scientific reports

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Mathematical model of the lumpy skin disease using Caputo fractional-order derivative via invariant point technique

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The aim of this paper is to study the fractional model of Lumpy Skin Disease, aiming to enhance our understanding of this disease. Specifically, we employ the recently introduced Caputo–Fabrizio fractional (CFF) derivative to analyze the Lumpy Skin Disease model in detail. To comprehensively study the model's solutions, we utilize the Picard-Lindelof approach to assess their existence and uniqueness. Furthermore, we employ numerical techniques, specifically the CFF derivative combined with the fundamental theorem of fractional calculus and fixed point theorem, to obtain the solutions of Lumpy Skin Disease in near form using fractional order. This innovative approach offers novel insights into the dynamics of the disease model that were previously unexplored. In addition, numerical simulations are conducted to explore the change in effects of control parameters on specific compartments within the model. The geometric representation of the model provides valuable insights into its complexity and reliability. By simulating each model compartment at various fractional orders and comparing them with integer-order simulations, we highlight the effectiveness of modern derivatives. Overall, our fractional analysis emphasizes the enhanced accuracy of non-integer order derivatives in capturing the dynamics of the Lumpy Skin Disease model. These findings contribute to advancing our understanding of the disease and may have implications for its control and management strategies.

Keywords Caputo–Fabrizio operator, Fractional order lumpy skin disease, Existence and uniqueness, Invariant point theory

Mathematics Subject Classification 26A33, 34A08

Lumpy Skin Disease, characterized by its distinct lesions, is attributed to the Lumpy Skin Disease Virus (LSDV)[1]. This virus, with a genome size of approximately 150 kb, is categorized as a double-stranded DNA virus and is one among the variant of the Capripoxvirus genus and another type of the virus called the Chordopoxviridae sub-family of the Poxviridae family[2]. Additionally, within the same genus are other notable members such as Goat poxvirus (GTPV) and Sheep poxvirus (SPPV). The morphological similarities of *LSDV* compared with the other members of the Poxviridae family like the vaccinia virus are observed through Electron microscopy[3]. Notably, LSDV is an infectious disease of non-zoonotic type and exhibts the display of host specificity, primarily infecting the cattle group namely (Bos indicus, Bos taurus) and tamed animals like water buffaloes (Bubalus bubalis)[4–6]. Moreover, LSDV has been documented to infect various mammalian species in the wild, including camels, giraffes, and wildebeests[7–9].

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Transmission of LSDV occurs through various routs, including direct contact via skin lesions, milk, and through parasitic insects like biting midges, acari, and mosquitoes[10–13]. On overview of the infection rate seems to be higher during warm and humid periods compared to winters, attributed to increased insect activity and mobility during the season of summer[14]. The transmission of the virus usually occurs from infected to healthy animals through the shedding of the virus via saliva, along the skin lesions, through the nasal discharge and lachrymal secretion[11,15]. The emergence of the Lumpy Skin Disease virus was first recorded in India in 2019, subsequently leading to severe outbreaks. The most recent outbreak began in May 2022, affecting nearly all states in the country. Among these, 15 Indian states suffered substantial economic losses, with a staggering death toll nearing 100,000 cattle[16]. Given that livestock production is a crucial means of livelihood in a developing nation like India, the emergence of a lethal disease like Lumpy Skin Disease has directly impacted the economy and hindered livestock production[16].

India harbors a vast cattle breeding with count of the population around 308 million, underscoring the significance to care and control the infectious disease that spreads among the cattle group[17]. Direct losses encompass cattle mortality and reduced milk production, while indirect losses involve constraints on cattle movement across the country[16]. Past research reveals facts about the organs and tissues of the infected animals possesses the pathological change. The changes include major occurrences of the hepatitis, cow mastitis, necrotic lymphadenitis, orchitis, and identifications of of the effect of myocardial damage[18]. The World Health Organization (WHO) has designated lumpy skin disease as a notifiable illness[19]. Initially discovered in Zambia in 1931[20], this disease remained largely confined to Sub-Saharan Africa until 1989, after which it began spreading beyond the region's borders into the Middle East and Asia[21,22]. LSDV was notably identified during the year 2016 in the largest country Russia and several Southeast countries of Europe[22]. In November 2019, the disease made its debut in India along with other Asian nations like Thailand, well developed giant nation China, boarder sharing nations such as Nepal, Bangladesh, and Bhutan[23,24]. While LSD has been present in India since 2019, its impact became notably severe in the year 2022, with more than two million infected cattle.

The symptomatic effect of Lumpy Skin Disease (LSD) differs from one individual to another individual animal and depend on the major cause of the infection. Typically, it takes almost one to four weeks to identify the traits apparently, which may include high fever, sight issues and nasal discharge, loss of appetite, and the development of nodular lesions on the skin[25]. Data suggest a mortality rate ranging from 5 to 45%[26–29]. Some of the states in India like Uttar Pradesh, Punjab, Haryana, Karnataka, West Bengal, Rajasthan, Gujarat and Maharashtra are the states most severely affected by Lumpy Skin Disease (LSD), experiencing high levels of both mortality and morbidity[30]. To combat the spread of LSD, the Indian government has implemented various measures, including mass vaccination campaigns, the establishment of quarantine like isolation of individuals to avoid contact, and block the movement of infected individual and this restriction applied to some susceptible animals. Nevertheless, the disease exists among the affected one need to be controlled remains a challenging task and due to inadequate awareness regarding the transmission of information and various options than can be applied to control, along with the early detection of infected group of animal.

The prevailing belief about the origin of LSDV projects that Lumpy Skin Disease Virus (LSDV) might have generated initially from the previous poxvirus species and got evolved through adaptation to various hosts. The LSDV which is a double-stranded DNA viruses are recognized for utilizing homologous recombination as an evolutionary mechanism, enabling them to broaden their host range and enhance virulence[31]. In this study, genome sequencing was utilized to identify LSDV variants that spreads in India. Phylogenetic analysis revealed two distinct classes of variants present in the country. Furthermore, mutation (SNP) analysis indicated significant differences in the number of mutations between these groups.

To model the Lumpy Skin Disease (LSD) study, the total population $\mathcal{N}(\mathrm{t})$ can be divided into five classes as follows::

1. Affected individuals, denoted by $\mathcal{S}_\mathrm{t}$, called Susceptible
2. Individuals, denoted by $\mathcal{V}_\mathrm{t}$, called as Vaccinated
3. Individuals, denoted by $\mathcal{E}_\mathrm{t}$, called as Exposed
4. Individuals, denoted by $\mathcal{I}_\mathrm{t}$, known as Infected
5. Individuals, denoted by $\mathcal{R}_\mathrm{t}$, known as Recovered

Thus, the collection of the entire population at any time moment is given by the sum of these classes:

$$\mathcal{N}_\mathrm{t} = \mathcal{S}_\mathrm{t} + \mathcal{V}_\mathrm{t} + \mathcal{E}_\mathrm{t} + \mathcal{I}_\mathrm{t} + \mathcal{R}_\mathrm{t}. \tag{1}$$

This model allows us to track the dynamics of LSD within a population over time, considering the transitions between these different classes.

The susceptible class, denoted as $\mathcal{S}_\mathrm{t}$, encompasses cattle vulnerable to the virus, which can lead to illness upon interaction with infected cattle. Cattle that have been vaccinated are called vaccinated group, denoted by $\mathcal{V}_\mathrm{t}$. When susceptible cattle come into contact with infected individuals, they transition to the exposed class, denoted as $\mathcal{E}_\mathrm{t}$. Thus, the exposed class comprises cattle that have been infected but are not yet infectious. Subsequently, the infected class, denoted by $\mathcal{I}_\mathrm{t}$, includes cattle in which the virus has established itself, rendering them infectious. An infectious cattle has the ability to spread the virus to others. Finally, the recovered class, $\mathcal{R}_\mathrm{t}$, comprises cattle with high immunity who recover quickly, possibly with the aid of medication.

