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A novel linear algebra-based method for complex interval linear systems in circuit analysis

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ABSTRACT

In this paper, systems of multivariate interval linear equations with complex interval coefficients are examined, and a novel linear algebra-based approach for locating all of their solutions is proposed. The key concept is to convert the system into a crisp polynomial system that is equivalent and allows for the use of the innovative computational features of Gröbner bases. It is possible to calculate all of the system's precise solutions at once after an appropriate Gröbner basis has been determined. Design is a condition for the presence of a solution in complex interval linear systems. In addition, an algorithm is devised to retrieve all solutions using the eigenvalue approach. In addition, a proportional case is solved using the provided approach to demonstrate its efficiency and efficacy. The given approach can locate all solutions for linear systems with complex intervals. Additionally, it determines the presence or absence of a solution for the system. We use the aforementioned technique in the context of circuit analysis to demonstrate the effectiveness of the findings obtained.

1. Introduction

Linear systems resulting from engineering challenges often include uncertain values. The practice of substituting acceptable intervals for the unknown variables is one of the well-known strategies for overcoming this ambiguity. Uncertain quantities produce complex values in key issues like electrical circuits [1,2], which causes complex variables and complex intervals to develop. In the primary topic of this essay, we presume that the issue is described by a linear system of equations with complicated interval equations that may be represented by a matrix of equations. Real interval arithmetic procedures may be used to determine the solutions when the system's coefficient matrix and the right-side vector are both inside the interval. An strategy for the outer interval solution of a parametrized linear system, for instance, was provided by Kolev et al. [3]. In relation with structural mechanics, Skalna et al. [4] have presented a few methods for handling interval parametric linear equations. In addition, Popova et al. [5] have proposed a technique for solving a set of linear equations that are parameterized. Visualizing and calculating the solutions of an interval linear system have been studied by Kraemer et al. [6]. Majumdar and Chakraverty [7] presented a novel method of solving an interval-form system of linear equations.

The aforementioned scientific articles have addressed some challenges that emerge while computing the solutions of a system of equations with interval coefficients. However, there are other concerns that have not been covered in these publications.

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1. The methods need to require a select initial point while it is often challenging.
2. Many approximate solutions for the system will be found.
3. There is no criterion or validation to check whether the systems have a solution.
4. The text does not provide any details on the quantity of solutions for a specific system.
5. When a system has no solution, the selected approach can be misleading.
6. Not all solutions of the systems can be simultaneously achieved.

This paper's major concept is to use the eigenvalue approach [8] to solve the aforementioned difficult issues (for more details see [9–15]). To do this, each interval coefficient is given a variable. The current system is transformed into a set of polynomial equations. The system solutions are then created by computing a Gröbner basis and creating a few so-called multiplication matrices using the system's eigenvalues.

Farahani et al. [16] employed the eigenvalue approach to effectively segregate the complex fuzzy linear system. This approach involves finding ways to solve of a system in a manner that is not reliant on each other. This approach employs deterministic features and transforms a matrix into a triangular matrix by straightforward row operations. Subsequently, Farahani et al. [17] endorsed Wu's method for resolving a Fuzzy Complex System of Linear Equations (FCSLE). The Wu's method serves as the fundamental basis for this technique for resolving FCSLE. Wu's technique may be used for resolving polynomial equation systems, including characteristic sets. The above sets have a triangular shape, which facilitates the use of forward substitution to estimate their range. The ultimate FCSLE solution is achieved by implementing the solutions that were previously mentioned. Farahmand Nejad et al. [18] recently employed the Gröbner basis approach to solve an FCSLE. This technique converts a complicated fuzzy linear system into a more straightforward system, allowing the creation of new solution systems to address novel system issues. The Gröbner base has a pronounced triangular shape as a result of its lexicographic ordering. The solution to many polynomial systems that utilize Gröbner theory is same. Since the initial equation in Gröbner's foundation involves just a single variable, it may be readily solved. Therefore, a traditional approach may be employed to compute the root of this variable polynomial. To get the solution to the second polynomial equation, one must first determine the root of the first equation and then substitute it into the bivariate polynomial equation. The project will persist as a substitute until all system alternatives are accessible. Solving an FCSLE results in the production of a system of univariate polynomial equations that has a root that is easier to locate compared to the original system.

The eigenvalue technique computes the elements of a system's solutions separately. As a result, the computation of the following solutions is unaffected by the likely errors and approximations that occurred in the computation of the prior solutions. In this strategy, let's assume that I is a zero-dimensional ideal in $\mathcal{K}[x_1,\dots,x_n]$. Consequently, the quotient ring is shown by $\frac{\mathcal{K}[x_1,\dots,x_n]}{I}$ is a finite dimensional vector space on $\mathcal{K}$. For each polynomial h, the matrix that represents the linear transformation described below

$$\varphi_h : \frac{\mathcal{K}[x_1,\dots,x_n]}{I} \to \frac{\mathcal{K}[x_1,\dots,x_n]}{I}$$
$$g + I \mapsto hg + I$$

with relation to a basis of $\frac{\mathcal{K}[x_1,\dots,x_n]}{I}$ is denoted by M_h. The eigenvalues of M_h are determined by evaluating the values of h at the solutions of the polynomial system. Specifically, assessing the polynomials $h = x_i, i = 1,...,n$, when we look at the set of all solutions to the polynomial equation system, we can get the coordinates of those solutions. By using this method, the difficulty of solving complex interval linear systems is reduced to the task of determining and computing a matrix's eigenvalues. Consequently, the matrix may be converted into a triangular matrix by the use of basic row operations, and the characteristics of the determinant can serve as a valuable tool in the field of linear algebra. The proposed method addresses the previously described problems, and it also addresses a system of n interval linear equations in n variables, where the right-hand sides and coefficients are both complex and complex interval, respectively. The primary concept behind this strategy is to convert the system into a precise polynomial system, leading to a system with $4n$ equations and $4n$ unknowns, from which an efficient scheme to solve systems may be used to determine the solution set of the novel system. As a consequence, the complicated interval linear system may be resolved using the eigenvalue approach. A need for the solutions' existence is also provided.

Modeling and simulation is a well recognized scientific technique that may be used to examine a system or forecast its behavior prior to its actual implementation [19–23]. In electrical circuits, changes in the value of a circuit component are caused by factors such as the method of production, temperature fluctuations, and uncertain device characteristics. When implemented in a real-world scenario, the input sources and circuit components inherently possess a certain level of uncertainty. Tolerance analysis is considered an essential phase in the circuit design process due to the potential for considerable uncertainties to affect the performance of the circuit. Interval analysis simplifies the solution of system equations by representing unknown parameters of a circuit as defined ranges of values. In order to guarantee the accurate functioning of the circuit in specific applications, it is necessary to have complete knowledge of the most unfavorable impact generated by unknown system parameters. The paper presents many techniques that use interval analysis to analyze linear analog circuits [24–26]. In order to evaluate or solve a circuit, it is necessary to ascertain the voltages and currents that are passing through each individual component. The methodology proposed in this research is suitable and dependable for circuit analysis.

The paper is structured as follows. After an introduction, Sections 2 and 3 go into the topic of interval arithmetic and polynomial rings, respectively, review fundamental definitions and findings. Section 3 explains how to resolve a polynomial problem using the eigenvalue approach. The complex interval linear system is presented in Section 4. In Section 5, the primary method for solving these sorts of systems is laid out and used to resolve a numerical example. We offer some discussions, in section 7. In Section 8, the conclusion is briefly discussed.

