

## RESEARCH ARTICLE

# Network dynamic model of epidemic transmission introducing a heterogeneous control factor

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Email: LwBeijing8@aliyun.com**Abstract**

The COVID-19 epidemic is not only a medical issue but also a sophisticated social problem. We propose a network dynamics model of epidemic transmission introducing a heterogeneous control factor. The proposed model applied the classical susceptible-exposed-infectious-recovered model to the network based on effective distance and was modified by introducing a heterogeneous control factor with temporal and spatial characteristics. International aviation data were approximately used to estimate the flux fraction matrix, and the effective distance was calculated. Through parameter estimation and simulation, the theoretical values of the modified model fit well with practical values. By adjusting the parameters and observing the change of the results, we found that the modified model is more in line with the actual needs and has higher credibility in the comprehensive analysis. The assessment shows that the number of confirmed cases worldwide will reach about 20 million optimistically. In severe cases, the peak value will exceed 80 million, and the late stage of the epidemic shows a long tail shape, lasting more than one and a half years. The effective way to control the global epidemic is to strengthen international cooperation and to impose international travel restrictions and other measures.

**KEYWORDS**

COVID-19, heterogeneous, modeling, network dynamics

## 1 | INTRODUCTION

Since the World Health Organization (WHO) declared coronavirus disease 2019 (COVID-19) a global pandemic on March 11, the whole world has been fought against the virus for almost 5 months. Positive effects have been achieved, although the pandemic is still prevalent. In many countries, the epidemic has been under control, but in certain countries, the number of confirmed cases is still growing rapidly. Now the pandemic is not only a medical issue but also a sophisticated social problem. It is more important to figure out how the pandemic transmits in different countries and the trend of the global epidemic situation in this interaction.

A lot of research was carried out on mathematical modeling of epidemic transmission.<sup>1-11</sup> The models not only help to estimate dynamics of epidemic transmission but also other significant forecasts.

The scientists and researchers have focused on dynamic models of epidemic transmission in network environment.<sup>12-18</sup> Many researchers describe the network topology of potential contacts as a random network, and use the susceptible-exposed-infectious-recovered (SEIR) model to analyze and predict the spread of epidemics.<sup>19-22</sup> These studies were based on a seminal work of modeling the SEIR epidemics on a random network in 2013.<sup>23</sup> Compared with the classical uniform mixture model, the stochastic network can describe the heterogeneity of contact quantity more realistically and accurately. Compared with classic methods, the network dynamics model reflects the complexity of the real world and can describe the characteristics and dynamics of virus transmission among different groups in different regions better. Recently mathematical modeling includes references 24-29. International air travel contributed to the international spread of the virus, and its elimination and control have got global attention.<sup>30</sup> It's unreasonable to

illustrate the influence of the pandemic propagation purely in regard to the geographical distance. Therefore, Brockmann et al.<sup>23</sup> proposed a notion called effective distance and its computation method to replace traditional geographical distance, which was successfully applied to explain the trend of the 2009 H1N1 influenza pandemic and 2003 SARS epidemic. Using the concept, complex spatiotemporal patterns can be reduced to surprisingly simple, homogeneous wave propagation patterns. Zhang and Dong<sup>29</sup> applied this method to analyze and predict the initial trend of the COVID-19 epidemic situation in China based on Baidu migration data. Pichit Boonkrong<sup>31</sup> proposed a multi-group SEIR epidemic model and a complicated network nodes' interaction algorithm to study the implication of epidemic transmission in the network structure quantitatively.

However, the current modeling methods are not suitable for the analysis of global epidemic transmission with heterogeneity. The speed of disease transmission is related to the national control strategies. This heterogeneity is mainly reflected in the different control strategies of different countries, which have obvious temporal and spatial characteristics. Using the existing modeling methods, there will be problems such as lack of refinement and parameter deviation, which is difficult to meet the needs of fine analysis of epidemic interaction between countries and the global epidemic trend.

Based on the above understanding, we in this study focus on a network dynamics model of epidemic transmission introducing a heterogeneous control factor. The proposed model applied the classical SEIR model to the network based on effective distance and was modified by introducing a heterogeneous control factor with temporal and spatial characteristics. The flux fraction matrix was approximately estimated by using international aviation data, and the effective distance was calculated. By comparing with the actual trends, we use numerical simulations to test the validity of the applied model and predict the future trend of countries and the world.