We assume that the state variables in time t such as $\mathcal{S}-\text{Susceptible}, \mathcal{V}-\text{Vaccinated}, \mathcal{E}-\text{Exposed}, \mathcal{I}-\text{Infected}$, and $\mathcal{R}-\text{Recovered}$ are differentiable functions continuously , where $\mathrm{t} \in [0, \infty)$. This formulation allows us to model the dynamics of LSD within a cattle population, considering the transitions between the states $\{\mathcal{S}, \mathcal{V}, \mathcal{E}, \mathcal{I}, \mathcal{R}\}$ over time. The flow pattern

for LSD, as depicted in Fig. 1, can be described by a nonlinear equations along with the ordinary differential equations (ODEs) as follows:

$$\begin{aligned}
\frac{d\mathcal{S}}{d\mathfrak{t}} &= \theta - \beta_1 \mathcal{S}\mathcal{I} - (\alpha_1 + \nu)\mathcal{S},\\
\frac{d\mathcal{V}}{d\mathfrak{t}} &= \alpha_1 \mathcal{S} - \alpha_2 \mathcal{V}\mathcal{I} - (\nu + \alpha_3)\mathcal{V},\\
\frac{d\mathcal{E}}{d\mathfrak{t}} &= \beta_1 \mathcal{S}\mathcal{I} + \alpha_2 \mathcal{V}\mathcal{I} - (\beta_2 + \nu)\mathcal{E},\\
\frac{d\mathcal{I}}{d\mathfrak{t}} &= \beta_2 \mathcal{E} - (\beta_3 + \nu + \delta_1)\mathcal{I},\\
\frac{d\mathcal{R}}{d\mathfrak{t}} &= \beta_3 \mathcal{I} + \alpha_3 \mathcal{V} - \nu\mathcal{R},
\end{aligned}$$

along with the following non-negative restrictions.

$$\mathcal{S}(0) = \mathcal{S}_0, \mathcal{V}(0) = \mathcal{V}_0, \mathcal{E}(0) = \mathcal{E}_0, \mathcal{I}(0) = \mathcal{I}_0, \mathcal{R}(0) = \mathcal{R}_0.$$

Fractional modeling concepts one can see[32–63]. Caputo and Fabrizio[64] proposed a new fractional derivative without any singularity in its kernel. Moore, Sirisubtawee and Koonprasert[65] studied Caputo–Fabrizio fractional differential equation model for HIV/AIDS with treatment compartment. Motivated by the above work, we propose and analyze a Caputo–Fabrizio fractional LSD model. The existence and uniqueness of the system of solutions of the model are established using a Banach fixed-point theorem. The power law derivative of the Riemann-Liouville fractional derivative or Caputo–Fabrizio fractional derivative, is associated with noisy information due to its unique memory qualities. The Caputo–Fabrizio fractional derivative produces less noise than the power law, whereas the Atangana-Baleanu fractional derivative provides a detailed explanation. The work carried in this paper is aligned as follows: In the introduction part under Section "Introduction", the objective of the study is outlined. Section "Lumpy Skin Disease with Caputo–Fabrizio": Caputo–Fabrizio Fractional LSD Model: Introduces the newly proposed Caputo–Fabrizio fractional LSD model, which details the various receptacle of the model along with parameters. This section also includes an analysis of the existence and uniqueness of solutions. Section "Ulam–Hyers stability of the Lumpy Skin Disease model": Stability Analysis: Explores the stability of the model to understand its long-term behaviors. Section "Numerical scheme": Numerical Simulations: Presents numerical simulations conducted to validate the theoretical findings and provide insights into the model's behavior under various scenarios. Section "Conclusions": Conclusion: Concludes the study by summarizing the main findings and their implications, as well as suggesting potential avenues for future research. This organizational structure guides the reader through the development, analysis, and validation of the proposed Caputo–Fabrizio fractional LSD model, leading to a comprehensive understanding of its dynamics and implications.

Lumpy skin disease with Caputo–Fabrizio

The lumpy skin disease model with Caputo–Fabrizio($\mathfrak{CF}$) derivative is given by

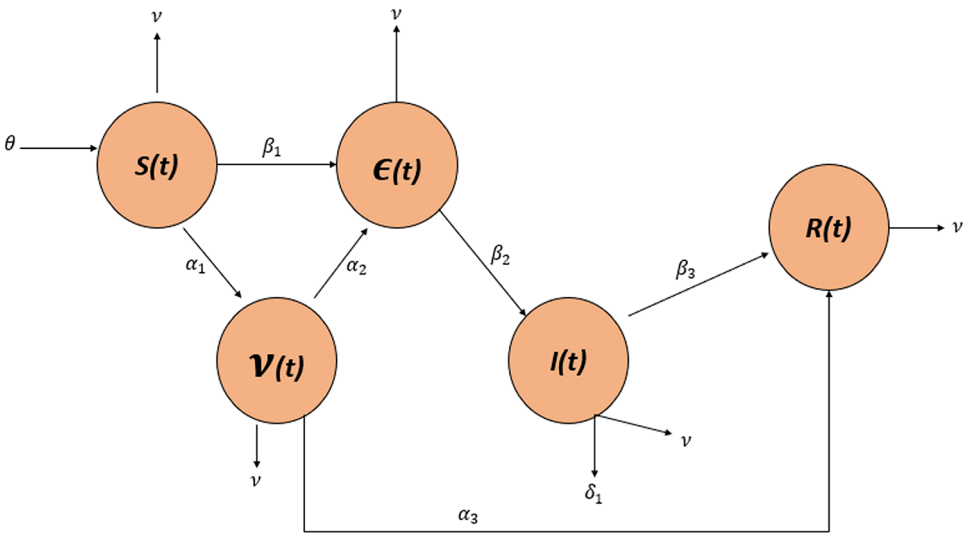

Fig. 1. Flowchart for the LSD.

$$\begin{cases} {}^{\mathfrak{CF}}\mathcal{D}_{\mathfrak{t}}^{\mathrm{v}}\mathcal{S} &= \theta - \beta_1\mathcal{S}\mathcal{I} - (\alpha_1+\nu)\mathcal{S}, \\ {}^{\mathfrak{CF}}\mathcal{D}_{\mathfrak{t}}^{\mathrm{v}}\mathcal{V} &= \alpha_1\mathcal{S} - \alpha_2\mathcal{V}\mathcal{I} - (\nu+\alpha_3)\mathcal{V}, \\ {}^{\mathfrak{CF}}\mathcal{D}_{\mathfrak{t}}^{\mathrm{v}}\mathcal{E} &= \beta_1\mathcal{S}\mathcal{I} + \alpha_2\mathcal{V}\mathcal{I} - (\beta_2+\nu)\mathcal{E}, \\ {}^{\mathfrak{CF}}\mathcal{D}_{\mathfrak{t}}^{\mathrm{v}}\mathcal{I} &= \beta_2\mathcal{E} - (\beta_3+\nu+\delta_1)\mathcal{I}, \\ {}^{\mathfrak{CF}}\mathcal{D}_{\mathfrak{t}}^{\mathrm{v}}\mathcal{R} &= \beta_3\mathcal{I} + \alpha_3\mathcal{V} - \nu\mathcal{R}, \end{cases} \tag{2}$$

with v denote the derivative order in the form of fractional with $0 < \mathrm{v} < 1$ subject to

$$\mathcal{S}_(0) = \mathcal{S}_0, \mathcal{V}(0) = \mathcal{V}_0, \mathcal{E}_(0) = \mathcal{E}_0, \mathcal{I}_(0) = \mathcal{I}_0, \mathcal{R}_(0) = \mathcal{R}_0. \tag{3}$$

Model basic preliminaries

Here are the mathematical preliminaries, presented as theorems, which will be applied to prove the positivity and uniqueness of the lumpy skin disease model with $\mathfrak{CF}$ derivative (2) as defined[64,66] respectively. Assume the function $\phi(\mathfrak{t})$ defined as in the space $\{\phi(\mathfrak{t}) \in \mathcal{C}([0,1] \to \mathbb{R})\}$ with $||\phi|| = \max_{[0,1]} |\phi(\mathfrak{t})|$.

Definition 2.1 For $\mathfrak{l}_2 > \mathfrak{l}_1$, assume that the function $\phi(\mathfrak{t}) \in \mathcal{H}^1(\mathfrak{l}_1, \mathfrak{l}_2)$ with $\tau \in [0,1]$. The $\mathfrak{CF}$ fractional operator is given as

$$\begin{aligned} \mathcal{D}_{\mathfrak{t}}^{\mathrm{v}}(\phi(\mathfrak{t})) &= \frac{\mathcal{M}(\mathrm{v})}{(1-\mathrm{v})}\int_{\mathfrak{l}_1}^{\mathfrak{l}_2} \phi^{'}(\sigma) exp\bigg(-\mathrm{v}\frac{\mathfrak{t}-\sigma}{1-\sigma}\bigg)d(\sigma), \quad 0 < \mathrm{v} < 1, \\ &= \frac{d\phi}{d\mathfrak{t}}, \quad \mathrm{v} = 1, \end{aligned} \tag{4}$$

where $\mathcal{M}(\mathrm{v})$ with the initial conditions, $\mathcal{M}(0) = \mathcal{M}(1) = 1$.

