



Research article

Is estimating the Capital Asset Pricing Model using monthly and short-horizon data a good choice?

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ABSTRACT

This research argued for estimating the Capital Asset Pricing Model (CAPM) using daily and medium-horizon data over monthly and short horizon-data. Using a Gibbs sample, the Bayesian framework via both parametric and non-parametric Bayes estimators, confidence interval approach, and six data sets (two daily, two weekly, and two monthly data) from a sample of 150 randomly selected S&P 500 stocks from 2007 – 2019, the empirical results showed that the CAPM using daily data yielded a statistically significant higher model fit and smaller Beta standard deviation, model error, and Alpha compared with monthly data. The CAPM using medium-horizon data yielded a statistically significant higher model fit, smaller Beta standard deviation and Alpha, and much less zeroed Betas compared with short-horizon data. These findings show 1) daily data is more reliable and efficient, has higher forecasting power, and fits better with the assumption of market efficiency compared with monthly data. 2) Medium-horizon data is more reliable and efficient, has more explanatory power, and fits better with the assumption of market efficiency compared with monthly data. Therefore, these findings challenge the common practices of using monthly (quarterly/annually) and short-horizon data among the practitioners and researchers in asset pricing work.

1. Introduction

In investment and corporate practices, a stock's systematic risk (Beta or beta) can be estimated using different asset pricing models (Bertomeu and Cheynel, 2016). However, the Capital Asset Pricing Model (CAPM) as in Eq. (1) is very often employed as shown in some studies (Bartholdy and Peare, 2005; Da et al., 2012; Fama and French, 1996b; Jacobs and Shivdasani, 2012; Zhang, 2017) even some recent studies (e.g., Binsbergen and Opp, 2019; Clementi and Palazzo, 2019; Doshi et al., 2019; Halim et al., 2019; Martin and Wagner, 2019; Zhang, 2019) showed its weaknesses. One of the possible reasons the CAPM is widely used in practice is that this model shows a very simple linear relationship between a stock's Beta and expected returns. The other reason is the CAPM components' data, such as the returns on the stocks and market, are easily accessed and available to all investors at all times.

In practice, the Beta is typically estimated using the parametric estimators such as the ordinary least square (OLS) because the OLS is the best linear unbiased estimator. Also, the monthly/quarterly/annual returns data are very often employed (Ang and Bekaert, 2007; Fama and French,

1992, 1993; 1996a, 2004; 2015, 2016; 2018; Kamara et al., 2016, 2018; Phuoc, 2018; Phuoc et al., 2018; Roy and Shijin, 2018; Zhang, 2006) because they are stable and likely normally distributed. However, the outliers – a very common problem in real stock returns (Phuoc, 2018) may seriously affect the performance of the parametric estimators compared with the non-parametric estimators as shown in some recent studies (Baissa and Rainey, 2018; Cenesizoglu et al., 2019; McDonald and Nelson, 1989; Reeves and Wu, 2013). Besides, the daily data may yield a more efficient Beta estimate compared with monthly data as suggested in some recent studies (Phuoc, 2018; Phuoc et al., 2018; Serra and Martelanc, 2013).

In literature, both long and short-horizon data have employed and advocated in asset pricing work. Some of the well-known studies employed the long horizon data (Boudoukh and Richardson, 1993, 1994; Campbell and Shiller, 1988; Cenesizoglu et al., 2017; Mehra and Prescott, 1985; Wong and Bian, 2000) and short-horizon data (Ang and Bekaert, 2007; Kamara et al., 2016, 2018) in their studies. Long-horizon data often yielded stable parameters estimation and higher model fit (Alexander and Chervany, 1980; Bartholdy and Peare, 2005; Levy, 1971;

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Levhari and Levy, 1977; Levy and Schwartz, 1997; Theobald, 1981; Valkanov, 2003), but high model errors or low forecasting power (Ang and Bekaert, 2007; Kamara et al., 2016; Kothari and Warner, 1997, 2001). Therefore, medium-horizon data, a balance of both very long and short-horizon data may yield good performance in the CAPM.

According to our research, we found one empirical study of Beta estimation and testing in the literature that conducted Monte Carlo simulations and simultaneously employed different data types and horizon but only one estimator (e.g. Valkanov, 2003). Some other studies used only different data types and estimators (e.g., Bian et al., 2013; Phuoc, 2018; Phuoc et al., 2018; Serra and Martelanc, 2013; Wong and Bian, 2000) and different horizon data (e.g., Ang and Bekaert, 2007; Kothari and Warner, 2001; Levy and Schwartz, 1997; Theobald, 1981). In addition, these studies did not employ the confidence interval approach and applied only one or two evaluation criteria. Therefore, this research tried to extend the knowledge of Beta estimation by questioning the common practice of using the CAPM using monthly and the short-horizon data in Beta estimation. Using a Gibbs sampler, the Bayesian framework via both the parametric and non-parametric Bayes estimators for consistency in results, confidence interval approach, and evaluation criteria such as Beta and the standard deviation, model error, model fit, and intercept coefficient Alpha, we simultaneously examined daily, weekly, and monthly data as well as medium and short-horizon data. The results argued for estimating the CAPM using daily and medium-horizon data over monthly and short horizon-data.

We organized this paper into 5 sections. Section 1 is the Introduction. Section 2 is the Literature Review. Section 3 is the Data, Approach and Estimators, and Evaluation Criteria. In this section, we presented and reasoned both the parametric and non-parametric Bayes estimators used in this study. Then, we set up the evaluation criteria to compare the performance between daily and monthly data as well as medium and short-horizon data. The details of the data were also provided in this section. Section 4 is for Empirical Results and Discussion. Conclusions are provided in section 5. The References also are provided at the end.

2. Literature review

The CAPM (e.g., Black, 1972; Jensen, 1967, 1969; Lintner, 1965; Sharpe, 1964, 1966) shows the relationship between the excess return at the time $t = 1, 2, \dots, n$ and on the stock $i, i = 1, 2, 3, \dots, N$ and the market risk premium as follows:

$$R_{it} - RF_t = \alpha_i + \beta_i(R_{Mt} - RF_t) + \varepsilon_{it} \quad (1)$$

where,

- R_{it} : the return on the stock i at the time t ,
- RM_t : the return of the market portfolio at the time t ,
- RF_t : the risk-free rate at the time t ,
- α_i : Jensen's alpha coefficient (Alpha) of stock i ,
- β_i : the stock i 's sensitivity to the market portfolio (Beta or beta),
- ε_{it} : the random error term that has mean zero and variance σ^2 (Sigma2).

From Eq. (1) above, we can easily see the linear relationship between stock and market returns. Therefore, we can regress $[(R_{it} - RF_t)]$ against $[(RM_t - RF_t)]$ to determine a stock's Beta. Unfortunately, different data types, horizon-data, and estimators used to estimate the beta in Eq. (1) often yield different results. In practice, monthly/quarterly/yearly data, very long-horizon data, and parametric estimators such as the OLS and Bayes estimators even with their deficiencies are the very popular choices.

Cornell and Dietrich (1978) conducted a study of Beta estimation of 100 stocks randomly selected from the S&P 500 index for 13 one-year periods with each set of annual betas estimated using weekly data. The authors realized the deficiencies of the OLS estimator in beta estimation.

Hence, they proposed the Mean Absolute Deviation (MAD) since this estimator gives less weight to outliers compared with the OLS estimator, even though they are both unbiased estimators. The authors hoped that MAD could generate a more efficient Beta estimation compared to the OLS approach. However, the empirical results showed that the MAD estimator did not necessarily produce a more efficient Beta estimation than the OLS estimator. This finding was consistent with another research (Sharpe, 1971). However, this research employed only one data type, horizon data, and criterion in comparison.

Chan and Lakonishok (1992) conducted a study of Beta estimation using a simulated and actual monthly returns data of 50 randomly selected stocks from the NYSE for 1983–1985. In this study, the authors compared the results of OLS and other estimators such as the minimum absolute deviations (MAD), the trimmed regression quantile (with trimming proportion a set to 0.10, 0.20, or 0.25), and the Tukey's trimmed and Gastwirth estimators because they believed that the assumptions of the OLS estimator may not be met. The empirical results showed that all estimators, except the MAD, substantially outperformed the OLS in terms of efficiency, especially when the distribution of residuals is the student-t distribution. These findings were consistent with other studies (e.g., Cornell and Dietrich, 1978; Sharpe, 1971). However, this research employed only one data type, horizon data, and criterion, the Beta standard deviation in comparison.

Fong (1997) conducted a study of Beta estimation of 22 stocks for 6 years from the Straits Times Industrial Index (SIX) with the natural logarithm of monthly price relatives, i.e., $\ln(p_t/p_{t-1})$ where p_t refers to the price of a stock at the end of month t . In this study, Fong proposed the Generalized Student-t (GET) to estimate the Beta since this GET approach was able to handle well both skewness and excess kurtosis in the data. The empirical results showed that the GET provided a significantly better fit to the data than the OLS estimator or the symmetric Student-t distribution and outperformed OLS and Student-t betas in terms of forecasting ability or mean square errors. However, this research employed only one data type, horizon data, and two criteria in comparison.

