Mathematical Biology

A game-theoretic model of rabies in domestic dogs with multiple voluntary preventive measures

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Abstract

Game theory is now routinely applied to quantitatively model the decision making of individuals presented with various voluntary actions that can prevent a given disease. Most models consider only a single preventive strategy and the case where multiple preventative actions are available is severely understudied. In our paper, we consider a very simple SI compartmental model of rabies in the domestic dog population. We study two choices of the dog owners: to vaccinate their dogs or to restrict the movements of unvaccinated dogs. We analyze the relatively rich patterns of Nash equilibria (NE). We show that there is always least one NE at which the owners utilize only one form of prevention. However, there can be up to three different NEs at the same time: two NEs at which the owners use exclusively only the vaccination or

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movement restriction, and the third NE when the owners use both forms of prevention simultaneously. However, we also show that, unlike the first two types of NEs, the third kind of NE is not convergent stable.

1 Introduction

Since the seminal paper Bauch and Earn (2004), game theory has been increasingly applied to quantitatively model the decision making of individuals presented with various voluntary interventions; see, for example Schecter (2021), Buonomo et al. (2019), Xin et al. (2019), Doutor et al. (2016), Reluga and Li (2013) or Verelst et al. (2016), Wang et al. (2016) and Chang et al. (2020) for recent reviews. The studies include recent COVID-19 prevention models (Choi and Shim 2020; Agusto et al. 2022; Piraveenan et al. 2021) as well as models of typhoid fever (Acosta-Alonzo et al. 2020), polio (Cheng et al. 2020; Reluga and Galvani 2011), smallpox (Bauch et al. 2003; Molina and Earn 2015), chickenpox (Liu et al. 2012), measles (Shim et al. 2012), influenza (Shim et al. 2012), cholera (Kobe et al. 2018) or Hepatitis B (Chouhan et al. 2020; Scheckelhoff et al. 2021). All of these models, similarly to a general model of Geoffard and Philipson (1997), demonstrate that voluntary prevention alone is not enough to eliminate the disease when the cost of prevention is high relative to the cost of the disease. On the other hand, there are models for vector-borne diseases such as malaria (Broom et al. 2016), dengue (Dorsett et al. 2016), Chagas (Han et al. 2020), chikungunya (Klein et al. 2020), visceral leishmaniasis (Fortunato et al. 2021) or zika (Angina et al. 2022) or diseases like Ebola (Brettin et al. 2018) that show that voluntary prevention can help with disease elimination once the cost of prevention is relatively low. Even in such cases, though, the elimination can never be achieved by the voluntary prevention alone (Bauch and Earn 2004).

Despite the variance of modeled diseases, the majority of the models have considered only a single preventive strategy and have not incorporated multiple options. The notable exceptions include Kobe et al. (2018) who developed a model involving protection against cholera by vaccination and the use of clean water, and Choi and Shim (2020) who studied protection against COVID-19 by vaccination and social distancing.

The aim of this paper is to analyze the multiple prevention options in more detail and to study the patterns of the Nash equilibria (NE). We minimize the number of compartments and parameters of the underlying disease transmission model and focus largely on the analysis of the game-theoretic component. We use a very simple Susceptible-Infected compartmental model of rabies in the dog population. We consider two choices of the dog owners: to vaccinate their dogs or to restrict the movements of unvaccinated dogs in order to prevent contact with potentially infected other dogs or wild animals. We determine the equilibria of the disease dynamics and then analyze the NEs of the population game. We demonstrate that even our simple model yields a

relatively rich structure of NEs. There is always at least one NE at which the owners utilize only one form of prevention. However, there can be up to three different NEs at the same time: the above two at which the owners use exclusively only one form of the prevention and the third NE when the owners use both forms of prevention simultaneously. We also show that, unlike the first two types of NEs, the third kind of NE is not convergent stable. Here, we adopt the notion of convergent stability as done in Bauch and Earn (2004).

2 Mathematical model

In this section we build a mathematical model for the voluntary prevention of rabies in dogs. We first introduce a simple compartmental model of rabies transmission. Then, we add the game-theoretic component that will allow us to investigate individuals' optimal decisions.

Our compartmental model is a simplification of the model considered in Zhang et al. (2011) who used Susceptible-Exposed-Infected-Vaccinated structure for both the dog population and human population. For simplicity, we consider only the dog population and omit the incubation period.

2.1 Compartmental model

We consider a very simple transmission model for rabies in domestic dogs. The dog population is split between susceptible, S, infected, I, and vaccinated, V. The dogs enter the population at a rate Λ. A fraction $\overline{v}$ of the new dogs get vaccinated and enter the compartment V while the rest, the fraction $(1 - \overline{v})$, of dogs remain susceptible and enter the compartment S. For simplicity, we assume that vaccination offers complete protection against rabies and that the owners re-vaccinate the dogs as the vaccine immunity wanes. The susceptible dogs become infected at rate $(1 - \overline{r})\beta \left(W + \frac{I}{N} \right)$ where $\overline{r}$ is the population level at which the dog owners restrict their dogs movement, β is the transmission rate (of unrestricted dogs), W represents the proportion of wild animals infected by rabies and $N = S + I + V$ is the total dog population size. For simplicity, we assume that W is constant. All dogs die of natural causes at a rate μ. Infected dogs die at an additional rate μ_R which stands for disease mortality as well as for intentional quarantine of infectious dogs by their owners. All rates are assumed positive and we also assume $W \geq 0$.

The schematic diagram of the model is shown in Fig. 1 and the model parameters are summarized in Table 1. The diagram yields the following differential equations.

$$\frac{dS}{dt} = (1 - \overline{v})\Lambda - (1 - \overline{r})\beta \left(W + \frac{I}{N} \right) S - \mu S \tag{2.1}$$

$$\frac{dV}{dt} = \overline{v}\Lambda - \mu V \tag{2.2}$$

$$\frac{dI}{dt} = (1 - \overline{r})\beta \left(W + \frac{I}{N} \right) S - (\mu + \mu_R)I. \tag{2.3}$$
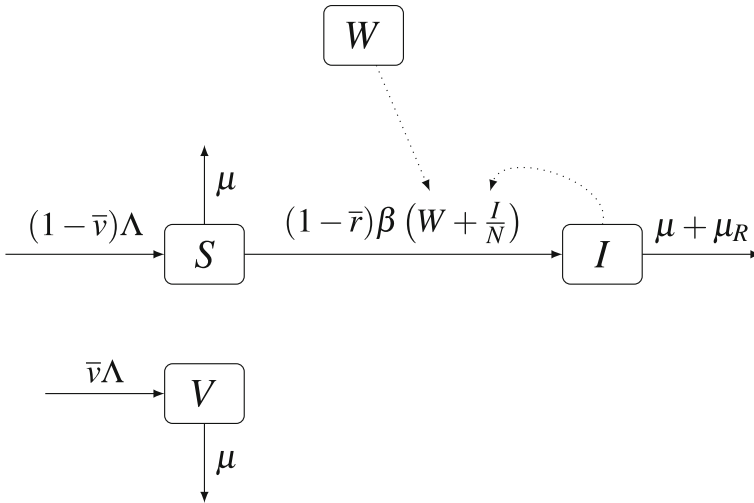
Fig. 1 Scheme of the compartmental ODE model for rabies transmission. The dog population is divided into susceptible, S, infected, I, and vaccinated, V. The wild animals infected by rabies are denoted by W. Solid arrows represent the transitions between compartments. The letters next to the arrows specify the per capita rates of the transitions. Dotted lines represent the influence of one compartment over the transmission rates

Table 1 Model parameters

Symbol	Description	Value
Λ	Birth rate	0.08
μ	Natural death rate	0.08
μ_R	Disease induced mortality rate	1
β	Transmission rate of unrestricted dogs	2.16
W	Prevalence of infection in the wild	0.04
$\bar{v}$	Proportion of dogs being vaccinated (in the whole population)	in [0, 1]
$\bar{r}$	Restriction of dog's movements (average amongst the unvaccinated population)	in [0, 1]
C_V	Cost of vaccination (relative to losing the dog)	> 0, variable
C_M	Cost of movement restriction (relative to losing the dog)	> 0, variable

Except Λ which is in individuals per year, other rates are per capita per year. Most values derived from Zhang et al. (2011); see Sect. 2.3

2.2 Game-theoretic component

We add a game-theoretic component to study individual prevention strategies and introduce the following game inspired by the framework introduced in Bauch and Earn (2004).