2. Interval arithmetic

In this part, we recall the arithmetical operations on real intervals [27]. Let $\circ \in \{+, -, \cdot, /\}$ be one of the natural algebraic operations, and let $[x] = [\underline{x}, \overline{x}]$ and $[y] = [\underline{y}, \overline{y}]$ be two real compact intervals. The corresponding operations due to the above intervals are characterized by

$$[x] \circ [y] = \{x \circ y | x \in [x], y \in [y]\}, \qquad [27]\,(1)$$

where, in the case of division, 0 does not belong to $[y]$.

Suppose $\mathbb{IR}$ be the set of all bounded and closed intervals on $\mathbb{R}$. For $[x], [y] \in \mathbb{IR}$, we have $[x] = [y] \iff \underline{x} = \underline{y}$ and $\overline{x} = \overline{y}$. It is easy to prove that $\mathbb{IR}$ is closed under addition, subtraction, multiplication and division. More importantly, we know that $[x] \circ [y]$ can be represented by only applying the bounds of $[x]$ and $[y]$. Moreover, the algebraic operations addition, subtraction and multiplication are given as follows, respectively:

- $+ : \mathbb{IR} \times \mathbb{IR} \to \mathbb{IR}, +([x], [y]) = [x] + [y] = [\underline{x} + \underline{y}, \overline{x} + \overline{y}]$.
- $- : \mathbb{IR} \times \mathbb{IR} \to \mathbb{IR}, -([x], [y]) = [x] - [y] = [\underline{x} - \overline{y}, \overline{x} - \underline{y}]$.
- $\cdot : \mathbb{IR} \times \mathbb{IR} \to \mathbb{IR}, .([x], [y]) = [\min\{\underline{x}\underline{y}, \underline{x}\overline{y}, \overline{x}\underline{y}, \overline{x}\overline{y}\}, \max\{\underline{x}\underline{y}, \underline{x}\overline{y}, \overline{x}\underline{y}, \overline{x}\overline{y}\}]$.

Every real number x may be conceptualized as an interval number $[x, x]$ with a width of zero. In other word, if $\underline{x} = \overline{x} = x$, $[x]$ just consists of the element x, i.e., $x \equiv [x, x]$.

Definition 2.1. ([28,29]) An interval $[z]$ that is complex is defined as

$$\begin{aligned}[z] &= [\underline{x} + i\underline{y}, \overline{x} + i\overline{y}] \\ &= \{x + iy \in \mathbb{C} | \underline{x} \le x \le \overline{x}, \underline{y} \le y \le \overline{y}\}.\end{aligned}$$

Suppose $\mathbb{IC}$ denote the set of all complex intervals. A complicated interval is defined in terms of its rectangular shape in Definition 2.1 It should be noted that two further types of complex intervals, polar form and circle form, are addressed in [30–32]. Moreover, in [33] besides visualization of complex arithmetic is presented.

Similar to [34], the equation may be expressed as $[z] = [x] + i[y]$, where $[x], [y] \in \mathbb{IR}$. Moreover,

$$\underline{z} = \underline{x} + i\underline{y}, \overline{z} = \overline{x} + i\overline{y}.$$

As clarified in [34], the following definition exhibits how the properties of real intervals are expandable to the complex ones.

Definition 2.2. [35] Consider two complex intervals $[z_1] = [x_1] + i[y_1]$ and $[z_2] = [x_2] + i[y_2]$, where $[x_j] = [\underline{x_j}, \overline{x_j}]$ and $[y_j] = [\underline{y_j}, \overline{y_j}]$, for $j = 1, 2$. Let also $c = a + ib$. Then,

$$\begin{aligned}[z_1] + [z_2] &= ([x_1] + i[y_1]) + ([x_2] + i[y_2]) \\ &= ([x_1] + [x_2]) + i([y_1] + [y_2]) \\ &= [\underline{x_1} + \underline{x_2}, \overline{x_1} + \overline{x_2}] + i[\underline{y_1} + \underline{y_2}, \overline{y_1} + \overline{y_2}],\end{aligned}$$

and

$$\begin{aligned}c \cdot [z_1] &= (a + ib) \cdot ([x_1] + i[y_1]) \\ &= (a[x_1] - b[y_1]) + i(a[y_1] + b[x_1]).\end{aligned}$$

Definition 2.3. [36] A complex interval vector is the vector $[\mathbf{z}] = ([z_1], [z_2], ..., [z_n])^T$, where $[z_j] \in \mathbb{IC}$, for $j = 1, 2, ..., n$.

3. Polynomial ring and Gröbner bases

This part aims to devote the essential definitions and statements on polynomial ring and Gröbner bases. Assume that $x_1, \ldots, x_n$ are n algebraically independent variables and $\mathcal{K}$ is a field. For $\alpha_1, \ldots, \alpha_n \in \mathbb{Z}_{\ge 0}$ a monomial is defined by the power product $\mathbf{x}^\alpha = x_1^{\alpha_1} \cdots x_n^{\alpha_n}$, where the sequence $x_1, \ldots, x_n$ is indicated by $\mathbf{x}$ and $\alpha = (\alpha_1, \ldots, \alpha_n)$. Monomial orderings, which are special types of total orderings, may be used to arrange the whole set of monomials over $\mathcal{K}$. In the next definition, a total ordering is defined.

Definition 3.1. On the set of monomials over $\mathcal{K}$, a monomial ordering $\succ$ is a total ordering that applies to each monomials $\mathbf{x}^\alpha, \mathbf{x}^\beta$ and $\mathbf{x}^\gamma$ for which the following conditions are satisfied:

- $\mathbf{x}^\alpha \succ \mathbf{x}^\beta \Longrightarrow \mathbf{x}^\gamma \mathbf{x}^\alpha \succ \mathbf{x}^\gamma \mathbf{x}^\beta$,

- $\succ$ is well-ordering.

Among infinite monomial orderings so that there is one convenient for any special kind of problem, graded reverse lexicographic ordering $\succ_{grevlex}$ is notable. Let $\mathbf{x}^\alpha$ and $\mathbf{x}^\beta$ be monomials in $\mathcal{K}[\mathbf{x}]$. Then $\mathbf{x}^\alpha \succ_{grevlex} \mathbf{x}^\beta$ if $\sum_{i=1}^n \alpha_i > \sum_{i=1}^n \beta_i$, or if $\sum_{i=1}^n \alpha_i = \sum_{i=1}^n \beta_i$, and in the difference $\alpha - \beta \in \mathbb{Z}^n$, the rightmost non-zero entry is negative.

A polynomial f in $x_1, \dots, x_n$ is a finite linear combination of monomials with coefficients in the field $\mathcal{K}$. We will write a polynomial f in the form $f(x_1, \dots, x_n) = \sum_\alpha c_\alpha \mathbf{x}^\alpha, c_\alpha \in \mathcal{K}$ where the sum is over a finite number of n-tuples $\alpha = (\alpha_1, \dots, \alpha_n)$. The set of all polynomials in $x_1, \dots, x_n$ with coefficients in the field $\mathcal{K}$ has the structure of a ring under addition and multiplication for polynomials is mentioned to be polynomial ring and is denoted $\mathcal{K}[x_1, \dots, x_n]$ or $\mathcal{K}[\mathbf{x}]$. Suppose $\mathcal{K}$ is a field and regard the polynomial ring $\mathcal{K}[\mathbf{x}]$. For any nonzero polynomial $f(x_1, \dots, x_n) = \sum_\alpha c_\alpha \mathbf{x}^\alpha$, the leading term of f (with respect to $\prec$) is the product $c_\alpha \mathbf{x}^\alpha$, where $\mathbf{x}^\alpha$ is the largest monomial appearing in f with respect to $\prec$. We will use the notation $LT(f)$ for the leading term of f. Moreover, if $LT(f) = c\mathbf{x}^\alpha$, then $LC(f) = c$ is the leading coefficient of f and $LM(f) = \mathbf{x}^\alpha$ is the leading monomial of f. Thus, when F is the set of polynomials, $LM(F) = \{LM(f) | f \in F\}$ and for the ideal I, the ideal generated by $LM(I)$, denoted $in(I)$, is regarded to be the initial ideal of I. In next paragraph, Gröbner basis theory briefly discussed which gives a deep insight into the ideal.

