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Effective Zitterbewegung of bosonic Bogoliubov quasi-particle with effective spin-orbital coupling

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Abstract

The conventional Zitterbewegung (ZB) refers to the jittering motion of a particle or wavepacket in space induced by the spin-orbital coupling. Here we show that the analogue of Zitterbewegung of bosonic Bogoliubov quasi-particle manifests itself in a form of periodic distortion of wave packet because of its dynamics is governed by a non-hermitian operator. The effective spin-orbital coupling (SOC) is due to the fact that the two modes (pseudo-spin) in the Bogoliubov de Gennes (BdG) equation are coupled to the spatial momentum. These results are illustrated with spin wave excitations in antiferromagnet and phonon excitations in BEC system.

Keywords: Condensed matter physics, Quantum mechanics

1. Introduction

Bosonic Bogoliubov quasiparticles arise in many different physical systems and have been studied extensively in condensed matter physics for their static properties [1, 2, 3, 4, 5, 6]. As the bosonic Bogoliubov operator is non-Hermitian, we have found in [7] that the dynamics is a continuous Lorentz transformation of a state in complex Minkowski space. In contrast, the usual quantum dynamics is a continuous unitary transformation of a state in Hilbert space. For this reason, we called the dynamics

of bosonic Bogoliubov quasiparticles governed by Bogoliubov de Gennes (BdG) equation the Lorentz quantum mechanics [7].

The spin-orbital coupling (SOC) in unitary quantum mechanics has been extremely studied both theoretically and experimentally [8, 9, 10, 11, 12, 13, 14, 15, 16, 17], where the trembling motion of various quantum wavepackets in space, defined Zitterbewegung (ZB) [18], were found and even detected as coherent phenomena.

In this paper, we study the possible ZB-like phenomenon in bosonic Bogoliubov quasi-particle system, where the SOC means the coupling between two modes (pseudo-spin) in the Bogoliubov de Gennes equation and the quasi-particle's spatial momentum. Because the non-hermitian BdG operator governs the Lorentzian evolution rather than the unitary one [7], ZB of bosonic Bogoliubov quasi-particle manifests itself as the periodic wavepacket's distortion rather than the original wavepacket's jittering motion in space. This complements the conventional ZB theory. In fact, the SOC defined in BdG dynamics appears in all of the bosonic Bogoliubov quasi-particles with spatial degrees of freedom. Here the spin wave excitations in antiferromagnet and phonon excitations in BEC system is employed to show the periodic wavepacket's distortion.

The dynamics of the Lorentz SOC system is governed by a non-Hermitian operator, the analogue of which has been extensively studied in the context of PT-symmetric quantum mechanics. The spectrum of PT-symmetric non-Hermitian operator in certain parameter regime is proved to be real [19], which has found extensively applications in phonon-laser (coupled-resonator) system [20, 21, 22, 23, 24]. In fact, the bosonic Bogoliubov operator studied here stands for a class of anti-PT symmetric Hamiltonian [25]. Specifically, an experimental proposal to observe the Berry phase effect on the dynamics of quasiparticles in a BEC with a vortex has been reported [6]. Thus the proposal of Lorentz SOC not only provides theoretical insights into the dynamical properties of quasiparticles, but also allows practical implementation using the present experimental techniques with ultracold quantum gases.

2. Theory

In this section we study the SOC in bosonic Bogoliubov quasi-particles and the corresponding effective ZB of periodic wavepacket distortion.

In the BdG representation, bosonic Bogoliubov quasi-particle satisfies the BdG equation [1],

$$i\hbar\frac{d}{dt}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \sigma_z H \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}, \tag{1}$$

where σ_z is the Pauli matrix in z direction,

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{2}$$

and H a Hermitian matrix. The dynamics is in the identical form with the unitary case,

$$\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = c_1 e^{-\frac{iE_1 t}{\hbar}} \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} + c_2 e^{-\frac{iE_2 t}{\hbar}} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}. \tag{3}$$

