OPEN

Dynamic Control of Collapse in a Vortex Airy Beam

Rui-Pin Chen[1], Khian-Hooi Chew[1] & Sailing He[1,2]

[1]Centre for Optical and Electromagnetic Research, Zhejiang Provincial Key Laboratory for Sensing Technologies, Zhejiang University, China; Joint Research Center of Photonics of the Royal Institute of Technology (Sweden) and Zhejiang University (China), [2]Department of Electromagnetic Engineering, School of Electrical Engineering, Royal Institute of Technology, S-100 44 Stockholm, Sweden.

Here we study systematically the self-focusing dynamics and collapse of vortex Airy optical beams in a Kerr medium. The collapse is suppressed compared to a non-vortex Airy beam in a Kerr medium due to the existence of vortex fields. The locations of collapse depend sensitively on the initial power, vortex order, and modulation parameters. The collapse may occur in a position where the initial field is nearly zero, while no collapse appears in the region where the initial field is mainly distributed. Compared with a non-vortex Airy beam, the collapse of a vortex Airy beam can occur at a position away from the area of the initial field distribution. Our study shows the possibility of controlling and manipulating the collapse, especially the precise position of collapse, by purposely choosing appropriate initial power, vortex order or modulation parameters of a vortex Airy beam.

N onlinear wave collapse has been investigated in many areas of physics, including optics, fluidics, plasma physics, and Bose-Einstein condensates[1]. In nonlinear optics, collapses of beams with various spatial distributions have been studied[1–4]. One of the challenges facing this field of interest is the possibility of controlling and manipulating the collapse dynamics. Recently, the Airy beam has attracted considerable attention after its experimental generation by Siviloglou et al.[5] due to its intriguing properties and potential applications such as weak-diffraction, transverse acceleration[6,7], self-healing[8], and sorting microscopic particles[9]. The evolutionary characteristics of an Airy beam in a nonlinear medium have been studied[10–17], such as plasma channel generation[13], laser filamentation[14,17], supercontinuum and solitary wave generation[15,16]. Vortices have been the subject of many studies and appear in many branches of physics[18], such as recent report of suppression of collapse for spiraling elliptic solitons[19]. A vortex Airy beam is formed by superposition of an Airy beam and a vortex optical field. The interesting propagation dynamics and non-classicality of a vortex Airy beam have been recently reported[20–22].

The spatial collapse dynamics of a vortex Airy beam are investigated in this work. The novel properties of the collapse dynamics of a vortex Airy beam in a Kerr medium shed light on how to control and manipulate the optical collapse in any spatial position. The coupling between the vortex and Airy beam strongly affects the nonlinear dynamic properties of the vortex and Airy beam in the Kerr medium. The diffraction against self-focusing of a vortex Airy beam is effectively enhanced compared to a non-vortex Airy beam, which results in the increase of the collapse power of the vortex Airy beam. In addition, the vortex of the beam always tends to suppress the collapse in a Kerr medium. For a non-vortex Airy beam in a Kerr medium, partial collapse can occur near the beam's center[12]. However, the collapse of the vortex Airy beam never occurs at the beam's center. This is because the center of the vortex is located at the center of the beam. Our findings reveal that the position of the partial collapse and the propagation distance for the appearance of partial collapse are dependent on the initial powers, vortex orders, and modulation parameters. The partial collapse may occur in the main lobe, side lobes, or outermost lobes, depending on the strength of initial powers, the order of vortex and the modulation parameters. The collapse can even occur in the position where the initial field is almost zero while no collapse appears in the position where the field originally exists with an appropriate power. We further show that the initial power, vortex order and modulation parameters can be exploited to control and manipulate the position of collapse in a vortex Airy beam. If the initial power is ultrahigh, the partial collapse occurs separately at the side lobes. Since the field distribution of a vortex Airy beam is modulated by some exponential factors, we also study the effect of these exponential factors on the nonlinear evolution of the beam in a Kerr medium. Finally, the evolution and collapse of a quasi-one-dimensional vortex Airy beam in a Kerr medium are analyzed as a limiting case. The comparison of the nonlinear dynamics between a vortex Airy beam and an Airy beam is given.

Results

The collapse power, evolution and partial collapse of vortex Airy beams in Kerr media. We first give the critical power of vortex Airy beams in a Kerr medium by the method of moments[23,24], which provides a rigorous and convenient way to obtain the evolution of the relevant quantities (without any assumption of the solution) and obtain some important information about the Kerr effect on the vortex Airy beam (see Methods below). Figure 1 shows the denary logarithmic vertical scale of the ratio of the critical power of a vortex Airy beam (with $m = 1, 2$) to the critical power $P_{cr}^G = \pi c \varepsilon_0 n_0^2 / (n_2 k^2)$ of a Gaussian beam, where c is the speed of light in a vacuum, ε_0 is the permittivity of free space, n_0 is the linear refraction index of the medium and n_2 is the third-order nonlinear coefficient. In Fig. 1 one sees that the value of the critical power of the vortex Airy beam depends mainly on the beam profile of the transverse distribution (besides the nonlinear parameters of the medium). The critical power of the vortex Airy beam increases as the order of vortex modes increases, but decreases as the modulation
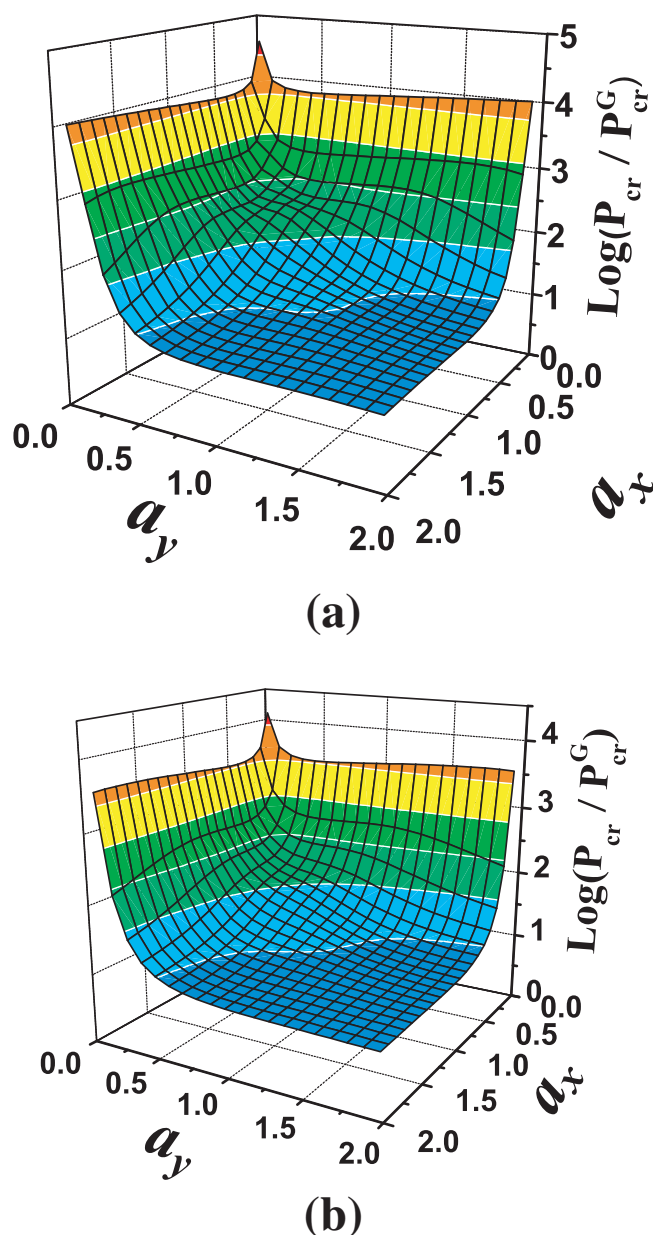
parameters a_x and a_y increase. If the initial power exceeds the critical power P_{cr}, the rms beam width goes to zero at a finite propagation distance, as predicted by the method of moments. Obviously, the critical power is the upper bound for the collapse power[25,26].