The players of the game are dog owners. In the fully general game, the owners can choose (a) to vaccinate their dogs, and/or (b) to limit the unvaccinated dogs' movement and thus influence the transmission rate β. Their strategy is given by a pair (v_{ind}, r_{ind}) where $v_{ind} \in \{0, 1\}$ specifies if their dog gets vaccinated (1) or not (0)

and $r_{ind} \in [0, 1]$ specifies the movement restriction for their unvaccinated dog (with 0 being no restriction and 1 being complete restriction). We will also consider cases when the owners can choose only to vaccinate or only to restrict their dog's movement.

There is a cost associated with the vaccination and movement restrictions. We assume the vaccine costs C_V while, for simplicity, the movement restrictions cost $C_M r_{ind}$. The cost is expressed relative to the cost of the dog contracting rabies (and effectively being lost to the disease), i.e., the cost of losing the dog is assumed to be 1.

The individual's benefits, or payoffs, depend on the individual's strategy but also on the prevalence of rabies in the population, i.e., on the strategies of other players. Following Bauch and Earn (2004), we assume that all individuals are provided with the same information such as the knowledge of W and the overall prevalence of rabies amongst dogs, I/N. We will also assume that they all use the information in the same and rational way to assess costs and risks.

The solution of the game, called the Nash equilibrium, is the population-level value (v_{NE}, r_{NE}) at which no individual can increase their own benefits by deviating from the population strategy. As in Bauch and Earn (2004), a Nash equilibrium is called a convergent stable Nash equilibrium (CSNE), if whenever the population is near but not exactly at the Nash equilibrium and a small number of focal individuals choose their optimal strategy, the population strategy (as an average) gets closer to the Nash equilibrium values.

2.3 Calibration

We base most of the demographic and epidemiological parameter values on Zhang et al. (2011). The values seem consistent with values used in other papers modeling rabies such as Lu et al. (2021), Zinsstag et al. (2009), Hampson et al. (2007). The natural death rate is $\mu = 0.08$, i.e., the dogs life for about 12.5 years. The disease induced mortality is $\mu_R = 1$. As in Hampson et al. (2007), Zhang et al. (2011), we will assume that $\mathcal{R}_e = 2$. Thus, if there is no vaccination and movement restriction, it gives $\beta = \mathcal{R}_e(\mu + \mu_R) = 2.16$. The value of Λ is irrelevant and we set it to $\Lambda = \mu = 0.08$ to achieve a unit population size in the disease-free equilibrium.

The percentage of rabies positive stray dogs in Bangkok was about 25% (Tepsumethanon and Sitprija 2005). The stray dogs account for about 17% of all dogs in Bangkok (Kasempimolporn et al. 2007) which gives $W \approx 0.25 * 0.17 \approx 0.04$.

3 Analysis of the ODE system

First, let us consider the equilibria of the ODE system (2.1)–(2.3), i.e., the solutions of

$$0 = (1 - \overline{v})\Lambda - (1 - \overline{r})\beta \left(W + \frac{I}{N} \right) S - \mu S \tag{3.1}$$

$$0 = \overline{v}\Lambda - \mu V \tag{3.2}$$

$$0 = (1 - \overline{r})\beta \left(W + \frac{I}{N} \right) S - (\mu + \mu_R)I. \tag{3.3}$$
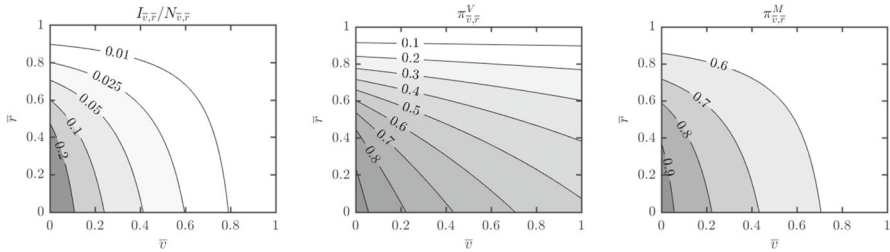
Fig. 2 The equilibrium values of the disease prevalence $I_{\bar{v},\bar{r}}/N_{\bar{v},\bar{r}}$ (left), the probability that an unvaccinated dog becomes infected, $\pi^V_{\bar{v},\bar{r}}$ (center), and the probability that an unvaccinated unrestricted dog becomes infected, $\pi^M_{\bar{v},\bar{r}}$ (center) as a function of $\bar{v}$ and $\bar{r}$ when the other parameter values are as in Table 1; the values of C_V and C_M do not matter. Note that $I_{\bar{v},\bar{r}}$ is decreasing in $\bar{v}$ and $\bar{r}$ as proved in Lemma 1

By (3.2), $V = \bar{v}\frac{\Lambda}{\mu}$. By adding (3.1) and (3.3), we get $S = (1-\bar{v})\frac{\Lambda}{\mu} - \left(1 + \frac{\mu_R}{\mu}\right)I$. Thus, $N = \frac{\Lambda}{\mu} - \frac{\mu_R}{\mu}I$. We also obtain I as a root of a function

$$q(I) = aI^2 + bI + c \tag{3.4}$$

with

$$a = \left(1 + \frac{\mu_R}{\mu}\right)\left((1-\bar{r})\beta - \mu_R - (1-\bar{r})\beta W\frac{\mu_R}{\mu}\right) \tag{3.5}$$

$$b = \frac{\Lambda}{\mu}\left(\mu + \mu_R - (1-\bar{r})\beta(1-\bar{v}) + (1-\bar{r})\beta W\left[1 + \frac{\mu_R}{\mu}(2-\bar{v})\right]\right) \tag{3.6}$$

$$c = -(1-\bar{r})(1-\bar{v})\left(\frac{\Lambda}{\mu}\right)^2\beta W. \tag{3.7}$$

The only biologically reasonable roots are between 0 and $\frac{\Lambda}{\mu+\mu_R}$. We have $q(0) = c \leq 0$ and $q\left(\frac{\Lambda}{\mu+\mu_R}\right) > 0$. Thus, there is only one such root of $q(I)$ and it is given by

$$I_{\bar{v},\bar{r}} = \begin{cases} \frac{-b+\sqrt{b^2-4ac}}{2a}, & \text{if } a \neq 0, \\ -\frac{c}{b}, & \text{otherwise.} \end{cases} \tag{3.8}$$

If $\bar{r} = 1$, we get $S = (1-\bar{v})\frac{\Lambda}{\mu}$ and $I_{\bar{v},1} = 0$. The same solution is also possible if $\bar{r} < 1$ and either $W = 0$ or $\bar{v} = 1$. When $\bar{r} < 1$, $W > 0$, and $\bar{v} < 1$, the disease-free equilibrium is no longer possible. When $W = 0$, we can easily obtain the effective reproduction number as $\mathcal{R}_e = (1-\bar{v})(1-\bar{r})\frac{\beta}{\mu+\mu_R}$ (van den Driessche and Watmough 2002)

The following lemma illustrates that increasing vaccination or movement restrictions decreases the number of infectious individuals in the endemic equilibrium. This is also shown in Fig. 2.