Definition 2.2 Fractional order derivative represented corresponding to the integral operator for the $\mathfrak{CF}$ fractional derivative is given as

$$\begin{aligned} \mathcal{J}_{\mathfrak{t}}^{\mathrm{v}}(\phi(\mathfrak{t})) &= \frac{2(1-\mathrm{v})}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}\phi(\mathfrak{t}) \\ &\quad + \frac{2\mathrm{v}}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}\int_0^{\mathfrak{t}} \phi(\xi)d\xi, \quad \mathfrak{t} \geq 0. \end{aligned} \tag{5}$$

Definition 2.3 The Laplace transform of $\mathfrak{CF}$ fractional derivative is ${}^{\mathfrak{CF}}\mathcal{D}_{\mathfrak{t}}^{\mathrm{v}}\phi(\mathfrak{t})$ is

$$\mathcal{L}[{}^{\mathfrak{CF}}\mathcal{D}_{\mathfrak{t}}^{\mathrm{v}}\phi(\mathfrak{t})] = \mathcal{M}(\mathrm{v})\frac{\kappa\mathcal{L}[-\phi(\mathfrak{t})] - \phi(0)}{\kappa + \mathrm{v}(1-\kappa)}. \tag{6}$$

Theorem 2.1 [68] *If $\mathcal{Q}$ be a closed subspace of a Banach space $\mathcal{X}$ and $\mathcal{M} : \mathcal{Q} \to \mathcal{Q}$ be a contraction mapping, then $\mathcal{M}$ has a unique fixed point in $\mathcal{Q}$.*

Positivity of solution of LSD model with $\mathfrak{CF}$

Theorem 2.2 *Initially at time* $\mathfrak{t} = 0$, $\mathcal{S} > 0$, $\mathcal{V} \geq 0$,$\mathcal{E} \geq 0$,$\mathcal{I} \geq 0$,$\mathcal{R} \geq 0$. The set $\Theta = \{(\mathcal{S}, \mathcal{V}, \mathcal{E}, \mathcal{I}, \mathcal{R}) \in \mathcal{R}_+^5\}$ *approaches positive solutions for all cases of the fractional order system* (2)*, with* $\mathfrak{t} \geq 0$.
We use Lemma 2.3 to prove Theorem 2.2.

Lemma 2.3 [32] *Suppose* $\mathfrak{f}(\mathfrak{t}) \in \mathcal{C}[\gamma, \zeta]$ *and* ${}^{\mathfrak{C}}\mathcal{D}_{\mathfrak{t}}^{\mathrm{v}}\mathfrak{f}(\mathfrak{t}) \in \mathcal{C}[\gamma, \zeta]$ *for all* $0 < \mathrm{v} \leq 1$, *then* $\mathfrak{f}(\mathfrak{t}) + \frac{1}{\tau(\mathrm{v})}{}^{\mathfrak{C}}\mathcal{D}_{\mathfrak{t}}^{\mathrm{v}}\mathfrak{f}(\xi)(\mathfrak{t}-\gamma)^{\mathrm{v}}$

, *where* $\gamma \leq \xi \leq \mathfrak{t}$, *for all* $\mathfrak{t} \in [\gamma, \zeta]$.

Following Lemma 2.3, we give obtain the following remark.

Remark 2.4 *For all* $0 < \mathrm{v} \leq 1$, *assume that* $\mathcal{K}(\mathfrak{x}) \in \mathcal{C}[\gamma, \zeta]$ *and* ${}^{\mathfrak{C}}\mathcal{D}_{\mathfrak{t}}^{\mathrm{v}}\mathcal{K}(\mathfrak{x}) \in \mathcal{C}[\gamma, \zeta]$. *If* ${}^{\mathfrak{C}}\mathcal{D}_{\mathfrak{t}}^{\mathrm{v}}\mathcal{K}(\mathfrak{x}) \geq 0$, *for all* $\mathfrak{x} \in (\gamma, \zeta)$, *then* $\mathcal{K}(\mathfrak{x})$ *is non decreasing and if* ${}^{\mathfrak{C}}\mathcal{D}_{\mathfrak{t}}^{\mathrm{v}}\mathcal{K}(\mathfrak{x}) \leq 0$ *for all* $\mathfrak{x} \in (\gamma, \zeta)$, *then* $\mathcal{K}(\mathfrak{x})$ *is non increasing.*

Proof The proof follows by using Lemma 2.3 and Remark 2.4 which shows that the solution of LSD model with $\mathfrak{CF}$ exist and has a unique solution. Here, the rate Θ is positively invariant for each hyperplane bonding, the positive octant of the vector field points in Θ. The model assumed in (2) becomes

$$\begin{cases} ({}^{\mathfrak{CF}}\mathcal{D}_{\mathfrak{t}}^{\mathrm{v}}\mathcal{S})_{\mathcal{S}=0} = \theta > 0, \\ ({}^{\mathfrak{CF}}\mathcal{D}_{\mathfrak{t}}^{\mathrm{v}}\mathcal{V})_{\mathcal{V}=0} = \alpha_1\mathcal{S} \geq 0, \\ ({}^{\mathfrak{CF}}\mathcal{D}_{\mathfrak{t}}^{\mathrm{v}}\mathcal{E})_{\mathcal{E}=0} = \beta_1\mathcal{S}\mathcal{I} + \alpha_2\mathcal{V}\mathcal{I} \geq 0, \\ ({}^{\mathfrak{CF}}\mathcal{D}_{\mathfrak{t}}^{\mathrm{v}}\mathcal{I})_{\mathcal{I}=0} = \beta_2\mathcal{E} \geq 0, \\ ({}^{\mathfrak{CF}}\mathcal{D}_{\mathfrak{t}}^{\mathrm{v}}\mathcal{R})_{\mathcal{R}=0} = \beta_3\mathcal{I} + \alpha_3\mathcal{V} \geq 0. \end{cases} \tag{7}$$

Thus, the system (2) has fixed positive value and all its solutions are positively attracting in terms of Θ with $t \geq 0$. □

Existence and uniqueness of the solutions

In this section, we prove the existence and uniqueness of the solution for the assumed model (2) by using the integral operator as defined by the authors Losada and Nieto[66] which yields:

$$\begin{cases} \mathcal{S}(t) &= \mathcal{S}_0 + {}^{\mathfrak{C}\mathfrak{F}}\mathcal{J}_t^{v}(\theta - \beta_1\mathcal{S}\mathcal{I} - (\alpha_1+\nu)\mathcal{S}), \\ \mathcal{V}(t) &= \mathcal{V}_0 + {}^{\mathfrak{C}\mathfrak{F}}\mathcal{J}_t^{v}(\alpha_1\mathcal{S} - \alpha_2\mathcal{V}\mathcal{I} - (\nu+\alpha_3)\mathcal{V}), \\ \mathcal{E}(t) &= \mathcal{E}_0 + {}^{\mathfrak{C}\mathfrak{F}}\mathcal{J}_t^{v}(\beta_1\mathcal{S}\mathcal{I} + \alpha_2\mathcal{V}\mathcal{I} - (\beta_2+\nu)\mathcal{E}), \\ \mathcal{I}(t) &= \mathcal{I}_0 + {}^{\mathfrak{C}\mathfrak{F}}\mathcal{J}_t^{v}(\beta_2\mathcal{E} - (\beta_3+\nu+\delta_1)\mathcal{I}), \\ \mathcal{R}(t) &= \mathcal{S}_0 + {}^{\mathfrak{C}\mathfrak{F}}\mathcal{J}_t^{v}(\beta_3\mathcal{I} + \alpha_3\mathcal{V} - \nu\mathcal{R}). \end{cases} \tag{8}$$

The authors in[66], uses the Eq (8) as

$$\begin{cases} \mathcal{S}(t) &= \mathcal{S}(0) + \frac{2(1-v)}{(2-v)\mathcal{M}(v)}(\theta - \beta_1\mathcal{S}(t)\mathcal{I}(t) - (\alpha_1+\nu)\mathcal{S}(t)), \\ &+ \frac{2v}{(2-v)\mathcal{M}(v)}\int_0^t(\theta - \beta_1\mathcal{S}(\xi)\mathcal{I}(\xi) - (\alpha_1+\nu)\mathcal{S}(\xi))d\xi, \\ \mathcal{V}(t) &= \mathcal{V}(0) + \frac{2(1-v)}{(2-v)\mathcal{M}(v)}(\alpha_1\mathcal{S}(t) - \alpha_2\mathcal{V}(t)\mathcal{I}(t) - (\nu+\alpha_3)\mathcal{V}(t)), \\ &+ \frac{2v}{(2-v)\mathcal{M}(v)}\int_0^t(\alpha_1\mathcal{S}(\xi) - \alpha_2\mathcal{V}(\xi)\mathcal{I}(\xi) - (\nu+\alpha_3)\mathcal{V}(\xi))d\xi, \\ \mathcal{E}(t) &= \mathcal{E}(0) + \frac{2(1-v)}{(2-v)\mathcal{M}(v)}(\beta_1\mathcal{S}(t)\mathcal{I}(t) + \alpha_2\mathcal{V}(t)\mathcal{I}(t) - (\beta_2+\nu)\mathcal{E}(t)), \\ &+ \frac{2v}{(2-v)\mathcal{M}(v)}\int_0^t(\beta_1\mathcal{S}(\xi)\mathcal{I}(\xi) + \alpha_2\mathcal{V}(\xi)\mathcal{I}(\xi) - (\beta_2+\nu)\mathcal{E}(\xi))d\xi \\ \mathcal{I}(t) &= \mathcal{I}(0) + \frac{2(1-v)}{(2-v)\mathcal{M}(v)}(\beta_2\mathcal{E}(t) - (\beta_3+\nu+\delta_1)\mathcal{I}(t)), \\ &+ \frac{2v}{(2-v)\mathcal{M}(v)}\int_0^t(\beta_2\mathcal{E}(\xi) - (\beta_3+\nu+\delta_1)\mathcal{I}(\xi))d\xi, \\ \mathcal{R}(t) &= \mathcal{R}(0) + \frac{2(1-v)}{(2-v)\mathcal{M}(v)}(\beta_3\mathcal{I}(t) + \alpha_3\mathcal{V}(t) - \nu\mathcal{R}(t)) \\ &+ \frac{2(1-v)}{(2-v)\mathcal{M}(v)}\int_0^t(\beta_3\mathcal{I}(\xi) + \alpha_3\mathcal{V}(\xi) - \nu\mathcal{R}(\xi))d\xi. \end{cases} \tag{9}$$

