From Euler disk to phonon bottleneck effect: Excited state physics

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Received: January 20, 2024; Accepted: March 16, 2024; Published Online: March 28, 2024; https://doi.org/10.1016/j.xinn.2024.100614

Citation: Lin X., Fu S., Zhai Y., et al., (2024). From Euler disk to phonon bottleneck effect: Excited state physics. The Innovation **5(3)**, 100614.

The Euler disk, known since 1765,[1] describes a solid disk that rotates on a smooth, firm surface, similar to a coin spun by a finger on a table, before its energy dissipates and it falls on the table.[2] It represents a typical nonequilibrium excited state in classical physics. The most striking characteristic of an Euler disk is that it exhibits a sharply enhanced sound frequency when approaching the critical time, t_c, after which the disk instantly ceases rotation and precession. At the excited-state-to-ground-state transition, its frequency abruptly changes from infinite to zero. This quasi-divergent and suddenly disappearing behavior is intriguing, revealing that this is a shared physical property in the microscopic quantum regime.

The phonon bottleneck effect (PBE) is a well-known phenomenon in quantum materials, which is caused by the closing or sharp shrinkage of an electronic energy gap[3] and has various effects. In ultrafast spectroscopy, the PBE plays the same role as the "U-shaped" double peaks in scanning tunneling spectroscopy when detecting a superconducting energy gap. When the PBE occurs at a critical temperature or pressure, the slow component lifetime of the photoexcited carriers sharply increases, manifesting quasi-divergent behavior (defined as an increasing rate of exhibiting an even higher value when approaching the critical point). Simultaneously, the amplitude, proportional to the density of the photocarriers, decreases noticeably, reaching a minimum value at identical critical state variables T_c or P_c. Similar to the Euler disk frequency, the carrier lifetime undergoes a sharp change immediately after the phase transition, reaching a constant value irrespective of the state before the phase transition. The amplitude reaches a constant value immediately after the phase transition.

In this work, the peculiar quasi-divergent behaviors of the classical regime Euler disk and quantum phenomenon PBE are compared, revealing a shared physical mechanism that bridges the classical Euler disk and quantum PBE, shedding light on the excited state physics.

The rolling Euler disk in the experiment is illustrated schematically in Figure 1A. The disk is made of aluminum alloy with a radius r = 33.5 mm, depth d = 7 mm, and mass m = 37.4 g. The Euler disk rotates fast when triggered by a finger, simultaneously precessing around the z axis with a speed of Ω for 12 s and then suddenly decreasing in frequency to zero. A microphone was used to record the sound signal caused by the collision with the glass surface. The signal is analyzed using Fourier transformation. A series of peak curves is identified, which becomes steeper near t_c (Figure 1B). One curve is selected for quantitative analysis (Figure 1D). The energy of the Euler disk steadily decreases, down to 0 at t_c (Figure 1C). The energy becomes 0 for $t > t_c$, and the definition of the frequency is no longer applicable. If one considers the period to be infinite, the frequency is 0. Thus, there is no connection between the $t < t_c$ and $t > t_c$ regimes. The physics and model of excited state relaxation only apply to $t < t_c$ for an Euler disk.

When approaching t_c, the Euler disk mechanical energy is given by

$$E = E_0(1 - t/t_c)^s, \qquad \text{(Equation 1)}$$

where E_0 is the energy at $t = 0$, and s is a coefficient jointly determined by energy dissipation channels, including both dry and viscous contour friction.[4] The precession frequency is given by

$$f = f_0(1 - t/t_c)^{-s/2}, \qquad \text{(Equation 2)}$$

where f_0 is the precession frequency at $t = 0$. Fitting the experimental data (red curves in Figures 1C and 1D) yields s = 0.564 and s = 0.552, respectively.

A similar peculiar behavior occurs in quantum materials. Ultrafast spectroscopy has revealed that the PBE in superconductors exhibits similar behavior. In the excited state above the Fermi surface, a microscopic quasi-equilibrium balance forms between photocarriers and phonons (Figure 1E). Photocarriers release high-frequency phonons (HFPs; defined as $\hbar\omega_{ph} > 2\Delta(T)$) to combine into Cooper pairs. Conversely, HFPs can break Cooper pairs to excite photocarriers. The closed-cycle process proceeds until the HFP propagates away or decays.[3] Rothwarf-Taylor equations[5] describe these behaviors well: (1) $dn/dt = \eta N - \beta n^2$ and (2) $dN/dt = -\eta N/2 + \beta n^2/2 - \Lambda(N - N_T)$, where Λ is the HFP decay rate.[3]

As the temperature approaches T_c, $\Delta(T)$ gradually becomes 0, strikingly enhancing the HFP density. Thus, the quasi-equilibrium is sustained, exhibiting a dramatically elongated lifetime (Figure 1G). This is the *slow* component lifetime τ_{slow},[3] which can only be detected by ultrafast experimental means. In parallel, the quasi particle density decreases, reaching a minimum at T_c. Simultaneously observing the behaviors in A_{slow} and τ_{slow} near T_c is necessary for identifying the transition (Figures 1F and 1G).