Lemma 1 *The equilibrium values of $I_{\bar{v},\bar{r}}$ and $\frac{I_{\bar{v},\bar{r}}}{N_{\bar{v},\bar{r}}}$ are non-increasing in $\bar{v}$ and $\bar{r}$. Moreover, $I_{\bar{v},\bar{r}}$ and $\frac{I_{\bar{v},\bar{r}}}{N_{\bar{v},\bar{r}}}$ are decreasing in $\bar{v}$ and $\bar{r}$ whenever $I_{\bar{v},\bar{r}} > 0$.*

Proof First, let us prove that $I_{\bar{v},\bar{r}}$ is non-increasing in $\bar{v}$. We have $\frac{\partial a}{\partial \bar{v}} = 0$, $\frac{\partial b}{\partial \bar{v}} = \frac{\Lambda}{\mu}(1-\bar{r})\beta[1 - W\frac{\mu_R}{\mu}]$, and $\frac{\partial c}{\partial \bar{v}} = \left(\frac{\Lambda}{\mu}\right)^2(1-\bar{r})\beta W$. By differentiating $q(I) = 0$ with respect to $\bar{v}$ and using $I_{\bar{v},\bar{r}} \leq \frac{\Lambda}{\mu+\mu_R}$, we get

$$0 = \frac{dq(I)}{d\bar{v}} = a' I_{\bar{v},\bar{r}}^2 + 2a \frac{\partial I_{\bar{v},\bar{r}}}{\partial \bar{v}} I_{\bar{v},\bar{r}} + b' I_{\bar{v},\bar{r}} + b \frac{\partial I_{\bar{v},\bar{r}}}{\partial \bar{v}} + c' \tag{3.9}$$

$$\geq (2a I_{\bar{v},\bar{r}} + b)\frac{\partial I_{\bar{v},\bar{r}}}{\partial \bar{v}} + \frac{\Lambda}{\mu}(1-\bar{r})\beta I_{\bar{v},\bar{r}} - \frac{\Lambda}{\mu}(1-\bar{r})\beta W \frac{\mu_R}{\mu}\frac{\Lambda}{\mu+\mu_R}$$

$$+ \left(\frac{\Lambda}{\mu}\right)^2 \beta W(1-\bar{r}) \tag{3.10}$$

$$= (2a I_{\bar{v},\bar{r}} + b)\frac{\partial I_{\bar{v},\bar{r}}}{\partial \bar{v}} + \frac{\Lambda}{\mu}(1-\bar{r})\beta I_{\bar{v},\bar{r}} + \left(\frac{\Lambda}{\mu}\right)^2 (1-\bar{r})\beta W \left[1 - \frac{\mu_R}{\mu+\mu_R}\right] \tag{3.11}$$

$$\geq (2a I_{\bar{v},\bar{r}} + b)\frac{\partial I_{\bar{v},\bar{r}}}{\partial \bar{v}}. \tag{3.12}$$

Thus, $\frac{\partial I_{\bar{v},\bar{r}}}{\partial \bar{v}} \leq 0$. Furthermore, the above inequalities are strict when $I_{\bar{v},\bar{r}} > 0$, i.e., $\frac{\partial I_{\bar{v},\bar{r}}}{\partial \bar{v}} < 0$ when $I_{\bar{v},\bar{r}} > 0$.

Similarly, $I_{\bar{v},\bar{r}}$ is non-increasing in $\bar{r}$. Indeed, $\frac{\partial c}{\partial \bar{r}} = -\frac{1}{1-\bar{r}}c$, $\frac{\partial b}{\partial \bar{r}} = -\frac{1}{(1-\bar{r})}[b - \frac{\Lambda}{\mu}(\mu+\mu_R)]$, and $\frac{\partial a}{\partial \bar{r}} = -\frac{1}{(1-\bar{r})}\left[a + \left(1 + \frac{\mu_R}{\mu}\right)\mu_R\right]$. Thus, by differentiating $q(I) = 0$ with respect to $\bar{r}$, we get

$$0 = \frac{df(I)}{d\bar{r}} = a' I_{\bar{v},\bar{r}}^2 + 2a \frac{\partial I_{\bar{v},\bar{r}}}{\partial \bar{r}} I_{\bar{v},\bar{r}} + b' I_{\bar{v},\bar{r}} + b \frac{\partial I_{\bar{v},\bar{r}}}{\partial \bar{r}} + c' \tag{3.13}$$

$$= (2a I_{\bar{v},\bar{r}} + b)\frac{\partial I_{\bar{v},\bar{r}}}{\partial \bar{r}} - \frac{1}{(1-\bar{r})}[a I_{\bar{v},\bar{r}}^2 + b I_{\bar{v},\bar{r}} + c] - \frac{1}{(1-\bar{r})}$$

$$\left[-\frac{\Lambda}{\mu}(\mu+\mu_R)I_{\bar{v},\bar{r}} + \left(1 + \frac{\mu_R}{\mu}\right)\mu_R I_{\bar{v},\bar{r}}^2\right] \tag{3.14}$$

$$= (2a I_{\bar{v},\bar{r}} + b)\frac{\partial I_{\bar{v},\bar{r}}}{\partial \bar{r}} + I_{\bar{v},\bar{r}}\frac{\mu+\mu_R}{\mu(1-\bar{r})}\left[\Lambda - \mu_R I_{\bar{v},\bar{r}}\right] \tag{3.15}$$

$$\geq (2a I_{\bar{v},\bar{r}} + b)\frac{\partial I_{\bar{v},\bar{r}}}{\partial \bar{r}}, \tag{3.16}$$

where the last inequality holds because $I_{\bar{v},\bar{r}} \leq \frac{\Lambda}{\mu+\mu_R}$. Thus, $\frac{\partial I_{\bar{v},\bar{r}}}{\partial \bar{r}} \leq 0$. Also, the inequality is strict if $I_{\bar{v},\bar{r}} > 0$ and thus $\frac{\partial I_{\bar{v},\bar{r}}}{\partial \bar{r}} < 0$ when $I_{\bar{v},\bar{r}} > 0$.

Finally, since $N_{\bar{v},\bar{r}} = S_{\bar{v},\bar{r}} + V_{\bar{v},\bar{r}} + I_{\bar{v},\bar{r}} = \frac{\Lambda}{\mu} - \frac{\mu_R}{\mu}I_{\bar{v},\bar{r}}$, it follows easily that $I_{\bar{v},\bar{r}}/N_{\bar{v},\bar{r}}$ is non-increasing (decreasing) in $\bar{v}$ and $\bar{r}$ (when $I_{\bar{v},\bar{r}} > 0$). $\qquad\square$

3.1 Social optimum

Aside from the individual decisions, we will also numerically compute the Pareto (socially) optimal solution, i.e., the levels of vaccination and movement restrictions that minimize the overall costs.

First, let us define the costs as

$$c(\bar{v}, \bar{r}) = V_{\bar{v},\bar{r}} C_V + S_{\bar{v},\bar{r}} \bar{r} C_M + I_{\bar{v},\bar{r}}, \tag{3.17}$$

where $V_{\bar{v},\bar{r}}$, $S_{\bar{v},\bar{r}}$, and $I_{\bar{v},\bar{r}}$ are equilibrium values of vaccinated, susceptible or infected dogs when the population vaccination and movement restriction levels are $\bar{v}$ and $\bar{r}$.

For fixed $\bar{v}$, the socially optimal value of $\bar{r}$ is always 0. Conversely, for fixed $\bar{r}$, the socially optimal value of $\bar{v}$ is 0 when either $C_V \geq C_M$ or $C_V < C_M$ and $\bar{r}$ smaller than a certain critical value; the optimal $\bar{v}$ is 1 otherwise. This corresponds to the fact that when $C_V < C_M$, it is cheaper to vaccinate the dogs than to restrict the movement of unvaccinated dogs. Overall, the Pareto optimal solution is $(0, 0)$, i.e., never vaccinate and never restrict.