For the BdG equation, it has also been proven that the interval of the two-mode state rather than the modulus is conserved [7]

$$\mathrm{In}((a(t), b(t))^T) = \mathrm{In}((a(0), b(0))^T), \tag{4}$$

where the interval function is defined as $\mathrm{In}((a, b)^T) = |a|^2 - |b|^2$. This is the analog of the Lorentz transformation in $1+1$ dimension Minkowski space (x, t) except that the current space is complex. As the interval defined in Eq. (4) does not change under Lorentz transformation, the complex Minkowski space has three subspaces up to the normalization constant, which are defined by $\mathrm{In}((a, b)^T) = |a|^2 - |b|^2 > 0$, $\mathrm{In}((a, b)^T) = |a|^2 - |b|^2 = 0$ and $\mathrm{In}((a, b)^T) = |a|^2 - |b|^2 < 0$. To set the convention, we call them space-like, light-like, and time-like, respectively. Physically, if the space-like states with $|a|^2 - |b|^2 > 0$ describe Bogoliubov quasiparticles, then the time-like states with $|a|^2 - |b|^2 < 0$ describe the corresponding anti-particles [5].

For the subsequent discussion of ZB, we review the basic properties of eigenstates [7]. Specifically, the eigenstates $|1\rangle$ and $|2\rangle$ satisfy,

$$\sigma_z H |1\rangle = E_1 |1\rangle, \tag{5}$$
$$\sigma_z H |2\rangle = E_2 |2\rangle. \tag{6}$$

Left multiplying the first equation by σ_z followed by projecting on the second, we obtain,

$$E_1 \langle 2|\sigma_z|1\rangle = E_2^* \langle 2|\sigma_z|1\rangle. \tag{7}$$

Thus $\langle 2|\sigma_z|1\rangle = 0$ so long as $E_1 \neq E_2^*$. The basic forms of eigen-spinors are then,

$$|1\rangle = \begin{pmatrix} u \\ v \end{pmatrix}, \quad |2\rangle = \begin{pmatrix} v^* \\ u^* \end{pmatrix}. \tag{8}$$

In bosonic Bogoliubov quasiparticle system [1, 7], it is convenient to normalize the eigen-spinors in Lorentz manner as $|u|^2 - |v|^2 = 1$. It is also apparent that any light-like spinor $(A, B)^T$ with $|A| = |B|$ is a superposition of two eigen-spinors with equal weight,

$$\begin{pmatrix} A \\ \\ B \end{pmatrix} = c(|1\rangle + |2\rangle). \tag{9}$$

When the spinor gets a spatial degree of freedom, it is endowed with a momentum (denoted k). Under SOC, due to the dependence of $H(k)$ on k, the eigenspinors and eigenvalues become k-dependent,

$$|1(k)\rangle = \begin{pmatrix} u(k) \\ \\ v(k) \end{pmatrix}; |2(k)\rangle = \begin{pmatrix} v^*(k) \\ \\ u^*(k) \end{pmatrix}. \tag{10}$$

To enhance coherent effect, we choose the initial spinor to be the superposition of two eigenspinors with equal weight, i.e., a light-like spinor,

$$\begin{pmatrix} a(0) \\ \\ b(0) \end{pmatrix} = \begin{pmatrix} 1 \\ \\ 1 \end{pmatrix} = \frac{1}{N}[e^{i\theta(k)}|1(k)\rangle + e^{-i\theta(k)}|2(k)\rangle], \tag{11}$$

where the angle θ is determined by,

$$ue^{i\theta} + v^*e^{-i\theta} = (ue^{i\theta} + v^*e^{-i\theta})^*, \tag{12}$$

and $N = ue^{i\theta} + v^*e^{-i\theta}$. For most of the practical bosonic Bogoliubov quasi-particle systems, the eigenstate is real,

$$|1(k)\rangle = \begin{pmatrix} |u(k)| \\ \\ |v(k)| \end{pmatrix}; |2(k)\rangle = \begin{pmatrix} |v(k)| \\ \\ |u(k)| \end{pmatrix}. \tag{13}$$

