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Research Article

Modeling the Parasitic Filariasis Spread by Mosquito in Periodic Environment

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In this paper a mosquito-borne parasitic infection model in periodic environment is considered. Threshold parameter R_0 is given by linear next infection operator, which determined the dynamic behaviors of system. We obtain that when $R_0 < 1$, the disease-free periodic solution is globally asymptotically stable and when $R_0 > 1$ by Poincaré map we obtain that disease is uniformly persistent. Numerical simulations support the results and sensitivity analysis shows effects of parameters on R_0, which provided references to seek optimal measures to control the transmission of lymphatic filariasis.

1. Introduction

Lymphatic filariasis is a parasitic disease caused by filarial nematode worms and is a mosquito-borne disease that is a leading cause of morbidity worldwide. Lymphatic filariasis affects 120 million humans in tropical and subtropical areas of Asia, Africa, the Western Pacific, and some parts of the Americas [1]. It is estimated that 40 million people are chronically disabled by lymphatic filariasis, making lymphatic filariasis the leading cause of physical disability in the world [2]. There are some clinical manifestations for infective individuals, such as acute fevers, chronic lymphedema, elephantiasis, and hydrocele [3].

W. bancrofti parasites, which account for 90% of the global disease burden, dwell in the lymphatic system, where the adult female worms release microfilariae (mf) into the blood. Mf are ingested by biting mosquitoes as a blood meal of a mosquito, through several developmental stages, that is, first into immature larvae and then L3 larvae. Infective stage larvae L3 actively escape from the mosquito mouthparts entering another human host at the next blood meal through skin [4]. These L3 larvae subsequently develop into worms in humans and the process continues. So in order to remove lymphatic filariasis from the society, not only are the infected persons to be recovered but also the infected vectors are to be killed or removed.

Mathematical models are powerful tools in disease control and may provide a powerful strategic tool for designing and planning control programs against infectious diseases [5]. Since 1960s, simple mathematical models of infection have been in existence for filariasis and provided useful insights into the dynamics of infection and disease in human populations [6–8]. Michael et al. describe the first application of the moment closure equation approach to model the sources and the impact of this heterogeneity for microfilarial population dynamics [9]. Simulation model for lymphatic filariasis transmission and control [10, 11] suggests that the impact of mass treatment depends strongly on the mosquito biting rate and on the assumed coverage, compliance, and efficacy; sensitivity analysis showed that some biological parameters strongly influence the predicted equilibrium pretreatment mf prevalence. References [12–14] take into account the complex interrelationships between the parasite and its human and vector hosts and provide the management decision support framework required for defining optimal intervention strategies and for monitoring and evaluating community-based interventions for controlling or eliminating parasitic diseases. Gambhir and Michael have shown a joint stability analysis of the deterministic filariasis transmission model [15]. All such models have proved to be of great value in guiding and assessing control efforts [16, 17].

Environmental and climatic factors play an important role for the transmission of vector-borne diseases and are researched in many articles [18, 19]. For lymphatic filariasis, proper temperature and humidity are more beneficial for mosquito population to give birth and propagate. For example, in temperate climates and in tropical highlands, temperature restricts vector multiplication and the development of the parasite in the mosquito, while in arid climates precipitation restricts mosquito breeding. Therefore, the transmission of lymphatic filariasis exhibits seasonal behaviors especially in the northern areas [20, 21]. Nonautonomous phenomenon in infectious disease often occurs, and basic reproductive number of periodic systems is described as the spectral radius of the next infection operator [22].

But the dynamics system considers the periodic environment between human and mosquito is little. How to make a comprehensive understanding of the role of periodic environment in the transmission of lymphatic filariasis and how to control the transmission of lymphatic filariasis efficiently are problems that provide motivation for our study. For the limitation of ecology environmental resources such as food and habitat, it is reasonable to adopt logistic growth for mosquito population. Nonautonomous logistic equations have been studied [23–28]. Based on above works and [29–34], we investigate a simple lymphatic filariasis model in periodic environment:

$$\begin{aligned} S_h'(t) &= \Lambda(t) - \frac{\beta_1(t) S_h(t) I_m(t)}{1+\alpha_1(t) S_h(t)} - \mu_1(t) S_h(t) \\ &\quad + \upsilon(t) I_h(t), \\ I_h'(t) &= \frac{\beta_1(t) S_h(t) I_m(t)}{1+\alpha_1(t) S_h(t)} - \mu_1(t) I_h(t) - \upsilon(t) I_h(t), \\ S_m'(t) &= r(t) S_m(t) \left(1 - \frac{S_m(t)}{K(t)}\right) - \beta_2(t) S_m(t) I_h(t), \\ I_m'(t) &= \beta_2(t) S_m(t) I_h(t) - \mu_2(t) I_m(t). \end{aligned} \tag{1}$$

In view of the biological background, system (1) has initial values

$$\begin{aligned} S_h^0(0) &> 0, \\ I_h^0(0) &> 0, \\ S_m^0(0) &> 0, \\ I_m^0(0) &> 0, \end{aligned} \tag{2}$$

where $S_h(t)$ and $I_h(t)$ separately denote the densities of the susceptible and the infective individuals for human population at time t; $S_m(t)$ and $I_m(t)$ represent the densities of the susceptible and the infected individuals for mosquito population at time t, respectively. It is easy to see that $N_h(t) = S_h(t) + I_h(t)$ and $N_m(t) = S_m(t) + I_m(t)$ are size of human population and mosquito population, respectively. $\Lambda(t)$ is the recruitment rates of human host at time t; $\mu_1(t)$ and $\mu_2(t)$ are the death rate of human host and infected mosquito, including the natural death rate and disease-induced death rate; $\beta_1(t)$ and $\beta_2(t)$ denote the contact rate of infected mosquito to humans or infected humans to mosquito; $\alpha_1(t)$ is the force of infection saturation at time t; $\upsilon(t)$ is the recovery rate of infectious human host at time t; $r(t)$ and $K(t)$ are the intrinsic growth rate and the carrying capacity of environment for mosquito population at time t, respectively.

In view of the biological background of system (1), we introduce the following assumptions:

(H_1) All coefficients are continuous, positive ω-periodic functions;

(H_2) $\int_0^\omega r(t)dt > 0$.

The organization of this paper is as follows. In Section 2, some preliminaries are given and compute the basic production number. In Section 3, we will study the globally asymptotical stability of the disease-free periodic solution and the uniform persistence of the model. In Section 4, simulations and sensitive analysis are given to illustrate theoretical results and exhibit different dynamic behaviors.

