Original Article

Determination of square equivalent field for rectangular field in electron therapy

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ABSTRACT

Equivalent field for electron beams is considered by using pencil beam theory. According to the Fermi-Eyges model the dose distribution of an electron pencil beam has a Gaussian profile. For this function determination of mean square radial displacement scattering of electrons is important. In this study the contribution of back scatter electron has been taken into account by using the multiple scattering theories for calculating mean square radial displacement scattering. The dimension of standard equivalent field depends on depth and shape of treatment field. Here the depth under study is the depth that mean square radial displacement scattering is extremum and the shape of treatment field is rectangular. In this study four energies were used 6, 9,12 and 15 MeV electron beams of 2100C/D Varian Linac. Findings of this study are based on analytical calculations, which are in good agreement with other experimental data. The findings of this study that were resulted from formula, shows, for all circular fields of radius ≥LSE (lateral scattering equilibrium) were considered broad field and equivalent. For validating the findings, Percentage Depth Dose (PDD) and Output factors were measured in 15 MeV electron beams for 7 × 3-cm, 6 × 4-cm and 4 × 2-cm and their equivalent squares and equivalent circular fields and compared.

Key words: Dosimetry, electron therapy, field equivalence, mass angular scattering power, pencil beam theory

Introduction

The pencil beam theory has been established in considering of the behavior of electron beams.[1,2] By using Fermi-Eyges model, one can show that the distribution of an electron pencil beam has a Gaussion profile.[3] It is well known that a fundamental parameter of the pencil beam method of electron beam treatment planning is mean square radial displacement scattering $\sigma_r^2(z)$. This parameter has been determined by several methods and tabulated.[4,5] In different studies they have been stated that the definition of equivalent field for electron beams is not generally possible.[1,6] On the other hand in several researches determination of equivalent field for electron beams are investigated by using Fermi-Eyges multiple scattering theory.[6,9] In this research the standard equivalent field of rectangular fields have been calculated by minimizing of $\sigma_r^2(z)$ in different energy electron beams of 2100C/D Varian Linac and compared with other studies.

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Materials and Methods

The dose distribution of electron pencil beam in a phantom looks like an onion. The lateral spread followed the Gaussian function in a depth. Gaussian function is characterized by $\sigma_r(z)$ which shows the raduis extending gaussian function, as well as scatter radius of electron pencil beam in water phantom. The scatter spread parameter $\sigma_r(z)$ was theoretically predicted by Eyges.[10]

$$\sigma_r^2(z) = \int_0^z \frac{\overline{\theta^2}}{\rho l}(z')\rho(z')(z-z')^2\,dz' \qquad .....(1)$$

$\frac{\overline{\theta^2}}{\rho l}$ Is the mass angular scattering power in the water, ρ is density of the slab phantom and z is depth. But there are limitations to the Eyges equation. As pointed out by Werner, Khan and Deible[3,5] σ that was given by Eyges equation, increases with depth infinitely, which is contrary to what is observed experimentally, also Eyges equation is based on small-angle multiple coulomb scattering and under estimate large angle scattering. One can solve this problem by changing the limitations of integral to 0 to practical range *Rp*.

$$\sigma_r^2(z) = \int_0^z \frac{\overline{\theta^2}}{\rho l}(z')\rho(z')(z-z')^2\,dz' \qquad .....(2)$$

By using the values of $\frac{\overline{\theta^2}}{\rho l}$ from ICRU report 21 one can solve the above integral for practical range *Rp* at 6,9,12 and 15 MeV energies of Varian 2100C-D linear accelerator. The limits of this integral should be 0 to *Rp*, because of the contribution of scattering electron underlying depth z has been taken into account at depth z. In first approximation of the quantity $\frac{\overline{\theta^2}}{\rho l}$ is almost independent of depth z[5]. From Eq.2 one can obtain.

$$\sigma_r^2(z) = \int_0^z \frac{\overline{\theta^2}}{\rho l}(z')\rho(z')(z-z')^2\,dz' = \qquad ..... (3)$$

$$-\frac{1}{3}\left[\frac{\theta^2}{\rho l}\right]\left[(z-R_p)^3 - z^3\right]$$

From Eq.3 it seems that $\sigma_r^2(z)$ has a maximum value at $z = \frac{R_p}{2}$, as the following:

$$\left[\sigma_{\max}\left(z = \frac{R_P}{2}\right)\right]^2 = \frac{1}{12}\frac{\overline{\theta^2}}{\rho l}R_p^3 \qquad .....(4)$$

The energy dependency of $\sigma_r(z)^2$ is obvious from Eq.4 by R_p.

