



Research article

Simulations of the Water Food Energy Nexus for policy driven intervention



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ABSTRACT

Water-Food-Energy (WFE) resources exert mutual influences upon each other and thus cannot be managed separately. Information on household WFE expenditures addresses knowledge that distinguishes between geo-spatial districts' social welfare. Social welfare and investment in districts' WFE resources are interconnected. District (node) product of WFE normalized expenditures (Volume) is considered as a representative WFE Nexus holistic quantity. This Volume is assumed to be a function of residents' knowledge of welfare level across districts. We prove that the Volume rate conforms to Boltzmann entropy, and this is the premise of our hypothesis for directed information from high to low welfare between network nodes. Welfare mass (WM) represents the district's Volume combined with its income and population density. This WM is used as input into a model balancing between all domain nodes that allows policymakers to simulate the effects of potential quantifiable policy decisions targeted to individual districts at a domain level while also considering influences between districts. Based on existing historic data, the established tool exemplifies its potential by providing outcomes for Israel districts showing the influence of imposing different temporal allocation/deallocation actions as managerial regulations to prescribed districts. It is found that districts with a high WM do not suffer when a defund is applied, but districts that have a low WM gain from subsidies.

1. Introduction

The questions of population dynamics, especially the question of whether the Earth has enough resources to sustain human population, have been considered for centuries and are still being addressed today (Malthus, 1798; Meadows et al., 1972; Motesharrei et al., 2014). In recent years, the concerns voiced in these works have started to enter the agendas of national governments worldwide. For example, Slovenia recently added a right to water for all citizens to its constitution (Lubl-jana, 2016), and Scotland is considering doing the same for food (Bews, 2016). The United Nations is also devoting significant attention to these issues, as exemplified by (Connor, 2015).

Within discussions on water resource management, there is a growing consensus that resources such as water, food, and energy exhibit mutual influence on each other. The term for the system consisting of those three components is the Water-Food-Energy (WFE) Nexus. Moreover, the benefits of managing these three mutually influencing resources in a coordinated way are being realized (FAO, 2014).

Previous works have taken a variety of approaches to discussing management in terms of the WFE Nexus. For example, Biggs et al. (2015) addressed the question by focusing on livelihoods, and create a

framework for measuring “environmental livelihood security of whole systems”. Hussien et al. (2018) combined risk-management techniques and seasonal variability considerations to evaluate resource conservation strategies in the context of the Nexus. White et al. (2018) evaluate East Asian trade and resource security in terms of virtual water and tradeoffs between resource consumption, economic growth, and environmental degradation. Villarroel Walker et al. (2014) conducted a city-scale analysis on the urban metabolism of greater London, with a focus on specific resources (nitrogen, carbon, etc.), and substantial opportunities for resource recovery and revenues. Garcia and You (2016) use a process systems engineering approach to analyze a pair of examples of the Nexus and propose directions for further research. Fuentes-cortes et al. (2019) use economic analysis to argue for more nuanced pricing schemes in regulation of WFE Nexus systems.

We are not aware of a work that addresses social welfare in terms of one combined quantity representing all three components of the WFE Nexus. Also, this work addresses the WFE Nexus in terms of household expenditures, i.e. an “end result” of all Nexus-related policies and events. Lastly, we are also unaware of another work that uses a system of ordinary differential equations (ODEs) to express Welfare Mass balance across nodes to allow for planning and prediction in a social welfare

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context. Thus this work aims to develop a macro-scale quantitative regulation methodology that is global in scope.

2. Management model

2.1. Goals

In what follows we develop a macro-management theory that uses as its premise the concept of Water-Food-Energy (WFE) Nexus consumption.

The model aims to influence social welfare implied consumption connected with resource security both influencing environmental sustainability in terms of the WFE Nexus, as well as to provide a quantitative framework which can be used to inform regulation and decision-making. In other words, the aim of this framework is to help decision-makers take into account the WFE Volume, representing the composite of the three components of the Nexus during their decision-making process.

2.2. WFE Expenditure Volume

The core idea of the model developed here is to create a combined representative measure for the WFE Nexus. The rationale is that if we can represent the Nexus using just one number, this should provide clarity and assist in the decision-making process.

The first step in creating the combined WFE Volume quantity for a given district i at k time level is to normalize each expenditure value by that district's initial-time value common to all districts. Namely,

$$\widehat{W}_i^k = \left(\frac{W^k}{W^{k_0}}\right)_i, \widehat{F}_i^k = \left(\frac{F^k}{F^{k_0}}\right)_i, \widehat{E}_i^k = \left(\frac{E^k}{E^{k_0}}\right)_i \quad (1)$$

where k_0 denotes the initial time level. It is important to note that a district can be a geospatial zone of any size. In this work, the data available was based on predefined districts within the state of Israel, each of which contained multiple municipalities (roughly analogous to a county in the United States). However, a district could be as small as a neighborhood or as large as a multinational region.

