



Nonlinear oscillation and acoustic scattering of bubbles

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ARTICLE INFO

Keywords:

Nonlinear Oscillation
Acoustic Scattering
Bubbles

ABSTRACT

The scattered acoustic pressure and scattered cross section of bubbles is studied using the scattered theory of bubbles. The nonlinear oscillations of bubbles and the scattering acoustic fields of a spherical bubble cluster are numerically simulated based on the bubble dynamic and fluid dynamic. The influences of the interaction between bubbles on scattering acoustic field of bubbles are researched. The results of numerical simulation show that the oscillation phases of bubbles are delayed to a certain extent at different positions in the bubble cluster, but the radii of bubbles during oscillation do not differ too much at different positions. Furthermore, directivity of the acoustic scattering of bubbles is obvious. The scattered acoustic pressures of bubbles are different at the different positions inside and outside of the bubble cluster. The scattering acoustic fields of a spherical bubble cluster depend on the driving pressure amplitude, driving frequency, the equilibrium radii of bubbles, bubble number and the radius of the spherical bubble cluster. These theoretical predictions provide a further understanding of physics behind ultrasonic technique and should be useful for guiding ultrasonic application.

1. Introduction

A large number of bubbles are formed in the liquid due to the dissolution of gas in the liquid. Air bubbles in water have a significant effect on the propagation of underwater acoustic due to their ability to efficiently scatter sound [1-4]. The acoustic scattering of bubbles has a wide application in ultrasonic imaging, ultrasonic detection and ultrasonic medical therapy [5-10]. Therefore the study of scattering of bubbles has attracted a great deal of attention. The researches on the acoustic scattering theory of bubbles mainly include the following two parts: the scattering of a single bubble and the scattering of multiple bubbles. Gompf and Kaplidiastrias et al. researched the scattering of a gas bubble experimentally and theoretically and provide a basis for the researches of multi-bubble scattering [1,11-15]. The theory of the scattering of multiple bubbles is more complexity. Maksimov researched the acoustic scattering of a pair of small spaced bubbles, the closed solution of the scattered acoustic field and its influencing factors [16]. Hasheminejad researched the acoustic scattering theory of a pair of two-phase dielectric spheres [17]. Gabrielli studied the acoustic scattering of two identical spheres theoretically, and showed quite in agreement with the experimental results [18]. Sage, Wang and Yong respectively studied the scattering resonance of the multi-bubble scattering acoustic field, the multi-bubble acoustic scattering model and the influence of surface tension on the multi-bubble scattering acoustic field [19-21]. However,

the researches of the scattering field and the influence factor of a bubble cluster are rather little, because the interaction between the bubbles is very complication [22-28], which has an important influence on the scattering of bubbles [29-31].

In this paper, we choose a spherical cluster as an acoustic scattering model. The scattered pressure, scattered acoustic intensity, scattered acoustic power and scattered cross section of bubbles is calculated based on the theory of scattering acoustic field. The scattering acoustic field of a spherical bubble cluster is numerically simulated, and the influence of the interaction between bubbles on the scattering acoustic field is researched. These theoretical predictions provide a theoretical basis for ultrasonic detection and other related fields.

2. Theory of the scattering of bubbles

As bubbles periodically expand and contract, the surrounding liquid is set into oscillation. The scattering of bubbles can be considered as the radiation of bubbles as the secondary acoustic source under the action of acoustic field [32], based on the equation of Euler, the radial velocity of a liquid particle at a distance r can be expressed as:

$$\frac{dv}{dt} = -\frac{1}{\rho} \frac{\partial P}{\partial r}. \quad (1)$$

The acoustic scattering of bubbles, which can be regarded as a

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secondary radiation of bubbles under the sound waves. Let us consider a bubble in a liquid driven by a stationary acoustic field for wavelength large compared with the radius of a bubble. According to the equation (1), the scattered pressure around a bubble can be expressed as:

$$p_{sc} = \frac{\rho}{r} \left(R^2 \ddot{R} + 2R\dot{R}^2 \right) \quad (2)$$

The scattered acoustic intensity of a bubble is

$$\left(1 - \frac{\dot{R}}{c} + \frac{3}{2}M \right) R\ddot{R} + \left(\frac{3}{2} - \frac{\dot{R}}{2c} + 3M \right) \dot{R}^2 = \left(1 + \frac{\dot{R}}{c} \right) \frac{1}{\rho} \left(P_l - P_0 - P_A \left(t + \frac{R}{c} \right) \right) + \frac{R}{\rho c} \frac{dP_l}{dt}. \quad (12)$$

$$I = \frac{1}{T} \int_0^T \text{Re}(p_{sc}) \text{Re}(v) dt, \quad (3)$$

where Re denotes a real component.