Definition 3.2. For an ideal $I \subset \mathcal{K}[\mathbf{x}]$, fix a monomial ordering. A finite set $G = \{g_1, ..., g_t\} \subset I$ is said to be a Gröbner basis for I, if every leading term of any element of I is divisible by one of the $LT(g_i)$.

We demonstrate that any polynomial ideal in $\mathcal{K}[\mathbf{x}]$ other than 0 has a Gröbner basis for each monomial ordering by using the Hilbert basis theorem [37]. A Gröbner basis may be computed from the generating set of any polynomial ideal using a method introduced by Bruno Buchberger in his 1965 dissertation. He provided the first method for calculating Gröbner bases. The Buchberger approach, developed concurrently with the concept of Gröbner basis, stands out as the most direct among all efficient algorithms, for calculating Gröbner bases. The Faugère F_5 algorithm [38] and other signature-based algorithms like G^2V [39] and GVW [40] are the most effective algorithms currently in use. It should be noted that a polynomial ideal's Gröbner basis is not always exclusive. In order to achieve unicity, the reduced Gröbner basis notion is introduced. A noteworthy observation is that in the worst situation, the complexity of the methods to calculate Gröbner bases is double exponential. When the input polynomials are in linear form, this high degree of complexity will, nevertheless, be reduced to a polynomial class [38]. The reduced Gröbner basis for a polynomial ideal is distinct up to each monomial ordering, it should be noted.

Definition 3.3. The Gröbner basis G is said to be reduced if for every $g \in G$,

- $LC(g) = 1$ and,
- No monomial of g appears in $LT(G - \{g\})$.

The topic is then further discussed by using an example to remind the audience how the Gröbner basis may be used to solve a system of polynomial equations. Gröbner bases have shown to be quite valuable in the field of discovering solutions to polynomial problems. Consider

$$\begin{cases} g_1(x_1, ..., x_n) &= 0 \\ &\vdots \\ g_s(x_1, ..., x_n) &= 0 \end{cases}$$

as a polynomial system. The set of all simultaneous solutions $(\alpha_1, ..., \alpha_n) \in \mathcal{K}^n$ is said to be the affine variety defined by $G = \{g_1, ..., g_s\}$ and is denoted by $\mathbf{V}(g_1, \dots, g_s)$ (or simply $\mathbf{V}(G)$). An affine variety is defined as a subset V in $\mathcal{K}^n$ that can be expressed as $V = V(g_1, ..., g_s)$ where g_i are polynomials in $\mathcal{K}[\mathbf{x}]$.

Assume that G is the ideal I's Gröbner basis with respect to a random monomial ordering. This is noteworthy because $I = \langle G \rangle$, this signifies $\mathbf{V}(I) = \mathbf{V}(G)$. A system of polynomial equations may have solutions found by employing this computational method. One is provided as follows:

Example 3.4. Suppose that

$$\begin{cases} x_1{}^2 - x_1 x_2 x_3 + 1 &= 0 \\ x_2{}^3 + x_3{}^2 - 1 &= 0 \\ x_1 x_2{}^2 + x_3{}^2 &= 0 \end{cases}$$

The reduced Gröbner basis of the ideal $I = \langle x_1{}^2 - x_1 x_2 x_3 + 1, x_2{}^3 + x_3{}^2 - 1, x_1 x_2{}^2 + x_3{}^2 \rangle$ in $\mathbf{Q}[x_1, x_2, x_3]$ with respect to the lexicographic ordering is as follows:

$$G = \{g_1(x_3), x_1 - g_2(x_3), x_2 - g_3(x_3)\}$$

with respect to $x_3 \prec_{lex} x_2 \prec_{lex} x_1$, where

$$\begin{cases} g_1(x_3) &=& {x_3}^{15} - 3{x_3}^{14} + 5{x_3}^{12} - 3{x_3}^{10} - {x_3}^9 - {x_3}^8 + 4{x_3}^6 - 6{x_3}^4 + 4{x_3}^2 - 1, \\ g_2(x_3) &=& 2{x_3}^{14} - 9{x_3}^{13} + 11{x_3}^{12} + 2{x_3}^{11} - 7{x_3}^{10} - 3{x_3}^9 + 2{x_3}^8 - {x_3}^7 + 4{x_3}^6 + \\ && +7{x_3}^5 - 10{x_3}^4 - 6{x_3}^3 + 11{x_3}^2 + 2x_3 - 4, \\ g_3(x_3) &=& {x_3}^{13} - 3{x_3}^{12} + {x_3}^{11} + 2{x_3}^{10} + {x_3}^9 - {x_3}^8 - 2{x_3}^6 + 2{x_3}^4 - {x_3}^3 - 3{x_3}^2 + 1. \end{cases}$$

Utilizing special form of the Gröbner basis for the original system, $\mathbf{V}(G)$ can be found by root-finding one-variable polynomial $g_1(x_3)$ then inserting the roots into the two last polynomials in G.

Theorem 3.5 *([11]). Let I be a polynomial ideal I and G be a reduced Gröbner basis for I with regard to any monomial order. $V(I)$ will be empty set if G be a singleton set with a single element $\{1\}$.*

Depending on the dimension of the ideal, univariate polynomials may or may not exist. We can explain what the term "dimension of an ideal" means in the definition that follows.

Definition 3.6. Let $I \subset \mathcal{K}[\mathbf{x}]$ be an ideal and also $\mathbf{u}$ is a set of variables, $\mathbf{u}$ is an independent set concerning, I, whenever $I \cap \mathcal{K}[\mathbf{u}] = \{0\}$. The cardinality of the largest independent set with respect to I is the dimension of I. In addition, I is referred to as a zero-dimensional ideal if its dimension is zero and a positive dimensional ideal in all other cases.

Zero-dimensional ideals provide significant advantages that enhance computational processes. For example, if I be a zero-dimensional ideal then the vector space $\mathcal{K}[\mathbf{x}]/I$ will be finite-dimensional. Consequently, a basis for $\mathcal{K}[\mathbf{x}]/I$ can be found by reading the leading monomials of a Gröbnr basis for I. Moreover, the set

$$A = \mathbb{M} \setminus in(I)$$

constructs a basis for $\mathcal{K}[\mathbf{x}]/I$, where $\mathbb{M}$ is the set of all monomials in $\mathcal{K}[\mathbf{x}]$. More precisely, to compute A it is enough to compute a Gröbner basis G at first. Next, do the operation of factoring out the monomials that are not divisible by $LM(g)$ for each $g \in G$. One of the key theorems proposed in this study, the following theorem, aims to characterize an important feature of a particular zero-dimensional ideal. It is required to consider the definition provided below before that.

Definition 3.7. Suppose I be a zero-dimensional polynomial ideal and B be a basis for the polynomial ring $\mathcal{K}[\mathbf{x}]/I$. For every polynomial h in $\mathcal{K}[\mathbf{x}]$, the linear transformation φ_h may be described as follows.

$$\varphi_h : \frac{\mathcal{K}[\mathbf{x}]}{I} \to \frac{\mathcal{K}[\mathbf{x}]}{I}$$
$$g + I \mapsto hg + I$$

Furthermore, let M_h be the matrix representation of φ_h concerning, for B. Then M_h is said to be the multiplication matrix of h concerning, for I.

Theorem 3.8 *([37]). Using the previous notations, the eigenvalues of the multiplication matrix M_h represent the values of h inside the set* $\mathbf{V}(I)$.