The next step is to combine the three normalized components of (1) by multiplying them together:

$$\widehat{U}_i^k = (\widehat{W}\widehat{F}\widehat{E})_i^k \quad (2)$$

As this quantity is the product of three components, we refer to it as the WFE Expenditure Volume. One appealing attribute of the WFE Volume is that it is holistic in that it captures the movement of any of its three components. Consider the example in which W_i changes from one time level to the next by a factor of α , e.g. $\widehat{W}_i^k = \alpha\widehat{W}_i^{k-1}$, while, the other two components remain constant over the same time span, i.e. $\widehat{E}_i^k = \widehat{E}_i^{k-1}$ and $\widehat{F}_i^k = \widehat{F}_i^{k-1}$. From (2) we obtain $\widehat{U}_i^k = \alpha\widehat{U}_i^{k-1}$. Hence, the whole U Volume will increase by a multiple of α , reflecting the change in the value of W .

Note that, for the rate of the \widehat{U} Volume from (2), for any district i , we obtain

$$\frac{d}{dt}(\widehat{W}\widehat{F}\widehat{E}) = (\widehat{W}\widehat{F}\widehat{E})\left(\frac{d\widehat{W}/dt}{\widehat{W}} + \frac{d\widehat{F}/dt}{\widehat{F}} + \frac{d\widehat{E}/dt}{\widehat{E}}\right) \quad (3)$$

Thus, by virtue of (3), $d\widehat{U}/dt$ will be dominated by the smallest value of the three individual components $\widehat{W}(I)$, $\widehat{F}(I)$, or $\widehat{E}(I)$, each a function of I information communicated across districts. In view of (3) we thus write

$$\frac{1}{\widehat{U}} \frac{d\widehat{U}}{dt} = \beta \frac{dI}{dt} \quad (4)$$

$$\beta \equiv \frac{d\widehat{W}/dI}{\widehat{W}} + \frac{d\widehat{F}/dI}{\widehat{F}} + \frac{d\widehat{E}/dI}{\widehat{E}} \quad (5)$$

We note that β in (4) and (5) represents the sum of “compressibility” measures of the relative change in response to change in information. For constant β value (4) conforms to Boltzmann entropy (Chakrabarti and Kajal, 2000) that reads

$$\ln \widehat{U} = \beta I \quad (6)$$

suggesting directional information from nodes (districts) with high \widehat{U} values to nodes with lower \widehat{U} values.

An additional optional Nexus measure is to consider \widehat{S} , the WFE surface area (Surface),

$$\widehat{S} = \widehat{S}_w + \widehat{S}_f + \widehat{S}_e, \widehat{S}_w \equiv \widehat{F}\widehat{E}, \widehat{S}_f \equiv \widehat{E}\widehat{W}, \widehat{S}_e \equiv \widehat{W}\widehat{F} \quad (7)$$

In view of (7) we account for the possibility of considering a zero value for one of the WFE components. Hence, for example, if $\widehat{E} = 0$ then by virtue of (7) we obtain

$$\widehat{S} = \widehat{S}_e \equiv \widehat{W}\widehat{F}, \quad \forall \widehat{E} = 0 \quad (8)$$

In view of (3), (4), (5), (6), and (8), $d\widehat{S}_e/dt$ (i.e. when $\forall \widehat{E} = 0$) will conform to Boltzmann entropy suggesting directional information from nodes with high \widehat{S}_e values to nodes with lower \widehat{S}_e values.

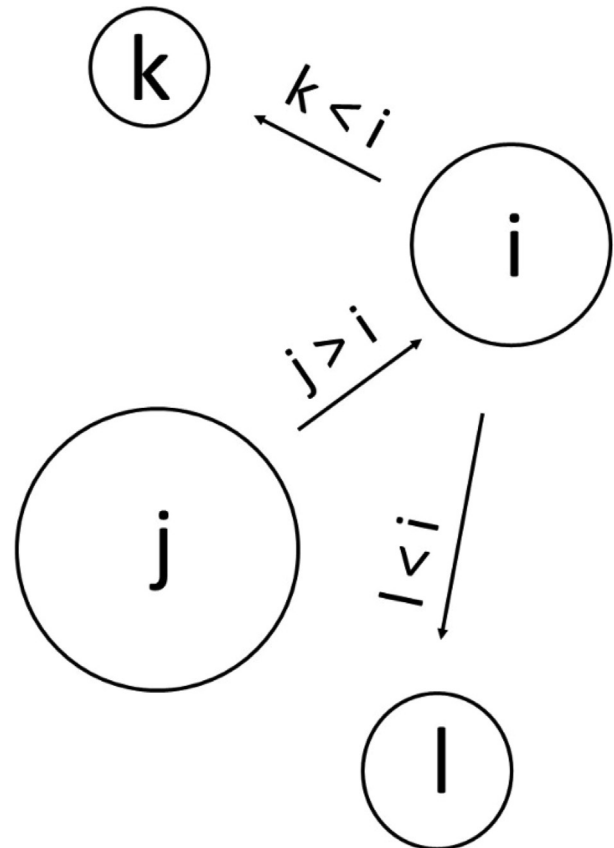


Figure 1. Illustration of flow of information from nodes having higher WM to those having lower WM (Value of WM for a given node is represented by the size of the node, with higher-WM nodes being bigger. Relative size of two nodes connected by a given directed information arrow is indicated by the label above the arrow, with e.g. the letter i used to denote WM_i , etc.)