Bowie and Bradfield (1998) studied the Beta estimation of small stock markets and small-cap stocks. In the small markets and small-cap stocks, the trading volume and trade frequencies are often low. Therefore, the flow of information on markets and stocks that are affecting the prices is slow as well. Hence, the outliers are very likely to exist in stock returns data. Therefore, the OLS estimator may not be a good estimator to estimate the Beta since it is known to be sensitive to outliers. In this study, the authors proposed the least absolute deviations (LAD), Tukey bi-weight (TBW), Gini, 15% Koenker-Basset trimmed mean, Lp-norm, to estimate the Beta of the randomly selected 110 stocks listed on the Johannesburg Stock Exchange (JSE) and monthly returns data over 15 years. The empirical results using the Jackknife resampling method showed that all estimators were more efficient than the OLS estimator, especially when the normal assumption was violated. This finding was consistent with another research (Chan and Lakonishok, 1992). However, this research employed only one data type, horizon data, and criterion in comparison.

Ho and Naugher (2000) and Shumacker et al. (2002) also conducted studies of Beta estimation. In their research, the authors pointed out the assumptions of OLS estimator are that 1) the errors should be normally distributed, 2) equal variance at all levels of the independent variable (homoscedasticity), and 3) uncorrelated with both the independent variable and with each other or independent and identical distributed (iid). Therefore, if the data contained the outliers which are common problems in real data, then the results from the OLS estimator could be unreliable. To overcome this problem, the authors of these two studies proposed the maximum likelihood type M (MM), least trimmed squares (LTS), least absolute residual (LAR), and robust simple regression by bi-weight (bi-square). To support their claims, Ho and Naugher (2000) used the student data from the Office of University Planning at the University of North Texas while Shumacker et al. (2002) used the High School and Beyond Data Set (as cited in Hinkle et al., 1998) to compare

the results of OLS and other estimators. The authors found that the other estimators provided a better fit than the OLS estimator. This finding was consistent with another research (Fong, 1997). However, these studies employed only one data type, horizon data, and criterion in comparison.

Wong and Bian (2000) conducted a study of Beta estimation of 12 U.S. industrial portfolios with annual and monthly returns data from 1926 - 1987. The authors chose the New York Stock Exchange (NYSE) as the market index. In this research, the authors proposed the robust Bayesian estimator, the Cauchy-type g-prior, introduced by Bian and Dickey (1996) because it works very well with the flat-tail returns data, the common problem with stock returns data. The empirical results showed that the robust Bayesian estimate was uniformly more efficient than the OLS estimator in terms of mean square error, especially for small samples. This finding was consistent with other studies (Fong, 1997; Martin and Simin, 2003). However, this research employed only one horizon data and criterion in comparison.

Shalit and Yitzhaki (2002) conducted a study of Beta estimation with two daily returns data for 10 years. The first data set is the 30 stocks in the Dow Jones Industrial Average (DJIA). The second data set is the 20 portfolios from the 100 largest traded stocks listed on the S&P 500 index. In this research, the authors proposed the Gini estimator, a type of robust estimator, to estimate beta because unlike the OLS estimator, the Gini estimator is non-parametric, not sensitive to outliers, and does not require any assumption of returns distribution. They conducted two experiments as follows: in the first one, they removed the highest four market performance observations based on the S&P 500 index from the sample, and then the betas are re-estimated. In the second experiment, they removed the highest four and the lowest four observations of the market, and then the Betas are re-estimated. The empirical results showed that in most cases, the Gini estimator was more efficient and consistent than the OLS estimator. This finding was consistent with other studies (Chan and Lakonishok, 1992; Bowie and Bradfield, 1998). However, this research employed only one data type, horizon data, and criterion in comparison.

Martin and Simin (2003) conducted a study of Beta estimation. In their research, the authors pointed out that the OLS estimator is known for its sensitivity to outliers in the data. Hence, beta estimation using the OLS can have bias beta estimates if outliers existed in the data, especially of the small stocks. The problem could become more serious for some locations in the data distribution where outliers existed. Therefore, the authors proposed the weighted least squares (WLS) with data-dependent weight. To prove their points, they collected the weekly returns for stocks listed on the NYSE, AMEX, and NASDAQ exchanges having all data for at least two-years between 1/1992 through 12/1996. The returns of the NYSE/AMEX/NASDAQ composite were treated as the market returns, and the one-month T-bill rate was used as the risk-free rate. The empirical results showed that the influential outliers in returns occur primarily for smaller capitalization stocks and the robust estimator outperformed the OLS in terms of predictive power or residual scale estimate when outliers existed in the data. These findings were consistent with another research (Fong, 1997). However, this research employed only one data type, horizon data, and criterion in comparison.

Bian et al. (2013) conducted a study of Beta estimation of 12 U.S. industrial portfolios with the same data used in Wong and Bian (2000). In this research, the authors employed the modified maximum likelihood (MML) estimator, introduced by Tiku et al. (1999), because this estimator handled the outliers better than the OLS estimator. The empirical results showed that the MML estimator is more efficient than the OLS estimator in terms of mean square errors in small samples. This finding was consistent with other studies (e.g., Fong, 1997; Martin and Simin, 2003; Wong and Bian, 2000). However, this research employed only one horizon data and criterion in comparison.

Serra and Martelanc (2013) conducted a similar study of Martin and Simin (2003), a study of Beta estimation for small stocks whose shares are not traded every day. In this study, the betas are estimated by three different methods: 1) repetition of the last quotation (RUC), 2)

trade-to-trade (TT), and 3) Scholes-Williams' adjustment (SW). Then the authors conducted a simulation with three levels of beta (0.75, 1.00, and 1.25), 5 levels of liquidity (60%, 70%, 80%, 90%, and 100%), three levels of period (daily, weekly, and monthly), and three different methods (RUC, TT, and SW). For each combination of 135 possible combinations in the simulation, they estimated 10,000 Betas. The empirical results showed that for shares not traded every day, the TT method is superior to the RUC and SW methods for estimation of the Beta. This TT method worked best in the case of daily periodicity because it produced the smallest beta standard deviation estimates compared with weekly and monthly periodicity. This finding was consistent with other studies (e.g., Chan and Lakonishok, 1992; Bowie and Bradfield, 1998; Shalit and Yitzhaki, 2002). However, this research employed only one horizon data and criterion in comparison.

Trzpiot (2013) conducted a study of both Jensen's Alpha and Beta estimation of stocks listed in the WIG20 index for July 2011–August 2012 using daily data. In this study, Trzpiot proposed using the robust least trimmed square (LTS) and quantile regression (QR) estimators instead of the OLS estimator due to the serious effects of outliers in the OLS estimator. The empirical results showed that the robust LTS and QR estimators outperformed the OLS in Beta estimation in terms of efficiency. This finding was consistent with other studies (e.g., Bowie and Bradfield, 1998; Chan and Lakonishok, 1992; Serra and Martelanc, 2013; Shalit and Yitzhaki, 2002). However, this research employed only one data type, horizon, and criterion in comparison.

Phuoc et al. (2018) conducted a study of 50 stocks listed on the S&P 500 for five-year horizon data. In this research, they wanted to reexamine the Beta estimation using 3 estimators from the frequentist approach: the OLS, maximum likelihood-type M (MM), and least trimmed squares (LTS) estimators and both daily and monthly returns data. The authors found that using each of the three estimators and daily returns data was more efficient than monthly returns data. They also found that the robust MM and LTS estimators and daily returns data outperformed the OLS estimator in terms of efficiency. This finding was then validated by the Jackknife resampling method and consistent with other studies (e.g., Bowie and Bradfield, 1998; Chan and Lakonishok, 1992; Serra and Martelanc, 2013; Shalit and Yitzhaki, 2002; Trzpiot, 2013). However, this research employed only one horizon, five-year data. Also, only one criterion, the Beta standard deviation was used to compare the performance between daily and monthly data. A similar study, Phuoc (2018) was conducted to estimate the Jensen's Alpha using the CAPM using both daily and monthly data, five-year horizon data, and the OLS, MM, parametric Bayes estimators. The author found the daily data yielded a more efficient Alpha estimate than monthly data using all three estimators and in line with another study (Trzpiot, 2013). More importantly, this finding was checked with and validated by Jackknife resampling results. Again, this research employed only one horizon data and criterion to compare the performance between daily and monthly data.

This study simultaneously employed the parametric and non-parametric Bayes estimators for consistency in results, daily, weekly, and monthly data, and short (three-year or 3 Yr) and medium-horizon (12-year or 12 Yr) data to find the well-suited data type and horizon data in the CAPM. Using the evaluation criteria such as the Beta and standard deviation, model error, model fit, and Alpha and a sample of 150 randomly selected S&P 500 stocks from 2007-2019, we found that the CAPM using daily data yielded a statistically significant higher model fit, smaller model error, beta standard deviation, and alpha, and shorter 95% confidence intervals of Beta for all stocks in the sample compared with monthly data. We also found that the CAPM using medium-horizon data yielded a statistically significant higher model fit, and smaller beta standard deviation and alpha, and shorter 95% confidence intervals of beta for all stocks in the sample compared with short-horizon data. Besides, the CAPM using medium-horizon data yielded a much less zeroed betas compared with short-horizon data. These findings question the common use of the CAPM (maybe other asset pricing models as well) using monthly (quarterly/annually) and short-horizon data in practice

and research. Hence, the results of this study argued for estimating the CAPM using daily and medium-horizon over monthly and short-horizon in practice and research.