The scattered acoustic power is:

$$\bar{W} = \int I ds. \quad (4)$$

The scattered cross section is usually regarded as an important characteristic parameter to measure the scattering effect of bubbles, which can be expressed as [33]:

$$S_s = \frac{\bar{W}}{I_A}. \quad (5)$$

I_A is [34]:

$$I_A = \frac{\langle P_A(t)^2 \rangle_\tau}{\rho c}, \quad (6)$$

and $\langle \cdot \rangle_\tau$ is the average of a quantity over a time interval τ .

3. Numerical method

3.1. Fluid dynamic

The finite element method (FEM) by COMSOL is used to simulate a spherical bubble cluster in an acoustic field, two phases of liquid and gas were considered. The liquid around bubbles should follow the continuity equation and Navier–Stokes Equation. The continuity equation of liquid is stated as:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \quad (7)$$

The Navier–Stokes Equation is [35]:

$$\rho \left[\frac{\partial v}{\partial t} + (v \cdot \nabla) v \right] = -\text{grad} p + \mu \Delta v. \quad (8)$$

The gas inside a bubble is assumed to obey a ploytropic law, which can be expressed as:

$$p_g = p_{g0} \left(\frac{V_0}{V} \right)^\gamma. \quad (9)$$

The boundary condition at the air–liquid interface of a gas bubble is [36]:

$$-\left(\mathbf{n} \cdot (p\mathbf{I} + \mu(\nabla \mathbf{v} + \nabla \mathbf{v}^T)) \cdot \mathbf{n} \right) = p_g - \sigma \kappa_m. \quad (10)$$

The boundary condition at far field is expressed as [37]:

$$P(\infty) = P_0 - P_A(t). \quad (11)$$

3.2. Dynamic of a spherical bubble cluster

If the bubble cluster may consist of N bubbles, these bubbles are concentrated in a roughly spherical region and we assume that all bubbles remain spherical for all time. For any bubble, the bubbles oscillate with approximately the same rhythm. The radial motion equation of a bubble at distance r_0 is [38]:

M is:

$$M = \frac{NR}{a} \left(1 - \frac{r_0^2}{3a^2} \right) \quad (13)$$

P_l can be expressed as:

$$P_l = \left(p_0 + \frac{2\sigma}{R_0} \right) \left(\frac{R_0}{R} \right)^{3\gamma} - \frac{2\sigma}{R} - \frac{4\mu\dot{R}}{R}. \quad (14)$$

All notation used in this work is summarized in table 1.

4. Numerical results

The oscillation of bubbles in a spherical bubble cluster is researched in an acoustic field. The main results are presented in Fig. 1(a) and Fig. 1(b). For illustration, the driving pressure is 0.9×10^5 Pa, the driving frequency is 50 kHz and the equilibrium radius of a bubble is 5 μm in the bubble cluster. Other parameter values used in computation are listed in Table 1. For a driving period, bubbles undergo the process of periodic expansion and contraction. Bubbles enter into a rapid contracted

Table.1

Notation and values used.

| Symbol | Definition |
|--------------|---|
| μ | Dynamic viscosity of the liquid(0.001kg/m·s) |
| σ | Surface tension coefficient (0.0725N/m) |
| ρ | Density of the liquid (1000 kg/m ³) |
| a | Radius of bubble cluster (1 mm) |
| c | Acoustic speed (1450 m/s) |
| p_0 | Ambient pressure (1.01 × 10 ⁵ Pa) |
| γ | Polytropic index of gas inside the bubble (1.33) |
| r_0 | Position in a bubble cluster |
| N | Bubble number |
| $P(\infty)$ | Pressure at far field |
| $P_A(t)$ | Driving pressure of an acoustic field |
| V | Volume of a bubble |
| p_{g0} | The gas pressure in a bubble in an initial state |
| V_0 | Volume of a bubble in an initial state |
| R_0 | Equilibrium radius of a bubble |
| R | The radius of a bubble |
| P | Pressure of fluid |
| p_{sc} | Scattered pressure |
| T | Driving period of sound |
| I_A | Intensity of incident wave |
| I | Scattered acoustic intensity |
| \bar{W} | Scattered acoustic power |
| \mathbf{n} | Unit normal vector of the interface of air and liquid |
| \mathbf{I} | Unit tensor |
| κ_m | Mean curvature |

