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ORIGINAL PAPER

The doctrine of normal tendency in active learning teaching methodology: investigations into probability distributions and averages

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Abstract
Traditional lecture and active learning methods of teaching a university course are compared. The particular course is university calculus. The lecture method was applied to two sections of calculus. The active learning method was applied to two other sections. In all cases students were given an examination near the beginning of the course and a final examination at the end of the course. The score averages for the active learning method were higher than for the lecture method. The distribution of scores for the lecture method were non-normal multimodal in the first and final examinations. The distribution for the active learning method went from non-normal multimodal in the first examination to unimodal normal in the final examination. A new undeceivable nature evidence-based method is presented for measuring teaching efficacy by probability distribution.

Keywords Active learning · Normal distribution · Didactic lecture · Multimodal distribution · Mental silos · Cognitive reintegration

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Introduction

Universities are always seeking to increase learning by students to improve their knowledge and proficiency. Americans are increasingly affluent, in search of higher education to perform jobs in science, technology, engineering and math (STEM). These jobs have replaced lower paying jobs that have declined due to globalization and outsourcing. A conundrum arises when the university is chartered to create opportunity for a larger number of students who might otherwise not be included, while at the same time maintaining or raising graduation rates. There are also concerns regarding a negative impact of those students on the high achieving population. At the same time, students are the victims of distracting classroom texting and lack of study habit. The information age has created a false impression that education can be obtained a la carte as opposed to through programmed instruction that follows a specified curriculum. Studies by Rosen (2013) and Beland and Murphy (2015) show that children attending schools where cell phones were permitted, attained 6% lower examination scores than those attending schools where cell phones were prohibited. In the case of what they called weak students the decline was 14%. Ansari and Khan (2020) claim that social media is a potentially useful tool for collaborative interactive learning. Aharony and Zion (2019) show that there is an adverse effect on pupils' working memory performance due to mobile instant messaging distractions. Digital immigrants may find it unbelievable that teenage natives would be allowed to bring cell phones to classrooms anywhere.

Significance

Didactic lectures and instructions segregate into incongruous mental silos of multimodal non-normal distributions of academic expression and underperformance. Empirical demonstration, computer simulation, and mathematical proof all show scientifically that active learning can reintegrate mental facilities into a congruent unimodal normal distribution that is a feature of self-efficacy, natural for healthy development.

Definition

Lecture is the continuous exposition by an instructor while student activity is limited to taking notes and/or asking occasional and impromptu questions of the instructor.

Definition

Active learning engages students in the process of learning through activities and/or discussion in class, as opposed to passively listening to an expert. It emphasizes higher order thinking and often involves group activity.

Definition

Normal refers to a distribution containing a most frequently occurring typical score at its peak (center) and atypical scores with lower and lower frequency as they occur further and further away from the mean.

Ways and means

One way to raise student grades is to recruit students that are better prepared for university level instruction. Or, hope that grade schools will find ways to improve the academic readiness of their graduates. Since that can take a very long time, another approach to consider is to help students focus by implementing study hall for all freshmen, sophomores, and underperforming students. This will be study habit forming until it becomes normal and natural. Regardless of whatever strategies are employed, students must study. Upperclassmen can graduate from study hall if they maintain a specified grade point average. The National Collegiate Athletic Association (NCAA) requires it for college athletes. Another idea is to block cell phone signals in classrooms. This is easily accomplished by a grounded ceiling metallic mesh over classrooms. A partial Faraday cage if you will. Since passive blocking does not involve active signal jamming, it will not require Federal Communications Commission (FCC) approval (in case jamming is preferred permission must be obtained from the FCC). The professor must be given access to a switch to turn on wifi when it is time to conduct some high technology higher learning activity that makes use of the internet or other smart phone experiment (Hochberg et al. 2020). The wifi can be turned off after the internet exercise. Another idea is to create a take-ownership-mindset by integrating entrepreneurial learning (de Silva et al. 2020; Llaugel and Ridley 2018a; Ngnepieba et al. 2018; Ridley 2018; Ridley 2020a, b; Ridley and Khan 2019) and the Golinkoff and Hirsh-Pasek (2016) 6Cs (collaboration, communication, content, critical thinking, creativity innovation, confidence). These and other methods (Bonwell and Eison1991; Brindley et al. 2009; Chickering and Gamson 1987; Kirschner et al. 2004; Rodriguez 2011; Swart and MacLeod 2020) are not mutually exclusive. They can be made inclusive. Prior to the experiment reported in this paper, a partial implementation of active learning was conducted. See Appendix A of the supplementary information.

The world is a most complicated place. One might think that the human being is curious to learn all about the world. That they wish to learn and experience all that is real. In fact, human beings are overwhelmed by the complexity and expanse of the world and often choose to avoid reality. To cope, they tend to build interfaces between themselves and the world. Interfaces are where human beings touch technology. They provide smarter ways to use simple things and simpler ways to use smart things. Interfaces release the potential of complex systems and technologies to the users who need them (*Xerox Corporation* 2020). And, every once in a while, they change everything. The most fundamental educational technologies have been speech, the abacus, the slate, the chalk and blackboard, and the erasable marker and white board. These are complemented by overhead projectors, power point slides and audiovisual recordings (Craig and Amernic 2006). The lecture method of teaching employs speech and white boards extensively. But the interface is limited in scope. And, the speech that it employs is unidirectional. It belongs to the constructivist classroom (Brooks and Brooks 1999). Attention span is also limiting. Active learning is designed to extend the interface to multiple technologies and learning styles. Multidirectional speech, when planned and orderly is designed to extend the student cognitive interface. We postulate that active learning along these lines can raise the score average of the class while changing the distribution of scores from multimodal non-normal to unimodal normal. It is expected that this disambiguation by means of histograms of the examination scores will serve to identify a clear path to better outcomes. See Freeman et al. (2004) for a meta-analysis of 225 studies, as well as Ruiz-Primo et al. (2011), Springer et al. (1999), Kogan and Laursen (2014), Lonka et al. (2020), and Ibarra-Sáiz et al. (2020).

