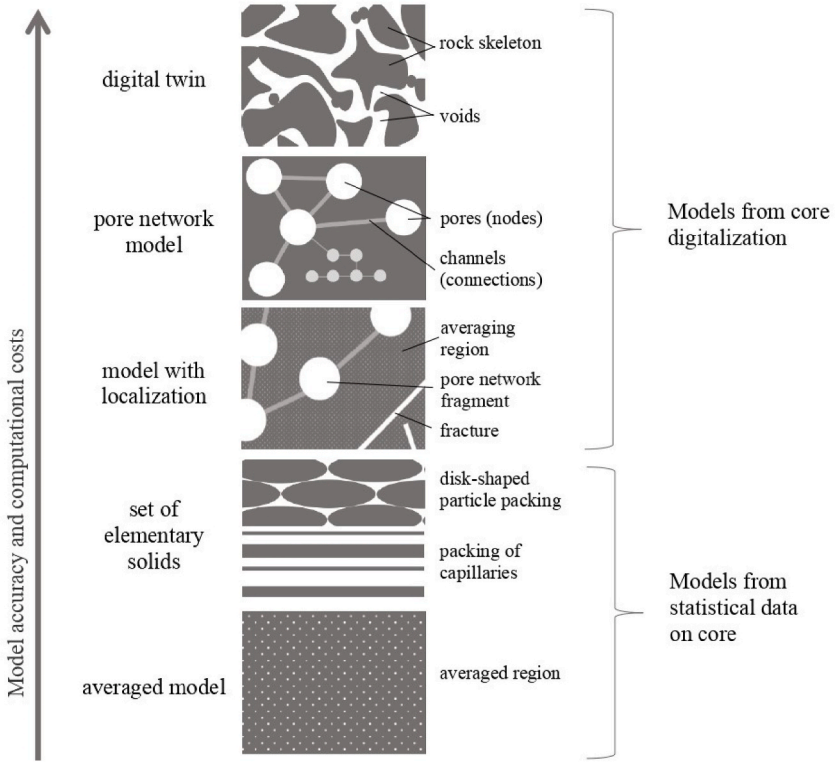

Fig. 1. Classification of porous medium models for fluid-flow problems.

In more recent studies, the classical approach that relies on the analysis of thin sections has increasingly been superseded by volume scanning on a CT scanner. The instrument patented by Hounsfield [23] was initially designed for medical applications. Later on, it gained a more widespread use, also in life science and technical disciplines, as it made it possible to effectively and invasively digitize objects of any complexity. Even though CT scan enabled high-quality and precise porous medium models to be generated, the application of such models is extremely expensive and not easily accessible.

The high costs for simulation of the digital twin have become the cause of searching for alternatives. The models derived by simplifying the actual geometry to a set of elementary solids can be considered as alternatives. From among the described group, the most precise reproducer of the well-core structure is the pore network model (PNM) in which the porous space is discretized into two groups of elements: pores and channels connected with each other on the principle of a graph or a network where the pores are network nodes and the channels are connections.

The PNM and digital twin enable the fluid-flow problems to be solved quite accurately but have two major drawbacks such as a construction difficulty and a high requirement for computing resources [8,14,21]. Nonetheless, they have significant merits for the problems of interest; therefore, combined models have been devised that, on one hand, take account of local phenomena (inhomogeneous medium, fractures, inclusions of other minerals and so on), while on the other hand, they are as simple and available as the averaged model. Such an idea has been supported by many researchers [19,24–31] and implemented in a group of porous medium localization models.

Attempts were made in a series of works to find a porous medium modeling method that would not require digitalization of actual rock samples–this would make the modeling much simpler. Such an approach for well core identifies grain packing patterns and statistical data on grain mineralogical composition, size and shape. The model medium in the said approach does no match the actual geometry but reflects its main features. Among instances of the described group are various packings of elementary solids or capillaries [32–36], as well as classical models of effective and perfect soils [20].

The main trends of using the models described above are to solve research problems and evaluate fluid-flow properties of reservoirs, which are further applied to engineering estimations in the hydrodynamic simulation of deposits by using the averaged model of porous medium.

The construction methods, merits and demerits of the porous medium models applied are considered in more detail hereinafter.

3. Digital twin of actual core

The digital twin of well core implies constructing a 3D model that would duplicate the actual structure of the porous medium sample as much as possible. As noted hereinabove, the modern studies often acquire such models by scanning the cores or parts thereof on a CT scanner, as a CT scanner enables rapid and qualitative data to be retrieved in a 3D format.

The pore structure of a rock can be 3D-modeled by placing the core into a CT scanner which consists basically of an X-ray source and a camera. The X-ray beam emitted from the scanner onto the sample is partially absorbed by the rock skeleton, and a shadow recorded by the camera originates on the reverse side of the source. The core is then displaced towards a small angle and x-rayed again to generate a series of 2D images of the shadows. The digitization is completed in a software program that renders the images into a 3D model of the rock skeleton and pore space.

The reconstruction of the rock sample pore space as a cube is exemplified in Fig. 2. The sample was cut out of the well core designated as “No. 1” and digitized on a micro-CT scanner by our research team. On the left are the sample images from three views, and on the right is a 3D pore model.

