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ORIGINAL ARTICLE

Optimal Implementation of Intervention Strategies for Elderly People with Ludomania

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Abstract

Objectives: Now-a-days gambling is growing especially fast among older adults. To control the gratuitous growth of gambling, well-analyzed scientific strategies are necessary. We tried to analyze the adequacy of the health of society mathematically through immediate treatment of patients with early prevention.
Methods: The model from Lee and Do was modified and control parameters were introduced. Pontryagin's Maximum Principle was used to obtain an optimal control strategy.
Results: Optimal control can be achieved through simultaneous use of the control parameters, though it varies from society to society. The control corresponding to prevention needed to be implemented in full almost all the time for all types of societies. In the case of the other two controls, the scenario was greatly affected depending on the types of societies.
Conclusion: Prevention and treatment for elderly people with ludomania are the main intervention strategies. We found that optimal timely implementation of the intervention strategies was more effective. The optimal control strategy varied with the initial number of gamblers. However, three intervention strategies were considered, among which, preventing people from engaging in all types of gambling proved to be the most crucial.

1. Introduction

Problem gambling or ludomania is a type of disorder that consists of an urge to continuously gamble despite harmful negative consequences or a desire to stop and that is associated with both social and family costs. Problem gambling and wider gambling-related harm constitute a significant health and social issue [1]. To study problems associated with gambling, Shaffer and Korn [2] used the classic public health model for communicable disease, which examines the interaction among host, agent, environment, and vector. Moreover, some sociologists [3–6] have shown that a significant predictor of the occurrence of ludomania is peer pressure; in the sense that the occurrence depends on the number of individuals involved, the number of individuals who might be involved, as well as the frequency, duration, priority, and intensity of association with peers. Therefore, ludomania might be considered as a contagious disease. Recently, from the point of view of a communicable disease, Lee and Do [7,8] used a mathematical modeling approach to study the dynamics of problem gambling.

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In this study, we adopted the optimal control theory to their model and tried to find optimal strategies for intervention. A variety of policies and services have been developed with the intent of preventing and reducing problem gambling and related harm. The prevalence and consequences of problem gambling as well as approaches to treatment can be found in the book by Petry [9]. We considered a basic model [7] to incorporate some important epidemiological features, such as time-dependent control functions. The extended model can then be used to determine cost-effective strategies for combating the spread of problem gambling in a given population; a mathematical modeling approach to study the dynamics of problem gambling.

2. Materials and methods

2.1. Basic model

We considered the model of Lee and Do [7] without demographic effect as follows:

$$\begin{aligned} \frac{dS}{dt} &= -\alpha S\frac{L+P}{N} \\ \frac{dL}{dt} &= \alpha S\frac{L+P}{N} - \beta L\frac{P}{N} - \phi L + \psi P \\ \frac{dP}{dt} &= \beta L\frac{P}{N} + \phi L - \psi P - \gamma P\frac{H}{N} - \theta P + \tau H \\ \frac{dH}{dt} &= \gamma P\frac{H}{N} + \theta P - \tau H \end{aligned} \tag{1}$$

The whole population $N(t)=S(t)+L(t)+P(t)+H(t)$ consisting of older adults aged 65−80 years was divided into four classes: susceptible $S(t)$, latent gamblers $L(t)$, pathological gamblers $P(t)$, and treated gamblers $H(t)$. The susceptible population $S(t)$ was a class of individuals who had never gambled more than five times in a single year in their life time. Using the per capita transition rate α, susceptible people entered the second compartment $L(t)$, which was composed of individuals who gamble frequently but had two or less symptoms of problem gambling in the previous year. The transition rate α might be understood as the peer pressure from people in $L(t)$ and $P(t)$. Latent people were pathological gamblers, with the peer pressure transition rate β from people in $P(t)$, or with the natural progression rate ϕ. The class of excessive gamblers $P(t)$ consisted of problem and pathological gamblers. When problem or pathological gamblers sought help, they transited to class $H(t)$ of individuals who were in treatment, with the peer pressure rate γ from people in$H(t)$, or with the voluntary transition rate θ. By attending several types of psychotherapy, including Gamblers Anonymous, cognitive behavioral therapy, behavioral therapy, psychodynamic therapy, and family therapy [10], people in $H(t)$ may have returned to $P(t)$ with the transition rate τ. The rate τ was closely related to the efficacy of a cognitive−behavioral treatment package for pathological gambling [11].