The solutions of a zero-dimensional polynomial equations system can be determined by calculating the eigenvalues of M_{x_i} for each variable x_i, as a direct result of Theorem 3.8. It should be noted that, the eigenvalues of M_{x_i} are the i-th component of $\mathbf{V}(J)$. The use of the eigenvalue method for locating zero-dimensional polynomial system solutions is shown in the following procedure.

Algorithm 1 Eigenvalue method.

Require: $A = \{f_1, \dots, f_k\}$ be a finite set which belongs to $\mathcal{K}[\mathbf{x}]$
Ensure: $\mathbf{V}(F)$
  $G :=$ a Gröbner basis of the polynomial ideal generated by A with regard to an arbitrary monomial ordering;
  $B :=$ a basis for $\mathcal{K}[\mathbf{x}]/\langle A\rangle$;
  for $j = 1, \dots, m$ **do**
    $E_j :=$ the eigenvalue set of M_{x_j};
  end for
  $V := E_1 \times \cdots \times E_m$;
  for $v \in V$ **do**
    if $f_i(v) \neq 0$ for an $i = 1, \dots, k$ **then**
      $V := V \setminus \{v\}$;
    end if
  end for
  Return (V);

It should be emphasized that the eigenvalue approach is a quick and easy way for determining a zero-dimensional polynomial system's solutions. The cartesian product of eigenvalues, which is another benefit of the solution set, makes it necessary, as stated in the procedure, such that we can determine if a tuple is a solution. For illustration, the eigenvalue technique must be used to examine 15^3 tuples to see whether they are part of the solution set in order to solve Example 3.4. This is why there are only 15 solutions to this system. As a result, the strategy outlined is useful when there are few univariates relative to the total number of variables.

The following illustrates the eigenvalue approach used to determine the actual solutions of a given polynomial problem.

Example 3.9. [16] Consider the following polynomial systems:

$$\begin{cases} {x_1}^2 + {x_2}^2 + {x_3}^2 = 6, \\ {x_1}^3 + {x_2}^3 + {x_3}^3 - x_1 x_2 x_3 = -4, \\ x_1 x_2 + x_1 x_3 + x_2 x_3 = -3. \end{cases}$$

Let I be the ideal generated by the polynomial in above system. In the first step, a Gröbner basis is computed for I with reference to the reverse graded lexicographic order and the basis of monomials

$$B = \{1, x_3, x_2, x_1, {x_3}^2, x_2 x_3, x_1 x_3, {x_2}^2, {x_3}^3, x_2 {x_3}^2, {x_2}^2 x_3, x_2 {x_3}^3\}$$

is obtained using the command

```
NormalSet(G,tdeg({x_{1}},{x_{2}},{x_{3}})).
```

Matrix representation of φ_{x_1} is obtained using the Gröbner basis and B as follows.

$$M_{x_1} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -3 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 2 & \frac{3}{2} & \frac{9}{2} & \frac{9}{2} & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ -2 & -\frac{9}{2} & -\frac{9}{2} & -\frac{9}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 \\ 2 & \frac{9}{2} & \frac{3}{2} & \frac{9}{2} & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 2 & 0 & 0 & \frac{3}{2} & \frac{9}{2} & \frac{9}{2} & 0 & 0 & 0 & 0 & -1 \\ 0 & -2 & 0 & 0 & -\frac{9}{2} & -\frac{9}{2} & -\frac{9}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{27}{2} & 0 & -2 & 0 & 0 & 0 & \frac{9}{2} & -\frac{9}{2} & 0 & 0 & 0 & 0 \\ -9 & -\frac{27}{4} & -\frac{81}{4} & -\frac{81}{4} & -2 & 0 & 0 & 0 & -\frac{9}{2} & 0 & 0 & 0 \end{bmatrix}$$

Furthermore, in Maple, we can compute this matrix applying the "`MultiplicationMatrix`" command. Using eigenvalue routine "*Eigenvalues*", two real eigenvalues $\{1, -2\}$ are provided. Similarly, we are able to obtain the values of x_2 and x_3. The eigenvalues of M_{x_2} and M_{x_3} are $\{1, -2\}$ and $\{1, -2\}$, respectively. The following solutions are derived from it:

$$V(I) = \{(1, -2, 1), (1, 1, -2), (-2, 1, 1)\}.$$

4. Complex interval linear systems

A complex interval linear system can be described as a $n \times n$ system.

$$\begin{cases} c_{11}[z_1] + c_{12}[z_2] + \dots + c_{1n}[z_n] = [w_1], \\ c_{21}[z_1] + c_{22}[z_2] + \dots + c_{2n}[z_n] = [w_2], \\ \vdots \\ c_{n1}[z_1] + c_{n2}[z_2] + \dots + c_{nn}[z_n] = [w_n], \end{cases} \quad [41] \ (2)$$

where the matrix consisting of the coefficients i.e. $\mathbf{C} = (c_{kj})_{n\times n}$, $c_{kj} = a_{kj} + i b_{kj}$, is an $n \times n$ complex-valued matrix, $[w_j] = [u_j] + i[v_j]$, $1 \le j \le n$ are complex intervals, and $[z_j] = [x_j] + i[y_j]$, $1 \le j \le n$ are complex interval unknowns [41]. System (2) can be represented as

$$\sum_{j=1}^{n} c_{kj}[z_j] = [w_k], \quad k \in \{1, 2, \dots, n\}. \quad [41] \ (3)$$

These equations can be represented as

$$\sum_{j=1}^{n} (a_{kj} + i b_{kj})([x_j] + i[y_j]) = [u_k] + i[v_k], \quad [41] \ (4)$$

and consequently,

$$\sum_{j=1}^{n}(a_{kj}[x_j]-b_{kj}[y_j])+i(a_{kj}[y_j]+b_{kj}[x_j])=[u_k]+i[v_k], \quad [41] \quad (5)$$

which shows that for each $k\in\{1,2,...,n\}$,

$$\{\sum_{j=1}^{n}(a_{kj}[\underline{x_j},\overline{x_j}]-b_{kj}[\underline{y_j},\overline{y_j}])\}+i\{\sum_{j=1}^{n}(a_{kj}[\underline{y_j},\overline{y_j}]+b_{kj}[\underline{x_j},\overline{x_j}])\}=[\underline{u_k},\overline{u_k}]+i[\underline{v_k},\overline{v_k}]. \quad [41] \quad (6)$$

In order to account for both positive and negative values of a_{kj} and b_{kj}, the given equation is expressed as follows:

$$\left\{\sum_{a_{kj}\geq 0} a_{kj}[\underline{x_j},\overline{x_j}]+\sum_{b_{kj}<0}-b_{kj}[\underline{y_j},\overline{y_j}]+\sum_{a_{kj}<0} a_{kj}[\overline{x_j},\underline{x_j}]+\sum_{b_{kj}\geq 0}-b_{kj}[\overline{y_j},\underline{y_j}]\right\}+i$$
$$\left\{\sum_{a_{kj}\geq 0} a_{kj}[\underline{y_j},\overline{y_j}]+\sum_{b_{kj}\geq 0} b_{kj}[\underline{x_j},\overline{x_j}]+\sum_{a_{kj}<0} a_{kj}[\overline{y_j},\underline{y_j}]+\sum_{b_{kj}<0} b_{kj}[\overline{x_j},\underline{x_j}]\right\} \quad [41] \quad (7)$$
$$=[\underline{u_k},\overline{u_k}]+i[\underline{v_k},\overline{v_k}],$$

for $k\in\{1,2,...,n\}$. Ultimately, System (2) may be expressed in the following manner