The remainder of the paper is organized as follows: A review of pertinent literature is given in the "Literature review" section. The experimental method is described in the "Method" section. The data and experimental results are given in the "Data and experimental results" section and in Appendix B of the supplementary information. The analysis of the data and results is given in the "Analysis of results" section. A psychological review that might account for the differences in the lecture and active learning methods is given in the "Psychological underpinnings" section. Some mathematical insights into how the scores may be combining when active learning occurs are explored in the "Mathematical underpinnings of central tendency" section. The possibilities for active learning impact on test score distribution is discussed in the "The doctrine of normal tendency" section. Some conclusions and suggestions for further research are given in the "Conclusions" section.

Literature review

For decades, the traditional form of lecturing has been viewed as an ineffective means of transferring knowledge to students, especially in Science, Technology, Engineering and Mathematics (STEM; Michael 2006). In his review, Michael (2006) noted that in the 1980s, the National Commission on Excellence in Education called for the reform of K-12 science education. The Association of American Medical Colleges similarly called for reform of medical students' science education. Since then, other bodies (e.g., the National Research Council) have also called for changes in the way in which STEM subjects have been taught at various levels from kindergarten to college levels. In fact, the use of the traditional lecture format in teaching STEM has been argued to explain why many students have been disinterested in pursuing STEM (Volpe 1984). Since the 1980s, it has been argued that "What is urgently needed is an educational program in which students become interested in actively knowing, rather than passively believing" (Volpe 1984). Such calls for the reform of teaching methods in STEM have continued to within the last few years. The Association of American Universities and the Research Corporation for Science Advancement Cottrell Scholars called in 2015 "for immediate change at all levels of research universities to improve the quality of university STEM education. It is no longer acceptable to blame primary- and secondary-school teachers for the deficits in STEM learning at the university level" (Bradforth et al. 2015).

Bradforth et al. (2015) argued that many students who intended to major in STEM subjects changed majors to non-STEM fields because of the traditional teaching practices employed. Such arguments have been supported by research showing that engaging in active learning techniques is associated with increased motivation and retention of students in mathematics and other sciences (Anthony et al. 2017).

Research generally has found strong support for active learning. Corkin et al. (2017) studied the effectiveness of active learning methods using a sample of 962 college students enrolled in a large US public university. Students were randomly assigned to a biology course utilizing active learning techniques or to a course utilizing traditional lecture. Results indicated that students who were taught using active learning techniques reported greater support from their instructors and that they had greater levels of understanding and interest in the material, than students in the traditional classroom. Ng et al. (2019) also found that active learning was associated with increased understanding of concepts taught and improved grades in calculus. Cicuto and Torres (2016) found an association between the use of active learning techniques and students' motivation in biochemistry. When students were taught using active learning techniques, those students had higher levels of achievement and valued science more than students taught using traditional methods. The researchers argued that students in active learning environments worked harder to learn the material than students in traditional classrooms.

While many studies (e.g., Cicuto and Torres 2016; Corkin et al. 2017) found benefits of active learning for students generally, some research has found that active learning techniques may be even more beneficial for students who are underrepresented in the sciences (Theobald et al. 2020). In their review of over 40 studies involving over 50,000 students, the authors found that the gap for underrepresented students in STEM was narrower in active learning classrooms than they were in classrooms using traditional lecture. The gap in examination scores for underrepresented students was reduced by a third, and gaps in passing rates were reduced by nearly half.

Active learning is also associated with increased positive perceptions of instructors' levels of care, understanding and acceptance, and in students' perceptions of their own intelligence (Cavanagh et al. 2018). Additionally, research has shown that active learning not only increased students' interest in the subject matter, but it also contributed to students' development of their own learning techniques and to their ability to apply what they have learned (Sivan et al. 2000). Students' themselves also highlight the benefits of active learning in a study reviewing the comments of over 260 undergraduate students in math and science that indicated that active learning helped reinforce the subject matter and highlighted areas where they misunderstood the lesson (Welsh 2012).

Despite the decades of various national bodies calling for STEM education to involve active learning techniques, many STEM educators seem reluctant to incorporate active learning in their STEM courses. Reasons for not engaging in active learning have been noted over the decades, as calls have continued to be made for

STEM to be taught using active methods. Faust and Paulson (1998) found that many university faculty members believed that while active learning may have been an effective strategy in some fields, such strategies would be ineffective for teaching STEM subjects. Michael (2006) also noted that some educators felt that by having homework and laboratories, their teaching was already active. Despite the advantages of teaching STEM using active learning techniques, even within the past few years, calls for teaching STEM subjects using active learning techniques have fallen on deaf ears (Bradforth et al. 2015; Braun et al. 2017). Bradforth et al. (2015) indicated that the majority of STEM instructors have yet to include active learning techniques in their teaching. They argued that such reluctance to engage with active learning may be due to a lack of incentives or support to include active learning components. They also argued that faculty members often do not have undergraduate teaching as a priority due to their emphasis in research, and therefore do not see value in spending time changing their methods.