2. Basic Reproduction Number

Denote

$$\begin{aligned} f^L &= \sup_{t\in[0,\omega)} f(t), \\ f^M &= \inf_{t\in[0,\omega)} f(t), \end{aligned} \tag{3}$$

where $f(t)$ is a continuous ω-periodic function.

Let (R^k, R_+^k) be the standard ordered k-dimensional Euclidean space with a norm $\|\cdot\|$. For $u, v \in R^k$, we denote $u \geq v$ if $u - v \in R_+^k$, $u > v$ if $u - v \in R_+^k \setminus \{0\}$, and $u \gg v$ if $u - v \in \mathrm{Int}(R_+^k)$, respectively.

Let $A(t)$ be a continuous, cooperative, irreducible, and ω-periodic $k \times k$ matrix function; we consider the following linear system:

$$\frac{dx(t)}{dt} = A(t) x(t). \tag{4}$$

Denote $\Phi_A(t)$ be the fundamental solution matrix of (4) and let $\rho(\Phi_A(\omega))$ be the spectral radius of $\Phi_A(\omega)$. Then by the Perron-Frobenius theorem, $\rho(\Phi_A(\omega))$ is the principle eigenvalue of $\Phi_A(\omega)$ in the sense that it is simple and admits an eigenvector $V^* \gg 0$.

Lemma 1 (see [35]). *Let $p = (1/\omega)\ln\rho(\Phi_A(\omega))$, where $A(t)$ is a continuous, cooperative, irreducible, and ω-periodic $k \times k$ matrix function. Then system (4) gives a solution $x(t) = e^{pt}v(t)$, where $v(t)$ is a positive ω-periodic function.*

When system (1) gives disease-free solution, obviously $I_h(t) \equiv 0$ and $I_m(t) \equiv 0$. So we get the following subsystem:

$$S_h'(t) = \Lambda(t) - \mu_1(t) S_h(t), \tag{5}$$

$$S_m'(t) = r(t) S_m(t) \left(1 - \frac{S_m(t)}{K(t)}\right). \tag{6}$$

From Lemma 2.1 of [33] and Lemma 2 of [23] we obtain the following lemma.

Lemma 2. *(i) System (5) has a unique positive ω-periodic solution $S_h^*(t)$ which is globally asymptotically stable. (ii) System (6) has a globally uniformly attractive ω-periodic solution $S_m^*(t)$.*

So, according to Lemma 2, system (1) has a unique disease-free periodic solution $(S_h^*(t), 0, 0, S_m^*(t))$.

In the following, we use the generation operator approach to define the basic reproduction number of (1). We check the assumptions (A1)–(A7) in [22] and denote $x = (I_h(t), I_m(t), S_h(t), S_m(t))^T$ and

$$\mathscr{F}(t,x) = \begin{pmatrix} \dfrac{\beta_1(t) S_h(t) I_m(t)}{1+\alpha_1(t) S_h(t)} \\ \beta_2(t) S_m(t) I_h(t) \\ 0 \\ 0 \end{pmatrix},$$

$$\mathscr{V}^-(t,x) = \begin{pmatrix} \mu_1(t) I_h(t) + \upsilon(t) I_h(t) \\ \mu_2(t) I_m(t) \\ \dfrac{\beta_1(t) S_h(t) I_m(t)}{1+\alpha_1(t) S_h(t)} + \mu_1(t) S_h(t) \\ r(t) \dfrac{S_m^2(t)}{K(t)} + \beta_2(t) S_m(t) I_h(t) \end{pmatrix}, \quad (7)$$

$$\mathscr{V}^+(t,x) = \begin{pmatrix} 0 \\ 0 \\ \Lambda(t) + \upsilon(t) I_h(t) \\ r(t) S_m(t) \end{pmatrix}.$$

So system (1) can be written as the following form:

$$x'(t) = \mathscr{F}(t, x(t)) - \mathscr{V}(t, x(t)) \equiv f(t, x(t)), \quad (8)$$

where $\mathscr{V}(t,x) = \mathscr{V}^-(t,x) - \mathscr{V}^+(t,x)$. From the expressions of $\mathscr{F}(t,x)$ and $\mathscr{V}(t,x)$, it is easy to see that conditions (A1)–(A5) are satisfied. We will check (A6) and (A7).

Obviously, $x^*(t) = (0, 0, S_h^*(t), S_m^*(t))$ is disease-free periodic solution of system (8). We define

$$M(t) = \left(\frac{\partial f_i(t, x^*(t))}{\partial x_j} \right)_{3 \le i,j \le 4}, \quad (9)$$

where $f_i(t, x^*(t))$ and x_i are the ith component of $f(t, x(t))$ and x, respectively. So we can get

$$M(t) = \begin{pmatrix} -\mu_1(t) & 0 \\ 0 & r(t) - \dfrac{2r(t)}{K(t)} S_m^*(t) \end{pmatrix}_{3 \le i,j \le 4}. \quad (10)$$

For $S_m^*(t)$ is the globally uniformly attractively ω-periodic solution of (6),

$$\int_0^\omega r(t) \left[1 - \frac{S_m^*(t)}{K(t)} \right] dt = 0. \quad (11)$$

Hence,

$$\exp\left\{ \int_0^\omega \left[r(t) - \frac{2r(t)}{K(t)} S_m^*(t) \right] dt \right\} = \exp\left\{ -\int_0^\omega \left[\frac{r(t)}{K(t)} S_m^*(t) \right] dt \right\} < 1. \quad (12)$$

It is easy to see that $\rho(\Phi_M(\omega)) < 1$, and condition (A6) holds.

Further, we define

$$\mathbf{F}(\mathbf{t}) = \left(\frac{\partial \mathscr{F}_i(t, x^*(t))}{\partial x_j} \right)_{1 \le i,j \le 2}, \quad \mathbf{V}(\mathbf{t}) = \left(\frac{\partial \mathscr{V}_i(t, x^*(t))}{\partial x_j} \right)_{1 \le i,j \le 2}. \quad (13)$$

$\mathscr{F}_i(t, x^*(t))$ and $\mathscr{V}_i(t, x^*(t))$ are the ith component of $\mathscr{F}(t, x^*(t))$ and $\mathscr{V}(t, x^*(t))$. So we obtain that

$$\mathbf{F}(\mathbf{t}) = \begin{pmatrix} 0 & \dfrac{\beta_1(t) S_h^*(t)}{1+\alpha_1(t) S_h^*(t)} \\ \beta_2(t) S_m^*(t) & 0 \end{pmatrix}, \quad \mathbf{V}(\mathbf{t}) = \begin{pmatrix} \mu_1(t) + \upsilon(t) & 0 \\ 0 & \mu_2(t) \end{pmatrix}. \quad (14)$$

Obviously $\rho(\Phi_{-V}(\omega)) < 1$; thus condition (A7) holds.