We thus defined the average costs as

$$\bar{c}(\bar{v}, \bar{r}) = \frac{c(\bar{v}, \bar{r})}{N_{\bar{v},\bar{r}}}, \tag{3.18}$$

where $N_{\bar{v},\bar{r}}$ is the equilibrium population size. Because of the relatively high mortality, the two notions of the costs are not equivalent to each other. We will see in Fig. 5 that the Pareto solution is unique, unless $C_M = C_V < 1$ in which case there are infinitely many of them.

4 Results

We will first consider a game when the dog owners only decide whether to vaccinate their dogs or not. Then, we will consider the game when the dog owners can only restrict the movement of their unvaccinated dogs. Finally, we will consider the game when the dog owners can use both preventive options.

4.1 Optimal vaccination decisions

In this part, we assume that the dog owners can only decide whether or not to vaccinate the dogs; all other model parameters including $\bar{r}$ are fixed.

In disease-free equilibrium, the optimal decision is not to vaccinate. Therefore, for the rest of the section, we assume that the population is in the endemic equilibrium and $I > 0$ (specifically, either $W > 0$, or $W = 0$ and $\frac{\beta}{\mu + \mu_R} > 1$, i.e., $\mathcal{R}_e > 1$ when $\bar{v} = \bar{r} = 0$).

When the owner decides to vaccinate their dog, the owner will pay the cost C_V. If the owner does not vaccinate, the dog will be susceptible and can become infected

with probability

$$\pi_{\bar{v},\bar{r}}^{V} = \frac{(1-\bar{r})\beta\left(W + \frac{I_{\bar{v},\bar{r}}}{N_{\bar{v},\bar{r}}}\right)}{(1-\bar{r})\beta\left(W + \frac{I_{\bar{v},\bar{r}}}{N_{\bar{v},\bar{r}}}\right) + \mu} \tag{4.1}$$

where $I_{\bar{v},\bar{r}}$ is given by (3.8). By Lemma 1, $I_{\bar{v},\bar{r}}$ and $I_{\bar{v},\bar{r}}/N_{\bar{v},\bar{r}}$ are decreasing in $\bar{v}$. Thus, $\pi_{\bar{v},\bar{r}}^{V}$ is decreasing in $\bar{v}$; see Fig. 2. As in Bauch and Earn (2004), the Nash equilibrium is thus unique and given by

$$v_{\text{NE}} = \begin{cases} 0, & \text{if } C_V > \pi_{\bar{v},\bar{r}}^{V} \text{ for } \bar{v} = 0, \\ \bar{v}, & \text{which solves } C_V = \pi_{\bar{v},\bar{r}}^{V}, \\ 1, & \text{if } C_V < \pi_{\bar{v},\bar{r}}^{V} \text{ for } \bar{v} = 1. \end{cases} \tag{4.2}$$

Also, v_{NE} is a convergent stable Nash equilibrium (CSNE). If $\bar{v} < v_{\text{NE}}$, then the best response is to vaccinate and thus increase the vaccination coverage in the population. Conversely, when $\bar{v} > v_{\text{NE}}$, then the best response is to not vaccinate, and thus decrease the vaccination coverage. In either case, the vaccination coverage in the population will tend towards v_{NE}.

Furthermore, by Lemma 1, as $\bar{r}$ increases, $I_{\bar{v},\bar{r}}$ and $\frac{I_{\bar{v},\bar{r}}}{N_{\bar{v},\bar{r}}}$ decrease. Thus, the risk of infection $\pi_{\bar{v},\bar{r}}^{V}$ decreases as well. Consequently, the equilibrium value of v_{NE} is decreasing in $\bar{r}$. This is shown in Fig. 3. Also, v_{NE} is a continuous function of $\bar{r}$.

When $\bar{r} = 1$, there is no disease in the dog population and $v_{\text{NE}} = 0$. Similarly, when $C_V > 0$ is fixed and $\bar{r} < 1$ is close enough to 1, $v_{\text{NE}} = 0$. Thus, there are only three possible shapes of the curve $v_{\text{NE}}(\bar{r})$ as illustrated in Fig. 3. The shapes depend on the relation of C_V to the following two critical values

$$\pi_{1,0}^{crit} = \frac{\beta W}{\beta W + \mu}, \tag{4.3}$$

$$\pi_{0,0}^{crit} = \frac{\beta\left(W + \frac{I_{0,0}}{N_{0,0}}\right)}{\beta\left(W + \frac{I_{0,0}}{N_{0,0}}\right) + \mu}. \tag{4.4}$$

$\pi_{1,0}^{crit}$ represents a risk of getting infected solely by wild animals (i.e. in a dog population with full prevention measures employed), while $\pi_{0,0}^{crit}$ is the risk of infection by wild animals and other dogs in a completely unprotected population. We have

1. If $C_V > \pi_{0,0}^{crit}$, then vaccination is too expensive and the Nash equilibrium is to not vaccinate even if $\bar{r} = 0$, i.e., to not vaccinate for any $\bar{r}$ and thus $v_{\text{NE}}(\bar{r}) = 0$ for all $\bar{r}$.
2. When $\pi_{1,0}^{crit} < C_V < \pi_{0,0}^{crit}$, then $0 < v_{\text{NE}}(0) < 1$ and $v_{\text{NE}}(1) = 0$, i.e., the owners should vaccinate (but not always) when $\bar{r} = 0$ and not vaccinate at all when $\bar{r} \approx 1$.
3. When $C_V < \pi_{1,0}^{crit}$, then $v_{\text{NE}}(\bar{r}) = 1$ for all $\bar{r}$ small enough while $v_{\text{NE}}(\bar{r}) = 0$ for $\bar{r}$ large enough, i.e., with no or only small movement restrictions, full vaccination is
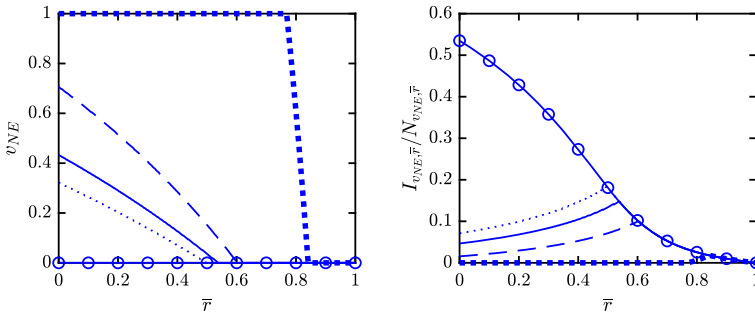
Fig. 3 Left: Dependence of the optimal vaccination decision, v_{NE}, on $\bar{r}$. Right: The equilibrium prevalence of infected dogs, I/N, in the population where the movement restriction is $\bar{r}$ and the vaccination level is v_{NE}. $C_V = 0.2$ (full squares), $C_V = 0.6$ (dashed line), $C_V = 0.7$ (solid line), $C_V = 0.75$ (dotted line) and $C_V = 1$ (empty circles). Value of other parameters as in Table 1

beneficial while nobody should vaccinate if the unvaccinated dogs are significantly restricted in the movement.

If $W = 0$, then $\pi_{1,0}^{crit} = 0$. Thus, there are effectively only two outcomes as we will never have $C_V < 0$.