Consider the evolution and collapse of a vortex Airy beam in a Kerr medium. Let us take $\lambda = 0.53$ μm, $x_0 = 100$ μm and $z_0 = kx_0^2/2 = 6$ cm. The results are obtained numerically by using a finite-difference method that combines the split-step local one-dimensional (LOD) approximation and the Crank–Nicolson method (see Methods below). The intensity distributions of the 1st and 2nd vortex Airy beams with $P_{in} = 5P_{cr}^G = 0.0023P_{cr}$ and $P_{in} = 15P_{cr}^G = 0.0024P_{cr}$ in the focusing nonlinear medium at different propagation distances are shown in Fig. 2, where hereafter the intensity distribution is normalized with respect to its initial peak intensity. One sees that the beams with different initial powers in the Kerr medium propagate along the same accelerating curve trajectory as an Airy beam in free space, as expected. The collapse will not occur during the propagation for these cases, but the evolution of intensity distribution of the beam, however, depends sensitively on the order of vortex and the initial power. In particular, the rms beam width increases as the propagation distance increases. The beam is stretched anisotropically as it propagates; however, no collapse occurs. The lateral beam-shift (resulting from the Airy function), diffraction, vortex and self-focusing lead to a redistribution of the intensity in the vortex Airy beam.

As the initial power increases, more energy accumulates at various locations of the beam due to the self-focusing effect, leading to partial collapse of the beam. If the initial power is $P_{in} = 15P_{cr}^G = 0.0069P_{cr}$, the 1st order vortex Airy beam collapses at the outermost main lobe of the beam, as illustrated in Fig. 3 (upper). No collapse of vortex Airy beam with the order of $m = 2$ is expected, if $P_{in} = 15P_{cr}^G = 0.0024P_{cr}$ (see Fig. 2 (lower)). The 2nd order vortex Airy beam, however, is expected to collapse at $P_{in} = 50P_{cr}^G = 0.0079P_{cr}$, as shown in Fig. 3(c) and 3(d). In this case, the partial collapse will occur separately in two lobes in the beam (however, not in the outermost lobes). Numerical simulations indicate that the actual collapse power for the 1st order vortex Airy beam with $a_x = a_y = 0.1$ is $P_{in} = 14P_{cr}^G = 0.0063P_{cr}$ and the actual collapse for the 2nd order vortex Airy beam with $a_x = a_y = 0.1$ is $P_{in} = 37P_{cr}^G = 0.0058P_{cr}$. It is interesting to see that the 1st order vortex Airy beam collapses at the outermost lobes, if $P_{in} = 50P_{cr}^G = 0.023P_{cr}$ (see Fig. 4(a) and (b)). A stronger initial power increases the number of lobes that collapse at the outermost region, and reduces the propagation distances of the beam (e.g. Fig. 4(c) and 4(d) with $P_{in} = 2500P_{cr}^G = 1.15P_{cr}$). In Fig. 5, we show the normalized peak intensities as a function of the propagation distance with different initial powers. Although the approach of moments predicts that the rms beam width will be broadened when $P_{in} < P_{cr}$, the results from the variation of peak intensities suggest a deformation and redistribution of the vortex Airy beams during the propagation. The intensity at some lobes will dominate and eventually collapse while the rms beam width increases or remains constant, such as for the cases $P_{in} = 15P_{cr}^G = 0.0069P_{cr}$ and $50P_{cr}^G$ (i.e., $0.023P_{cr}$) for the 1st order vortex Airy beam and $P_{in} = 50P_{cr}^G = 0.0079P_{cr}$ for the 2nd order vortex Airy beam. This indicates that when the initial power reaches a certain threshold but is still less than the critical power, the self-focusing effect of a certain part of the vortex Airy beam will dominate, which results in a compression and eventually gives rise to a partial collapse in the corresponding position, though the total rms beam width is still broadening as predicted by the method of moments. The numerical results indicate that a higher initial power leads to a shorter propagation distance before collapse. When the initial power begins to exceed the collapse power, the flow of transverse energy of the Airy beam and vortex lead to a collapse occurring at the outermost main lobe of the beam. It is interesting to see that the energy accumulates to the position where the initial field is almost

(a)

(b)

Figure 1 | The critical powers of vortex Airy beams for different a_x and a_y. (a) m = 1; (b) m = 2.
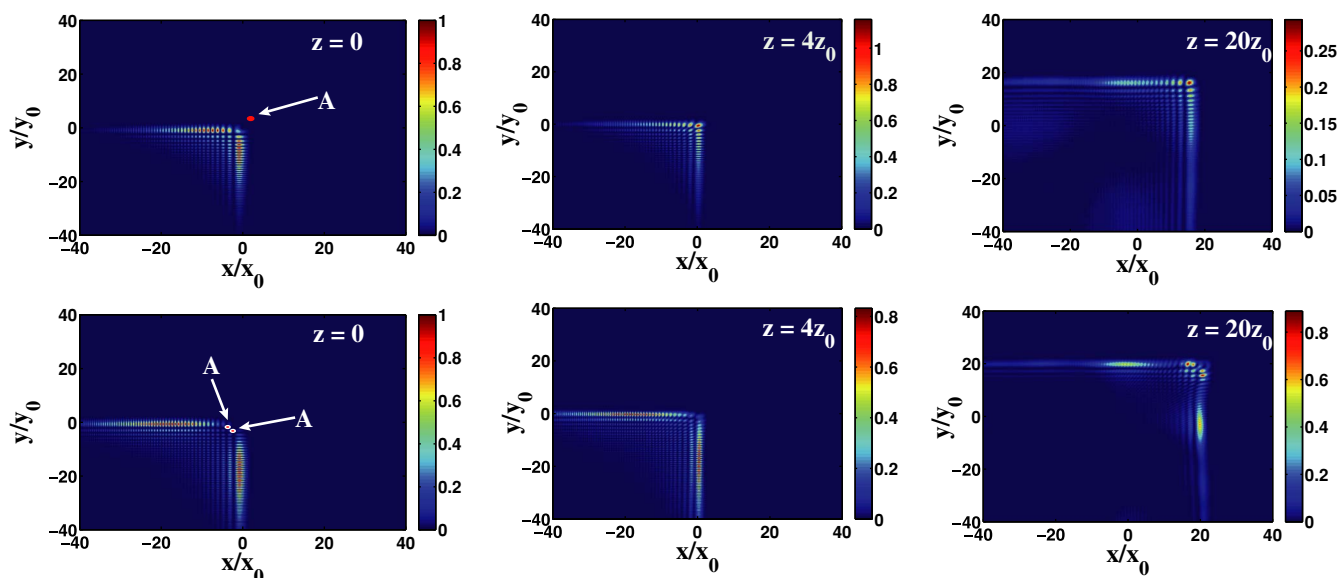
Figure 2 | The intensity distribution of vortex Airy beam ($a_x = a_y = 0.1$) at different propagation distances with initial powers in focusing Kerr medium. Upper row: m = 1, $P_{in} = 5P_{cr}^G = 0.0023P_{cr}$; and lower row: m = 2, $P_{in} = 15P_{cr}^G = 0.0024P_{cr}$.