Without loss of generality we have

$$\begin{cases} \Psi_1(t,\mathcal{S}) &= \theta - \beta_1\mathcal{S}\mathcal{I} - (\alpha_1+\nu)\mathcal{S}, \\ \Psi_2(t,\mathcal{V}) &= \alpha_1\mathcal{S} - \alpha_2\mathcal{V}\mathcal{I} - (\nu+\alpha_3)\mathcal{V}, \\ \Psi_3(t,\mathcal{E}) &= \beta_1\mathcal{S}\mathcal{I} + \alpha_2\mathcal{V}\mathcal{I} - (\beta_2+\nu)\mathcal{E}, \\ \Psi_4(t,\mathcal{I}) &= \beta_2\mathcal{E} - (\beta_3+\nu+\delta_1)\mathcal{I}, \\ \Psi_5(t,\mathcal{R}) &= \beta_3\mathcal{I} + \alpha_3\mathcal{V} - \nu\mathcal{R}. \end{cases} \tag{10}$$

Denoting the above set of equations as $(\mathcal{G})$, we consider the state variables $\mathcal{S}(t), \mathcal{S}^\star(t), \mathcal{V}(t), \mathcal{V}^\star(t), \mathcal{E}(t), \mathcal{E}^\star(t), \mathcal{I}(t), \mathcal{I}^\star(t), \mathcal{R}(t), \mathcal{R}^\star(t) \in \mathcal{L}[0,1]$ be continuous such that $||\mathcal{S}(t)|| \leq \mathfrak{c}_1$, $||\mathcal{V}(t)|| \leq \mathfrak{c}_2$, $||\mathcal{E}(t)|| \leq \mathfrak{c}_3$, $||\mathcal{I}(t)|| \leq \mathfrak{c}_4$ and $||\mathcal{R}(t)|| \leq \mathfrak{c}_5$, for some positive constants $\mathfrak{c}_1, \mathfrak{c}_2, \mathfrak{c}_3, \mathfrak{c}_4, \mathfrak{c}_5 > 0$.

Theorem 2.5 *Assuming* $(\mathcal{G})$ *with each kernel* $(\Psi_1, \Psi_2, \Psi_3, \Psi_4, \Psi_5)$ *satisfying the Lipschitz condition*

$$\mathfrak{j}_i < 1, \text{ for } i = 1,2,3,4,5.$$

Proof Now,

$$\begin{aligned} ||\Psi_1(t,\mathcal{S}) - \Psi_1(t,\mathcal{S}_1)|| &= ||(\mathcal{S}-\mathcal{S}_1)(\beta_1\mathcal{I}) + (\mathcal{S}-\mathcal{S}_1)(\alpha_1+v))|| \\ &\leq ||\beta_1\mathcal{I})||||\mathcal{S}-\mathcal{S}_1|| + (\alpha_1+v)||\mathcal{S}-\mathcal{S}_1|| \\ &\leq (||\beta_1\mathcal{I}|| + \alpha_1 + v)||\mathcal{S}-\mathcal{S}_1|| \\ &\leq (\beta_1\mathfrak{c}_4 + \alpha_1 + v)||\mathcal{S}-\mathcal{S}_1|| \\ &= \mathfrak{j}_1||\mathcal{S}-\mathcal{S}_1||, \end{aligned}$$

where $\mathfrak{j}_1 = \beta_1\mathfrak{c}_4 + \alpha_1 + v$. Hence,

$$||\Psi_1(t,\mathcal{S}) - \Psi_1(t,\mathcal{S}_1)|| \leq \mathfrak{j}_1||\mathcal{S}-\mathcal{S}_1||.$$

Thus, Ψ_1 satisfies the Lipschitz condition. Continuing in this manner, we can prove that $(\Psi_2, \Psi_3, \Psi_4, \Psi_5)$ satisfy the Lipschitz conditions,

$$\begin{aligned} ||\Psi_2(t,\mathcal{V}) - \Psi_2(t,\mathcal{V}_1)|| &\leq \mathfrak{j}_2||\mathcal{V}-\mathcal{V}_1|| \\ ||\Psi_3(t,\mathcal{E}) - \Psi_3(t,\mathcal{E})|| &\leq \mathfrak{j}_3||\mathcal{E}-\mathcal{E}_1|| \\ ||\Psi_4(t,\mathcal{I}) - \Psi_4(t,\mathcal{I}_1)|| &\leq \mathfrak{j}_4||\mathcal{I}-\mathcal{I}_1|| \\ ||\Psi_5(t,\mathcal{R}) - \Psi_5(t,\mathcal{R}_1)|| &\leq \mathfrak{j}_5||\mathcal{R}-\mathcal{R}_1||. \end{aligned}$$

□

Now, from (9), we have

$$\begin{cases} \mathcal{S}(t) &= \mathcal{S}(0) + \frac{2(1-v)}{(2-v)\mathcal{M}(v)}\Psi_1(t,\mathcal{S}) + \frac{2v}{(2-v)\mathcal{M}(v)}\int_0^t \Psi_1(\xi,\mathcal{S})d\xi, \\ \mathcal{V}(t) &= \mathcal{V}(0) + \frac{2(1-v)}{(2-v)\mathcal{M}(v)}\Psi_2(t,\mathcal{V}) + \frac{2v}{(2-v)\mathcal{M}(v)}\int_0^t \Psi_2(\xi,\mathcal{V})d\xi, \\ \mathcal{E}(t) &= \mathcal{E}(0) + \frac{2(1-v)}{(2-v)\mathcal{M}(v)}\Psi_3(t,\mathcal{E}) + \frac{2v}{(2-v)\mathcal{M}(v)}\int_0^t \Psi_3(\xi,\mathcal{E})d\xi, \\ \mathcal{I}(t) &= \mathcal{I}(0) + \frac{2(1-v)}{(2-v)\mathcal{M}(v)}\Psi_4(t,\mathcal{I}) + \frac{2v}{(2-v)\mathcal{M}(v)}\int_0^t \Psi_4(\xi,\mathcal{I})d\xi, \\ \mathcal{R}(t) &= \mathcal{R}(0) + \frac{2(1-v)}{(2-v)\mathcal{M}(v)}\Psi_5(t,\mathcal{R}) + \frac{2v}{(2-v)\mathcal{M}(v)}\int_0^t \Psi_5(\xi,\mathcal{R})d\xi, \end{cases}$$

with the initial conditions

$$\mathcal{S}(0) = \mathcal{S}_0, \mathcal{V}(0) = \mathcal{V}_0, \mathcal{E}(0) = \mathcal{E}_0, \mathcal{I}(0) = \mathcal{I}_0, \mathcal{R}(0) = \mathcal{R}_0.$$

Suppose, we define the iterative recursive forms below,

$$\begin{cases} \mathcal{S}_\ell(t) &= \mathcal{S}(0) + \frac{2(1-v)}{(2-v)\mathcal{M}(v)}\Psi_1(t,\mathcal{S}_{\ell-1}) + \frac{2v}{(2-v)\mathcal{M}(v)}\int_0^t \Psi_1(\xi,\mathcal{S}_{\ell-1})d\xi, \\ \mathcal{V}_\ell(t) &= \mathcal{V}(0) + \frac{2(1-v)}{(2-v)\mathcal{M}(v)}\Psi_2(t,\mathcal{V}_{\ell-1}) + \frac{2v}{(2-v)\mathcal{M}(v)}\int_0^t \Psi_2(\xi,\mathcal{V}_{\ell-1})d\xi, \\ \mathcal{E}_\ell(t) &= \mathcal{E}(0) + \frac{2(1-v)}{(2-v)\mathcal{M}(v)}\Psi_3(t,\mathcal{E}_{\ell-1}) + \frac{2v}{(2-v)\mathcal{M}(v)}\int_0^t \Psi_3(\xi,\mathcal{E}_{\ell-1})d\xi, \\ \mathcal{I}_\ell(t) &= \mathcal{I}(0) + \frac{2(1-v)}{(2-v)\mathcal{M}(v)}\Psi_4(t,\mathcal{I}_{\ell-1}) + \frac{2v}{(2-v)\mathcal{M}(v)}\int_0^t \Psi_4(\xi,\mathcal{I}_{\ell-1})d\xi \\ \mathcal{R}_\ell(t) &= \mathcal{R}(0) + \frac{2(1-v)}{(2-v)\mathcal{M}(v)}\Psi_5(t,\mathcal{R}_{\ell-1}) + \frac{2v}{(2-v)\mathcal{M}(v)}\int_0^t \Psi_5(\xi,\mathcal{R}_{\ell-1})d\xi. \end{cases}$$

Theorem 2.6 *There is at least a solution of the Lumpy Skin Disease model with* $\mathfrak{CF}(2)$ *if* $\delta = \max\{j_1, j_2, j_3, j_4, j_5\} < 1$.