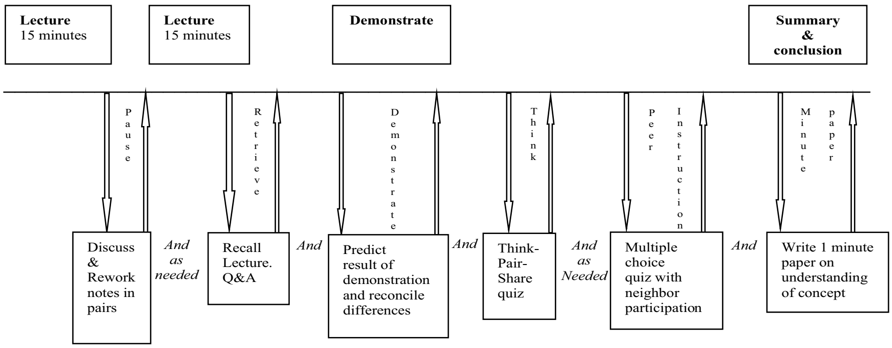
The reliance on traditional methods can also be explained due to the difficulty in understanding the definition of active learning. Research has had mixed definitions of what constitutes active learning (Michael 2006). Michael defined active learning broadly as any teaching method that engages students. Michael (2006) also argued that the type of active learning technique also needs to be specified in order to limit confusion around active learning. Michael argued that collaborative learning, in which students work together in small groups, should be distinguished from cooperative learning, which also involves group work but in which students are assessed individually. Michael also argued that problem-based learning should also be defined specifically as a teaching method in which students are introduced to relevant problems early in the instructions, such that the learning goals of the instruction are centered around the problem. Michael proposed such specific definitions to aid educators in distinguishing the specific types of active learning techniques, since while overlap exists between them, each has its core elements. Through such specification, educators may be better able to incorporate active learning techniques in STEM.

Reassuringly, in their study of different active-learning approaches, LoPresto and Slater (2016) found that students in their astronomy courses did not indicate preferences for any specific active learning approach tested and that the impact of the different approaches were generally indistinguishable from each other. LoPresto and Slater (2016) however did find that despite there not being a specific active learning technique that was superior to the others, they noted that all of the approaches were found to be more effective than traditional lecture alone. Such findings, coupled with the literature showing benefits of active learning for all students, support the importance of incorporating active learning techniques in STEM classrooms.

Method

The particular method of active learning investigated in this research is that described in Ngnepieba et al. (2018) as “active learning, in which students solve problems, answer questions, formulate questions of their own, discuss, explain, debate, or brainstorm during class (Hacisalihoglu et al. 2018). Active learning refers to activities that are introduced into the classroom. The core elements of active learning are student activity and engagement in the learning process." The six active learning techniques listed by Brame (2016) and employed in Ngnepieba et al. (2018) are (1) The pause procedure, (2) Retrieval practice, (3) Demonstration, (4) Think-pair-share, (5) Peer instruction and (6) Minute paper. These are depicted in the below active learning schematic. The schematic shown may be executed once per concept that is to be learned. One or more concepts may be taught in any class period. A concept that is started in one class period may be finished in a subsequent class period. The learning activities are performed on an as needed basis since some concepts are easily grasped while others require more activity. For example, if the think-pair-share quiz results in a perfect quiz score, there is no need for peer instruction. But, if only one student makes a high score, that student will be asked to tell the class what he or she knows. Peer instruction is expected to produce higher order thinking (Mahoney and Harris-Reeves 2019).

Active learning schematic



Our experiment was conducted at an American university in the Fall 2019 and Spring 2020 semesters. Both semesters comprised 16 weeks each. The recorded data are examination scores and averages for university calculus section 1, section 2, section 3 and section 4. Student assignment to sections are quasi-random designs where students self-sorted into classes, blind to the treatment at the time of registering for the class. The sections were independent of each other. The student academic backgrounds and ages were similar and random with respect to the sections. The professors were three tenured holders of the Ph.D. degree in mathematics. One professor was assigned to teach both sections 1 and 2. The other two professors were assigned separately to teach sections 3 and 4. They were directed to instruct the students by traditional didactic lecture and by active learning methods, respectively, regardless of their personal teaching philosophies. The setting for the lecture method was the traditional classroom teacher-centered arrangement shown in Fig. 1a. The setting for the active learning method was the student-centered arrangement shown in Fig. 1b.

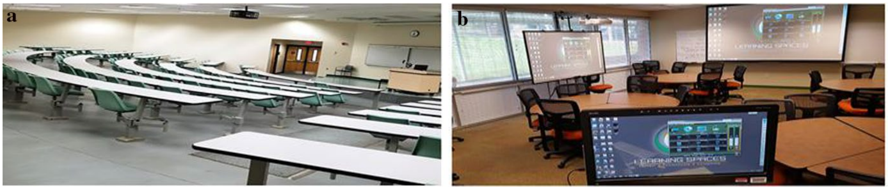


Fig. 1 **a** Traditional lecture classroom setting. **b** Active leaning classroom setting

The classrooms were centrally heated and cooled equally, and the lighting and no noise levels were the same. All sections received the same examinations. Examinations were problem based and were scored on a scale of 0 to 100%. The exams and grades were reviewed for quality and consistency by the chairman of the mathematics department.

Data and experimental results

The data for the Spring semester are given in Table 1. The examination scores were used to prepare histograms as shown in Figs. 2a, b and 3a, b for the standard lecture method and Figs. 4a, b and 5a, b for the active learning method.