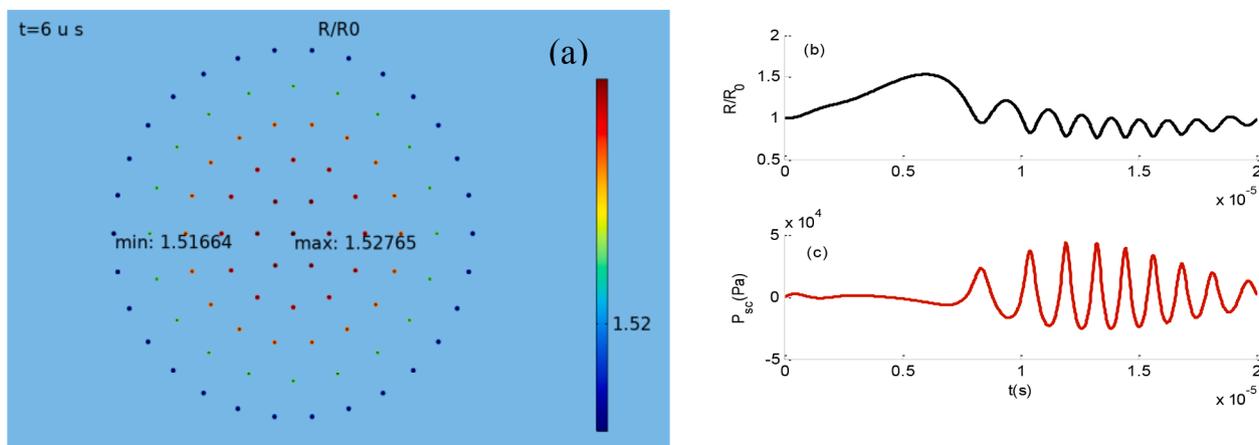


Fig. 1. The oscillation of bubbles with equilibrium radius of $5 \mu\text{m}$ (a) The max radii of a spherical bubble cluster (b) The oscillation of a bubble in bubble cluster with time (c) The scattering pressure of a bubble with time ($P_A = 0.9 \times 10^5 \text{ Pa}$, $f = 50 \text{ kHz}$ and $N = 87$).

oscillation after expanding to the maximum radii. The oscillation phases of bubbles at different positions in the radial bubble cluster have a certain delay, but the radii of bubbles differ little. In the Fig. 1 (a), the radii of bubbles of different positions in a spherical bubble cluster are shown at $6 \mu\text{s}$. The results show that when the bubbles expand to the maximum, bubbles near the center of the spherical bubble cluster have the maximum oscillation radii, which were 1.527 times of the equilibrium radius. The radii of bubbles which are farther away from the center of the spherical bubble cluster were 1.516 times of the equilibrium radius. The difference of radii of bubbles in different position was not significant.

The radius of a bubble in a spherical bubble cluster with the time in the first driving period is shown in Fig. 1 (b). Fig. 1 (c) is the scattered pressure of a bubble in the spherical bubble cluster with time in the first driving period. The results show that the scattered pressure of a bubble is related to the oscillation of the bubble. It is seen that a bubble in a spherical bubble cluster expands to the maximum radius at $6 \mu\text{s}$ and contracts to the minimum radius at $8.4 \mu\text{s}$. Then the bubble expands and contracts around the equilibrium radius, and the bubble contracts to the minimum radius at $12 \mu\text{s}$ in the first driving period. The scattered pressure of a bubble also reaches to the maximum at $12 \mu\text{s}$, which is shown in Fig. 1(c). The above results show that when the bubble expands to the maximum radius, the acceleration of the bubble reaches the minimum, while the radial velocity of the bubble is small. In this case, the surrounding liquid is affected at least by the bubble oscillation, so the scattered acoustic pressure of the bubble is smaller. When the bubble shrinks to the minimum radius, the acceleration of the bubble reaches the maximal value, and the radial oscillation velocity of the bubble also reaches a maximal value. Therefore, the scattered pressure of the bubble reaches the maximum.