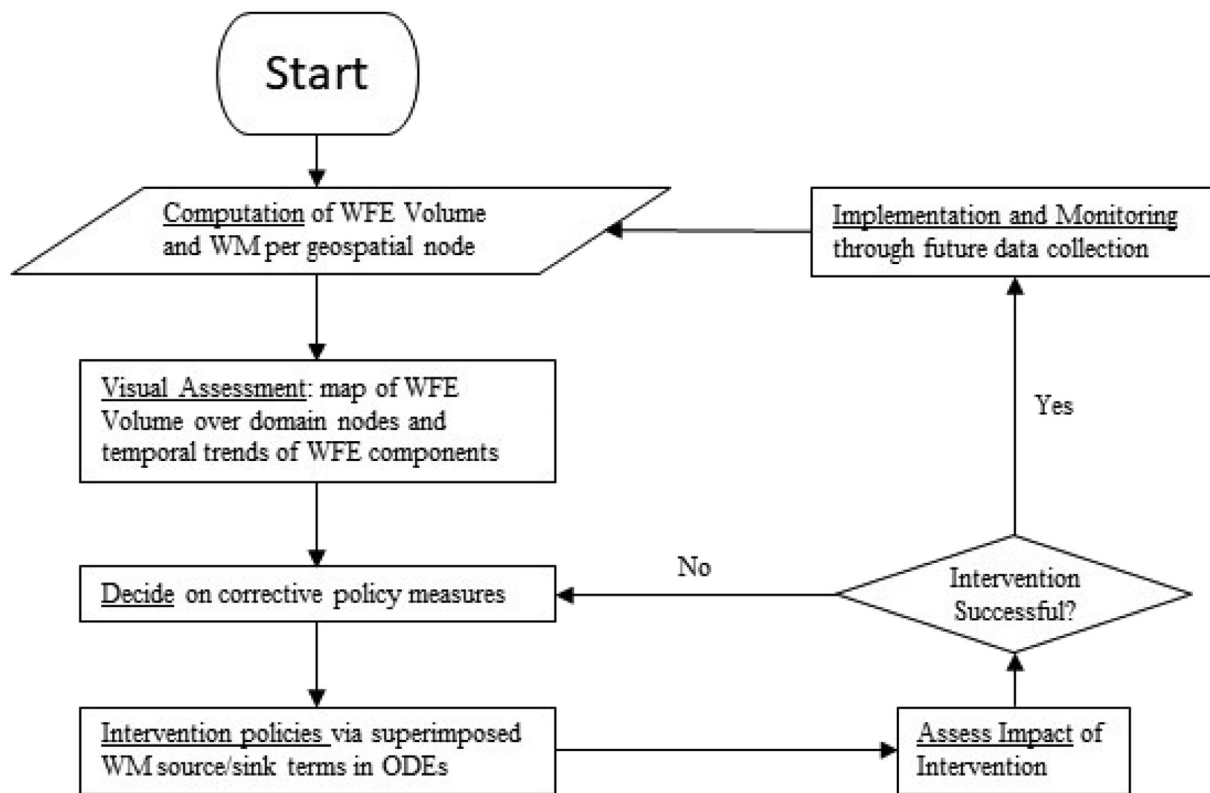


Figure 2. Schematic diagram illustrating the management approach.