Let $Y(t,s)$ be 2×2 matrix solution of the following initial value problem:

$$\frac{dY(t,s)}{dt} = -V(t) Y(t,s) \quad \forall t \ge s;\ Y(s,s) = I. \quad (15)$$

I is identity matrix. Let C_ω be the ordered Banach space of all ω-periodic functions from $R \to R^2$, which is equipped with maximum norm $\|\cdot\|_\infty$ and the positive cone $C_\omega^+ = \{\phi \in C_\omega : \phi(t) \ge 0,\ \forall t \in R\}$. By the approach in [22], we consider the following linear operator $L : C_\omega \to C_\omega$. Suppose that $\phi(s) \in C_\omega$ is the initial distribution of infectious individuals in this periodic environment. $F(s)\phi(s)$ is the distribution of new infections produced by the infected individuals who were introduced at time s, and $Y(t,s)F(s)\phi(s)$ represents the distributions of those infected individuals who were newly infected at time s and remain in the infected compartment at time t. Then

$$\psi(t) = \int_{-\infty}^0 Y(t,s) F(s) \phi(s)\, ds = \int_0^{+\infty} Y(t, t-a) F(t-a) \phi(t-a)\, da \quad (16)$$

denotes the distribution of accumulative new infections at time t produced by all those infected individuals $\phi(s)$ introduced at previous time to t.

$$(L\phi)(t) = \int_0^{+\infty} Y(t, t-a) F(t-a) \phi(t-a)\, da, \quad \forall t \in R,\ \phi \in C_\omega. \quad (17)$$

As in [22], L is the next infection operator, and the basic reproduction number of system (1) is given by

$$R_0 = \rho(L), \tag{18}$$

where $\rho(L)$ is the radius of L. Next we show that R_0 serves as a threshold parameter for the local stability of the disease-free periodic solution.

Theorem 3 (see Wang and Zhao [22], Theorem 2.2). *Assume that (A1)–(A7) hold; then the following statements are valid:*

(i) $R_0 = 1$ *if and only if* $\rho(\Phi_{F-V}(\omega)) = 1$;

(ii) $R_0 > 1$ *if and only if* $\rho(\Phi_{F-V}(\omega)) > 1$;

(iii) $R_0 < 1$ *if and only if* $\rho(\Phi_{F-V}(\omega)) < 1$.

So the disease-free periodic solution $(S_h^*(t), 0, 0, S_m^*(t))$ is asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

3. Global Stability of Disease-Free Periodic Solution

Denote

$$\begin{aligned}\Omega = \Big\{ &(S_h, I_h, S_m, I_m) : S_h > 0,\ I_h \geq 0,\ S_m \geq 0,\ I_m \\ &\geq 0,\ 0 < S_h + I_h \leq \frac{\Lambda^L}{\mu_1^M} < +\infty,\ 0 \leq S_m + I_m \\ &\leq M_m^* \Delta < +\infty \Big\}.\end{aligned} \tag{19}$$

Ω is a positively invariant set with respect to system (1) and a global attractor of all positive solutions of system (1).

$$N_h'(t) = \Lambda(t) - \mu_1(t) N_h(t) \leq \Lambda^L - \mu_1^M N_h(t), \tag{20}$$

where $\Lambda^L = \sup_{t>0}\Lambda(t)$ and $\mu_1^M = \inf_{t>0}\mu_1(t)$. So it is easy to obtain $N_h(t) \leq \Lambda^L/\mu_1^M$.

$$\begin{aligned}N_m'(t) &= r(t) S_m(t) \left(1 - \frac{S_m(t)}{K(t)}\right) - \mu_2(t) I_m(t) \\ &\leq \left(r(t) + \mu_2(t)\right) S_m(t) - \mu_2(t) N_m(t) \\ &\leq M_m - \mu_2(t) N_m(t),\end{aligned} \tag{21}$$

where $M_m = \sup_{t\in[0,\omega)}(r(t) + \mu_2(t))S_m(t)$.

From the third equation of (1), for all $t \geq 0$ we have

$$S_m'(t) \leq r(t) S_m(t) \left(1 - \frac{S_m(t)}{K(t)}\right); \tag{22}$$

by the comparison principle and Lemma 2, we obtain

$$\lim_{t\to\infty} \sup S_m(t) \leq \lim_{t\to\infty} \sup S_m^*(t) \leq M_m^*, \tag{23}$$

where $S_m^*(t)$ is the globally uniformly attractively positive ω-periodic solution and $M_m^* = \max_{t\in[0,\omega]} S_m^*(t)$. So, for any small ϵ existing a t_0, for all $t \geq t_0$ we have

$$S_m(t) \leq S_m^*(t) + \varepsilon \leq M_m^* + \epsilon. \tag{24}$$

So we obtain

$$\begin{aligned}N_m'(t) \leq &\sup_{t\geq 0} \left(r(t) + \mu_2(t)\right) \left(M_m^* + \epsilon\right) \\ &- \mu_2(t) N_m(t),\end{aligned} \tag{25}$$

and $\lim_{t\to\infty} \sup N_m(t) \leq (M_m^* + \epsilon)\Delta$, where $\Delta = \sup_{t>0}(r(t) + \mu_2(t))/\inf_{t>0}\mu_2(t)$. For ϵ small enough, $N_m(t) \leq M_m^*\Delta$.

Theorem 4. *If* $R_0 < 1$, *the disease-free periodic solution* $(S_h^*(t), 0, S_m^*(t), 0)$ *is globally asymptotically stable. And if* $R_0 > 1$, *it is unstable.*

Proof. By Theorem 3 we obtain that if $R_0 < 1$, $(S_h^*(t), 0, S_m^*(t), 0)$ is locally stable. Next we prove that when $R_0 < 1$ the disease-free solution $(S_h^*(t), 0, S_m^*(t), 0)$ has global attractivity.