At temperatures near T_c, A_{slow} and τ_{slow} are given by[3]

$$A_{slow} \propto \left[\sqrt{\Delta(T)T}\exp\left(-\Delta(T)\big/k_B T\right)+C\right]^{-1}, \qquad \text{(Equation 3)}$$

and

$$\tau_{slow} \propto \left\{\left[\delta+2n_T(T)\right]\left[\Delta(T)+\alpha T\Delta(T)^4\right]\right\}^{-1}, \qquad \text{(Equation 4)}$$

where C is related to η and β; n_T is the density of thermal carriers, δ and α; and $\Delta(T) = \Delta_0\tanh(\Theta\sqrt{T_c/T-1})$ is the empirical SC gap,[5] with Θ being related to the coupling strength. The SC Δ_0 can be obtained precisely by fitting a large number of data points. No coefficient is assumed between Δ_0 and T_c, enabling ultrafast spectroscopy to detect the strong coupling in superconductors.

The critical order parameters, peculiar characteristic behaviors, and physical processes of the Euler disk and PBE exhibit similarities. For example, phase transitions occur in both, with well-defined order parameters. Both *extensive quantities* (Euler disk order parameter energy and PBE order parameter carrier density) decrease with increasing state variable and reach their minimum value at the critical state variable. Both *intensive quantities* (Euler disk order parameter frequency and PBE order parameter lifetime) exhibit quasi-divergent enhancement. Both the Euler disk and the quantum material enter a new regime that cannot be described by excited state physics when it exceeds the critical state variable.

These similarities reflect the shared physics: both are determined by how their excited states relax to ground states, where a microscopic dynamic quasi-equilibrium exists. The key enabling gradient is the existence of the excited state. Near the transition, the gap shrinks, leading to a higher frequency of the energy exchange (Figure 1H). The lifetime is also enhanced because more carriers exist in the excited state (Figure 1I)—these are two faces of one coin. Accordingly, a similar example is proposed: bouncing a ball on a surface. During the excited-state-to-ground-state relaxation, the bouncing frequency is enhanced, and the mechanical energy gradually dissipates (Figure 1J).

The formulae describing the excited states no longer apply to systems at $T > T_c$. All shared properties disappear with the disappearance of the microscopic quasi-equilibrium, which only exists because of the excited state. At $T > T_c$, all the physical quantities are determined by their new states. The excited state is spectacular, because when it is gone, a phase transition occurs.

In conclusion, a correlation was discovered between the classical Euler disk and quantum material dynamics PBE. The Euler disk frequency and PBE lifetime increase sharply when approaching the critical point, accompanied by a decrease