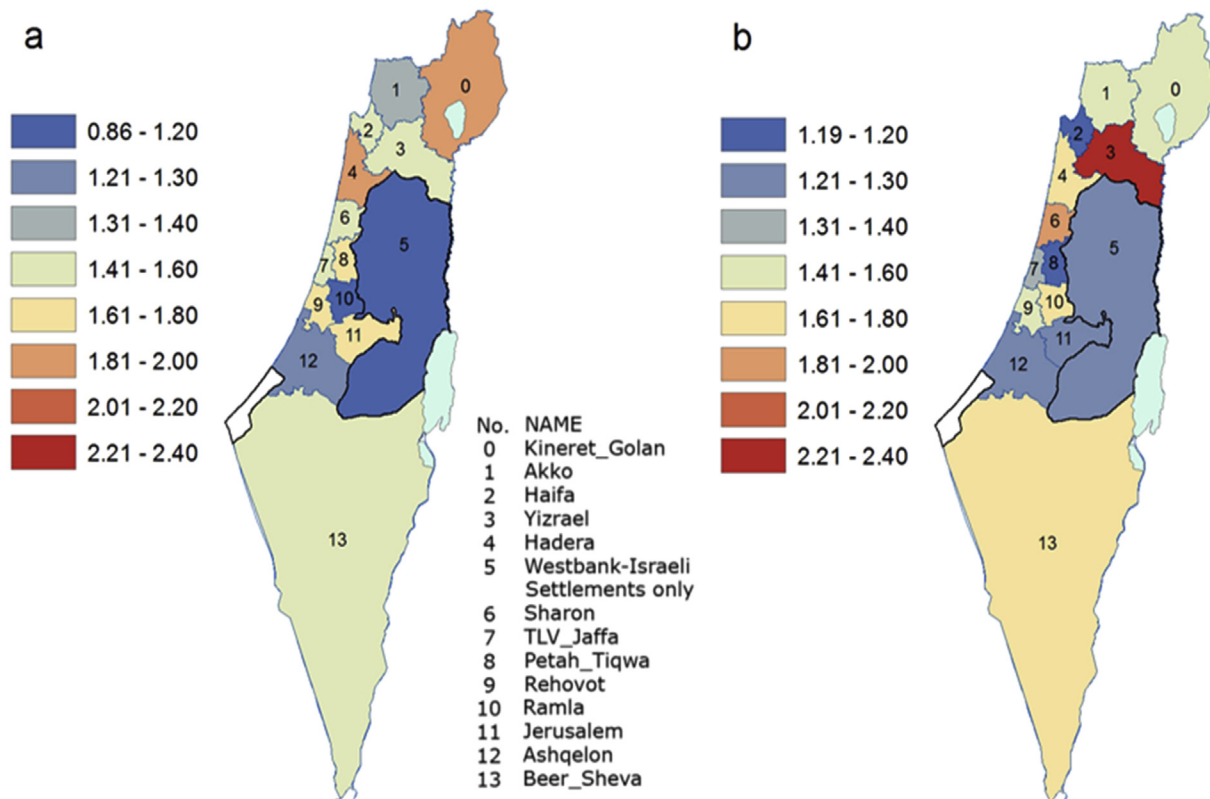


Figure 3. WFE Volume per districts as indicated for: (a) 2003–2007; (b) 2007–2011. Note in (b) that Yizrael district stands out in self-evolution.

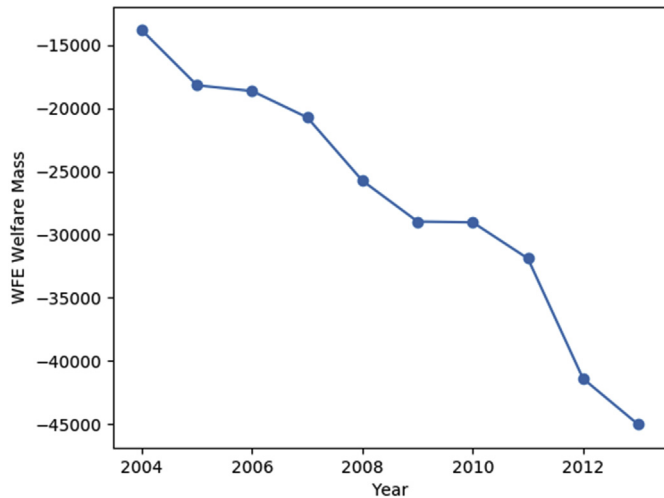


Figure 4. Calibrated temporal sink values evaluated from the same CBS historical data as Figure 3 for the solution of the WFE Volume balance ODE at the Beer-Sheva district (district 13 in Figure 3).

2.3. Income-Density Characteristic

Under the current model, socioeconomic status is addressed through the definition of what we call the “Income-Density Characteristic.”

We define the areal density for district i at time level k as

$$\rho_i^k \equiv (N^k/A)_i \tag{9}$$

in which N_i^k denotes the number of households in district i at time level k , and A_i denotes the area of the i district. Let $(IP)_i^k$ denote per-household income for district i at time k as defined by

$$(IP)_i^k \equiv (IM/N)_i^k \tag{10}$$

in which IM_i^k denotes the total amount of money earned at time level k by all residents of district i . By virtue of (10) let \overline{IP}^k denote the representative domain income at time level k given by the areal mean of all per-district household income values, namely