The scattered acoustic field of a spherical bubble cluster is shown in Fig. 2. Fig. 2 (a), (b) and (c) show the total acoustic pressure, acoustic pressure level and absolute pressure generated in the surrounding space respectively when the bubbles shrink to the minimum radii for the first time in a driving period ($t = 8.4 \mu\text{s}$). Fig. 2 (d), (e) and (f) are the total acoustic pressure, acoustic pressure level and absolute pressure diagram respectively when the bubbles expand to the maximum radii for the second time ($t = 9.4 \mu\text{s}$). Fig. 2 (g), (h) and (j) are the total acoustic pressure, acoustic pressure level and absolute pressure diagram respectively when the bubbles contract to the minimum radii ($t = 12 \mu\text{s}$) in the first driving period.

The numerical simulation results show that the scattered acoustic field of bubbles is different at different position in the space. The scattered acoustic field at the position of the bubble cluster is the strongest. The scattered acoustic pressure reaches the maximum when bubbles shrink to the minimum radius, and reaches the minimum when the

bubbles expand to the maximum radii which are shown in Fig. 2 (c), (f) and (l). In the external space of bubble cluster, the scattered acoustic field at different positions is also different. The distribution of scatter pressure is complex, which depends on the oscillation of bubbles. The distribution of total acoustic pressure outside bubbles decreases with bubble cluster spherically when bubbles expand. The total acoustic pressures outside of bubble cluster increase with the constriction of bubbles. When the bubbles shrink to the minimum radii in the first driving period, the distribution of total acoustic pressure outside bubbles increases to the maximum value.

Comparing to the acoustic pressure level diagrams Fig. 2 (b), (e) and (h), a larger area of scattered shadows formed on the back side of the bubble cluster when the bubbles expanded and contracted. When the bubbles contracted to the minimum radii for the first time in a period, the areas of scattered shadows on the back side of the bubble cluster were small. When the bubbles next expanded to the maximum radii and contracted to the minimum radii, the areas of scattered shadows decreased and then increased. The results show that the scattered acoustic field of bubble cluster is closely related to the oscillation state of bubbles.

In addition, the factors of driving acoustic pressure, the driving frequency, the equilibrium radius of bubbles in bubble cluster, and the size of bubble cluster could affect the scattering of bubbles. Figures are omitted due to the limited space.

The amplitude of driving pressure is an important factor for the scattered acoustic field of a spherical bubble cluster. For illustration, let us consider section of Fig. 3, $R_0 = 5 \mu\text{m}$. Fig. 3 (a), (b) and (c) show the variation curve of the maximum scattered acoustic pressure with the driving frequency of a single bubble in the spherical bubble cluster when the driving acoustic pressure amplitude is $0.9 \times 10^5 \text{ Pa}$, $1.1 \times 10^5 \text{ Pa}$ and $1.3 \times 10^5 \text{ Pa}$ respectively. The numerical simulation results show that the maximum scattered acoustic pressure of bubbles P_{max} varies with the driving frequency, which is different under different driving pressure amplitudes. When the driving acoustic pressure amplitude P_A is lower, P_{max} increases with the increasing of driving frequency, which is shown in Fig. 3 (a) and (b). When the pressure P_A increases from $1.1 \times 10^5 \text{ Pa}$ to $1.3 \times 10^5 \text{ Pa}$, the maximum scattered pressure of a bubble is bigger at low frequencies. With the increasing of driving frequency, P_{max} decreases, which is shown in Fig. 3(c). The results show that bubble cluster is more likely to generate a stronger scattered acoustic field in a low frequency and strong acoustic field.

The equilibrium radius of a bubble in the bubble cluster also has a certain influence on the scattered pressure of bubbles. The variation curves of the maximum scattered acoustic pressure of a bubble P_{max} with the driving frequency are shown in Fig. 4 (a), (b) and (c) when the equilibrium radius of a bubble is $2 \mu\text{m}$, $5 \mu\text{m}$ and $10 \mu\text{m}$, respectively.