The material shown in Fig. 2 was acquired by our Academic Geochemistry Laboratory of the Tyumen Industrial University. The

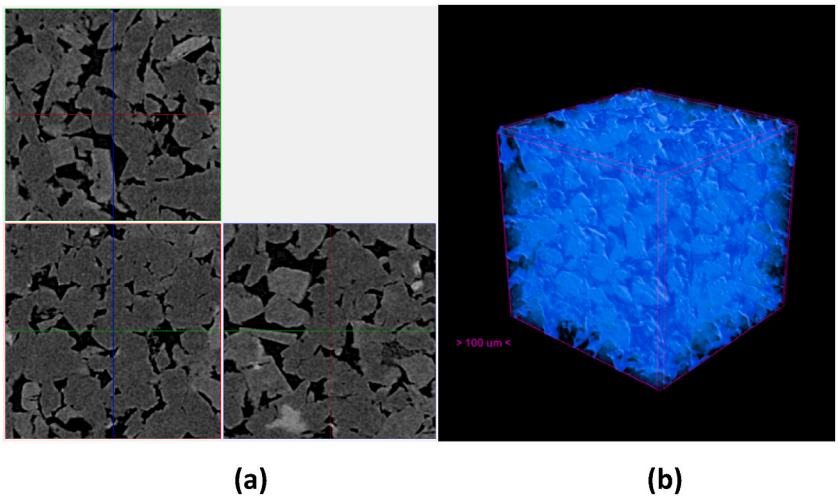

Fig. 2. An exemplified reconstruction of the 1-mm cube. (**a**) CT-scans of the specimen and (**b**) 3D reconstruction of the same specimen.

scanned specimen was taken from the West-Siberian petroleum basin. The specimen was scanned on a SkyScan 1172 X-ray micro-CT scanner. When the specimen was analyzed by the micro-CT technique, the following scan settings were employed: 100 kV voltage and 100 μA current strength on X-ray tube, 26.7 μm/pixel scanning resolution, aluminum and copper filter, 0.6° rotation angle, number of frames: 3, number of random frames: 15, scanning at 180°, 1 h scanning time. The reconstruction was performed using special NRecon software. More details on the procedure used to retrieve the data are reported in our previous study [37] in which we carried out a similar but more in-depth CT-scan examination of other specimens from the West-Siberian petroleum basin.

CT-scan can quite broadly examine the core structure, as well as acquire statistical data on pore sizes and estimate the sample porosity [38].

With the adoption of computerized tomography, it has become possible not only to evaluate the core structure but also the core composition. Using 3D simulation by CT scan, the researchers have learnt to reproduce the actual spatial position of minerals, as well as to track mineral inclusions [19], liquid droplets [12] and gas hydrates [4,6]. The instances are the left-hand images in Fig. 2 in which mineral inclusions of the core distinct form the basic material can be delineated from the depth of black color.

In Russia, the X-ray tomography technique was first employed [39] on humus rocks and has recommended itself to be a quality alternative to conventional morphological methods and thin-section analysis methods.

3.1. Merits of CT scan

A distinctive merit of the CT technique in constructing a digital twin of the well core is that it is invasive: the integrity of a sample when scanned is not impaired and hence the sample or part thereof can be used repeatedly. For instance, the researchers of gas hydrates used the CT scan to assess the cementation features of xenon hydrate and the impact it has on relative phase permeabilities (RPP). The CT scanner was coupled with a gas-charged accumulator that stimulated the generation of hydrates in well-core pores [17, 38]. Analyzing the images of the hydrate formation process, combined with mathematical modeling, those authors derived a correlation between the gas hydrate saturation and relative permeability to gas—a key parameter in the recovery task at gas hydrate fields.

The noninvasiveness of the technique also matters when digitizing rock samples having no stable cemented structure, such as upper geologic sediments and soils. A Canadian research team [40,41] examined peats by using X-ray tomography for modeling the transport of pollutants and nutrients. Within the objective set, generating a 3D model of peat by CT scan is the only way for the precise modeling of the porous structure of peat because using thin sections of such rocks proves to be impossible.

Another feature of 3D CT models is that complex processes occurring at the pore scale can be explored, some of which cannot be examined in the lab. Hazlett [42] was the first to employ X-ray images of well core for direct simulation of the multiphase flow and for estimation of relative permeabilities. The recent international [21,43–47] and Russian studies [29,48–52] broaden the knowledge about the mutual expulsion of fluids in pores, and their evaluation and the assessment of microphenomena (capillary forces, adhesion, wettability, slip effects) are often possible only on precise 3D models, that is, on digital twins. The computer-aided investigation methods of transport processes such as heat transfer, diffusion, and single- and multiphase flow were evaluated by Chen et al. [43]. These authors note the promising outlook of the pore-scale modeling by precise digitized models of porous media, especially when exploring how the pores mutually influence each other.

Three-dimensional CT models are employed not only as the computational region, but also in second-priority problems. They are more preferable in selecting optimum mathematical equations suitable for a phenomenon of interest. As regards fluid flow, there are a range of methods to describe the fluid flow at different scales. The methods were reviewed in papers [32,43,53,54], and the analysis demonstrates that the existent approaches are not versatile and have a selective efficiency; therefore, a question arises before the researchers how to select an optimum approach in a specific case. A series of studies address this problem analytically through the input of parameters comparing the efficiency of solutions with respect to convergence, error and so on [55]. Owing to the CT scan, an alternative can be the direct comparison when a range of methods are applied to the digitized core, and the results are matched against the experiment performed on the same sample. Such a study was conducted by Yan et al. [12] who compared five models (pore network model, stationary and nonstationary models of pore morphology, and lattice Boltzmann model) on a 3D copy of the core from the experiment.

To the second-priority problems solvable via the CT scan can also be referred the training of neural networks and the adjustment of algorithms that are created for quasi-experimental modeling of the porous medium or for its upscaling from the core to the oilfield level [39,55].