The depth dose at central axis for irregular shaped field is calculated as the following form.

$$D_{irreg}(0,0,z) = D_\infty(0,0z)\frac{\Delta\theta}{2\pi}\sum_{i=1}^{n}\left[1 - Exp\left(\frac{-R_i^2}{\sigma_r^2(z)}\right)\right] =$$

$$D_\infty(0,0,z)\frac{1}{2\pi}\int_0^{2\pi}\left[1 - Exp\left(\frac{r^2}{\sigma_r^2(z)}\right)\right]d\theta. \qquad .....(5)$$

Where the field is divided into *n* sectors at angular intervals of $\Delta\theta$ and R_i is the radius of the i[th] sectors and $D_\infty(0,0,z)$ is the central axis depth dose per unit incident flounce for an infinitely wide parallel beam($2a, 2b \geq \sigma(z)$). $\sigma_r(z)$ is the root mean square radial spread of the Gaussian pencil beam as a function of depth.

For a beam of circular cross section of radius R, the central axis depth dose distribution Eq.5 is given by.

$$D_R(0,0,z) = D_\infty(0,0,z)\left[1 - exp\left(\frac{-R_{eq}{}^2}{\sigma_r^2(z)}\right)\right]. \qquad .....(6)$$

With equalizing Eqs.5 and 6 and solving the integrals for square and rectangular fields, the equivalent radius for rectangular fields (2a × 2b) of Figure 1 becomes:

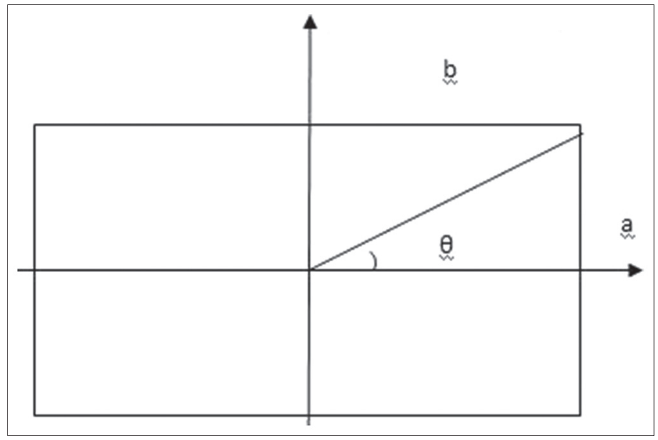

Figure 1: Rectangular field (2a × 2b)

$$R_{eq}^2 = -\sigma^2 \ln\frac{2}{\pi}\times \left[\int_0^{\tan^{-1}(\frac{b}{a})} \exp\left(\frac{-a^2}{\sigma^2\cos^2(\theta)}\right)d\theta + \int_{\tan^{-1}(\frac{b}{a})}^{\frac{\pi}{2}} \exp\left(\frac{-b^2}{\sigma^2\sin^2(\theta)}\right)d\theta\right] \qquad .....(7)$$

Lateral scattering equilibrium was calculated by khan formula to determine of limits to define the equivalent fields.[8]

Jaws were opened 14 × 14 cm for 10 × 10 applicator and Nine cerrobend cutouts fields (7 cm × 3 cm, 6 cm × 4 cm and 4 cm × 2 cm Rectangular fields, their equivalent circular and square fields) were used to perform dosimetry.

Dosimetric measurements were performed for a 15 MeV beam from Varian 2100 C/D linac, at 100 cm source-to-surface distance (SSD), using CC13 ionization chamber (0.13 cm^3 volume, total active length 5.8 mm, cylinder length 2.8 mm, inner diameter of cylinder 6.0 mm, wall thickness 0.4 mm, diameter of inner electrode 1.0 mm and length of inner electrode 3.3 mm) in a 50 × 50 × 50 cm^3 Scanditronix water phantom. The isodose curves, PDD and Output factor for each cutout were drawn and tabulated by omnipro-accept and Excel software.