Proof Let

$$\begin{aligned} \lambda_{1\ell}(t) &= \mathcal{S}_{\ell+1}(t) - \mathcal{S}(t), \\ \lambda_{2\ell}(t) &= \mathcal{V}_{\ell+1}(t) - \mathcal{V}(t), \\ \lambda_{3\ell}(t) &= \mathcal{E}_{\ell+1}(t) - \mathcal{E}(t), \\ \lambda_{4\ell}(t) &= \mathcal{I}_{\ell+1}(t) - \mathcal{I}(t), \\ \lambda_{5\ell}(t) &= \mathcal{R}_{\ell+1}(t) - \mathcal{R}(t). \end{aligned}$$

Then, we have

$$\begin{aligned} ||\lambda_{1\ell}(t)|| = ||\mathcal{S}_{\ell+1}(t) - \mathcal{S}(t)|| &= \left\| \frac{2(1-v)}{(2-v)\mathcal{M}(v)}(\Psi_1(t,\mathcal{S}_\ell) - \Psi_1(t,\mathcal{S})) \right. \\ &\quad \left. + \frac{2v}{(2-v)\mathcal{M}(v)}\int_0^t (\Psi_1(\xi,\mathcal{S}_\ell) - \Psi_1(\xi,\mathcal{S}))d\xi \right\| \\ &\leq \frac{2(1-v)}{(2-v)\mathcal{M}(v)}\left\|\Psi_1(t,\mathcal{S}_\ell) - \Psi_1(t,\mathcal{S})\right\| \\ &\quad + \frac{2v}{(2-v)\mathcal{M}(v)}\int_0^t \left\|\Psi_1(\xi,\mathcal{S}_\ell) - \Psi_1(\xi,\mathcal{S}))\right\|d\xi \\ &\leq \frac{2(1-v)}{(2-v)\mathcal{M}(v)}j_1||\mathcal{S}_\ell - \mathcal{S}|| \\ &\quad + \frac{2v}{(2-v)\mathcal{M}(v)}\int_0^t j_1||\mathcal{S}_\ell - \mathcal{S}||d\xi \\ &= \frac{2(1-v)}{(2-v)\mathcal{M}(v)}j_1||\mathcal{S}_\ell - \mathcal{S}|| \\ &\quad + \frac{2v}{(2-v)\mathcal{M}(v)}j_1\int_0^t ||\mathcal{S}_\ell - \mathcal{S}||d\xi \\ &\leq \left(\frac{2(1-v)}{(2-v)\mathcal{M}(v)} + \frac{2vt}{(2-v)\mathcal{M}(v)}\right)j_1||\mathcal{S}_\ell - \mathcal{S}|| \\ &\leq \left(\frac{2(1-v)}{(2-v)\mathcal{M}(v)} + \frac{2vt}{(2-v)\mathcal{M}(v)}\right)^n j_1^n||\mathcal{S}_1 - \mathcal{S}||. \end{aligned}$$

Since $j_1 < 1$. As $n \to \infty$, we have $\mathcal{S}_n \to \mathcal{S}$. Similarly,

$$||\lambda_{2\mathfrak{k}}(\mathfrak{t})|| \leq \left(\frac{2(1-\mathfrak{v})}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})} + \frac{2\mathfrak{v}\mathfrak{t}}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}\right)^{\mathfrak{n}} \mathfrak{j}_2^{\mathfrak{n}}||\mathcal{V}_1 - \mathcal{V}||$$
$$||\lambda_{3\mathfrak{k}}(\mathfrak{t})|| \leq \left(\frac{2(1-\mathfrak{v})}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})} + \frac{2\mathfrak{v}\mathfrak{t}}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}\right)^{\mathfrak{n}} \mathfrak{j}_3^{\mathfrak{n}}||\mathcal{E}_1 - \mathcal{E}||$$
$$||\lambda_{4\mathfrak{k}}(\mathfrak{t})|| \leq \left(\frac{2(1-\mathfrak{v})}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})} + \frac{2\mathfrak{v}\mathfrak{t}}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}\right)^{\mathfrak{n}} \mathfrak{j}_4^{\mathfrak{n}}||\mathcal{I}_1 - \mathcal{I}||$$
$$||\lambda_{5\mathfrak{k}}(\mathfrak{t})|| \leq \left(\frac{2(1-\mathfrak{v})}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})} + \frac{2\mathfrak{v}\mathfrak{t}}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}\right)^{\mathfrak{n}} \mathfrak{j}_5^{\mathfrak{n}}||\mathcal{R}_1 - \mathcal{R}||.$$

As $\mathfrak{n} \to \infty$, we get $\lambda_{\mathfrak{i}\mathfrak{k}}(\mathfrak{t}) \to 0$ with $\mathfrak{j}_\mathfrak{i} < 1$ for $\mathfrak{i} = 1, 2, 3, 4, 5$. Hence, Lumpy Skin Disease model with $\mathfrak{CF}$ (2) has a solution. □

Theorem 2.7 *The Lumpy Skin Disease model with* $\mathfrak{CF}$ (2) *has a unique solution if*

$$\left(\frac{2(1-\mathfrak{v})}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})} + \frac{2\mathfrak{v}\mathfrak{t}}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}\right)\mathfrak{j}_\mathfrak{i} \leq 1, \text{ for } \mathfrak{i} = 1, 2, 3, 4, 5.$$

Proof Assume that there exists another solution $\mathcal{S}^\star(\mathfrak{t}), \mathcal{V}^\star(\mathfrak{t}), \mathcal{E}^\star(\mathfrak{t}), \mathcal{I}^\star(\mathfrak{t}), \mathcal{R}^\star(\mathfrak{t})$ with initial values such that

$$\mathcal{S}^\star(\mathfrak{t}) = \mathcal{S}(0) + \frac{2(1-\mathfrak{v})}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}\Psi_1(\mathfrak{t}, \mathcal{S}^\star) + \frac{2\mathfrak{v}}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}\int_0^\mathfrak{t} \Psi_1(\xi, \mathcal{S}^\star)d\xi,$$
$$\mathcal{V}^\star(\mathfrak{t}) = \mathcal{V}(0) + \frac{2(1-\mathfrak{v}^\star)}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}\Psi_2(\mathfrak{t}, \mathcal{V}^\star) + \frac{2\mathfrak{v}}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}\int_0^\mathfrak{t} \Psi_2(\xi, \mathcal{V}^\star)d\xi,$$
$$\mathcal{E}^\star(\mathfrak{t}) = \mathcal{E}(0) + \frac{2(1-\mathfrak{v})}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}\Psi_3(\mathfrak{t}, \mathcal{E}^\star) + \frac{2\mathfrak{v}}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}\int_0^\mathfrak{t} \Psi_3(\xi, \mathcal{E}^\star)d\xi,$$
$$\mathcal{I}^\star(\mathfrak{t}) = \mathcal{I}(0) + \frac{2(1-\mathfrak{v})}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}\Psi_4(\mathfrak{t}, \mathcal{I}^\star) + \frac{2\mathfrak{v}}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}\int_0^\mathfrak{t} \Psi_4(\xi, \mathcal{I}^\star)d\xi$$
$$\mathcal{R}^\star(\mathfrak{t}) = \mathcal{R}(0) + \frac{2(1-\mathfrak{v})}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}\Psi_5(\mathfrak{t}, \mathcal{R}^\star) + \frac{2\mathfrak{v}}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}\int_0^\mathfrak{t} \Psi_5(\xi, \mathcal{R}^\star)d\xi.$$

Now,

$$\begin{aligned}
||\mathcal{S} - \mathcal{S}^\star|| &= \left\| \frac{2(1-\mathfrak{v})}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}(\Psi_1(\mathfrak{t}, \mathcal{S}) - \Psi_1(\mathfrak{t}, \mathcal{S}^\star)) \right. \\
&\quad \left. + \frac{2\mathfrak{v}}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}\int_0^\mathfrak{t}(\Psi_1(\xi, \mathcal{S}) - \Psi_1(\xi, \mathcal{S}^\star))d\xi \right\| \\
&\leq \frac{2(1-\mathfrak{v})}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}\left\|\Psi_1(\mathfrak{t}, \mathcal{S}) - \Psi_1(\mathfrak{t}, \mathcal{S}^\star)\right\| \\
&\quad + \frac{2\mathfrak{v}}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}\int_0^\mathfrak{t}\left\|\Psi_1(\xi, \mathcal{S}) - \Psi_1(\xi, \mathcal{S}^\star))\right\|d\xi \\
&\leq \frac{2(1-\mathfrak{v})}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}\mathfrak{j}_1||\mathcal{S} - \mathcal{S}^\star|| \\
&\quad + \frac{2\mathfrak{v}}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}\int_0^\mathfrak{t}\mathfrak{j}_1||\mathcal{S} - \mathcal{S}^\star||d\xi \\
&= \frac{2(1-\mathfrak{v})}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}\mathfrak{j}_1||\mathcal{S} - \mathcal{S}^\star|| \\
&\quad + \frac{2\mathfrak{v}}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}\mathfrak{j}_1\int_0^\mathfrak{t}||\mathcal{S} - \mathcal{S}^\star||d\xi \\
&\leq \left(\frac{2(1-\mathfrak{v})}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})} + \frac{2\mathfrak{v}\mathfrak{t}}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}\right)\mathfrak{j}_1||\mathcal{S} - \mathcal{S}^\star|| \\
&\leq \left(\frac{2(1-\mathfrak{v})}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})} + \frac{2\mathfrak{v}\mathfrak{t}}{(2-\mathfrak{v})\mathcal{M}(\mathfrak{v})}\right)\mathfrak{j}_1||\mathcal{S} - \mathcal{S}^\star||,
\end{aligned}$$

which implies that

$$\left(1-\left(\frac{2(1-\mathrm{v})}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}+\frac{2\mathrm{v}\mathrm{t}}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}\right)\mathrm{j}_1\right)||\mathcal{S}-\mathcal{S}^\star||\leq 0.$$

