RESEARCH ARTICLE

Modeling of linear programming and extended TOPSIS in decision making problem under the framework of picture fuzzy sets

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Abstract

Picture fuzzy sets (PFSs) are comparatively a new extension of fuzzy sets which describe the human opinions that has more answers like acceptance, rejection, neutral and desist, which cannot be correctly presented in fuzzy sets (FSs) and intuitionistic fuzzy sets (IFSs). The PFSs are categorized by three objects, the degree of belonging, the degree of neutral belonging and the degree of non- belonging such that the total of these three degrees must not be more than one. So far, there is no such work presented in the literature which deals with unknown weights of criteria based on PFSs. In the present work, we have developed a linear programming (LP) model to find the exact weights from the given constraints of weights for the criteria and construct a modified distance based on similarity measure between picture fuzzy sets. Then we have utilized this similarity measure to achieve the best option in the multiple criteria decision making (MCDM) problem. Lastly, two practical examples for the selection of alternatives are presented to compare the obtained results with the existing similarity measures.

Introduction

Zadeh presented the idea of fuzzy sets [1] in the middle of 1960s, which has opened the new horizon for the researchers. Fuzzy sets (FSs) generally exhibit uncertainty and ambiguity in real life problems. Most of the specialists have focused on the extensions of fuzzy sets (FSs) and its applications. The idea of intuitionistic fuzzy sets (IFSs) was presented by Atanassov [2] in 1986, the one of the important extension of FSs. In 2013, Cuong [3] introduced a novel concept of picture fuzzy sets (PFSs) that answered the human's opinions which consist of more than two answers like, yes, no, refusal and neutral. Casting a vote is an excellent example of such situations because the voters can be divided into four groups such as vote for, vote neutral, vote against and vote refusal. Later on, Cuong and Kreinovich [4] presented some operations for PFSs. The PFSs are described by three components, the level of belonging, the level of non-belonging and the neutral level. The characteristic of these components is that the sum of the three levels must not be more than one.

The recent developments of PFSs included: Singh [5] found out the correlation coefficient of PFSs, Son [6] developed the distance measure and applied it to picture fuzzy clustering and Wei [7] presented the cross-entropy measure of PFSs and then implemented it for multiple attribute decision making problems. Son et al. [8] introduced a novel fuzzy inference structure on PFSs to improve the induction execution of the conventional fuzzy derivation framework. Thong and Son [9, 10] connected a novel picture fuzzy grouping methods for complex information. Wei [11] displayed the picture fuzzy accumulation operators strategy and used it to multi-attribute decision making (MADM) for positioning of enterprise resource planning (ERP). Garg [12] presented a few actions on PFSs and utilized it to multiple criteria decision making (MCDM) issues.

Furthermore, Jana et al. [13] presented some aggregation operators called Dombi operators for PFSs situations and implemented these actions to MADM process. Ashraf et al. [14] introduced a novel concept of cubic picture fuzzy sets, the extended form of PFSs. Wang and Li [15] extended the hesitant fuzzy sets to picture hesitant fuzzy sets and use it in MCDM. Wang et al. [16] formulated a hybrid fuzzy multiple criteria decision making framework with picture fuzzy information to rank the risk features of energy performance contracting (EPC) projects. Wang et al. [17] developed a novel comparison technique between two distance measures under the probabilistic linguistic term sets (PLTSs). Wang et al. [18] used the picture fuzzy numbers (PFNs) data in muirhead mean and weighted muirhead mean operators for multiple attribute decision making (MADM) problems. Moreover, Wei and Zhang [19] utilize power aggregation operators and Bonferroni mean to develop some single-valued neutrosophic Bonferroni power aggregation operators and single-valued neutrosophic geometric Bonferroni power aggregation operators to choose the best strategic suppliers. Recently, Zhang et al. [20] developed the score, accuracy functions and action rules for picture 2-tuple linguistic numbers (P2TLNs), implemented these for multiple criteria group decision making by using evaluation based on distance from average solution (EDAS). Wei et al. [21] extended the Maclaurin symmetric mean (MSM) operator and the dual MSM operator to q—rung orthopair fuzzy sets and deliberated their some properties in detail. Peng [22] constructed the picture fuzzy ordered weighted geometric (PFOWG) operator and picture fuzzy induced OWG (PFIOWG) operators to examine the multiple attribute decision making problems under picture fuzzy information.

In our daily life, we all are required to make distinct decisions intentionally and unintentionally which make us DMs. The data we assemble are to enable us to get our goals accurate and efficient. Not all the data are helpful for enhancing our comprehension and decisions. Decision making, for which we accumulate most of our data, has turned into a mathematical science nowadays [23]. DMs are inclined to use MCDM approaches in order to cope with multiple criteria problems more effectively. MCDM is a field of operational research where alternatives are assessed to select the most suitable alternative that satisfies an ideal objective from a set of multiple and often conflicting criteria [24, 25]. MCDM plays a dominant role in decision making and operational research. It is a collection of strategies and methodology by which different and conflicting criteria can be joined into a decision process.

Linear programming (LP) [26] is utilized to get ideal answers for task investigations. Utilizing LP enables scientists to locate the best, the most conservative answer to an issue inside the majority of its restrictions, or limitations. In order to increase the proficiency, LP procedures are preferably applied in the fields of agriculture, engineering, transportation, manufacturing and energy. LP permits characterizing factors, discovering imperatives and developing the objective function, or what should be optimized. Many researchers applied the LP technique in their proposed work for example, Wang and Chen [27] presented a new MCDM method based on linear programming methodology which provided new score and accuracy function

of interval-valued intuitionistic fuzzy values (IVIFVs), Su et al. [28] presented an input-output LP model, to study energy-economic recovery resilience of an economy, Aliyev [29] presented interval LP where the ambiguous location is termed by interval numbers and Wang and Chen [30] presented LP methodology and the extended TOPSIS method for interval-valued intuitionistic fuzzy numbers for the selection of the best alternative, which deals with two interval values: a belonging and a non belonging. Recently, Sindhu et al. [31] applied the LP model to calculate the unknown weights of the criteria and utilize these weights on a certain MCDM problem.

TOPSIS technique introduced by Hawang and Yoon [32], that handles the MCDM issues with crisp information and the extended TOPSIS strategy [33] are generally utilized by the DMs with regards to different extensions of fuzzy sets. Kuo [34] figured out a flaw in TOPSIS and modified it for the different ranking index. Zhoua et al. [35] improved the TOPSIS with weighted hesitant vague information. Tian et al. [36] used the best-worst method to evaluate the weights of criteria and then utilized these weights in TOPSIS to resolve the MCGDM issues under intuitionistic fuzzy environment.