For simplicity and clarity, we here consider only this case. The initial state $(1, 1)^T$ evolves as (neglecting an k-independent overall phase),

$$\begin{pmatrix} a(t) \\ \\ b(t) \end{pmatrix} = \begin{pmatrix} \dfrac{u(k)+v(k)e^{-i\frac{(E_2-E_1)}{\hbar}t}}{u(k)+v(k)} \\ \\ \dfrac{v(k)+u(k)e^{-i\frac{(E_2-E_1)}{\hbar}t}}{u(k)+v(k)} \end{pmatrix}. \tag{14}$$

In the ZB-typical first-order approximation, $E_1 - E_2$ is assumed to be independent of k (the practical example will validate this point), resulting in the periodic evolution with period $T = \frac{2\pi\hbar}{|E_1-E_2|}$. Let's now study the wavepacket directly,

$$|\psi(x,0)\rangle = G(x)\begin{pmatrix} a(0) \\ \\ b(0) \end{pmatrix} = G(x)\begin{pmatrix} 1 \\ \\ 1 \end{pmatrix}. \tag{15}$$
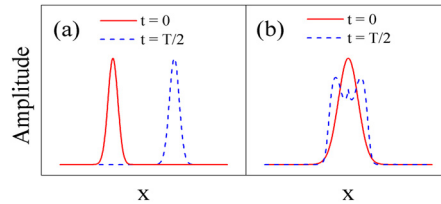
At half period, the state becomes,

Figure 1. Schematic plots of the initial wavepacket and the wavepacket at half of the period when a general ZB is induced by SOC. (a) The conventional unitary ZB is induced by the conventional SOC; (b) generalized ZB of periodic wavepacket distortion arises for bosonic Bogoliubov quasiparticles, resulting from the Lorentz SOC.

$$
\begin{pmatrix} a(\frac{T}{2}) \\ \\ b(\frac{T}{2}) \end{pmatrix} = \begin{pmatrix} \frac{u(k)-v(k)}{u(k)+v(k)} \\ \\ \frac{v(k)-u(k)}{u(k)+v(k)} \end{pmatrix},
\tag{16}
$$

which demonstrates that the wavepacket's overall phase does not depend on k but the modulus does. The k-dependence of $|\frac{u(k)-v(k)}{u(k)+v(k)}|$ is extremely different from that of $(a(0), b(0))^T = (1, 1)^T$. By Fourier analysis, the k-dependence of the overall phase (modulus) determines the wavepacket's position (shape), so a deformation of the wavepacket occurs at half period without a spatial displacement. We define here this generalized oscillation as Lorentz ZB. A comparison between the unitary ZB and Lorentz ZB is made in Figure 1.

It can be readily verified that, during the evolution

$$
|\psi(x,t)\rangle = \begin{pmatrix} \xi(x,t) \\ \\ \zeta(x,t) \end{pmatrix},
\tag{17}
$$

the interval is conserved,

$$
\int dx[|\xi(x,t)|^2 - |\zeta(x,t)|^2] = \int dx[|\xi(x,0)|^2 - |\zeta(x,0)|^2] = 0,
\tag{18}
$$

indicating that the Lorentz SOC system are undergoing an overall Lorentz evolution.

To demonstrate the periodic wavepacket deformation, we first take a toy model for a heuristic discussion,

$$
\sigma_z H = E \begin{pmatrix} -\frac{k^4+1}{k^4-1} & \frac{2k^2}{k^4-1} \\ -\frac{2k^2}{k^4-1} & \frac{k^4+1}{k^4-1} \end{pmatrix}.
\tag{19}
$$

The eigenstates read,

$$
|1(k)\rangle = \frac{1}{\sqrt{1-k^4}} \begin{pmatrix} 1 \\ k^2 \end{pmatrix}; \quad |2(k)\rangle = \frac{1}{\sqrt{1-k^4}} \begin{pmatrix} k^2 \\ 1 \end{pmatrix},
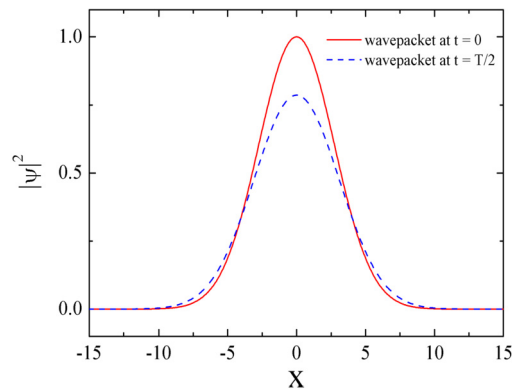\tag{20}
$$