3. Data, approach and estimator, and evaluation criteria

3.1. Data

This research used six data sets (daily and short-horizon, daily and medium-horizon, weekly and short-horizon, weekly and medium-horizon, monthly and short-horizon, and monthly and medium-horizon data) from a random sample of 150 S&P 500 stocks because these stocks are big and efficient. We chose these stocks because they were known to be very efficient, a needed-assumption of the CAPM. We believed that if our proposals worked on these S&P 500 stocks then they would also hold on the other U.S. stocks and in other less-efficient markets. Besides, the S&P 500 stocks are not volatile stock, so we used the stock returns, not logarithm returns to measure their performance as recommended in another study (Shafer and Vovk, 2019). The other selection criteria included up-to-date, ups-and-downs included data, the bias of the very long/long-horizon data (e.g., Ang and Bekaert, 2007; Kothari and Warner, 2001; Valkanov, 2003), and the stability in parameter estimations in the long horizon-data (Levy and Schwartz, 1997; Theobald, 1981). We finalized with medium-horizon data (12-year or 12 Yr) from July 2007 to May 2019. This horizon was consistent with what other studies suggested (e.g., Campbell and Shiller, 1988; Fama and French, 1988, 1989). The short-horizon data (3-year or 3 Yr) were from May 2016 to May 2019. The risk-free rate was the three-month U.S. T-Bill's secondary market rate from the Federal Reserve Economic Data (FRED) because it was stable and employed in other studies (e.g., Fama and French, 1993; Kothari et al., 1995; Martin and Simin, 2003; Moore, 2016; Phuoc et al., 2018; Phuoc and Pham, 2020). The market was the S&P 500 index which was consistent with other studies (e.g., Brotherson et al., 2013; MacKinlay, 1995; Phuoc, 2018; Phuoc et al., 2018; Phuoc and Pham, 2020). The stocks and index returns were calculated based on their adjusted close prices.

3.2. Approach and estimator

The practitioners and researchers often used the frequentist approach in the CAPM as shown in the Literature Review. Importantly, very few of these studies applied the simulations to reduce the misspecification as recommended in some studies (e.g., Barillas and Shaken, 2018; Kothari and Warner, 1997, 2001). In this study, we applied the Bayesian approach to utilize the advantages of our past knowledge of the parameters. It is also much easier to derive the posterior distributions of the parameters, especially the model error and fit, compared with the frequentist approach,

Using the Bayesian approach, the posterior distribution of a parameter can be estimated as follows:

$$p(\theta|x) \propto p(\theta)p(x|\theta) \quad (2)$$

where,

- $p(\theta|x)$: the posterior distribution of the parameter theta,
- $p(\theta)$: the prior distribution of the parameter theta,
- $p(x|\theta)$: the likelihood distribution

We employed two Bayes estimators to check for the consistency in the results. The first one was the parametric Bayes (Bayes) using the normal likelihood, $N(\alpha + \beta(RM_t - RF_t), \sigma^2)$ because it is the most used estimator among Bayesian practitioners. Also, this Bayes and the ordinary least square (OLS) estimators performed similarly (Barry, 1980; Phuoc, 2018; Zellner, 1971). The weakly informative normal priors, $N(0, 0.001)$ of both Alpha and Beta of this Bayes estimator were chosen to match with

the frequentist approach and our past knowledge. Then, Eq. (2) would become:

$$p(\alpha, \beta|x) \propto N(0, 0.001) * N(\alpha + \beta(RM_t - RF_t), \sigma^2) \quad (3)$$

The second estimator was the mild non-parametric estimator (t.Bayes) using the Student's t likelihood with three degrees of freedom, $t((\alpha + \beta(RM_t - RF_t)), 3, 3)$ because the t-distribution is flatter and handles outliers better than the normal distribution or when the variance of stock returns is unknown. Some studies (e.g., Blattberg and Gonedes, 1974; Bollerslev, 1987; Kim and Kon, 1994; Kon, 1984; Phuoc and Pham, 2020) successfully applied this Student's t in practice. Again, the weakly informative normal priors, $N(0, 0.001)$ for both Alpha and Beta of this t.Bayes estimator were chosen to match with the frequentist approach and past knowledge. Then, Eq. (2) would become:

$$p(\alpha, \beta|x) \propto N(0, 0.001) * t((\alpha + \beta(RM_t - RF_t)), 3, 3) \quad (4)$$

Using the Gibbs sampler in WINBUGS with 200,000 iterations to minimize the Markov chain error, we can easily estimate the posterior distribution of the parameters. Besides, some studies discussed and successfully applied the Bayes and non-parametric-type Bayes estimators in asset pricing models in finance and economics literature (e.g., Lange et al., 1989; Phuoc, 2018; Phuoc and Pham, 2020; Wong and Bian, 2000).

3.3. Evaluation criteria

In the beginning, we evaluated and compared the performance between the CAPM using daily and weekly data; daily and monthly data via both advance and common criteria. In this section, we employed medium-horizon (12-year) data and both Bayes and t.Bayes estimators to have a stable parameter estimate and consistency in results, respectively.

Theoretically, the main contribution of the CAPM is the linear relationship between market risk and stock returns through the Beta as in Eq. (1). In other words, the CAPM holds if it yielded a non-zero Beta coefficient. As stated in the Introduction, the CAPM is very often used to estimate the stock's beta in practice. Then the firm's managers and investors will use this beta to estimate their stock's cost of equity and expected returns, respectively. This means that the firm's managers and investors must make sure their stock's Beta is a non-zero coefficient first before they can apply this model further. Hence, the first evaluation criterion that we employed was the Beta and the standard deviation (Beta.Std). The ideal data would yield non-zeroed and stable/efficient beta estimates. Hence, the data that yielded less zeroed and stable/efficient Beta estimates would be a preferred one. Some recent studies applied this criterion in the work (e.g., Bowie and Bradfield, 1998; Chan and Lakonishok, 1992; Phuoc, 2018; Phuoc et al., 2018; Phuoc and Pham, 2020; Serra and Martelanc, 2013; Shalit and Yitzhaki, 2002; Trzpiot, 2013; Valkanov, 2003).

The second evaluation criterion was the model error using both mean square error (MSE) and model error variance (Sigma2) since these indicators provided information on the model precision and forecasting power. The data that yielded smaller MSE and Sigma2 would be considered in practice. Some researchers applied these indicators in their work (e.g., Gelman et al., 2014; Martin and Simin, 2003; Phuoc and Pham, 2020; Wu et al., 2010, 2011).

The third evaluation criterion was the model fit using the Bayesian r-squared (R2B) since it measured the explanatory power of the model. Also, some studies (e.g., Buse, 1973; Lee, 1991; Phuoc and Pham, 2020; Trzpiot, 2013) applied this criterion in the work. The data that yielded higher R2B would be a preferred one.

The last evaluation criterion was the intercept coefficient Alpha since the CAPM considered a non-traded factor model. One of the CAPM's assumptions is the market efficiency and the stock return only depended on the market return, i.e., the Alpha is zero. However, in the real stock returns data, the Alpha is not always zero (e.g., Barillas and Shanken, 2017, 2018; Fama and French, 2018; Hou et al., 2015; Kozak et al., 2018;

Phuoc, 2018; Phuoc and Pham, 2020; Stambaugh and Yuan, 2016). Hence, the data that yielded smaller Alpha (in absolute value) would be a preferred one since it fitted better with the CAPM's assumption.

Next, we compared and evaluated the performance between the CAPM using the 12 and 3-year data. In this section, we employed daily, weekly, and monthly data to have consistency in results. We also used the t.Bayes estimator since it was more efficient than the parametric Bayes estimator (Phuoc and Pham, 2020), especially in the case of outliers or unequal variance, a very common problem in the real and short-term stock returns data. Similarly, we used the Beta and standard deviation, R2B, and Alpha as evaluation criteria.

In the analyses, we did not use p-value as usual due to its serious shortcomings in hypothesis testing as suggested in other studies (e.g., Halsey, 2019; Ranstam, 2012). Instead, we applied the confidence interval approach. We also graphed and examined the difference between two parameters since the hypotheses themselves may not yield enough information in decision makings.

4. Empirical results and discussion

4.1. The CAPM using daily vs. weekly data and daily vs. monthly data

First of all, we examined Beta. All Panels (a), (b), and (c) of both Figures 1 and 2 showed that 95% confidence interval of Beta using daily, weekly, and monthly data and both Bayes and t.Bayes estimators yielded only one or none zeroed Beta (less than 1%) of all stocks in the sample. So, we could claim that the CAPM as in Eq. (1) using daily, weekly, and monthly data and both Bayes and t.Bayes estimators did not work for only one stock in the sample. These findings also reconfirmed a fact that the CAPM works well with daily, weekly, and monthly data.