2.2. Optimal control

Using sensitivity analysis, Lee and Do [7] showed that the best way to reduce gambling problems among elderly people is to minimize the value of α, which is similar to the claim of Shaffer and Korn [2] that primary prevention is most important. We considered three interventions to reduce gambling problems among elderly people: reducing α and β, and urging the pathological gamblers to take medical services, which resulted in increasing θ. Although we may have gained some insights from such constant controlling of the parameters, it is unrealistic to have constant controls to α, β, and θ over time. The goal was to show that it was possible to implement time-dependent control techniques while minimizing the cost of implementation of such control measures.

We formulated an optimal control problem for the transmission dynamics of gambling by adding control terms to the basic model (1) as follows:

$$\begin{aligned} \frac{dS}{dt} &= -\alpha(1-u_1(t))S\frac{L+P}{N} \\ \frac{dL}{dt} &= \alpha(1-u_1(t))S\frac{L+P}{N} - \beta(1-u_2(t))L\frac{P}{N} - \phi L + \psi P \\ \frac{dP}{dt} &= \beta(1-u_2(t))L\frac{P}{N} + \phi L - \psi P - \gamma P\frac{H}{N} \\ &\quad -(\theta+\rho u_3(t))P + \tau H \\ \frac{dH}{dt} &= \gamma P\frac{H}{N} + (\theta+\rho u_3(t))P - \tau H \end{aligned} \tag{2}$$

Here, we noted that $N(t)=S(t)+L(t)+P(t)+H(t)$ was constant.

The control variables $u_1(t)$, $u_2(t)$, and $u_3(t)$ represent the amount of intervention related to the parameters α, β, and θ at time t, respectively. The factor of $1-u_1(t)$ and $1-u_2(t)$ reduced the per capita transition rate α from S to L and β from L to$P$, respectively. It was also assumed that the per capita transition rate θ from P to H increased at a rate proportional to $u_3(t)$; where $\rho > 0$ was a rate constant.

We defined our control set to be:

$$U=\{(u_1(t),u_2(t),u_3(t)) : u_i(t) \text{is Lebesgue measurable on } [0,T], 0 \le u_i(t) \le 1, i=1,2,3\}.$$

An optimal control problem with the objective cost functional can be given by

$$J(u_1,u_2,u_3)=\int_0^T \Big(A_L L(t) + A_P P(t) + \frac{B_1}{2}u_1^2(t) + \frac{B_2}{2}u_2^2(t) + \frac{B_3}{2}u_3^2(t)\Big)dt \tag{3}$$

subject to the state system given by (2). In the objective cost functional, the quantities A_L, A_P, B_1, B_2 and B_3 represented the weight constants. The costs associated with controls of transition rates were described in the terms $B_1u_1^2(t), B_2u_2^2(t)$ and $B_3u_3^2(t)$. The goal was to minimize the populations $L(t)$ and $P(t)$ of problem gamblers and the cost of implementing the controls. Therefore, optimal control functions (u_1^*, u_2^*, u_3^*) needed to be found such that:

$$J(u_1^*, u_2^*, u_3^*) = \min\{J(u_1, u_2, u_3) : (u_1, u_2, u_3) \in U\} \quad (4)$$

subject to the system of equations given by (2). In order to find an optimal solution, first we should define the Hamiltonian function H for the problems (2) and (3), and then use Pontryagin's Maximum Principle [12] to derive the characterization for an optimal control. The principle converts (2) and (3) into a problem of minimizing point wise a Hamiltonian, H, with respect to u_1,u_2 and u_3. The Hamiltonian for our problem was the integrand of the objective functional coupled with the four right-hand sides of the state equations, where

$$\mathbf{X}(t) = (S(t), L(t), P(t), H(t)), \mathbf{u}(t) = (u_1(t), u_2(t), u_3(t))$$

and $\boldsymbol{\Lambda}(t) = (\lambda_1(t), \lambda_2(t), \lambda_3(t), \lambda_4(t))$. Then