Figure 2. The wavepacket's profiles at $t = 0$ and $t = T/2$ in the toy model (19), with the initial wavepacket assuming the Gaussian form (15). The width of the initial wavepacket is set 5.46 in units of $\hbar/k$.

with the eigenvalues being E and $-E$, respectively. Under the BdG equation (1), an initial wavepacket in the form given in (15) evolves as,

$$|\psi(t)\rangle = g(k) \left[\cos\left(\frac{Et}{\hbar}\right) \begin{pmatrix} 1 \\ 1 \end{pmatrix} + i \sin\left(\frac{Et}{\hbar}\right) \begin{pmatrix} \frac{1-k^2}{1+k^2} \\ \frac{k^2-1}{1+k^2} \end{pmatrix} \right],$$

demonstrating a periodic wavepacket distortion. In Figure 2 the wavepacket's profiles at $t = 0$ and $t = T/2$ (with T being the period of the effective ZB oscillation), i.e., $|\langle \psi(0)|\psi(0)\rangle|^2$ and $|\langle \psi(T/2)|\psi(T/2)\rangle|^2$, are depicted.

3. Example

Spin-wave in one dimensional antiferromagnet

Let's now consider a practical physical system, namely, the spin wave in the one dimensional antiferromagnet, which supports effectively the Bosonic Bogoliubov quasiparticles and Lorentz SOC. For antiferromagnet in sold state physics, the lattice is usually divided into two sublattices, denoted A and B, regarding the positive and negative magnetic moment near zero temperature. In the Heisenberg's description, the overall Hamiltonian reads,

$$H = J \sum_{i,\delta} [S_{ai}^z S_{b,i+\delta}^z + \frac{1}{2}(S_{ai}^+ S_{b,i+\delta}^- + S_{ai}^- S_{b,i+\delta}^+)]$$

$$+ J \sum_{j,\delta} [S_{bj}^z S_{a,j+\delta}^z + \frac{1}{2}(S_{bj}^+ S_{a,j+\delta}^- + S_{bj}^- S_{a,j+\delta}^+)], \tag{21}$$

where $\delta = \pm 1$ stands for the nearest neighbor site, $J > 0$ is the antiferromagnetic exchange integral, $\mathbf{S}_{ai}$ and $\mathbf{S}_{bj}$ are the spin operators in the ith site on A sublattice and jth site on B sublattice, respectively. Without loss of generality, we assume that

at low temperature limit the spin direction in A takes the positive z direction and that in B the $-z$ direction.

After the Holstein-Primakoff transformation [26] and Fourier transformation to momentum space,

$$a_i = N^{-\frac{1}{2}} \sum_k e^{ikR_i} a_k, \quad a_i^\dagger = N^{-\frac{1}{2}} \sum_k e^{-ikR_i} a_k^\dagger,$$
$$b_j = N^{-\frac{1}{2}} \sum_k e^{-ikR_j} b_k, \quad b_j^\dagger = N^{-\frac{1}{2}} \sum_k e^{ikR_j} b_k^\dagger, \tag{22}$$

the Hamiltonian for a spin-S system expressed in creation operators $a_k^\dagger$ (decreasing z component of the spins in A), $b_k^\dagger$ (increasing z component of the spins in B) and annihilation operators b_k (decreasing z component of the spins in B) a_k (increasing z component of the spins in A) assumes (drop a constant),

$$H = 2ZSJ \sum_k (a_k^\dagger a_k + b_k^\dagger b_k + \gamma_k a_k^\dagger b_k^\dagger + \gamma_k b_k a_k) \tag{23}$$

$$= 2ZSJ \sum_k \begin{pmatrix} a_k^\dagger & b_k \end{pmatrix} \begin{pmatrix} 1 & \gamma_k \\ \gamma_k & 1 \end{pmatrix} \begin{pmatrix} a_k \\ b_k^\dagger \end{pmatrix} + \text{Con.},$$

where $Z = 2$ is the coordination number for one dimensional system; $\gamma_k = \frac{1}{Z} \sum_\delta e^{i\mathbf{k}\cdot\delta} = \cos(ka_l)$ is the structure factor of the one dimensional lattice (we set the lattice constant to be one $a_l = 1$ throughout such that the momentum k is in fact in units of $\hbar/a_l$).