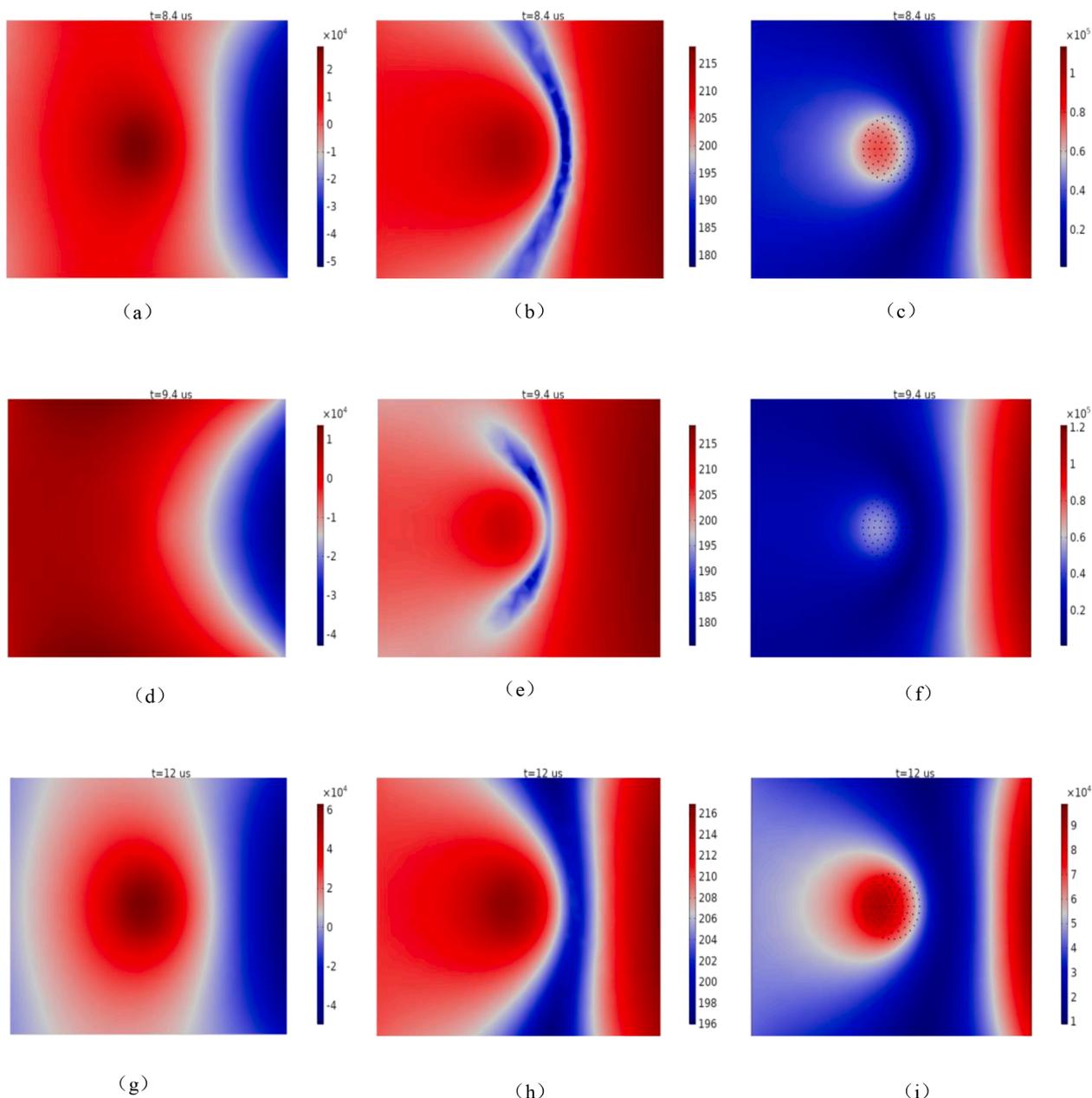


Fig.2. The scattered acoustic field of a spherical bubble cluster with time ($P_A = 0.9 \times 10^5$ Pa, $f = 50$ kHz, $N = 87$ and $R_0 = 5$ μm).

The numerical simulation results show that the pressure P_{\max} with a small equilibrium radius is smaller in an acoustic field. On the contrary, a bubble with a large equilibrium radius in a spherical bubble cluster will generate a large scattered acoustic pressure in a lower frequency and strong acoustic field. For bubbles with the same equilibrium radii, the scattered acoustic field is stronger at low frequencies, and a weaker scattered acoustic field will be generated at a high frequency acoustic field.

