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Evidence for quantized magnetic flux in an axon

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Abstract
In December of 2018 I published my consolidated findings of a closed-form description of propagated signaling phenomena in the membrane of an axon [1]. Those results demonstrate how intra-cellular conductance, the thermodynamics of magnetization, and current modulation, function together in generating an action potential in a unified differential equation. At present, I report on a subsequent finding within this model. Namely, evidence of quantized magnetic flux Φ_0 in an axon.

Keywords: Action potential; axon; cable theory; field-dependent current modulation; fluxoid; Hodgkin; Huxley; Langevin; intracellular magnetization; magnetic flux quantum; membrane depolarization; membrane electric field; membrane magnetic field; myelinated nerve; quantized magnetic flux.

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I. Scope of this Article
To present evidence that the natural constant magnetic flux quantum Φ_0 [4,5] is built-into the fabric of the action potential as per my recently published model [1].

II. Method
Melendy [1] demonstrated that the differential equation:

$$\left[\frac{\partial V_m}{\partial(\Delta r)}\right]^2 - \frac{1}{\Gamma}V_m - u = 0 \tag{1a}$$

is a novel, closed-form quantification of the membrane action potential, V_m. (1a) is in contrast to the Hodgkin-Huxley quantification of V_m [6] which requires numerically integrating four differential equations to solve for the membrane voltage. Here, $u = (67.9 \times 10^{-3})\Gamma^{-1}$, where:

$$\Gamma = \frac{\varepsilon_0 \Delta r\left(t^{0.5e}\sin\pi t\right)}{(0.5\pi)^{\tanh\left(4\pi\frac{\mu B}{kT}\right)}\left(G_{in}\cosh\pi X\right)} \tag{1b}$$

In review [1] (p. 107), G_{in} is the leaky cable input conductance along the longitudinal length of neuronal fiber (Ω^{-1}) and X (Chi) is a normalized length (dimensionless) [7,8]. The hyperbolic tangent term (p. 108) is Langevin's thermodynamic relation [9]. The sine term (p. 108-109) is the current-modulation function and is a magnetic field-dependent current. On p. 110 [1] the myelinated thickness of the axon is given as Δr and ε_0 is the vacuum permittivity of free space (8.854 x 10^{-12} F/m). The relationship between V_m and Γ is given as $V_m = \Gamma E_m^2 - 67.9 \times 10^{-3}$ V (p. 112), where E_m is the membrane electric field (V/m).

From classical electrodynamics [10], Melendy showed that (p. 110) $E_m(\pi a^2)\rho_m^{-1}$ (t sin $n\pi t$) = $B^2{}_m(2\pi a/\mu_0)$ (t sin $n\pi t$), where a is the axon radius (µm), ρ_m is the longitudinal membrane resistivity (Ω-m), B_m is the membrane magnetic field (T or Wb/m^2), and μ_0 is the vacuum permittivity of free space (4π x 10^{-7} H/m). From this relationship between E_m and B_m is Melendy's origianly-derived model (p. 109) but that includes identical numerical parameters as in (1b):

$$V_m = \frac{B_m^2\left(\frac{2\pi a}{\mu_0}\right)\left(t^{0.5e}\sin\pi t\right)}{\left(0.5\pi\right)^{\tanh\left(4\pi\frac{\mu B}{kT}\right)}\left(G_{in}\cosh\pi X\right)} - 67.9\times10^{-3}\,\text{V} \qquad (1c)$$

In other words, (1c) produces identical results to (4b) (p. 111) [1] for B_m = 2.964(74) × 10^{-5} Wb/m^2 and a = 4.710(94) µm.

The laws of electrodynamics state that $B_m = \Phi_m/\pi a^2$, where Φ_m is the membrane magnetic flux (Wb). The axon cross-sectional area [1] is πa^2 = 6.972(15) × 10^{-11} m^2, resulting in a flux of Φ_m = 2.067(06) × 10^{-15} Wb. This is very nearly the value of the magnetic flux quantum, $\Phi_0 = h/2e$ [11-13], where h = 6.626 069 934 x 10^{-34} J/Hz is the currently reported measured value of Planck's constant [14] and e = 1.602 176 634 x 10^{-19} C is elementary charge [15].