$$\mathsf{H}(\mathbf{X}(t), \mathbf{u}(t), \boldsymbol{\Lambda}(t)) = A_L L(t) + A_P P(t) + \frac{B_1}{2}u_1^2(t) + \frac{B_2}{2}u_2^2(t) + \frac{B_3}{2}u_3^2(t) + \boldsymbol{\Lambda}(t)\left(\frac{d\mathbf{X}(t)}{dt}\right)^T \quad (5)$$

$$\begin{aligned}
&= A_L L(t) + A_P P(t) + \frac{B_1}{2}u_1^2(t) + \frac{B_2}{2}u_2^2(t) + \frac{B_3}{2}u_3^2(t) + \lambda_1(t)\left(-\alpha(1-u_1(t))S\frac{L(t)+P(t)}{N(t)}\right) \\
&\quad + \lambda_2(t)\left(\alpha(1-u_1(t))S\frac{L(t)+P(t)}{N(t)} - \beta(1-u_2(t))L(t)\frac{P(t)}{N(t)} - \phi L(t) + \psi P(t)\right) \\
&\quad + \lambda_3(t)\left(\beta(1-u_2(t))L(t)\frac{P(t)}{N(t)} + \phi L(t) - \psi P(t) - \gamma P(t)\frac{H(t)}{N(t)} - (\theta+\rho u_3(t))P(t) + \tau H(t)\right) \\
&\quad + \lambda_4(t)\left(\gamma P(t)\frac{H(t)}{N(t)} + (\theta+\rho u_3(t))P(t) - \tau H(t)\right)
\end{aligned} \quad (6)$$

3. Results

Let $S^*(t), L^*(t), P^*(t), H^*(t)$ be optimal state solutions with associated optimal control variables $u_1^*(t), u_2^*(t)$ and $u_3^*(t)$ for the optimal control problem (2) and (3). Then, there were adjoint variables $\lambda_1(t), \lambda_2(t), \lambda_3(t), \lambda_4(t)$ that satisfied

$$\begin{aligned}
\lambda_1'(t) &= (\lambda_1(t)-\lambda_2(t))\alpha(1-u_1^*(t))\frac{L^*(t)+P^*(t)}{N(t)} \\
\lambda_2'(t) &= -A_L + (\lambda_1(t)-\lambda_2(t))\alpha(1-u_1^*(t))\frac{S^*(t)}{N^*(t)} + (\lambda_2(t)-\lambda_3(t))\left(\beta(1-u_2^*(t))\frac{P^*(t)}{N^*(t)} + \phi\right) \\
\lambda_3'(t) &= -A_P + (\lambda_1(t)-\lambda_2(t))\alpha(1-u_1^*(t))\frac{S^*(t)}{N^*(t)} + (\lambda_2(t)-\lambda_3(t))\left(\beta(1-u_2^*(t))\frac{L^*(t)}{N^*(t)} - \psi\right) \\
&\quad + (\lambda_3(t)-\lambda_4(t))\left(\gamma\frac{H^*(t)}{N^*(t)} + \theta + \rho u_3^*(t)\right) \\
\lambda_4'(t) &= (\lambda_3(t)-\lambda_4(t))\left(\gamma\frac{P^*(t)}{N^*(t)} - \tau\right)
\end{aligned}$$

with the transversality condition (or boundary condition)

$$\lambda_j(T) = 0, j = 1, 2, 3, 4. \quad (7)$$

Furthermore, the optimal controls $u_1^*(t), u_2^*(t)$ and $u_3^*(t)$ were given by

$$\begin{aligned}
u_1^*(t) &= \min\left\{1, \max\left\{0, \frac{1}{B_1}\left(\frac{\alpha S^*(L^*+P^*)(\lambda_2-\lambda_1)}{N}\right)\right\}\right\} \\
u_2^*(t) &= \min\left\{1, \max\left\{0, \frac{1}{B_2}\left(\frac{\beta L^* P^*(\lambda_3-\lambda_2)}{N}\right)\right\}\right\} \\
u_3^*(t) &= \min\left\{1, \max\left\{0, \frac{\rho P^*(\lambda_3-\lambda_4)}{B_3}\right\}\right\}
\end{aligned} \quad (8)$$

(See Appendix 1 for the detailed derivation)

For numerical simulation, the forward−backward sweep method [13] based on 4th order Runge−Kutta algorithm was used to treat the problem. The problem consisted of eight ordinary differential equations describing states and adjoint variables along with three

Table 1. Parameter values for the model.