When $R_0 < 1$ and by (iii) of Theorem 3, we have $\rho(\Phi_{F-V}(\omega)) < 1$. So there exists a small enough constant $\varepsilon_1 > 0$ such that $\rho(\Phi_{F-V+\varepsilon_1 N}(\omega)) < 1$, where

$$N(t) = \begin{pmatrix} 0 & \dfrac{\beta_1(t)}{1 + \alpha_1(t)\left(S_h^*(t) + \varepsilon_1\right)} \\ \beta_2(t) & 0 \end{pmatrix}. \tag{26}$$

From Lemma 2 and nonnegativity of the solutions, for any $\varepsilon_1 > 0$ there exists $t_1 > 0$ such that $S_h(t) \leq S_h^*(t) + \varepsilon_1$ and $S_m(t) \leq S_m^*(t) + \varepsilon_1$, so for all $t > t_1$ we have

$$\begin{aligned}I_h'(t) \leq &\frac{\beta_1(t)\left(S_h^*(t) + \varepsilon_1\right) I_m(t)}{1 + \alpha_1(t)\left(S_h^*(t) + \varepsilon_1\right)} - \mu_1(t) I_h(t) \\ &- \upsilon(t) I_h(t), \\ I_m'(t) \leq &\beta_2(t)\left(S_m^*(t) + \varepsilon_1\right) I_h(t) - \mu_2(t) I_m(t).\end{aligned} \tag{27}$$

Considering the auxiliary system

$$\begin{aligned}\widetilde{I_h'(t)} = &\frac{\beta_1(t)\left(S_h^*(t) + \varepsilon_1\right) \widetilde{I_m'(t)}}{1 + \alpha_1(t)\left(S_h^*(t) + \varepsilon_1\right)} - \mu_1(t) \widetilde{I_h'(t)} \\ &- \upsilon(t) \widetilde{I_h'(t)}, \\ \widetilde{I_m'(t)} = &\beta_2(t)\left(S_m^*(t) + \varepsilon_1\right) \widetilde{I_h'(t)} - \mu_2(t) \widetilde{I_m'(t)}.\end{aligned} \tag{28}$$

From Lemma 1, it follows that there exists a positive ω-periodic solution $v_1(t)$ such that $J(t) \leq e^{pt} v_1(t)$, where $J(t) = (I_h(t), I_m(t))^T$ and $p = (1/\omega)\ln\rho(\Phi_{F-V+\varepsilon_1 N}(\omega)) < 0$. Then $\lim_{t\to\infty} J(t) = 0$; that is, $\lim_{t\to\infty} I_h(t) = 0$ and $\lim_{t\to\infty} I_m(t) = 0$.

Moreover, from the equations of $S_h(t), S_m(t)$, we get

$$\begin{aligned}\lim_{t\to\infty} S_h(t) &= S_h^*(t), \\ \lim_{t\to\infty} S_m(t) &= S_m^*(t).\end{aligned} \tag{29}$$

Hence, disease-free periodic solution of system (1) is globally attractive. This completes the proof. □

Define

$$X = \{(S_h, I_h, S_m, I_m) : S_h > 0,\ I_h \geq 0,\ S_m \geq 0,\ I_m \geq 0\}, \quad (30)$$
$$X_0 = \{(S_h, I_h, S_m, I_m) \in X : I_h > 0,\ I_m > 0\}.$$

We have

$$\partial X_0 = X \setminus X_0 = \{(S_h, I_h, S_m, I_m) \in X : I_h I_m = 0\}. \quad (31)$$

From system (1), it is easy to see that X and X_0 are positively invariant, and ∂X_0 is also a relatively closed set in X.

Let $P : X \to X$ be the Poincaré map associated with system (1), satisfying

$$P(x^0) = u(\omega, x^0), \quad \forall x^0 \in X; \quad (32)$$

$u(t, x^0)$ is the unique solution of system (1) satisfying initial condition $u(0, x^0) = x^0$. P is compact for the continuity of solutions of system (1) with respect to initial value, and P is point dissipative on X.

We further define

$$M_\partial = \{(S_h^0, I_h^0, S_m^0, I_m^0) \in \partial X_0 : P^m(S_h^0, I_h^0, S_m^0, I_m^0) \in \partial X_0\ \forall m > 0\}, \quad (33)$$

where $P^m = P(P^{m-1})$ for all $m > 1$ and $P^1 = P$. Now, prove

$$M_\partial = \{(S_h^0, 0, S_m^0, 0) : S_h^0 > 0,\ S_m^0 \geq 0\}. \quad (34)$$

Obviously $\{(S_h, 0, S_m, 0) : S_h > 0,\ S_m \geq 0\} \subseteq M_\partial$.

If $M_\partial \setminus \{(S_h, 0, S_m, 0) : S_h > 0,\ S_m \geq 0\} \neq \emptyset$, then there exists at least a point $(S_h^0, I_h^0, S_m^0, I_m^0) \in M_\partial$ satisfying $I_h^0 > 0$ or $I_m^0 > 0$. We consider two possible cases.

If $I_h^0 = 0$ and $I_m^0 > 0$, then it is clear that from system (1) $I_m(t) \geq 0$ for any $t > 0$. From the second equation of system (1) and $S_h > 0$, we obtain

$$I_h(t) = I_h^0 e^{-\int_0^t [\mu_1(s)+\upsilon(s)]ds} + \int_0^t \frac{\beta_1(s) S_h(s) I_m(s)}{1 + \alpha_1(s) S_h(s)} e^{\int_s^t [\mu_1(\tau)+\upsilon(\tau)]d\tau} ds > 0, \quad (35)$$

for all $t > 0$.

If $I_m^0 = 0$ and $I_h^0 > 0$, then $I_h(t) = I_h^0 e^{-\int_0^t [\mu_1(\tau)+\upsilon(\tau)]d\tau} > 0$. From the third equation of system (1) and $S_m > 0$, we obtain

$$I_m(t) = I_m^0 e^{-\int_0^t \mu_2(s)ds} + \int_0^t \beta_2(s) S_m(s) I_h(s) e^{\int_s^t \mu_2(\tau)d\tau} ds > 0, \quad (36)$$

for all $t > 0$. Hence, for any case, it follows that $(S_h(t), I_h(t), S_m(t), I_m(t)) \notin \partial X_0$, so $(S_h^0, I_h^0, S_m^0, I_m^0) \notin M_\partial$. This leads to a contradiction; there exists one fixed point $E_0 = (S_h^*(t), 0, S_m^*(t), 0)$ of P in M_∂.