Analysis of results

The promise of normal aptitude

The College Board is America's largest college-going organization, helping millions of students navigate the transition from high school to college each year through programs like the scholastic aptitude test (SAT), and advanced placement (AP) test. The College Board (2019) reports SAT scores that are normally distributed (Table 2 in Appendix B of the supplementary information). Both writing and math scores are normally distributed. The SAT purports to be a measure of aptitude. This may be an indication of expectations, and potential that lives within the student population. The College Board (2011) reports a total student AP score distribution that falls short of normally distributed (Table 3 of Appendix B of the supplementary information) (It is possible that if the bin size were smaller the scores could be approximately normal. This was the case with the posttest data from this experiment). English scores are approximately normal. Physics and calculus scores tend to be multimodal, not unlike the university calculus pretest scores that were observed in this experiment. Psychology scores are also multimodal. The AP results are our first look that we have at how well grade schools have developed the potential suggested by the SAT. It is possible that the differences in distributions between SAT and AP are due to relevant AP subject matter specific aptitude (or interest) not being tested in the SAT. Subject matter and other features related to learning are taken up later in the section on discussion and normalization in our calculus experiment. In any case, these AP score departures from normality may be the beginning signs of failure in school grades k-12.

Active learning by doing is the only real meaningful option in kindergarten. As kids grow into elementary school children they insist on more independence. Middle school teachers instruct on a broad range of subjects. High school teachers instruct on subjects that are more selective as children contemplate future areas of specialization, career jobs and professions. For any number of reasons, some children fail and dropout. In the final two college preparatory grades, children are weaned off the teacher as they are sent more often to the library. The non-normal AP scores imply these departures from general normal scholastic aptitude to special interest and performance skills. Silos of thought enclaves begin to form. One might assume that these continue into the university freshman classes.

One of the functions of university faculty is to perform didactic lectures, where efficiency is sought to deliver large volumes of information in higher complexity to large numbers of students by expert professors. Science laboratory exercises to supplement the lectures are conducted by various teaching assistants. Instead of all students demonstrating greater learning and competence, some of them become increasingly frustrated, often ending in failure and dropout.

The didactic lecture method

The first remarkable observation is that the starting histograms (Fig. 2a for section 1 in Fall 2019 and Fig. 3a for section 2 in Spring 2020) of pretests at the beginning of the semester are approximately multimodal with the possibility of two or more underlying approximately normal subpopulations. Normality is what we expect for numerous data such as economic data, population characteristics, climate data, etc. All of these are commonly occurring in nature. One might therefore assume that our creator intended these data to be normally distributed. To use a synecdoche where normality is one of many possible distributions, and normality as a metonym of social union. The next observation is not so remarkable. After lectures and posttests at the end of the semester, the ending histogram (Figs. 2b, 3b) is also multimodal. The histograms shift to the right. The mean class score for section 1 increases from 53.61 to 70.10%. The mean class score for section 2 increases from 46.88 to 61.44%. These persistent non-normalities in distributions may be evidence of a university level institutional deficiency. An unfulfilled promise, one might say.

In this particular case, the averages for the Spring semester could be adversely affected by the disruption due a corona virus pandemic outbreak, and a switch to online instruction between examination 1 and the final examination. The blank entries in Table 1 represent no shows. The zeros represent a score of “0.” If the zeros are removed from the average because they indicate a final examination disruption, the average increase for section 2 is from 46.88 to 85.33%. Of course, there is no telling what the scores would have been had the disruption not occurred. In any case, if there were a change of operations effect (due to corona), it would affect both methods of teaching. One might attribute the final examination increase in score

Table 1 Examination scores for three sections of calculus taught by lecture and active learning

Calculus I Section 2 (see Fig. 3) Lecture Spring Semester 2020		Calculus I Section 3 (see Fig. 4) Active Learning Spring Semester 2020		Calculus I Section 4 (see Fig. 5) Active Learning Spring Semester 2020	
Examination 1	Final examination	Examination 1	Final examination	Examination 1	Final examination
65	87	50		24	0
23	0	14		69	67
49	80	76	82.67	59	78
63	93	67	70.33	70	84
36	80	38	62.10	66	90
66	90	80	76.10	87	90
88	90	78	84.00	49	
0	0	83	82.00	57	81
53	90	57	85.33	47	84
60	93	17	55.19	37	74
45	87	49	78.86	41	
49	77	69	69.00	33	
66	97	92	70.00	47	
32	0	74		53	77
65	73	55	80.86	95	100
85	80	93	70.76	7	
47	0	93		68	
14	90	84	84.53	95	100
25	83	0	86.86	77	88
42	53	64	87.43	100	100
17	0	38	86.00	13	
38	0	65	91.00	30	6
26	0	72	92.00	85	77
55	93	26	72.29	91	81
63	100	53	53.52	73	83
		40	0		
		67	60.52		
Mean					
46.88	61.44	59.04	73.10	58.92	75.56
With drop outs omitted:					
Mean	85.33		76.43		84.63
Standard deviation		24.97	11.32	26.16	9.63
Skewness (S)		-0.7	-0.62	-0.24	0.28
Kurtosis (K)		2.88	2.41	2.25	2.62

Table 1 (continued)

	Calculus I Section 2 (see Fig. 3) Lecture Spring Semester 2020		Calculus I Section 3 (see Fig. 4) Active Learning Spring Semester 2020		Calculus I Section 4 (see Fig. 5) Active Learning Spring Semester 2020	
	Examination 1	Final examination	Examination 1	Final examination	Examination 1	Final examination
$\chi^2 = \sum (fo - fe)^2 / fe$ Frequency: fo = observed fe = expected			9.1	3.6	7.6	1.6
Jarque–Bera statistic $JB = (n/6)(S^2 + (1/4)(K-3)^2)$			2.23	1.75	0.82	0.31