Parameters	α	β	ϕ	ψ	γ	θ	τ
Value	0.095	0.011	0.0039	0.11	0.79	0.019	0.47

controls. The parameters in Table 1 were adopted from a previous study [7] and used for our simulation. A natural shortcoming was that the controls were not 100% effective, so the upper boundary of the controls u_1, u_2, and u_3 was chosen to be 0.6. The rate constant ρ for u_3 was chosen to be 1.

Figure 1 depicts the variation of the maximum for the three controls, which illustrates that the control u_2 became useless for $A_L \geq A_P$, and the control u_2 came into action only for $A_P > A_L$.

Figure 2 depicts the numerical simulation that was carried out in the time interval $[0, 20]$(years) with initial conditions $S(0) = 68500, L(0) = 21000, P(0) = 9000, H(0) = 1500$, so that $N(0) = 100000$ with the weight values $A_L = 1, A_P = 20, B_1 = 5000, B_2 = 500, B_3 = 50000$. The solid lines in the four graphs on the left show populations in different compartments in the absence of control efforts and the dotted line shows the states with implementation of the optimal controls. These graphs reveal the impact of control by the reduced number of gamblers and pathological gamblers, and increased number of susceptible gamblers. The rightmost graph shows the control profile, which says that we need a full three controls almost all the time.

On the other hand, the control scenario would not be similar in all societies. The control scenario might be greatly affected by the number of gamblers and pathological gamblers, that is to say, the control scenario may vary depending on the initial conditions. To analyze the effect of the number of gamblers in society, keeping the total population unchanged, we varied the total number of gamblers and pathological gamblers from 5% to 35%, among which gamblers and pathological gamblers were in the ratio 7:3, and the proportion of treated gamblers was 5% of the total gambling population. Simulation results have been plotted in Figure 3, which illustrates that the control u_1 is implemented in full for almost all the time in all types of societies. In the case of u_2 and u_3, the scenario was more dramatic. Both of the controls had maximum implementation for a long time in a highly-gambling society only. As the percentage of gamblers fell, maximum implementation of u_2 shrank gradually. However, in the case of u_3, it reduced slowly up to $\sim$20%, after which it fell abruptly. For u_2 and u_3, if the gambling populations were $<$11% and $<$10%, respectively, maximum implementation was not necessary at all. However, in societies with a low percentage of gamblers u_3 is used more than u_2.

4. Discussion

An optimal control strategy was analyzed with the help of Pontryagin's Maximum Principle for three control factors. The control scenario would not be similar in all societies. The control scenario might have been affected by its impact on society, and the impact of gamblers on society was introduced into the model through the coefficients A_L and A_P in the cost functional. The inequality $A_P > A_L$ means that pathological gamblers are more detrimental than gamblers. The controls