III. Summary

The development of an original, quantitative model of the membrane (action) potential V_m was presented in [1]. This is a conductance-based model rooted in cable theory. It is independent of the chemistry and physics behind the contribution of sodium, potassium, and leakage ions to the action potential cycle. In this article, the electric field model (1a) was re-stated in terms of the membrane magnetic field B_m and subsequently, the membrane magnetic flux, Φ_m. It was discovered that Φ_m = 2.067(06) × 10^{-15} Wb, which is very nearly the value of the magnetic flux quantum, $\Phi_0 = h/2e$. This is evidence for quantized magnetic flux in the membrane of an axon.

IV. Ethical Approval

The conducted research reported in this article is not related to either human or animal use.

V. Compliance with Ethical Standards

Conflict of Interests: The author Robert F. Melendy, Ph.D. declares that I have no conflict of interest(s).

References

1. R.F. Melendy, A single differential equation description of membrane properties underlying the action potential and the axon electric field. Journal of Electrical Bioimpedance, 9(1), (2018). https://doi.org/10.2478/joeb-2018-0015
2. R.F. Melendy, Resolving the biophysics of axon transmembrane polarization in a single closed-form description. Journal of Applied Physics, 118(24), (2015). https://doi.org/10.1063/1.4939278
3. R.F. Melendy, A subsequent closed-form description of propagated signaling phenomena in the membrane of an axon. AIP Advances, 6(5), (2016). https://doi.org/10.1063/1.5052550
4. U. Schollwöck et al., Quantum Magnetism (Springer, 2004).
5. Ş. Erkoç, Fundamentals of Quantum Mechanics (Taylor and Francis, 2006).
6. A.L. Hodgkin, A.F. Huxley, A quantitative description of membrane current and its application to conduction and excitation in nerve. Journal of Physiology, 117, 500-544 (1952). https://doi.org/10.1113/jphysiol.1952.sp004764
7. W. Rall, Core Conductor Theory and Cable Properties of Neurons: Handbook of Physiology, the Nervous System, Cellular Biology of Neurons (American Physiological Society, 1977), pp. 39-93. https://doi.org/10.1002/cphy.cp010103
8. R. Hobbie, Intermediate Physics for Medicine and Biology (AIP Press, New York, 1997).
9. W.T. Coffey, Y.P. Kalmykov, J.T. Waldron, The Langevin Equation, with Applications in Physics, Chemistry, and Electrical Engineering (World Scientific, River Edge, NJ, 1996). https://doi.org/10.1142/2256
10. R.L. Armstrong, J.D. King, The Electromagnetic Interaction (Prentice Hall, Englewood Cliffs, NJ, 1973).
11. The Nation Institute of Standards and Technology (NIST) Reference on Constants, Unit, and Uncertainty [Internet]. [Cited June 26, 2019]; Available from https://physics.nist.gov/cgi-bin/cuu/Value?flxquhs2e.
12. P. Hawkes (Ed.) Advances in Imaging and Electron Physics (Academic Press, 2011).
13. L. Deecke, J. Eccles, and V. Mount castle (Eds.), From Neuron to Action: An Appraisal of Fundamental and Clinical Research (Springer-Verlag, 1990).
14. The Nation Institute of Standards and Technology (NIST) New Measurement will Help Redefine International Unit of Mass [Internet]. [Cited June 26, 2019]; Available from https://www.nist.gov/news-events/news/2017/06/new-measurement-will-help-redefine-international-unit-mass.
15. The Nation Institute of Standards and Technology (NIST) Reference on Constants, Unit, and Uncertainty [Internet]. [Cited June 26, 2019]; Available from https://physics.nist.gov/cgi-bin/cuu/Value?e.