zero, causing a collapse to occur. On the other hand, no collapse occurs at a position where the initial field mainly exists. In the present study, however, an unusual collapse is observed for the present vortex Airy beam. The vortex Airy beam may collapse at a location where the initial field is nearly zero, e.g., the collapse point in Fig. 3(b) (cf. the zero initial field at this point, marked as point A on the first subfigure of Fig. 2). During the propagation, the energy accumulates

at this collapse position. The higher the initial power value, the closer the collapse position to the beam's center and the shorter the propagation distance before the collapse. These results, therefore, provide useful information on how to spatially manipulate a collapse in an experiment. If the initial power increases further to a certain threshold value, the beam collapses at many side lobes simultaneously, but not in the outermost lobes (see e.g. the collapse position
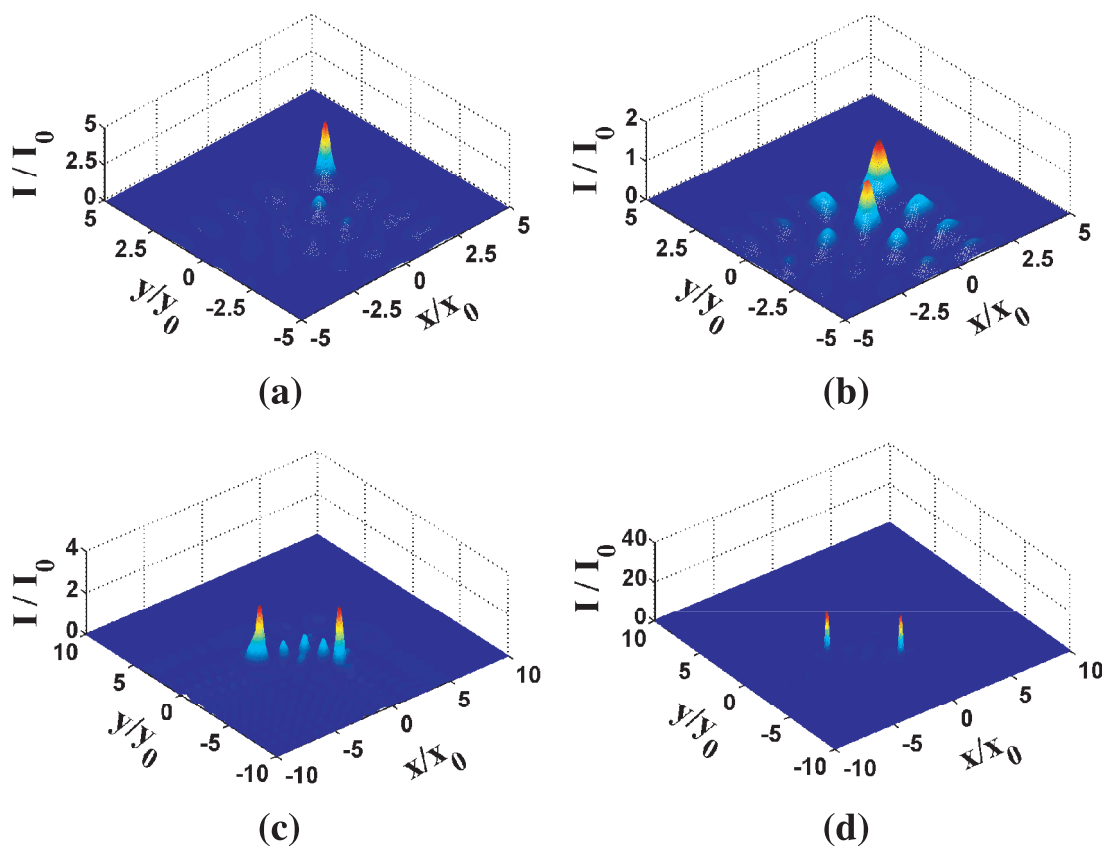

Figure 3 | The intensity distribution of the vortex Airy beam ($a_x = a_y = 0.1$) in focusing medium. (a) m = 1, $P_{in} = 15P_{cr}^G = 0.0069P_{cr}$, z = $5z_0$; (b) m = 1, $P_{in} = 15P_{cr}^G = 0.0069P_{cr}$, z = $6z_0$; (c) m = 2, $P_{in} = 50P_{cr}^G = 0.0079P_{cr}$, z = $7z_0$; (d) m = 2, $P_{in} = 50P_{cr}^G = 0.0079P_{cr}$, z = $7.3z_0$.
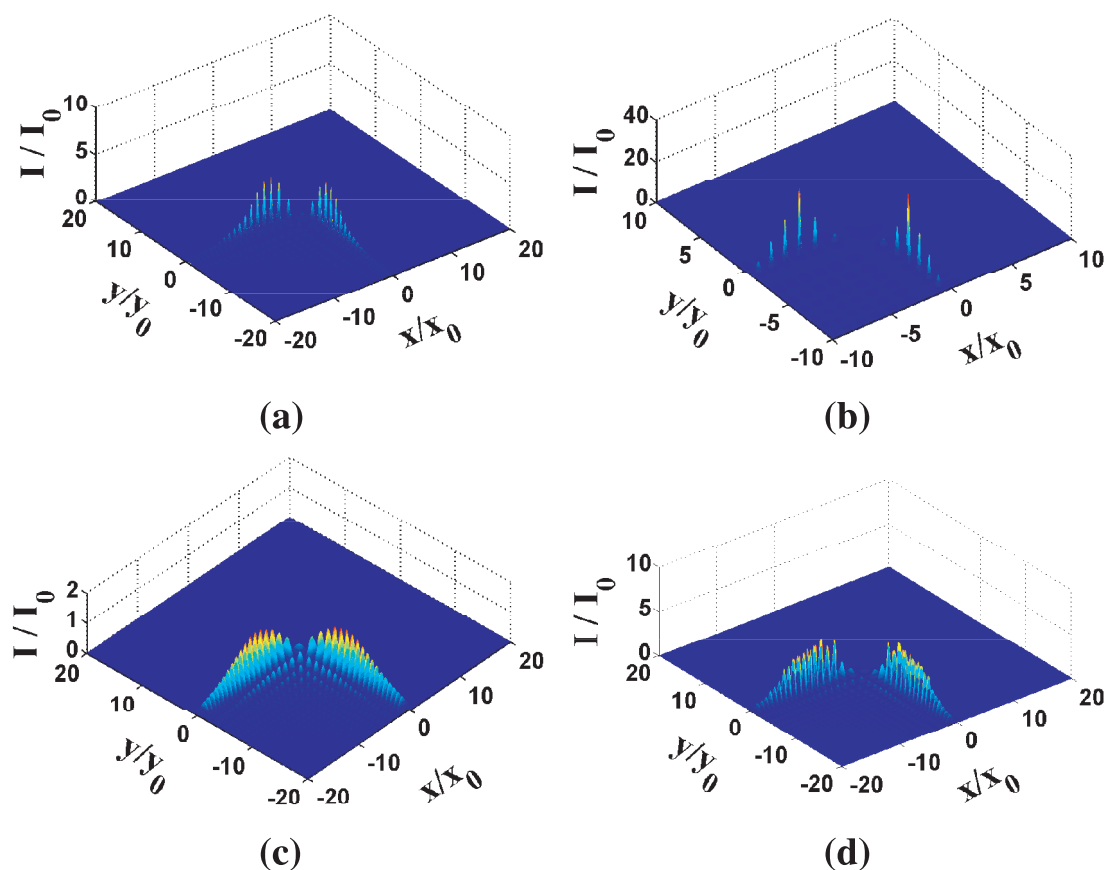
Figure 4 | The intensity distribution of the vortex Airy beam ($a_x = a_y = 0.1$) in a focusing medium. (a) $m = 1$, $P_{in} = 50P_{cr}^G = 0.023P_{cr}$, $z = 0.7z_0$; (b) $m = 1$, $P_{in} = 50P_{cr}^G = 0.023P_{cr}$, $z = 0.8z_0$. (c) $m = 1$, $P_{in} = 2500P_{cr}^G = 1.15P_{cr}$, $z = 0.02z_0$; (d) $m = 1$, $P_{in} = 2500P_{cr}^G = 1.15P_{cr}$, $z = 0.03z_0$.