$$\begin{cases}\underline{u_k}=\sum_{a_{kj}\geq 0} a_{kj}\underline{x_j}+\sum_{b_{kj}<0}-b_{kj}\underline{y_j}+\sum_{a_{kj}<0} a_{kj}\overline{x_j}+\sum_{b_{kj}\geq 0}-b_{kj}\overline{y_j},\\ \overline{u_k}=\sum_{a_{kj}\geq 0} a_{kj}\overline{x_j}+\sum_{b_{kj}<0}-b_{kj}\overline{y_j}+\sum_{a_{kj}<0} a_{kj}\underline{x_j}+\sum_{b_{kj}\geq 0}-b_{kj}\underline{y_j},\\ \underline{v_k}=\sum_{a_{kj}\geq 0} a_{kj}\underline{y_j}+\sum_{b_{kj}\geq 0} b_{kj}\underline{x_j}+\sum_{a_{kj}<0} a_{kj}\overline{y_j}+\sum_{b_{kj}<0} b_{kj}\overline{x_j},\\ \overline{v_k}=\sum_{a_{kj}\geq 0} a_{kj}\overline{y_j}+\sum_{b_{kj}\geq 0} b_{kj}\overline{x_j}+\sum_{a_{kj}<0} a_{kj}\underline{y_j})+\sum_{b_{kj}<0} b_{kj}\underline{x_j}.\end{cases} \quad [41] \quad (8)$$

The following definition elucidates the significance of an algebraic solution for the complex interval linear system (2).

Definition 4.1. A complex interval vector $[\mathbf{z}]=([z_1],[z_2],...,[z_n])^T$, where $[z_j]=[\underline{z_j},\overline{z_j}]$, is said to be an algebraic solution for the complex linear system (2) if

$$\sum_{j=1}^{n} c_{kj}[z_j]=[w_k],\ \ k\in\{1,2,...,n\}.$$

Theorem 4.2. *The complex linear system* (2) *and the System* (8) *have the same solutions.*

Proof. The proof is straightforward (see Definition 2.2).

5. The main idea

The primary concept of this study is described in this part. The system (2) may be converted into a polynomial system with $4n$ equations and $4n$ variables in the polynomial ring, as detailed in Section 4.

$$\mathbf{R}=\mathbb{R}[\underline{x_1},\overline{x_1},\underline{x_2},\overline{x_2},...,\underline{x_n},\overline{x_n},\underline{y_1},\overline{y_1},\underline{y_2},\overline{y_2},...,\underline{y_n},\overline{y_n}].$$

Regarding the reverse graded lexical ordering, the system (8) generates an ideal, we calculate the reduced Gröbner basis G in this case. The system may then be resolved using the eigenvalue approach after that (2). A test for determining if System (2) has a solution or not is given in the following theorem.

Theorem 5.1. *System* (2) *has a solution if the computed Gröbner basis does not contain* 1.

Proof. The solutions of the complex linear system (2) and the system (8) are identical, as stated by Theorem 4.2. Therefore, regarding Theorem 3.5, the sufficient and necessary condition for the absence of a solution is $G=\{1\}$.

The followings express the resolution processes of a complex interval linear system applying the Gröbner basis with regards to the algorithm below.

Algorithm 2 Main algorithm.

Require: The complex interval linear system (2)
Ensure: The set of solutions, i.e., S for system (2)
[1] Convert System (2) into System (2)
[2] Calculate the Gröbner basis for the ideal given by system (8) in the ring **R** using the reverse graded lexicographical ordering
[4] If $G = \{1\}$ then $S := \emptyset$ else go to 5
[5] $S := EIGENVALUE\ METHOD(G)$
[7] End

6. Numerical examples and applications

The above discussion is illuminated in the following example.

Example 6.1. Let us examine the above system, comprising of three complicated interval linear equations with three unknowns:

$$\begin{cases} (2+i)[z_1] + (2-i)[z_2] + (-1+i)[z_3] = [-9,-1] + i[-5,3], \\ (-2+2i)[z_1] + (1-3i)[z_2] + (3+i)[z_3] = [-1,11] + i[-9,3], \\ (1+i)[z_1] + (3-2i)[z_2] + (1+3i)[z_3] = [-7,4] + i[-4,7]. \end{cases}$$

Let $[z_j] = [x_j] + i[y_j]$, where $[x_j] = [\underline{x_j}, \overline{x_j}]$ and $[y_j] = [\underline{y_j}, \overline{y_j}]$ for $j = 1,2,3$. Using suggested method that is the contribution of this article, the original system can be written as the following equivalent system:

$$\begin{cases} (2+i)([\underline{x_1},\overline{x_1}] + i[\underline{y_1},\overline{y_1}]) + (2-i)([\underline{x_2},\overline{x_2}] + i[\underline{y_2},\overline{y_2}]) + (-1+i)([\underline{x_3},\overline{x_3}] + i[\underline{y_3},\overline{y_3}]) \\ = [-9,-1] + i[-5,3], \\ (-2+2i)([\underline{x_1},\overline{x_1}] + i[\underline{y_1},\overline{y_1}]) + (1-3i)([\underline{x_2},\overline{x_2}] + i[\underline{y_2},\overline{y_2}]) + (3+i)([\underline{x_3},\overline{x_3}] + i[\underline{y_3},\overline{y_3}]) \\ = [-1,11] + i[-9,3], \\ (1+i)([\underline{x_1},\overline{x_1}] + i[\underline{y_1},\overline{y_1}]) + (3-2i)([\underline{x_2},\overline{x_2}] + i[\underline{y_2},\overline{y_2}]) + (1+3i)([\underline{x_3},\overline{x_3}] + i[\underline{y_3},\overline{y_3}]) \\ = [-7,4] + i[-4,7]. \end{cases}$$

We can rewrite the equations of the above system in the following form:

$$\begin{cases} \left(2[\underline{x_1},\overline{x_1}] + 2[\underline{x_2},\overline{x_2}] - [\underline{x_3},\overline{x_3}] - [\underline{y_1},\overline{y_1}] + [\underline{y_2},\overline{y_2}] - [\underline{y_3},\overline{y_3}]\right) + \\ i\left(2[\underline{y_1},\overline{y_1}] + 2[\underline{y_2},\overline{y_2}] - [\underline{y_3},[\overline{y_3}] + \underline{x_1},\overline{x_1}] - [\underline{x_2},\overline{x_2}] + [\underline{x_3},\overline{x_3}]\right) = [-9,-1] + i[-5,3], \\ \left(-2[\underline{x_1},\overline{x_1}] + [\underline{x_2},\overline{x_2}] + 3[\underline{x_3},\overline{x_3}] - 2[\underline{y_1},\overline{y_1}] + 3[\underline{y_2},\overline{y_2}] - [\underline{y_3},\overline{y_3}]\right) + \\ i\left(-2[\underline{y_1},\overline{y_1}] + [\underline{y_2},\overline{y_2}] + 3[\underline{y_3},\overline{y_3}] + 2[\underline{x_1},\overline{x_1}] - 3[\underline{x_2},\overline{x_2}] + [\underline{x_3},\overline{x_3}]\right) = [-1,11] + i[-9,3], \\ \left([\underline{x_1},\overline{x_1}] + 3[\underline{x_2},\overline{x_2}] + [\underline{x_3},\overline{x_3}] - [\underline{y_1},\overline{y_1}] + 2[\underline{y_2},\overline{y_2}] - 3[\underline{y_3},\overline{y_3}]\right) + \\ i\left([\underline{y_1},\overline{y_1}] + 3[\underline{y_2},\overline{y_2}] + [\underline{y_3},\overline{y_3}] + [\underline{x_1},\overline{x_1}] - 2[\underline{x_2},\overline{x_2}] + 3[\underline{x_3},\overline{x_3}]\right) = [-7,4] + i[-4,7]. \end{cases}$$