3.2. Demerits CT scan

It is not always possible to apply a precise porous medium model derived from the core digitization to oilfield-related and, sometimes, research problems. This is because the model creation entails high expenditures, and supercomputers are required for the estimations.

First, the model generation is high-cost because it requires access to expensive equipment and specific post-rendering of results into a format fit for the estimated model [9].Each new problem requires that a new rock sample be digitized, hence time and financial expenditures for such a model in stream challenges are considerable and can exceed the cost of a fluid-flow experiment, which does not satisfy the Digital Core concept.

Second, in research problems pursued to evaluate physical phenomena, beyond the calculation of engineering indicators, a precise model is also touted as the most expensive, which is attributed to the computing costs. Many problems of the transport at the core scale are solved by numerical methods which are implemented on a grid pursuant to the finite-element or finite-volume technique [14]. The

grid is generated through discretization of the computational region by elementary solid/volumes, hence the inhomogeneous and complex computational region requires that the grid be made more frequent, which increases the scope of computations. Besides, the direct simulation methods, for example using lattice Boltzmann equations, requires that high-performance computers be used. And, while computations by simulation of the single-phase flow are accelerated by additional means (for example, algorithm parallelization [51]), this is not enough for multiphase flow problems and the computational region needs to be replaced by a simpler one.

Third, the CT scan accuracy is a finite and will never allow a perfect digital twin of the actual core to be created. In this aspect, on one hand, the accuracy depends on the scanned sample size, as is illustrated in Fig. 3. Smaller samples enable more quality and well-defined images to be obtained; however, in that case, the computational region diminishes and a concern arises whether the sample chosen for scanning is representative of the whole core.

On the other hand, the scanning resolution of the instrument is finite, wherefore the definition of the resultant images does not always suffice for univocally delineating the boundary between the solid body and the pore (a blurred boundary observed) [14,21]. In some cases, the whole regions, for example microporous sites, can be blurred and unresolved. This requires the use of dedicated procedures of signal processing (smoothing), which in turn reduce the accuracy of the model created. Our example of smoothing an image of the sample from Fig. 2 is depicted in Fig. 4.

The images shown in Figs. 3 and 4 were obtained likewise those displayed in Fig. 2. The smoothing illustrated in Fig. 4 was applied in reconstructing the images in NRecon software. More details on the smoothing procedure in reconstructing a specimen were reported in our previous study [56] in which we noted the effects of smoothing and CT scanner resolving power on the evaluation of the specimen porosity.

The factor described above is also noted by the authors [19,24], who examined low-porosity samples of rocks similar to the shale ones. The blurred regions therein could not be univocally assigned to the solid skeleton or void space, as it required that either the scanning resolution should be enhanced or an additional problem of their identification solved.

The insufficient CT scanner resolving power and difficulties interpreting the results affect not only the 3D model quality and computation regions but also the evaluation accuracy of the core permeability parameters. Our research team measured the effective porosity of three rock samples by two techniques: from CT images, and experimentally by the standard procedure, as shown in Table 1.

The methodology for data acquisition in Table 1 is similar to that described in our earlier study [56], and the conclusions made herein coincide with the previous ones.

The porosity measurement from the flat cross-sectional image, which is akin to thin-section analysis, gave the least precise result. The porosity from the analysis of 3D reconstruction of the pore space when the sample was scanned in the CT scanner was close to the experimental, but was 2.5 % lower, which might be due to both the scanning resolution and model reproduction method and the choice of the element from the whole core that was scanned.

Another important drawback of the precise model is the problem of its representativeness at several scales. The computational region of this problem cannot exceed the sizes of the digitized sample that corresponds to the core (for medium- and highly productive reservoirs) or part thereof (for low-productive reservoirs). That said, as noted by Sadeghnejad et al. [14], the core itself is often significantly smaller than the representative elementary volume (REV) of the deposit–the least fragment thereof–to which the averaging tool is applicable. In such a case, some questions arise [14] as to how to take account of the effect of different-scale processes

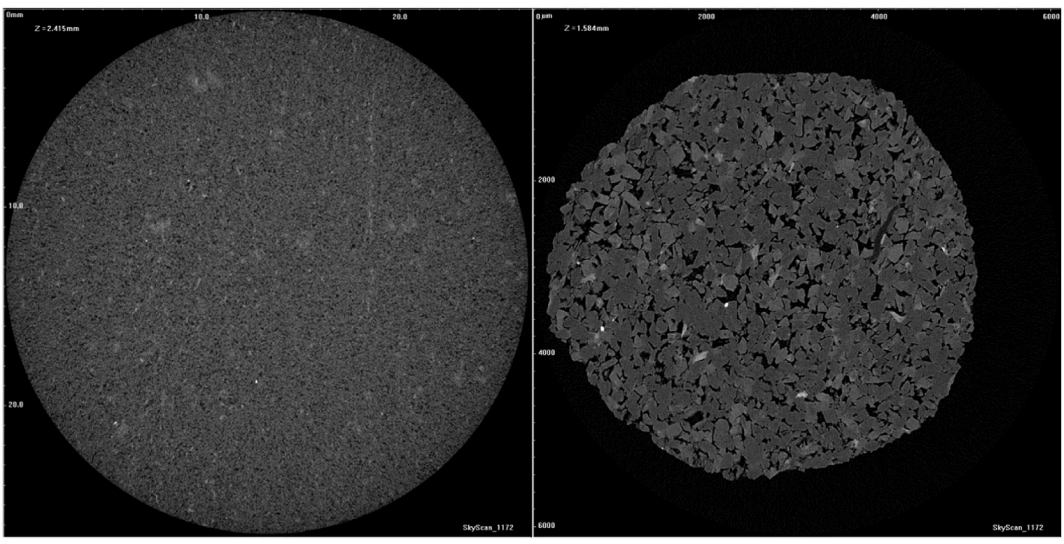

Fig. 3. Quality of a CT image subject to sizes of core sample No. 1: (**a**) a 30-mm sample, 7 μm/pixel scanning resolution; and (**b**) a 5-mm sample, 1.5 μm/pixel scanning resolution.