For the use in Sect. 4.3, let $\bar{r}_0(C_V)$ be the smallest value of $\bar{r}$ such that $v_{NE}(\bar{r}) = 0$. It follows from (4.2) that $\bar{r}_0(C_V)$ solves $C_V = \pi_{0,\bar{r}}^V$, i.e., more explicitly

$$C_V = \frac{(1 - \bar{r})\beta \left(W + \frac{I_{0,\bar{r}}}{N_{0,\bar{r}}} \right)}{(1 - \bar{r})\beta \left(W + \frac{I_{0,\bar{r}}}{N_{0,\bar{r}}} \right) + \mu}. \tag{4.5}$$

We can see in Fig. 5 that the Pareto optimal value of $\bar{v}$ is smaller or equal to v_{NE} when $C_V \geq C_M$ or when $\bar{r}$ is small enough. If $C_V < C_M$ and $\bar{r}$ is large enough, the Pareto optimal solution is larger than v_{NE}. This is because when $\bar{r}$ is high, it is cheaper to vaccinate than to restrict the movement of unvaccinated dogs.

4.2 Optimal movement restrictions

Here, we consider a game in which the dog owners can only decide how much they restrict movement of their unvaccinated dogs and all other model parameters including $\bar{v}$ are fixed.

Assume that the rest of the population is using $\bar{r}$ while the focal owner uses r_{ind}. The risk of infection for the (unvaccinated) dog of a focal individual is given by

$$\pi(r_{ind}) = \frac{(1 - r_{ind})\beta \left(W + \frac{I_{\bar{v},\bar{r}}}{N_{\bar{v},\bar{r}}} \right)}{(1 - r_{ind})\beta \left(W + \frac{I_{\bar{v},\bar{r}}}{N_{\bar{v},\bar{r}}} \right) + \mu}. \tag{4.6}$$

The owner will choose $r_{ind} \in [0, 1]$ that minimizes the overall costs, i.e., minimizes the function

$$h(r_{ind}) = C_M r_{ind} + \pi(r_{ind}). \tag{4.7}$$

We have

$$h' = C_M - \frac{\mu\beta\left(W + \frac{I_{\bar{v},\bar{r}}}{N_{\bar{v},\bar{r}}}\right)}{\left((1 - r_{ind})\beta\left(W + \frac{I_{\bar{v},\bar{r}}}{N_{\bar{v},\bar{r}}}\right) + \mu\right)^2}, \tag{4.8}$$

and thus $h'' < 0$. Hence, the minimum of h must occur at the endpoints of the interval $[0, 1]$, i.e., either at 0 or 1. The condition $h(1) < h(0)$ is equivalent to

$$C_M < \pi_{\bar{v},\bar{r}}^M, \tag{4.9}$$

where

$$\pi_{\bar{v},\bar{r}}^M = \frac{\beta\left(W + \frac{I_{\bar{v},\bar{r}}}{N_{\bar{v},\bar{r}}}\right)}{\beta\left(W + \frac{I_{\bar{v},\bar{r}}}{N_{\bar{v},\bar{r}}}\right) + \mu}. \tag{4.10}$$

Thus, the optimal choice for the focal individual is

$$r_{ind} = \begin{cases} 1, & \text{if (4.9) is true, i.e., if } C_M < \pi_{\bar{v},\bar{r}}^M, \\ 0, & \text{if } C_M > \pi_{\bar{v},\bar{r}}^M, \\ 0 \text{ or } 1, & \text{if } C_M = \pi_{\bar{v},\bar{r}}^M. \end{cases} \tag{4.11}$$

In other words, while the dog owners can choose any value r from $[0, 1]$, their optimal choice is given by either 0 or 1. Also, by Lemma 1, as $\bar{r}$ increases, $\pi_{\bar{v},\bar{r}}^M$ decreases. Thus r_{ind} is a non-increasing function of $\bar{r}$. Hence, as in the previous section, and similarly to the social distancing game considered in Choi and Shim (2020), the NE is obtained in the scenario where the individual payoff is consistent regardless of whether or not one adopts a movement restricting strategy. Thus, the NE is unique and given by

$$r_{NE} = \begin{cases} 0, & \text{if } C_M > \pi_{\bar{v},0}^M, \\ \bar{r}, & \text{which solves } C_M = \pi_{\bar{v},\bar{r}}^M, \\ 1, & \text{if } C_M < \pi_{\bar{v},1}^M. \end{cases} \tag{4.12}$$

Also, by Lemma 1, $\pi_{\bar{v},\bar{r}}^M$ is decreasing in $\bar{v}$; see Fig. 2. Thus r_{ind} and consequently r_{NE} is a non-increasing function of $\bar{v}$ This is shown in Fig. 4.

When $\bar{v} = 1$, every dog is vaccinated and the movement restriction option is irrelevant as it applies only to unvaccinated dogs. For consistency, we will still define r_{NE} even in this instance. We will set $r_{NE} = 1$ if $C_M < \pi_{1,0}^{crit} = \frac{\beta W}{\beta W + \mu}$, and $r_{NE} = 0$
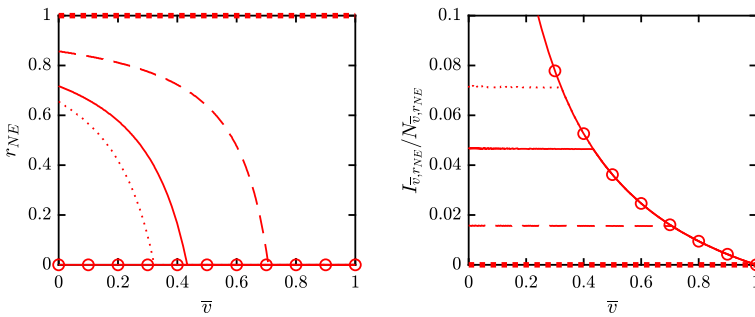
Fig. 4 Left: Dependence of the optimal movement restrictions, r_{NE}, on $\overline{v}$. Right: The equilibrium prevalence of infected dogs, I/N, in the population where the movement restriction is r_{NE} and the vaccination level is $\overline{v}$. $C_M = 0.2$ (full squares), $C_M = 0.6$ (dashed line), $C_M = 0.7$ (solid line), $C_M = 0.75$ (dotted line) and $C_M = 1$ (empty circles). Value of other parameters as in Table 1. On the right, the line with empty circles starts above 0.5 but the figure is truncated to better show what happens for $C_M < 1$

otherwise. This will preserve the continuity of r_{NE} as a function of $\overline{v}$. Note that then $r_{\mathrm{NE}} = 1$ (for any value of $\overline{v}$) if and only if $C_M < \pi_{1,0}^{crit}$.

Similarly to the analysis in the previous section, the r_{NE} is CSNE and the outcomes depend on the relationship between C_M and the two critical values $\pi_{1,0}^{crit}$ and $\pi_{0,0}^{crit}$ defined in (4.3) and (4.4). We again have the following three possibilities for the function $r_{\mathrm{NE}}(\overline{v})$ as illustrated in Fig. 4.

1. If $C_M > \pi_{0,0}^{crit}$, then $r_{\mathrm{NE}}(\overline{v}) = 0$ for all $\overline{v}$, i.e., the cost of movement restriction is prohibitively high to restrict unvaccinated dogs in any fashion no matter what is the vaccination coverage in the population.
2. If $\pi_{1,0}^{crit} < C_M < \pi_{0,0}^{crit}$, then $1 > r_{\mathrm{NE}}(0) > 0$ and $r_{\mathrm{NE}}(1) = 0$, i.e., the owners should somewhat restrict (but never fully) their unvaccinated dog if the vaccination coverage is relatively low, but do not restrict at all once the vaccination coverage is above a certain threshold.
3. If $C_M < \pi_{1,0}^{crit}$, then $r_{\mathrm{NE}}(\overline{v}) = 1$ for all $\overline{v} \in [0, 1]$, i.e., the owners should fully restrict the dog movement regardless of the vaccination coverage in the population.

As before, the third option is possible only if $W > 0$.