PFSs are applied in such environment in which the opinions of the decision makers (DMs) have more answers: acceptance, neutral, rejection and refusal which can not be accommodated precisely in FSs and IFSs. In order to find the weights of criteria, LP model is modest and user friendly, and responds rapidly as compared to other techniques. TOPSIS plays an important role to rank the alternatives in different fields of MCDM problems. Inspired by the merits and advantages of LP model, TOPSIS, as far as we know, LP model has not been considered under the framework of PFSs. It is noteworthy that the decision making under PFSs environment may acquire more attention and further research. Thus, we extend the LP model for PFSs to calculate the weights of criteria and then use these weights in TOPSIS to obtain the best alternative from the information provided by the DMs in the form of PFSs.

The remaining of the paper is organized as: In Section 2, we briefly describe the core concepts of fuzzy sets, intuitionistic fuzzy set, picture fuzzy sets and some operations about the PFSs. In Section 3, we developed a modified distance formula and a similarity measure based on the distance measure for PFSs. Section 4 contains a proposed MCDM method based on picture fuzzy TOPSIS (PF-TOPSIS). In Section 5, we utilize PF-TOPSIS on the practical examples to analyze the experimental results of the proposed technique. A comprehensive comparative analysis is discussed in Section 6. Conclusions and the future work direction are given in the Section 7.

Some basic concepts

In the present section, a brief overview is given about the basic ideas associated to fuzzy sets (FSs), intuitionistic fuzzy set (IFSs), picture fuzzy sets (PFSs) and some operations like union, intersection and complement of PFSs.

Definition 1. [1] Let $X = \{x_1, x_2, \ldots, x_n\}$ be a universe of discourse set, then a fuzzy set $\mathcal{A}$ defined on X can be written as a collection of ordered pairs as:

$$\mathcal{A} = \{(x, \mu_{\mathcal{A}}(x))|x \in X\},$$

where $\mu_{\mathcal{A}}(x)) : X \rightarrow [0, 1]$ is a membership function so that $x \in X$ to the set $\mathcal{A}$.

Definition 2. [2] Let X be a fixed set, an intuitionistic fuzzy set $\mathcal{A}$ on X is defined as:

$$\mathcal{A} = \{\langle x, \alpha_{\mathcal{A}}(x), \beta_{\mathcal{A}}(x)\rangle|x \in X\},$$

where $\alpha_{\mathcal{A}}(x), \beta_{\mathcal{A}}(x) \in [0, 1]$ are called the membership and non membership degrees of $x \in X$ to the set $\mathcal{A}$, respectively with the condition: $0 \leq \alpha_{\mathcal{A}}(x) + \beta_{\mathcal{A}}(x) \leq 1$, for all $x \in X$.

For all $x \in X$, $\gamma_A(x)$ is called the degree of hesitancy of $x \in \mathcal{A}$ where $\gamma_A(x) = 1 - \alpha_{\mathcal{A}}(x) - \beta_{\mathcal{A}}(x)$.

Definition 3. [3] Let $X = \{x_1, x_2, \ldots, x_n\}$ be a fixed set, a picture fuzzy set $\mathcal{P}$ on X is defined as:

$$\mathcal{P} = \{\langle x, \alpha_{\mathcal{P}}(x), \gamma_{\mathcal{P}}(x), \beta_{\mathcal{P}}(x)\rangle | x \in X\},$$

where $\alpha_{\mathcal{P}}(x), \beta_{\mathcal{P}}(x), \gamma_{\mathcal{P}}(x) \in [0, 1]$ are called the acceptance membership, neutral and rejection membership degrees of $x \in X$ to the set $\mathcal{P}$, respectively and $\alpha_{\mathcal{P}}(x), \gamma_{\mathcal{P}}(x)$ and $\beta_{\mathcal{P}}(x)$ fulfil the condition: $0 \leq \alpha_{\mathcal{P}}(x) + \gamma_{\mathcal{P}}(x) + \beta_{\mathcal{P}}(x) \leq 1$, for all $x \in X$. Also $\eta_{\mathcal{P}}(x) = 1 - \alpha_{\mathcal{P}}(x) - \gamma_{\mathcal{P}}(x) - \beta_{\mathcal{P}}(x)$, then $\eta_{\mathcal{P}}(x)$ is said to be a degree of refusal membership of $x \in X$ in $\mathcal{P}$. For our convenience, the picture fuzzy sets over a fixed set X is written as PFSs(X).

Definition 4. [4] Let $\mathcal{A}$ and $\mathcal{B}$ be two PFSs on X, then the union, intersection and complement are described as follows:

(1). $\mathcal{A} \subseteq \mathcal{B}$ iff $\alpha_{\mathcal{A}}(x) \leq \alpha_{\mathcal{B}}(x)$, $\gamma_{\mathcal{A}}(x) \leq \gamma_{\mathcal{B}}(x)$ and $\beta_{\mathcal{A}}(x) \geq \beta_{\mathcal{B}}(x)$ such that for all $x \in X$;

(2). $\mathcal{A} = \mathcal{B}$ iff $\mathcal{A} \subseteq \mathcal{B}$ and $\mathcal{B} \subseteq \mathcal{A}$;

(3). $\mathcal{A} \sqcup \mathcal{B} = \{x, \max(\alpha_{\mathcal{A}}(x), \alpha_{\mathcal{B}}(x)), \min(\gamma_{\mathcal{A}}(x), \gamma_{\mathcal{B}}(x)), \min(\beta_{\mathcal{A}}(x), \beta_{\mathcal{A}}(x)) | x \in X\}$;

(4). $\mathcal{A} \sqcap \mathcal{B} = \{x, \min(\alpha_{\mathcal{A}}(x), \alpha_{\mathcal{B}}(x)), \min(\gamma_{\mathcal{A}}(x), \gamma_{\mathcal{B}}(x)), \max(\beta_{\mathcal{A}}(x), \beta_{\mathcal{A}}(x)) | x \in X\}$;

(5). $\mathcal{A}^{\mathcal{C}} = \{\langle x, \beta_{\mathcal{A}}(x), \gamma_{\mathcal{A}}(x), \alpha_{\mathcal{A}}(x)\rangle | x \in X\}$.

Definition 5. [26]. A LP model is formulated as follows:

$$\text{Maximize}: \quad S = c_1 x_1 + c_2 x_2 + c_3 x_3 + \ldots + c_n x_n$$

$$\text{Subject to}: \quad a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + \ldots + a_{1n} x_n \leq b_1$$

$$a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + \ldots + a_{2n} x_n \leq b_2$$

$$\vdots$$

$$a_{m1} x_1 + a_{m2} x_2 + a_{m3} x_3 + \ldots + a_{mn} x_n \leq b_m$$

$$x_1, x_2, \ldots, x_n \geq 0,$$

where m and n denotes the number of constraints and the number of decision variables $x_1, x_2, \ldots, x_n$. A solution $(x_1, x_2, \ldots, x_n)$ is called feasible if it satisfies all of the constraints. The purpose of the LP methodology is to find the optimal values of the decision variables $x_1, x_2, \ldots, x_n$ for maximizing the linear function S.

The distance is a quite essential idea in the instinctive fuzzy set theory. It can reveal the variance between two instinctive fuzzy sets.