Results

The values of σ have been calculated from Eq.4 by using $\frac{\overline{\theta^2}}{\rho l}$ from ICRU report 21 and practical range R_p for 6, 9, 12 and 15 MeV energies of electron beam for Varian 2100C/D Linac were determined. The data are tabulated in Table 1.

From Eq.7 the equivalent radiuses for rectangular field at 6, 9, 12 and 15 MeV energies are calculated and tabulated in Tables 2-5.

An equation was fitted to main diagonals of data

Table 1: Values of σ² for 6, 9, 12 and 15 MeV

E (MeV)	*6*	*9*	*12*	*15*
$\frac{\theta^2}{\rho l}(Radian^2 cm^2 gr^{-1})$	1.87×10^{-1}	9.45×10^{-2}	6.03×10^{-2}	3.67×10^{-2}
R_p(cm)	2.9	4.4	5.9	7.5
σ^2 (Radian² cm²)	0.38	0.67	1.03	1.3

Table 2: R_{eq} for rectangle field sizes, E=6 MeV and σ² =0.38 (*radian² cm²*). 2a and 2b (cm) are the sides of rectangle

2b	*2a*			
	1	*2*	*3*	*4*
1	0.56			
2	0.71	1.09		
3	0.72	1.20	1.60	
4	0.72	1.21	1.68	2.10

Table 3: R_{eq} for rectangle field sizes, E=9 MeV and σ² =0.67 (*radian² cm²*) 2a and 2b (cm) are the sides of rectangle

2b	*2a*				
	1	*2*	*3*	*4*	*5*
1	0.56				
2	0.74	1.11			
3	0.79	1.26	1.63		
4	0.80	1.29	1.75	2.14	
5	0.80	1.29	1.76	2.24	2.64

Table 4: R_{eq} for rectangle field sizes, E=12 MeV and σ² =1.03 (*radian² cm²*) 2a and 2b (cm) are the sides of rectangle

2b	*2a*					
	1	*2*	*3*	*4*	*5*	*6*
1	0.56					
2	0.76	1.11				
3	0.84	1.30	1.65			
4	0.86	1.36	1.81	2.16		
5	0.86	1.36	1.84	2.30	2.67	
6	0.86	1.37	1.85	2.32	2.79	3.17

Table 5: R_{eq} for rectangle field sizes, E=15 MeV and σ² =1.3 (*radian² cm²*). 2a and 2b (cm) are the sides of rectangle

2b	2a						
	1	2	3	4	5	6	7
1	0.56						
2	0.77	1.12					
3	0.86	1.32	1.65				
4	0.89	1.39	1.83	2.18			
5	0.9	1.41	1.89	2.34	2.69		
6	0.9	1.41	1.9	2.37	2.83	3.19	
7	0.9	1.41	1.9	2.37	2.85	3.32	3.69

Tables 2-5 by Matlab software with license No.161052. This equation shows the simple relation between R_{eq} and a_{eq} as the following form:

$$a_{eq}(cm) = 2R_{eq}(cm) - (0.0134E(Mev) + 0.13) \quad .....(8)$$

Equation 8 would be true if 2a is less than Lateral Scattering Equilibrium (2a ≥ LSE) because when the sides of a rectangular field were larger than LSE, all fields were considered broad fields and are equivalent.[8]

For example, according to Table 5 for a 6 cm × 4 cm rectangular field size, equivalent radius derived R_{eq}= 2.4 and from Eq.8 a_{eq}= 4.4 cm.

For validating the tabulated data, dosimetric measurements (PDD and Output factors) were performed in 15 MeV electron beam for 7 cm × 3 cm, 6 cm × 4 cm and 4 cm × 2 cm Rectangular field and their equivalent circular and equivalent square fields, for comparing measured data were tabulated in Tables 6-9. Also this comparison is achieved for percentage depth dose by the PDD curve [Figures 2-4].