In the following, we will discuss the uniform persistence of the disease, and R_0 serves as a threshold parameter for the extinction and the uniform persistence of the disease.

Theorem 5. *If $R_0 > 1$, then system (1) is uniformly persistent. There exists a positive constant ε, such that for all initial conditions (1) satisfies*

$$\liminf_{t\to\infty} I_h(t) \geq \varepsilon, \quad \liminf_{t\to\infty} I_m(t) \geq \varepsilon. \quad (37)$$

When $R_0 > 1$, system (1) admits at least one positive periodic solution.

Proof. From Theorem 3, if $R_0 > 1$ then we obtain $\rho(\Phi_{F-V}(\omega)) > 1$. For an arbitrary small constant $\eta > 0$, that $\rho(\Phi_{F-V-\eta N}(\omega)) > 1$, $N(t)$ is the same as in Theorem 3. From assumption (H_2), we obtain any small enough $\varepsilon > 0$, $\int_0^\omega [r(t) - \alpha(t)\varepsilon]dt > 0$. Consider perturbed equations

$$S'_{\varepsilon h}(t) = \Lambda(t) - \frac{\varepsilon\beta_1(t) S_{\varepsilon h}(t)}{1 + \alpha_1(t) S_{\varepsilon h}(t)} - \mu_1(t) S_{\varepsilon h}(t), \quad (38)$$

$$S'_{\varepsilon m}(t) = r(t) S_{\varepsilon m}(t)\left(1 - \frac{S_{\varepsilon m}(t)}{K(t)}\right) - \varepsilon\beta_2(t) S_{\varepsilon m}(t). \quad (39)$$

Using Lemma 2 in [25] and Lemma 1 of [27], we obtain (38) and (39) that admit globally uniformly attractive positive ω-periodic solutions $S^*_{\varepsilon h}(t)$ and $S^*_{\varepsilon m}(t)$. For the continuity of solutions with respect to ε, and for $\eta > 0$ there exists $\varepsilon_1 > 0$ for all $t \in [0, \omega]$; thus we have

$$S^*_{\varepsilon_1 m}(t) > S^*_m(t) - \eta, \quad S^*_{\varepsilon_1 h}(t) > S^*_h(t) - \eta. \quad (40)$$

Denote $x^0 = (S_h^0, I_h^0, S_m^0, I_m^0) \in X_0$, according to the continuity of the solution with respect to the initial condition; there exists δ for given ε_1, for all $x^0 \in X_0$ with $\|x^0 - E_0\| < \delta$; it follows $\|u(t, x^0) - u(t, E_0)\| < \varepsilon_1$ for all $t \in [0, \omega]$.

Following, we prove

$$\limsup_{m\to\infty} d(P^m(x^0), E_0) \geq \delta. \quad (41)$$

We suppose the conclusion is not true; then following inequality holds:

$$\limsup_{m\to\infty} d(P^m(x^0), E_0) < \delta, \quad (42)$$

for some $x^0 \in X_0$. Without loss of generality, we can assume that

$$d(P^m(x^0), E_0) < \delta \quad \forall m \geq 0. \quad (43)$$

So we obtain

$$\|u(t, P^m(x^0)) - u(t, E_0)\| < \varepsilon_1 \quad \forall m \geq 0,\ t \in [0, \omega]. \quad (44)$$

For any $t \geq 0$, $t = m\omega + t'$, where $t' \in [0, \omega]$ and $m = [t/\omega]$ is the greatest integer less than or equal to t/ω, so we have

$$\|u(t, x^0) - u(t, E_0)\| = \|u(t', P^m(x^0)) - u(t', E_0)\| < \varepsilon, \quad \forall t \geq 0. \quad (45)$$

Hence, it follows that $0 \le I_h(t) \le \varepsilon_1$ and $0 \le I_m(t) \le \varepsilon_1$ for all $t \ge 0$. Then from the first and third equations of (1),

$$S_h'(t) \ge \Lambda(t) - \frac{\varepsilon_1 \beta_1(t) S_h(t)}{1 + \alpha_1(t) S_h(t)} - \mu_1(t) S_h(t),$$
$$S_m'(t) = r(t) S_m(t) \left(1 - \frac{S_m(t)}{K(t)}\right) - \varepsilon_1 \beta_2(t) S_m(t). \quad (46)$$

By the comparison principle, we obtain for any $t \ge 0$

$$S_h(t) \ge S_{\varepsilon_1 h}(t),$$
$$S_m(t) \ge S_{\varepsilon_1 m}(t). \quad (47)$$

Consider (38); there exists $t_1 > 0$; for all $t > t_1$ we have

$$S_{\varepsilon_1 m}(t) > S^*_{\varepsilon_1 m}(t) - \eta,$$
$$S_{\varepsilon_1 h}(t) > S^*_{\varepsilon_1 h}(t) - \eta. \quad (48)$$

By (38) and (48) we obtain

$$S_m(t) > S^*_m(t) - \eta,$$
$$S_h(t) > S^*_h(t) - \eta. \quad (49)$$

Then for all $t > t_1$ we have

$$I_h(t) \ge \frac{\beta_1(t)\left(S^*_h(t) - \eta\right) I_m(t)}{1 + \alpha_1(t)\left(S^*_h(t) - \eta\right)} - \mu_1(t) I_h(t) - \upsilon(t) I_h(t),$$
$$I_m(t) \ge \beta_2(t)\left(S^*_m(t) - \eta\right) I_h(t) - \mu_2(t) I_m(t). \quad (50)$$

Consider the following auxiliary system:

$$\overline{I_h(t)} = \frac{\beta_1(t)\left(S^*_h(t) - \eta\right)\overline{I_m(t)}}{1 + \alpha_1(t)\left(S^*_h(t) - \eta\right)} - \mu_1(t)\overline{I_h(t)} - \upsilon(t)\overline{I_h(t)},$$
$$\overline{I_m(t)} = \beta_2(t)\left(S^*_m(t) - \eta\right)\overline{I_h(t)} - \mu_2(t)\overline{I_m(t)}. \quad (51)$$

From Lemma 1, it follows that there exists a positive ω-periodic function $v_2(t)$ such that (51) has a solution $J(t) = v_2(t)e^{p_1 t}$, where $p_1 = (1/\omega)\ln(\rho(\Phi_{F-V-\eta N}(\omega)))$. For $\rho(\Phi_{F-V-\eta N}(\omega)) > 1$,

$$\lim_{t\to\infty} I_h(t) = +\infty,$$
$$\lim_{t\to\infty} I_m(t) = +\infty. \quad (52)$$

This leads to a contradiction. □

That is to say, $M_\partial \setminus \{(S_h, 0, S_m, 0) : S_h > 0, S_m \ge 0\} = \emptyset$ and $\{M_1\}$ is globally attractive in M_∂, and all orbit in M_∂ converges to $\{M_1\}$. By [22], we obtain that P is weakly uniformly persistent with respect to $(X_0, \partial X_0)$. All solutions are uniformly persistent with respect to $(X_0, \partial X_0)$; thus we have $\lim_{t\to\infty} I_h(t) \ge \varepsilon$, $\lim_{t\to\infty} I_m(t) \ge \varepsilon$.