A modified distance measure between PFSs

In the present section, we construct a modified distance measure between two PFSs by including an extra term, the neutral belonging degree term of the PFSs in the Wang and Xin's formula [37].

Definition 6. Let $\mathcal{A}$ and $\mathcal{B}$ be two PFSs defined on a fixed set $X = \{x_1, x_2, \ldots, x_n\}$, then the distance $D_p(\mathcal{A}, \mathcal{B})$ is defined as:

$$D_p(\mathcal{A}, \mathcal{B}) = \frac{1}{3n} \sum_{i=1}^{n} \left(\begin{array}{c} [|\alpha_{\mathcal{A}}(x_i) - \alpha_{\mathcal{B}}(x_i)| + |\gamma_{\mathcal{A}}(x_i) - \gamma_{\mathcal{B}}(x_i)| + |\beta_{\mathcal{A}}(x_i) - \beta_{\mathcal{B}}(x_i)|] + \\ \max[|\alpha_{\mathcal{A}}(x_i) - \alpha_{\mathcal{B}}(x_i)|, |\gamma_{\mathcal{A}}(x_i) - \gamma_{\mathcal{B}}(x_i)|, |\beta_{\mathcal{A}}(x_i) - \beta_{\mathcal{B}}(x_i)|] \end{array} \right) \quad (1)$$

Example 1. Let $\mathcal{A}$ and $\mathcal{B}$ be two PFSs defined on a set $X = \{x_1, x_2, x_3\}$ given by $\mathcal{A} = \{(x_1, (0.8, 0.1, 0)), (x_2, (0.4, 0.2, 0.3)), (x_3, (0.5, 0.3, 0))\}$ and $\mathcal{B} = \{(x_1, (0.3, 0.3, 0.2)), (x_2, (0.7, 0.1, 0.1)), (x_3, (0.4, 0.3, 0.2))\}$, then by using the distance formula defined in Definition 6, we get, $D_p(\mathcal{A}, \mathcal{B}) = 0.2556$.

Theorem 1. Suppose that, $\mathcal{D}$ is a mapping $\mathcal{D} : PFSs(X) \times PFSs(X) \rightarrow [0, 1]$, then $\mathcal{D}_p(\mathcal{A}, \mathcal{B})$ is a distance measure if the conditions below hold:

(1). $0 \leq \mathcal{D}_p(\mathcal{A}, \mathcal{B}) \leq 1$;

(2). $\mathcal{D}_p(\mathcal{A}, \mathcal{B}) = 0$ iff $\mathcal{A} = \mathcal{B}$;

(3). $\mathcal{D}_p(\mathcal{A}, \mathcal{B}) = \mathcal{D}_p(\mathcal{B}, \mathcal{A})$;

(4). $\mathcal{D}_p(\mathcal{A}, \mathcal{C}) \geq \mathcal{D}_p(\mathcal{A}, \mathcal{B})$ and $\mathcal{D}_p(\mathcal{A}, \mathcal{C}) \geq \mathcal{D}_p(\mathcal{B}, \mathcal{C})$, for any $\mathcal{A}, \mathcal{B}, \mathcal{C} \in PFSs(X)$.

Proof. As, (1)–(3) are obvious, we thereby, prove the last condition (4). For any $A, B, C \in PFSs(X)$, and $A \subseteq B \subseteq C$, then based on Definition 6, we see that

$$|\alpha_{\mathcal{A}}(x_i) - \alpha_{\mathcal{C}}(x_i)| \geq |\alpha_{\mathcal{A}}(x_i) - \alpha_{\mathcal{B}}(x_i)| \qquad (2)$$

$$|\alpha_{\mathcal{A}}(x_i) - \beta_{\mathcal{C}}(x_i)| \geq |\alpha_{\mathcal{A}}(x_i) - \beta_{\mathcal{B}}(x_i)| \qquad (3)$$

$$|\alpha_{\mathcal{A}}(x_i) - \gamma_{\mathcal{C}}(x_i)| \geq |\alpha_{\mathcal{A}}(x_i) - \gamma_{\mathcal{B}}(x_i)| \qquad (4)$$

By adding Eqs (3)–(5), we get

$$|\alpha_{\mathcal{A}}(x_i) - \alpha_{\mathcal{C}}(x_i)| + |\alpha_{\mathcal{A}}(x_i) - \beta_{\mathcal{C}}(x_i)| + |\alpha_{\mathcal{A}}(x_i) - \gamma_{\mathcal{C}}(x_i)|$$
$$\geq \alpha_{\mathcal{A}}(x_i) - \alpha_{\mathcal{B}}(x_i)| + |\alpha_{\mathcal{A}}(x_i) - \beta_{\mathcal{B}}(x_i)| + \alpha_{\mathcal{A}}(x_i) - \gamma_{\mathcal{B}}(x_i)|$$

$\Rightarrow$

$$|\alpha_{\mathcal{A}}(x_i) - \alpha_{\mathcal{C}}(x_i)| + |\alpha_{\mathcal{A}}(x_i) - \beta_{\mathcal{C}}(x_i)| + |\alpha_{\mathcal{A}}(x_i) - \gamma_{\mathcal{C}}(x_i)| + \max\{|\alpha_{\mathcal{A}}(x_i) - \alpha_{\mathcal{C}}(x_i)|,$$
$$|\alpha_{\mathcal{A}}(x_i) - \beta_{\mathcal{C}}(x_i)|, |\alpha_{\mathcal{A}}(x_i) - \gamma_{\mathcal{C}}(x_i)|\} \geq |\alpha_{\mathcal{A}}(x_i) - \alpha_{\mathcal{B}}(x_i)| + |\alpha_{\mathcal{A}}(x_i) - \beta_{\mathcal{B}}(x_i)|$$
$$+ |\alpha_{\mathcal{A}}(x_i) - \gamma_{\mathcal{B}}(x_i)| + \max\{|\alpha_{\mathcal{A}}(x_i) - \alpha_{\mathcal{B}}(x_i)|, |\alpha_{\mathcal{A}}(x_i) - \beta_{\mathcal{B}}(x_i)|, |\alpha_{\mathcal{A}}(x_i) - \gamma_{\mathcal{B}}(x_i)|\}$$

$\Rightarrow D_P(\mathcal{A}, \mathcal{C}) \geq D_P(\mathcal{A}, \mathcal{B})$, on the same way, we can show that, $D_P(\mathcal{A}, \mathcal{C}) \geq D_P(\mathcal{B}, \mathcal{C})$.