Although the ground state and the excited ones predicted by the spin-wave method is not exact [27], the spin-wave method is a reliable approximation at low temperatures, which has become a standard method in a solid state textbook.

Suppose $|E\rangle$ is an overall eigenstate of the system, then a rather arbitrary spinor-like two-mode state (ρ is set as the normalization constant),

$$\begin{pmatrix} u \\ v \end{pmatrix} \equiv \frac{1}{\rho}(ua_k^\dagger + vb_k)|E\rangle \tag{24}$$

will not be stationary. According to definition of creation operator the z-component of the spins in A sublattice in state $a_k^\dagger|E\rangle$ will be lower than that in state $|E\rangle$; on the contrary, the z-component of the spins in B sublattice in state $b_k|E\rangle$ will be lower than that in state $|E\rangle$. However, the states $a_k^\dagger|E\rangle$ and $b_k|E\rangle$ will not be orthogonal to each other. In fact, the state $|E\rangle$ is a superposition of enormous Fock states in the particle number representation $a_k^\dagger a_k$, $b_k^\dagger b_k$, so it is quite natural that the states $a_k^\dagger|E\rangle$ and $b_k|E\rangle$ contain the same Fock states, leading to $\langle E|a_k b_k|E\rangle \neq 0$. For this Bosonic Bogoliubov quasiparticle systems, the representation taken in (24) depicts vividly the "particle" and "hole" on top of an overall eigenstate.

From the overall Hamiltonian (23), the spinor-like state (24) will evolve under the BdG equation,

$$i\hbar \frac{d}{dt} \begin{pmatrix} u(t) \\ v(t) \end{pmatrix} = 2ZSJ\sigma_z \begin{pmatrix} 1 & \gamma_k \\ \gamma_k & 1 \end{pmatrix} \begin{pmatrix} u(t) \\ v(t) \end{pmatrix}, \tag{25}$$

the eigenspinors $|1\rangle = (u,v)^T$ and $|2\rangle = (v,u)^T$ are found to be real in this system,

$$u = \sqrt{\frac{1}{2}\left(\frac{1}{|\sin(k)|} + 1\right)},$$

$$v = \text{sign}(\cos(k))\sqrt{\frac{1}{2}\left(\frac{1}{|\sin(k)|} - 1\right)} \tag{26}$$

with $\text{sign}(\cos(k)) = 1(-1)$ as $\cos(k) > (<)0$.

To enhance the ZB effect, we select a light-like initial state,

$$\langle j|\psi(0)\rangle = G(j)e^{ik_0 j} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \tag{27}$$

where $G(x)$ is a broad Gaussian in real space centered at $j = 0$. The dynamics can be formulated according to (25),

$$\langle k|\psi(t)\rangle = \frac{g(k-k_0)}{u+v} \begin{pmatrix} u \\ v \end{pmatrix} + \frac{g(k-k_0)}{u+v} e^{i\omega t} \begin{pmatrix} v \\ u \end{pmatrix}, \tag{28}$$

where the frequency $\omega = 4ZJS|\sin(k)|/\hbar$, $g(k)$ is the Fourier transformation of $G(j)$.

After some simple algebra we can prove that the spinor will always be light-like,

$$\int dk[|u(k,t)|^2 - |v(k,t)|^2] = 0, \tag{29}$$

proving the evolution is Lorentzian.