The bubble number N in the bubble cluster will also have a certain influence on the scattered acoustic field of a spherical bubble cluster. The curves in Fig. 5 (a), (b) and (c) are the maximum scattered pressures of a bubble with frequencies when the bubble number is 87, 300 and 500 respectively in a certain driving acoustic field. When the bubble number N is small, the scattered acoustic pressure is large. With the increase of bubble number from 87 to 300 and 500, the scattered pressure decrease, which are shown in Fig. 5 (a), (b) and (c). Because there is an interaction between bubbles in the bubble cluster, and the interaction between bubbles is relatively complex, which depends on the bubble distribution,

the location of bubbles, and the distance between two bubbles. On the whole, when the radius of the spherical bubble cluster is a certain value and the bubble number N is small, the distance between bubbles is large and the interaction between bubbles is small. When the bubble number N increases, the distance between bubbles decreases, and the interaction between bubbles also increases. The above factor can restrain the contraction and expansion of bubbles. Therefore, the scattered pressure of bubbles with big bubble number is smaller than that of small bubble number.

The scattered cross section is usually regarded as an important characteristic parameter to measure the scattering effect of bubbles. The variation curve of the scattered cross section of a bubble S_s with the driving frequencies in an acoustic driving period are shown in Fig. 6 (a), (b) and (c) when the bubble equilibrium radius is 2 μm , 5 μm and 10 μm , respectively. The results show that the equilibrium radius of a bubble in bubble cluster has a certain effect on the cross section of bubbles. The S_s is larger when the bubble is driven at low frequencies in a strong acoustic field, and S_s decreases with the increasing of the driving

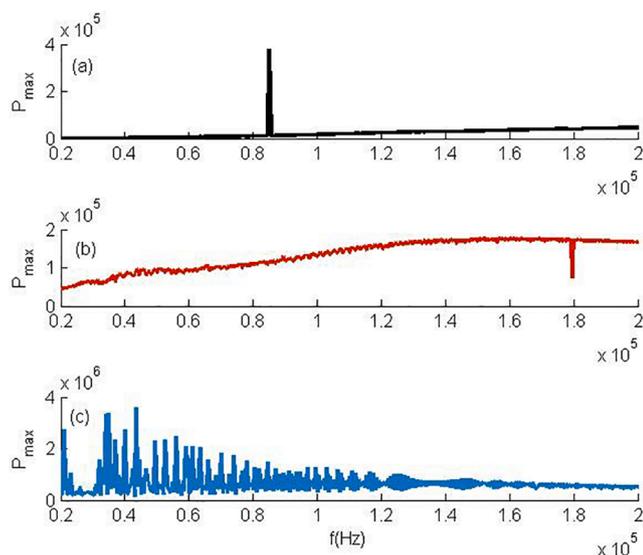


Fig. 3. The maximum scattered pressure with driving frequency under different driving pressure amplitudes (a) $P_A = 0.9 \times 10^5$ Pa (b) $P_A = 1.1 \times 10^5$ Pa (c) $P_A = 1.3 \times 10^5$ Pa.

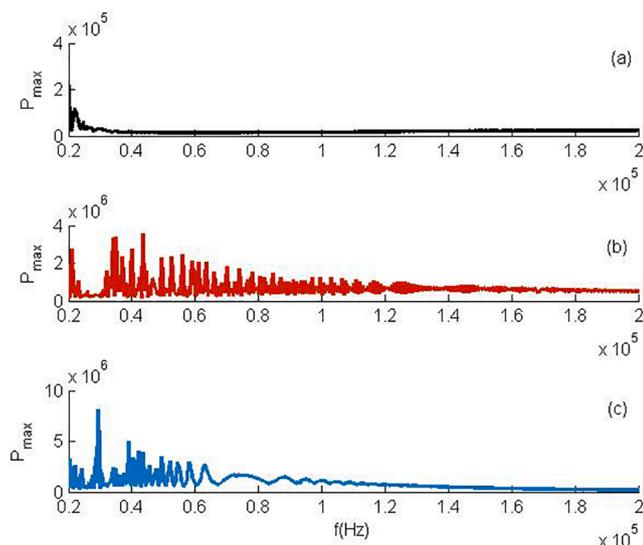


Fig. 4. The maximum scattered pressure of different equilibrium radii of bubbles with the driving frequency (a) $R_0 = 2 \mu\text{m}$ (b) $R_0 = 5 \mu\text{m}$ (c) $R_0 = 10 \mu\text{m}$ ($P_A = 1.3 \times 10^5$ and $N = 87$).