The system mentioned above may be reformulated in the following manner:

$$\begin{cases} [2\underline{x_1} + 2\underline{x_2} - \overline{x_3} - \overline{y_1} + \underline{y_2} - \overline{y_3}, 2\overline{x_1} + 2\overline{x_2} - \underline{x_3} - \underline{y_1} + \overline{y_2} - \underline{y_3}] + \\ i[2\underline{y_1} + 2\underline{y_2} - \overline{y_3} + \underline{x_1} - \overline{x_2} + \underline{x_3}, 2\overline{y_1} + 2\overline{y_2} - \underline{y_3} + \overline{x_1} - \underline{x_2} + \overline{x_3}] = [-9,-1] + i[-5,3], \\ [-2\overline{x_1} + \underline{x_2} + 3\underline{x_3} - 2\overline{y_1} + 3\underline{y_2} - \overline{y_3}, -2\underline{x_1} + \overline{x_2} + 3\overline{x_3} - 2\underline{y_1} + 3\overline{y_2} - \underline{y_3}] + \\ i[-2\overline{y_1} + \underline{y_2} + 3\underline{y_3} + 2\underline{x_1} - 3\overline{x_2} + \underline{x_3}, -2\underline{y_1} + \overline{y_2} + 3\overline{y_3} + 2\overline{x_1} - 3\underline{x_2} + \overline{x_3}] = [-1,11] + i[-9,3], \\ [\underline{x_1} + 3\underline{x_2} + \underline{x_3} - \overline{y_1} + 2\underline{y_2} - 3\overline{y_3}, \overline{x_1} + 3\overline{x_2} + \overline{x_3} - \underline{y_1} + 2\overline{y_2} - 3\underline{y_3}] + \\ i[\underline{y_1} + 3\underline{y_2} + \underline{y_3} + \underline{x_1} - 2\overline{x_2} + 3\underline{x_3}, \overline{y_1} + 3\overline{y_2} + \overline{y_3} + \overline{x_1} - 2\underline{x_2} + 3\overline{x_3}] = [-7,4] + i[-4,7]. \end{cases}$$

As seen in Section 4, the precise polynomial system may be expressed as

$$\begin{cases} 2\underline{x_1} + 2\underline{x_2} - \overline{x_3} - \overline{y_1} + \underline{y_2} - \overline{y_3} = -9, \\ 2\overline{x_1} + 2\overline{x_2} - \underline{x_3} - \underline{y_1} + \overline{y_2} - \underline{y_3} = -1, \\ 2\underline{y_1} + 2\underline{y_2} - \overline{y_3} + \underline{x_1} - \overline{x_2} + \underline{x_3} = -5, \\ 2\overline{y_1} + 2\overline{y_2} - \underline{y_3} + \overline{x_1} - \underline{x_2} + \overline{x_3} = 3, \\ -2\overline{x_1} + \underline{x_2} + 3\underline{x_3} - 2\overline{y_1} + 3\underline{y_2} - \overline{y_3} = -1, \\ -2\underline{x_1} + \overline{x_2} + 3\overline{x_3} - 2\underline{y_1} + 3\overline{y_2} - \underline{y_3} = 11, \\ -2\overline{y_1} + \underline{y_2} + 3\underline{y_3} + 2\underline{x_1} - 3\overline{x_2} + \underline{x_3} = -9, \\ -2\underline{y_1} + \overline{y_2} + 3\overline{y_3} + 2\overline{x_1} - 3\underline{x_2} + \overline{x_3} = 3, \\ \underline{x_1} + 3\underline{x_2} + \underline{x_3} - \overline{y_1} + 2\underline{y_2} - 3\overline{y_3} = -7, \\ \overline{x_1} + 3\overline{x_2} + \overline{x_3} - \underline{y_1} + 2\overline{y_2} - 3\underline{y_3} = 4, \\ \underline{y_1} + 3\underline{y_2} + \underline{y_3} + \underline{x_1} - 2\overline{x_2} + 3\underline{x_3} = -4, \\ \overline{y_1} + 3\overline{y_2} + \overline{y_3} + \overline{x_1} - 2\underline{x_2} + 3\overline{x_3} = 7. \end{cases}$$

Next, the Gröbner basis for the ideal created by the polynomials in the previously given system, using the reverse graded lexicographic order, is as follows:

$$G = \{\underline{x_1} + 2, \overline{x_1} + 1, \underline{x_2}, \overline{x_2} - 1, \underline{x_3} - 1, \overline{x_3} - 2, \underline{y_1}, \overline{y_1} - 1, \underline{y_2} + 1, \overline{y_2}, \underline{y_3}, \overline{y_3} - 1\}.$$

The standard monomial basis $B = \{1\}$ is provided. Applying the monomial basis B, the eigenvalues of the matrices of the full multiplication operator $m_{\underline{x_1}}$, $m_{\overline{x_1}}$, $m_{\underline{x_2}}$, $m_{\overline{x_2}}$, $m_{\underline{x_3}}$, $m_{\overline{x_3}}$, $m_{\underline{y_1}}$, $m_{\overline{y_1}}$, $m_{\underline{y_2}}$, $m_{\overline{y_2}}$, $m_{\underline{y_3}}$ and $m_{\overline{y_3}}$ can be provided equal to $-2, -1, 0, 1, 1, 2, 0, 1, -1, 0, 0$ and 1 respectively. Ultimately, the equations associated with this basis may be resolved, yielding the solution for the aforementioned system as follows:

$$\begin{aligned} z_1 &= [x_1] + i[y_1] \\ &= [\underline{x_1}, \overline{x_1}] + i[\underline{y_1}, \overline{y_1}] \\ &= [-2, -1] + i[0, 1] \end{aligned}$$

and

$$\begin{aligned} z_2 &= [x_2] + i[y_2] \\ &= [\underline{x_2}, \overline{x_2}] + i[\underline{y_2}, \overline{y_2}] \\ &= [0, 1] + i[-1, 0] \end{aligned}$$

and

$$\begin{aligned} z_3 &= [x_3] + i[y_3] \\ &= [\underline{x_3}, \overline{x_3}] + i[\underline{y_3}, \overline{y_3}] \\ &= [1, 2] + i[0, 1]. \end{aligned}$$

The proposed technique is used in the following example.

Example 6.2. The circuit shown in Fig. 1 consists of two loops where the nominal values of the circuit components were shown. The electric components include resistance, inductance and capacitance are considered fixed quantities. However, the source voltages are assumed interval parameters with the nominal (center) values as $V_1 = j10$, $V_2 = 4$ and $V_3 = 1.7 + j$, where $j = \sqrt{-1}$.

In the absence of source tolerance, it is possible to simply calculate loop currents using the loop analysis approach. Considering the nominal values, the loop analysis approach yields the following system of polynomial equations:

$$\begin{cases} j30I_1 + 55(I_1 - I_2) + V_2 - V_1 = 0, \\ -j20I_2 - 55(I_1 - I_2) + V_3 - V_2 = 0 \end{cases}$$

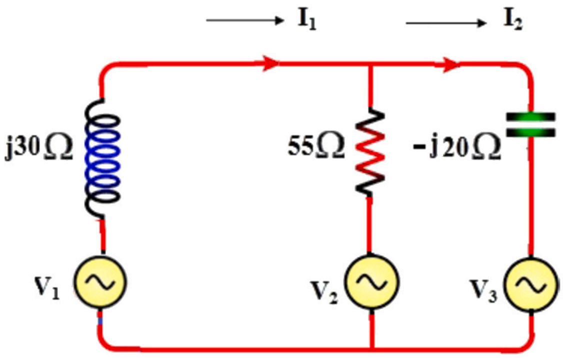

Fig. 1. RLC circuit with interval voltage sources.