4. Sensitivity Analysis and Prevention Strategy

We conducted numerical simulation to this model and computed the reproductive numbers R_0. It was confirmed that using the basic reproduction number of the time-averaged autonomous systems of a periodic epidemic model overestimates or underestimates infection risks in many other cases. Bacaer and Guernaoui give methods to compute R_0, such as method of discretization of the integral eigenvalue [36] and Fourier series method for general periodic case and sinusoidal case and application of Floquet Theory method [37]. In [22] Wang and Zhao propose that in order to compute R_0 we only need to compute the spectrum of evolution operator of the following system (53):

$$\frac{dw}{dt} = \left[-V(t) + \frac{F(t)}{\lambda}\right] w, \quad w \in R^n, \ \lambda \in (0, \infty); \quad (53)$$

here system (53) is ω-periodic equation, and $W(t, s, \lambda)$ is the evolution operator of system (53) with $t \ge s$, $s \in R$. By Perron-Frobenius theorem $\rho(W(\omega, 0, \lambda))$ is an eigenvalue of $W(t, 0, \lambda)$, $t \ge 0$. Next, using Theorem 2.1 in [22] to compute R_0 numerically, R_0 serves as threshold parameter in periodic circumstances.

Firstly, by the means of the software Matlab we compute R_0. We choose parameters $\Lambda(t) = 0.6 + 0.4\sin(2\pi t/12)$, $\mu_1(t) = 0.5 + 0.1\sin(2\pi t/12)$, $\mu_2(t) = 0.8 + 0.1\sin(2\pi t/12)$, $\beta_1(t) = 0.6 + 0.1\sin(2\pi t/12)$, $\beta_2(t) = 0.7 + 0.1\sin(2\pi t/12)$, $\alpha_1(t) = 0.2 + 0.1\sin(2\pi t/12)$, $\upsilon(t) = 0.02 + 0.03\sin(2\pi t/12)$, $r(t) = 0.5 + 0.4\sin(2\pi t/12)$, $K(t) = 0.9 + 0.3\sin(2\pi t/12)$. By numerical calculations, we obtain $R_0 = 0.9243 < 1$; then the disease will be extinct; see Figure 1(a). If we choose $\beta_1(t) = 0.9 + 0.1\sin(2\pi t/12)$, $\beta_2(t) = 1.2 + 0.1\sin(2\pi t/12)$, then $R_0 = 1.4662 > 1$; the disease is permanent; see Figure 1(b). The evolution trajectory in spaces (S_h, I_h) and (S_m, I_m) are in Figures 2(a) and 2(b), respectively.

In order to perform sensitivity analysis of parameters $\beta_1(t)$, $\beta_2(t)$, $K(t)$, and $\alpha_1(t)$, we fix all parameters as in Figure 1, except that we choose the composite functions as follows:

$$\beta_1(t) = \beta_{01} + 0.1\sin\left(\frac{2\pi t}{12}\right),$$
$$\beta_2(t) = \beta_{02} + 0.1\sin\left(\frac{2\pi t}{12}\right),$$
$$K(t) = k_0 + 0.3\sin\left(\frac{2\pi t}{12}\right),$$
$$\alpha_1(t) = \alpha_0 + 0.1\sin\left(\frac{2\pi t}{12}\right), \quad (54)$$

where $\beta_{01} = (1/12)\int_0^{12}\beta_1(t)dt$, $\beta_{02} = (1/12)\int_0^{12}\beta_2(t)dt$, $k_0 = (1/12)\int_0^{12}K(t)dt$, and $\alpha_0 = (1/12)\int_0^{12}\alpha_1(t)dt$.

We first fix other parameters and detect the effect of parameters of k_0 and α_0 on R_0. From Figure 3(a), we see that with the increase of α_0, R_0 decreases, and the gradient also decreases, so this strengthens the psychological hint of susceptible human individuals to be benefit for the extinction of the disease. In Figure 3(b), with the increasing of k_0 the