in Fig. 3(d); cf. the initial field at these points around point A in Fig. 2 with $z = 0$). In this case, we see that the position of collapse, the number of collapse lobes, and the propagation distance (before the collapse) also depend on the initial power. As the initial power reaches a considerably high value, the collapse will appear at many outermost lobes with a very short propagation distance (a higher initial power gives a larger number of collapsed lobes). Even if the approach of moments predicts that the rms beam width will go to be zero at some propagation distance when the initial power exceeds the critical power P_{cr}, the vortex Airy beam collapses separately at the outermost lobes as shown in Fig. 4(c) and 4(d). The presence of critical power P_{cr} indicates that there exists a competition between the self-focusing effect of the beam and the defocusing effect of diffraction. However, we cannot explore the evolution of the beam in each position and the occurrence of the partial collapse during its propagation in the Kerr medium due to the presence of an irregular beam such as a vortex Airy beam. Since all the positions in the beam are considered in the calculation, the results obtained using the finite-difference method can correctly reflect the evolution of the beam, as well as the occurrence of the partial collapse of the beam. In a general case, the partial collapse occurs in a certain position before the total rms beam width becomes constant, or decreases and eventually goes to zero when the initial power reaches and exceeds the critical power. Thus, the critical power P_{cr} predicted by the method of moments is obviously the upper bound for the actual collapse. Only for the Townes profile, the critical power P_{cr} predicted by the method of moments can correctly predict the occurrence of the actual collapse[25–27].

The collapse of quasi-one-dimensional vortex Airy beams. We now investigate the effect of modulation parameters on the

nonlinear evolution of the vortex Airy beam. When $a_x = 1.5$ and $a_y = 0.05$, the beam becomes a quasi-one-dimensional vortex Airy beam, as shown in Fig. 6. Under a low initial power of $P_{in} = 1.5P_{cr}^G = 0.00043P_{cr}$, the beam propagates along the accelerating trajectory with a feature similar to the propagation behavior of an Airy beam in free space, the field distribution is evidently different from that of the beam in free space, and the beam does not collapse as shown in Fig. 6(a). By increasing the initial power from $P_{in} = 1.5P_{cr}^G$ to $5.5P_{cr}^G$, the off-center part of the beam partially collapses at a certain propagation distance although the propagation trajectory is the same as that of $P_{in} = 1.5P_{cr}^G$ as shown in the second subfigure of Fig. 6(b) [cf. the zero initial field at point A with the same x and y coordinates in the first subfigure of Fig. 6(b) as the collapse point]. On the other hand, no collapse occurs at the position where the field originally exists, as shown in Fig. 6(b). However, the collapse of the beam occurs at the initial power of $P_{in} = 5.5P_{cr}^G = 0.00157P_{cr}$. This value is more deviated from the critical power predicted by the method of moments than that of a vortex Airy beam. This can be explained by the fact that the total extension of the quasi-one-dimensional vortex Airy beam is larger than that of a vortex Airy beam even if the partial collapse has occurred. However, the critical power predicted by the method of moments just indicates the total balance between the diffraction and the self-focusing effect. The nonlinear dynamics of an Airy beam have been intensively studied recently, which include collapse[12], multi-filaments[17], curve filament[14], curved plasma channel[13], and self-trapped Airy beams[15]. The collapse dynamics of a quasi-one-dimensional Airy beam with the same parameters as that of a quasi-one-dimensional vortex Airy beam are also studied here. Figure 6(c) shows that the energy accumulates at the center of the Airy beam due to the self-focusing effect during the propagation when $P_{in} = 3P_{cr}^G = 0.111P_{cr}$, but no
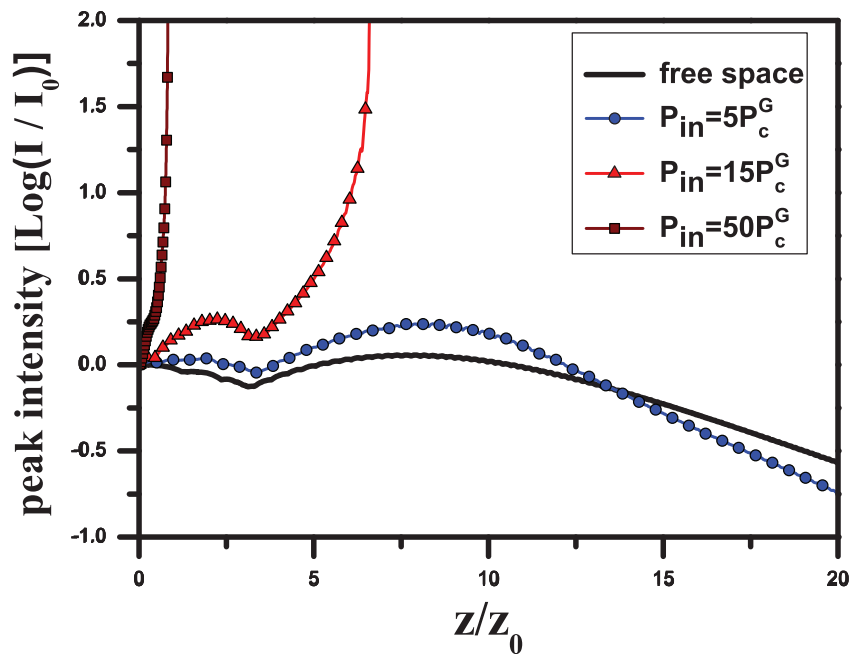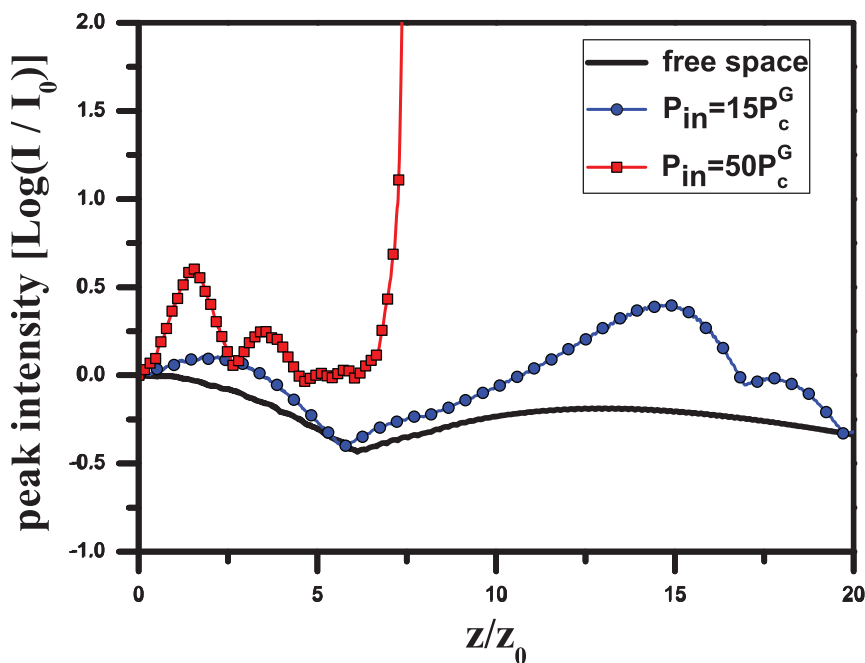
Figure 5 | The peak intensity of vortex Airy beam $a_x = a_y = 0.1$ as a function of the propagation distance with different initial powers (a) m = 1; (b) m = 2.