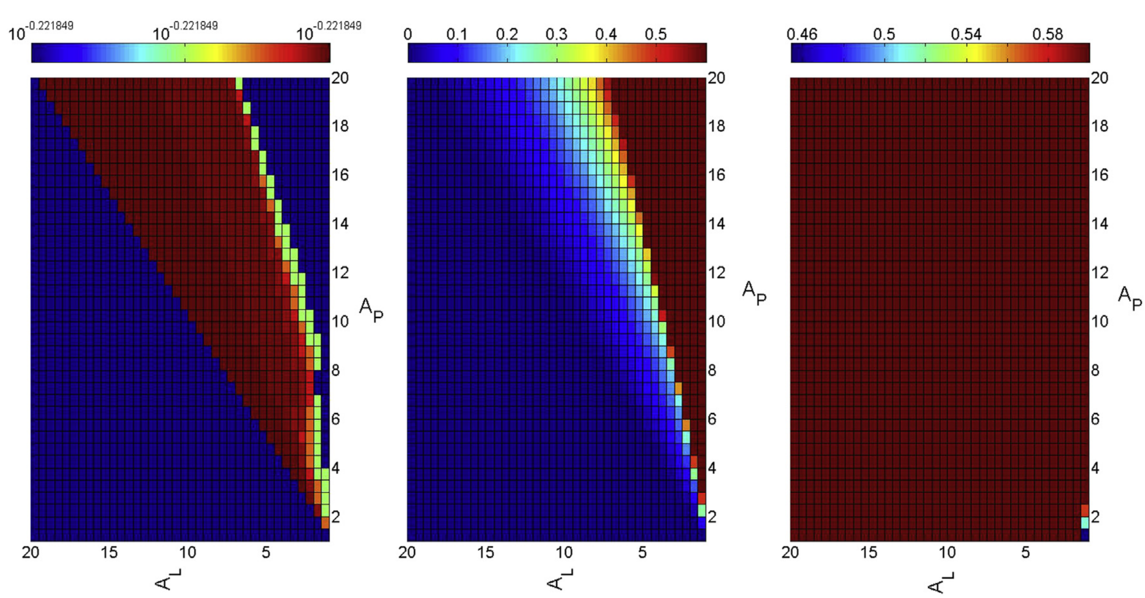

Figure 1. Variation of maximum controls subject to social structure. $A_L = 1, A_P = 20, B_1 = 5000, B_2 = 500$, and $B_3 = 50{,}000$.

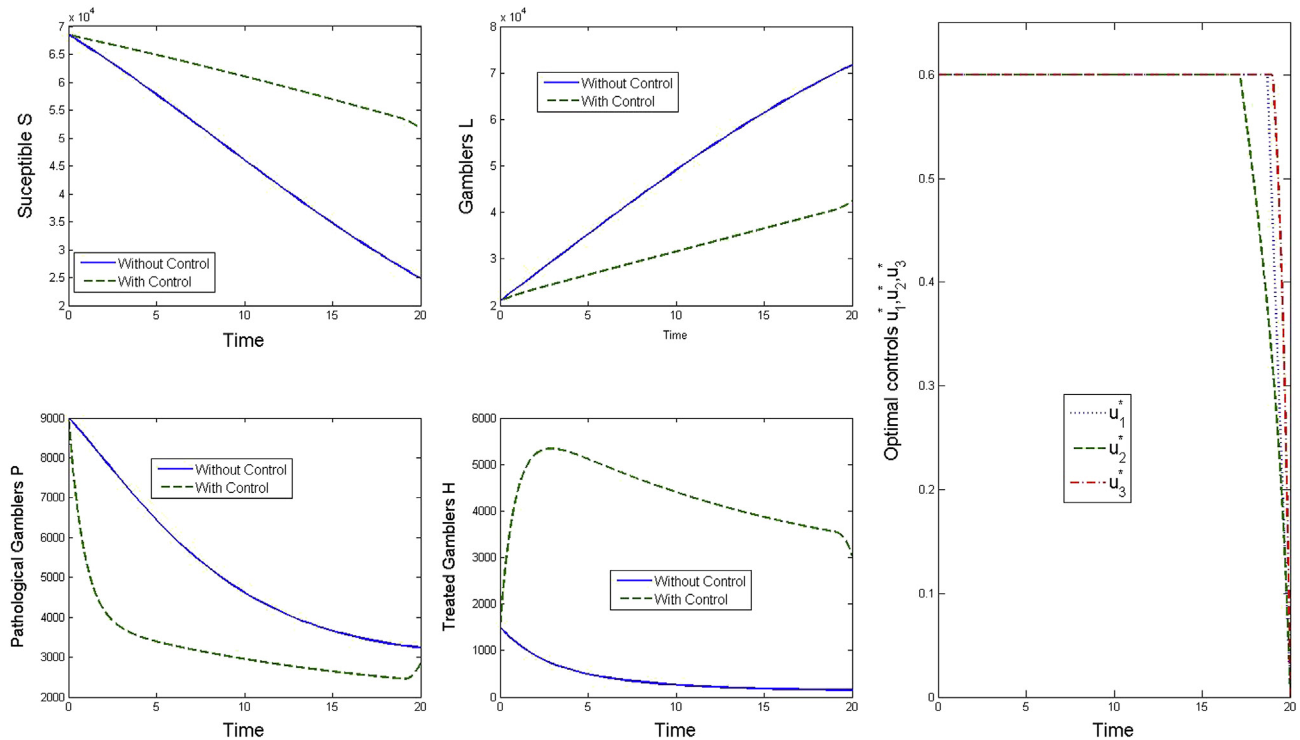

Figure 2. Optimal control scenario with $A_L = 1$, $A_P = 20$, $B_1 = 5000$, $B_2 = 500$, and $B_3 = 50{,}000$.

