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Research article

Teaching strategies, cognitive factors and mathematics

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A R T I C L E I N F O

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A B S T R A C T

Mathematics teaching strategies have a positive impact on learning. However, there is a lack of studies on non-traditional approaches to early mathematics education in the specialized scientific literature. In this theoretical framework, a study to connect teaching methodology with the various cognitive processes implicated in learning has been designed. A total of 114 primary school students aged 74 and 84 months who were taught mathematics either with the method called Open Algorithm Based on Numbers or with the more traditional Closed Algorithm Based on Ciphers, participated in the study. After conducting a thorough examination of cognitive processes and early math performance using well-established assessment instruments, a comparative analysis was undertaken to explore the relationship between cognitive predictors of mathematical performance, while considering the mathematics teaching strategies used. Students were distributed according to their level of mathematical competence and teaching methodology and the type of schools (Charter or Public). The results from the multivariate statistical test showed that the teaching strategy was inconclusive for most of the cognitive factors studied. Significant differences according to mathematical performance were found for fluid intelligence, verbal short-term memory, and visuospatial working memory. Finally, no significant differences were found in the cognitive variables studied when considering the interaction between the teaching approach, school characteristics, and mathematical achievement as a reference.

1. Introduction

Interest in mathematics has been increasing in recent years, mainly due to the consequences for society of mathematical learning difficulties [1,2]. Primary school students should reach a level of mathematical abilities that ensures success in future stages of their education and eventually allows them to become skilled and competent adults for the benefit and development of the community [3].

There are several ways to improve mathematical skills [4]. The teaching approaches in mathematics have shown different theoretical and applied backgrounds, distinguishing, among others, those based on principles like active-discovery learning [5], dialogic teaching [6], or questioning-developing teaching [7]. Numerous studies offer different methods using new teaching-learning techniques [8–12] or novel instruments [13–15].

Among the procedures for teaching mathematics in the active-discovery learning domain, approaches have been developed and are regarded within the scope of Realistic Mathematics Education (RME) [16]. One such innovative mathematics teaching approach is the so-called open Algorithm Based on Numbers (ABN) based on the principles of RME [17]. ABN acronym means: (1) it is 'open' because each student can solve mathematical problems according to their own pace, abilities, and strategies; and (2) it is 'based on numbers' in

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that the students work with numbers as a whole (as a global quantity), instead of working with figures (more abstractly) as is done in traditional mathematical teaching methodologies [17,18].

ABN relates written numbers and operations on them to existing/familiar mathematical knowledge (e.g., real situations, concrete representations with manipulatives, verbal and nonverbal number knowledge). The ABN method is an example of 'realistic mathematics' theory [17,19,20]. It involves more than simply solving calculations with a novel algorithm, also aiming to allow students to approach the basic concepts in a more specific and useful manner. All students can benefit from this method as it is capable of adapting to the student's characteristics, making it very effective for those at risk of developing mathematical learning difficulties (MLD). A brief overview of the differences between the ABN and CBC methods used in this study can be seen in Table 1.[1]

Some studies have highlighted its benefits compared to the more traditional Closed Algorithms Based on Ciphers (CBC) teaching methodology [8,22–25].

Aragón et al. [23] analysed the current differences in the specific precursors of mathematical competence in students taught using the ABN and CBC methods. The results showed better performance by ABN students in Piagetian-type and estimation tasks and in terms of their ability to generally apply basic mathematical knowledge to daily activities. These findings align with previous studies underlining the differences in mathematical achievement of students who learn with other teaching methodologies. Adamuz-Povedano & Bracho-López [22] analysed the differences between the ABN and CBC methods in terms of the performance and formal or informal mathematical knowledge of 4th grade primary school students. The differences were statistically significant for calculation and numbering of informal mathematical thinking, that is when dealing with numbers in simple real-life situations using basic data involving number sequencing skills and enumeration tasks associated with cardinality. However, concerning formal mathematical knowledge, they differed significantly in terms of basic mathematical concepts and the ability to read and write quantities.