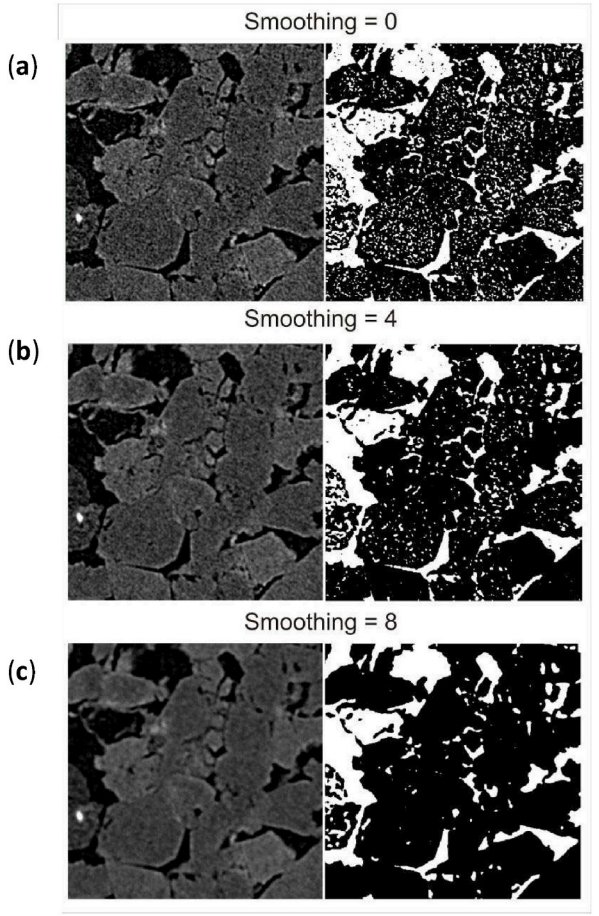

Fig. 4. Pore space reconstruction and smoothing: (**a**) smoothing = 0, (**b**) smoothing = 4 and (**c**) smoothing = 8.

Table 1
A comparison between sample porosity data obtained from measurement tests and CT images.

Sample #	Lithologic description	Porosity coefficient, %		
		Standard tests on a setup	3D reconstruction of pore space (Fig. 2)	2D image (Fig. 3)
1	Oil-saturated, fine-grained, aleurolite sandstone (in Fig. 2)	19.0	16.29	5.0
2	Oil-saturated, fine-grained, aleurolite sandstone.	19.2	17.75	5.0

on each other (for instance, the impact of adhesion or capillary phenomena on the total flow), how to incorporate heterogeneities whose sizes exceed the core into the hydrodynamic model (for example, extended fractures after hydrofracturing), and how to acquire an estimated model at the REV scale from digitization of one or several cores.

Thus, the direct numerical simulation of fluid flow using the digital twin is hardly applicable in the pure form to solving engineering problems, as corroborated by the study [8] on modern approaches to the digital modeling of well core. In addition, the digital twin cannot be a complete alternative to the experimental method for estimating permeability coefficients because the accuracy of the resultant data does not always justify expenditures for data acquisition. As for applied problems, it is most reasonable to use the described model in pointwise adjustment of mathematical approaches and program algorithms or in a combined approach that employs several models of porous medium.

Nonetheless, the direct numerical simulation applying the digital twin has gained wide use in research on micro- and nano-phenomena like the flow and transport at the pore scale. Solving such problems on a precise porous medium model holds promise on account of the disseminated idea, suggested by Sadeghnejad et al. [14], on creating an open 3D-scan database for cores from various laboratories. The localization of core models would appreciably cut down the expenses for equipment procurement and maintenance, and hence for exploration surveys as well.

4. Pore network model

The pore network model (PNM), which is quite close to the actual geometry, is measurably less costly, on one hand, and can be reckoned the nearest alternative to the digital twin of the actual core, on the other hand.

As noted hereinabove, the PNM idea consists in simplifying the pore geometry to simple volumes with the actual spatial position of pores preserved (Figs. 5 and 6). The source data is the digital twin that is processed and rendered into a network of spherical pores and channels connecting the same. As an example, Fig. 5(a and b) illustrates a pore network (on the right) acquired from the digital twin (on the left) digitized by Gharedaghloo et al. [41] on a micro-CT scanner.

The main objective of PNM is to keep the balance between the computing costs and the model proximity to the actual geometry of pores, therefore the pore space elements are averaged by shape but their spatial position is invariable and matches the actual one. Therefore, the pore network model can be specified by a set of data on geometry (radius, length), coordinates and ties of pores and channels to adjacent elements.

The three methods are best known for porous network construction from CT or SEM images [14,21,58]: medial axis transform or skeletonization, maximum ball transform and watershed transform methods.

The first method identifies a porous space skeleton made up of medial axes of pores. Each axis is replaced by a channel, while the branch point by a spherical pore [59]. In the second method, a small sphere contacting with adjacent grains is inscribed into the image element [60]. In the third approach, the image is processed as a topology map [61,62].