## 2 | MODEL FORMULATION

### 2.1 | Effective distance

In network topology structure, distance is used to define the mutual influence between nodes. The farther the distance, the less mutual influence. The distance between two nodes is usually represented by the number of edges in the shortest path or the actual geography distance. In the case of epidemic transmission, using the above two methods to define the distance will lead to the research results inconsistent with the real evolution process of infectious diseases.

The concept of effective distance was first proposed by Brockmann and Helbing in 2013.<sup>23</sup> They found that in the network with information flow as interaction, intuitive geographical distance and the number of edges on the shortest path cannot effectively measure the mutual influence between node pairs, and effective distance can solve this problem.

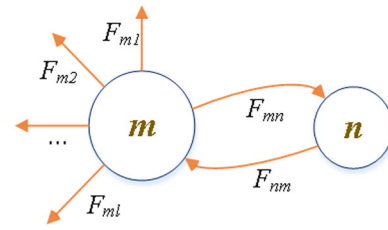


FIGURE 1 Information flow among nodes

As is shown in Figure 1, the information flow between node  $m$  and node  $n$  is represented by  $F_{mn}$ . We define flux fraction from node  $m$  to node  $n$  as

$$p_{mn} = F_{mn}/F_m, F_m = \sum_{m \neq n} F_{mn}, 0 \leq p_{mn} \leq 1, \quad (2.1)$$

where  $F_m$  is the sum of the information fluxes from node  $m$  to all others nodes. Therefore, flux fraction  $p_{mn}$  is the proportion of the information flow from node  $m$  to node  $n$  in the sum of information flow from node  $m$  to all other nodes.

The effective length from node  $m$  to node  $n$  can be calculated by the flux fraction, and then the effective distance can be calculated. The effective distance is the sum of the effective lengths on the shortest path from node  $m$  to node  $n$ . We define the effective distance  $d_{mn}$  from node  $m$  to connected node  $n$  as follows:

$$d_{mn} = 1 - \log(p_{mn}), d_{mn} \geq 1. \quad (2.2)$$

The effective distance  $D_{mn}$  is the shortest path from node  $m$  to indirectly connected node  $n$

$$D_{mn} = \min_{\Gamma} (\Gamma), \quad (2.3)$$

where  $\Gamma$  is the sum of all the paths from node  $m$  to node  $n$ .

In directed networks, the effective distance between two nodes is usually not equal and needs to be treated differently. Moreover, we can see that the larger the flux fraction  $p_{mn}$ , the smaller the effective length  $D_{mn}$ , which means that  $p_{mn}$  is inversely proportional to  $D_{mn}$ .

### 2.2 | Network dynamics model of epidemic transmission based on effective distance

According to Yang et al.,<sup>2</sup> we add the mobility of different people in regions to the classic SEIR model, constructing the network dynamic epidemic transmission model based on effective distance, as is shown in Figure 2.

The mathematical model is given by the following set of coupled differential equations:

$$\begin{cases} \partial s_n / \partial t = -\lambda_n s_n i_n + \sum_{m \neq n} (\omega_{nm} s_m - \omega_{mn} s_n), \\ \partial e_n / \partial t = \lambda_n s_n i_n - \varphi_n e_n + \sum_{m \neq n} (\omega_{nm} e_m - \omega_{mn} e_n), \\ \partial i_n / \partial t = \varphi_n e_n - (\mu_n + \eta_n) i_n + \sum_{m \neq n} (\omega_{nm} i_m - \omega_{mn} i_n), \\ \partial r_n / \partial t = (\mu_n + \eta_n) i_n + \sum_{m \neq n} (\omega_{nm} r_m - \omega_{mn} r_n), \\ s_n + e_n + i_n + r_n = 1. \end{cases} \quad (2.4)$$

In a certain country  $n$ , the susceptible population  $s$  is transformed into the exposed population  $e$  by probability  $\lambda$ , the exposed