Commonly, the weights of the criteria have significant features in decision making, so we formulate the distance measure presented in Definition 6 into the weighted distance measure between two PFSs as:

Definition 7. Let $\mathcal{A}$ and $\mathcal{B}$ be two PFSs defined on a fixed set $X = \{x_1, x_2, \ldots, x_n\}$ and w_j be the weights of the m criteria such that $\sum_{j=1}^{m} w_j = 1$. Then the weighted distance measure $D_p^w(\mathcal{A}, \mathcal{B})$ is defined as

$$D_p^w(\mathcal{A}, \mathcal{B}) = \frac{1}{3} \sum_{i=1}^{n} w_j \left(\begin{array}{c} [|\alpha_{\mathcal{A}}(x_i) - \alpha_{\mathcal{B}}(x_i)| + |\gamma_{\mathcal{A}}(x_i) - \gamma_{\mathcal{B}}(x_i)| + |\beta_{\mathcal{A}}(x_i) - \beta_{\mathcal{B}}(x_i)|] + \\ \max[|\alpha_{\mathcal{A}}(x_i) - \alpha_{\mathcal{B}}(x_i)|, |\gamma_{\mathcal{A}}(x_i) - \gamma_{\mathcal{B}}(x_i)|, |\beta_{\mathcal{A}}(x_i) - \beta_{\mathcal{B}}(x_i)|] \end{array} \right) \qquad (5)$$

Example 2. Let $\mathcal{A}$ and $\mathcal{B}$ be two PFSs on a set $X = \{x_1, x_2, x_3\}$ described in Example 1 and the weights of x_1, x_2 and x_3 are $w_1 = 0.25$, $w_2 = 0.35$ and $w_3 = 0.4$, respectively. Hence the weighted distance between $\mathcal{A}$ and $\mathcal{B}$ by using Definition 7 is $D_p^w(\mathcal{A}, \mathcal{B}) = 0.2883$.

Theorem 2. Let $X = \{x_1, x_2, \ldots, x_n\}$ be a fixed set, then $D_p^w(\mathcal{A}, \mathcal{B})$ is the level of weighted distance measure between two PFSs $\mathcal{A}$ and $\mathcal{B}$ satisfy the following conditions:

(1). $0 \leq \mathcal{D}_p^w(\mathcal{A}, \mathcal{B}) \leq 1$;

(2). $\mathcal{D}_p^w(\mathcal{A}, \mathcal{B}) = 0$ iff $\mathcal{A} = \mathcal{B}$;

(3). $\mathcal{D}_p^w(\mathcal{A}, \mathcal{B}) = \mathcal{D}_p(\mathcal{B}, \mathcal{A})$;

(4). $\mathcal{D}_p^w(\mathcal{A}, \mathcal{C}) \geq \mathcal{D}_p^w(\mathcal{A}, \mathcal{B})$ and $\mathcal{D}_p^w(\mathcal{A}, \mathcal{C}) \geq \mathcal{D}_p^w(\mathcal{B}, \mathcal{C})$, for any $\mathcal{A}, \mathcal{B}, \mathcal{C} \in PFSs(X)$.

Proof. Follow the same procedure as Theorem 1.

Definition 8. Let $\mathcal{A}$ and $\mathcal{B}$ be two PFSs defined on a fixed set $X = \{x_1, x_2, \ldots, x_n\}$. Then a similarity measure $S_p(\mathcal{A}, \mathcal{B})$ based on Definition 7 is defined as

$$S_p(\mathcal{A}, \mathcal{B}) = 1 - \frac{1}{3} \sum_{i=1}^{n} w_j \left(\frac{[|\alpha_\mathcal{A}(x_i) - \alpha_\mathcal{B}(x_i)| + |\gamma_\mathcal{A}(x_i) - \gamma_\mathcal{B}(x_i)| + |\beta_\mathcal{A}(x_i) - \beta_\mathcal{B}(x_i)|] +}{\max[|\alpha_\mathcal{A}(x_i) - \alpha_\mathcal{B}(x_i)|, |\gamma_\mathcal{A}(x_i) - \gamma_\mathcal{B}(x_i)|, |\beta_\mathcal{A}(x_i) - \beta_\mathcal{B}(x_i)|]} \right) \quad (6)$$

where $w_j (1 \leq j \leq m)$ denotes the weights of the m criteria such that $\sum_{j=1}^{m} w_j = 1$.

Definition 9. A mapping $\mathcal{S} : PFSs(X) \times PFSs(X) \rightarrow [0, 1]$. $\mathcal{S}_p(\mathcal{A}, \mathcal{B})$ is said to be a similarity measure if, $\mathcal{S}_p(\mathcal{A}, \mathcal{B})$ satisfy the conditions below:

(1). $0 \leq \mathcal{S}_p(\mathcal{A}, \mathcal{B}) \leq 1$;

(2). $\mathcal{S}_p(\mathcal{A}, \mathcal{B}) = 1$ iff $\mathcal{A} = \mathcal{B}$;

(3). $\mathcal{S}_p(\mathcal{A}, \mathcal{B}) = \mathcal{S}_p(\mathcal{B}, \mathcal{A})$;

(4). $\mathcal{S}_p(\mathcal{A}, \mathcal{C}) \leq \mathcal{S}_p(\mathcal{A}, \mathcal{B})$ and $\mathcal{S}_p(\mathcal{A}, \mathcal{C}) \leq \mathcal{S}_p(\mathcal{B}, \mathcal{C})$, for any $\mathcal{A}, \mathcal{B}, \mathcal{C} \in PFSs(X)$ and $\mathcal{A} \subseteq \mathcal{B} \subseteq \mathcal{C}$.

Picture fuzzy TOPSIS (PF-TOPSIS) for MCDM

In this section, we proposed an MCDM with picture fuzzy information based on TOPSIS by using LP metrology, LP model is adopted to evaluate the weights of criteria under various constraints. Let $A = \{A_1, A_2, \ldots, A_n\}$ be a discrete set of alternatives, and $U = \{U_1, U_2, \ldots, U_m\}$ be the collection of criteria with $w = \{w_1, w_2, \ldots, w_m\}$, where $\sum_{j=1}^{m} w_j = 1$ as the weighing vector of the criteria U_j where $j = 1, 2, 3, \ldots, m$. A picture fuzzy decision matrix denoted by $\mathcal{R} = [\nabla_{ij}]_{n \times m} = [(\alpha_{ij}, \gamma_{ij}, \beta_{ij})]_{n \times m}$ with α_{ij} as degree of acceptance, γ_{ij} degree of neutral and β_{ij} degree of rejection that the alternatives $A_i(i = 1, 2, \ldots, n)$ fulfils respectively. In order to make the best decision, the procedure to find the MCDM is as follow:

Step 1. Developed a picture fuzzy decision matrix denoted by $\mathcal{R} = [\nabla_{ij}]_{n \times m}$ according to the given information presented by the DM.