The modern approaches to pore network modeling enable pore networks to be adapted to a heterogeneous geometry—for example, one that contains microporous sites or fractures. The paper on pore network models by Sadeghnejad et al. [14] categorizes them by the scale of key elements into single-scale pore network models (S-PNMs), dual-scale PNMs (D-PNMs), triple-scale PNMs (T-PNMs), and fractured rock network model. In the latter model, subnetworks are introduced to evaluate different-scale effects on the flow through the inclusion of clay microporosity, pore cement or carbonate rocks [57,63,64].

The pore network modeling allows a more qualitative study on fluid flow in shale rocks that exhibit a low porosity and permeability. In the problems of capillary-dominated media, the classical averaged models furnish a significant error because the impact of microphenomena like slippage or adsorption is not initially factored in therein. That said, the application of a precise digitized model of porous medium, that is, the digital twin of core, is also hindered because the computing costs rise as the capillary number increases [53]. The pore network model proves to be a fairly rough averaging of the actual geometry, but its application has been recognized in the studies [65,66] to be an acceptable compromise. For instance, Afsharpoor and Javadpour [65] proposed a modified Darcy's law for shale rocks taking into account the channel geometry and flow slip phenomena on the channel walls:

$$Q = \frac{A^2}{\mu L}\left[a + bL_{sd} + cG + dL_{sd}^2 + eG^2 + fL_{sd}G\right]\Delta p \tag{2}$$

where a-f are the empirical constants [65], Q is the volume flow rate, A is the cross-sectional area, μ is the fluid viscosity, L is the channel length, Δp is the pressure differential, and G is the constant governing the channel cross-section shape.

Eq. (2) was later upgraded by the shale rock research team [66–70] who applied approaches based on molecular dynamics principles in their early studies. Yang et al. [66] improved Afsharpoor' model and took into account the impact of the adsorbed layer when hydrocarbons are flowing in shales.

Despite the PNM requiring significantly less computing time and computing resources, it has the same drawbacks as the digital twin of porous medium: cores also need to be digitized by a CT scanner or SEM; different-scale networks (D-PNMs and T-PNMs) can still be used only on high-performance computers; the identification problem of core images/scans of low resolution is not rare; the computational region is also limited by the digitized sample size, and the problem of increasing the computational region at least to the representative elementary volume (REV) needs to be handled isolatedly.

Besides the above-listed, an issue arises regarding the generation of the network itself and pore−channel connections at different

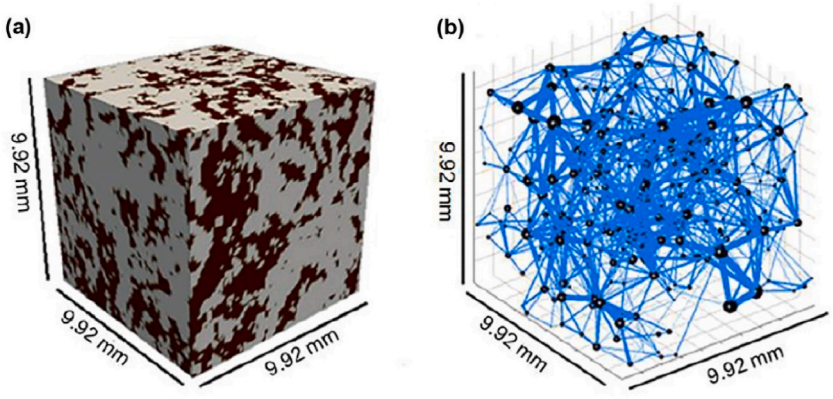

Fig. 5. (**a**) Digital twin generated by CT scan and (**b**) pore network model from digital twin (adapted from Gharedaghloo et al., 2018 [41]).

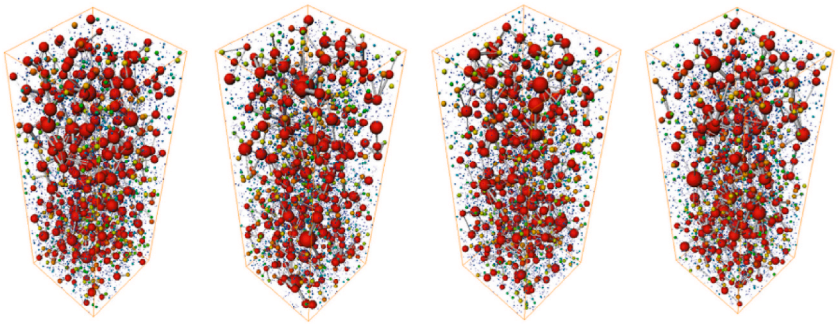

Fig. 6. Instances of pore network models for the whole core (adapted from Zhang et al., 2022 [57]).

scales, in line with Sadeghnejad et al. [14].

Thus, the pore network can be considered only as a simplified variation of the core digital twin because using the PNM requires less computing costs; however, it is yet hardly applicable in terms of the Digital Core concept.

5. Localization models

The search for simpler and more accessible solutions that would however allow for capillary and local phenomena (heterogeneous medium, fractures, inclusions of other minerals and so on) has resulted in a developed localization model whose schematic is depicted in Fig. 7.

As noted hereinabove, not all the scans/images of core allow the univocal assignment of the image element to the void space or rock skeleton. In some cases, such unresolved regions are not dominant fluid-flow elements and can therefore be represented by a local averaged region in which the Darcy's law variations or the Brinkman equation are active. The studies [19,24] employed a similar approach to create a pore network from images deficient in high resolution. The unresolved regions, as well as microporous sites, were specified by a generalized model. Consequently, a pore network was created on an averaged continuum, as in Fig. 7, for which a combined Navier-Stokes-Brinkman method was further employed.

