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Counter-diabatic driving for fast spin control in a two-electron double quantum dot

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The techniques of shortcuts to adiabaticity have been proposed to accelerate the "slow" adiabatic processes in various quantum systems with the applications in quantum information processing. In this paper, we study the counter-diabatic driving for fast adiabatic spin manipulation in a two-electron double quantum dot by designing time-dependent electric fields in the presence of spin-orbit coupling. To simplify implementation and find an alternative shortcut, we further transform the Hamiltonian in term of Lie algebra, which allows one to use a single Cartesian component of electric fields. In addition, the relation between energy and time is quantified to show the lower bound for the operation time when the maximum amplitude of electric fields is given. Finally, the fidelity is discussed with respect to noise and systematic errors, which demonstrates that the decoherence effect induced by stochastic environment can be avoided in speeded-up adiabatic control.

ELECTRON spins in quantum dots (QDs)[1–5] have been extensively investigated for potential applications in quantum information processing, as spins in QDs are expected as a possible realization of qubit in quantum information science and technology[6]. Especially, a two-electron double QD can be further regarded as the smallest network to implement quantum computation, in which the highly entangled spin state, *i.e.* the singlet, can be generated. Requirements of precisely controlled qubits have intensively stimulated the detailed studies of the interactions in double-dot systems[7,8] and the observations of phenomena thereby, such as Pauli spin blockade[8] and Coulomb blockade[9]. Furthermore, the demands for achieving efficient quantum computations and avoiding decoherence motivate us to manipulate spin states in double QDs in a fast and robust way. There are several methods to manipulate spin in QDs, such as electron spin resonance induced by magnetic field oscillating at the Zeeman transition frequency[1] and electric control with spin-orbit (SO) coupling[2]. Recently, conventional "rapid" adiabatic passages in quantum optics, for example, Landau-Zener scheme, have been extensively used to spin control in single QD[10], coupled double QD[11], tripled QD[12], which can be applied to prepare entanglement states[13] and quantum logical gates, such as NOT[14] and CNOT[15] gates.

Shortcuts to adiabaticity[16,17] have been proposed to speed up the adiabatic process without final excitation with many applications in atomic, molecular, optical physics, many-body physics, and even spintronics, see recent review[18]. In a single QD, we applied the inverse engineering method[19] to design a fast and robust protocol of spin flip in the nanosecond timescale[20], based on the Lewis-Riesenfeld invariant theory[21]. Furthermore, in a two-electron QD, more freedom in the applied electric fields provides the flexibility to control spin states by the invariant dynamics and controllable Lewis-Riesenfeld phases[22]. An alternative shortcut is provided by counter-diabatic control proposed by Demirplak and Rice[23], equivalent to tansitionless quantum driving[24]. This technique was originally utilized to fast adiabatic control in two-level quantum systems theoretically[17,23,24] and experiment-ally[25,26]. Short afterwards, it has been extended to multi-level systems[17,27], and even many-body systems[28–31].

In this Report, we propose a fast and reliable protocol to generate the entangled spin states by using counter-diabatic driving. The external electric fields are designed for rapid spin control in a two-electron double QD in the presence of a static magnetic field and SO coupling. We apply the electric fields, instead of magnetic fields, and take advantage of SO coupling, since the time-dependent electric fields are easy to be generated on the nanoscale by adding local electrodes[3]. In addition, as comparing to a single QD, counter-diabatic driving is applicable in a two-electron double QD, as there exists more freedom with four controllable parameters, x and y components of the external electric fields for each dot. To simplify the experimental setup and reduce the device-dependent noise, we further apply the concept of multiple Schrödinger pictures[32] to find an alternative shortcut with only x
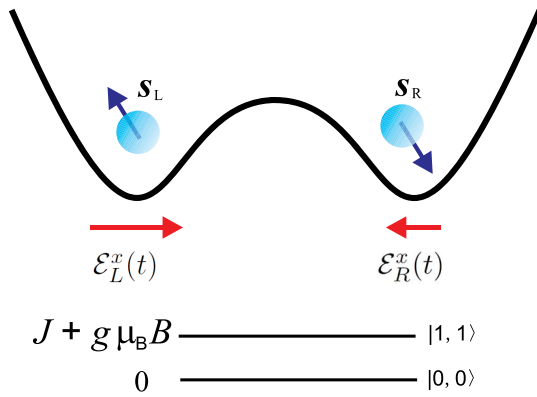
Figure 1 | Schematic diagram of a two-electron double quantum dot in the presence of external electric fields and spin-orbit coupling, where the singlet state and the lowest one of triplet states are considered as effective two-level system, when $|J + \Delta| \ll J$ with Zeeman term $\Delta = g\mu_B B$.