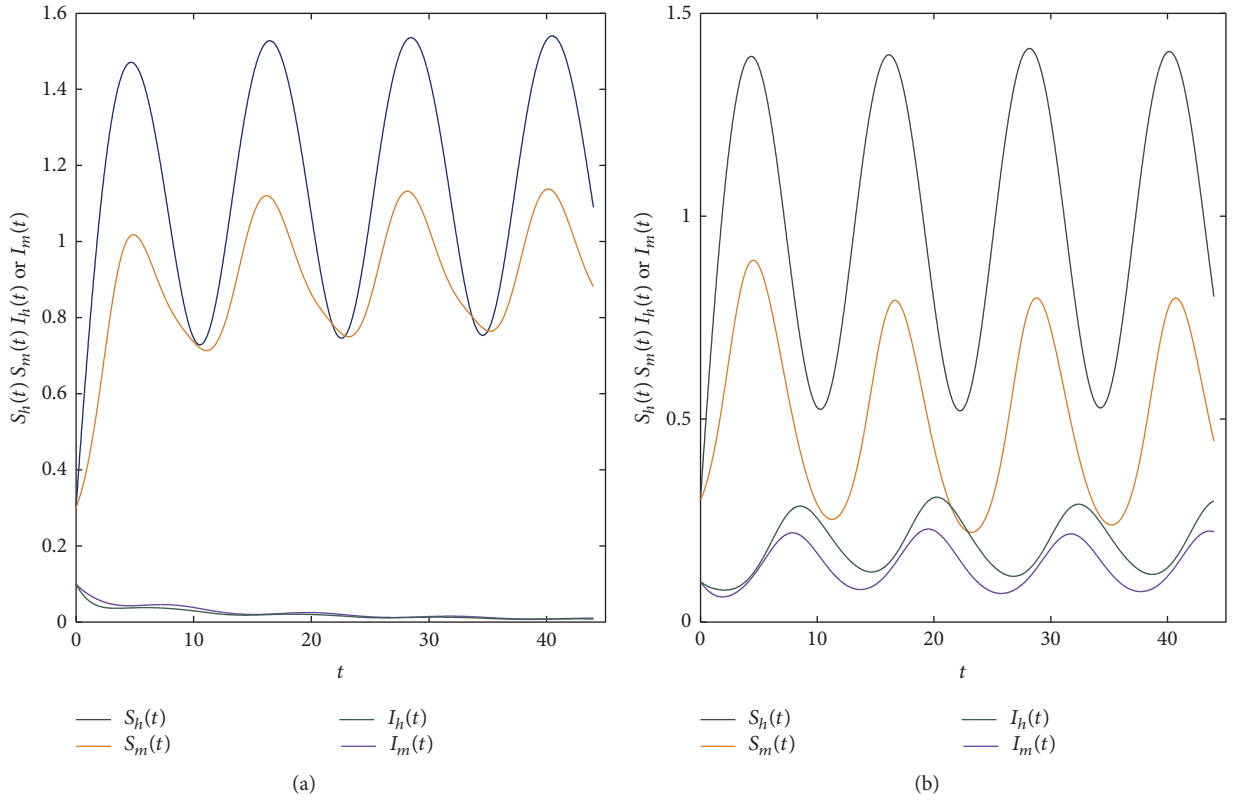

FIGURE 1: Plot the evolution tendency of four populations. (a) Fixed parameters $\beta_1(t) = 0.6 + 0.1\sin(2\pi t/12)$, $\beta_2(t) = 0.7 + 0.1\sin(2\pi t/12)$; then $R_0 = 0.9243 < 1$; (b) Parameters $\beta_1(t) = 0.9 + 0.1\sin(2\pi t/12)$, $\beta_2(t) = 1.2 + 0.1\sin(2\pi t/12)$; then $R_0 = 1.4662 > 1$.

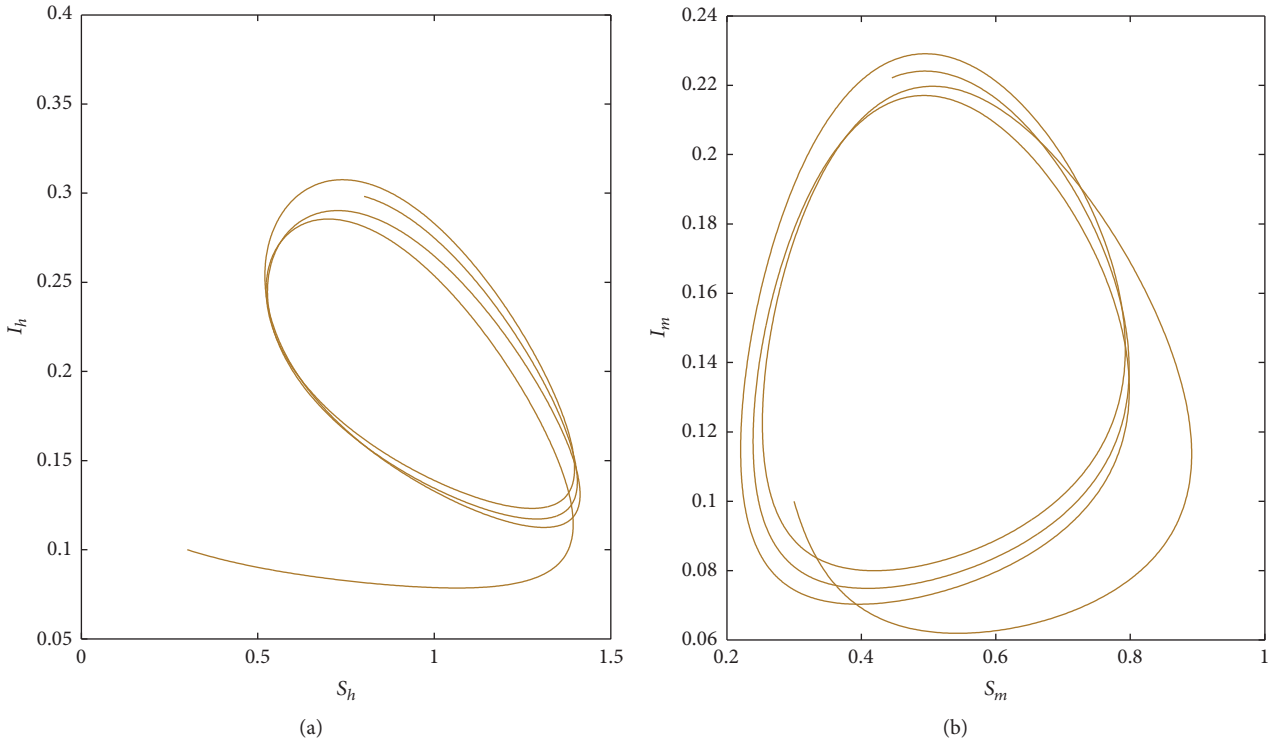

FIGURE 2: When $R_0 = 1.4662$, we graph the trajectory of two populations in spaces (S_h, I_h) and (S_m, I_m), respectively.