To justify the ZB-typical first-order approximation, the frequency ω determined by the difference of eigenenergies must be approximated to a constant, which calls for a narrow wavepacket in k space. For this purpose we take $k_0 = \pi/2$ and employing the first-order approximation to get,

$$\sin\left(\frac{\pi}{2} + \delta k\right) = 1 - \frac{(\delta k)^2}{2!} + \ldots + (-1)^n \frac{(\delta k)^{2n}}{(2n)!} + \ldots$$
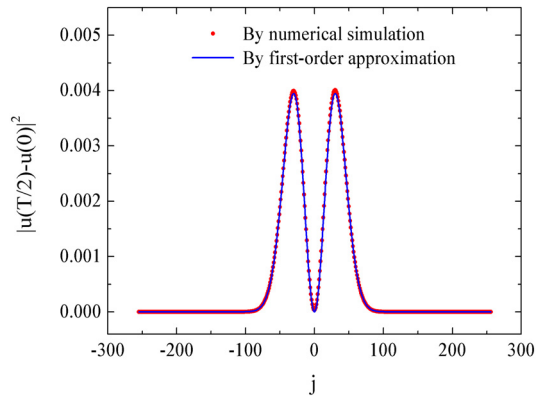
$$\simeq 1 \tag{30}$$

Figure 3. Difference between $u(j, 0)$ and $u(j, T/2)$ in coordinate lattice space associated with the first component of the wavefunction within the representation (24). The analytic result (blue solid line) by first-order approximation almost coincides with the numerical simulation (red dots) without any approximation. In the simulation, the width of the wavepacket is set to be 43 lattice site; the height of the initial Gaussian wavepacket in real space is set to be unity.

with all the orders of δk higher than one being neglected. From Eq. (28), the frequency (period) of the quantum evolution of a narrow wavepacket will be $\omega_0 = 4ZJS/\hbar$ ($T = 2\pi/\omega_0$).

Substituting Eq. (26) into (28) and employing the ZB-typical first-order approximation, the time-evolving state reads (neglecting an overall k-independent phase),

$$|\psi_k(t)\rangle = g(\delta k) \cos(\omega_0 t/2) \begin{pmatrix} 1 \\ 1 \end{pmatrix} - ig(\delta k) \sin(\omega_0 t/2) \begin{pmatrix} 1 + \delta k \\ -(1 + \delta k) \end{pmatrix}, \tag{31}$$

which shows a periodic wavepacket distortion as the modulus depends on time periodically. The real space wavepacket can be obtained by the inverse Fourier transformation of (31).

The comment is in order. With the initial wavepacket prepared appropriately (in the form of (27)), the profile will neither decay fast nor induce higher excitation; instead, it undergoes periodic distortion (at least during the first several period when taking account to the ZB-typical approximation). This constitutes an unexpected coherent dynamic phenomenon. A numerical simulation of the Lorentz ZB, i.e., periodic wavepacket distortion, is given in Figure 3.

It is worthwhile to note that the conventional Hermitian SOC has been found in the higher-spin (rather than spin-1/2) antiferromagnet chain with easy-axis when linearizing the Landau-Lifshitz-Gilbert equation regarding the classical spin vector, where the conventional Hermitian ZB manifesting itself as trembling motion of the spin vectors occurs [17]. Here the Lorentz ZB is found instead in the representation of Bogoliubov quasi-particles.

Phonon excitation in one dimensional BEC system

In this subsection we study the phonon excitation in one dimensional BEC system. Suppose the BEC is composed of interacting bosons and can be described by the following Gross Pitaevskii (GP) equation (without an optical lattice),

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2}\frac{\partial^2}{\partial x^2}\psi + c|\psi|^2\psi, \tag{32}$$

with c characterizing the interacting strength between bosons. For the purpose of ZB, we take the attractive interaction $c < 0$. The state $\psi = \phi_k \exp(ikx)$ (without an optical lattice the Bloch state reduces to a plane-wave and we then take $\phi_k \equiv \phi$) is apparently the nonlinear eigenstate of the GP operator since the nonlinear term in this case contributes a constant.