It should be noted that the implementation of ABN/CBC methods means that students learn mathematics with either method throughout the entire period of early and primary education. This is not a short-term, incidental program but a methodology chosen throughout the entire schooling process. These methods of teaching mathematics have been implemented in both public and charter schools. A charter school is a privately owned educational institution. In the social context of this study, both charter and public schools receive state funding, making them tuition-free for parents.

Likewise, in a recent study of early childhood education (five-year-old participants), Cerda et al. [8] found statistically significant differences in support of the ABN method for number line estimation, as well as in most of the informal thinking dimensions evaluated. However, in the case of four-year-old children, the differences were limited to computational tasks and informal mathematical concepts. In addition to the impact of teaching methods on specifically mathematical variables, it is interesting to analyse whether there are also differences in terms of the domain-general precursors.

Along these lines, Aragón-Mendizábal et al. [24] found that the cognitive profile of students who learned mathematics with the CBC method differed from those who were taught with the ABN method. Although they shared common features, the weight of the statistical variables in the explanation of early mathematical competence varied both qualitatively and quantitatively. The main similarity was that fluid intelligence was the factor that best explained mathematical performance for both methods. However, in the case of ABN students, this was followed by visuospatial working memory, while in the CBC group, verbal short-term memory was more significant. The authors attributed this potential difference to the fact that the demands of the teaching methods enhance different general skills, operating as a cognitive basis when solving the mathematical tasks proposed.

Consequently, it is appropriate to analyse the cognitive parameters that affect early mathematics learning to develop actions that contribute to improving teaching and learning experiences. The practical aim is to get the most out of the students' abilities to enhance the acquisition of number sense [26–30].

2. Aims of the present study

Considering the potential significance of the teaching method in the development of domain-general skills and its relationship with mathematical performance, it is relevant to analyse the relationship of these cognitive factors and variables both separately and in interaction. Our aim was to determine whether or not current differences in cognitive variables are a function of the teaching method, mathematical performance or, an interaction of both factors. The main research target was to analyse the relationship between the cognitive variables of fluid intelligence, verbal and visuospatial short-term memory, and verbal and visuospatial working memory according to the teaching method, mathematical performance, and the interaction of both aspects. Additionally, we were interested in determining the extent to which the school setting could impact the results (Fig. 1). The hypotheses proposed are as follows:

H1. The general domain cognitive variables (verbal short-term memory, visuospatial short-term memory, verbal working memory, visuospatial working memory, and fluid intelligence) explain the variability in mathematical competence, considering the teaching methodology (ABN/CBC) and the type of school (public/charter).

H2. The characteristics of the school (public/charter) and the instructional method (ABN/CBC) influence the mathematical performance evaluated with the Utrecht Early Numeracy Test-Revised (UENT-R).

[1] A more extensive explanation of the ABN method and its distinctions from the CBC approach exceeds the space available in the article. However, a comprehensive understanding can be found in the works of Cerda et al. and by Martínez-Montero [8,17], and in the PhD dissertation by Canto [21]. An extended summary of both methods can be accessed at: https://nube.uca.es/index.php/s/4Q3KvA913LejiBE.

Table 1
General differences between the Open Algorithm Based on Numbers (ABN) and Closed Algorithms Based on Ciphers (CBC) mathematics teaching methods.

Closed Algorithms Based on Ciphers (CBC)	Open Algorithm Based on Numbers (ABN)
Relies on rote memorization of rules	Processes stem from the child's reasoning
Calculations follow a fixed method	Allows flexibility with algorithms in calculations
Presents hypothetical scenarios	Real-world situations drawn from student experiences
Operations performed opaquely	Transparent format for operations
Students often struggle in written tests	Enhances problem-solving with an intuitive approach
Mathematic is frequently a disliked subject with high failure rates	Improves math motivation, leading to better results
Students follow opaque methods for solving unknowns	Encourages freedom in choosing methods, emphasizing reasoning