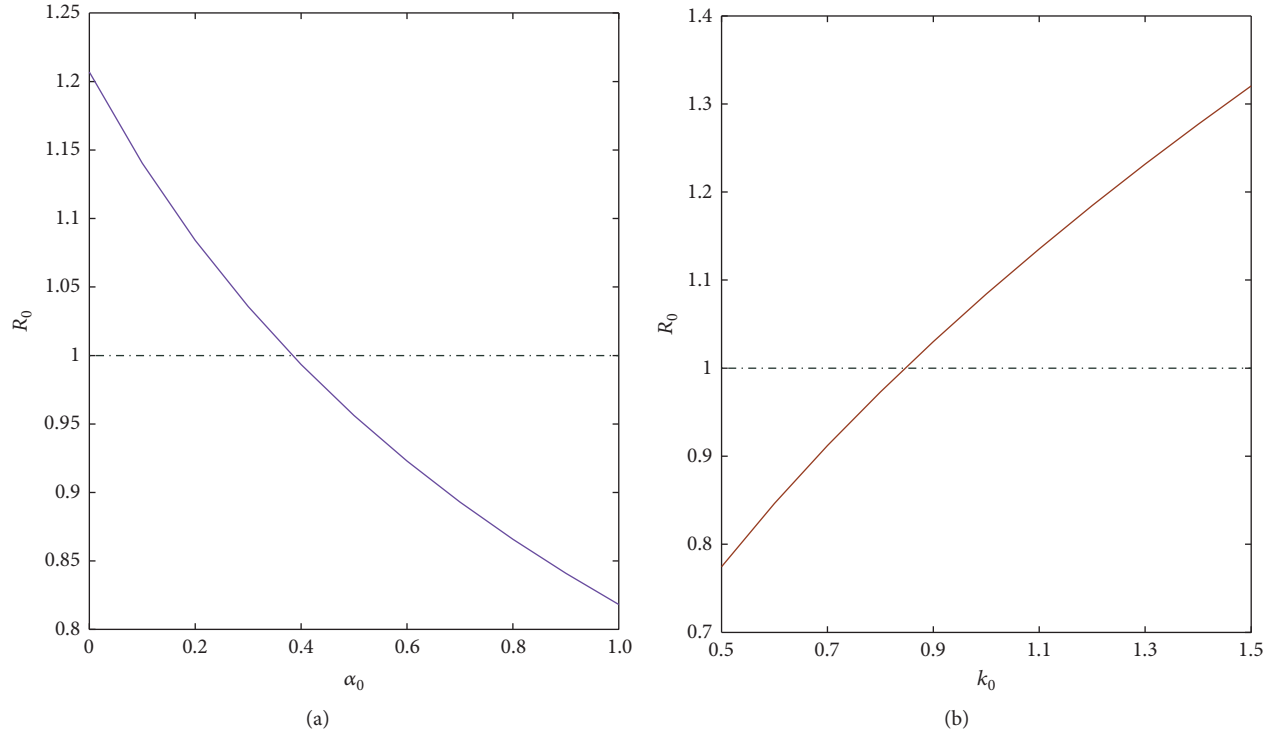

FIGURE 3: Sensitivity analysis of the basic reproduction R_0 with parameter k_0 or α_0.

sensitivity of R_0 increases. That is to say, the carrying capacity of environment for mosquito is bigger and the disease is widespread more easily, so decreasing the circumstance fit survival for mosquitoes, such as contaminated pool or puddle and household garbage, is a necessary method for the extinction of disease.

Next, we consider the combined influence of parameters β_{10} and β_{20} on R_0; in Figure 4 we can see that the basic reproduction number R_0 may be less than 1 when β_{10} and β_{20} are small; the smaller β_{20} the more sensitive the effect on R_0.

In Figure 5, the basic reproduction number R_0 is affected by β_{10} and k_0; with the increasing of k_0 the sensitivity of R_0 increases; if we fix β_{10} as a constant the case will be similar to Figure 3(b). And the similar trend of β_{10} on the sensitivity of R_0, so in the season in which temperature and humidity are more beneficial for mosquito population to give birth and propagate taking measures to avoid more bites is necessary.

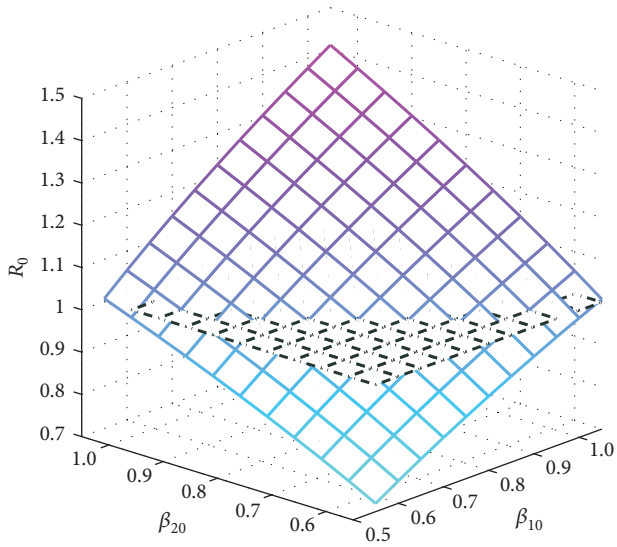

FIGURE 4: Sensitivity analysis of the basic reproduction R_0 with parameters β_{01} and β_{02}.

5. Conclusion

In this paper, we have studied the transmission of lymphatic filariasis; lymphatic filariasis is a mosquito-borne parasitic infection that occurs in many parts of the developing world. In order to systematically investigate the impact that vector genus-specific dependent processes may have on overall lymphatic filariasis transmission, we, according to the nature characteristic of lymphatic filariasis and considering the logistic growth in periodic environments of mosquito, model the transmission of lymphatic filariasis. The dynamic behavior of system (1) is determined by the threshold parameter R_0; when $R_0 < 1$ disease-free periodic solution is globally asymptotically stable and when $R_0 > 1$ disease is uniformly persistent. We also give some numerical simulations which support the results we prove, confirming that R_0 serves as a threshold parameter. Sensitivity analysis show effects of parameters on R_0, which contribute to providing a decision

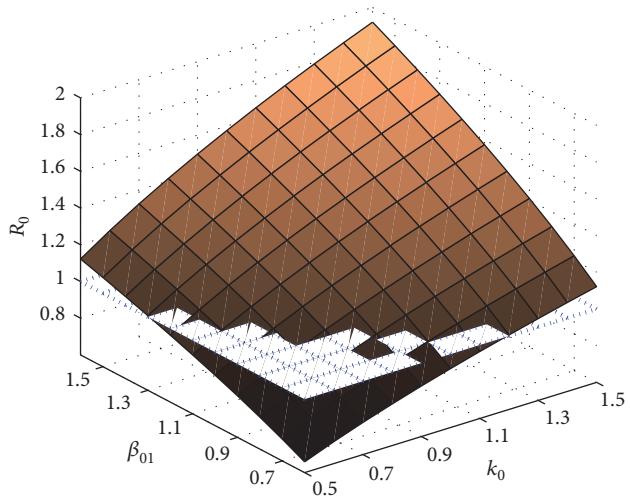

FIGURE 5: Sensitivity analysis of the basic reproduction R_0 with parameters β_{01} and k_0.

support framework for determining the optimal coverage for the successful prevention programme.

Competing Interests

The authors declare that they have no competing interests.

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