Discussion

The calculated σ² for 6, 9, 12 and 15 MeV at $z = \frac{R_p}{2}$ are in good agreement by practical data that was resulted by Khan and Brunivis[1,11] By applying the pencil beam theory and Equations (5), (6) for irregular and circular field one can derive R_{eq} and a_{eq} for any arbitrary shaped electron field.

In this study for deriving equivalent field, the sector integration method for rectangular field is applied. Khan and Higgins have applied Gaussian pencil beam theory to this problem and derived an equation that can be used to

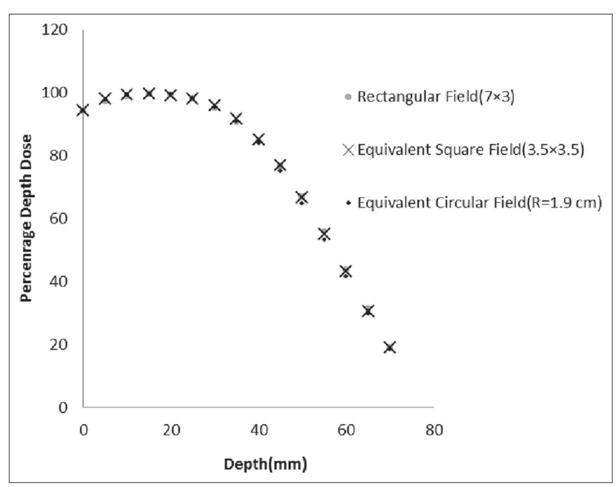

Figure 2: Comparison of depth dose distribution of rectangular field 7 × 3 cm and equivalent circular field R = 1.9 cm and equivalent square field 3.5 × 3.5 cm at 15 MeV

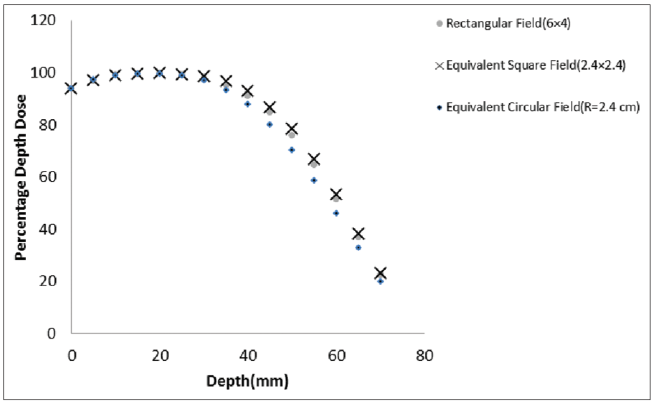

Figure 3: Comparison of depth dose distribution of rectangular field 6 × 4 cm and equivalent circular field R = 2.4 cm and equivalent square field 4.4 × 4.4 cm at 15 MeV

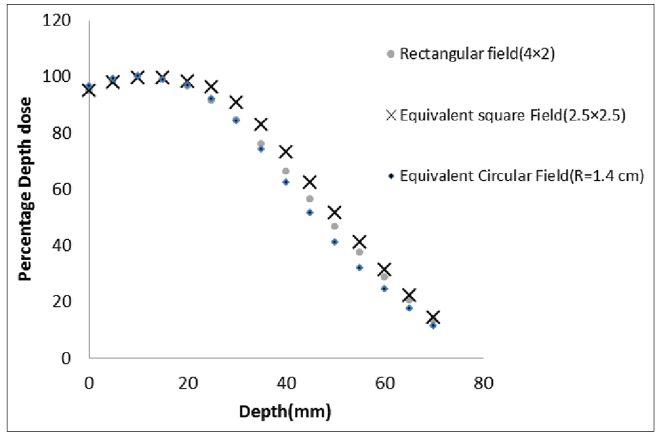

Figure 4: Comparison of depth dose distribution of rectangular field 4 × 2 cm and equivalent circular field R = 1.4 cm and equivalent square field 2.5 × 2.5 cm at 15 MeV

Table 6: Measured depth dose distribution of rectangular field 7×3 cm and equivalent circular field r=2 cm and equivalent square field 3.5×3.5 cm for 15 MeV