component of the applied electric fields. Moreover, we also quantify how the electric fields increase with shortening the time, to provide the lower bound of operation time for a given maximal amplitude of electric fields. Finally, the stability of designed shortcuts are discussed with respect to decoherence and systematic errors. Our approach presents a simple way to manipulate the singlet-triplet transition, which could be useful for rapid entanglement state preparation.
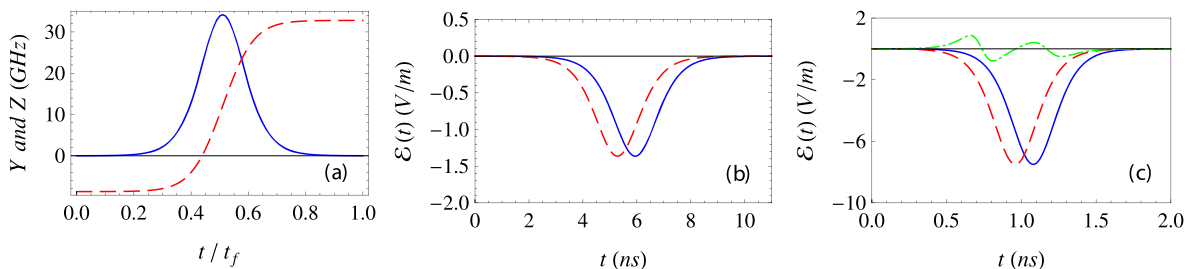
Results

Two electrons are confined in a double QD, described as a quartic potential in Fig. 1, where they are isolated by Coulomb blockade[9]. In the presence of the applied magnetic fields, the lowest four eigenstates of the system can be expressed by singlet and triplet for $S = 0$ and $S = 1$ in the basis of $|S, S_z\rangle$. This report presents a method to achieve fast adiabatic transition between the triplet and the singlet. We design the electric fields in $x - y$ plane to manipulate spin states with static magnetic fields along z direction in each dot, considering structure-related Rashba (α) and bulk-originated Dresselhaus (β) for [110] growth axis. If the energy difference between the singlet and the lowest one of the triplet is much less than the gap between the singlet and the triplet, we focus on the state transition between these lowest two, as shown in Fig. 1, where Landé factor $g < 0$ like in GaAs and InAs QDs.

By choosing $|1\rangle = (1, 0)^T$ and $|-1\rangle = (0, 1)^T$, referring to the states $|0, 0\rangle$ and $|1, 1\rangle$, respectively, we may first take the reference Hamiltonian as

$$H_0 = \frac{\hbar}{2} \begin{pmatrix} Z & iY \\ -iY & -Z \end{pmatrix}, \qquad (1)$$

where $Y = -\sqrt{2}\alpha e (A_L^x - A_R^x)/\hbar c$, $Z = (-J - \Delta)/\hbar + e\beta(A_L^x + A_R^x)/\hbar c$, and A_j^x is determined by the electric fields, $\mathcal{E}_j(t) = -(1/c)\partial A_j/\partial t$.