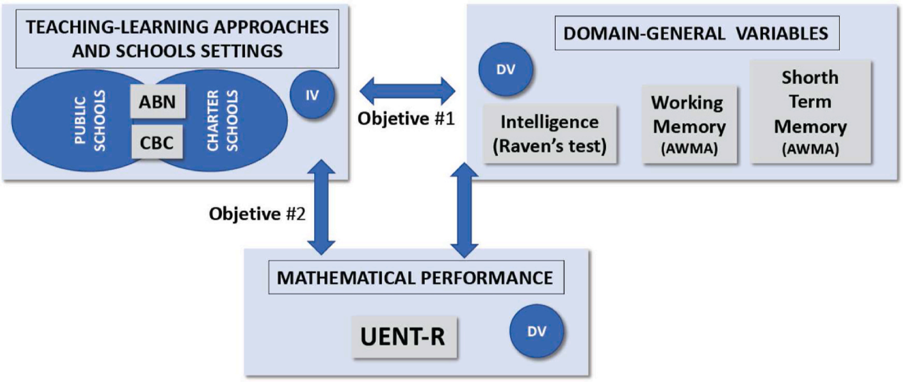

Fig. 1. Overview of the study's experimental structure and goals. AWMA: Automated Working Memory Assessment; UENT-R: Utrecht Early Numeracy Test-Revised; ABN: Open Algorithm Based on Numbers; CBC: Closed Algorithm Based on Ciphers; IV: Independent Variables; DV: Dependent Variables; CV: Control Variables; Raven's Test: Raven's Coloured Progressive Matrices.

3. Methods

In this section, the study's sample, and the different evaluations performed on the independent/dependent variables are detailed. Next, the intervention procedure is described, and, finally, the type of statistical analysis conducted on the data is outlined.

3.1. Participants

The students who participated in the study consisted of 114 first-grade primary school children aged between 74 and 84 months ($M = 77.84$, $SD = 3.24$). The participants were students from entire intact classrooms and were divided into two groups. The ABN experimental group consisted of students taught mathematics with the ABN method, ($n = 68$; $M = 76.88$, $SD = 3.33$). The CBC control group consisted of 46 students ($M = 79.26$, $SD = 2.52$) who were taught with the CBC method. Of the total sample, 53 were boys ($M = 77.74$, $SD = 3.14$) and 61 were girls ($M = 77.93$, $SD = 3.35$). The students taught with ABN comprised 31 boys ($M = 76.87$, $SD = 3.17$) and 37 girls ($M = 76.89$, $SD = 3.51$). Those taught with CBC consisted of 22 boys ($M = 78.95$, $SD = 2.73$) and 24 girls ($M = 79.54$, $SD = 2.3$). The groups demonstrated homogeneity in age and gender, and no statistically significant differences were found in these variables. The homogeneity calculations were performed through multivariate tests ($F = 0.611$; p *ns*). Intelligence assessed through the Raven test did not indicate statistically significant differences between both groups.

The students belonged to two charter schools and two public schools located in environments with equivalent socioeconomic levels. Charter and public schools have no differences in admission standards. The sampling carried out was non-probabilistic since it was necessary to select schools and classrooms that applied the ABN or CBC teaching methods from the beginning of early childhood education. Students with special educational needs according to the educational supervision teams were also evaluated, but their data were not included for statistical calculation. Six students were excluded due to having a psycho-educational assessment implemented by the school's orientation team for severe special educational needs.

3.2. Materials

Participants were assessed with different tests to determine their mathematical skills, memory, and intelligence. An overview of the assessment instruments is summarized in Table 2.