where can be represented in matrix form as

$$\begin{pmatrix} 55+j30 & -55 \\ -55 & 55-j20 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} V_1 - V_2 \\ V_2 - V_3 \end{pmatrix} = \begin{pmatrix} -4+j10 \\ 2.3-j \end{pmatrix}$$

Therefore, the loops currents are obtained as: $I_1 = 3.0877 + j2.5329$ and $I_2 = -2.0219 + j1.2918$. Thus, in real condition the value of a voltage source varies within a tolerance interval where for the circuit depicted in Fig. 1 intervals can be assumed as: $V_1 = j[9.9, 10.1]$, $V_2 = 4$ and $V_3 = [1.53, 1.87] + j[0.9, 1.1]$. Corresponding the complex interval linear system for this circuit problem can be represented as

$$\begin{cases} (55+j30)[I_1] - 55[I_2] = -4 + j[9.9, 10.1], \\ -55[I_1] + (55-j20)[I_2] = [2.13, 2.47] + j[-1.1, -0.9]. \end{cases}$$

Let $[I_n] = [x_n] + j[y_n]$, where $[x_n] = [\underline{x}_n, \overline{x}_n]$ and $[y_n] = [\underline{y}_n, \overline{y}_n]$ for $n = 1, 2$. The aforementioned technique can be written as the following equivalent system:

$$\begin{cases} (55+j30)([\underline{x}_1, \overline{x}_1] + j[\underline{y}_1, \overline{y}_1]) - 55([\underline{x}_2, \overline{x}_2] + j[\underline{y}_2, \overline{y}_2]) = -4 + j[9.9, 10.1], \\ \\ -55([\underline{x}_1, \overline{x}_1] + j[\underline{y}_1, \overline{y}_1]) + (55-j20)([\underline{x}_2, \overline{x}_2] + j[\underline{y}_2, \overline{y}_2]) = [2.13, 2.47] + j[-1.1, -0.9]. \end{cases}$$

The aforementioned system may be rephrased in the following style:

$$\begin{cases} \left(55[\underline{x}_1, \overline{x}_1] - 30[\underline{y}_1, \overline{y}_1] - 55[\underline{x}_2, \overline{x}_2]\right) + j\left(55[\underline{y}_1, \overline{y}_1] + 30[\underline{x}_1, \overline{x}_1] - 55[\underline{y}_2, \overline{y}_2]\right) \\ = -4 + j[9.9, 10.1], \\ \\ \left(-55[\underline{x}_1, \overline{x}_1] + 55[\underline{x}_2, \overline{x}_2] + 20[\underline{y}_2, \overline{y}_2]\right) + j\left(-55[\underline{y}_1, \overline{y}_1] + 55[\underline{y}_2, \overline{y}_2] - 20[\underline{x}_2, \overline{x}_2]\right) \\ = [2.13, 2.47] + j[-1.1, -0.9]. \end{cases}$$

As shown in Section 4, the precise polynomial system may be expressed as

$$\begin{cases} 55\underline{x}_1 - 30\overline{y}_1 - 55\overline{x}_2 = -4, \\ \\ 55\overline{x}_1 - 30\underline{y}_1 - 55\underline{x}_2 = -4, \\ \\ 55\underline{y}_1 + 30\underline{x}_1 - 55\overline{y}_2 = 9.9, \\ \\ 55\overline{y}_1 + 30\overline{x}_1 - 55\underline{y}_2 = 10.1, \\ \\ -55\overline{x}_1 + 55\underline{x}_2 + 20\underline{y}_2 = 2.13, \\ \\ -55\underline{x}_1 + 55\overline{x}_2 + 20\overline{y}_2 = 2.47, \\ \\ -55\overline{y}_1 + 55\underline{y}_2 - 20\overline{x}_2 - 1.1, \\ \\ -55\underline{y}_1 + 55\overline{y}_2 - 20\underline{x}_2 = -0.9. \end{cases}$$

We use Maple to calculate a Gröbner basis for the given system using the reverse graded lexicographic order. As a result, we get a basis consisting of monomials. Using the monomial basis the eigenvalues of the matrices of the full multiplication operator $m_{\underline{x_1}}$, $m_{\overline{x_1}}$, $m_{\underline{x_2}}$, $m_{\overline{x_2}}$, $m_{\underline{y_1}}$, $m_{\overline{y_1}}$, $m_{\underline{y_2}}$, $m_{\overline{y_2}}$ can be provided. Finally, the solution of the mentioned system is as follows:

$$\begin{aligned} I_1 &= [x_1] + i[y_1] \\ &= [\underline{x_1}, \overline{x_1}] + i[\underline{y_1}, \overline{y_1}] \\ &= [0.5732004776, 0.5744253449] + i[0.4351451522, 0.4295311769] \end{aligned}$$

and

$$\begin{aligned} I_2 &= [x_2] + i[y_2] \\ &= [\underline{x_2}, \overline{x_2}] + i[\underline{y_2}, \overline{y_2}] \\ &= [0.4098007166, 0.4116380171] + i[0.5592177288, 0.5677999582]. \end{aligned}$$

7. Discussion and results

The general numerical algorithms which are designed for systems of linear and nonlinear equations work also for polynomial systems. The disadvantages of these approaches are as follows:

1. To follow the procedures, you must be aware that answers might be either positive or negative. The approaches will be useless until this is addressed.
2. Identifying an appropriate starting point for the procedures is challenging.
3. These approaches only provide a subset of the approximate answers.
4. Within the techniques, we lack specific criteria or definitive requirements to determine the presence of solutions for the systems.
5. The approaches do not provide information on the quantity of solutions for the systems. Therefore, the techniques are laborious and possess a significant computing burden.
6. If the systems are unsolvable, then the approaches may be deceptive.

Through the implementation of the suggested methodology, it is possible to identify all precise solutions of intricate interval linear systems without resorting to approximation techniques, requiring the selection of an appropriate starting point, or imposing value restrictions. Consequently, we may efficiently solve the system and acquire all the solutions of the system of polynomial equations, in a universal scenario. The suggested approach reliably merges to all solutions of the systems, assuming they exist. Moreover, a condition that is both essential and enough is given for the existence and uniqueness of the solution of complex interval linear systems. With this strategy, we are free from the aforementioned constraints.

8. Conclusion

Actual simulation of real world is the primary challenge in some engineering application fields. Simulation of actual current and voltage is the primary challenge in circuit analysis and design. We attempted to offer a technique based on complex intervals for simulating voltage and current sources, as well as complex interval current and voltage in circuit equations. Complex interval linear systems were developed for this method. In this paper, complicated interval linear systems are examined, and a novel linear algebra-based approach for locating all of their solutions is proposed. Design is a condition for the presence of a solution in complicated interval linear systems. In addition, an algorithm is devised to retrieve all solutions using the eigenvalue approach. In addition, a proportional case is solved using the provided approach to demonstrate its efficiency and efficacy. The given approach can locate all solutions for linear systems with complex intervals. It also determines the existence of a solution for the system. The subject for future investigation entails the development of learning algorithms for the polynomial systems that have more generality, fully complex interval linear systems.