To be used in the next section, note that

$$\pi_{\overline{v},\overline{r}}^{V} \leq \pi_{\overline{v},\overline{r}}^{M} \tag{4.13}$$

with the equality happening only when $\overline{r} = 0$. Also, let us define a critical value, $C = C(C_V)$, by

$$C = \pi_{0,\overline{r}_0(C_V)}^{M} = \frac{\beta\left(W + \frac{I_{0,\overline{r}_0(C_V)}}{N_{0,\overline{r}_0(C_V)}}\right)}{\beta\left(W + \frac{I_{0,\overline{r}_0(C_V)}}{N_{0,\overline{r}_0(C_V)}}\right) + \mu}, \tag{4.14}$$

where $\overline{r}_0(C_V)$ solves (4.5). Note that when $C_M = C$, then $r_{\mathrm{NE}}(0)$ is the smallest value of $\overline{r}$ for which $v_{\mathrm{NE}}(\overline{r}) = 0$; see Fig. 7.

We can see in Fig. 5 that the Pareto optimal value of $\bar{r}$ is smaller or equal to r_{NE} in all circumstances.

4.3 Vaccination and movement restriction

Here, we consider the general game in which the owners decide both (a) whether to vaccinate their dogs or not and (b) how much they restrict unvaccinated dogs' movement.

The NE (v_{NE}, r_{NE}) in this instance must satisfy that v_{NE} is the NE of the game when $\bar{r} = r_{NE}$ is fixed (as discussed in Sect. 4.1) and r_{NE} is the NE of the game when $\bar{v} = v_{NE}$ is fixed (as discussed in Sect. 4.2). Graphically, the NE happens at the intersection of the two curves from Figs. 3 and 4 (or when the end point of one of the curves lies on the other curve) as illustrated in Fig. 5. The fact that there is always at least one NE follows from the continuity of $v_{NE}(\bar{r})$ and $r_{NE}(\bar{v})$. However, theoretically there can be more than one NE as the curves can intersect more than once.

Since there are three shapes of the functions $v_{NE}(\bar{r})$ and three shapes of $r_{NE}(\bar{v})$, there are at least nine different scenarios for us to investigate in general. However, the results will depend also on the relationship between C_V and C_M with each other.

Assume that the parameters Λ, μ, μ_R, β are fixed. We will investigate the possible outcomes depending on the values of C_V and C_M and their relationship to $\pi_{0,0}^{crit}$ and $\pi_{1,0}^{crit}$ and themselves. The schematic results are summarized in Fig. 6.

1. When $C_M > \pi_{0,0}^{crit}$, then there is only one NE in the form $(v_{NE}, 0)$ where v_{NE} can take any value in $[0, 1]$. This is illustrated in the top row of Fig. 5.
2. When $C_M < \pi_{1,0}^{crit}$, then there is only one NE in the form $(0, 1)$. This is illustrated in the bottom row of Fig. 5. This option is not possible if $W = 0$ because then $\pi_{1,0}^{crit} = 0$.
3. When $C_V > \pi_{0,0}^{crit}$, then there is only one NE in the form $(0, r_{NE})$ where r_{NE} can take any value in $[0, 1]$. This is illustrated in the right column of Fig. 5.
4. When $\pi_{1,0}^{crit} < C_V < \pi_{0,0}^{crit}$ and $\pi_{1,0}^{crit} < C_M < C_V$, then there is only one NE in the form $(0, r_{NE})$ where $r_{NE} \in (0, 1)$. This is illustrated in Fig. 5 when $C_M = 0.05$ and $C_V = 0.1$.
5. When $\pi_{1,0}^{crit} < C_V < \pi_{0,0}^{crit}$ and $C_V < C_M < C(C_V)$ for the critical value $C(C_V)$ defined by (4.14), then there are three NEs $(v_{NE}^{(i)}, r_{NE}^{(i)})$, $i = 1, 2, 3$ where $0 = v_{NE}^{(1)} < v_{NE}^{(2)} < v_{NE}^{(3)} < 1$ and $1 > r_{NE}^{(1)} > r_{NE}^{(2)} > r_{NE}^{(3)} = 0$. This is illustrated in Fig. 5 when $C_M = 0.15$ and $C_V = 0.1$.
6. When $\pi_{1,0}^{crit} < C_V < \pi_{0,0}^{crit}$ and $C(C_V) < C_M$ for the critical value $C(C_V)$ defined by (4.14), then there is only one NE in the form $(v_{NE}, 0)$ where $v_{NE} \in (0, 1)$. This is illustrated in Fig. 5 when $C_M = 0.5$ and $C_V = 0.1$.
7. When $C_V < \pi_{1,0}^{crit}$ and $\pi_{1,0}^{crit} < C_M < C(C_V)$ for the critical value $C(C_V)$ defined by (4.14), then there are three NEs $(v_{NE}^{(i)}, r_{NE}^{(i)})$, $i = 1, 2, 3$ where $0 = v_{NE}^{(1)} < v_{NE}^{(2)} < v_{NE}^{(3)} = 1$ and $1 > r_{NE}^{(1)} > r_{NE}^{(2)} > r_{NE}^{(3)} = 0$. This is illustrated in Fig. 5 when $C_M = 0.05$ and $C_V = 0.01$. This option is not available if $W = 0$ because then $\pi_{1,0}^{crit} = 0$.
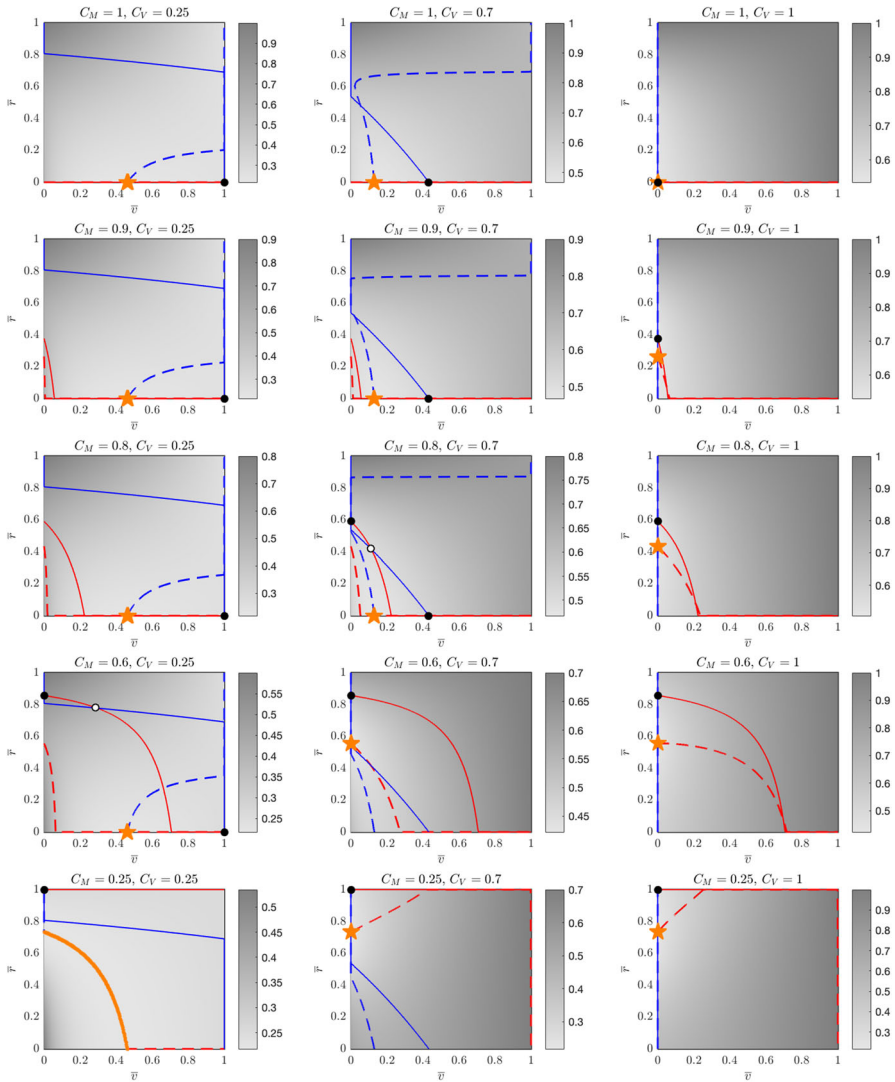
Fig. 5 The Nash equilibria solid curves (red for r_{NE} as a function of $\bar{v}$ and blue for v_{NE} as a function of $\bar{r}$) and black (CSNE) or white (NE that is not CSNE) circles at the intersection of the curves. Pareto optimal solutions as dashed curves or orange stars (if $C_V = C_M = 1$, the orange star is under the black circle; if $C_V = C_M = 0.25$, there are infinitely many Pareto optimal solutions shown by a thick orange curve that is overlapping with the blue and red dashed curves). The underlying shades correspond to the average societal cost $\bar{c}$ given in (3.18). The parameter values are as in Table 1 and the values of C_V and C_M change as indicated in the figures. $C_V = 0.25 < \pi_{1,0}^{crit}$ in the left column, $\pi_{1,0}^{crit} < C_V = 0.7 < \pi_{0,0}^{crit}$ in the center column, and $C_V = 1 > \pi_{0,0}^{crit}$ in the right. Also, $C_M = 0.25 < \pi_{1,0}^{crit}$ at the bottom row, $\pi_{1,0}^{crit} < C_M \in \{0.6, 0.8, 0.9\} < \pi_{0,0}^{crit}$ in the middle rows and $C_M = 1 > \pi_{0,0}^{crit}$ in the top row
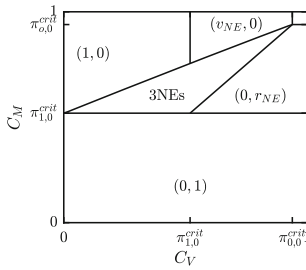
Fig. 6 Schematic diagram of the patterns of NEs as they depend on the values of C_M and C_V for fixed $\Lambda, \mu, \mu_R, \beta$ as in Table 1. $(1, 0)$ denote the NE "always vaccinate, never restrict", $(0, 1)$ is "never vaccinate, fully restrict", $(0, 0)$ is " never vaccinate, never restrict", etc. 3NEs denote a situation where we have three different Nash equilibria as shown in Fig. (5) for $C_M = 0.6, C_V = 0.25$ or $C_M = 0.8, C_V = 0.7$. The empty square in the top right corner is $(0, 0)$, i.e., "never vaccinate and never restrict"