3.2.1. Utrecht Early Numeracy Test-Revised (UENT-R) [31])

This assesses nine aspects of mathematical competence in different subtests: (1) comparison of the quantitative and qualitative characteristics of objects; (2) classification of objects into classes or subclasses; (3) one-to-one correspondence; (4) seriation of objects into classes or subclasses based on criteria by using counting words forwards and backwards; (5) using numerals: structured counting (the child must count a sequence that appears disorganized or unstructured. This determines whether the child can show coordination between counting and pointing); (6) synchronous counting (in this component, while using material such as pawns, it is examined whether children master synchronous counting of quantities. In this component, children are allowed to point to the objects with their fingers while counting); (7) shortened counting from the dice structure (it is investigated whether certain dice structures are recognized immediately), and resultative counting: with this component, it is checked whether children are capable of determining the total number of both structured and unstructured set of objects) as well as counting hidden quantities; (8) Applying knowledge of numbers, ability to use knowledge of the number system in simple problems; and (9) estimation on number lines. Each subtest has five items. The UENT-R was individually administered in a 20-min session. Afterwards, the answers were scored with the aid of the UENT-R scoring key. According to Cronbach's α, the reliability of the UENT-R was α = 0.92.

3.2.2. Raven's Coloured Progressive Matrices (CPM) [33]

This classic non-verbal intelligence test measures children's fluid intelligence, assessing their ability to make sense of disorganized or confusing material by managing non-verbal constructs, that allow for the comprehension of a complex structure, and their ability to reason based on figurative stimuli regardless of cultural influences. The children are required to complete a total of 36 geometrical figure items by choosing the missing piece from among six possible drawings. Raven's test distributes its 36 items into three groups of 12 items each, and all of them are individually administered. The Cronbach's alpha was 0.82.

3.2.3. Automated Working Memory Assessment (AWMA) [32]

The AWMA evaluates verbal working memory and visuospatial memory using simultaneous information storage and processing tasks. Tasks involving only the storage of information are used to measure visuospatial short-term verbal memory. The AWMA provides standardis ed scores for three measurements for each of the four categories of short-term verbal memory, visuospatial short-term memory, verbal working memory, and visuospatial working memory. The whole test has a total of 12 tasks and is aimed at people between 4 and 22 years of age. An adaptation of the AWMA into Spanish was used [34]. Although the AWMA has a total of 12 tasks, in this study only four tasks were used as described below: (1) Verbal short-term memory (digit recall): Participants have to recall a sequence of numbers in the correct order. Cronbach's alpha was 0.88; (2) Verbal working memory (backward digit recall): Participant must remember the items in reverse order. (Cronbach = 0.89), (3) Visuospatial short-term memory (dot matrix): In the dot matrix, the child is asked to point to the squares of a 4-by-4 matrix where a sequence of red dots appears in the same order that they are presented. (Cronbach = 0.91); and (4) Visuospatial working memory (odd-one-out). The child is asked to indicate shapes in a three-square matrix as the 'odd one out' (Cronbach = 0.91).

3.3. Procedure

Two individualised evaluation sessions with a duration of 20 and 30 min respectively were carried out by trained professionals. The tests were distributed to avoid participant fatigue and were also carried out in optimal conditions in quiet places without potential distractions to allow the participants to focus on the tasks. The assessment sessions were implemented within the first three months of the academic year (September to November).

Two groups of participants were defined. Firstly, the control group consisted of students who were taught mathematics using traditional education methods (CBC), beginning in their first year of early childhood education at three years of age. This is the method

Table 2
Summary of assessment instruments used in the study.

Assessment Instruments	Abilities Assessed (# Items)	
Utrecht Early Numeracy Test UENT-R [31]	Relational subtests	Comparison (5)
		Classification (5)
		One-to-one Correspondence (5)
		Seriation (5)
	Numerical Subtests	Using numerals (5)
		Synchronous counting (5)
		Shortened counting and resultative counting (5)
		Applying knowledge of numbers (5)
		Estimation (5)
		Total Items UENT-R = 45
Automated Working Memory Assessment (AWMA) [32]	1. Verbal short-term memory (digit recall)	
	2. Verbal working memory (backward digit recall)	
	3. Visuospatial short-term memory (dot matrix)	
	4. Visuospatial working memory (odd-one-out)	
Raven's Coloured Progressive Matrices (CPM) [33]	Fluid intelligence	
	Total Items CPM = 36	

used by most schools in Spain. It focuses on teaching content based on the official curriculum and the use of books provided by publishers and adapted to the content required by the educational authorities.