The subscriptions $j = L, R$ represent the left and the right dots, respectively. Here we assume the ansatz of the vector potentials is $A_j^x = A_0 \{\tanh[(t - a_j t_f)/(w_j t_f)] + 1\}$, where $a_L = 0.54$, $a_R = 0.48$, $w_L = w_R = 0.1$. The ansatz of vector potentials satisfies the condition $A_j^x(0) \simeq 0$ and guarantees that the electric fields $\mathcal{E}_j^x$ start to be driven from $t = 0$, that is, $\mathcal{E}_j^x \equiv 0$, when $t \leq 0$. When the adiabatic condition

$$\left| \frac{Z\dot{Y} - Y\dot{Z}}{(Y^2 + Z^2)^{3/2}} \right| \ll 1 \qquad (2)$$

is fulfilled, the spin state will evolves from $|-1\rangle$ to $|1\rangle$ adiabatically along one of instantaneous eigenstates. When the final time is $t_f = 11$ ns, the spin state is completely inverted, and the final population of $|1\rangle$ is larger than 0.9999.

Shortening the manipulation time to $t_f = 2$ ns, shrinking A_j^x into this time duration and keeping the same amplitude, we can find the state evolution is no longer adiabatic and the final state cannot reach $|1\rangle$ at the final time. The same profiles of time-dependent Y and Z terms in H_0 are shown in Fig. 2 (a) for different operation times, t_f.

Counter-diabatic driving, equivalent to transitionless quantum driving[17,23,24], provides supplementary time-dependent interactions H_1 to cancel the diabatic couplings of H_0, and make the process fast and adiabatic, where H_1 is[17]

$$H_1 = \frac{\hbar}{2} \begin{pmatrix} 0 & X \\ X & 0 \end{pmatrix}, \qquad (3)$$

with $X = \sqrt{2}\alpha e (A_L^y - A_R^y)/\hbar c$, driven by $\mathcal{E}_D^y$, the difference between y component of two electric fields. As a result, the exact dynamical evolution of total Hamiltonian $H = H_0 + H_1$ coincides with adiabatic approximation of the reference Hamiltonian H_0. However, to implement accelerated adiabatic transitions more energy price has to pay, that is, the maximal amplitude of A_j^y in the X term increases when the finally time t_f is shortened. This can be intuitively understood from time-energy uncertainty principle, that is, A_j^y is proportional to $1/t_f$. Since $\mathcal{E}_j(t) = -(1/c)\partial \mathbf{A}_j/\partial t$, the larger value of $\mathcal{E}_j^x$ and $\mathcal{E}_D^y$ are finally required for the shorter time, t_f, as shown in Fig. 2 (c).

In reality, the electron spin is subject to the device-dependent noise, which could be the amplitude noise of the electric fields[20]. It can be quite important, especially when the electric fields are relatively weak. From the above analysis, we find that four controllable parameters, $\mathcal{E}_j^x$ and $\mathcal{E}_j^y$, x and y components of the electric fields for each electron in a double QD should be applied. If y component of the electric fields can be reduced, we can remove the amplitude noise from y component of the electric field. In addition to decreasing the total decoherent effects resulting from the device-dependent noise, the cancellation of y component of the electric field might be also useful to simplify the setup. To this end, we apply the concept of multiple Schrödinger pictures to find an alternative way to implement the shortcuts. Making unitary transformation of

Figure 2 | (a) Time dependence of Y (solid blue line) and Z (dashed red line) terms of H_0. (b) The applied electric fields $\mathcal{E}_L^x$ (solid blue line) and $\mathcal{E}_R^x$ (dashed red line) drive the state transition of H_0 adiabatically, with $t_f = 11$ ns. (c) The applied electric fields $\mathcal{E}_L^x$ (solid blue line), $\mathcal{E}_R^x$ (dashed red line) and $\mathcal{E}_D^y$ (dot-dashed green line) drive the state transition of H in a fast adiabatic way with shorter time $t_f = 2$ ns.

Hamiltonian $H^{32,33}$ by a rotation around z axis with the angle $\pi/2 - \phi$, we obtain

$$\tilde{H} = \frac{\hbar}{2}\begin{pmatrix} Z+\dot{\phi} & iQ \\ -iQ & -Z-\dot{\phi} \end{pmatrix}, \qquad (4)$$

without σ_x term, where $\tan\phi = Y/X$ and $Q = \sqrt{X^2+Y^2}$. Again, the maximal amplitude of Q will increase when decreasing time t_f, due to the fact that X becomes dominant (the maximal amplitude of Y is unchanged). The Hamiltonian $\tilde{H}$ is equal to the original one H at $t = 0$ and t_f, which guarantees that the initial (final) states of H and $\tilde{H}$ coincide. However, the dynamics is not same during the intermediate process, although the populations are always equal. Accordingly, we may acquire two new controllable x component of the electric fields, $\mathcal{E}_L^{xn}$ and $\mathcal{E}_R^{xn}$, calculated from Eq. (4), see Fig. 3.