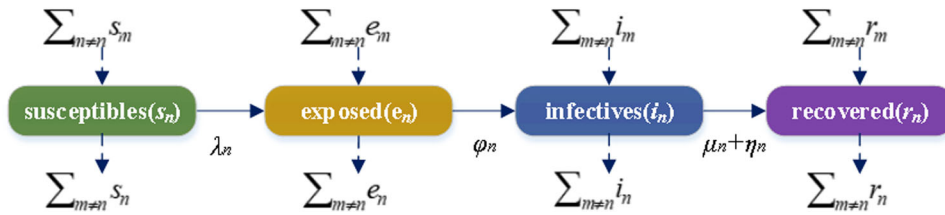


FIGURE 2 Network dynamics model of epidemic transmission based on effective distance

TABLE 1 Parameter estimation for the general case

Parameter	Definitions	Estimated mean value
$\lambda$	Infection rate	0.38
$\varphi$	Diagnosis rate	1/7
$\mu$	Cured rate	0.1
$\eta$	Death rate	0.03
$\gamma$	Average emigration population ratio	0.12

population  $e$  is transformed into the infectious population  $i$  by probability  $\varphi$ , and the infectious population is transformed into the withdrawal population (recovered population and death population) by probability  $\mu + \eta$  after treatment. Furthermore, we also consider the population flowing out of the country with probability  $\omega_{nm}$  and the population flowing into the country with probability  $\omega_{mn}$ .  $s_n, e_n, i_n, r_n$  is, respectively, the proportion of the susceptible, exposed, infectious, and recovered in the sum population in country  $n$ .  $\lambda_n, \varphi_n, \mu_n$  and  $\eta_n$  represent the infection rate, diagnosis rate, cured rate, and death rate in country  $n$ , respectively.  $\omega_{nm}$  is the proportion of the population flowing from node  $n$  to node  $m$  in the sum population of node  $n$ . Considering Lie algebra<sup>32,33</sup> and other methods, the Runge Kutta method is used in Python language to get the analytical solution of the equation.

After substituting Equation (2.1) into Equation (2.4), we can get the updated equations as follows:

$$\begin{cases} \partial s_n / \partial t = -\lambda_n s_n i_n + \gamma_n \sum_{m \neq n} P_{mn} (s_m - s_n), \\ \partial e_n / \partial t = \lambda_n s_n i_n - \varphi_n e_n + \gamma_n \sum_{m \neq n} P_{mn} (e_m - e_n), \\ \partial i_n / \partial t = \varphi_n e_n - (\mu_n + \eta_n) i_n + \gamma_n \sum_{m \neq n} P_{mn} (i_m - i_n), \\ \partial r_n / \partial t = (\mu_n + \eta_n) i_n + \gamma_n \sum_{m \neq n} P_{mn} (r_m - r_n), \\ s_n + e_n + i_n + r_n = 1, \end{cases} \quad (2.5)$$

where  $\gamma_n$  is the average emigration population ratio of node  $n$ , which ranges from 0 to 1.

Basic reproduction number ( $R_0$ ) is the most important parameter in epidemic dynamics. It can describe the internal transmission ability of infectious diseases and can be used for public health policy analysis, epidemic evaluation at home and abroad, and disease transmission inflection point prediction. It refers to the average number of people infected by an infective person in a susceptible environment. (1) when  $R_0 < 1$ , COVID-19 will gradually disappear; (2) when  $R_0 > 1$ ,

it indicates that COVID-19 will spread rapidly in an exponential manner; (3) when  $R_0 = 1$ , COVID-19 will reach a balance, and it will always exist as a local disease.

The basic reproduction number of COVID-19 was estimated as 3.77.<sup>34</sup> In the SEIR model,  $R_0$  is computed as  $R_0 = \lambda / \mu$ .<sup>35</sup> So we can estimate the related parameters for the general case, as is shown in Table 1.