$$\overline{IP}^k = \sum_i [(IP)^k A]_i / \sum_i A_i \tag{11}$$

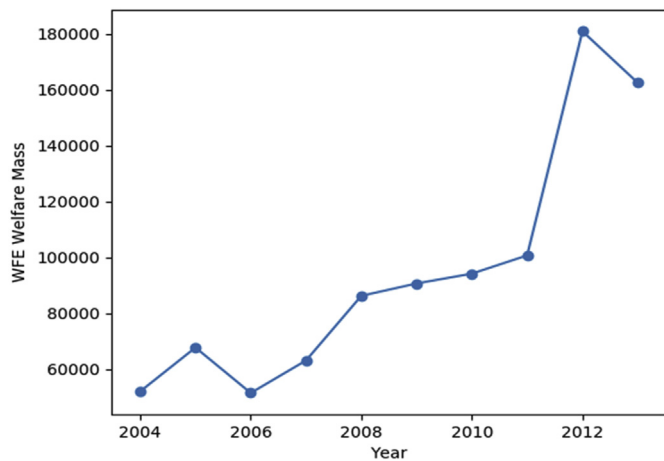


Figure 5. Calibrated temporal source values evaluated from the same CBS historical data as Figure 3 for the solution of the WFE Volume balance ODE at the RamatGan district (similar to District 7 in Figure 3).

By virtue of (11) let D_i^k denote the district income index for a district i at time k defined by

$$D_i^k \equiv [(IP)_i / \overline{IP}]^k \tag{12}$$

in which the D_i^k denotes the ratio that indicates how district i compares to the other districts in terms of per-household income. In light of (9), (10), (11), and (12) we can now write the Income-Density Characteristic $(\rho^*)_i^k$ for district i at time level k as

$$(\rho^*)_i^k = (\rho D)_i^k \tag{13}$$

2.4. Welfare mass

Consider that $(\rho^*)_i^k$ in (13) represents the Income-Density Characteristic (i.e., a density) and that \hat{U}_i^k of (2) can be interpreted as the volume of a box having sides of lengths \hat{W}_i^k , \hat{F}_i^k , and \hat{E}_i^k associated with the i district at time k . Therefore, $(\rho^* \hat{U})$ has an interpretation as the district welfare mass. We thus define the WFE Welfare Mass (WM) quantity for a district i at time k as

$$(WM)_i^k \equiv (\rho^* \hat{U})_i^k \tag{14}$$

because the three components of the Nexus are interconnected and thus the differences in quantity will arise naturally through the Volume, which is one holistic quantity. Therefore, if investing in one, an investor may cause a shock to the others.

2.5. District mass balance ODE

In light of the WM as in (14), we use a mass balance as the model underlying the interactions between the domain districts. I.e. the model is a system of first-order ODEs, built around the assumption of conservation of mass. For every node (district) i in the domain, for WM_i we write

$$\frac{d}{dt} WM_i = \sum_{j \neq i} \alpha_{ij} \Delta_{ij} + Q_i, \quad \Delta_{ij} \equiv WM_j - WM_i \tag{15}$$

in which, in light of (6) and the fact that WM corresponds to U, Δ_{ij} denotes the information divergence (or relative entropy) expressing the increase or decrease in information associated with WM for node i , addressing transfers between node i and other nodes j (Figure 1). Q_i denotes the i node source/sink term and α_{ij} denotes a transfer coefficient. Both α_{ij} and Q_i are calibration factors. For future predictions on the basis of (15), we assume that α_{ij} are constant over time. Q_i is the input parameter, the lever that this model provides to regulators, and through our choice of Q_i we impose a value for the $k+1$ time level. Hence, we apply an implicit numerical scheme for simulation of (15).

By virtue of (15), summing over all domain nodes, we have for any time level the condition

$$\sum_i \frac{d}{dt} WM_i = \sum_i Q_i \tag{16}$$

3. Management overview

The schematic diagram shown in Figure 2 illustrates the managerial concept behind the developed management theory. The goal is to enable management by considering spatial resolution at a coarse scale, i.e. “macro-management”. The model enables simulation of potential policy decisions, and quantitative comparison of both simulated and observed values (asking, “to what extent, in comparison to”). The user can make such comparisons through assessment of graphical visualization aids