Scores are measured in percent where the maximum is 100%

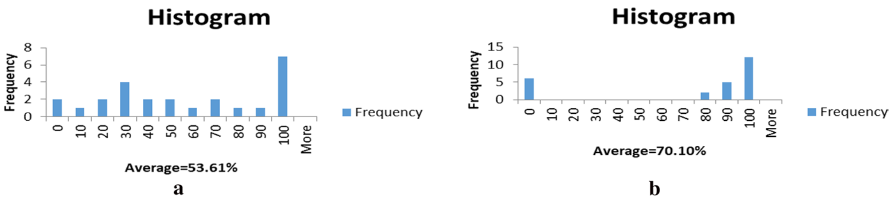


Fig. 2 **a** Lecture Section 1 Fall 2019 Exam 1. **b**. Lecture Section 1 Fall 2019 Final Exam

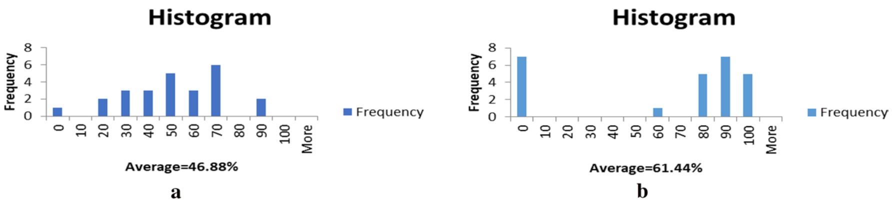


Fig. 3 **a** Lecture Section 2 Spring 2020 Exam 1. **b** Lecture Section 2 Spring 2020 Final Exam

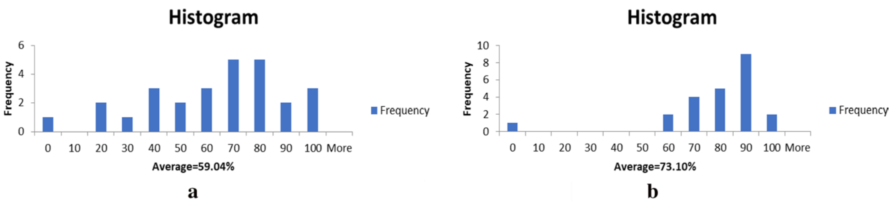


Fig. 4 **a** Active learning Section 3 Spring 2020 Exam 1. **b** Active learning Section 3 Spring 2020 Final Exam

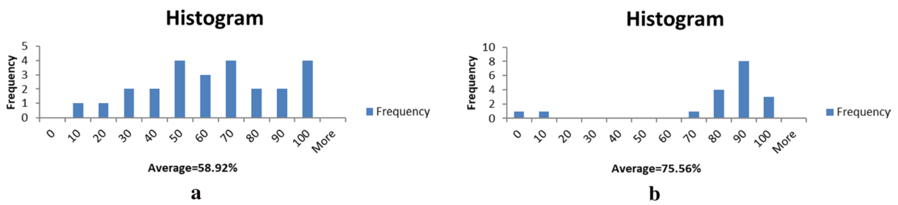


Fig. 5 **a** Active learning Section 4 Spring 2020 Exam 1. **b** Active learning Section 4 Spring 2020 Final Exam

average to the lecture content and the ability of the student to learn and regurgitate the course information. The score average on the final examination is not the focus of this research as it might be apocryphal since much depends on the level of difficulty and the grading standard of the professor (Jephcote et al. 2020). We are more interested in the distribution of the scores. Surprised by the multimodal pattern, one might speculate on the cause, and the possibility of obtaining better outcomes. More normal ones if you will.

An in depth and enlightening discovery might lie in the cause and explanation of the non-normal grade distribution. One might conjecture that there are two student populations in the university. One population is of students with a low mean test score. The students in this population are underprepared for university level learning and have low incoming test scores at their time of matriculation into the university. For them, the university appears to be accepting of low performing students with the intension of creating an opportunity for them. They are aware of high performing students but do not think of them as being inside their own classroom. The high performing students are recruited by other institutions but attend the university because of encouragement from a family member who may have attended the university in the past. They might have attended high performing k-12 schools and have little or no classroom experience with low performing students. In any case, non-normal student distributions are a recipe for a difunctional community. The high performing students quickly recognize that they can outperform the low performing students and reduce their personal study efforts dramatically, possibly at their peril should they slack off too much or get into the wrong company. Therefore, it is better to attack the root cause of the non-normality of the distribution.

The simple activities of asking questions or calling on students, and pop quizzes, are barely engaging. Group activity is promising but when students are allowed to select their own teams the results are strong teams, weak teams, and more of the same. Showing students how to enter data into computer programs and reading and interpreting the outputs serve as job training, not mind training. Without knowing how the computations are performed, understanding is limited, and critical thinking is bypassed. Critical thinking cannot develop. They do not provide the mind training that is required for a professional to become a lifelong learner. These methods barely qualify as active learning. And, the commitment to diligence is minimal compared to deep learning performance-based activities. Furthermore, the non-normal grade distribution persists (Figs. 2a, b, 3a, b).

Learning is a deeply personal process. No professor can learn on behalf of any student. Nor can any fellow student. A student who is learning effectively can leave a lecture with the correct knowledge and understanding and will be better off for it. But a student who misunderstood the lecture may be worse off and is likely to misunderstand all subsequent lectures that depends on that knowledge. A class discussion could lead to a correction of the misunderstanding. Therein lies the potential for active learning to demonstrate some value.