Step 2. Figure out the picture fuzzy positive ideal solution (PFPIS), ∇_p^+ and picture fuzzy negative ideal solution (PFNIS), ∇_p^- as:

$$\nabla_p^+ = \{(\alpha_{ij}^+, \gamma_{ij}^+, \beta_{ij}^+)\} = \left(\begin{array}{l} \left\{ (\max_j(\alpha_{ij}), \min_j(\gamma_{ij}), \min_j(\beta_{ij})) \right\} : U_j \in J_1 \\ \left\{ (\max_j(\alpha_{ij}), \max_j(\gamma_{ij}), \min_j(\beta_{ij})) \right\} : U_j \in J_2 \end{array} \right) \quad (7)$$

$$\nabla_p^- = \{(\alpha_{ij}^-, \gamma_{ij}^-, \beta_{ij}^-)\} = \begin{pmatrix} \left\{ (\min_j(\alpha_{ij}), \min_j(\gamma_{ij}), \max_j(\beta_{ij})) \right\} : U_j \in J_1 \\ \\ \left\{ (\min_j(\alpha_{ij}), \max_j(\gamma_{ij}), \max_j(\beta_{ij})) \right\} : U_j \in J_2 \end{pmatrix} \quad (8)$$

where J_1 is a subset of benefit criteria and J_2 is a subset of cost criteria, and $J_1 \cap J_2 = \emptyset$.

Step 3. Calculate the degree of weighted similarity $S_{p_i}^+$ between PFPIS ∇_p^+ and each alternative as well as the degree of weighted similarity $S_{p_i}^-$ between PFNIS ∇_p^- by using Eq (6), respectively:

$$S_{wpi}^+(A_i, \nabla_p^+) = 1 - \frac{1}{3}\sum_{j=1}^{m} w_j \begin{pmatrix} [|\alpha_{\mathcal{A}}(x_i) - \alpha_{ij}^+| + |\gamma_{\mathcal{A}}(x_i) - \gamma_{ij}^+| + |\beta_{\mathcal{A}}(x_i) - \beta_{ij}^+|] + \\ \\ \max[|\alpha_{\mathcal{A}}(x_i) - \alpha_{ij}^+|, |\gamma_{\mathcal{A}}(x_i) - \gamma_{ij}^+|, |\beta_{\mathcal{A}}(x_i) - \beta_{ij}^+|] \end{pmatrix} \quad (9)$$

$$S_{wpi}^-(A_i, \nabla_p^-) = 1 - \frac{1}{3}\sum_{j=1}^{m} w_j \begin{pmatrix} [|\alpha_{\mathcal{A}}(x_i) - \alpha_{ij}^-| + |\gamma_{\mathcal{A}}(x_i) - \gamma_{ij}^-| + |\beta_{\mathcal{A}}(x_i) - \beta_{ij}^-|] + \\ \\ \max[|\alpha_{\mathcal{A}}(x_i) - \alpha_{ij}^-|, |\gamma_{\mathcal{A}}(x_i) - \gamma_{ij}^-|, |\beta_{\mathcal{A}}(x_i) - \beta_{ij}^-|] \end{pmatrix} \quad (10)$$

where, $1 \le i \le n$.

Step 4. Based on Eqs (9) and (10), construct the model to find the objective function Z for the weights of criteria as:

$$Z = (S_{pi}^+(A_i, \nabla_p^+) - S_{pi}^-(A_i, \nabla_p^-)) \quad (11)$$

Step 5. By solving the LP model presented in [26], we get the weights w_j of the criteria U_j where $j = 1, 2, 3, \ldots, m$, so that the objective function Z obtained in Step 4 is maximized.

Step 6. Based on Eqs (9) and (10), calculate the degree of similarity $S_{p_i}^+$ and $S_{p_i}^-$ between each alternative and the elements obtained in PFPIS ∇_p^+ and PFNIS ∇_p^-, respectively.

Step 7. Evaluate the relative closeness $\mathcal{CR}_i$ of alternative A_i with respect to the PFPIS ∇_p^+ as:

$$\mathcal{CR}_i = \frac{S_{p_i}^+}{S_{p_i}^+ + S_{p_i}^-} \quad (12)$$

The larger the value of the relative closeness $\mathcal{CR}_i$ of the alternatives with regard to the PFPIS ∇_p^+ means that, we get the best alternative from different alternative A_i, where $1 \le i \le n$.

We consider two practical examples of MCDM problems from the literature [13] and [11] to show the visibility and benefits of the proposed method.

Practical examples

In this section, two practical examples are established to implement the proposed MCDM approach under the environment of PFSs.

Example 3

An organization wants to hire a technical firm to manage the technicalities of the organization. For this purpose, decision maker call five technicians $T = \{S_1, S_2, S_3, S_4, S_5\}$ from different firms to set up an interview under the four criteria $C = \{C_1, C_2, C_3, C_4\} \in J_1$, that is all these criteria are beneficial criteria such that:

C_1 (advancement in technology), C_2 (market potential), C_3 (ability of vendors) and C_4 (formation of employment and the innovations in technology and of science). The numerical data is adopted from [13]. To evade any conflict, the DM gave the weights to the criteria under some traits accordingly.

Subject to:

$$
\begin{aligned}
&-0.3w_1 + 0.2w_2 + 0.5w_3 + 0.6w_4 \leq 0.55; \\
&0.2w_1 - 0.1w_2 + 0.2w_3 - 0.25 + w_4 \leq 0.26; \\
&0.1w_1 + 0.2w_2 - 0.3w_3 0.4 + w_4 \leq 0.3; \\
&w_1 + w_2 + w_3 + w_4 = 1; \\
&0.1 \leq w_1 \leq 0.2; \\
&0 \leq w_2 \leq 0.1; \\
&0.2 \leq w_3 \leq 0.3; \\
&0.3 \leq w_4 \leq 0.4.
\end{aligned}
\tag{13}
$$

Step 1. A matrix $\mathcal{R}$ is constructed according to provided information provided by the DM under the PFSs environment in Table 1.

Step 2. Based on Eqs (7) and (8), evaluate the picture fuzzy positive ideal solution (PFPIS), ∇_p^+ and picture fuzzy negative ideal solution (PFNIS), ∇_p^-, respectively:

$$\nabla_p^+ = [(0.8800, 0.0600, 0.0300), (0.9000, 0.0700, 0.0300), (0.4000, 0.3300, 0.0500), (0.7200, 0.1400, 0.0300)];$$

$$\nabla_p^- = [(0.5600, 0.0600, 0.1000), (0.0800, 0.0700, 0.2200), (0.0300, 0.3300, 0.1900), (0.0700, 0.1400, 0.0900)].$$

Step 3. Evaluate the level of similarity $S_{p_i}^+$ between PFPIS ∇_p^+ and each alternative as well as the degree of similarity $S_{p_i}^-$ between PFNIS ∇_p^-, respectively, by using Eqs (9) and (10).

$$
S_{pi}^+(A_i, \nabla_p^+) = 1 - \frac{1}{3} \sum_{j=1}^{4} w_j \left(\frac{[|\alpha_{\mathcal{A}}(x_i) - \alpha_{ij}^+| + |\gamma_{\mathcal{A}}(x_i) - \gamma_{ij}^+| + |\beta_{\mathcal{A}}(x_i) - \beta_{ij}^+|] +}{\max[|\alpha_{\mathcal{A}}(x_i) - \alpha_{ij}^+|, |\gamma_{\mathcal{A}}(x_i) - \gamma_{ij}^+|, |\beta_{\mathcal{A}}(x_i) - \beta_{ij}^+|]} \right)
\tag{14}
$$

Table 1. PFSs matrix $\mathcal{R}$ given by the DM's information.