Panel (a) of Figure 1 showed that the CAPM using daily data and the Bayes estimator yielded the shortest 95% confidence interval of Beta for all stocks in the sample compared with both weekly and monthly data from Panels (b) and (c) of Figure 1, respectively. Similarly, Panel (a) of Figure 2 showed that the CAPM using daily data and the t.Bayes estimator yielded the shortest 95% confidence interval of

Beta for all stocks in the sample compared with both weekly and monthly data from Panels (b) and (c) of Figure 2, respectively. Besides, Panels (a)–(d) of Figure 3 showed that the difference in Beta.Std. between daily and weekly data and between daily and monthly data using both Bayes and t.Bayes estimators were less than zero for all stocks in the sample. Importantly, Table 1 showed that the mean and 95% confidence interval of the difference in Beta.Std. between daily and weekly data using the Bayes estimators were -0.034 and (-0.037, -0.031), respectively; between daily and monthly data using the Bayes estimators were -0.113 and (-0.122, -0.104), respectively; between daily and weekly data using the t.Bayes estimator were -0.03 and (-0.032, -0.028), respectively; between daily and monthly data using the t.Bayes estimator were -0.104 and (-0.111, -0.098), respectively. Also, the difference in Beta.Std between daily and weekly data was greater than between daily and monthly data. In addition, Table 2 showed that the mean of Beta.Std. using the daily data and both Bayes and t.Bayes estimators were 0.022 and 0.020, respectively; weekly data and both Bayes and t.Bayes estimators were 0.056 and 0.050, respectively; monthly data and both Bayes and t.Bayes estimators were 0.135 and 0.125, respectively. All these findings showed that the CAPM using daily data and both Bayes and t.Bayes estimators yielded a statistically significant smallest Beta.Std., hence, most stable and accurate Beta estimate compared with both weekly and monthly data. Also, the monthly data yielded the highest Beta.Std. or performed worst in terms of efficiency. Hence, these findings were against the common practices of using CAPM and monthly data while supporting daily data in practice and research.

Next, we examine the model error using the mean square error (MSE). Panels (a)–(d) of Figure 4 showed that the difference in MSE between daily and weekly data and between daily and monthly data using both Bayes and t.Bayes estimators were less than zero for all stocks in the sample except one. Importantly, Table 1 showed that the mean and 95% confidence interval of the difference in MSE between daily and weekly data using the Bayes estimators were -11.616 and

(-14.27, -8.965), respectively; between daily and monthly data using the Bayes estimators were -54.822 and (-65.74, -43.90), respectively;

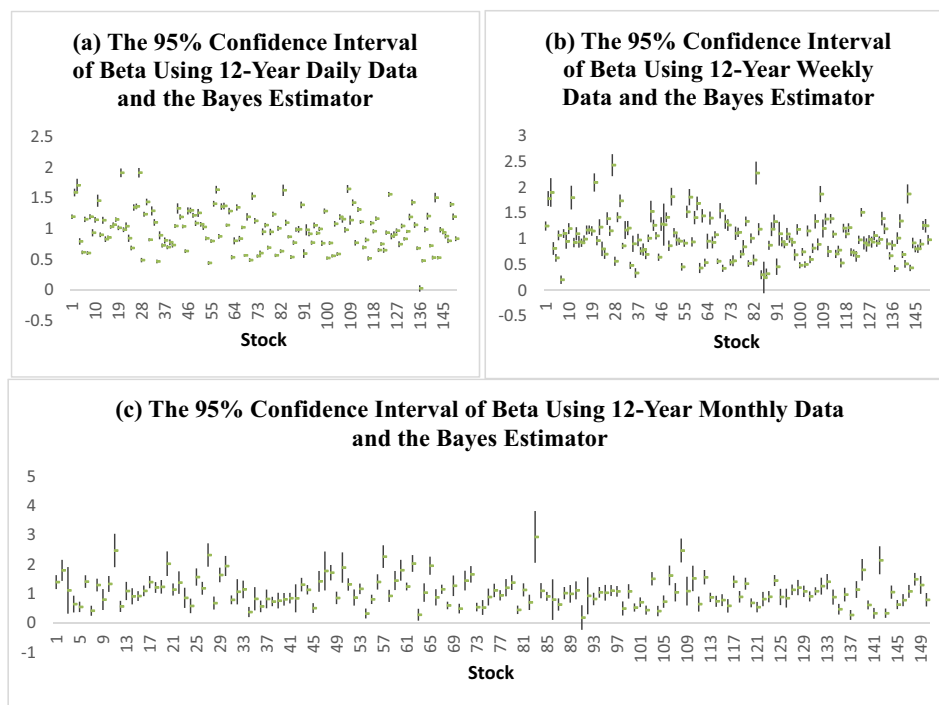


Figure 1. The 95% confidence interval of Beta using 12-year daily, weekly, and monthly data and the Bayes estimator. Notes: All panels of Figure 1 show one stock with zeroed Beta. Also, Panel (a) shows the daily data yields the shortest 95% confidence interval of Beta for all stocks compared with weekly and monthly data.

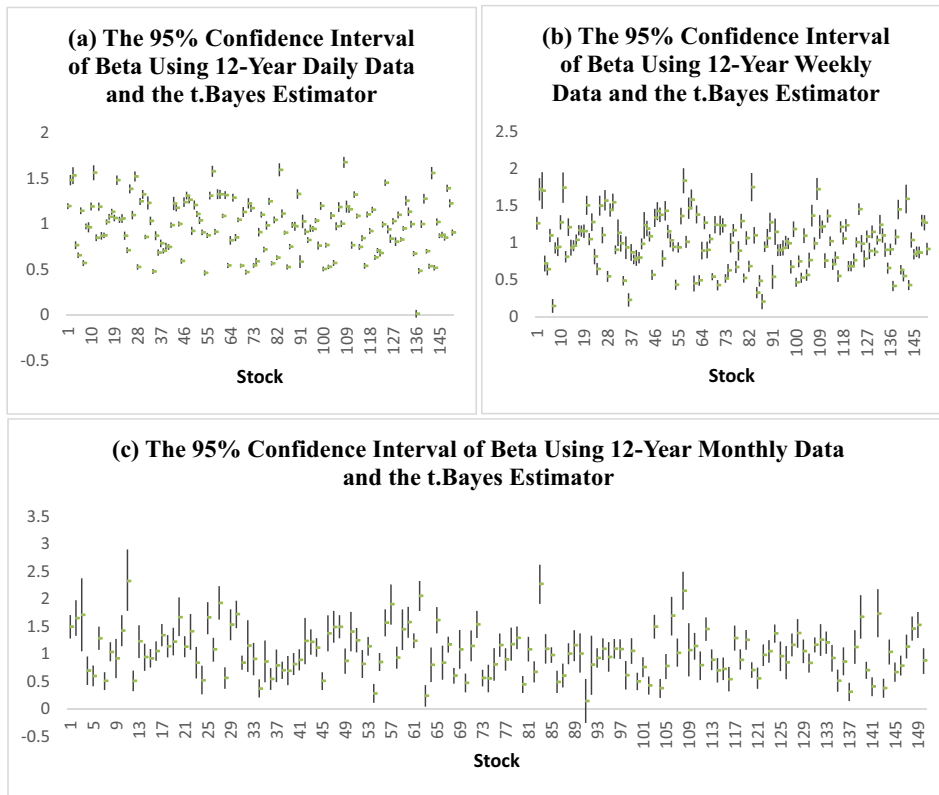


Figure 2. The 95% confidence interval of Beta using 12-year daily, weekly, and monthly data and the t.Bayes estimator. Notes: Both Panels (a) and (c) of Figure 2 show one stock with zeroed beta. Panel (b) shows none of the stock with zeroed Beta. Also, Panel (a) shows the daily data yields the shortest 95% confidence interval of Beta for all stocks compared with weekly and monthly data.

between daily and weekly data using the t.Bayes estimator were -11.712 and (-14.38, -9.04), respectively; between daily and monthly data using the t.Bayes estimator were -55.296 and (-66.42, -44.18), respectively. Also, the difference in MSE between daily and weekly data was greater

than between daily and monthly data. Also, Table 2 showed that the mean of MSE using the daily data and both Bayes and t.Bayes estimators were 2.663 and 2.675, respectively; weekly data and both Bayes and t.Bayes estimators were 14.279 and 14.387, respectively; monthly data

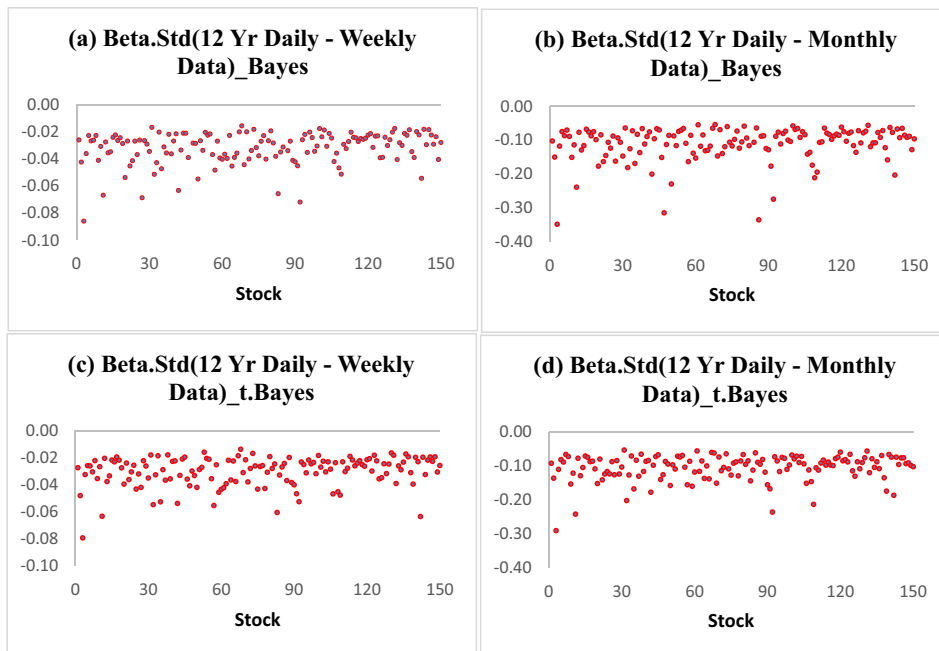


Figure 3. The difference in Beta.Std between 12-year daily and weekly data; between 12-year daily and monthly data using both Bayes and t.Bayes estimators. Note: All panels of Figure 3 show that the differences in Beta standard deviation are negative for all stocks.