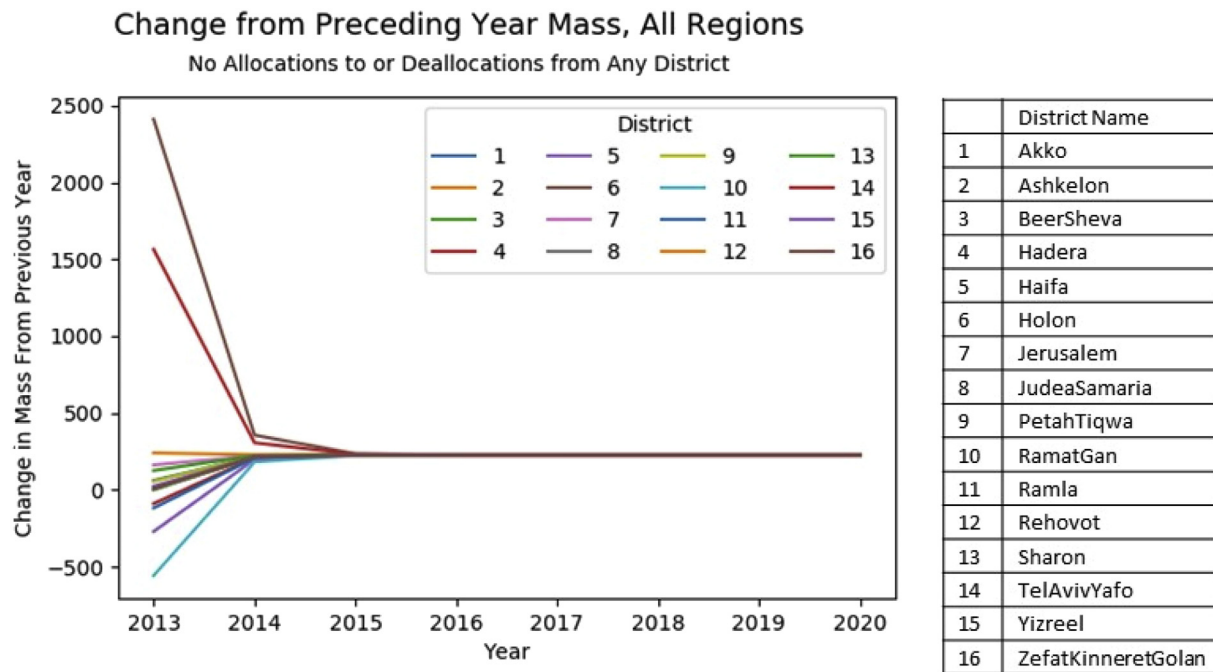


Figure 6. Typical predicted temporal districts changes without imposing any policy action. It can be seen that the system undergoes a transient phase until 2015, after which it behaves in a steady-state manner.

Predicted 2013 and 2020 Mass Values, Vs. 2012 Observations

No Allocations to or Deallocations from Any District

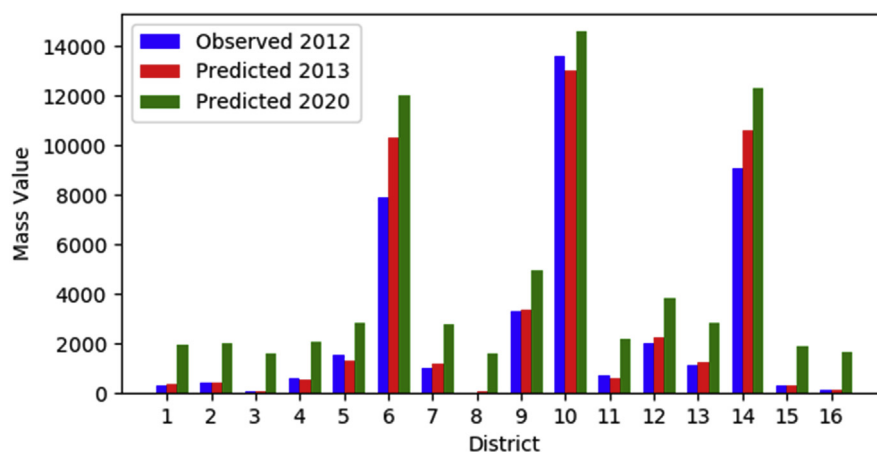


Figure 7. Observed 2012 values, and predicted values in 2013 and 2020, under no simulated policy action. District numbers are as in Figure 6.

following domain maps of historical data displaying districts' distribution of mean Volume values and model code predictions following (15) and (16).

In what follows we will exemplify simulations of our developed model for the macro management theory based on the database of Israel's Central Bureau of Statistics (CBS) (<http://www.cbs.gov.il/reader>) gathered across the country districts.

4. Implementation of methodology

4.1. Simulation

In what follows we use available historic data from CBS to demonstrate the potential of the developed management tool as a possible future working procedure. We demonstrate exemplifications in Figures 3,

4, 5, 6, 7, 8, and 9 with some examples of sensitivity due to different interventions of subsidy and defund loads (Figures 7, 8, and 9), namely Q values in (15). Sensitivity due to fluctuations in expenditures on the individual components is not the issue at hand because we represent the Nexus using the combined Volume, which takes individual component fluctuations into account. Sensitivity to e.g. different grouping of geo-spatial administrative districts could be examined but is not considered here. Demonstration will rely on statistics addressing per-household WFE Volume expenditures, based on the historical database of CBS gathered across the country districts. In all simulations/calibrations, the a_{ij} denoting the transfer coefficients were chosen as constants being of unit value.