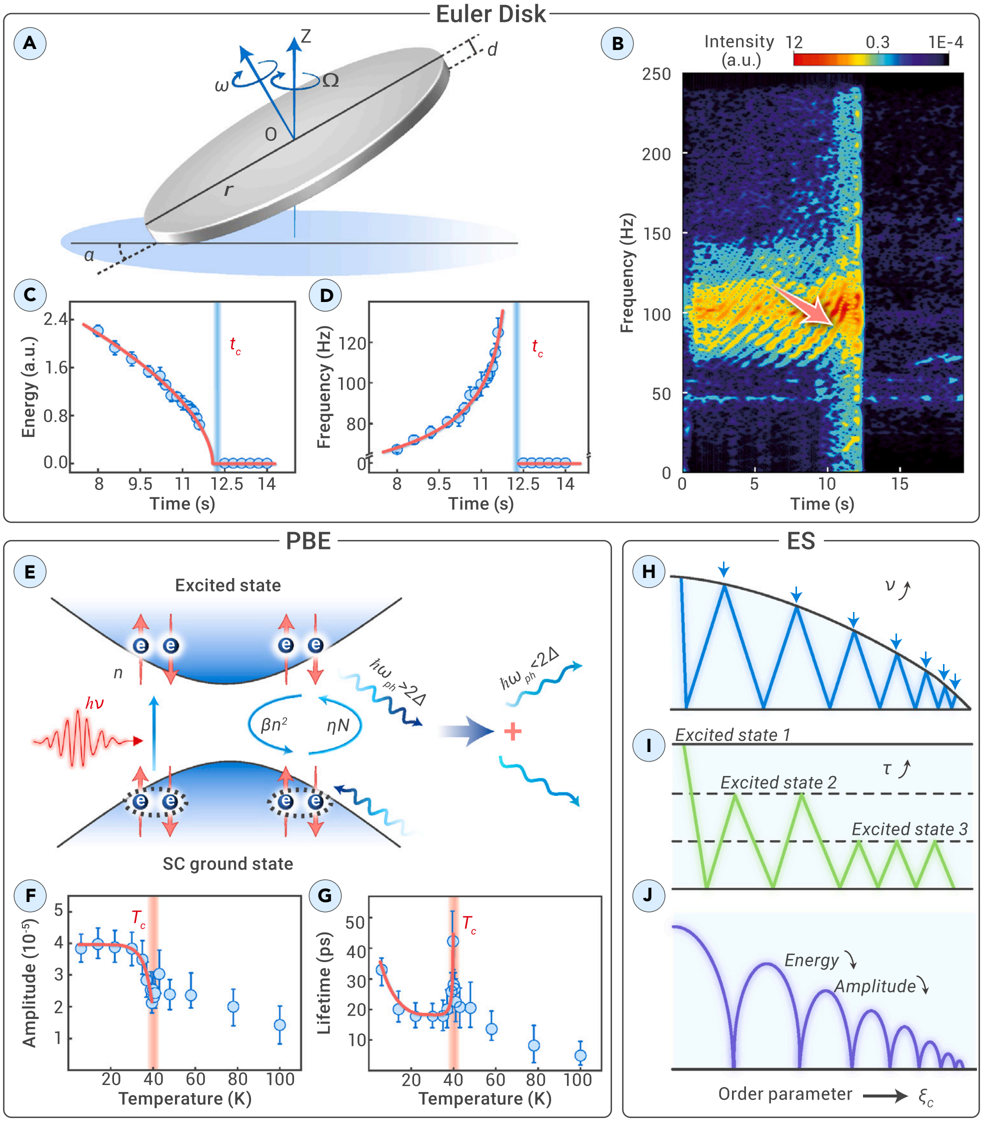

Figure 1. Comparison of the Euler disk with the PBE (A) Schematic of a rolling Euler disk. (B) Experimentally recorded sound signal of the Euler disk from 0 to 19.38 s in the frequency domain. The color (from black to red) represents the intensity of the sound signal. (C and D) Time-dependent Euler disk energy and frequency of a typical curve marked in (B). Blue dots, experimental data; red curves, fitting curves. The error bars in (C) and (D) represent the energy fluctuation and full width at half maxima of the frequency, respectively. (E) Illustration of the microscopic equilibrium between photocarriers and high-frequency phonons (HFPs). Here, ***N*** denotes the density of HFPs and ***n*** of photocarriers, η and β are parameters, and Δ is the superconducting gap. (F and G) The amplitude and lifetime of the quasiparticle slow component, adapted from a previous work.[3] (H–J) Illustration of the increasing frequency ν (H), lifetime τ (I), and bouncing (relaxation) of a falling ball in its excited state (J).

in their energy and quasiparticle density. Critical extensive and intensive order parameters are identified. The findings reveal that excited state physics has shared properties in the classical and quantum regimes. The Euler disk is a classical visualization of the PBE, and the PBE is a quantum-regime Euler disk. The shared properties between the Euler disk and PBE exist ubiquitously in all excited states.

REFERENCES

1. Leonhard, E. (1765). Theory of the motion of solid or rigid bodies (Euler Archive - All Works by Eneström Number. 289). Joseph Bendik from Hughes Aircraft Company named the disk as "Euler disk" because Leonhard Euler was very interested in the math and physics of spolling (*i.e.*, spinning and rolling) rigid bodies, and the Laplace equation needed to solve the problem was also first derived by Euler in 1752. http://www.eulersdisk.com/history.html.
2. Moffatt, H.K. (2000). Euler's disk and its finite-time singularity. Nature ***404***(6780): 833–834. (See also http://m.toutiao.com/is/RbaYvvp/). https://doi.org/10.1038/35009017.
3. Wu, Q., Zhou, H., Wu, Y., et al. (2020). Ultrafast quasiparticle dynamics and electron-phonon coupling in $(Li_{0.84}Fe_{0.16})OHFe_{0.98}Se$. Chin. Phys. Lett. ***37***(9): 097802. https://doi.org/10.1088/0256-307X/37/9/097802.
4. Leine, R.I. (2009). Experimental and theoretical investigation of the energy dissipation of a rolling disk during its final stage of motion. Arch. Appl. Mech. ***79***(11): 1063–1082. https://doi.org/10.1007/s00419-008-0278-6.
5. Tian, Y.C., Zhang, W.H., Li, F.S., et al. (2016). Ultrafast dynamics evidence of high temperature superconductivity in single unit cell FeSe on $SrTiO_3$. Phys. Rev. Lett. ***116***(10): 107001. https://doi.org/10.1103/PhysRevLett.116.107001.

ACKNOWLEDGMENTS

This work was supported by the National Key Research and Development Program of China (grants 2021YFA1400201 and 2017YFA0303603), the Beijing Natural Science Foundation (grant 4191003), the CAS Project for Young Scientists in Basic Research (grant YSBR-059), the Strategic Priority Research Program of CAS (grant XDB30000000), the National Natural Science Foundation of China (grant 12174304), the International Partnership Program of Chinese Academy of Sciences (grant GJHZ1826), and the CAS Interdisciplinary Innovation Team.

DECLARATION OF INTERESTS

The authors declare no competing interests.