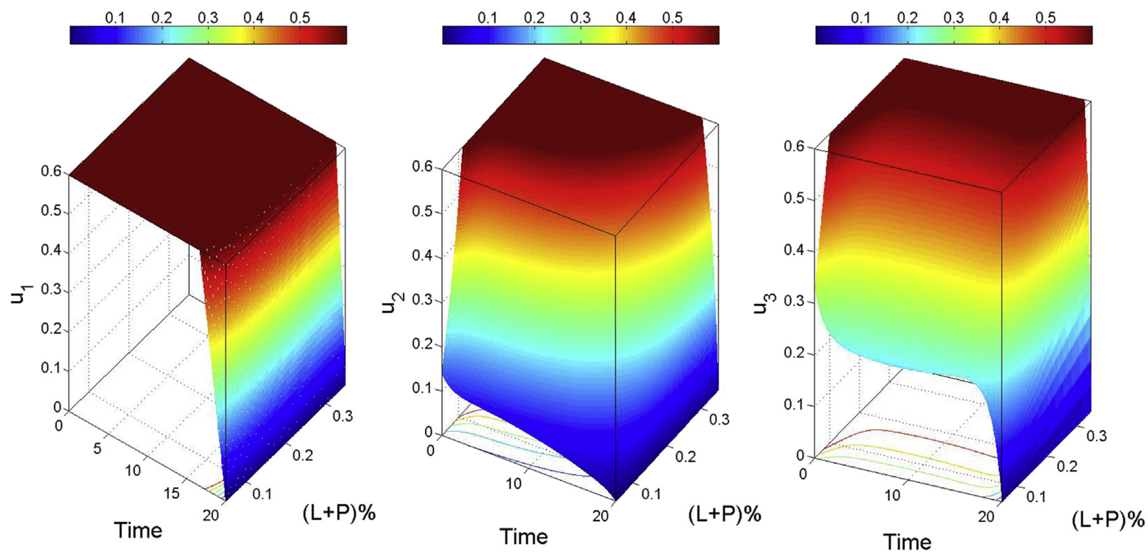

Figure 3. Variation of optimal controls subject to social structure for $B_1 = 5000$, $B_2 = 500$ and $B_3 = 50{,}000$.

u_1 and u_3 were not affected by this, but the control u_2 showed an important response to A_L, A_P, and the control u_2 became useless for $A_L \geq A_P$, and the control u_2 came into action only for $A_P > A_L$. According to Figure 2, we need a full three controls during almost all the time. However, the control scenario might also be greatly affected by the total number of gamblers and pathological gamblers in society. If the control u_1 needs to be implemented in full for almost all the time for all types of societies, in the case of u_2 and u_3, the scenario is greatly dependent on types of societies.

In conclusion, it was conspicuous that simultaneous implementation of all the controls gave the most effective result. However, the control u_1 corresponding to peer pressure on the susceptible gamblers was more crucial than the control u_2 corresponding to peer pressure on the gamblers and u_3 to pressure towards an urge for medical services. In addition, for $A_L \geq A_P$, the control u_2 became totally ineffective. Therefore, strategies should be taken to keep people away not only from problem gambling, but rather from gambling altogether.

Conflicts of interest

All contributing authors declare no conflicts of interest.

Acknowledgments

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Appendix 1. The derivation of optimal controls.

Theorem 1.

Let $S^*(t), L^*(t), P^*(t), H^*(t)$ be optimal state solutions with associated optimal control variables $u_1^*(t), u_2^*(t)$, and $u_3^*(t)$ for the optimal control problem (2) and (3). Then, there were adjoint variables $\lambda_1(t), \lambda_2(t), \lambda_3(t), \lambda_4(t)$ that satisfied

$$\lambda_1'(t) = (\lambda_1(t) - \lambda_2(t))\alpha\left(1 - u_1^*(t)\right)\frac{L^*(t) + P^*(t)}{N(t)}$$