collapse occurs. When the initial power reaches the collapse power $P_{in} = 4.15P_{cr}^G = 0.153P_{cr}$, the collapse occurs at the position near the center (i.e., the major lobe) of the Airy beam as shown in Fig. 6(d). The phenomena can be extended to the collapse of an Airy beam[12,17]. Figure 6(e) shows the evolution of the peak intensity of a quasi-one-dimensional vortex Airy beam and a quasi-one-dimensional Airy beam during the propagation in a Kerr medium. It shows that a deformation and redistribution of a quasi-one-dimensional vortex

Airy beam and a quasi-one-dimensional Airy beam occur during the propagation. The underlying physics is similar for the collapse behavior of a vortex Airy beam and a non-vortex Airy beam[12]. Some differences can be found by comparing the collapse dynamics between a vortex Airy beam and a non-vortex Airy beam. First, the initial power for the collapse of a vortex Airy beam is larger than that of a non-vortex Airy beam. Second, unlike the collapse of a non-vortex Airy beam, the energy never accumulates at
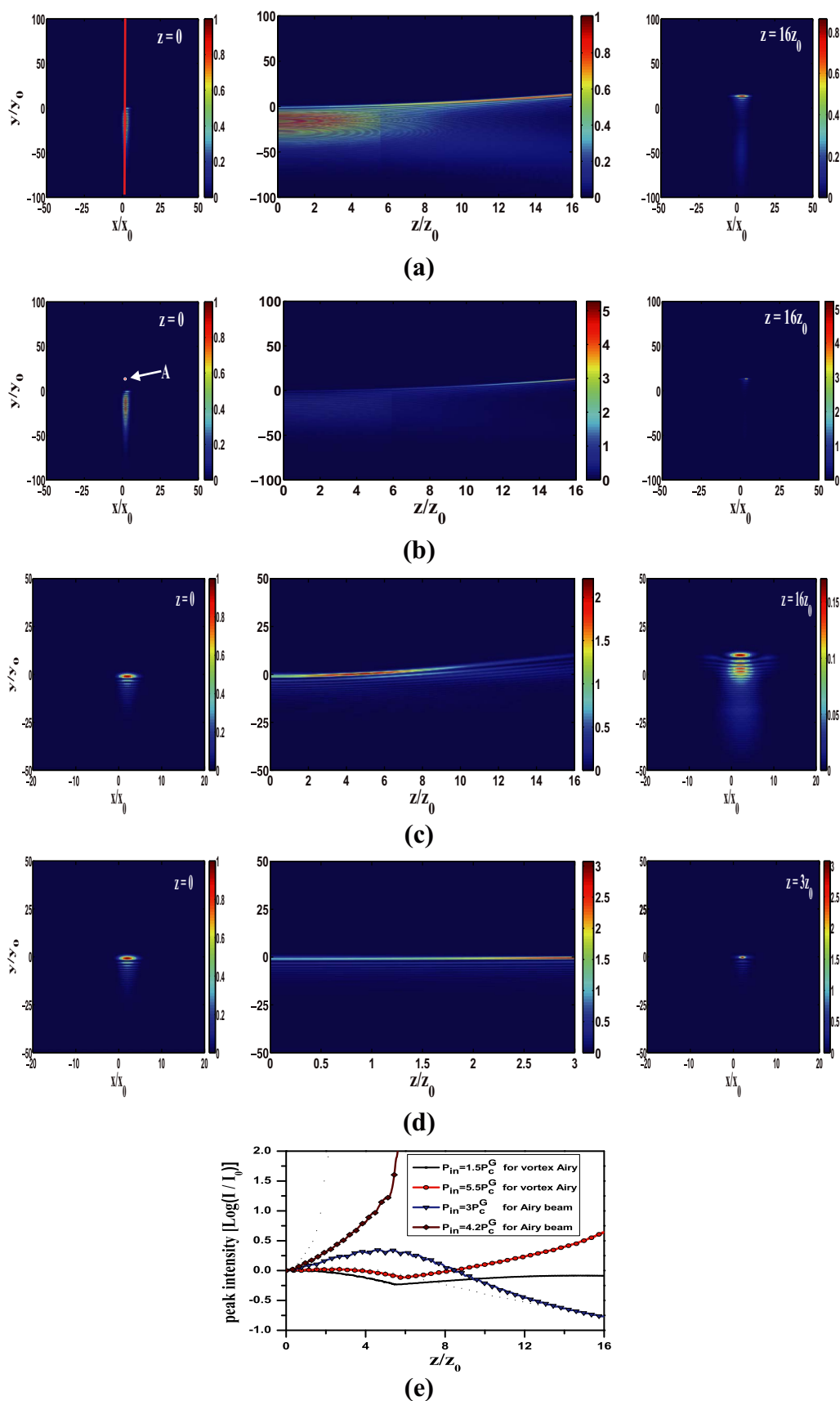
Figure 6 | The intensity distribution of a quasi-one-dimensional vortex Airy beam and a non-vortex Airy beam ($a_x = 1.5$, $a_y = 0.05$) at different propagation distances with different initial powers (a)$P_{in} = 1.5 P_{cr}^G = 0.00043 P_{cr}$ for a vortex Airy beam; (b) $P_{in} = 5.5 P_{cr}^G = 0.00157 P_{cr}$ for a vortex Airy beam; (c) $P_{in} = 3 P_{cr}^G = 0.111 P_{cr}$ for a non-vortex Airy beam; (d) $P_{in} = 4.2 P_{cr}^G = 0.153 P_{cr}$ for a non-vortex Airy beam; The thin red line in the first plot indicates the position of the longitudinal cross-section. (e) The peak intensity of the quasi-one-dimensional vortex Airy beam and non-vortex Airy beam as a function of the propagation distance with different initial powers.
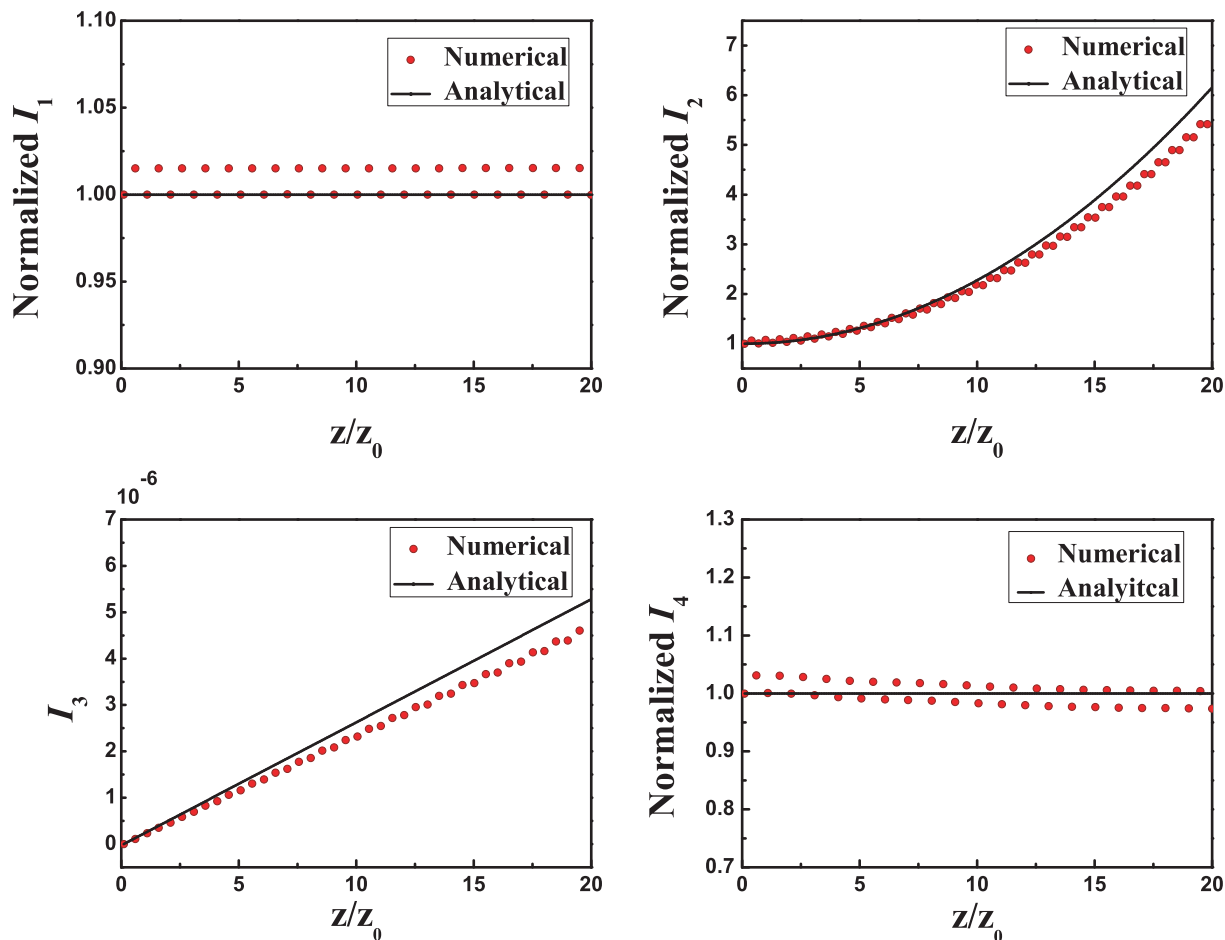
Figure 7 | Comparison of the evolution results for the moments of a vortex Airy beam in a Kerr medium obtained by an analytical method (using Eqs.(2a)–(2d)) and the present numerical approach. The parameters are the same as in Fig. 2 for m = 1 case.