Secondly, the experimental group consisted of students receiving mathematics instruction using the ABN method since their first year of early childhood education. The ABN teaching method is not a specific experience in an academic course, but rather a methodological change in teaching that must be adopted as a collaborative decision by the School Board for all educational courses and levels. One of the peculiarities of the method is that it requires specific training for the teachers who put it into practice, always keeping in mind the specific content, competencies, and objectives that the educational authorities require for each course.

A sector of the Spanish school system is currently implementing the ABN method [35]. This approach focuses on the natural path of each of the stages of the exploratory process that children go through to understand numbers and their characteristics [8]. In short, both groups of participants received the teaching of the mandatory school curriculum for each course with the same teaching timing contemplated by the Spanish Ministry of Education, with the only difference being the type of methodology used. Both ABN and CBC students received an equivalent number of hours of mathematics instruction, following the regular schedule of their classes and per the official school curriculum for school mathematics. A more extensive comparative description of ABN/CBC mathematics teaching approaches can be found in the supplementary material to this article.

When carrying out the research the pertinent ethical considerations were taken into account and the informed consent of parents, principals, and school teachers involved in the study was obtained. The study was approved by the University of Cadiz Ethics Committee for non-biomedical experimentation (Ref. 003–2022). Once the research was completed, a brief report was provided to the schools based on the results obtained to assist teachers, with pertinent educational information such as evaluation results and recommendations.

3.4. Data analysis

A mixed model was calculated using SPSS. 21. This model considers the nested structure of the data since the subjects are not independent and are grouped in contexts with particular characteristics.

4. Results

In order to confirm the hypotheses, a linear mixed model was calculated. These models consider that the early mathematical competence (EMC) as a dependent variable, evaluated with the UENT-R scales, is linearly related to fixed, random, and covariate factors. As modelling was carried out according to data hierarchy, two contextual variables were considered: 1) the instructional approach (ABN/CBC); and 2), the type of school (charter and public schools). To calculate the model, EMC was taken as the dependent variable, and fluid intelligence, verbal short-term memory, verbal working memory, visuospatial short-term memory, and visuospatial working memory scores were considered as covariates (level 1 subject variables). Table 3 shows the descriptive statistics of the variables in the calculated model.

Taking the average scores as a reference, it is observed that the students in the experimental group outperformed those in the control group in all variables studied except for verbal working memory ($M_{control}$ = 104.94; $M_{experimental}$ = 112.26).

Within the control group, students in charter schools outperformed students in public schools in four out of the six variables studied: mathematical competence ($M_{charter}$ = 32.87; M_{public} = 32.54), visuospatial short-term memory ($M_{charter}$ = 118.25; M_{public} = 112.86), verbal working memory ($M_{charter}$ = 113.33; M_{public} = 111.09), and fluid intelligence ($M_{charter}$ = 23.79; M_{public} = 23.73).

Table 3
Descriptive statistics, including the mean and standard deviation (sd), were computed, taking into account the hierarchical data structure related to the Experimental (ABN)/Control (CBC) mathematical teaching methodologies and the distinction between Public or Charter schools.