Therefore, $||\mathcal{S}-\mathcal{S}^\star||=0$. Hence, $\mathcal{S}=\mathcal{S}^\star$. Similarly, we can prove

$$\mathcal{V}=\mathcal{V}^\star,\mathcal{E}=\mathcal{E}^\star,\mathcal{I}=\mathcal{I}^\star,\mathcal{R}=\mathcal{R}^\star.$$

Hence, Lumpy Skin Disease model with $\mathfrak{CF}$ (2) has a unique solution. □

Ulam–Hyers stability of the lumpy skin disease model

In this section, we obtain the Ulam–Hyers stability of the Lumpy Skin Disease model with $\mathfrak{CF}$ (2). We state the required definition.

Definition 3.1 The Lumpy Skin Disease model with $\mathfrak{CF}$ (2) has Ulam–Hyers stability if there exist constants $\Psi_\mathrm{i}>0,\mathrm{i}=1,2,3,4,5$ satisfying: For every $\epsilon_\mathrm{i}>0,\mathrm{i}=1,2,3,4,5$, if

$$\begin{cases}\left|{}^{\mathfrak{CF}}\mathcal{D}_\mathrm{t}^\mathrm{v}\mathcal{S}(\mathrm{t})-\Psi_1(\mathrm{t},\mathcal{S})\right|\leq\epsilon_1,\\ \left|{}^{\mathfrak{CF}}\mathcal{D}_\mathrm{t}^\mathrm{v}\mathcal{V}(\mathrm{t})-\Psi_2(\mathrm{t},\mathcal{V})\right|\leq\epsilon_2,\\ \left|{}^{\mathfrak{CF}}\mathcal{D}_\mathrm{t}^\mathrm{v}\mathcal{E}(\mathrm{t})-\Psi_3(\mathrm{t},\mathcal{E})\right|\leq\epsilon_3,\\ \left|{}^{\mathfrak{CF}}\mathcal{D}_\mathrm{t}^\mathrm{v}\mathcal{I}(\mathrm{t})-\Psi_4(\mathrm{t},\mathcal{I})\right|\leq\epsilon_4,\\ \left|{}^{\mathfrak{CF}}\mathcal{D}_\mathrm{t}^\mathrm{v}\mathcal{R}(\mathrm{t})-\Psi_5(\mathrm{t},\mathcal{R})\right|\leq\epsilon_5,\end{cases}\tag{11}$$

and there exists a solution of the Lumpy Skin Disease model with $\mathfrak{CF}$ (2), $\mathcal{S}^\star(\mathrm{t}),\mathcal{V}^\star(\mathrm{t}),\mathcal{E}^\star(\mathrm{t}),\mathcal{I}^\star(\mathrm{t})$ and $\mathcal{R}^\star(\mathrm{t})$ that statisfying the given model, such that

$$||\mathcal{S}-\mathcal{S}^\star||\leq\eta_1\epsilon_1,||\mathcal{V}-\mathcal{V}^\star||\leq\eta_2\epsilon_2,||\mathcal{E}-\mathcal{E}^\star||\leq\eta_3\epsilon_3,$$
$$||\mathcal{I}-\mathcal{I}^\star||\leq\eta_4\epsilon_4,||\mathcal{R}-\mathcal{R}^\star||\leq\eta_5\epsilon_5.$$

Remark 3.1 The function $\mathcal{S}$ satisfies the first inequality (11) if and only if there exists a continuous function $\mathfrak{h}_1$, which is dependent on $\mathcal{S}_1$, such that

(A1) $|\mathfrak{h}_1(\mathrm{t})|<\epsilon_1$, and
(A2) ${}^{\mathfrak{CF}}\mathcal{D}_\mathrm{t}^\mathrm{v}\mathcal{S}(\mathrm{t})=\Psi_1(\mathrm{t},\mathcal{S})+\mathfrak{h}_1(\mathrm{t})$.

Similarly, we can define for other classes of the model (11) for some $\mathfrak{h}_\mathrm{i}$ where $\mathrm{i}=2,3,4,5$.Theorem 3.2 *Assume that the hypothesis $(\mathcal{G})$ holds true. Then the Lumpy Skin Disease model with $\mathfrak{CF}$ (2) is Ulam–Hyers stable if*

$$\left(\frac{2(1-\mathrm{v})}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}+\frac{2\mathrm{v}\mathrm{t}}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}\right)\mathrm{j}_\mathrm{i}\leq 1,\ \text{for } \mathrm{i}=1,2,3,4,5.$$

Proof Let $\epsilon_1>0$ and the function $\mathcal{S}$ be arbitary such that

$$\left|{}^{\mathfrak{CF}}\mathcal{D}_\mathrm{t}^\mathrm{v}\mathcal{S}(\mathrm{t})-\Psi_1(\mathrm{t},\mathcal{S})\right|\leq\epsilon_1.$$

By the Remark 3.1, we have

$${}^{\mathfrak{CF}}\mathcal{D}_\mathrm{t}^\mathrm{v}\mathcal{S}(\mathrm{t})=\Psi_1(\mathrm{t},\mathcal{S})+\mathfrak{h}_1(\mathrm{t}).$$

Consequently, we get

$$\mathcal{S}(\mathrm{t})=\mathcal{S}(0)+\frac{2(1-\mathrm{v})}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}\Psi_1(\mathrm{t},\mathcal{S}(\mathrm{t}))+\frac{2\mathrm{v}}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}\int_0^\mathrm{t}\Psi_1(\xi,\mathcal{S}(\mathrm{t}))d\xi$$
$$+\frac{2(1-\mathrm{v})}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}\mathfrak{h}_1(\mathrm{t})+\frac{2\mathrm{v}}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}\int_0^\mathrm{t}\mathfrak{h}_1(\xi)d\xi.$$

Let $\mathcal{S}^\star$ be the unique solution of the Lumpy Skin Disease model with $\mathfrak{CF}$ (2). Then,

$$\mathcal{S}^{\star}(\mathrm{t}) = \mathcal{S}(0) + \frac{2(1-\mathrm{v})}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}\Psi_1(\mathrm{t}, \mathcal{S}^{\star}(\mathrm{t})) + \frac{2\mathrm{v}}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}\int_0^{\mathrm{t}} \Psi_1(\xi, \mathcal{S}^{\star}(\mathrm{t}))d\xi.$$

Hence,

$$\begin{aligned}
|\mathcal{S}(\mathrm{t}) - \mathcal{S}^{\star}(\mathrm{t})| &\leq \frac{2(1-\mathrm{v})}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}|\Psi_1(\mathrm{t}, \mathcal{S}(\mathrm{t})) - \Psi_1(\mathrm{t}, \mathcal{S}^{\star}(\mathrm{t}))| \\
&\quad + \frac{2\mathrm{v}}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}\int_0^{\mathrm{t}} |\Psi_1(\xi, \mathcal{S}(\mathrm{t})) - \Psi_1(\xi, \mathcal{S}^{\star}(\mathrm{t}))|d\xi \\
&\quad + \frac{2(1-\mathrm{v})}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}|\mathfrak{h}_1(\mathrm{t})| + \frac{2\mathrm{v}}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}\int_0^{\mathrm{t}} |\mathfrak{h}_1(\xi)|d\xi \\
&\leq \left[\frac{2(1-\mathrm{v})}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})} + \frac{2\mathrm{v}\mathrm{t}}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}\right]\mathrm{j}_1|\mathcal{S}(\mathrm{t}) - \mathcal{S}^{\star}(\mathrm{t})| \\
&\quad + \left[\frac{2(1-\mathrm{v})}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})} + \frac{2\mathrm{v}\mathrm{t}}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}\right]\epsilon_1 \\
||\mathcal{S}(\mathrm{t}) - \mathcal{S}^{\star}(\mathrm{t})|| &\leq \frac{\left[\frac{2(1-\mathrm{v})}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})} + \frac{2\mathrm{v}\mathrm{t}}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}\right]\epsilon_1}{1 - \left[\frac{2(1-\mathrm{v})}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})} + \frac{2\mathrm{v}\mathrm{t}}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}\right]\mathrm{j}_1}.
\end{aligned}$$