the center of a vortex Airy beam and the collapse never occurs at the center of a vortex Airy beam due to the existence of the vortex. Furthermore, the collapse of a vortex Airy beam can occur at a position away from the area of the initial field distribution whereas the collapse of a non-vortex Airy beam always occurs near the center (i.e., the major lobe) of the Airy beam. This is because that the lateral beam-shift resulting from the Airy function cannot compensate or exceed the self-focusing effect of the Airy beam when the collapse occurs. On the other hand, the lateral beam shift and vortex effect of the vortex Airy beam can push the collapse position away from the initial field area. These results indicate that the vortex Airy beam can provide a more flexible and effective way to control the collapse, especially the position of collapse.

Discussion

Can the nonlinear wave collapse be controlled and managed by using a specially distributed spatial field? Our study indicates that the position of collapse can be manipulated in a vortex Airy beam by purposely choosing appropriate initial powers, vortex orders or modulation parameters of the vortex Airy beam. The interaction of the transverse acceleration of the nondiffraction Airy beam embedded with a vortex with the Kerr self-focusing effect leads to the novel nonlinear dynamic phenomenon.

In summary, the collapse of a vortex Airy beam requires a higher energy than that of a non-vortex beam. Although vortex Airy beams with different initial powers propagate along a similar accelerating curve trajectory in the Kerr medium as an Airy beam propagates in free space, the evolution and appearance of the collapse in a vortex Airy beam exhibits novel features due to the vortex, Airy function,

and modulations parameters with different initial powers. Our study shows that the beam collapse can occur at different locations other than the center, and the locations can be controlled by the initial powers, the order of vortex, or the modulation parameters. Based on the method of moments, the evolution of the total rms beam width and the critical power for balancing between the self-focusing effect and the diffraction have been studied and discussed. The occurrence of the partial collapse of the vortex Airy beam has been numerically analyzed by taking into account the evolution of the whole cross section of the beam in the Kerr medium.

This study provides an efficient and flexible way to manipulate the position of the nonlinear optical wave collapse, through a specially selected spatial field distribution. We found that the collapse can be manipulated to occur in any desired position (even if the initial field is nearly zero), by carefully choosing appropriate initial powers, vortex orders or modulation parameters of the vortex Airy beam. These findings can have implications in the management of the nonlinear wave collapse and optical breakdown.

Methods

Derivation of the critical power and the evolution of rms beam width of a vortex Airy beam. In our analytical studies for the collapse powers and the evolution of the rms beamwidth of a vortex Airy beam, the method of moments[23,24] is performed. The propagation of a light beam in a Kerr medium is described in the paraxial approximation by the following nonlinear Schrödinger (NLS) equation:

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} - 2ik\frac{\partial E}{\partial z} + \frac{2n_2 k^2}{n_0}|E|^2 E = 0, \qquad (1)$$

where k is the linear wave number, x and y are the transverse coordinates, z is the longitudinal coordinate, n_0 is the linear refraction index of the medium, and n_2 is the third order nonlinear coefficient. Due to the complexity of the evolution of a vortex