Discussion

Comparisons of $\mathcal{E}_L^{xn}$ and $\mathcal{E}_R^{xn}$ provided by different times suggest that stronger electric fields have to be used for shorter times, though the amplitude of electric fields might be optimized by using superadiabatic iterations[32]. However, the amplitude of electric fields cannot be arbitrarily large simply because strong fields may destroy the systems. In order to quantify the energy price mentioned above, we demonstrate the relation between the maximal values of electric fields and the operation time t_f, see Fig. 4. The maximal amplitude of electric fields, $\mathcal{E}_{max} = \max(|\mathcal{E}_L^{xn}|, |\mathcal{E}_R^{xn}|)$, fulfills the scaling law at very short times,

$$\mathcal{E}_{max} \propto \frac{1}{t_f^2}, \qquad (5)$$

since $\mathcal{E}_j^{xn} \propto A_j^y / t_f$ and $A_j^y \propto 1/t_f$ go to infinity in the limit of $t_f \to 0$. The asymptotic exponent of t_f implies that the minimal time should be $\propto \mathcal{E}_{max}^{-1/2}$, which provides the lower bound of operation time when the maximal amplitude of electric fields is given. If the spin system in quantum dot, rather than the atom in harmonic trap, is considered as working medium in the cooling cycles of quantum refrigerator, the minimal time for the (accelerated) adiabatic process, bounded by the energy, could be relevant to the third law of thermodynamics and the unattainability principle[34,35].

For a realistic setup, the coupling to the stochastic environment is a general scenario to be considered, where the hyperfine interactions with the nuclear spin could play important role at low temperature. To study the decoherence effect, we present the master equation for the density matrix[36] in a generic form:

$$\dot{\rho} = -\frac{i}{\hbar}[\tilde{H},\rho] - \frac{\gamma}{2}\sum_i [\sigma_i,[\sigma_i,\rho]] \qquad (6)$$
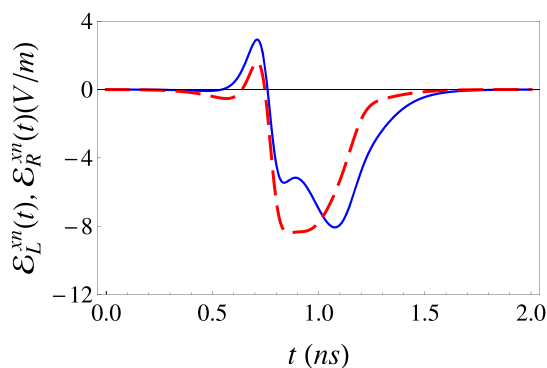
Figure 3 | Electric fields of $\mathcal{E}_L^{xn}$ (solid blue line) and $\mathcal{E}_R^{xn}$ (dashed red line), designed from the Hamiltonian $\tilde{H}$, see Eq. (4).
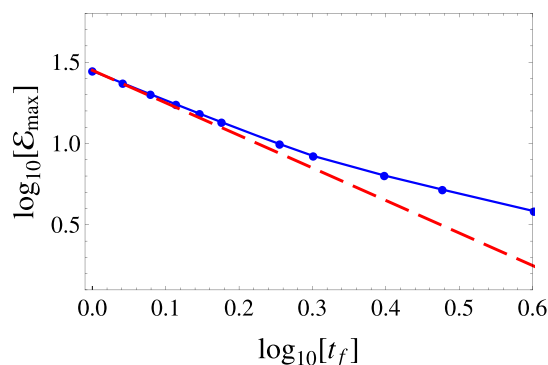
Figure 4 | Dependence of $\mathcal{E}_{max}$ on short time t_f (solid blue line), where the dashed straight line shows the asymptotic exponent of t_f i.e. $\mathcal{E}_{max} \propto 1/t_f^2$.

where γ is the dephasing rate. Solving the Bloch equation, we can obtain the final fidelity ($F = \rho_{11}$) for different times, see Fig. 5, and demonstrate that the faster manipulation increases the fidelity with less influences attributed by decoherence.