The active learning method

While the focus of this research is not on the absolute score averages, it is noted that the Spring semester active learning score averages of 73.1% and 75.76% are greater than 61.44% for the lecture method. The net result of active teaching is the grade distribution in Fig. 4a and b for section 3, and Fig. 5a, b for section 4. In the case of section 3 (Fig. 4a, b) the score average increased from 59.04 to 73.10%. If the zeros are removed from the average, the increase is from 59.04 to 76.43% while the standard deviation goes from 24.97 to 11.32%, the skewness (S) goes from -0.7 to -0.62 and the kurtosis (K) goes from 2.88 to 2.41. In the case of section 4 (Fig. 5a, b) the score average increased from 58.92 to 75.76%. If the zeros are removed from the average, the increase is from 58.92 to 84.63% while the standard deviation goes from 26.16 to 9.63%, the skewness goes from − 0.24 to 0.28 and the kurtosis goes from 2.25 to 2.62. In both cases, $K < 3$ imply that the distributions are platykurtic.

The active learning score distributions are closer to being normal in the sense that they are unimodal. When compared to a normal distribution with the same mean and standard deviation, the sum of Chi square per unit differences between the observed and expected frequencies ($\chi^2 = \sum (fo - fe)^2/fe$)) and the Jarque–Bera (1980, 1987) test statistic ($JB = (n/6)(S^2 + (1/4)(K\text{-}3)^2)$) will be smaller the closer the data are to being normal. The section 3 posttest $\chi^2 = 3.6 <$ the pretest $\chi^2 = 9.1$, and the posttest $JB = 1.75 <$ the pretest $JB = 2.23$. The section 4 posttest $\chi^2 = 1.6 <$ the pretest $\chi^2 = 7.6$, and the posttest $JB = 0.31 <$ the pretest $JB = 0.82$. That is, the posttest scores are closer to being normal. Active learning also implies greater levels of student retention since many students taking mathematics courses must pass if they are to matriculate into various other majors. The precise reasons for this preferred outcome from active learning could be the subject of further study in a larger and more in-depth research project.

Psychological underpinnings

Many different reasons for why active learning techniques are advantageous for students over traditional lecture have been posited over the years. One of the earliest arguments for active learning involves students' ability to focus. Since the 1970s, researchers have argued that students have limited attention spans and are only able to focus on learning for 10 to 15 min, after which attention falters and they are unable to retain information (Hartley and Davis 1978). Such arguments have persisted into the 2000s (e.g. Wankat 2002). Thus, traditional teaching styles, which involve professors lecturing at students for approximately 50 min, are problematic as students are incapable of engaging for such a length of time.

Based on such theories about 15-min attention spans, it could be assumed then that in a typical lecture, the first 15-min would be the most important and students would retain the most information from that time block. In an argument against the idea of a 15-min attention span for students, however, Giles et al. (1982) found that retention of lecture material by a sample of medical residents was poor for material presented during the first 15-min of the lecture. The material most retained was that presented during the 15 to 30-min interval. They also found that where students sat, interestingly, had as much of an impact on student retention as did the time when the material was presented.

The benefits of active learning do not seem to be due to students only having a 15-min attention span. As indicated by Bradbury (2016), few studies have empirically supported the argument that students have 15-min attention spans with robust experimentation and analysis. Despite the lack of evidence that students' have only 15-min attention spans, though, many arguments for the benefits of active learning indicate that active learning is beneficial due to the segmenting of lecture into smaller units to maintain student engagement, by changing the format through which information is conveyed. In fact, many argue that active learning methods are characterized by the introduction of student activity into traditional lecture, with the intention of promoting student engagement, which has been touted as one of the most important predictors of student success (Astin 1993; Hake 1998).

In their call for science professors to engage in active learning, representatives from the Association of American Universities (AAU) and the Research Corporation for Science Advancement Cottrell Scholars argued that it is through active learning that students increase engagement in learning, because it is their position that "students learn better when they participate in and reflect on their own learning process" (Bradforth et al. 2015). Indeed, research has shown that active learning does increase students' interest in the subject matter (Sivan et al. 2000). Rather than due to students only having 15-min attention spans, however, it may be as has been also argued that the high levels of engagement due to active learning techniques may be because active learning is more student-centered compared to more passive traditional lecture, which is teacher centered (Michael 2006).

Michael (2006) also posited several other reasons for why active learning works. He argued that the benefits of active learning may also be related to how we learn, since we learn by actively constructing our own ideas of the topic. We make connections between the material and our previous knowledge and experiences, which is facilitated by active learning techniques. Additionally, in order to develop procedural knowledge that involves knowing the "how" to do things, students need practice through active learning, since only facts and declarative knowledge, which is the "what" of things can be learned through passive means like traditional lecture. Michael (2006) also argued that active learning facilitates learning through encouraging students to explain topics to others, which helps them develop a deeper understanding of the topic and highlights areas of misunderstanding, as suggested in other work (Welsh 2012). Many active learning techniques involve cooperative learning techniques in which students learn by explaining lessons to each other. That helps develop critical thinking skills and fosters greater support among students (Slavin 1996). Such cooperative active learning techniques are also particularly beneficial in larger class sizes, where professors have greater difficulty in engaging with students (Faust and Paulson 1998).

More recently, arguments for the benefits of active learning have involved theories of motivation (e.g., Bradbury 2016). Motivation theories have been used to explain student's engagement, persistence, decision making, help seeking behaviors, and school performance (Meece et al. 2006). Active learning is a major theme for motivation theorists, as students who lack motivation, generally are unable to engage in learning. Additionally, through active learning, teachers are better able to create goal structures in the classroom (Kaplan et al. 2002).

Achievement goal theory is a theory often used by motivation scholars to explore reasons for engaging and persisting with different learning methods (Meece et al. 2006). Under achievement goal theory, two opposing motivations for learning are typically argued. Mastery goal orientation describes students' focus on learning with the goal of accomplishing and understanding challenging material, whereby mastering the material is the source of satisfaction for students. Performance goals, however, describe a goal orientation in which students' aim to be high performing for the satisfaction in outperforming others. With performance goals, social comparison drives students' motivation to succeed.