$$\lambda_2'(t) = -A_L + (\lambda_1(t) - \lambda_2(t))\alpha\left(1 - u_1^*(t)\right)\frac{S^*(t)}{N^*(t)} + (\lambda_2(t) - \lambda_3(t))\left(\beta\left(1 - u_2^*(t)\right)\frac{P^*(t)}{N^*(t)} + \phi\right)$$

$$\lambda_3'(t) = -A_P + (\lambda_1(t) - \lambda_2(t))\alpha\left(1 - u_1^*(t)\right)\frac{S^*(t)}{N^*(t)} + (\lambda_2(t) - \lambda_3(t))\left(\beta\left(1 - u_2^*(t)\right)\frac{L^*(t)}{N^*(t)} - \psi\right)$$
$$+ (\lambda_3(t) - \lambda_4(t))\left(\gamma\frac{H^*(t)}{N^*(t)} + \theta + \rho u_3^*(t)\right)$$

$$\lambda_4'(t) = (\lambda_3(t) - \lambda_4(t))\left(\gamma\frac{P^*(t)}{N^*(t)} - \tau\right)$$

with the transversality condition (or boundary condition)

$$\lambda_j(T)=0, j=1,2,3,4. \quad (7)$$

Furthermore, the optimal controls $u_1^*(t), u_2^*(t)$ and $u_3^*(t)$ were given by

$$u_1^*(t)=\min\left\{1,\max\left\{0,\frac{1}{B_1}\left(\frac{\alpha S^*(L^*+P^*)(\lambda_2-\lambda_1)}{N}\right)\right\}\right\}$$
$$u_2^*(t)=\min\left\{1,\max\left\{0,\frac{1}{B_2}\left(\frac{\beta L^*P^*(\lambda_3-\lambda_2)}{N}\right)\right\}\right\}$$
$$u_3^*(t)=\min\left\{1,\max\left\{0,\frac{\rho P^*(\lambda_3-\lambda_4)}{B_3}\right\}\right\} \quad (8)$$

Proof.

To determine the adjoint equations and the transversality conditions, we used the Hamiltonian (6). By Pontryagin's Maximum Principle [12], setting $S(t)=S^*(t), L(t)=L^*(t), P(t)=P^*(t), H(t)=H^*(t)$ and also differentiating the Hamiltonian (6) with respect to $S(t), L(t), P(t), H(t)$, we obtained:

$$\lambda_1'(t)=-\frac{\partial \mathsf{H}}{\partial S}=\lambda_1(t)\alpha\left(1-u_1^*(t)\right)\frac{L^*(t)+P^*(t)}{N(t)}-\lambda_2(t)\alpha\left(1-u_1^*(t)\right)\frac{L^*(t)+P^*(t)}{N(t)}$$
$$\lambda_2'(t)=-\frac{\partial \mathsf{H}}{\partial L}=-A_L+\lambda_1(t)\alpha\left(1-u_1^*(t)\right)\frac{S^*(t)}{N^*(t)}-\lambda_2(t)\alpha\left(1-u_1^*(t)\right)\frac{S^*(t)}{N^*(t)}+\lambda_2(t)\beta\left(1-u_2^*(t)\right)\frac{P^*(t)}{N^*(t)}$$
$$+\lambda_2(t)\phi-\lambda_3(t)\beta\left(1-u_2^*(t)\right)\frac{P^*(t)}{N^*(t)}-\lambda_3(t)\phi$$
$$\lambda_3'(t)=-\frac{\partial \mathsf{H}}{\partial P}=-A_P+\lambda_1(t)\alpha\left(1-u_1^*(t)\right)\frac{S^*(t)}{N^*(t)}-\lambda_2(t)\alpha\left(1-u_1^*(t)\right)\frac{S^*(t)}{N^*(t)}+\lambda_2(t)\beta\left(1-u_2^*(t)\right)\frac{L^*(t)}{N^*(t)}$$
$$-\lambda_2(t)\psi-\lambda_3(t)\beta\left(1-u_2^*(t)\right)\frac{L^*(t)}{N^*(t)}+\lambda_3(t)\psi+\lambda_3(t)\left(\gamma\frac{H^*(t)}{N^*(t)}+\theta+\rho u_3^*(t)\right)-\lambda_4(t)\left(\gamma\frac{H^*(t)}{N^*(t)}+\theta+\rho u_3^*(t)\right)$$
$$\lambda_4'(t)=-\frac{\partial \mathsf{H}}{\partial H}=\lambda_3(t)\left(\gamma\frac{P^*(t)}{N^*(t)}-\tau\right)-\lambda_4(t)\left(\gamma\frac{P^*(t)}{N^*(t)}-\tau\right)$$