To demonstrate the feasibility of our protocol, we also check the stability with respect to systematic errors in $\mathcal{E}_j^{xn}$. The real electric fields can be $\mathcal{E}_j^{real} = \mathcal{E}_j^{xn}(1+\lambda)$, where λ is the relative deviation. The dependence of fidelity F on λ is exhibited in Fig. 6 for different times. Different from decoherence affected by the stochastic environment, fidelity is more stable with larger t_f, since the systematic error considered here depends on the amplitude of electric fields. In general, the speeded-up adiabatic protocol has different stability with respect to different types of noise and systematic errors. Alternatively, one can combine the inverse engineering and optimal control theory to pick up the most robust protocol in quantum two-level systems in presence of different noise and errors[37–39].

Methods

Effective Hamiltonian. The total spin-dependent Hamiltonian consists of Heisenberg term, Zeeman term, and interactions between the electric fields and the electrons, expressed as

$$H_{total} = J\boldsymbol{s}_L\cdot\boldsymbol{s}_R + \sum_j \Delta_j s_j^z - \frac{e}{c}\sum_j \mathbf{A}_j\cdot\mathbf{v}_j, \qquad (7)$$

The subscripts $j = L, R$ represent the left dot and the right one, respectively. Zeeman term is $\Delta = g\mu_B B$ with the equal magnetic fields B applied to the left dot and the right one in z direction, and $\mathbf{A}_j$ are the vector potentials of the electric fields. The spin operators of two electrons confined in each dot are $\boldsymbol{s}_j = \boldsymbol{\sigma}_j/2$ with z component s_j^z. The Heisenberg term $J\boldsymbol{s}_L\cdot\boldsymbol{s}_R$ describes the exchange coupling J between two spins. The example of a double QD of GaAs-based structure ($g = -0.44$) is taken with $B = 3.7$ T. The energy gap between the singlet and the triplet is $J = 0.1$ meV, so that
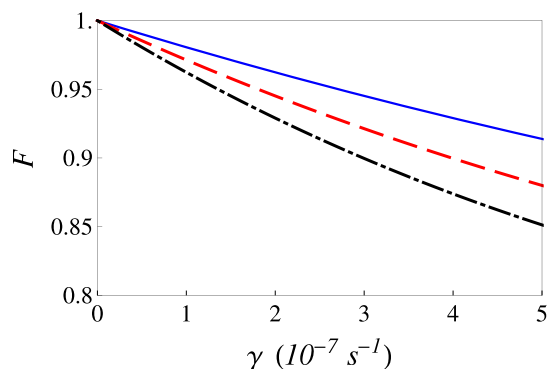
Figure 5 | Fidelity F versus dephasing rate γ with respect to $t_f = 2$ ns (solid blue line), $t_f = 3$ ns (dashed red line), $t_f = 4$ ns (dot-dashed black line).
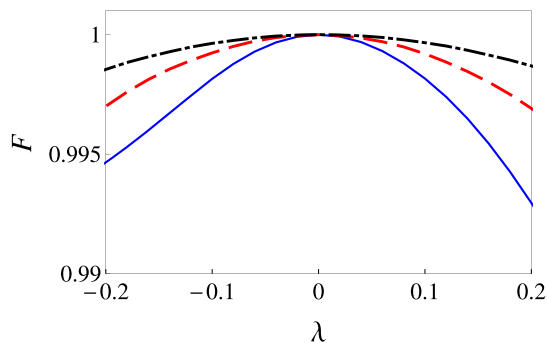
Figure 6 | Fidelity F versus λ with respect to $t_f = 2$ ns (solid blue line), $t_f = 3$ ns (dashed red line), $t_f = 4$ ns (dot-dashed black line).