Dependent Variables	Experimental (ABN)			Control (CBC)			Total		
	Charter (n = 47)	Public (n = 21)	Total (n = 68)	Charter (n = 24)	Public (n = 22)	Total (n = 46)	Charter (n = 71)	Public (n = 43)	Total (n = 114)
	Mean (sd)			Mean (sd)			Mean (sd)		
1	32.53 *(5.97)*	33.80 *(5.07)*	32.92 *(5.70)*	32.87 *(3.31)*	32.54 *(3.47)*	32.71 *(3.35)*	32.64 *(5.20)*	33.16 *(4.32)*	32.84 *(4.87)*
2	95.37 *(12.27)*	95.70 *(11.82)*	95.48 *(12.04)*	93.58 *(9.73)*	93.63 *(9.98)*	93.60 *(9.74)*	94.77 *(11.43)*	94.64 *(10.84)*	94.72 *(11.16)*
3	115.44 *(14.75)*	119.14 *(12.66)*	116.58 *(14.15)*	118.25 *(13.86)*	112.86 *(9.56)*	115.67 *(12.18)*	116.39 *(14.42)*	115.93 *(11.49)*	116.211 *(3.34)*
4	105.80 *(11.94)*	103.00 *(13.18)*	104.94 *(12.31)*	113.33 *(10.44)*	111.09 *(11.40)*	112.26 *(10.85)*	108.35 *(11.93)*	107.13 *(12.82)*	107.89 *(12.23)*
5	116.82 *(11.62)*	117.66 *(11.87)*	117.08 *(11.62)*	114.87 *(12.04)*	116.86 *(9.85)*	115.82 *(10.97)*	116.16 *(11.72)*	117.25 *(10.76)*	116.57 *(11.33)*
6	25.5 *(14.23)*	24.90 *(2.77)*	25.32 *(3.83)*	23.79 *(4.16)*	23.73 *(4.03)*	23.76 *(4.05)*	24.93 *(4.25)*	24.30 *(3.48)*	24.69 *(3.98)*

Dependent Variables: 1. Mathematical Competence; 2. Verbal short-term memory; 3. Visuospatial short-term memory; 4. Verbal working memory; 5. Visuospatial working memory; 6. Fluid intelligence; Charter: Charter schools; Public: Public schools.

However, in the experimental group, there were only two variables in which students from charter schools outperformed those from public schools: verbal working memory ($M_{charter} = 105.80$; $M_{public} = 103.00$) and fluid intelligence ($M_{charter} = 25.50$; $M_{public} = 24.90$).

The calculation of parameter estimates employed the restricted maximum likelihood method (REML), with adjustments made for fixed effects. This procedure made smaller standard errors than a method of maximum likelihood and considered the degrees of freedom used to estimate the fixed effects, unlike the one mentioned above (Table 4).

Considering the fixed effects, variables with a higher statistical explanatory power on mathematical competence were visuospatial working memory, verbal short-term memory, and fluid intelligence, considering a confidence interval of 95 %.

On the other hand, Table 5 shows the estimation effect of the variables associated with the type of school and the instruction method. These variables were introduced as grouping variables, and considered as a random effect, respecting the hierarchical data structure to evaluate the fixed effects of the covariates. The findings suggest that there is no significant influence of the null hypothesis, type of school, and teaching method on mathematical performance, and thus, it can be accepted. They also did not have a determining effect on the covariates and their fixed effects.

5. Discussion

5.1. Summary of key ideas discussed

Studies of the methods for mathematics teaching and the predictors affecting its learning have been a common research topic in recent years. This study pursued to address issues related to both topics, with potential benefits for students at risk of suffering MLD. Regarding the hypotheses posed (H1 & H2), it was determined that the general domain variables fluid intelligence, verbal short-term memory, and visuospatial working memory explained the variability of mathematical competence considering the teaching methodology (ABN/CBC) and the type of school (public/charter). However, visuospatial short-term memory and verbal working memory were not explanatory of mathematical competence.

5.2. Discussion of results (hypotheses 1 and 2)

Concerning the first hypothesis (H1), students from the ABN group who had a high mathematical performance also had a higher performance for the domain-general precursors evaluated. Their results were higher than the ABN group for most of the variables (except fluid intelligence). Thus, these differences between the two groups of children concerning verbal working memory are noticeable. In this sense, it can be considered that perhaps the strength of the associations between verbal working memory and mathematical abilities has weakened [36]. Responding to arithmetic computations requires several threads: temporary storage of phonological information, retrieval of calculation rules from long-term memory, preserving intermediate values, and, finally, outputting the results. These processes are supported by the working memory system to temporarily store information while performing mental operations. It therefore seems that the differences between the groups (in the components of the UENT-R) cannot be explained by verbal working memory.