Table 1. The mean and 95% confidence interval of the mean difference between 12-year daily and weekly data; between 12-year daily and monthly data using both Bayes and t.Bayes estimators.

Evaluation criterion	Estimator applied	Difference between 12-year daily and weekly data			Difference between 12-year daily and monthly data		
		No. less than zero	Mean	95% confidence interval	No. less than zero	Mean	95% confidence interval
Beta.Std	Bayes	150	-0.034	(-0.037, -0.031)	150	-0.113	(-0.122, -0.104)
	t.Bayes	150	-0.03	(-0.032, -0.028)	150	-0.104	(-0.111, -0.098)
MSE	Bayes	150	-11.616	(-14.27, -8.965)	150	-54.822	(-65.74, -43.90)
	t.Bayes	150	-11.712	(-14.38, -9.04)	149	-55.296	(-66.42, -44.18)
Sigma2	Bayes	150	-11.663	(-14.32, -9.00)	150	-55.644	(-66.73, -44.56)
	t.Bayes	150	-4.216	(-4.75, -3.68)	150	-24.082	(-27.31, -20.85)
R2B	Bayes	38	0.041	(0.02, 0.06)	24	0.089	(0.07, 0.11)
	t.Bayes	11	0.049	(0.04, 0.06)	7	0.133	(0.12, 0.147)
Alpha	Bayes	150	-0.129	(-0.145, -0.113)	150	-0.639	(-0.719, -0.558)
	t.Bayes	145	-0.122	(-0.137, -0.108)	149	-0.65	(-0.72, -0.57)

Note: This table reports the number of negative difference, mean difference, and 95% confidence interval of the mean difference between 12-year daily and weekly data as well as 12-year daily and monthly data in terms of Beta standard deviation, model error (MSE and Sigma2), model fit/adequacy (R2B), and Alpha using both Bayes and t.Bayes estimators of the S&P 500 stocks.

Table 2. The mean of each evaluation criterion using daily, weekly, and monthly data and both Bayes and t.Bayes estimators.

Evaluation criterion	Estimator applied	Mean		
		12-yr daily data	12-yr weekly data	12-yr monthly data
Beta.Std	Bayes	0.022	0.056	0.135
	t.Bayes	0.020	0.050	0.125
MSE	Bayes	2.663	14.279	57.485
	t.Bayes	2.675	14.387	57.925
Sigma2	Bayes	2.665	14.328	58.309
	t.Bayes	0.922	5.138	25.004
R2B	Bayes	0.404	0.364	0.316
	t.Bayes	0.783	0.734	0.650
Alpha	Bayes	0.028	0.1415	0.571
	t.Bayes	0.008	0.084	0.398

Note: This table reports the mean of Beta standard deviation, model error (MSE and Sigma2), model fit/adequacy (R2B), and Alpha using 12-year daily, weekly, and monthly data and both Bayes and t.Bayes estimators of the S&P 500 stocks.

and both Bayes and t.Bayes estimators were 57.485 and 57.925, respectively. These findings showed that the CAPM using daily data and both Bayes and t.Bayes estimators yielded a statistically significant smallest MSE, hence, highest forecasting power compared with both weekly and monthly data. Also, the monthly data yielded the highest MSE among three data and hence, performed worst in terms of forecasting power.

Then, we further examined the model error by looking into its variance (Sigma2). Panels (a)–(d) of Figure 5 showed that the difference in Sigma2 between daily and weekly data and between daily and monthly data using both Bayes and t.Bayes estimators were less than zero for all stocks in the sample. Importantly, Table 1 showed that the mean and 95% confidence interval of the difference in Sigma2 between daily and weekly data using the Bayes estimators were -11.663 and (-14.32, -9.00), respectively; between daily and monthly data using the Bayes estimators were -55.644 and (-66.73, -44.56), respectively; between daily and weekly data using the t.Bayes estimator were -4.216 and (-4.75, -3.68), respectively; between daily and monthly data using the t.Bayes estimator were -24.082 and (-27.31, -20.85), respectively. Besides, the difference in Sigma2 between daily and weekly data was greater than between daily and monthly data. Also, Table 2 showed that the mean of Sigma2 using the daily data and both Bayes and t.Bayes estimators were 2.665 and 0.922, respectively; weekly data and both Bayes and t.Bayes estimators were 14.3281 and 5.1383, respectively; monthly data and both Bayes

and t.Bayes estimators were 58.309 and 25.004, respectively. All these findings showed that the daily data and both Bayes and t.Bayes estimators yielded a statistically significant smallest model error variance or most efficient compared with the other two data. Also, the monthly data yielded the highest MSE among three data and hence, performed worst in terms of efficiency.

Again, all findings related to model error and variance supported daily over monthly data to be applied in the CAPM in practice and research.

Next, we examined the model fit using Bayesian r-squared (R2B). Panels (a)–(d) of Figure 6 showed that the difference in R2B between daily and weekly data and between daily and monthly data using both Bayes and t.Bayes estimators were greater than zero for most stocks in the sample. Importantly, Table 1 showed that the mean and 95% confidence interval of the difference in R2B between daily and weekly data using the Bayes estimators were 0.041 and (0.02, 0.06), respectively; between daily and monthly data using the Bayes estimators were 0.089 and (0.07, 0.11), respectively; between daily and weekly data using the t.Bayes estimator were 0.049 and (0.04, 0.06), respectively; between daily and monthly data using the t.Bayes estimator were 0.133 and (0.12, 0.147), respectively. Also, the difference in R2B between daily and weekly data was less than between daily and monthly data for most stocks in the sample. Besides, Table 2 showed that the mean of R2B using the daily data and both Bayes and t.Bayes estimators were 0.404 and 0.783, respectively; weekly data and both Bayes and t.Bayes estimators were 0.364 and 0.734, respectively; monthly data and both Bayes and t.Bayes estimators were 0.316 and 0.650, respectively. All these findings showed that the daily data and both Bayes and t.Bayes estimators yielded a statistically significant the highest model fit or explanatory power compared with both weekly and monthly data. Also, the monthly data yielded the smallest model fit and hence, performed worst in terms of explanatory power compared with the other two data. All findings related to model fit were against the common practices of using the CAPM and monthly data but favored daily data in practice and research.

Last, we examined the CAPM's intercept coefficient Alpha. Panels (a)–(d) of Figure 7 showed that the difference in Alpha between daily and weekly data and between daily and monthly data using both Bayes and t.Bayes estimators were less than zero for most stocks in the sample. Importantly, Table 1 showed that the mean and 95% confidence interval of the difference in Alpha between daily and weekly data using the Bayes estimators were -0.129 and (-0.145, -0.113), respectively; between daily and monthly data using the Bayes estimators were -0.639 and (-0.719, -0.558), respectively; between daily and weekly data using the t.Bayes estimator were -0.122 and (-0.137, -0.108), respectively; between daily and monthly data using the t.Bayes estimator were -0.65 and (-0.72,

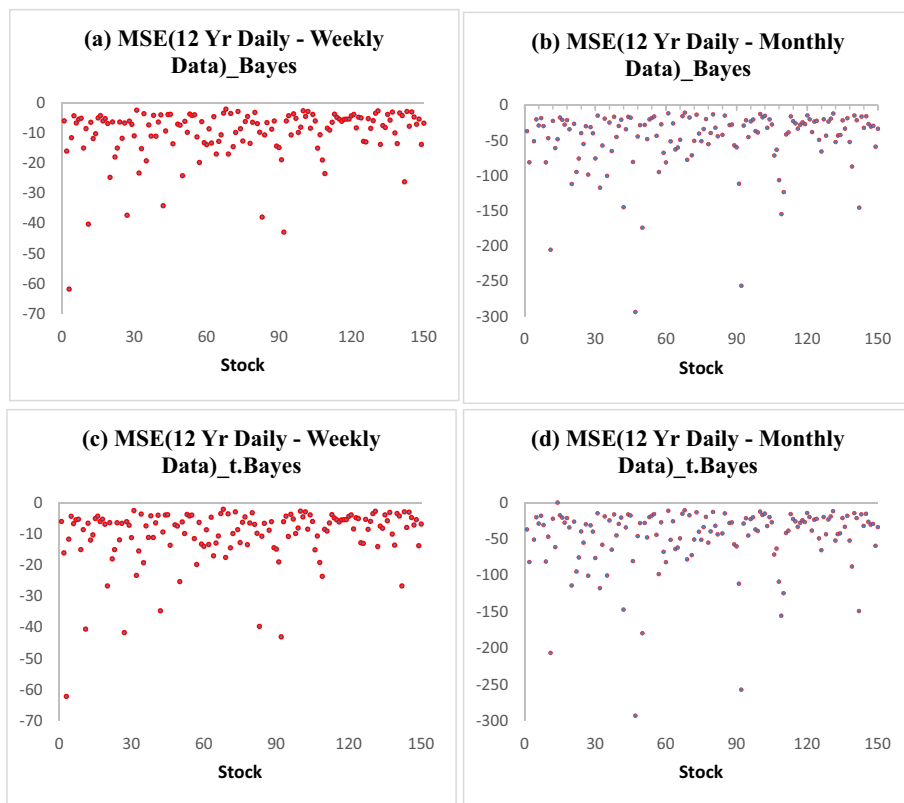


Figure 4. The difference in MSE between 12-year daily and weekly data; between 12-year daily and monthly data using both Bayes and t.Bayes estimators. Note: All panels of Figure 4 show that the differences in MSE are negative for all stocks except one in the case of between daily and monthly data using the t.Bayes estimator.