We then perturb the system around a plane-wave state,

$$\psi(x) = e^{ikx}[\phi(x) + \delta\phi(x)]. \tag{33}$$

Due to the translational symmetry of the system, these perturbations can be decomposed into different modes labelled by q,

$$\delta\phi(x,q) = u(q)e^{iqx} + v^*(q)e^{-iqx}, \tag{34}$$

where q labels wave-number of the phonon excitation. Substitute (33) and (34) into the dynamics (32), we get the BdG equation for the dynamics of the Bogoliubov parameters,

$$i\hbar\frac{d}{dt}\begin{pmatrix} u(q) \\ v(q) \end{pmatrix} = \sigma_z M(q)\begin{pmatrix} u(t) \\ v(t) \end{pmatrix}, \tag{35}$$

where,

$$M(q) = \begin{pmatrix} q^2/2 + kq + c - \mu & c \\ c & q^2/2 - kq + c - \mu \end{pmatrix}, \tag{36}$$

with μ denoting the chemical potential of the nonlinear BEC eigenstate [5]. The eigenvalues of the operator $\sigma_z M(q)$ are calculated as,

$$E_\pm = kq \pm \sqrt{q^4/4 + \mu^2 + q^2 c - q^2\mu - 2c\mu}. \tag{37}$$

To minimize the decaying of ZB effect, we take the center of the phonon wavepacket of q_0 where the derivative of the energy difference $(E_+ - E_-)$ with respect to q vanishes. For this purpose, we take $q_0 = \sqrt{-2c}$ ($q = q_0 + \delta q$). The eigenstates of $\sigma_z M(q)$ are calculated as $|+\rangle = (u, v)^T$ and $|-\rangle = (v, u)^T$, with,

$$u = \frac{1}{B}\left(-\sqrt{q^4/4 + \mu^2 + q^2 c - q^2 \mu - 2c\mu} - q^2/2 - c + \mu\right),$$

$$v = \frac{c}{B}, \tag{38}$$

with B the normalization constant. As in the previous discussion, we take the light-like initial narrow phonon wavepacket in momentum space as $(u(0), v(0))^T = g(\delta q)(1, 1)^T$, with $g(\delta q)$ a narrow Gaussian. To simplify the formulae, we take $\mu = -\sqrt{2}c$. As in the convention of ZB research, we finally derive the dynamics of the wavepacket within the first order approximation with respect to δq,

$$\begin{pmatrix} u(t) \\ v(t) \end{pmatrix} = g(\delta q)\cos(\omega_0 t/2)\begin{pmatrix} 1 \\ 1 \end{pmatrix} - ig(\delta k)\sin(\omega_0 t/2)\begin{pmatrix} -A \\ A \end{pmatrix}, \tag{39}$$

with

$$A = \frac{\sqrt{2}}{2 - \sqrt{2}} + \frac{\sqrt{-2c}}{c}\frac{2(1 - \sqrt{2})}{(2 - \sqrt{2})^2}\delta q, \tag{40}$$

and $\omega_0 = -2c$. Equation (39) shows a periodic wavepacket distortion as the modulus depends on time periodically. The real space wavepacket can be obtained by the inverse Fourier transformation of (39).

4. Conclusion

In summary, the generalized Zitterbewegung oscillation under spin-orbital coupling in Bosonic Bogoliubov quasiparticle systems is studied, where the spin-orbital coupling means that the two modes of the Bogoliubov de Gennes equation is coupled to the spatial momentum. Because the basic evolution for a quasiparticle's state is Lorentzian, Zitterbewegung manifests itself as a wavepacket's periodic distortion, instead of the periodic oscillation in space. The result shall enrich the concept of quantum evolution and spin-orbital coupling. Generalizing the result to spinors beyond two-mode should be of considerable interest.

Declarations

Author contribution statement

Qi Zhang: Conceived and designed the analysis; Analyzed and interpreted the data; Contributed analysis tools or data; Wrote the paper.

Competing interest statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

References

[1] N.N. Bogoliubov, On the theory of superfluidity, J. Phys. (USSR) 11 (1947) 23.

[2] T. Giamarchi, Quantum Physics in One Dimension, Oxford University Press, Oxford, 2004.

[3] C.J. Pethick, H. Smith, Bose–Einstein Condensation in Dilute Gases, Cambridge University Press, Cambridge, 2002.

[4] G.E. Volovik, The Universe in a Helium Droplet, Clarendon Press, Oxford, 2003.

[5] B. Wu, Q. Niu, Superfluidity of Bose–Einstein condensate in an optical lattice: Landau–Zener tunnelling and dynamical instability, New J. Phys. 5 (2003) 104.