A similar approach was implemented by Soulaine et al. [26] to simulate the flow in shale rocks, taking into consideration the features of the multiphase fluid flow and effects that distinguish it from the Darcy's flow. The localization was also employed in a series of studies [27,28] on examining the flow in tight sandstones in which clay inclusions were documented and further averaged by numerical modeling. The permeability estimation data showed a good convergence with the experimental data derived from helium pycnometry of the core.

An approach described in the studies [19,24–28] allowed the fluid-flow problem to be settled without enhancing the image resolution. Besides, as compared to the digitized models, the required computational scope in localization models is significantly smaller. This factor predetermined their application in the macro-scale problems, for example, when the fluid flow in fractured rocks and wellbore zone post-hydrofracturing was explored [58]. The complexity of such problems, on one hand, is that the computational region is much greater than the core sizes, wherefore only a generalized model is applicable. On the other hand, fractures cannot be averaged, as these increase the total permeability and are dominant elements in the pore space structure [52,58]. Furthermore, inside the fractures the flow is considered free, a model with an explicit delineation of the void space is therefore required for them. The optimum porous medium model in the said problems can be a localization model in which a fracture network is explicitly discernible in the averaged region. The fluid flow in rocks with regular and irregular one-dimensional system of fractures was studied in Refs. [29–31] and is depicted in Fig. 8 wherein the fluid-flow features were evaluated from the numerical experimental results, with the fracture geometry and frequency of estimated networks factored in.

Thus, the approach for the localization of the averaged model can serve as an example of a successful combination of precise digitized models with the averaged one. This approach is applicable to the different-scale problems, staring from simulation of fluid flow in shale rocks to the flow in regions containing a system of extended fractures.

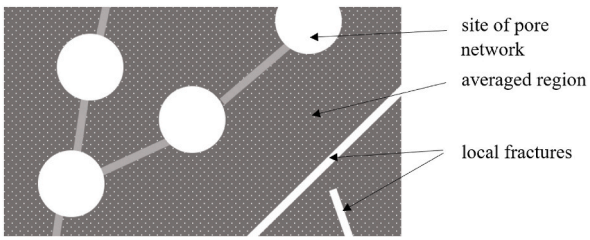

Fig. 7. A schematic of porous medium localization model.

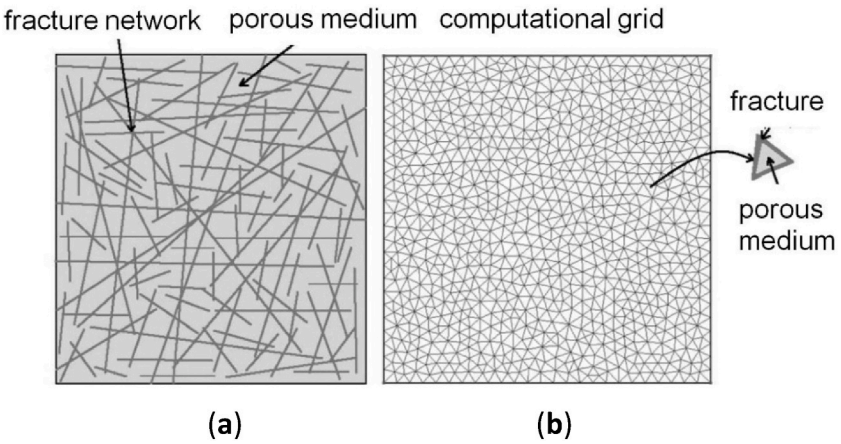

Fig. 8. (**a**) The fracture system localization model and (**b**) the computational region for that problem (adapted from Vasilyev et al., 2018 [52]).

6. A set of primitive solids

In the period of time where a qualitative digitization of cores was unavailable due to low instrumental resolution, the most accurate way of modeling porous media was by representing the same by different sets of elements. Today, this approach is applicable to problems in which the previous models are too costly and only statistical data on the core is known.

Assuming that any sedimentary rock a reservoir bed is often made up of represents compacted and cemented particles, the computational region for fluid-flow problems can be replaced by the packing of elementary figures (spheres, disks, channels of