Verbal and visuospatial working memory are closely related, yet they may be differentiated [37]. Visuospatial working memory has a greater influence when the task has clear visual features related to the handling and manipulation of data [38]. This is related to evidence suggesting that different brain areas are activated in tasks requiring the handling of data [39]. Although a neurological appraisal is beyond the scope of the current study, this characteristic may be a means of differentiating between students taught with ABN or CBC. The CBC method exposes the students to more abstract content tasks and language is a critical facilitator, potentially contributing to the development of greater relevance of verbal working memory. Similarly, in problem-solving the characteristic of superficial processing typical of traditional tasks and problems requires a certain verbal ability to analyse the problem and search for expressions and keywords [40]. This effect may be due to the instructional tasks carried out under the ABN method. In their early years, schoolchildren work with objects to allow them to grasp mathematical concepts and reduce their abstract character [4,18]. Students with better results in mathematics taught with the ABN method stand out in terms of visuospatial skills, more closely linked to representations of mathematical concepts, which is likely due to the intrinsic characteristics of the ABN method [4].

Hamamouche et al. [41] studied the advantages of sharing scenarios for problem-solving with students between 37 and 72 months of age. Complex arithmetic tasks such as partitioning are particularly sensitive to this procedure. Likewise, they underlined the importance of generating action-based rather than verbal responses. When tasks are contextualised, action-based responses are more

Table 4
Fixes effects estimates.

Parameter	Estimation	Standard Error	*t*
Intersection			−0.359
Verbal short- term memory	−1.83	5.10	2.85*
Visuoespatial short- term memory	0.109	0.038	0.036
Verbal working memory	0.001	0.031	0.007
Visuoespatial working memory	0.000	0.031	3.66**
Fluid intelligence	0.135	0.037	3.23*

*p < 0.01; **p < 0.001; Dependent Variable: Mathematical Competence (UENT-R).

Table 5
Estimates of covariance parameters.

Parameters		Estimation	Standard error	Z
Residual		15.138	2.079	7.281**
Intersection [subject = group * type of school]	Variance	0.109	0.476	0.229

**$p < 0.001$; Note. Dependent Variable: Mathematical Competence (UENT-R).

precise than verbal responses. The ABN method widens the differences between the students based on their performance in terms of visuospatial working memory and verbal short-term memory. However, these parameters were lower for the CBC group with a different mathematical outcome. On the other hand, fluid intelligence was also a differentiating variable between the ABN and CBC groups. Fluid intelligence establishes analogies between spatial representations to solve problems without acquired knowledge. This may be especially relevant in the case of the ABN methodology. Fluid intelligence may be enhanced by contextualised teaching that draws from the student's everyday life, and its general benefits may be extended to other school competencies. In addition, as children receive more formal education and acquire more learning experiences, mathematical improvement can promote the development of fluid intelligence [42].

Results indicated that at the beginning of primary school (1st grade), students who had high mathematical achievement also excelled in most of the cognitive variables studied, regardless of the teaching method. These results are comparable to those of the longitudinal study over eight years (preschool to 8th grade of primary school) by Geary et al. [43]. In this study, domain-general skills were strong predictors throughout the entire schooling period. Furthermore, these cognitive predictors were more significant than domain-specific predictors in the early years, matching each other in terms of relevance as schooling progressed. In the current study, children with the lowest scores in UENT-R (they should be considered at risk of MLD) showed weaker performance in these cognitive abilities, in contrast to their peers with higher mathematical performance [44]. More specifically, fluid intelligence skills, verbal short-term memory and visuospatial working memory emerged as variables associated with variability in mathematical performance [37,45,46].