8. When $C_V < \pi_{1,0}^{crit}$ and $C < C_M$ for the same critical value C as above, then $(1, 0)$ is the only NE. This is illustrated in Fig. 5 when $C_V = 0.01$ and $C_M = 0.5$ or $C_M = 0.15$. This option is not available if $W = 0$ because then $\pi_{1,0}^{crit} = 0$.

When $\pi_{1,0}^{crit} < C_M = C_V < \pi_{0,0}^{crit}$, then there are two NEs, $(0, r_{NE})$ and $(v_{NE}, 0)$ where $r_{NE}, v_{NE} \in (0, 1)$. This is illustrated in Fig. 7. If $\bar{r} = 0$, then v_{NE} is such a value of $\bar{v}$ for which $C_V = \pi_{\bar{v},0}^V = \dfrac{\beta\left(W + \frac{I_{\bar{v},0}}{N_{\bar{v},0}}\right)}{\beta\left(W + \frac{I_{\bar{v},0}}{N_{\bar{v},0}}\right) + \mu}$. Similarly, the smallest value of $\bar{v}$ for which $r_{NE} = 0$ has to satisfy $C_M = \pi_{\bar{v},0}^M = \dfrac{\beta\left(W + \frac{I_{\bar{v},0}}{N_{\bar{v},0}}\right)}{\beta\left(W + \frac{I_{\bar{v},0}}{N_{\bar{v},0}}\right) + \mu}$. This means that the two values coincide if $C_M = C_V$. Conversely, when we consider the situation at $v_{NE} = 0$, note that r_{NE} is such a value of $\bar{r}$ for which $C_M = \dfrac{\beta\left(W + \frac{I_{0,\bar{r}}}{N_{0,\bar{r}}}\right)}{\beta\left(W + \frac{I_{0,\bar{r}}}{N_{0,\bar{r}}}\right) + \mu}$. However, then we get

$$C_V = C_M = \pi_{0,\bar{r}}^M = \frac{\beta\left(W + \frac{I_{0,\bar{r}}}{N_{0,\bar{r}}}\right)}{\beta\left(W + \frac{I_{0,\bar{r}}}{N_{0,\bar{r}}}\right) + \mu} > \frac{(1-\bar{r})\beta\left(W + \frac{I_{0,\bar{r}}}{N_{0,\bar{r}}}\right)}{(1-\bar{r})\beta\left(W + \frac{I_{0,\bar{r}}}{N_{0,\bar{r}}}\right) + \mu} = \pi_{0,\bar{r}}^V.$$

(4.15)

Thus, $v_{NE} = 0$ even for lower values of $\bar{r}$ since decreasing $\bar{r}$ increases $(1 - \bar{r})$ as well as $\frac{I_{0,\bar{r}}}{N_{0,\bar{r}}}$. In the same way, we see that the red curve is always above the corresponding blue curve and they intersect only at the point where $\bar{r} = 0$ or $\bar{v} = 0$.

There can also be two NEs $(0, r_{NE})$ and $(v_{NE}, 0)$ (where $r_{NE} \in (0, 1)$ and $v_{NE} \in (0, 1]$) if $C_M = C(C_V)$ as illustrated in Fig. 7 for $C_M = 0.35$ and $C_V = 1$.

To see that the above are the only possibilities and that the NEs shown in Fig. 5 are indeed representatives of all possible NEs, let us consider what happens the red and blue curve intersect. By using the implicit differentiation on $C_M = \pi_{\bar{v},\bar{r}}^M = \dfrac{\beta\left(W + \frac{I_{\bar{v},\bar{r}}}{N_{\bar{v},\bar{r}}}\right)}{\beta\left(W + \frac{I_{\bar{v},\bar{r}}}{N_{\bar{v},\bar{r}}}\right) + \mu}$,
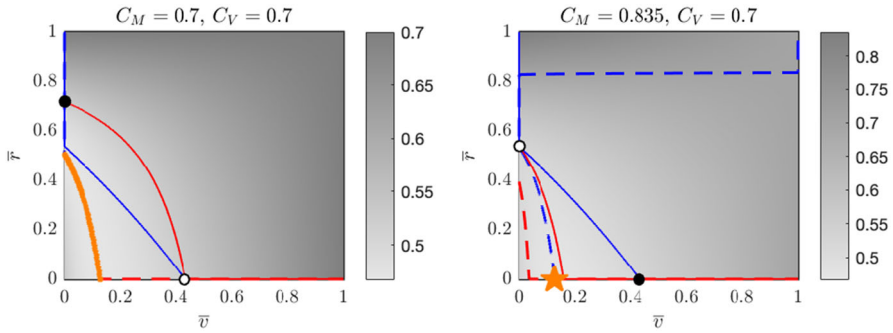
Fig. 7 For non-generic parameter values requiring precise equalities, there can be two NEs and infinitely many Pareto optimal solutions. Left: $C_M = C_V$. Right $C_M = C$ for the critical value C defined by (4.14). Other parameter values as in Table 1. As in Fig. 5, the underlying colors corresponds to the average societal cost $\bar{c}$ given in (3.18)

we get that the slope of the red curve is $-\left(\frac{\partial f}{\partial \bar{v}}\right)/\left(\frac{\partial f}{\partial \bar{r}}\right)$ where $f = \beta\left(W + \frac{I_{\bar{v},\bar{r}}}{N_{\bar{v},\bar{r}}}\right)$.