Following the WFE Expenditure Volume of (2), Figure 3 describes two mean temporal WFE Volume maps of the CBS observed historical data

Predicted 2013 and 2020 Mass Values, Vs. 2012 Observations
 One-Time Alloc./Dealloc. of 9000 to BS/RG Respectively

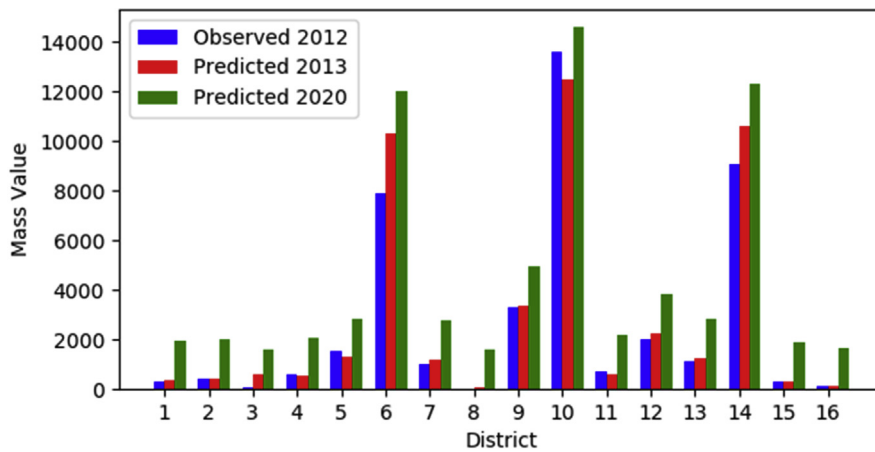


Figure 8. The impact of simultaneous and equal initial (observed vs altered) one-time impulses of allocation to Beer-Sheva (District 3) and deallocation from RamatGan (District 10). The simulated actions were imposed between the years 2012 and 2013, and this graph shows the observed 2012 values, predicted 2013 values, and predicted 2020 values. District numbers are as in Figure 6.

Predicted 2013 and 2020 Mass Values, Vs. 2012 Observations
 Deallocation from RG, Allocation to BS by 9000 Each Per Year

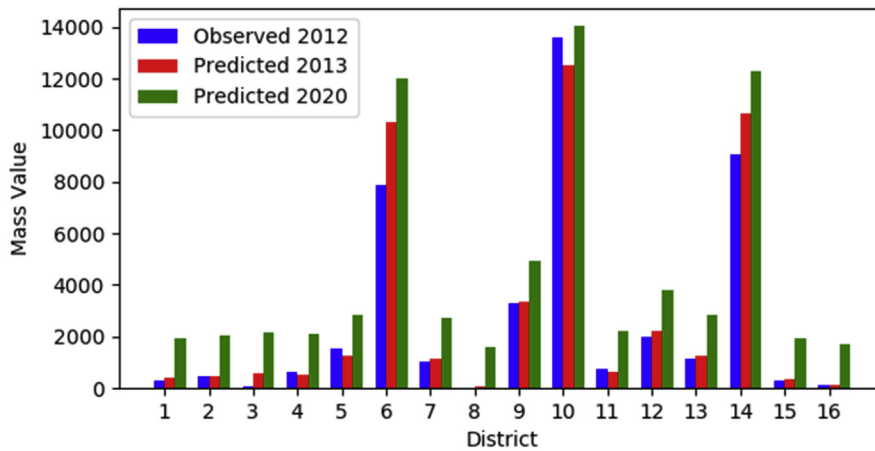


Figure 9. The impact of equal annual impulses of allocation to Beer-Sheva (District 3) and deallocation from RamatGan (District 10). The simulated actions were imposed between the years 2012 and 2013, and this graph shows the observed 2012 values, predicted 2013 values, and predicted 2020 values. District numbers are as in Figure 6.

over two periods. We note how this leads to a simple managerial tool in terms of pinpointing “to what extent in comparison to”.

Based on the CBS historical data and commencing from the assessment of different welfare levels in Figure 3, Figures 4 and 5 describe an example of two evaluated temporal source/sink terms for means of calibrating (15) the WFE mass balance ODEs across domain districts.

Figure 6 delineates the predicted outcome after solving (15) the mass balance ODEs across districts, with no imposed policy action. Figure 7 shows the net effect of the per-year changes shown in Figure 6, including observed values for 2012, predicted values for 2013, and predicted values for 2020. We note (Figure 6) the quick subsidence of the incremental transient excitation across all domain districts, as well as the fact that all domain districts converge to the same constant annual growth. Thus, according to this model, a policymaker who wants to increase or decrease the Welfare Mass of one district in comparison to others will need to impose a sustained (multi-year) subsidy or defund, respectively, on that district. Monitoring and followup to determine the effectiveness of this policy can then be carried out as per Figure 2.