	C_1	C_2	C_3	C_4
S_1	(0.56, 0.34, 0.10)	(0.90, 0.07, 0.03)	(0.40, 0.33, 0.19)	(0.09, 0.79, 0.03)
S_2	(0.70, 0.10, 0.09)	(0.10, 0.66, 0.20)	(0.06, 0.81, 0.12)	(0.72, 0.14, 0.09)
S_3	(0.88, 0.09, 0.03)	(0.08, 0.10, 0.06)	(0.05, 0.83, 0.09)	(0.65, 0.25, 0.07)
S_4	(0.80, 0.07, 0.04)	(0.70, 0.15, 0.11)	(0.03, 0.88, 0.05)	(0.07, 0.82, 0.05)
S_5	(0.85, 0.06, 0.03)	(0.64, 0.07, 0.22)	(0.06, 0.88, 0.05)	(0.13, 0.77, 0.09)

$$S_{pi}^{-}(A_i, \nabla_p^-) = 1 - \frac{1}{3}\sum_{j=1}^{4} w_j \left(\begin{array}{c} [|\alpha_{\mathcal{A}}(x_i) - \alpha_{ij}^-| + |\gamma_{\mathcal{A}}(x_i) - \gamma_{ij}^-| + |\beta_{\mathcal{A}}(x_i) - \beta_{ij}^-|] + \\ \max[|\alpha_{\mathcal{A}}(x_i) - \alpha_{ij}^-|, |\gamma_{\mathcal{A}}(x_i) - \gamma_{ij}^-|, |\beta_{\mathcal{A}}(x_i) - \beta_{ij}^-|] \end{array} \right) \quad (15)$$

where, $1 \leq i \leq 5$.

Step 4. Substituting the values of S_{pi}^+ and S_{pi}^- obtained from Eqs (14) and (15) in Eq (11), construct a model to find the objective function Z as:

$$Z = (S_{pi}^+(A_i, \nabla_p^+) - S_{pi}^-(A_i, \nabla_p^-))$$

where, $1 \leq i \leq 5$, we get, $Z = 0.0720w_1 + 0.0413w_2 - 0.0347w_3 + 0.0247w_4$

Step 5. Based on the objective function Z obtained in Step 4 and the constraints given by the DM in system of Eq (13), solve the LP model by maximizing Z presented in [26] to get the exact weights of the criteria as:

$$w_1 = 0.2000; w_2 = 0.1000; w_3 = 0.3000; w_4 = 0.4000.$$

Step 6. After substituting the weights of criteria obtained in Step 5, evaluate the level of similarity $S_{p_i}^+$ and $S_{p_i}^-$ between each alternative and the elements obtained in PFPIS ∇_p^+ and PFNIS ∇_p^-, respectively, which are:

$$S_{p_1}^+ = 0.4007; S_{p_2}^+ = 0.3257; S_{p_3}^+ = 0.3637; S_{p_4}^+ = 0.2060; S_{p_5}^+ = 0.2333;$$
$$S_{p_1}^- = 0.8067; S_{p_2}^- = 0.8017; S_{p_3}^- = 0.7857; S_{p_4}^- = 0.7573; S_{p_5}^- = 0.7833.$$

Step 7. Based on Eq (12), evaluate the relative closeness $\mathcal{CR}_i$ of alternative A_i, where, $1 \leq i \leq 5$, such that: $\mathcal{CR}_1 = 0.3319; \mathcal{CR}_2 = 0.2889; \mathcal{CR}_3 = 0.3164; \mathcal{CR}_4 = 0.2138$ and $\mathcal{CR}_5 = 0.2295$, which gives the ranking order as: $S_1 \succ S_3 \succ S_2 \succ S_5 \succ S_4$, shows that the best alternative is S_1.

Example 4

Another practical example is established to implement the suggested MCDM approach under the PFSs environment. The information given in the article presented by Wei [11] is adopted. The same proposed MCDM technique is applied to evaluate the best alternative.

Follow the same Steps as given in Example 3, we get the values of $S_{p_i}^+$ and $S_{p_i}^-$, respectively as:

$$S_{p_1}^+ = 0.3350; S_{p_2}^+ = 0.3360; S_{p_3}^+ = 0.3753; S_{p_4}^+ = 0.2220; S_{p_5}^+ = 0.2757;$$
$$S_{p_1}^- = 0.7987; S_{p_2}^- = 0.8117; S_{p_3}^- = 0.7890; S_{p_4}^- = 0.7403; S_{p_5}^- = 0.7787,$$

and the corresponding values of relative closeness are: $\mathcal{CR}_1 = 0.2955; \mathcal{CR}_2 = 0.2928; \mathcal{CR}_3 = 0.3224; \mathcal{CR}_4 = 0.2307$ and $\mathcal{CR}_5 = 0.2615$, which gives the preference order as: $A_3 \succ A_1 \succ A_2 \succ A_5 \succ A_4$ that is A_3 is the desired alternative.

Comparative analysis

In order to illustrate the strength and validity of the proposed method (PF-TOPSIS), we applied it on the information adopted from Jana et al. [13] and Wei [11]. The results obtained by using the proposed technique are then compared with existing methods. The technique proposed (PF-TOPSIS) in the present work deals with the picture fuzzy environment. The LP model is introduced to evaluate the unknown weights of criteria under the given constraints. The comparative results of the outcome achieved by the said technique

Table 2. Comparison with Jana et al. [13] and proposed technique.

Techniques	$\mathcal{CR}_1$	$\mathcal{CR}_2$	$\mathcal{CR}_3$	$\mathcal{CR}_4$	$\mathcal{CR}_5$	Ranking arrangement
Jana et al. [13]	0.8991	0.7945	0.8939	0.8415	0.8746	$S_1 \succ S_3 \succ S_5 \succ S_4 \succ S_2$
Proposed Technique	0.3319	0.2889	0.3164	0.2138	0.2295	$S_1 \succ S_3 \succ S_2 \succ S_5 \succ S_4$

Table 3. Comparison with Wei [11] (PFWG) and proposed technique.