[6] C. Zhang, A.M. Dudarev, Q. Niu, Berry phase effects on the dynamics of quasiparticles in a superfluid with a vortex, Phys. Rev. Lett. 97 (2006) 040401.

[7] Qi Zhang, Biao Wu, Lorentz quantum mechanics, New J. Phys. 20 (2018) 013024.

[8] L. Lamata, J. Le'on, T. Schätz, E. Solano, Dirac equation and quantum relativistic effects in a single trapped ion, Phys. Rev. Lett. 98 (2007) 253005.

[9] R. Gerritsma, G. Kirchmair, F. Zähringer, E. Solano, R. Blatt, C.F. Roos, Quantum simulation of the Dirac equation, Nature 463 (2010) 68.

[10] G. Juzeliunas, J. Ruseckas, M. Lindberg, L. Santos, P. Ohberg, Quasirelativistic behavior of cold atoms in light fields, Phys. Rev. A 77 (2008) 011802(R).

[11] J.Y. Vaishnav, C.W. Clark, Observing Zitterbewegung with ultracold atoms, Phys. Rev. Lett. 100 (2008) 153002.

[12] M. Merkl, et al., Atomic Zitterbewegung, Europhys. Lett. 83 (2008) 54002.

[13] Y.-J. Lin, R.L. Compton, K. Jimenez-Garcia, J.V. Porto, I.B. Spielman, Synthetic magnetic fields for ultracold neutral atoms, Nature 462 (2009) 628.

[14] K.S. Novoselov, et al., Electric field effect in atomically thin carbon films, Science 306 (2004) 666.

[15] J. Larson, S. Levin, Effective Abelian and non-Abelian gauge potentials in cavity QED, Phys. Rev. Lett. 103 (2009) 013602.

[16] L.J. LeBlanc, M.C. Beeler, K. Jiménez-García, A.R. Perry, S. Sugawa, R.A. Williams, I.B. Spielman, Direct observation of zitterbewegung in a Bose–Einstein condensate, New J. Phys. 15 (2013) 073011.

[17] Weiwei Wang, Chenjie Gu, Yan Zhou, Hans Fangohr, Magnonic analog of relativistic Zitterbewegung in an antiferromagnetic spin chain, Phys. Rev. B 96 (2017) 024430.

[18] E. Schrödinger, Non powerful motion in the relativistic quantum mechanics, Sitz.ber. Preuss. Akad. Wiss., Phys.-Math. Kl. 24 (1930) 418.

[19] C.M. Bender, S. Boettcher, Real spectra in non-Hermitian Hamiltonians having PT symmetry, Phys. Rev. Lett. 80 (1998) 5243;
C.M. Bender, S. Boettcher, P.N. Meisinger, PT-symmetric quantum mechanics, J. Math. Phys. 40 (1999) 2201.

[20] Zhong-Peng Liu, et al., Metrology with PT-symmetric cavities: enhanced sensitivity near the PT-phase transition, Phys. Rev. Lett. 117 (2016) 110802.

[21] H. Jing, et al., PT-symmetric phonon laser, Phys. Rev. Lett. 113 (2014) 053604.

[22] Xin-You Lü, Hui Jing, Jin-Yong Ma, Ying Wu, PT-symmetry-breaking chaos in optomechanics, Phys. Rev. Lett. 114 (2015) 253601.

[23] H. Jing, et al., Optomechanically-induced transparency in parity-time-symmetric microresonators, Sci. Rep. 5 (2015) 9663.

[24] Jing Zhang, et al., Giant nonlinearity via breaking parity-time symmetry: a route to low-threshold phonon diodes, Phys. Rev. B 92 (2015) 115407.

[25] P. Peng, et al., Anti-parity–time symmetry with flying atoms, Nat. Phys. 12 (2016) 1139.

[26] T. Holstein, H. Primakoff, Field dependence of the intrinsic domain magnetization of a ferromagnet, Phys. Rev. 58 (1940) 1098.

[27] Jacques des Cloizeaux, J.J. Pearson, SpinWave spectrum of the antiferromagnetic linear chain, Phys. Rev. 128 (1962) 2131.