The activities that teachers utilize in the classroom help promote different goal orientations in students (Kaplan et al. 2002). Mastery goals are supported when classrooms are perceived as having goals tied to students' effort and understanding. Performance goals, however, are supported when students' abilities are compared and schools are seen to emphasize competition for grades (Meece et al. 2006).

Although mixed research has supported whether mastery or performance goals are related to better grades for students, active learning may be related to mastery goals in students, which may be related to better learning outcomes for students (Cicuto and Torres 2016). Cicuto and Torres (2016) found that Biochemistry students had high levels of mastery goals and low levels of performance goals in courses employing active learning strategies, and that they had high levels of self-efficacy and motivation to learn science. Additionally, the researchers found that students' motivation to learn was higher in the classes using active learning techniques, compared to other courses that utilized traditional lectures. The researchers concluded that active learning positively impacted students' motivation to learn.

Mathematical underpinnings of central tendency

We are interested to know if there are any underpinnings of central tendency that might be responsible for combining the efforts of two subpopulations of students when directed by the active learning method. Figure 6a shows a simulation of a bimodal normal distribution obtained from the concatenation ($X = X_1 \mathbin{+\!\!\!+} X_2$) % of two components $X_1 \sim \mathbb{N}(25,25)$ % and $X_2 \sim \mathbb{N}(75,25)$ %. Figure 6b shows the bimodal distribution shifted to the right by 15% to reflect what the student would learn regardless of method of teaching. The two components are then combined by arithmetic averaging $X = 0.5(X_1 + X_2) + 30\%$ and plotted in Fig. 7b. A content contribution of 15% is added to reflect what the student would learn regardless of method of teaching, and a 15% payoff bonus to reflect the increased efficacy of the active learning method over standard lecture. The combined distribution $0.5(X_1 + X_2) + 30\%$ is normally distributed $\sim \mathbb{N}(0.5(25+75)+30, 0.25(25+25)) \sim \mathbb{N}(80, 12.5)$.

The two components are also combined by geometric averaging $X = (X_1X_2)^{0.5} + 30\%$ and plotted in Fig. 8b. In this simulation we are careful to avoid negative values of X_1 and X_2, and thereby avoid complex numbers. A theoretical accounting of how these combinations can occur is given in Appendix C of the supplementary information.

The doctrine of normal tendency

The question remains. Why does active learning convert non-normal scores to normal scores? How is the brain rewired? How do synapses reconnect? What is the cognitive metamorphosis? Each time a student learns something new and practices it, their brain will either change the structure of its neurons (cells) or increase the number of synapses between their neurons, allowing them to send and receive information faster (Klemm 2020; Steven 2014). The operative word here is practice. And practice makes perfect. Doing something over and over does not make it any easier. But the brain changes to become better at it. Active learning may be playing a role. Learning how to solve one kind of task in a set of similar tasks (learning set) makes it easier to learn new tasks in the set (Levine and Harlow 1959). This can lead to higher final examination mean scores within one course and in all courses for which it is a prerequisite. But it does not automatically explain the change in the distribution towards normality. Also, why does normality and therefore the rehabilitation of normality matter (Trafimow et al. 2019), and what are its consequences for communities within a society? For this we might look to Tomasello (2001) who in explaining the social nature of mankind and learning wrote that collaboration is the basis of all human culture.

This mystery of active learning can be studied in future research. In the interim we speculate as follows. The process of active learning effectiveness leading to higher

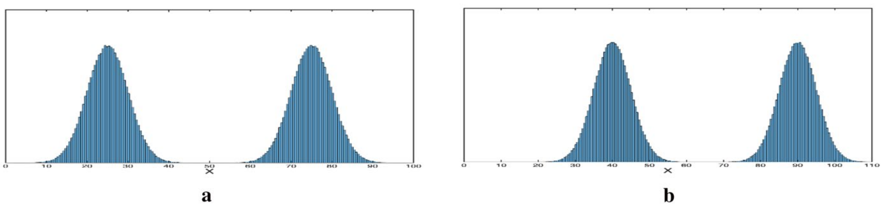


Fig. 6 **a** $X = (X_1 ╫ X_2)$ %. **b** $X = (X_1 ╫ X_2) + 15\%$

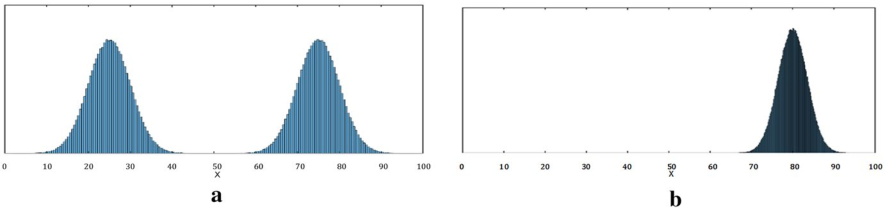


Fig. 7 **a** $X=(X_1 \mathbin{+\!\!+} X_2)$ %. **b** $X=0.5(X_1+X_2)+30\%$

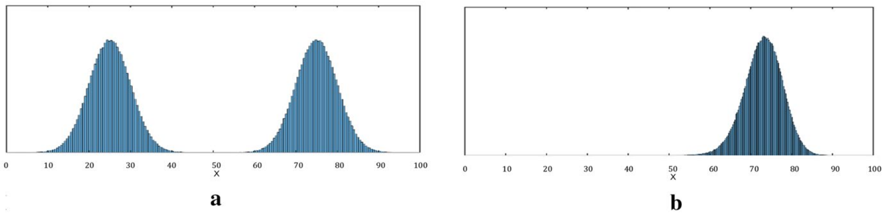


Fig. 8 **a** $X=(X_1 \mathbin{+\!\!+} X_2)$ %. **b** $X=(X_1X_2)^{0.5}+30\%$

mean scores also results in a normal distribution. The higher mean score is the objective, and the normal distribution is the evidence-based indicator of success. Similarly, a team of employees whose members graduated from high schools, colleges or universities where they learned to work towards normality, might be recognizable from its relatively high achievement and normal distribution of member performances, were they to be measured.