-0.57), respectively. Besides, the difference in Alpha between daily and weekly data was greater than between daily and monthly data. Also, Table 2 showed that the mean of Alpha using the daily data and both Bayes and t.Bayes estimators were 0.028 and 0.008, respectively; weekly data and both Bayes and t.Bayes estimators were 0.1415 and 0.084,

respectively; monthly data and both Bayes and t.Bayes estimators were 0.571 and 0.398, respectively. All these findings showed that the daily data and both Bayes and t.Bayes estimators yielded a statistically significant smallest Alpha or best match with the CAPM's assumption of market efficiency compared with the other two data. Also, the monthly

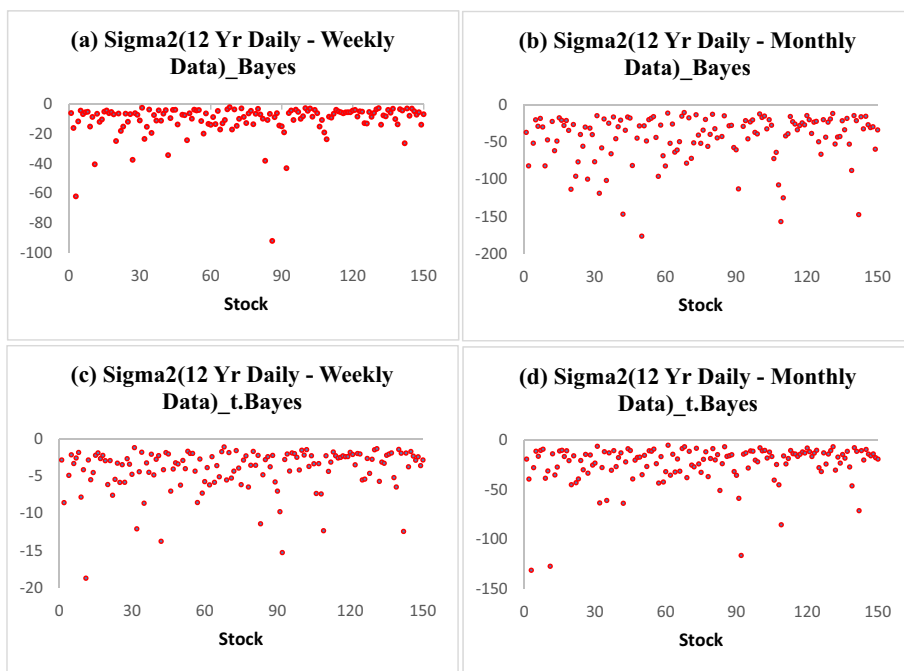


Figure 5. The difference in Sigma2 between 12-year daily and weekly data; between 12-year daily and monthly data using both Bayes and t.Bayes estimators. Note: All panels of Figure 5 show that the differences in Sigma2 are negative for all stocks.

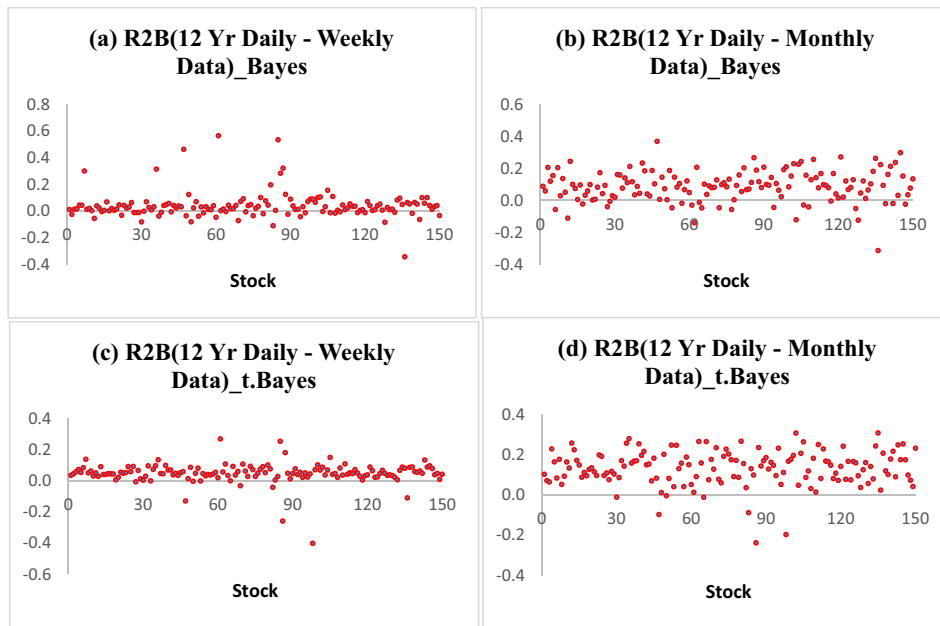


Figure 6. The difference in R2B between 12-year daily and weekly data; between 12-year daily and monthly data using both Bayes and t.Bayes estimators. Note: All panels of Figure 6 show that the differences in R2B are positive for most stocks.

data yielded the highest Alpha among three data and hence, performed worst in terms of market efficiency hypothesis.

4.2. The CAPM using 12 vs. 3-year data and the t.Bayes estimator

Similar to the previous section, we examined Beta first. All Panels (a), (c), and (e) of Figure 8 showed that 95% confidence interval of Beta using 12-year daily, weekly, and monthly data yielded either one or no zeroed Beta (less than 1%) of all stocks in the sample. However, panels

(b), (d), and (f) of Figure 8 showed that 95% confidence interval of Beta using 3-year daily, weekly, and monthly data yielded 1, 5, and 30 zeroed Betas, respectively. Hence, we could claim that the CAPM as in Eq. (1) using 12-year data did not work for only one stock while 3-year data did not work for many more stocks in the sample, especially in the case of 3-year monthly data (30 stocks). Besides, Table 4 showed that the means of Beta using 12-year daily, weekly, and monthly data of 0.974, 0.991, and 1.040, respectively, were greater than the means of Beta using 3-year daily, weekly, and monthly data of 0.912, 0.921, and 1.009,

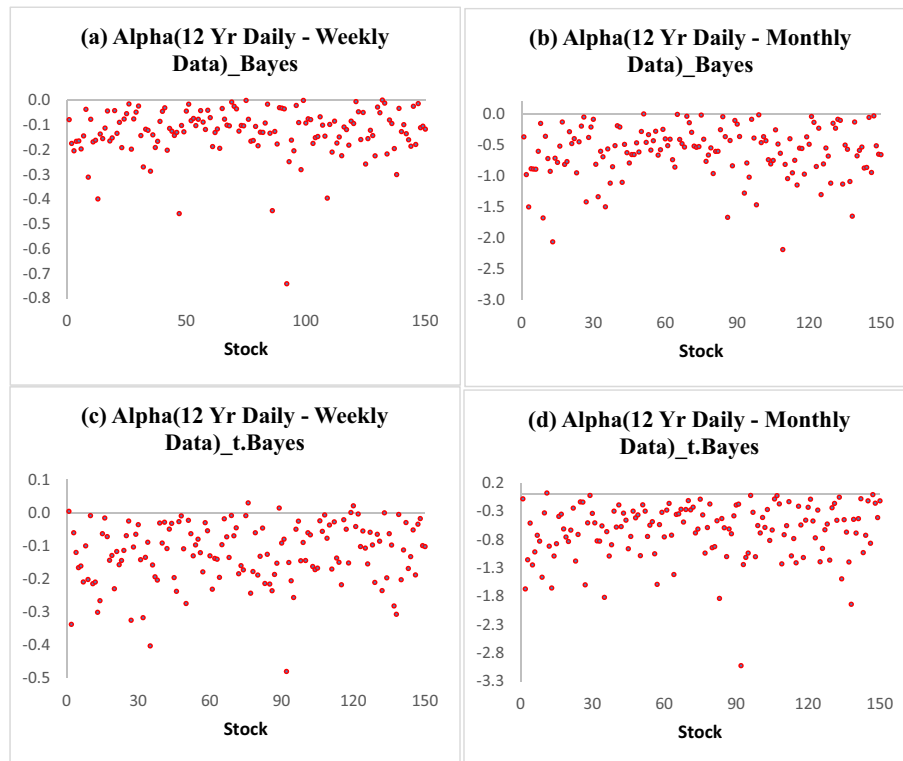


Figure 7. The difference in Alpha between 12-year daily and weekly data; between 12-year daily and monthly data using both Bayes and t.Bayes estimators. Note: All panels of Figure 7 show that the differences in Alpha are negative for most stocks.