Techniques	$\mathcal{CR}_1$	$\mathcal{CR}_2$	$\mathcal{CR}_3$	$\mathcal{CR}_4$	$\mathcal{CR}_5$	Ranking arrangement
Wei [11] (PFWG)	0.174	0.140	0.185	0.055	0.093	$A_3 \succ A_1 \succ A_2 \succ A_5 \succ A_4$
Proposed Technique	0.2955	0.2928	0.3224	0.2307	0.2615	$A_3 \succ A_1 \succ A_2 \succ A_5 \succ A_4$

with the Jana et al. [13] and Wei [11] are shown in Tables 2 and 3. In the practical Example 3, the preference order obtained by the proposed technique and picture fuzzy Dombi weighted average (PFDWA) presented by Jana et al. [13] are slightly different in arrangement but the desired best alternative is same, that is, S_1 which shows the effectiveness of the proposed technique. However, in Example 4, the results obtained by the proposed technique and picture fuzzy weighted geometric (PFWG) operator provided by the Wei [11] are totally agreed to each other which also shows the usefulness of our proposed technique. Moreover, the techniques based on the aggregations have some limitations, like, its calculations are complex and hard. The complexity of evaluation can be increased rapidly if more elements are handled. But on the other hand, our proposed technique is based on the distance measure which is easy to calculate the intricate problems. The graphical representation of the preference order of the alternatives received by the proposed method, Wei [11] and Jana et al. [13] are shown in Figs 1 and 2.
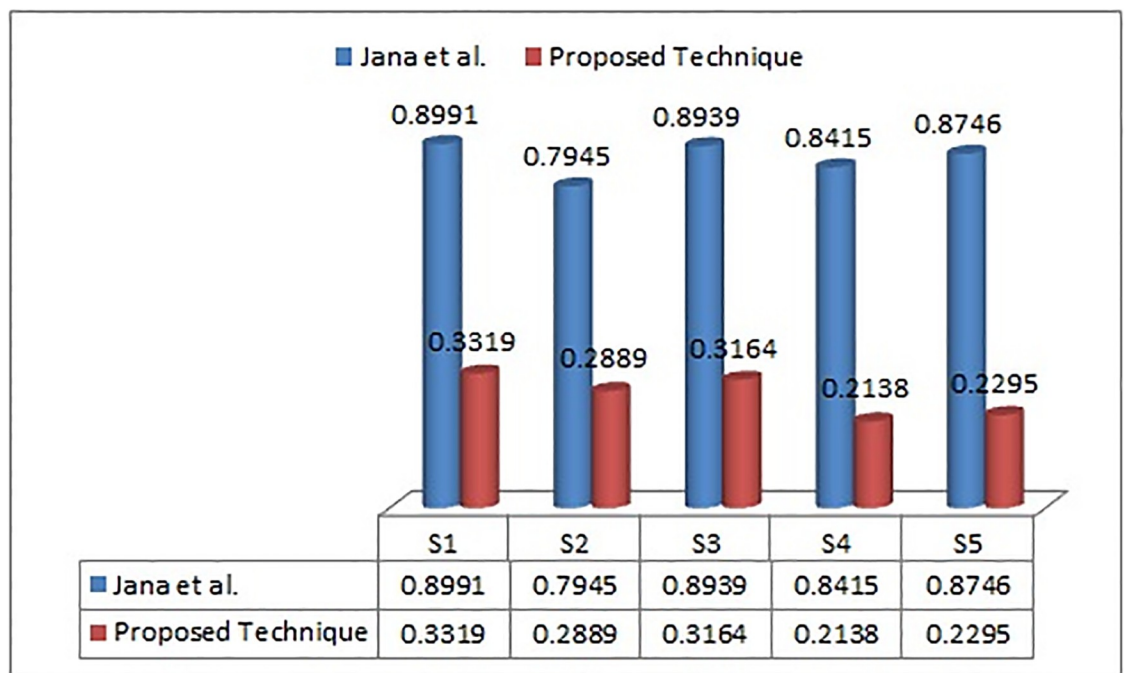

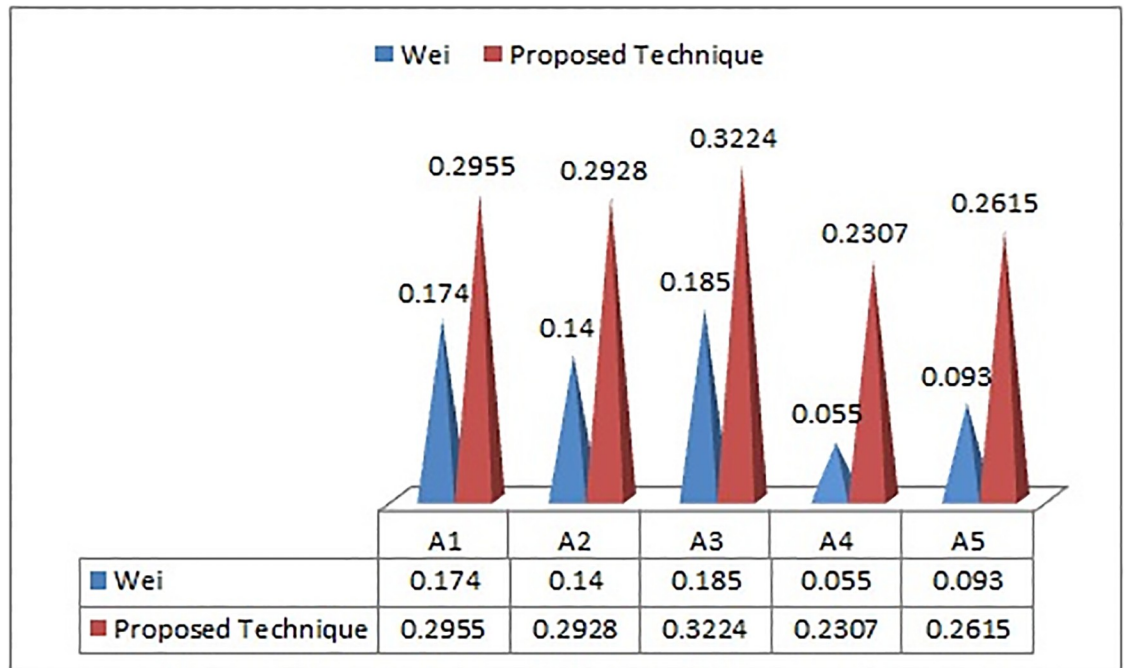**Fig 1. Graphical comparison of Jana et al. [13] and proposed technique.**

Fig 2. Graphical comparison of Wei and proposed technique.

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Conclusions

Assigning the weights to the criteria is a difficult task for the DMs. Mostly, DMs feel hesitation or have less information to assign the weights to the criteria. LP technique is a useful tool that answers rapidly trough the MATLAB. In the present work, we focus to evaluate the weights of the criteria by using the linear programming model which is defined in Definition 5 that needs minimum appraisal information, yet leads to more reliable assessments as compared to other existing techniques. Furthermore, we utilized these weights in PF-TOPSIS to attain the best technical firm and enterprise resource planning (ERP) system. The comparative analysis exhibit the importance and efficiency of the suggested technique. The future research direction is to implement the suggested technique which can be extended in decision making problems under the framework of polygonal fuzzy sets and other vague situations.

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References

1. Zadeh LA. Fuzzy sets. Inform Control. 1965 June; 8(3): 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X

2. Atanassov K. Intuitionistic fuzzy sets. Fuzzy Sets Syst. 1986 Aug; 20(1): 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3

3. Cuong BC. Picture fuzzy sets, a new concept for computational intelligence problems. Published in proceedings of the third world congress on information and communication technologies. Hanoi, Vietnam. 2013 Dec; 1–6.