Figure 8 shows the results of having imposed simultaneous one-time initial allocation of resources (source) impulse to BeerSheva (District 3 in Figure 6) together with the same value initial deallocation (sink) impulse to RamatGan (District 10 in Figure 6), so as to maintain unchanged country overall budget allocation, as suggested by $\sum_i Q_i^{k+1} = Const.$ in reference to (16). In the simulation, these actions were taken between the years 2012 and 2013, i.e. 2012 served as the starting time level. It can be seen (Figure 8) that in 2013, in the first year after the simulated actions, the WM values of BeerSheva and RamatGan are higher and lower, respectively, than without the simulated action. The extent (magnitude) of the difference in WM (unitless) is 529.41 in both cases. However, the extent in time of these effects is short-lived, as by 2020 they have disappeared.

Figure 9 shows the effect of imposing the same policy actions as Figure 8 every year between 2012 and 2020, instead of just as one-time impulses between 2012 and 2013. In this case, WM values in 2020 for BeerSheva and RamatGan show an increase and decrease (respectively)

of 562.5 in comparison to the corresponding values for the one-time action shown in Figure 8. In other words, the extent of the effect has increased in both magnitude and time.

4.2. Theoretical management

Let \bar{U} define a variant of the Volume for a district i commencing from the outcome at the $k+1$ time level of the model (15) with $\Delta\hat{W}$, $\Delta\hat{F}$ and $\Delta\hat{E}$ denoting, respectively, the increments in the WFE components from the Volume at the k previous time level,

$$\bar{U} = \bar{W} \bar{F} \bar{E} \tag{17}$$

in which, e.g.,

$$\bar{W} = (\hat{W} + \Delta\hat{W}) = \hat{W} \left(1 + \frac{\Delta\hat{W}}{\hat{W}} \right) \tag{18}$$

and similar for \bar{F} and \bar{E} . From (17) and (18) it can be seen that

$$\bar{U} = (\bar{W} \bar{F} \bar{E}) = \hat{U} \left(1 + \frac{\Delta\hat{W}}{\hat{W}} \right) \left(1 + \frac{\Delta\hat{F}}{\hat{F}} \right) \left(1 + \frac{\Delta\hat{E}}{\hat{E}} \right) \tag{19}$$

and let

$$f = \frac{\bar{U}}{\hat{U}} = \left(1 + \frac{\Delta\hat{W}}{\hat{W}} \right) \left(1 + \frac{\Delta\hat{F}}{\hat{F}} \right) \left(1 + \frac{\Delta\hat{E}}{\hat{E}} \right) \tag{20}$$

Expanding f as a Taylor series for the three variables $\frac{\Delta\hat{W}}{\hat{W}}$, $\frac{\Delta\hat{F}}{\hat{F}}$, $\frac{\Delta\hat{E}}{\hat{E}}$ in (20) around the point (1,1,1) reads

$$f \cong \frac{\bar{U}}{4\hat{U}} = \frac{\Delta\hat{W}}{\hat{W}} + \frac{\Delta\hat{F}}{\hat{F}} + \frac{\Delta\hat{E}}{\hat{E}} - 1 \tag{21}$$

Hence although the solution of (21) enables the prediction of future WFE Volume values as a whole, in view of (21) the decision maker can regulate these into specific allocations per each of the WFE components. This ability is important because it could be used to attempt to compensate for uneven distributions of resources throughout the domain. Taking Israel as an example, the climate of Israel ranges from temperate in the north to desert in the south. Therefore, one policy goal can be to reduce water resource stresses on southern Israel, with the intended policy outcome being to elevate water consumption in that region. Now, suppose the change in \hat{W} , \hat{F} and \hat{E} in a given district at the resolved $k+1$ proceeding time level, are to be regulated on the basis of \hat{W} . The decision maker can then, e.g., decide

$$\lambda = \frac{\Delta\hat{W}}{\hat{W}}; \frac{\Delta\hat{F}}{\hat{F}} = \frac{\lambda}{2}; \frac{\Delta\hat{E}}{\hat{E}} = \frac{\lambda}{2} \tag{22}$$

and obtain

$$\frac{\bar{U}}{4\hat{U}} = 2\lambda - 1 \tag{23}$$

enabling the solution for λ .

5. Summary

The WFE Nexus concept states that water, food, and energy are three interdependent resources that must be managed together. We have formulated a method for doing so. This method includes both the

calculation of per-district Welfare Mass (one combined holistic quantity representing the social welfare of a given district in terms of the WFE Nexus Volume) and prediction/simulation of the propagation of this quantity using a mass-balance model. It is important to note that this work is just the demonstration of a concept using available historic data, intended to become a management working tool. Development of a working tool would require follow-up data acquisition with evaluation and recalibration, and simulation for the subsequent year.

Declarations

Author contribution statement

Yoni Teitelbaum: Performed the experiments; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Alex Yakirevich, Amit Gross: Analyzed and interpreted the data.

Shaul Sorek: Conceived and designed the experiments; Contributed reagents, materials, analysis tools or data.

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Competing interest statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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