frequency. It can be seen from Fig. 6 (a), (b) and (c), the smaller the bubble equilibrium radius is, the smaller the bubble scattered cross section is under the same driving conditions. The reason is that the bubble resonance frequency is related to the bubble equilibrium radius, the smaller the bubble equilibrium radius is, the larger the bubble resonance frequency is. The resonance frequency of a bubble with a small equilibrium radius is much larger than the driving frequency in a low frequency acoustic field. The scattered acoustic power is only a small part of the incident acoustic power. With the increasing of the equilibrium radius of the bubble, the scattered power also increases. Therefore, when the driving frequency is far lower than the resonance frequency of the bubble, the bubble cluster which is composed of large bubbles has a larger scattering cross section and a better scattering effect.

The bubble number is an important factor which affects the scattered cross section of bubbles. The curves in Fig. 7 (a), (b) and (c) are the

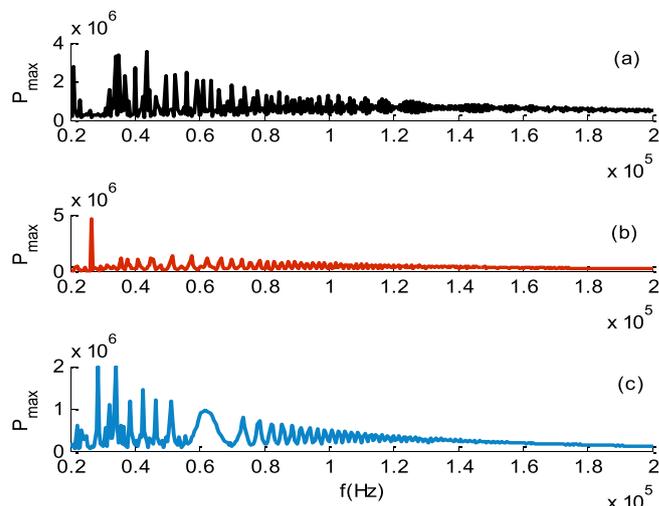


Fig. 5. The maximum scattered pressure of different bubble numbers with the driving frequency (a) $N = 87$ (b) $N = 300$ (c) $N = 500$ ($P_A = 1.3 \times 10^5$ Pa and $R_0 = 5 \mu\text{m}$).

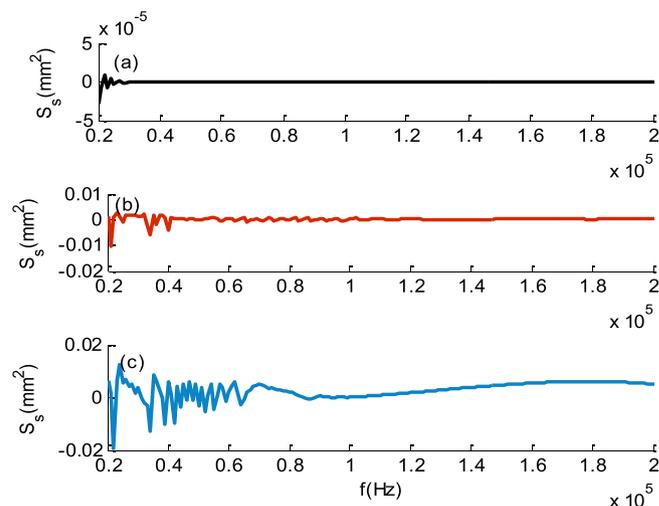


Fig. 6. The scattered cross section of different equilibrium radii of bubbles with the driving frequency (a) $R_0 = 2 \mu\text{m}$ (b) $R_0 = 5 \mu\text{m}$ (c) $R_0 = 10 \mu\text{m}$ ($P_A = 1.3 \times 10^5$ Pa and $N = 87$).