Regarding the second hypothesis (H2), and as a general conclusion derived from the linear mixed model, no significant differences were observed resulting from the interaction of both categorical factors. The model applied indicated that when the teaching method (ABN/CBC) and the type of school (public/charter) were used as grouping factors, they did not exert a significant influence on mathematical competence assessed with the UENT-R at an early age. However, evidence regarding the impact of the method has been documented in prior studies involving students aged 5 [8]. These disparities persist into primary education (1st grade) when the variable 'type of school' is utilized as a control variable in the mixed model. Nevertheless, when the type of school (public/charter) is introduced as a 'contextual variable,' the model reduces the influence of the teaching approach on academic performance.

5.3. Practical implications, outlook, and future research

In the current study, domain-general predictors were positively related when mathematical performance was high at an early age, granting them the role of a learning facilitator [30,47]. Consequently, training in domain-general skills at these early ages would improve mathematical competence. Therefore, it would be relevant to include them as a general pedagogical tool jointly with specific mathematical skills in order to obtain positive academic results [48,49]. Likewise, early mathematical skills clustered under the 'number sense' construct constitute a difficulty index in mathematics at higher academic levels [50]. According to Geary et al. [43] they increase in significance as students' schooling progresses [51]. This supports the implementation of preventive educational interventions or programmes for students at risk of mathematical difficulties to avoid them suffering academic problems in the future [52,53]. These intervention programmes should include activities focusing on the development of both domain-general and domain-specific skills [14]. Future studies could include children matched in verbal working memory as a control variable to examine differences between students with different teaching methods (ABN vs CBC).

Another practical consequence derived from the study concerns the implications it may have for teacher training. The development of a professional training program for future teachers should be based on innovative elements extracted from evidence [54]. Rather than mechanically and uncritically applying early mathematics intervention programs, they should be grounded in scientific evidence [55]. The teacher professional development program design may be useful in other (research) contexts to improve mathematics teaching-learning approaches, and evidence-based tools developed could help build a framework for dynamic early mathematics teaching.

5.4. Study limitations

Finally, certain study limitations should be highlighted. Firstly, the procedure for selection of the sample was intentional given that a selection of students taught using the ABN and CBC methods was required. Another limitation was the small number of first-grade participants. We are aware that if the sample had been larger and with a greater age range the results would be more generalised. This opens up a possibility for future research.

5.5. Conclusions

Different methods of teaching mathematics at an early age are being experimented with in different school settings. This study has revealed the following conclusions:

The characteristics of the school (public/charter) and the method of instruction (ABN/CBC) were not determinants of mathematical performance. Therefore, regardless of the school grouping, students' mathematical competence was influenced by cognitive factors, such as verbal short-term memory, visuospatial working memory, and fluid intelligence. These differences were notable in verbal short-term memory and visuospatial working memory in favor of students following the ABN approach. However, students learning with the CBC methodology showed better performance in fluid intelligence. Consequently, both methods can support either cognitive ability as they stimulate differential aspects in the variables contributing to early mathematics learning.

Therefore, the educational actions or strategies to be followed must be aimed at strengthening general skills that have a strong influence not only in mathematical competence but also in other school abilities. The benefits derived from this cognitive stimulation may also spread improvements in the performance of schoolchildren beyond the educational measures related to the pedagogical or institutional field.

Ethics statement

Consent from parents, principals, and school teachers involved in the study was obtained. The study received approval from the University of Cadiz Ethics Committee for non-biomedical experimentation (Ref. 003–2022) and adheres to the Declaration of Helsinki.

Data availability statement

The data are accessible through the University of Cadiz repository at: https://nube.uca.es/index.php/s/PGzGT8wTIdJ6tr3 (File: "Learning114.sav")

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CRediT authorship contribution statement

Estívaliz Aragón: Writing – original draft, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Inmaculada Menacho:** Writing – original draft, Methodology, Investigation, Conceptualization. **José I. Navarro:** Writing – original draft, Investigation, Funding acquisition, Formal analysis, Conceptualization. **Manuel Aguilar:** Writing – original draft, Methodology, Investigation, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.heliyon.2024.e29831.

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