LETTER

Aspect ratio dependence of the ultimate-state transition in turbulent thermal convection

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Iyer et al. (1) report heat (Nu) and momentum (Re) transport results for turbulent Rayleigh-Bénard convection (RBC) for a Prandtl number $\mathrm{Pr}=1$ from direct numerical simulation (DNS) for a cylindrical sample of aspect ratio (diameter D/height H) $\Gamma=1/10$. The data show the classic scaling $\mathrm{Nu}=0.0525\mathrm{Ra}^{0.331}$ in the range $10^{10}\le \mathrm{Ra}\le 10^{15}$. The authors emphasize that their data do not reveal a transition Rayleigh number Ra^* to the RBC ultimate state (2, 3), but neglect to point out that sidewall stabilization, and thus Ra^*, is expected to increase with decreasing Γ. Here, we point out that experimental Ra^* values do indeed show a strong Γ dependence with Ra^* well above 10^{15} for $\Gamma=0.1$.

Fig. 1 shows the Ra dependence of $\mathrm{Nu}/\mathrm{Ra}^{0.331}$ for $\Gamma=0.50$ and 1.00 (4–6). The data are from experiments using compressed SF_6 gas with $\mathrm{Pr}\simeq 0.8$ at Ra up to 1.1×10^{15}. Each dataset reveals a transition range $\mathrm{Ra}_1^*\lesssim \mathrm{Ra}\lesssim \mathrm{Ra}_2^*$ of Nu(Ra). For $\mathrm{Ra}\lesssim \mathrm{Ra}_1^*$, we found the classic scaling $\mathrm{Nu}\propto \mathrm{Ra}^{\gamma_{eff}}$ with $\gamma_{eff}=0.312$ (0.321) for $\Gamma=0.50$ (1.00). For $\mathrm{Ra}\gtrsim \mathrm{Ra}_2^*$, we found $\gamma_{eff}\simeq 0.37$ for both Γ, consistent with the predicted scaling for the ultimate state (2, 3). Over the transition range, γ_{eff} was close to 0.33. One sees that the transition range increased as Γ decreased. The values found for Ra_1^* and Ra_2^* were confirmed also by Reynolds number measurements (5, 7).

Fig. 2 shows the measured Ra_1^* and Ra_2^* as a function of Γ. While the Γ dependence of Ra_1^* is weak, Ra_2^* changes by one decade over the data range and can be described by the power law $\mathrm{Ra}_2^*=a\Gamma^b$, with $a=8.13\times 10^{13}$ and $b=-3.04$. Extrapolating to $\Gamma=1/10$ indicates that the transition Ra is near 10^{17} for such a slender sample. A similar Γ dependence $\mathrm{Ra}^*\sim\Gamma^{-2.5\pm 0.5}$ was found in cryogenic experiments for $\mathrm{Pr}\simeq 1.5$ over the range $0.23\le\Gamma\le 1.14$ (8). However, the reported values of Ra^* (also shown in Fig. 2), a transition Rayleigh number defined by Roche et al. (8), are much lower than our results. Extrapolating them to $\Gamma=1/10$, one finds that the transition should occur in the range $2\times 10^{13}\lesssim \mathrm{Ra}^*\lesssim 10^{14}$. This disagrees with the DNS result by Iyer et al. (1).

Thus, the conclusion by Iyer et al. (1) is incomplete, since they did not consider the strong influence of Γ on Ra^*. For $\Gamma\simeq 1$, a number of experiments (4, 5, 7, 9) have revealed that the transition occurs near $\mathrm{Ra}=10^{14}$, which is consistent with the prediction by Grossmann and Lohse (GL) (3). Note that the GL prediction does not apply for Γ much less than 1, since the parameters in the model are all from experimental data for $\Gamma=1$. Our results show that $\mathrm{Ra}_2^*\sim\Gamma^{-3.04}$, leading to a much higher transition Ra for a slender sample. For $\Gamma=1/10$, our data suggest that the ultimate-state transition will occur near $\mathrm{Ra}=10^{17}$, which is well above the Ra limit of the DNS data in ref. 1. That is why the authors found that "classic 1/3 scaling of convection holds up to $\mathrm{Ra}=10^{15}$."

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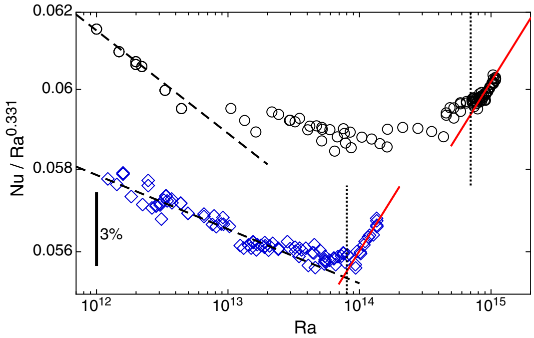

Fig. 1. Reduced Nusselt number $Nu/Ra^{0.331}$ as a function of Ra for $\Gamma = 0.50$ (black circles) and 1.00 (blue diamonds). Solid lines denote power laws $Nu = Nu_0 Ra^{\gamma_{eff}}$, with $\gamma_{eff} = 0.37$ and Nu_0 adjusted to fit the data for $Ra > Ra_2^*$ for each Γ. Dashed lines are the power-law fits to the data for $Ra < Ra_1^*$, with $\gamma_{eff} = 0.312$ for $\Gamma = 0.50$ (*Upper*) and $\gamma_{eff} = 0.321$ for $\Gamma = 1.00$ (*Lower*). Vertical dotted lines are $Ra_2^* = 8 \times 10^{13}$ and 7×10^{14}.

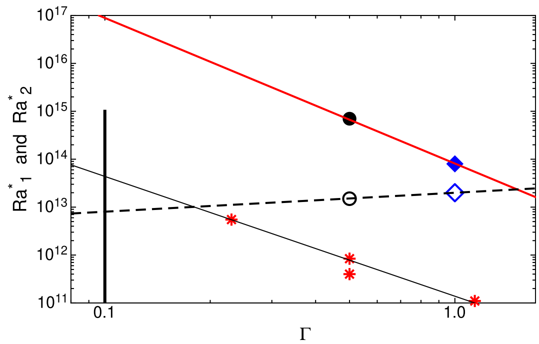

Fig. 2. Ra_1^* (open symbols) and Ra_2^* (solid symbols) as a function of Γ. The black dashed line and red solid line represent the power function $y = ax^b$ with the exponent $b = 0.40$ and -3.04, respectively. Red stars are the Ra^* data from ref. 8. The black solid line corresponds to $Ra^* \sim \Gamma^{-2.5}$. Vertical black solid line indicates the Ra range of the DNS data in ref. 1.

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