The mechanics of the distribution might be proposed as follows. In the strictly didactive lecture method, each student utilizes learning features that seem appropriate to them. For example, learning styles (observation, imitation, trial and error, insight) or methods (visual, auditory, kinesthetic, reading/writing, logic, social, interpersonal, physical, naturalistic) or human brain facility (frontal lobe, temporal lobe, occipital lobe, parietal lobe, hypothalamus, cerebrum, brain stem, cerebellum). These are but a few of the features that relate to consciousness (Crick and Koch 1990; Gardner 1983). Assume that there are M students in a class and that each student deploys only one feature of learning through interaction with the professor. The maximum number of features is M. The number of features expressed in their M test scores is at most M. Now assume that there are $N>1$ such learning features distributed throughout a class of M students, and that active learning invokes these N features in each of the M students. The deployment of N features of learning can only occur by student–student interaction, not solely by student-professor interaction. The maximum number of features is now MxN. The number of ways that these can be expressed in M aggregate test scores is MxN. We know from the central limit theorem (de Moivre 1738) that as the number of expressions increases from M to MxN, the distribution of sample means approaches a normal distribution. This is so regardless of the distribution of the population values.

Consider also, the effect of active learning increasing the test scores. The test score range is from 0 to 100%. If the test scores range from 0 to 100% with a mean of let us say 50%, there are many opportunities for the scores to be bimodal or even multimodal. If the impact of active learning is to raise the test scores such that they fall in the upper half of the range from 50 to 100%, the chances of multiple modes decrease, and a unimodal distribution is more likely. Is it enigmatic or even paradoxical that the relaxation of learning strictures can lead to distributions that are strictly normal? Herein lies the doctrine of normal tendency in active learning. The impact of active learning, ceteris paribus, may be to make test scores more normally distributed but never less.

Regarding the invocation of professor-student–student interactions, consider the following two scenarios. (1) Professor-student interaction: Professor "Tom, how is the sine calculated?" Tom "Hypotenuse over opposite." Professor "Good try, but it is opposite over hypotenuse." This interaction may raise Tom's score and the class average. (2) Professor-student–student interaction: Professor "Tom, how is the sine calculated?" Tom "Hypotenuse over opposite." Professor "*Okay, Hmmm*,…, so Mary, what do you think of Tom's answer?" Mary "I think it is opposite over hypotenuse." Professor "Correct." The student–student element of this interaction may make the score distribution more normal.

Conclusions

In recent years the student pre-college preparation appears to be less and less than adequate. This might be attributable to k-12 educational outcomes. One might think that the students would exploit an opportunity to raise their understanding and performance to give themselves the best possible chances. Instead, they have been consumed with cell phone texting. It is as if it is an epidemic disease. It has made learning nearly impossible (Beland and Murphy 2015; Rosen 2013). In the past, 80% of students participated as intended in some active learning methods. Now, only 20% participate. A non-participating 80% are enough to disrupt the process and defeat the goals. If the Pareto law is in force, the 80/20 elements have reversed. Furthermore, the active learning method exposes the professor to disruptive students that will go to any length to text on their phones, including reporting any attempt to stop them as disrespectful to them! They claim that their cell phone is their property to do with as they wish. One question is: will the university administration support that student claim or support the professor? In case of the former, the professor is unlikely to deploy active learning as it will be systematically defeated and will not serve any useful purpose.

The results from this research are that the spring 2020 semester active learning score averages of 73.1% and 75.76% are greater than the lecture method score average of 61.44%. Also, the distributions of active learning scores are closer to being unimodal normal. A new evidence-based method for measuring teaching efficacy by probability distribution is presented. Active learning is an opportunity to correct the residual non-normalities that arose from a less than rigorous k-12 grade education. A review of k-12 grade education might reveal opportunities for active learning methods that are appropriate at the grade level. Active learning also implies greater potential for student retention and ultimate graduation rate. The precise reasons for this preferred outcome from active learning could be the subject of further study in a larger and more in-depth research project. It is now well known that diversity makes for better decision making in problem solving. Even if certain limiting acuity were obstacles in some institutions, active learning is salutary to academic achievement in terms of making the best of what is possible. Surowiecki (2005) explains how the wisdom of crowds can exceed that of the smartest individual amongst them.

The distributions obtained from active learning are approximately normal. While they are unimodal, they tend to be asymmetrically skewed to the left. But, they are moving in the direction of normality. A doctrine of normal tendency. While professor-student interaction can raise test scores and their averages, professor facilitated student–student interactions makes them more normal. If active learning were to be deployed in all courses throughout the university, there is a possibility that the approximately normal distribution achieved here may become fully symmetrical unimodal normal. It is a principle of management, founded in behavioral psychology, to reward activities that lead to desired outcomes (Kerr 1995; Lunenburg 2011; Maslow 1943). A methodology to measure how well each professor prepares their students to perform in all the other professor's classes is given in Korovyakovskaya et al. (2020), Llaugel and Ridley (2018b), and Ridley and Collins (2015). This can be investigated further in future research.

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Data availability All data are given in the paper.

Declarations

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