Then,

$$||\mathcal{S}(\mathrm{t}) - \mathcal{S}^{\star}(\mathrm{t})|| \leq \eta_1\epsilon_1,$$

where

$$\eta_1 = \frac{\left[\frac{2(1-\mathrm{v})}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})} + \frac{2\mathrm{v}\mathrm{t}}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}\right]}{1 - \left[\frac{2(1-\mathrm{v})}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})} + \frac{2\mathrm{v}\mathrm{t}}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}\right]\mathrm{j}_1}.$$

Similarly, we have

$$\begin{aligned}
||\mathcal{V} - \mathcal{V}^{\star}|| \leq \eta_2\epsilon_2, ||\mathcal{E} - \mathcal{E}^{\star}|| \leq \eta_3\epsilon_3, \\
||\mathcal{I} - \mathcal{I}^{\star}|| \leq \eta_4\epsilon_4, ||\mathcal{R} - \mathcal{R}^{\star}|| \leq \eta_5\epsilon_5.
\end{aligned}$$

Thus, Lumpy Skin Disease model with $\mathfrak{CF}$ (2) is Ulam–Hyers stable. □

Numerical scheme

Here, a numerical scheme for the Lumpy Skin Disease model with $\mathfrak{CF}$ (2) is developed. For this, we use the approach related to Lagrange interpolation polynomials. Consider a general Cauchy problem with fractal fractional differential operator as:

$$\begin{cases} {}^{\mathfrak{CF}}\mathcal{D}_{\mathrm{t}}^{\mathrm{v}}\phi(\mathrm{t}) = \Psi(\mathrm{t}, \phi(\mathrm{t})) \\ \phi(0) = \phi_0. \end{cases} \tag{12}$$

Utilizing the fractal fractional integral operator, we obtain

$$\phi(\mathrm{t}) = \phi(0) + \frac{2(1-\mathrm{v})}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}\Psi(\mathrm{t}, \phi(\mathrm{t})) + \frac{2\mathrm{v}}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}\int_0^{\mathrm{t}} \Psi(\xi, \phi(\xi))d\xi.$$

Putting by $\mathrm{t}_{\mathrm{n}+1}$, which gives

$$\phi_{\mathrm{n}+1} = \phi(0) + \frac{2(1-\mathrm{v})}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}\Psi(\mathrm{t}_{\mathrm{n}}, \phi(\mathrm{t}_{\mathrm{n}})) + \frac{2\mathrm{v}}{(2-\mathrm{v})\mathcal{M}(\mathrm{v})}\int_0^{\mathrm{t}_{\mathrm{n}+1}} \Psi(\mathrm{t}, \phi(\mathrm{t}))d\mathrm{t}.$$

The successive terms difference is given as follows:

$$\begin{aligned}\phi_{n+1}-\phi_n &= \frac{2(1-v)}{(2-v)\mathcal{M}(v)}(\Psi(t_n,\phi_n)-\Psi(t_{n-1},\phi_{n-1}))\\ &\quad+\frac{2v}{(2-v)\mathcal{M}(v)}\int_{t_n}^{t_{n+1}}\Psi(t,\phi(t))dt.\end{aligned} \tag{13}$$

Over the closed interval $[t_m, t_{m+1}]$, the function $\Psi(\xi,\phi(\xi))$ can be approximated by the Lagrange polynomial interpolation

$$\theta_m(t)\cong\frac{g(t_m,\eta_m)}{\hbar}(t-t_{m-1})-\frac{g(t_{m-1},\eta_{m-1})}{\hbar}(t-t_m), \tag{14}$$

where $\hbar = t_m - t_{m-1}$. Consequently,

$$\begin{aligned}\int_{t_n}^{t_{n+1}}\Psi(t,\phi(t))dt &= \int_{t_n}^{t_{n+1}}\left(\frac{\Psi(t_n,\phi_n)}{\hbar}(t-t_{n-1})-\right.\\ &\quad\left.\frac{\Psi(t_{n-1},\phi_{n-1})}{\hbar}(t-t_n)\right)dt\\ &= \frac{3\hbar}{2}\Psi(t_n,\phi_n)-\frac{\hbar}{2}\Psi(t_{n-1},\phi_{n-1}).\end{aligned} \tag{15}$$

Putting (15) in (13) and after simplification, we get

$$\begin{aligned}\phi_{n+1} &= \phi_n+\left(\frac{2(1-v)}{(2-v)\mathcal{M}(v)}+\frac{3v\hbar}{(2-v)\mathcal{M}(v)}\right)\Psi(t_n,\phi_n)\\ &\quad-\left(\frac{2(1-v)}{(2-v)\mathcal{M}(v)}+\frac{v\hbar}{(2-v)\mathcal{M}(v)}\right)\Psi(t_{n-1},\phi_{n-1}).\end{aligned}$$

Hence, the numerical scheme for (9) is obtained as the following:

$$\begin{aligned}\mathcal{S}_{n+1} &= \mathcal{S}_n+\left(\frac{2(1-v)}{(2-v)\mathcal{M}(v)}+\frac{3v\hbar}{(2-v)\mathcal{M}(v)}\right)\Psi_1(t_n,\mathcal{S}_n)\\ &\quad-\left(\frac{2(1-v)}{(2-v)\mathcal{M}(v)}+\frac{v\hbar}{(2-v)\mathcal{M}(v)}\right)\Psi_1(t_{n-1},\mathcal{S}_{n-1}),\\ \mathcal{V}_{n+1} &= \mathcal{V}_n+\left(\frac{2(1-v)}{(2-v)\mathcal{M}(v)}+\frac{3v\hbar}{(2-v)\mathcal{M}(v)}\right)\Psi_2(t_n,\mathcal{V}_n)\\ &\quad-\left(\frac{2(1-v)}{(2-v)\mathcal{M}(v)}+\frac{v\hbar}{(2-v)\mathcal{M}(v)}\right)\Psi_2(t_{n-1},\mathcal{V}_{n-1}),\\ \mathcal{E}_{n+1} &= \mathcal{E}_n+\left(\frac{2(1-v)}{(2-v)\mathcal{M}(v)}+\frac{3v\hbar}{(2-v)\mathcal{M}(v)}\right)\Psi_3(t_n,\mathcal{E}_n)\\ &\quad-\left(\frac{2(1-v)}{(2-v)\mathcal{M}(v)}+\frac{v\hbar}{(2-v)\mathcal{M}(v)}\right)\Psi_3(t_{n-1},\mathcal{E}_{n-1}),\\ \mathcal{I}_{n+1} &= \mathcal{I}_n+\left(\frac{2(1-v)}{(2-v)\mathcal{M}(v)}+\frac{3v\hbar}{(2-v)\mathcal{M}(v)}\right)\Psi_4(t_n,\mathcal{I}_n)\\ &\quad-\left(\frac{2(1-v)}{(2-v)\mathcal{M}(v)}+\frac{v\hbar}{(2-v)\mathcal{M}(v)}\right)\Psi_4(t_{n-1},\mathcal{I}_{n-1}),\\ \mathcal{R}_{n+1} &= \mathcal{R}_n+\left(\frac{2(1-v)}{(2-v)\mathcal{M}(v)}+\frac{3v\hbar}{(2-v)\mathcal{M}(v)}\right)\Psi_5(t_n,\mathcal{R}_n)\\ &\quad-\left(\frac{2(1-\varpi)}{(2-\varpi)\mathcal{M}(\varpi)}+\frac{\varpi\hbar}{(2-\varpi)\mathcal{M}(\varpi)}\right)\Psi_5(t_{n-1},\mathcal{R}_{n-1}).\end{aligned}$$

Numerical results and discussion

In the current study, we deliberated the model developed by (2) for observing the dynamic spread of the disease in the population and obtained some results for the considered model. The purpose of this extensive model is to observe what happens when fractional order changes in the model. Here, the numerical outcomes are presented for model employing the parameters values defined in Table 1, which are compatible with Lumpy Skin Disease for different v. Now, by taking initial values as

Parameter	Description	Values	Source
θ	Rate of birth	0.6	Assumed
β_1	Transmission rate from $\mathcal{S}$ to $\mathcal{E}$ due to infected state $\mathcal{I}$	0.032	Assumed
β_2	Translation rate from $\mathcal{E}$ to $\mathcal{I}$	0.59	[67]
β_3	Translation rate from $\mathcal{I}$ to $\mathcal{R}$	0.3	[67]
α_1	Rate of the susceptible case gets vaccinated	0.9	Assumed
α_2	Rate of the vaccinated cattle that are exposed due to contact with infected individuals $\mathcal{I}$	0.055	[67]
α_3	Rate at of the vaccinated cattle that are getting recovered	0.3	Assumed
δ_1	Death rate due to the cause of disease in the infected state $\mathcal{I}$	0.03	Assumed
ν	death rate that happened naturally	0.07	Assumed

Table 1. Description of the variables and parameters used in the model.