scattered cross section of a bubble in bubble cluster with frequencies, when the bubble number N is 87, 300 and 500 respectively. The numerical simulation results show that the bubble number has a great influence on the scattered cross section of a single bubble. When the radius of the spherical bubble cluster is a certain value, the fewer bubbles in the spherical bubble cluster, the larger the bubble cross section is. On the contrary, the more bubbles in the spherical bubble cluster, the smaller the bubble cross section is. Because the bubble number is smaller, the distance between bubbles is greater in a certain driving acoustic field. The interaction between bubbles is related to the distance between bubbles. The larger the distance between bubbles, the smaller the interaction between bubbles is. With the increasing of N in the spherical bubble cluster, the distance between bubbles decrease and the interaction between bubbles increases. The influence of other bubbles on the scattered acoustic power of a bubble increases, which caused the scattered acoustic power and the scattered cross section of a bubble to decrease.

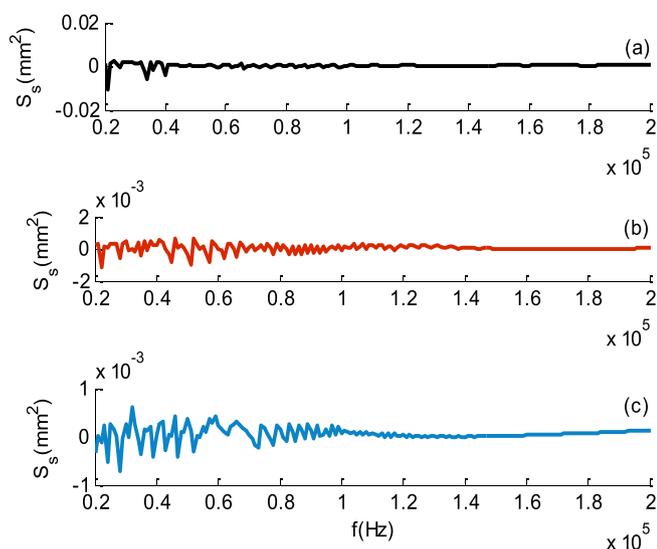


Fig. 7. The scattered cross section of different bubble numbers with the driving frequency (a) $N = 87$ (b) $N = 300$ (c) $N = 500$ ($P_A = 1.3 \times 10^5$ Pa and $R_0 = 5 \mu\text{m}$).

5. Conclusion

The existence of bubbles in liquids has a considerable influence on the propagation of acoustic waves. The scattering of acoustic waves by bubbles is one of the most important effects. The nonlinear oscillation and scattered acoustic fields of a spherical bubble cluster are studied in this paper. The numerical simulation results showed that the oscillation phase of the bubbles at different positions of the bubble cluster has a certain delay, but the magnitude of contraction and expansion of the bubbles differed little. The scattered acoustic field of the bubble cluster has an obvious directivity. The scattered acoustic pressures of the bubble cluster are different at different positions inside and outside of the spherical bubble cluster, and the distribution is complex, which depends on the driving acoustic pressure amplitude, driving frequency, the equilibrium radius of bubble and the radius of the spherical bubble cluster. The scattered acoustic field of bubble cluster is related to the oscillation state of bubbles in a certain driving acoustic field. The scattered pressures of bubble cluster reach the maximum when the bubbles shrink to the minimum radius, while reach the minimum when the bubbles expand to the maximum radius. When the amplitude of driving pressure is lower, the scattered pressure of bubbles increased with the increase of driving frequency. When the pressure of driving acoustic field increased, bubbles can generate more scattered acoustic pressure at a low frequency acoustic field. Bubble cluster constituted with small equilibrium radii of bubbles will produce a smaller scattering acoustic field. With increasing of the number of bubbles in the bubble cluster, the scattered pressure of bubbles decreased.

The cross section of a bubble in a spherical bubble cluster is larger at a low frequency and a strong acoustic field, and the cross section of the bubble decreased with the increasing of driving frequency. Under the same driving conditions, the equilibrium radius of a bubble in a spherical bubble cluster is smaller, the smaller the scattered cross section of the bubble is. The fewer bubbles in a spherical bubble cluster, the larger the bubble cross section is.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

The authors wish to acknowledge the Project of Ning Xia Normal University (NXSFZDA2003), Key Research , Development Project of Ning Xia of China (2020BEB04038, 2019BEB04019), Project of Higher Schools of Ning Xia of China (NGY2020078), and General Industrial Project of Ning Xia City (2019GKGY034, 2019GKGY036) for their financial supports.

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