Similarly, by implicit differentiation of $C_V = \pi_{v,r}^V = \frac{(1-\bar{r})\beta\left(W + \frac{I_{\bar{v},\bar{r}}}{N_{\bar{v},\bar{r}}}\right)}{(1-\bar{r})\beta\left(W + \frac{I_{\bar{v},\bar{r}}}{N_{\bar{v},\bar{r}}}\right)+\mu}$, we get that

the slope of the blue curve is $-\left((1-\bar{r})\frac{\partial f}{\partial \bar{v}}\right)/\left((1-\bar{r})\frac{\partial f}{\partial \bar{r}} - f\right)$. Because, by Lemma 1, $\frac{\partial f}{\partial \bar{v}} < 0$ and $\frac{\partial f}{\partial \bar{r}} < 0$, we thus have that the slope of the blue curve is larger than the slope of the red curve. Consequently, the red and blue curve can intersect only once.

Finally, let us observe that the NEs in the form $(0, r_{NE})$ or $(v_{NE}, 0)$ are CSNEs (for generic parameter values) while the NE in the form $(v_{NE}^{(2)}, r_{NE}^{(2)}) \in (0, 1)^2$ is not. This is illustrated in Fig. 8. Indeed, first consider the NE in the form $(0, r_{NE})$. In the game when players can only choose whether to vaccinate or not $v_{NE} = 0$ is CSNE; similarly, r_{NE} is a CSNE in the game where the players could only choose how much to restrict the movement. Thus, for any strategy $(\bar{v}, \bar{r})$ close to $(0, r_{NE})$, the players will prefer to vaccinate less and they will prefer to restrict less if $\bar{r} > r_{NE}$ and restrict more if $\bar{r} < r_{NE}$. The proof that $(v_{NE}, 0)$ is a CSNE is similar. To show that $(v_{NE}^{(2)}, r_{NE}^{(2)}) \in (0, 1)^2$ is not a CSNE, we note that the intersecting curves split the neighborhood of the intersection into four regions and while the intersection is "attracting" from two of the regions, it is "repelling" from the other two regions—when $(\bar{v}, \bar{r})$ is left from the intersection, right from the blue curve (representing CSNEs from the vaccination only game) but below the red curve (representing CSNEs from the restrictions only game), the individuals will prefer to vaccinate even less and restrict even more which will drive the population away from the equilibrium point. Note that the red and blue curves intersect only in the way illustrated in Fig. 8 because as proved above, the red line is below the blue line left from the intersection while it is above right from the intersection.

In generic cases when there is a unique Pareto optimal solution, the solution is always to use only one of the prevention methods (and never more than the CSNE level). Since the multiple CSNEs happen only when $C_V < C_M$, the Pareto optimal solution in that case is to not restrict dog movements at all and use vaccination only.
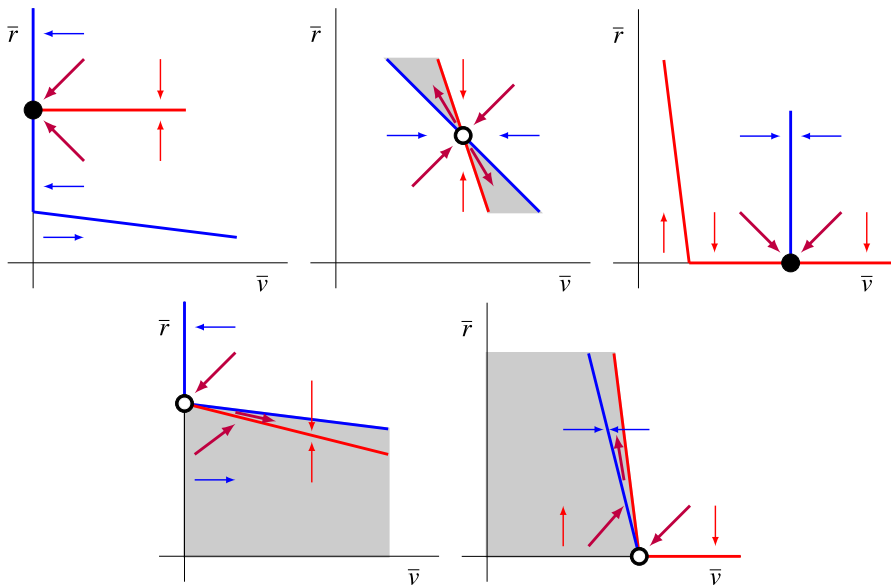
Fig. 8 Schematic diagrams of convergence stability of different NEs. The NEs $(0, r_{NE})$ (top left) and $(v_{NE}, 0)$ (top right) denoted by full black circles are convergent stable as long the point does not fall on the corner of the blue or red curve as in the bottom row. The NE (v'_{NE}, r'_{NE}) (top center) or the NEs in the bottom row denoted by an empty circle are not convergent stable because once we are in the gray area, the population strategy will tend to go away from the NE

5 Conclusions and discussion

We applied the game-theoretic framework (Bauch and Earn 2004) to a simple compartmental ODE model of rabies transmission. We studied the effects of having two different preventive strategies the dog owners can use—vaccination or movement restriction. We have seen that there is a relatively rich structure of possible patterns of NEs. There can be only one NE in the form $(v_{NE}, 0)$, i.e., the individuals should only vaccinate (to a various degree, including not at all) but not limit the movement of unvaccinated dogs. Similarly, there can be only one NE in the form $(0, r_{NE})$, i.e., the individuals should not vaccinate at all but should limit the movement of unvaccinated dogs to a various degree. There can also be three NEs at the same time, the two "pure" ones as described above together with the "mixed" NE.

Our model can be extended in several ways. First, we focused on the game-theoretic aspect and largely simplified the actual disease transmission dynamic. Considering SEI, SIS, SIR, or SIRS dynamics would be the next natural step. We believe that our results, in particular the existence of multiple NEs, will persist even for those dynamics. However, it is conceivable that when the underlying dynamics is significantly more complex, such as the one considered in Choi and Shim (2020), the structure of NEs can become even more complicated.

Second, we assumed that vaccines offer complete protection. This is not entirely realistic (Wu et al. 2011; Magori and Park 2014; Arino and Milliken 2022). Thus even

vaccinated dogs can become infected their owners should be deciding on how to limit their movement. Third, we focused only on the dog population while rabies easily spread from canines to humans (Zhang et al. 2011; Bolzoni et al. 2014; Neilan and Lenhart 2011; Lu et al. 2021; Layan et al. 2021). The dog owners as well as individuals without dogs thus face an additional decision whether or not to vaccinate themselves.

Finally, we note that the two protective actions we considered in our model were qualitatively different. Vaccination is a strictly binary option, one either vaccinates the dog or does not. One may argue that vaccine waning and the need for re-vaccination makes the decision a little more complex; but even with these complexities, the strategy choice is essentially discrete. On the other hand, movement restriction offers a continuity of options from none to full restriction. We have shown that even with this strategy continuum, the optimal choice for individual dog owners is to either restrict the movement completely or not at all, making the vaccination and movement restriction somewhat comparable. However, from the game-theoretic perspective, the options are not equivalent and interchangeable. This is, for example, demonstrated by the fact that "never vaccinate but fully restrict" cannot be NE with any other NE while "always vaccinate and never restrict" can be NE alone or with other NEs. It would be interesting to study the situation when the two protective actions are qualitatively comparable from the beginning. Specifically, one can create and analyze a model where susceptible individuals have various vaccination options available. This is the case for COVID-19 for which there are now numerous vaccines that offer various levels of protection while having different costs and side-effects (Beatty et al. 2021) that range from relatively mild (Riad et al. 2021) to more serious (Ciccarese et al. 2022).

Data Availibility Statement The manuscript has no associated data.

Declarations

Conflict of interest The authors declare no conflict of interest.

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