4. Cuong BC, Kreinovich V. Picture Fuzzy Sets. Journal of Computer Science and Cybernetics. 2014; 30(4): 409–420.

5. Singh P. Correlation coefficients for picture fuzzy sets. J Intell Fuzzy Syst. 2015 Dec; 28: 1–12.

6. Son LH. Generalized picture distance measure and applications to picture fuzzy clustering. Appl Soft Comput. 2016; 46: 284–295. https://doi.org/10.1016/j.asoc.2016.05.009

7. Wei GW. Picture fuzzy cross-entropy for multiple attribute decision making problems. J Bus Econ and Manag. 2016 July; 17(4): 491–502. https://doi.org/10.3846/16111699.2016.1197147

8. Son LH, Viet P, Hai P. Picture inference system: a new fuzzy inference system on picture fuzzy set. Appl Intell. 2017 Apr; 46(3): 652–669. https://doi.org/10.1007/s10489-016-0856-1

9. Thong PH, Son LH. Picture fuzzy clustering for complex data. Eng Appl Artif Intel. 2016 Sep; 56: 121–130. https://doi.org/10.1016/j.engappai.2016.08.009

10. Thong PH, Son LH. A novel automatic picture fuzzy clustering method based on particle swarm optimization and picture composite cardinality. Knowl Based Syst. 2016 June; 109: 48–60. https://doi.org/10.1016/j.knosys.2016.06.023

11. Wei GW. Picture fuzzy aggregation operators and their application to multiple attribute decision making. J Intell Fuzzy Syst. 2017 July; 33: 713–724.

12. Garg H. Some picture fuzzy aggregation operators and their applications to multicriteria decision-making. Arab J Sci Eng. 2017 Dec; 42(12): 1–16. https://doi.org/10.1007/s13369-017-2625-9

13. Jana C, Senapati T, Pal M, Yager RR. Picture fuzzy Dombi aggregation operators: Application to MADM process. Appl Soft Comput. 2018 Oct; 74: 99–109. https://doi.org/10.1016/j.asoc.2018.10.021

14. Ashraf S. Abdullah S, Qadir A, Novel concept of cubic picture fuzzy set. Journal of New Theory. 2018 Sep; 24: 59–72.

15. Wang R, Li Y. Picture hesitant fuzzy set and its application to multiple criteria decision making. Sym. 2018 July; 10(7).

16. Wang L, Peng J, Wang J. A multi-criteria decision-making framework for risk ranking of energy performance contracting project under picture fuzzy environment. J Clean Prod. 2018 Aug; 191(1): 105–118. https://doi.org/10.1016/j.jclepro.2018.04.169

17. Wang X, Wang J, Zhang H. Distance based multicriteria group decision making approach with probabilistic linguistic term sets. Expert Syst. 2018 Dec; 6.

18. Wang R, Wang J, Gao H, Wei G. Methods for MADM with picture fuzzy muirhead mean operators and their application for evaluating the financial investment risk. Sym. 2018 Dec; 11(6).

19. Wei G, Zhang Z. Some single-valued neutrosophic Bonferroni power aggregation operators in multiple attribute decision making. J Amb Intel Hum Comp. 2019 Mar; 10(3): 863–882. https://doi.org/10.1007/s12652-018-0738-y

20. Zhang S, Gao H, Wei G, Wei Y, Wei C. Evaluation based on distance from average solution method for multiple criteria group decision making under picture 2-tuple linguistic environment. Mathematics. 2019 Mar; 7(2).

21. Wei G, Wei C, Wang J, Gao H, Wei Y. Some q—rung orthopair fuzzy Maclaurin symmetric mean operators and their applications to potential evaluation of emerging technology commercialization. Int J Intell Syst. 2019 Nov; 34: 50–81. https://doi.org/10.1002/int.22042

22. Peng SM. Study on enterprise risk management assessment based on picture fuzzy multiple attribute decision-making method. J Intell Fuzzy Syst. 2017 Nov; 33(6): 3451–3458.

23. Figuera J, Greco S, Ehrgott M. Multiple criteria decision analysis, state of the art surveys. New York: Springer.; 2005.

24. Ishizaka A, Siraj S. Are multi-criteria decision-making tools useful? An experimental comparative study of three methods. Eur J Oper Res. 2018 Jan; 264(2): 462–71. https://doi.org/10.1016/j.ejor.2017.05.041

25. Wang L, Zhang H, Wang J, Wu G. Picture fuzzy multi-criteria group decision making method to hotel building energy efficiency retrofit project selection. RAIRO—Operations Research. 2018.

26. Vanderbei RJ. Linear Programming: Foundations and Extensions. Springer-Verlag, Berlin, Heidelberg.; 2014.

27. Wang C, Chen S. A new multiple attribute decision making method based on linear programming methodology and novel score function and novel accuracy function of interval-valued intuitionistic fuzzy values. Inform Sciences. 2018 Feb; 429(C): 421–432.

28. Su B, He P, Sheng Ng T. Energy economic recovery resilience with input-output linear programming models. Energy Econ. 2015 June; 50: 177–191.

29. Aliyev RR. Interval linear programming based decision making on market allocations. Procedia Comput Sci. 2017 Aug; 120: 47–52. https://doi.org/10.1016/j.procs.2017.11.209

30. Wang CY, Chen SM. Multiple attribute decision making based on interval-valued intuitionistic fuzzy sets, linear programming methodology, and the extended TOPSIS method. Inform Sciences. 2016 Nov; 397-398: 155–167. https://doi.org/10.1016/j.ins.2017.02.045

31. Sindhu MS, Rashid T, Kashif A, Guirao JL. Multiple criteria decision making based on probabilistic interval-valued hesitant fuzzy Sets by using LP methodology. Discrete Dyn Nat and Soc. 2019 Apr; Article ID 1527612, 12 pages.

32. Hwang C, Yoon K. Multiple attributes decision making methods and applications, berlin, heidelberg. Berlin, Heidelberg, Springer.; 1981.

33. Kacprzak D. A doubly extended TOPSIS method for group decision making based on ordered fuzzy numbers. Expert Syst Appl. 2019 Jan; 116: 243–254. https://doi.org/10.1016/j.eswa.2018.09.023

34. Kuo T. A modified TOPSIS with a different ranking index. Eur J Oper Res. 2017 July; 260(1): 152–160. https://doi.org/10.1016/j.ejor.2016.11.052

35. Zhou S, Liu W, Chang W. An improved TOPSIS with weighted hesitant vague information. Chaos Solution Fract. 2015 Nov; 89: 47–53.

36. Tian ZP, Zhang HY, Wang JQ, Wang TL, Green supplier selection using improved TOPSIS and best-worst method under intuitionistic fuzzy environment. Informatica. 2018 July; 29(4): 773–800. https://doi.org/10.15388/Informatica.2018.192

37. Wang W, Xin X. Distance measure between intuitionistic fuzzy sets. Pattern Recogn Lett. 2005 Oct; 26 (13): 2063–2069. https://doi.org/10.1016/j.patrec.2005.03.018