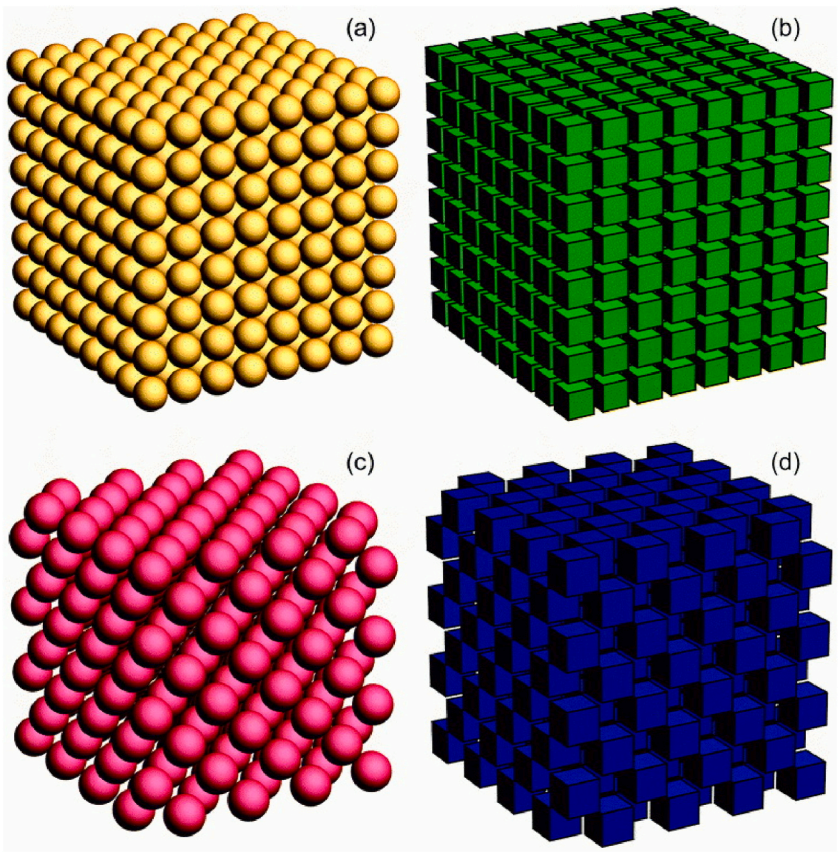

Fig. 9. Porous medium models examined by Röding: (**a**) a simple cubic lattice of spheres, (**b**) a simple cubic lattice of cubes, (**c**) a face-centered cubic lattice of spheres and (**d**) a face-centered cubic lattice of cubes (adapted from Vasilyev, 2017 [35]).

invariable and variable cross-section, as exemplified in Fig. 9(a–d) and modifications thereof (Figs. 10 and 11). The best-known models from this group are the perfect and effective soils [20] that are specified by a set of parallel capillaries of constant cross-section and by identical spheres, respectively.

The fluid flow in models of elementary solids was explored in a series of studies [32–36]. These studies employed different equations such as modified Navier-Stokes and Kozeny–Carman equations to describe the fluid flow.

On one hand, the described approach to simulating porous media enables the pore space to be delineated and hence a wide range of fluid-flow models to be applied to evaluate micro-phenomena. For example, Fatikhov [32] introduced an upgraded model of perfect soil that was utilized for a more qualitative estimation of permeability coefficients of the capillary-dominated oil-saturated reservoir. In this model, the porous medium was represented as the packing of constant cross-section cylinders whose radius variation conformed with some distribution law (Fig. 12). Consequently, a model of capillary displacement of oil by water was derived that reproduces lab-scale experimental results more accurately and hence allows the permeability for two-phase flow to be estimated with a greater reliability.

On the other hand, the figure packing model is less costly for the approach implementation and less demanding for core data quality than the digitized models [35]. Also, same as the averaged model, it has no difficulties rescaling the fluid-flow problems because the model is not bound to a specific sample and can be broadened.

This factor enables one to perform multiscale studies on fluid-flow patterns that can be reduced to new models of flow in porous media. One of such models was proposed in Ref. [36], where a regular packing of identical spheres connected via capillaries served as the computational region. Using the model, those authors employed averaging methods of fundamental equations for the flow in pores. Consequently, a new approach was suggested for the estimation of basic reservoir parameters (porosity and permeability). The resultant approach requires no empirical studies and its results are applicable to macroscopic equations for flow.

The major limiting complexity of particle packing models is the selection of the structure generation mechanism, that is, the shape of elements and distribution patterns thereof. The model must fit to the actual rock, otherwise its application will be unjustified.

7. Discussion

Significant efforts are being put into the idea of rendering fluid-flow experiments into a digitized form. Many large oil and gas producers and service companies have project on this subject. The largest universities and research centers are engaged in the work. But despite a more-than-20-year history, a real, fairly accurate solution available for the subsurface users has not been found. Digital twins of actual cores are expensive to create; digital twin-based estimations are energy-consuming and, most importantly, digital twins do not always coincide with experimental data. The simpler models have applicability limits and require evidence of their justified use in each case. The petroleum industry's demands for enhanced hydrodynamic simulation accuracy of hydrocarbon fields and for replacement of empirical studies of core by digital simulation remain unsatisfied.

A parallel can be drawn between the development of well-core digitalization projects and the history of moving vehicle design in aerohydrodynamics. From the physical standpoint, the parallel relies on the fact that the equations describing the motion are the same–simplistic–in either case (because equations for heat transfer, diffusion and so on need to be added to the system of equations being solved), these are Navier-Stokes equations [71]. The ways and methods for solving those equations are the same for the problems of both aerodynamics and underground hydrodynamics. For that, one needs to know information on moving fluids and conditions at the flowage zone boundaries and have necessary computational capacity. Of course, considerable differences in these two problems are that the shape of the aerodynamic body is known in aerodynamics and a substantial role is played by inertial forces that cause turbulence, while the pore space structure, composition and physical properties of capillary walls are unknown for certain in the fluid flows and the capillary forces play an essential role. But, the approaches to solving the problems faced are alike in many respects.

At the initial stage of aerodynamics when the designers, for example, of aircrafts had no access to computational resources, all input data for the design were derived empirically from a physical experiment in wind tunnels. Such experiments are justified by the use of the similarity and dimensional methods in mechanics [72]. Indeed, at the one-phase flow the Reynolds number ($\mathrm{Re} = d \bullet u/\nu$, where d is the channel diameter or characteristic dimension, u is the flow velocity in the channel, and ν is the kinematic viscosity of the fluid) is the only criterion of similarity enabling the model dimensions to be scaled up (of course, to the limits until the compressibility of the medium flowing around the model begins to exert a considerable impact on the motion). Before the emergence of sufficiently powerful computing machines, all the data required for aircraft design were derived in wind tunnels and these installations were loaded with stream experiments on measuring the lift force, drag force, pitch moments, etc. In today's hydrodynamic field modeling, all the requisite data (porosity, permeability, relative phase permeability) are retrieved from stream experiments in fluid-flow benches.