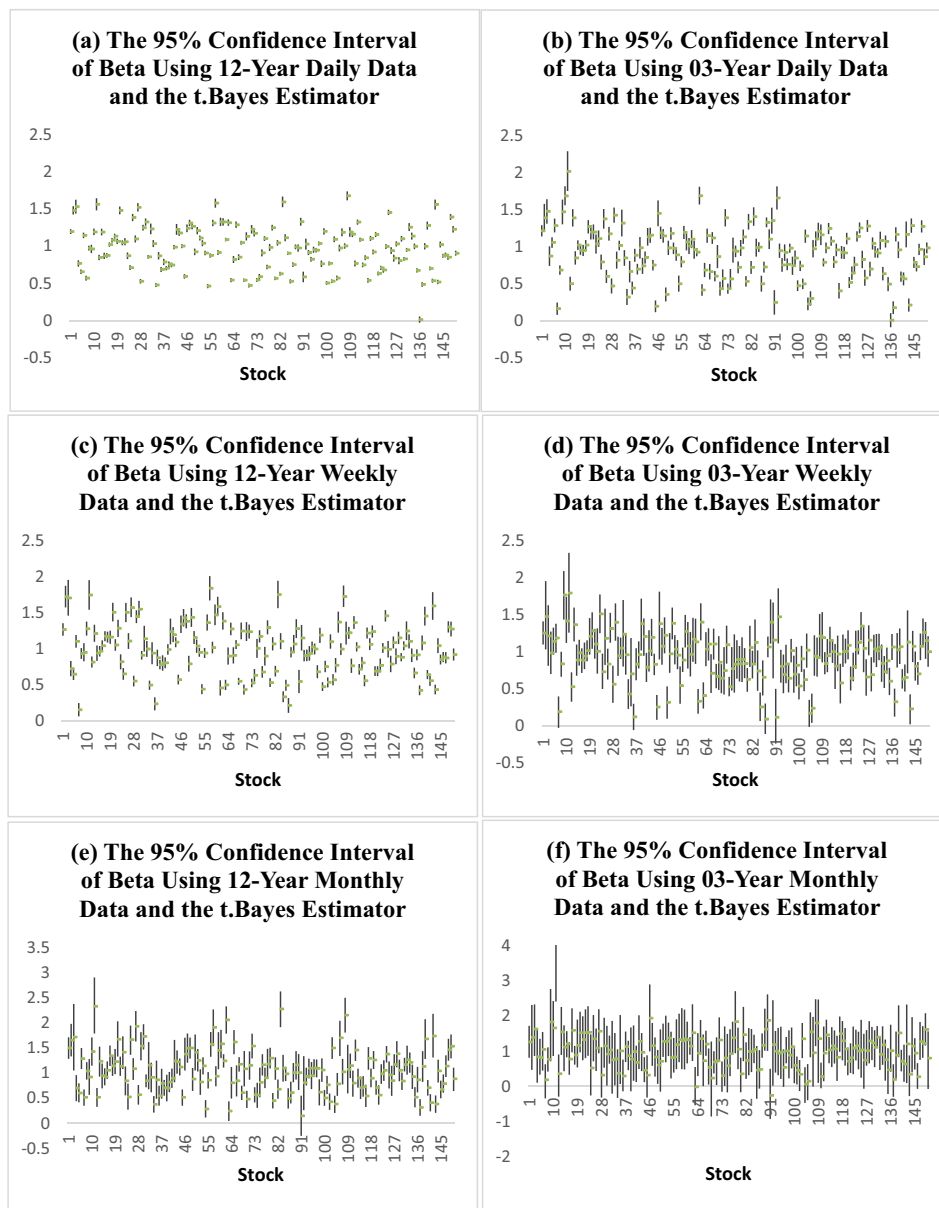


Figure 8. The 95% confidence intervals of beta using 12and 3-year daily, weekly, and monthly data and the t.Bayes estimator. Notes: All Panels (a), (b), and (e) of Figure 8 show either one stock with zeroed Beta. Panel (c) shows no stock with zero Beta. In contrast, Panels (d) and (f) show 5 and 30 stocks with zeroed Beta, respectively. Besides, Panels (a), (c), and (e) show the 12-year data yield the shorter 95% confidence interval of Beta for all stocks compared with 3-year data.

respectively. This finding showed that the CAPM using 12-year data yielded higher Beta or more explanation of the relationship between market and stock returns compared with 3-year data. Hence, the CAPM using 12-year data matched better with the CAPM's assumption of the stock returns depend only on the market returns.

Panel (a), (c), and (e) of both Figure 8 also showed that the CAPM using 12-year daily, weekly, monthly data yielded the shorter 95% confidence interval of Beta for all stocks in the sample compared with 3-year daily, weekly, and monthly data from Panels (b), (d), and (f) of Figure 8, respectively. Besides, all Panels (a)–(c) of Figure 9 showed that the difference in Beta.Std. between 12 and 3-year daily, 12 and 3-year weekly, and 12 and 3-year monthly data, respectively, were less than zero for all stocks in the sample. Importantly, Table 3 showed that the mean and 95% confidence interval of the difference in Beta.Std. between 12 and 3-year daily data were -0.03 and (-0.032, -0.029), respectively; between 12 and 3-year weekly data were -0.071 and (-0.076, -0.067), respectively; between 12 and 3-year monthly data were -0.190 and (-0.203, -0.177), respectively. Also, Table 4 showed that the mean of

Beta.Std. using the 12 and 3-year daily data were 0.020 and 0.051, respectively; 12 and 3-year weekly data were 0.050 and 0.122, respectively; 12 and 3-year monthly data were 0.125 and 0.315, respectively. All these findings showed that the CAPM using 12-year data yielded a statistically significant smaller Beta.Std., hence, more stable and accurate Beta estimate compared with 3-year data. Hence, these findings supported the CAPM using medium-horizon (12-year) data over and short-horizon (3-year) data in practice. Among all six scenarios that we invested in this study, the CAPM medium-horizon and daily data worked best in Beta estimation since it yielded the smallest Beta.Std. and hence, most stable Beta estimates.

Next, we examined the model fit using R2B. Panels (a)–(c) of Figure 10 showed that the difference in R2B between 12 and 3-year daily, 12 and 3-year weekly, and 12 and 3-year monthly data, respectively, were greater than zero for either all or most stocks in the sample. Importantly, Table 3 showed that the mean and 95% confidence interval of the difference in R2B between 12 and 3-year daily were 0.706 and (0.693, 0.720), respectively; between 12 and 3-year weekly data were

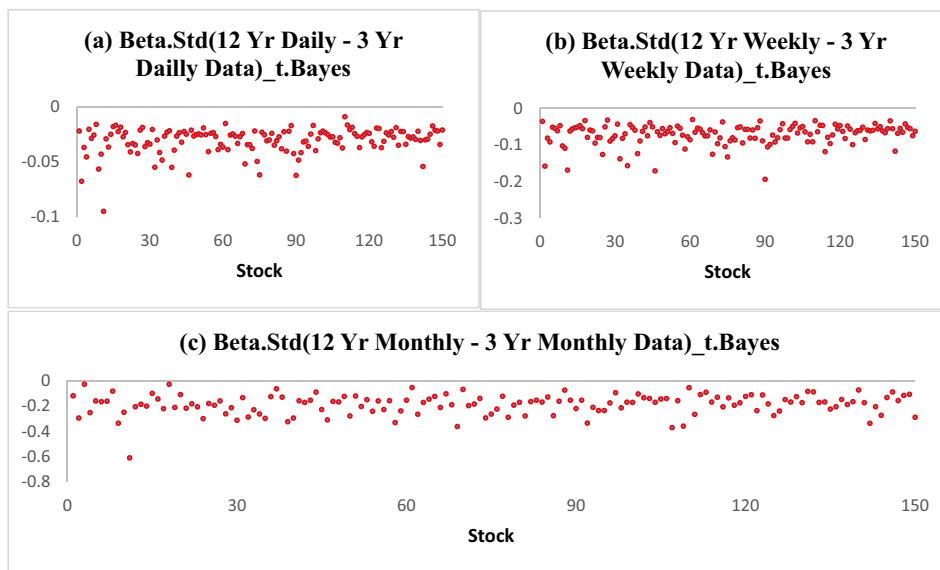


Figure 9. The difference in Beta.Std. between 12 and 3-year daily, weekly, and monthly data. Note: All panels of Figure 9 show that the differences in Beta standard deviation are negative for all stocks.

0.091 and (0.078, 0.105), respectively; between 12 and 3-year monthly data were 0.114 and (0.097, 0.132), respectively. Besides, Table 4 showed that the mean of R2B using the 12 and 3-year daily data were 0.783 and 0.706, respectively; 12 and 3-year weekly data were 0.734 and 0.64296, respectively; 12 and 3-year monthly data were 0.650 and 0.535, respectively. All these findings showed that the CAPM using 12-year data yielded a statistically significant greater model fit or explanatory power compared with 3-year data. All these findings supported the CAPM using

medium-horizon data over short-horizon data in practice. Again, among all six scenarios that we invested in this study, the CAPM using medium-horizon and daily data yielded the highest R2B or explanatory power.

Last, we examined the Alpha. Panels (a)–(c) of Figure 11 showed that the difference in Alpha between 12 and 3-year daily, 12 and 3-year weekly, and 12 and 3-year monthly data, respectively, were less than zero for the majority of stocks in the sample. Importantly, Table 3 showed that the mean and 95% confidence interval of the difference in Alpha

Table 3. The means and 95% confidence intervals of difference between 12 and 3-year daily data, 12 and 3-year weekly data, and 12 and 3-year monthly data using the t.Bayes estimators.