CRediT authorship contribution statement

Maryam Farahmand Nejad: Writing – original draft, Methodology, Conceptualization. **Hamed Farahani:** Writing – review & editing, Writing – original draft, Data curation. **Rahele Nuraei:** Resources, Data curation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

References

[1] L.V. Kolev, Interval Methods for Circuit Analysis, World Scientific, Singapore, 1993.
[2] L.V. Kolev, S.S. Vladov, Linear circuit tolerance analysis via systems of linear interval equations, in: ISYNT89 6th International Symposium on Networks, Systems and Signal Processing, June 28–July 1, Zagreb, 1989, pp. 57–60.
[3] L.V. Kolev, A method for outer interval solution of linear parametric systems, Reliab. Comput. 10 (2004) 227–239.
[4] I. Skalna, Methods for solving systems of linear equations of structure mechanics with interval parameters, Comput. Assist. Mech. Eng. Sci. 10 (3) (2003) 281–293.
[5] E.D. Popova, On the solution of parametrised linear systems, in: Scientific Computing, Validated Numerics, Interval Methods, 2001, pp. 127–138.
[6] W. Kraemer, Computing and Visualizing Solution Sets of Interval Linear Systems, University of Wuppertal, BUW-WRSWT, 42119 Wuppertal, Germany, 2006.
[7] S. Chakraverty, S. Majumdar, A method for the solution of interval system of linear equation, Int. J. Mod. Math. Sci. 4 (2) (2012) 67–70.
[8] H.J. Stetter, Multivariate polynomial equations as matrix eigenproblems, in: Contributions to Numerical Mathematics, in: World Scientific Series in Applicable Analysis, vol. 2, World Scientific, 1993, pp. 355–371.
[9] R.M. Corless, P.M. Gianni, B.M. Trager, S.M. Watt, The singular value decomposition for polynomial systems, in: A.H.M. Levelt (Ed.), Proceedings of the 1995 International Symposium on Symbolic and Algebraic Computation, Montreal, ACM, New York, 1995, pp. 195–207.
[10] R.M. Corless, P.M. Gianni, B.M. Trager, A reordered Schur factorization method for zero-dimensional polynomial systems with multiple roots, in: W. Kuechlin (Ed.), Proceedings of the 1997 International Symposium on Symbolic and Algebraic Computation, Kihei, HI, ACM, New York, 1997, pp. 133–140.
[11] D. Cox, J. Little, D. O'Shea, Ideal, Varieties, and Algorithms: An Introduction to Computational Algebra Geometry and Commutative Algebra, third edition, Springer-Verlag, New York, 2007.
[12] I.Z. Emiris, On the complexity of sparse elimination, J. Complex. 12 (2) (1996) 134–166.
[13] H.M. Möller, H.J. Stetter, Multivariate polynomial equations with multiple zeros solved by matrix eigenproblems, Numer. Math. 70 (3) (1995) 311–329.
[14] B. Mourrain, Computing the isolated roots by matrix methods, J. Symb. Comput. 6 (1998) 715–738.
[15] B. Sturmfels, Solving Systems of Polynomial Equations, CBMS Regional Conference Series in Mathematics, vol. 97, American Mathematical Society, Providence, RI, 2002.
[16] H. Farahani, H. Mishmast Nehi, M. Paripour, Solving fuzzy complex system of linear equations using eigenvalue method, J. Intell. Fuzzy Syst. 31 (2016) 1689–1699.
[17] H. Farahani, M. Paripour, Resolution of fuzzy complex systems of linear equations via Wu's method, Int. J. Ind. Math. 12 (2020) 135–146.
[18] M. Farahmand Nejad, H. Farahani, R. Nuraei, A. Gilani, Gröbner basis approach for solving fuzzy complex system of linear equations, New Math. Nat. Comput. (2023), https://doi.org/10.1142/S1793005724500297.
[19] M. Haowei, U.A.R. Hussein, Z. Haleem Al Qaim, F. Altalbawy, H. AI Sadi, A.A. Fadhil, M.M. Al Taee, S. Hadrawi, R.M. Khalaf, I.H. Jirjees, M. Zarringhalam, M. Hekmatifar, Employing Sisko non-Newtonian model to investigate the thermal behavior of blood flow in a stenosis artery: effects of heat flux, different severities of stenosis, and different radii of the artery, Alex. Eng. J. 68 (2023) 291–300.
[20] M. Samadi, H. Sarkardeh, E. Jabbari, Prediction of the dynamic pressure distribution in hydraulic structures using soft computing methods, Soft Comput. 25 (2021) 3873–3888.
[21] M. Shafagh Loron, M. Samadi, A. Shamsai, Predictive explicit expressions from data-driven models for estimation of scour depth below ski-jump bucket spillways, Water Supply 23 (1) (2023) 304–316.
[22] M.A. Diop, J.L. Polleux, C. Algani, S. Mazer, M. Fattah, M. EL Bekkali, Design electrical model noise and perform nonlinearities of SiGe bipolar phototransistor, Int. J. Innov. Res. Sci. Stud. 6 (4) (2023) 731–740.
[23] S.A. Fayaz, M. Zaman, M.A. But, Numerical and experimental investigation of meteorological data using adaptive linear M5 model tree for the prediction of rainfall, Rev. Comput. Eng. Res. 9 (1) (2022) 1–12.
[24] A. Dreyer, Interval analysis of linear analog circuits, in: 12th GAMM - IMACS International Symposium on Scientific Computing, Computer Arithmetic and Validated Numerics (SCAN 2006), 2006, p. 14.
[25] L. Hedrich, E. Barke, A formal approach to verification of linear analog circuits with parameter tolerances, in: Proceedings Design, Automation and Test in Europe, 1998, pp. 649–654.
[26] T. Rahgooy, H. Sadoghi Yazdi, R. Monsefi, Fuzzy complex system of linear equations applied to circuit analysis, Int. J. Comput. Electr. Eng. 1 (5) (2009) 535–541.
[27] G. Alefeld, G. Mayer, Interval analysis: theory and applications, J. Comput. Appl. Math. 121 (2000) 421–464.
[28] G. Alefeld, Intervallrechnung uber den komplexen Zahlen undeinige Anwendungen, Ph.D. thesis, University Karlsruhe, Karlsruhe, 1968.
[29] R. Boche, Complex interval arithmetic with some applications, Technical report LMSC4-22-66-1, Lockheed Missiles & Space Company, Sunnyvale, 1966.
[30] Y. Candau, T. Raissi, N. Ramdani, L. Ibos, Complex interval arithmetic using polar form, Reliab. Comput. 12 (2006) 1–20.
[31] I. Gargantini, P. Henrici, Circular arithmetic and the determination of polynomial zeros, Numer. Math. 18 (4) (1971) 305–320.
[32] P. Henrici, Circular arithmetic and the determination of polynomial zeros, in: Conference on Applications of Numerical Analysis, Dundee, in: Lecture Notes in Mathematics, vol. 228, 1971, pp. 86–92.
[33] E. Garajova, M. Meciar, Solving and visualizing nonlinear set inversion problems, Reliab. Comput. 22 (2016) 104–115.
[34] B.S. Djanybekov, Interval Householder method for complex linear systems, Reliab. Comput. 12 (2006) 35–43.
[35] R. Nuraei, M. Ghanbari, Algebraic solving of complex interval linear systems by limiting factors, Int. J. Ind. Math. 11 (1) (2019) 11–24.
[36] M. Ghanbari, A new idea for exact solving of the complex interval linear systems, Casp. J. Math. Sci. 19 (1) (2020) 39–55.
[37] T. Becker, V. Weispfenning, Gröbner Bases, Springer-Verlag, New York, 1993.
[38] J.-C. Faugère, A new efficient algorithm for computing Gröbner bases without reduction to zero (F_5), in: T. Mora (Ed.), Proceedings of ISSACS, ACM Press, 2002.
[39] S. Gao, Y. Guan, F. Volny IV, A new incremental algorithm for computing Gröbner bases, in: Proceedings of the 2010 International Symposium on Symbolic and Algebraic Computation, ISSAC'10, ACM, 2010, pp. 13–19.
[40] S. Gao, F. Volny IV, M. Wang, A new framework for computing Gröbner bases, Math. Comput. 85 (2016) 449–465.
[41] M. Ghanbari, An estimation of algebraic solution for a complex interval linear system, Soft Comput. 22 (2017) 2881–2890.