Depth (mm)	*PDD for rectangular field 7×3 (cm)*	*PDD for equivalent square field 3.5×3.5 (cm)*	*% error*	*PDD for equivalent circular field (radius=2 cm)*	*% error*
0	93.98	94.36	0.40	94.35	0.39
5	97.33	98.12	0.81	97.72	0.40
10	98.95	99.39	0.44	99.37	0.42
15	99.51	99.7	0.19	99.6	0.09
20	99.29	99.21	0.08	99.29	0
25	98.01	98.14	0.13	98.31	0.30
30	95.26	95.89	0.66	95.42	0.16
35	90.72	91.82	1.21	90.87	0.16
40	84.3	85.12	0.97	84.03	0.32
45	76.37	77.01	0.83	75.26	1.45
50	66.87	66.63	0.35	64.78	3.12

PDD: Percentage depth dose

Table 7: Measured depth dose distribution of rectangular field 4×2 cm and equivalent circular field r=1.4 cm and equivalent square field 2.5×2.5 cm for 15 MeV

Depth (mm)	*PDD for rectangular field 4×2 (cm)*	*PDD for equivalent square field 2.5×2.5 (cm)*	*% error*	*PDD for equivalent circular field (radius=1.4 cm)*	*% error*
0	95.04	95.15	0.11	96.82	1.87
5	98.22	98.12	0.10	99.33	1.13
10	99.8	99.57	0.23	100.27	0.47
15	98.91	99.5	0.59	99.01	0.10
20	96.56	98.36	1.86	97.02	0.47
25	91.28	96.34	5.54	92.1	0.89
30	84.7	90.92	7.34	84.18	0.61
35	75.98	82.91	9.12	74.08	2.50
40	66.3	73.1	10.25	62.49	5.74
45	56.47	62.34	10.39	51.47	8.85
50	46.68	51.68	10.71	41.2	11.73

PDD: Percentage depth dose

Table 8: Measured depth dose distribution of rectangular field 6×4 cm and equivalent circular field r=2.4 cm and equivalent square field 4.4×4.4 cm for 15 MeV

Depth (mm)	*PDD for rectangular field 6×4 (cm)*	*PDD for equivalent square field 4.4×4.4 (cm)*	*% error*	*PDD for equivalent circular field (radius=2.4 cm)*	*% error*
0	93.78	93.75	0.03	93.66	0.12
5	96.64	96.96	0.33	97.13	0.50
10	99.01	98.76	0.25	98.83	0.18
15	99.39	99.45	0.06	99.37	0.02
20	99.9	99.7	0.20	99.34	0.56
25	99.01	99.21	0.20	98.78	0.23
30	97.99	98.48	0.50	96.83	1.18
35	95.18	96.53	1.41	93.09	2.19
40	91.02	92.8	1.95	87.72	3.62
45	84.56	86.71	2.54	79.93	5.47
50	75.72	78.45	3.60	70.3	7.15

PDD: Percentage depth dose

find approximation equivalence circular or square fields of any shaped.

According to the definition of equivalent field, each two fields have the same percentage depth dose on central axis are equivalent. From the Tables 2-5 one can obtain the equivalent radius and finally by applying Eq.8, equivalent square field was derived. The dosimetric measurement (PDD and Output factor) validated the tabulated data with good agreement.

Conclusions

Findings of this study are based on analytical which is in good agreement with other semi empirical.[8] This method was suggested to use in treatment planning system (TPS) for calculating the equivalent square field for rectangular treatment field in electron therapy.

Table 9: Measured output factors of rectangular, equivalent square and equivalent circular field for 15 MeV electron beam at 3.7 cm

Rectangular field	*Output factor*	*Square equivalent field*	*Output factor*	*% error*	*Radius of equivalent circular field*	*Output factor*	*% error*
7×3	0.88	3.5×3.5	0.88	0	2	0.89	1.13
4×2	0.70	2.5×2.5	0.72	2.85	1.4	0.72	2.85
6×4	0.95	4.4×4.4	0.94	1.05	2.4	0.94	1.05

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