$|J + \Delta|/J = 0.06 \ll 1$. SO coupling term of Hamiltonian includes structure-related Rashba (α) term and bulk-originated Dresselhaus (β) term for [110] growth axis,

$$H_{soc} = \sum_j \alpha \left(\sigma_j^x p_j^y - \sigma_j^y p_j^x \right) + \sum_j \beta \sigma_j^z p_j^x, \tag{8}$$

so that the spin-dependent velocity operators become

$$v_j^{x(y)} = \frac{i}{\hbar} \left[H_{soc}, x(y)_j \right]. \tag{9}$$

As a result, after shifting some quantity of H_{total}, we can derive a 2×2 Hamiltonian

$$H = \frac{\hbar}{2} \begin{pmatrix} Z & X + iY \\ X - iY & -Z \end{pmatrix}, \tag{10}$$

where Z, Y are A_j^x-dependent while X is A_j^y-dependent, seen in the section above.

Counter-diabatic driving and Z-axis rotation. Naturally, we separate the Hamiltonian H into two parts, H_0 and H_1, where H_0 includes the Y and Z terms driven by the x components of electric fields applied in each dot, and H_1 includes only X term driven by the y components. The strategy of counter-diabatic driving in a two-electron double QD is to set H_0 as reference first, which could be not adiabatic at all. Next, we calculate and add the complementary interaction H_1 to cancel the diabatic couplings of H_0 and make the spin control fast and adiabatic[17,23,24]. Actually, the separation of Hamilton H (10) into H_0 and H_1 depends strongly on the choice of growth axis [110]. For instance, if the growth axis [111] is chosen, the SO coupling term should be modified as

$$H_{soc} = \sum_j \alpha \left(\sigma_j^x p_j^y - \sigma_j^y p_j^x \right) + \sum_j \beta \sigma_j^y p_j^x, \tag{11}$$

and the 2×2 Hamiltonian (10) becomes

$$H = \frac{\hbar}{2} \begin{pmatrix} -(J+\Delta)/\hbar & X + iY \\ X - iY & (J+\Delta)/\hbar \end{pmatrix}, \tag{12}$$

with $X = \sqrt{2} \alpha e \left(A_L^y - A_R^y \right) / \hbar c$ and $Y = -\sqrt{2}(\alpha + \beta) e \left(A_L^x - A_R^x \right) / \hbar c$. Therefore, the approach presented here is not valid, since the reference H_0 and the counter-diabatic driving H_1 can not be naturally separated and calculated.

Here counter-diabatic driving is applicable in a two-electron double QD, as in Hamiltonian H (10) there exists freedom with four controllable parameters, x and y components of the external electric fields for each dot. This is different from the Hamiltonian in a single QD where there are only two controllable parameters, x and y components of the electric field, so that it is impossible to produce the required all-electrical interaction by counter-diabatic driving[20].

Furthermore, using multiple Schrödinger pictures to describe various physical settings sharing the same dynamics is helpful to find alternative shortcuts, when the counter-diabatic term is difficult or impossible to implement[32]. One can transform the Hamiltonian based on Lie algebra to cancel the unwanted component of Hamiltonian[40]. Applying this concept, we make unitary transformation of Hamiltonian H by z-axis rotation. While the original dynamics satisfies $i\hbar \partial_t \Psi(t) = H\Psi(t)$, the new dynamics is given by $i\hbar \partial_t \tilde{\Psi}(t) = \tilde{H}\tilde{\Psi}(t)$, where $\tilde{\Psi}(t) = U^\dagger \Psi(t)$, $\tilde{H} = U^\dagger (H - K) U$ and $K = i\hbar \dot{U} U^\dagger$. In our case, we use the unitary operator $U = (\hbar/2) e^{-i(\pi/2 - \phi)\sigma_z}$, to obtain the Hamiltonian (4).

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Acknowledgments

Author contributions

Y.B. carried out the theoretical and numerical calculation; X.C. analyzed the theoretical results. Both authors wrote and reviewed the manuscript.

Additional information