Airy beam in a Kerr medium, we apply the method of moments to study the nonlinear dynamics by analyzing the evolution of several integral quantities derived from the NLS. These quantities are defined by

$$I_1(z) = \iint_s |E|^2 \, dxdy, \tag{2a}$$

$$I_2(z) = \iint_s (x^2 + y^2)|E|^2 \, dxdy, \tag{2b}$$

$$I_3(z) = \frac{i}{2k} \iint_s \left[x \left(E \frac{\partial E^*}{\partial x} - E^* \frac{\partial E}{\partial x} \right) + y \left(E \frac{\partial E^*}{\partial y} - E^* \frac{\partial E}{\partial y} \right) \right] dxdy, \tag{2c}$$

$$I_4(z) = \frac{1}{2k^2} \iint_s \left(\left| \frac{\partial E}{\partial x} \right|^2 + \left| \frac{\partial E}{\partial y} \right|^2 - \frac{k^2 n_2}{n_0} |E|^4 \right) dxdy. \tag{2d}$$

These quantities are associated with the beam power (I_1), beam width (I_2), momentum (I_3), and Hamiltonian (I_4), and satisfy a closed set of coupled ordinary differential equations[23]: $dI_1(z)/dz = 0$, $dI_2(z)/dz = I_3(z)$, $dI_3(z)/dz = 4I_4(z)$, $dI_4(z)/dz = 0$, and the important invariant under evolution, $Q = 2I_4 I_2 - I_3^2/4$. Therefore, the following Ermakov-Pinney equation describing the dynamics of the scaled beam width can be obtained:

$$\frac{d^2 I_2^{1/2}(z)}{dz^2} = \frac{Q}{I_2^{3/2}(z)}. \tag{3}$$

For a vortex Airy beam, an initial field distribution can be described by[19,20]

$$E(x,y; z=0) = A_0 Ai(x/x_0) \exp(a_x x/x_0) Ai(y/x_0) \exp(a_y y/x_0)(x+iy)^m, \tag{4}$$

where A_0 is the amplitude of the complex amplitude $E(x, y, z = 0)$ and x_0 is an arbitrary transverse scale. Exponential factors a_x and a_y are positive in order to ensure finite energy of the infinite Airy tail in the $-x$ and $-y$ directions, respectively. The azimuthal index m represents the order of vortex. The general solution to Eq. (3) with the vortex Airy beam as an initial field distribution can be given by

$$I_2(z) = I_2(z=0) + \frac{Q}{I_2(z=0)} z^2. \tag{5}$$

Eq. (4) describes the variation of the scaled beam width of the vortex Airy beam in a Kerr medium. When $Q = 0$, the rms beam width remains constant as recognized from Eq. (5), indicating that the self-focusing effect totally compensates for the diffraction, and the critical value, I_1^{cr}, is derived from $2I_4 I_2 - I_3^2/4 = 0$ and Eq. (2). The corresponding power can be found through $P_{cr} = n_0 c \varepsilon_0 I_1^{cr}/2$. The detailed expression of P_{cr} is not presented here due to its lengthy expression. The denary logarithmic vertical scale of the ratio of the critical power of a vortex Airy beam (with $m = 1, 2$) to the critical power of a Gaussian beam is shown in Fig. 1. Compared with some widely used methods such as the variational method, collective coordinate method, and averaged Lagrangian method, the method of moments provides a much more convenient and rigorous way to obtain the evolution of some relevant quantities without any assumption of the solution of the NLS equation. Important information about the nonlinear dynamics of a vortex Airy beam in a Kerr medium can be obtained by the evolution of the corresponding quantities.

Finite-difference method for solving the nonlinear schrödinger (NLS) equations. In this work, the evolution of the vortex Airy beam in a Kerr medium is numerically solved by a finite-difference method that combines the local one-dimensional (LOD) approximation and the Crank–Nicolson method[28–30]. This scheme is unconditionally stable and numerically accurate to the second order as it uses the Crank–Nicolson algorithm[28,30]. Based on the LOD approximation[30], Eq. (1) can be written as the following two half-steps:

$$\frac{\partial E}{\partial z} = \frac{1}{ik} \frac{\partial^2 E}{\partial x^2} + \frac{n_2 k}{in_0} |E|^2 E, \tag{6a}$$

$$\frac{\partial E}{\partial z} = \frac{1}{ik} \frac{\partial^2 E}{\partial y^2} + \frac{n_2 k}{in_0} |E|^2 E, \tag{6b}$$

which can be discretized into the following finite-difference equations using the Crank–Nicolson method[28]:

$$(1+r_x)E_{s,t}^{j+1/2} - c_x E_{s-1,t}^{j+1/2} - c_x E_{s+1,t}^{j+1/2} = (1-r_x)E_{s,t}^j + c_x E_{s-1,t}^j + c_x E_{s+1,t}^j, \tag{7a}$$

$$(1+r_y)E_{s,t}^{j+1} - c_y E_{s-1,t}^{j+1} - c_y E_{s+1,t}^{j+1} = (1-r_y)E_{s,t}^{j+1/2} + c_y E_{s,t-1}^{j+1/2} + c_y E_{s,t+1}^{j+1/2}, \tag{7b}$$

where $1 < s < M_x - 1$, $1 < t < M_y - 1$. Here M_x and M_y denote the maximum step numbers in the x- and y- directions, respectively, with

$$c_x = \frac{\Delta z}{ik\Delta x^2}, \quad c_y = \frac{\Delta z}{ik\Delta y^2}, \tag{8a}$$

$$r_x = 2c_x - \frac{n_2 k}{2in_0} \left| E_{s,t}^j \right|^2 \Delta z, \quad r_y = 2c_y - \frac{n_2 k}{2in_0} \left| E_{s,t}^{j+1/2} \right|^2 \Delta z. \tag{8b}$$

In Eq. (8), Δx, Δy and Δz are the spatial step sizes in the x-, y- and z-directions with $\Delta x = L_x/M_x$, $\Delta y = L_y/M_y$, where L_x, L_y are the maximum calculation ranges in the

x- and y- directions, respectively. Δz is the longitudinal step size (along z-direction). In this work, we take $L_x = L_y = 200x_0$, $M_x = M_y = 1024$, $\Delta z = 0.05z_0$. As the initial power increases, we increase M_x and M_y (the total numbers of steps) to 2048, and reduce accordingly the longitudinal step size Δz. The discrete field at lattice point ($x = s\Delta x, y = t\Delta y, z = j\Delta z$) is represented by $E_{s,t}^j$. If the electric field at z step j (i.e., $E_{s,t}^j$) and boundary conditions are known, we can use Eq. (7a) to get the electric field at z step $j + 1/2$ (i.e., $E_{s,t}^{j+1/2}$) and then use Eq. (7b) to get the electric field at z step $j + 1$ (i.e., $E_{s,t}^{j+1}$). The procedures are repeated until the appearance of beam collapse in the nonlinear medium is obtained. This numerical approach has been used in the analysis of the nonlinear dynamics of an Airy beam[12], a sinh-Gaussian beam[31] and a vortex[32], as well as the analysis of the z-scan technique with an arbitrary beam[33]. Here the evolution of the moments of a vortex Airy beam in a Kerr medium obtained by using this numerical approach is verified by the analytical result obtained from Eqs. (2a)–(2d). The comparison is shown in Fig. 7 (the corresponding parameters are the same as in Fig. 2 for m = 1 case, and the corresponding intensity distributions at different propagation distances have been shown in Fig. 2). We note that one still sees some difference (particularly I_2 and I_3 at a large propagation distance), which is due to the numerical error in differentiations $\partial E/\partial x$, $\partial E/\partial y$ and numerical integration for the moments. However, the difference/error will decrease as the discretization step decreases. We also note that such a difference/error (in the moments), which arises from the numerical differentiation of the field and the numerical integration, does not exist in all the other figures where we are interested in the distribution of the field intensity (instead of the moments).