Later on, with the advent of computing machines, the mistaken belief arose that all flow regimes were computable. The computations encountered the problem of laminar–turbulent transition, the turbulence. It turned out that the correct computation of energy dissipation required taking account of the scales ranging from the molecular one to those exceeding manyfold the aircraft dimensions. And this requires huge computational resources. The assessment of the necessary computational resources for direct numerical simulation was reported by Belov and Isaev [73]. Such resources are hardly accessible and expensive, therefore the pathway of direct numerical simulation as applied to constructing aircrafts was recognized to be dead-end. In the hydrodynamic simulation of fluid flows, even if all complete information on the pore space structure is available, the direct numerical simulation requires not less resources, and therefore the direct numerical modeling in the deposit volume is of little promise.

In aerodynamics, the problem of flow simulation is currently being solved by building semi-empirical turbulence models and incorporating the turbulent viscosity into the Navier-Stokes equation [74]. Over a dozen models have now been built and are briefly reviewed by Belotserkovsky et al. [75]. What all the semi-empirical models have in common is that their governing parameters are

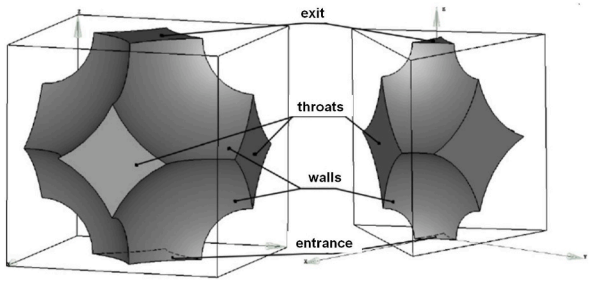

Fig. 10. A porous medium's element (adapted from Igoshin, 2015 [33]).

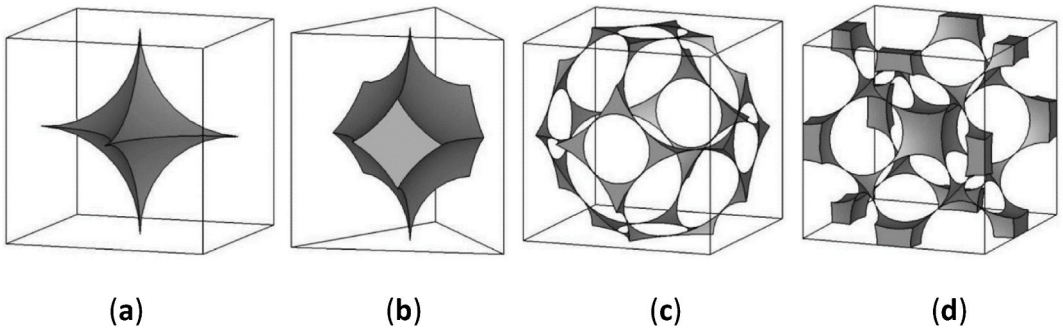

Fig. 11. Representation options of porous medium by elements of different geometry: (**a**) simple cubic, (**b**) simple hexagonal, (**c**) body-centered cubic and (**d**) face-centered cubic structures (adapted from Gubaidullin et al., 2016 [34]).

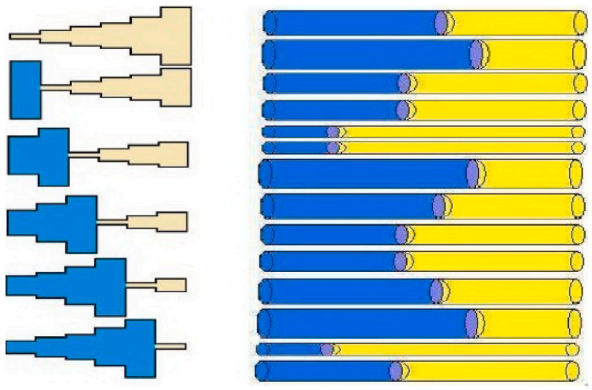

Fig. 12. An illustration of the porous medium model (adapted from Fatikhov, 2016 [32]).

selected using experimental data derived on the basis of in-depth basic research into the laminar–turbulent transition physics and turbulence nature. The use of semi-empirical models made it possible to modify the Navier-Stokes equation and sufficiently cheaply obtain characteristics of the flow around aircrafts with an accuracy sufficient for engineering problems. Nowadays, wind tunnels are basically being used for verification and adjustment of semi-empirical models and for aerophysical research. For example, Afanas'ev et al. [76] made estimations using a series of vortex viscosity models and compared them with experimental data obtained in wind tunnels, while Mikheev et al. [77] reported a method for evaluating the turbulence model parameters using experimental data.

8. Conclusion

The analysis performed herein demonstrates that none of the existent models of porous media cannot satisfy in full the petroleum industry's request for accurate hydrodynamic modeling of the fluid flow in the reservoir. First of all, this is due to the lack of information on the pore space structure and due to the huge requirement for computational resources. More simplified models cannot provide reliable results in most problems whose solution is required in practice; for example, the application of any particular methods for oil recovery enhancement. Hence, there is a need for proposing new approaches to hydrodynamic simulation of multiphase fluid

flow, which are straightforward and available for engineering application. We believe that building a model porous medium must rely on a detailed study of the multiphase flow at the pore level and on an in-depth study of phenomena at the micro-level. Then, it is essential to modify the equations describing multiphase flows and incorporate terms adjustable by experimental data thereto, and propose multiphase-flow estimation procedures.

Data availability

The data will be made available on request.
No additional information is available for this paper.

CRediT authorship contribution statement

Oleg A. Simonov: Conceptualization, Data curation, Formal analysis, Writing – original draft. **Yulia Yu Erina:** Resources, Software, Validation, Writing – original draft. **Andrey A. Ponomarev:** Project administration, Supervision, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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