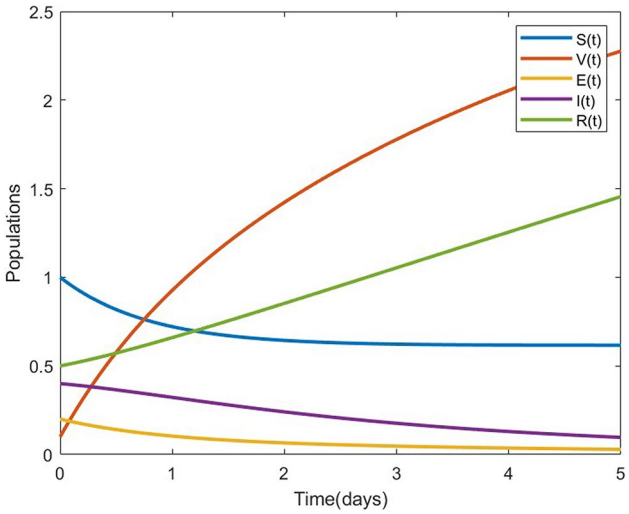

Fig. 2. Population of SVEIR.

$$\mathcal{S}(0) = 1, \mathcal{V}(0) = 0.1, \mathcal{E}(0) = 0.2, \mathcal{I}(0) = 0.4, \mathcal{R}(0) = 0.5,$$

Fig. 2 exemplifies the effect of derivative order and behavior on achieved outcomes by the proposed solution procedure for $\mathcal{S}(t), \mathcal{V}(t), \mathcal{E}(t), \mathcal{I}(t), \mathcal{R}(t)$. The outcomes have been plotted for different fractional orders $v = 0.8, 0.85, 0.9$ and 0.95 in Fig. 2 of the model (2). From Fig. 3, it is observed that the group of susceptible people $\mathcal{S}(t)$ varies continuously with time derivative. From Fig. 4, it is observed that the group of vaccinated population varies continuously with time derivative. From Fig. 5, it is observed that the group of exposed population varies continuously with time derivative. From Fig. 6, it is observed that the group of infected population varies continuously with time derivative. From Fig. 7 it is clear that the group of recovered people increases with time.

Conclusions

This study presents a model for Lumpy Skin Disease (LSD) within the framework of the Caputo–Fabrizio fractional derivative. Initially, we derive the existence and uniqueness theory for the assumed model, by establishing the existence and uniqueness of solutions using a fixed point approach. Subsequently, we investigate the stability of the solutions, by employing the Ulam–Hyers stability criterion. To validate our theoretical findings, we develop a numerical scheme and apply it to obtain graphical results. Our simulations yield realistic graphs, which are thoroughly explained in the numerical section of the paper. We analyze the behavior of the model under various orders of fractional derivatives, providing insights into its dynamics and implications for LSD management. We encourage readers to explore the model further by employing alternative numerical techniques and considering different fractional operators. Such investigations may offer additional perspectives and deepen our understanding of LSD dynamics and control strategies. It is an interesting open problem to study LSD dynamics under ABC fractional derivative and piecewise Caputo–Fabrizio fractional derivative.

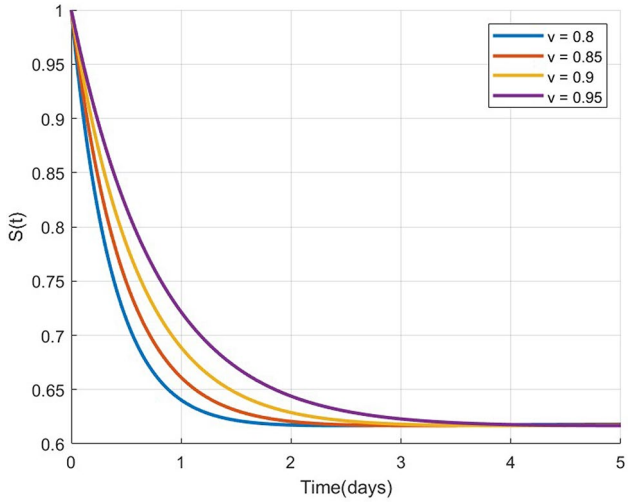

Fig. 3. Susceptible class.

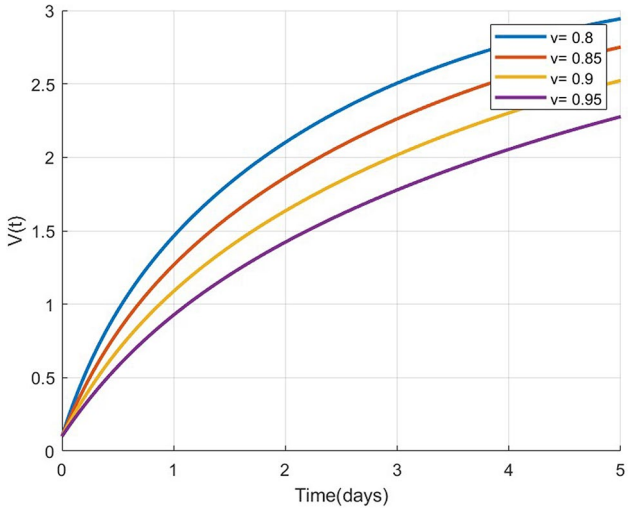

Fig. 4. Population of Vaccinated class.

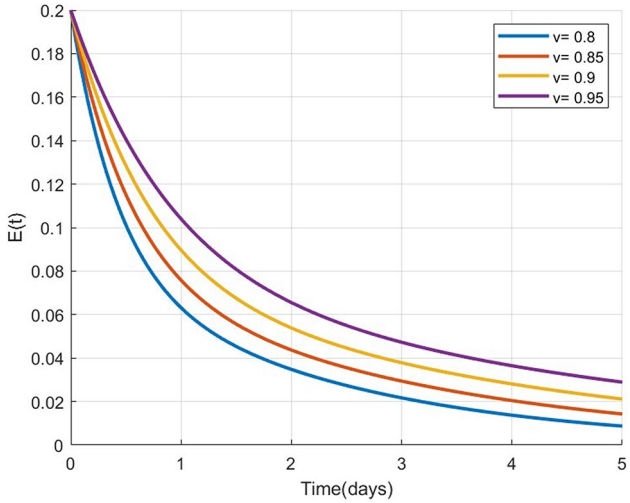

Fig. 5. Exposed class.

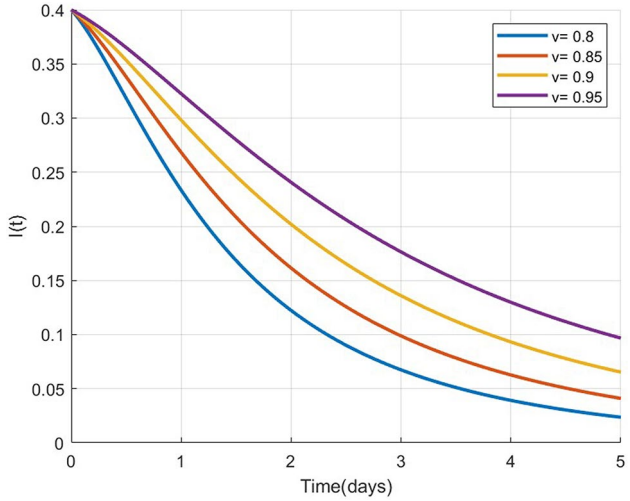

Fig. 6. Population of Infected class.

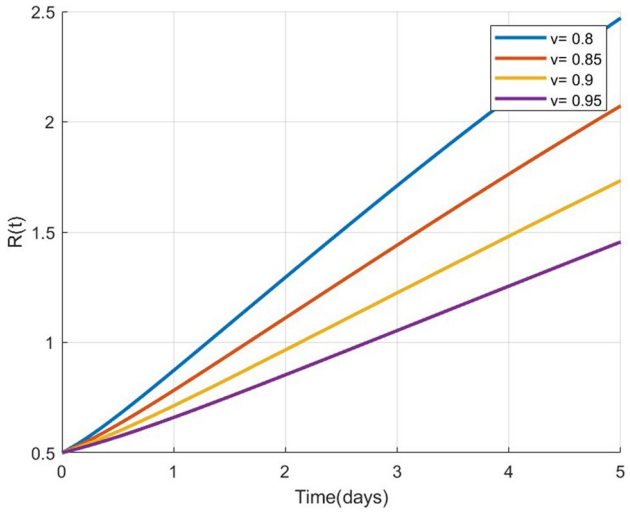

Fig. 7. Population of Recovered class.

Data availability

Data used in this work is available from the corresponding author based on a reasonable request.

Received: 31 August 2024; Accepted: 3 March 2025
Published online: 17 March 2025

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Acknowledgments

The author Abdoalrahman S.A. Omer extends the appreciation to the Deanship of Postgraduate Studies and Scientific Research at Majmaah University for funding this research work through the project number R-2025-1650.

Author contributions

G.N. Conceptualization, Methodology, Software. Writing—Original draft preparation. A.J.G: Simulations, Visualization, Investigation. S.R.: Revision, Reviewing and Editing. A.S.A. Omer: Numerical computations, results and discussion. I.K. Methodology, Software. Writing—Original draft preparation. All the authors reviewed the manuscript.

Declarations

Conflicts of interest

The authors do not have any conflict or competing interests.

Additional information

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