1. Cross, M. C. & Hohenberg, P. C. Pattern formation outside of equilibrium. *Rev. Mod. Phys.* **65**, 851–1112 (1993).
2. Robinson, P. A. Nonlinear wave collapse and strong turbulence. *Rev. Mod. Phys.* **69**, 507–573 (1997).
3. Bergé, L., Gouédard, C., Schjødt-Eriksen, J. & Ward, H. Filamentation patterns in Kerr media vs. beam shape robustness, nonlinear saturation and polarization states. *Physica (Amsterdam) D* **176**, 181 (2003).
4. Vuong, L. T., Grow, T. D., Ishaaya, A., Gaeta, A. L., Hooft, G. W. & Eliel, E. R. Collapse of optical vortices. *Phys. Rev. Lett.* **96**, 133901 (2006).
5. Siviloglou, G. A., Brokly, J., Dogariu, A. & Christodoulides, D. N. Observation of accelerating Airy beams. *Phys. Rev. Lett.* **99**, 213901 (2007).
6. Kasparian, J. & Wolf, J. P. Laser beams take a curve. *Science* **324**, 194 (2009).
7. Bandres, M. A. & Gutierrez-Vega, J. C. Airy-Gauss beams and their transformation by paraxial optical systems. *Opt. Express* **15**, 16719–16728 (2007).
8. Broky, J., Siviloglou, G. A., Dogariu, A. & Christodoulides, D. N. Self-healing properties of optical Airy beams. *Opt. Express* **16**, 12880–12891 (2008).
9. Baumgartl, J., Mazilu, M. & Dholakia, K. Optically mediated particle clearing using Airy wavepackets. *Nature Photon.* **2**, 675–678 (2008).
10. Ellenbogen, T., Voloch-Bloch, N., Ganany-Padowicz, A. & Arie, A. Nonlinear generation and manipulation of Airy beams. *Nature Photonics* **3**, 395–398 (2009).
11. Jia, S., Lee, J., Fleischer, J. W., Siviloglou, G. A. & Christodoulides, D. N. Diffusion-trapped Airy beams in photorefractive media. *Phys. Rev. Lett.* **104**, 253904 (2010).
12. Chen, R. P., Yin, C. F., Chu, X. X. & Wang, H. Effect of Kerr nonlinearity on an Airy beam. *Phys. Rev. A* **82**, 043832 (2010).
13. Polynkin, P., Kolesik, M., Moloney, J., Siviloglou, G. A. & Christodoulides, D. N. Curved plasma channel generation using ultraintense Airy beams. *Science* **324**, 229 (2009).
14. Polynkin, P., Kolesik, M. & Moloney, J. Filamentation of femtosecond laser Airy beams inwater. *Phys. Rev. Lett.* **103**, 123902 (2009).
15. Kaminer, I., Segev, M. & Christodoulides, D. N. Self-Accelerating self-trapped optical beams. *Phys. Rev. Lett.* **106**, 213903 (2011).
16. Lotti, A., Faccio, D., Couairon, A., Papazoglou, D. G., Panagiotopoulos, P., Abdollahpour, D. & Tzortzakis, S. Stationary nonlinear Airy beams. *Phys. Rev. A* **84**, 021807 (2011).
17. Panagiotopoulos, P., Abdollahpour, D., Lotti, A., Couairon, A., Faccio, D., Papazoglou, D. G. & Tzortzakis, S. Nonlinear propagation dynamics of finite-energy Airy beams. *Phys. Rev. A* **86**, 013842 (2012).
18. Pismen, L. M. *Vortices in Nonlinear Fields* (Clarendon Press, Oxford, 1999).
19. Desyatnikov, A. S., Buccoliero, D., Dennis, M. R. & Kivshar, Y. S. Suppression of collapse for spiraling elliptic solitons. *Phys. Rev. Lett.* **104**, 053902 (2010).
20. Mazilu, M., Baumgartl, J., Cizmar, T. & Dholakia, K. Accelerating vortices in Airy beams. *Proc. SPIE* **7430**, 74300C–1 (2009).
21. Dai, H. T., Liu, Y. J., Luo, D. & Sun, X. W. Propagation dynamics of an optical vortex imposed on an Airy beam. *Opt. Lett.* **35**, 4075–4077 (2010).
22. Chen, R. P. & Ooi, C. H. R. Nonclassicality of vortex Airy beams in the Wigner representation. *Phys. Rev. A* **84**, 043846 (2011).
23. Pérez-García, V. M., Torres, P. & Montesinos, G. D. The method of moments for nonlinear schrödinger equations: theory and applications. *SIAM J. Appl. Math.* **67**, 990–1015 (2007).
24. García–Ripoll, J. J., Pérez–García, V. M. & Torres, P. Extended parametric resonances in nonlinear Schrödinger systems. *Phys. Rev. Lett.* **83**, 1715 (1999).
25. Weinstein, M. Nonlinear Schrödinger equations and sharp interpolation estimates. *Commun. Math. Phys.* **87**, 567–576 (1983).
26. Fibich, G. & Gaeta, A. L. Critical power for self-focusing in bulk media and in hollow waveguides. *Opt. Lett.* **25**, 335–337 (2000).
27. Chiao, R., Garmire, E. & Townes, C. Self-trapping of optical beams. *Phys. Rev. Lett.* **13**, 479–482 (1964).

28. Press, W. H., Teukolsky, S. A., Vetterling, W. T. & Flannery, B. P. *Numerical Recipes: The Art of Scientific Computing,* 3rd ed. (Cambridge University Press, New York, 2007), Chap. 20.
29. Zang, W. P., Tian, J. G., Liu, Z. B., Zhou, W. Y., Zhang, C. P. & Zhang, G. Y. Comparison of the solutions from a novel variational method with numerical results for the study of beam propagation in a Kerr medium with nonlinear absorption. *Opt. Lett.* **28**, 722–724 (2003).
30. Lapidus, L. & Pinder, G. F. *Numerical Solution of Partial Differential Equationals in Science and Engineering* (Wiley, New York, 1982), Chap. 4.
31. Chen, R. P., Zheng, H. P. & Chu, X. X. Propagation properties of a sinh-Gaussian beam in a Kerr medium. *Appl. Phys. B* **102**, 695–698 (2011).
32. Chen, R. P. & Wang, H. T. Propagation of Laguerre–Gaussian beams in cubic–quintic nonlinear media by variational approach. *Opt. & Laser Tech.* **42**, 1318–1322 (2010).
33. Zang, W. P., Tian, J. G., Liu, Z. B., Zhou, W. Y., Song, F. & Zhang, C. P. Local one-dimensional approximation for fast simulation of Z-scan measurements with an arbitrary beam. *Appl. Opt.* **43**, 4408–4414 (2004).

Author contributions

R.P.C. initiated the work and performed most of the computations. S.H. supervised the finding of the present work, and was responsible for scientific explanation for the finding and revision. K.H.C. contributed to the discussions and computational methods. S.H and R.P.C. wrote the paper.

Additional information

How to cite this article: Chen, R.P., Chew, K.-H. & He, S.L. Dynamic Control of Collapse in a Vortex Airy Beam. *Sci. Rep.* **3**, 1406; DOI:10.1038/srep01406 (2013).