Difference between horizons		Beta.Std	R2B	Alpha
Difference between 12 and 3-year daily data	No. less than zero	150	0	101
	Mean	-0.03	0.706	-0.013
	95% confidence interval	(-0.032, -0.029)	(0.693, 0.720)	(-0.018, -0.007)
Difference between 12 and 3-year weekly data	No. less than zero	150	19	95
	Mean	-0.071	0.091	-0.050
	95% confidence interval	(-0.076, -0.067)	(0.078, 0.105)	(-0.074, -0.026)
Difference between 12 and 3-year monthly data	No. less than zero	150	17	83
	Mean	-0.190	0.114	-0.212
	95% confidence interval	(-0.203, -0.177)	(0.097, 0.132)	(-0.344, -0.081)

Note: This table reports the number of negative difference, mean difference, and 95% confidence interval of the mean difference between 12 and 3-year daily, weekly, and monthly data in terms of Beta standard deviation, model fit/adequacy (R2B), and Alpha using the t.Bayes estimators of the S&P 500 stocks.

Table 4. The mean of each evaluation criterion using 12-year (medium-horizon) and 3-year (short-horizon) data.

Evaluation criterion	Horizon Data	Mean		
		Daily data	Weekly data	Monthly data
Beta	12-year	0.974	0.991	1.040
	3-year	0.912	0.921	1.009
Beta.Std	12-year	0.020	0.050	0.125
	3-year	0.051	0.122	0.315
R2B	12-year	0.783	0.734	0.650
	3-year	0.706	0.64296	0.535
Alpha	12-year	0.008	0.084	0.398
	3-year	0.019	0.059	0.144

Note: This table reports the mean of Beta, Beta standard deviation, model fit/adequacy (R2B), and Alpha using 12 and 3-year daily, weekly, and monthly data and the t.Bayes estimators of the S&P 500 stocks.

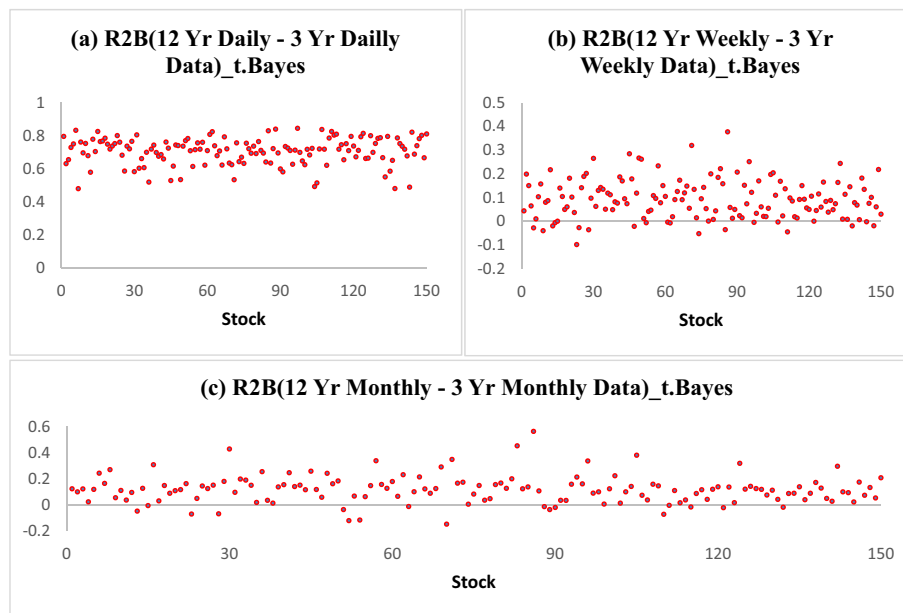


Figure 10. The difference in R2B between 12 and 3-year daily, weekly, and monthly data. Notes: Panel (a) of Figure 10 shows that the differences in R2B are positive for all stocks. Panels (b) and (c) show that the differences in R2B are positive for most stocks.

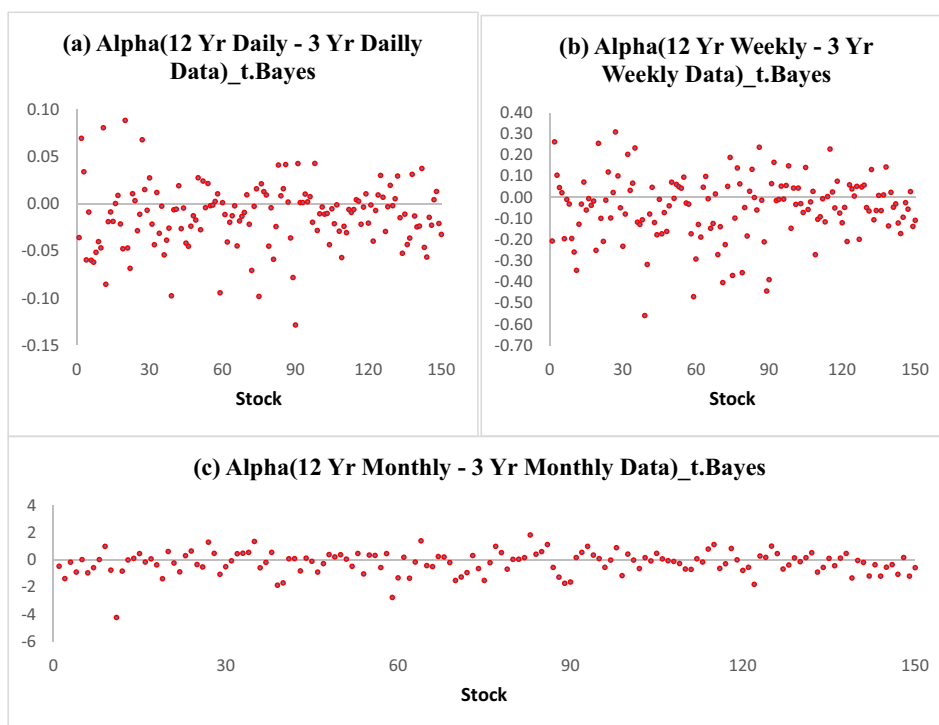


Figure 11. The difference in Alpha between 12 and 3-year daily, weekly, and monthly data. Note: All panels of Figure 11 show that the differences in Alpha are negative for the majority of stocks.

between 12 and 3-year daily were -0.013 and (-0.018, -0.007), respectively; between 12 and 3-year weekly data were -0.050 and (-0.074, -0.026), respectively; between 12 and 3-year monthly data were -0.212 and (-0.344, -0.081), respectively. These findings showed that the CAPM as in Eq. (1) using 12-year data yielded a statistically significant smaller Alpha or better match with the CAPM's assumption of market efficiency compared with 3-year data. Similarly, among all six scenarios that we invested in this study, the CAPM using medium-horizon and daily data yielded the smallest Alpha.

5. Conclusions

The Capital Asset Pricing Model is the most used asset pricing model in practice. The practitioners very often employ the CAPM using monthly and/or short-horizon data to estimate the stock's Beta and a firm's cost of equity. Hence, this research questioned this common practice. Through the use of a Gibbs sampler, the Bayesian approach via both parametric and non-parametric Bayes estimators, confidence interval approach, evaluation criteria such as the Beta and standard deviation, model error,

model fit, and Alpha, and six data sets (daily and short-horizon, daily and medium-horizon, weekly and short-horizon, weekly and medium-horizon, monthly and short-horizon, and monthly and medium-horizon data) of 150 randomly selected S&P 500 stocks from 2007-2019, this research argued for estimating the CAPM using daily and medium-horizon data over monthly and short horizon-data, respectively, in practice and research.

The empirical results showed that CAPM using the medium-horizon and daily data yielded only one zeroed Beta, a statistically significant smaller Beta standard deviation, and the shorter 95% confidence interval of Beta for all stocks compared with the short-horizon and monthly data, respectively. These findings are consistent with other studies (Bowie and Bradfield, 1998; Chan and Lakonishok, 1992; Phuoc, 2018; Phuoc et al., 2018; Phuoc and Pham, 2020; Serra and Martelanc, 2013; Wong and Bian, 2000), but against the others (Bartholdy and Peare, 2005; Valkanov, 2003). Besides, the CAPM using daily data yielded a statistically significant smaller model error and variance, higher model fit, and smaller Alpha compared with monthly data. These findings are consistent with other studies (Cenesizoglu et al., 2016; Phuoc, 2018). Also, the empirical results showed that the CAPM using medium-horizon a statistically significant higher model fit and smaller Alpha. These findings are consistent with other studies (Bartholdy and Peare, 2005; Levy, 1971; Levhari and Levy, 1977; Levy and Schwartz, 1997; Valkanov, 2003). Besides, the CAPM using medium-horizon yielded a much less zeroed Betas compared with short-horizon data. This finding contradicts another study (Valkanov, 2003).

All these findings implied that daily and medium-horizon worked much better than monthly and short-horizon data, respectively, in the CAPM. We expect that similar results would hold if we compared the performance between daily and quarterly or year data in the CAPM as well as in other asset pricing models. Hence, we strongly suggest that the practitioners and researchers consider using the CAPM using daily data and the medium-horizon data simultaneously in their asset pricing work and the stock's beta and cost of equity estimations. Unfortunately, daily data are not available for many other asset pricing models (e.g., Fama and French, 1996a, 2015; 2018; Hou et al., 2015; Roy and Shijin, 2018).

The optimum horizon data deserves more attention in the literature. Therefore, we will focus on this direction in our next studies.

Declarations

Author contribution statement

L.T. Phuoc: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

C.D. Pham: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data.

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Competing interest statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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