To obtain the optimality conditions (8), we also differentiated the Hamiltonian H, with respect to $u_1(t), u_2(t), u_3(t)$ and set it equal to zero.

$$0=\frac{\partial \mathsf{H}}{\partial u_1}=B_1u_1^*(t)+\lambda_1(t)\alpha S^*(t)\frac{L^*(t)+P^*(t)}{N(t)}-\lambda_2(t)\alpha S^*(t)\frac{L^*(t)+P^*(t)}{N(t)}$$
$$0=\frac{\partial \mathsf{H}}{\partial u_2}=B_2u_2^*(t)+\lambda_2(t)\beta L^*(t)\frac{P^*(t)}{N^*(t)}-\lambda_3(t)\beta L^*(t)\frac{P^*(t)}{N^*(t)}$$
$$0=\frac{\partial \mathsf{H}}{\partial u_3}=B_3u_3^*(t)-\lambda_3(t)\rho P^*(t)+\lambda_4(t)\rho P^*(t)$$

Solving for optimal controls, we obtained:

$$u_1^*(t)=\frac{\alpha S^*(t)(L^*(t)+P^*(t))(\lambda_2(t)-\lambda_1(t))}{B_1N(t)}$$
$$u_2^*(t)=\frac{\beta L^*(t)P^*(t)(\lambda_3(t)-\lambda_2(t))}{B_2N(t)}$$
$$u_3^*(t)=\frac{\rho P^*(t)(\lambda_3(t)-\lambda_4(t))}{B_3}$$

To determine an explicit expression for the optimal controls for $0\le u_i^*(t)\le 1, (i=1,2,3)$, a standard optimality technique was utilized. We considered the following three cases.

On the set $\{t: 0<u_1^*(t)<1\}$, we had $\partial\mathsf{H}/\partial u_1=0$. Hence, the optimal control was:

$$u_1^*(t)=\frac{\alpha S^*(t)(L^*(t)+P^*(t))(\lambda_2(t)-\lambda_1(t))}{B_1N(t)}$$

On the set $\{t: u_1^*(t)=0\}$, we had $\partial\mathsf{H}/\partial u_1\ge 0$. This implies that:

$$\lambda_1(t)\alpha S^*(t)\frac{L^*(t)+P^*(t)}{N(t)}-\lambda_2(t)\alpha S^*(t)\frac{L^*(t)+P^*(t)}{N(t)}\ge 0$$

when we had that $\alpha S^*(t)(L^*(t)+P^*(t))(\lambda_2(t)-\lambda_1(t))/B_1N(t) \leq 0 = u_1^*(t)$

On the set $\{t : u_1^*(t) = 1\}$, we had $\partial H/\partial u_1 \leq 0$. This implied that

$$\lambda_1(t)\alpha S^*(t)\frac{L^*(t)+P^*(t)}{N(t)} - \lambda_2(t)\alpha S^*(t)\frac{L^*(t)+P^*(t)}{N(t)} \leq -B_1$$

when we had that $\alpha S^*(t)(L^*(t)+P^*(t))(\lambda_2(t)-\lambda_1(t))/B_1N(t) \geq 1 = u_1^*(t)$

Combining these three cases above, we found a characterization of u_1^*

$$u_1^*(t) = \min\left\{1, \max\left\{0, \frac{1}{B_1}\left(\frac{\alpha S^*(L^*+P^*)(\lambda_2-\lambda_1)}{N}\right)\right\}\right\}$$

Using similar arguments, we also obtained the second and third optimal control function

$$u_2^*(t) = \min\left\{1, \max\left\{0, \frac{1}{B_2}\left(\frac{\beta L^* P^*(\lambda_3-\lambda_2)}{N}\right)\right\}\right\}$$

$$u_3^*(t) = \min\left\{1, \max\left\{0, \frac{\rho P^*(\lambda_3-\lambda_4)}{B_3}\right\}\right\}$$

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