### 2.3 | Introducing a heterogeneous control factor

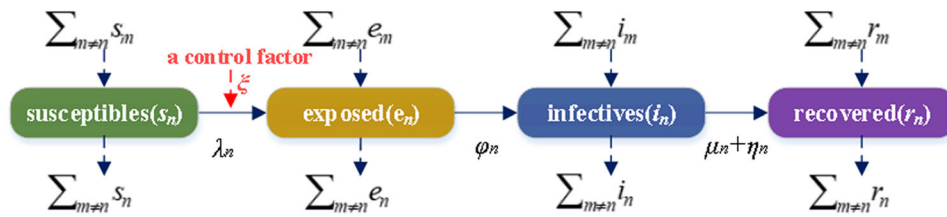
Different countries have different control strategies. The intensity of control varies in different times and places. Based on the heterogeneity of control strategies in different countries, the above model can be optimized, and we construct a network dynamic model of epidemic transmission introducing a heterogeneous control factor with temporal and spatial characteristics, as is shown in Figure 3.

Based on Equation (2.5), we introduced a control factor  $\xi$ , and propose the following optimal mode:

$$\begin{cases} \partial s_n / \partial t = -\lambda_n \xi_n s_n i_n + \gamma_n \sum_{m \neq n} P_{mn} (s_m - s_n), \\ \partial e_n / \partial t = \lambda_n \xi_n s_n i_n - \varphi_n e_n + \gamma_n \sum_{m \neq n} P_{mn} (e_m - e_n), \\ \partial i_n / \partial t = \varphi_n e_n - (\mu_n + \eta_n) i_n + \gamma_n \sum_{m \neq n} P_{mn} (i_m - i_n), \\ \partial r_n / \partial t = (\mu_n + \eta_n) i_n + \gamma_n \sum_{m \neq n} P_{mn} (r_m - r_n), \\ s_n + e_n + i_n + r_n = 1, \\ \xi(t, t_0, t_m, \delta) = \frac{1}{1 + \exp(\delta(t - t_0 - t_m/2))}. \end{cases} \quad (2.6)$$

Here, we define  $\xi$  to simulate the heterogeneous control factor, reflecting the intensity of disease prevention and control. At the initial stage of the epidemic, the government's macro-control is not enough, and it has not taken strict control measures. At this time, the value of  $\xi$  is at a high level. With the continuous improvement of epidemic prevention measures and the enhancement of people's awareness of epidemic prevention, the  $\xi$  value continues to decrease until the epidemic situation becomes stable. The whole process can be described by an improved Logistic model.<sup>5</sup> The trend of  $\xi$  is shown in Figure 4.

The heterogeneous control factor  $\xi$  plays the same role as the infection rate  $\lambda$ , which directly affects the interaction probability between susceptible people and infected people.  $\xi$  has temporal and spatial characteristics and can be described by a function, which can be represented by the initial control time  $t_0$ , the control lasting time  $t_m$ , the proportionality coefficient  $\delta$ .



**FIGURE 3** Network dynamics model of epidemic transmission introducing a heterogeneous control factor The trend of the control factor. There is no precautionary action at the initial stage of the pandemic, the value of  $\xi$  is 1

The smaller the value of  $\xi$ , the greater the intensity of epidemic prevention and control. When the epidemic situation is stable, the intensity of epidemic prevention and control reaches its maximum, and the minimum value of  $\xi$  is obtained. Suppose that the minimum value of  $\xi$  is  $\varepsilon$ , we can calculate the value of  $\delta$  as follows:

$$\delta = 2 \frac{\log\left(\frac{1-\varepsilon}{\varepsilon}\right)}{t_m} \tag{2.7}$$

The parameters of the control factor are estimated in a certain range and shown in Table 2.

### 3 | NUMERICAL SIMULATION

#### 3.1 | Calculation and verification of effective distance

According to the current epidemic situation, ten representative countries were selected for research, their basic information is shown in Table 3.

World Tourism Cities Federation and Tourism Research Centre, Chinese Academy of Social Sciences jointly published the world tourism economics trend report (2019), which ranked the top 20 countries in tourism revenue. It is not difficult to find that these countries are also the most severe countries affected by COVID-19. We used data on the tourism income among selected countries<sup>36</sup> to

approximately estimate the matrix of connectivity components and to calculate effective distance among selected countries.

After normalization of the data, the matrix of connectivity components was obtained. According to Equations (2.2, 2.3), the effective distance among selected countries in the world was calculated. As shown in Figure 5, in terms of geographical distance, the distance between China and other European countries such as Italy, France, and Germany is similar, but the effective distance between China and Italy is obviously smaller than that of other European countries.

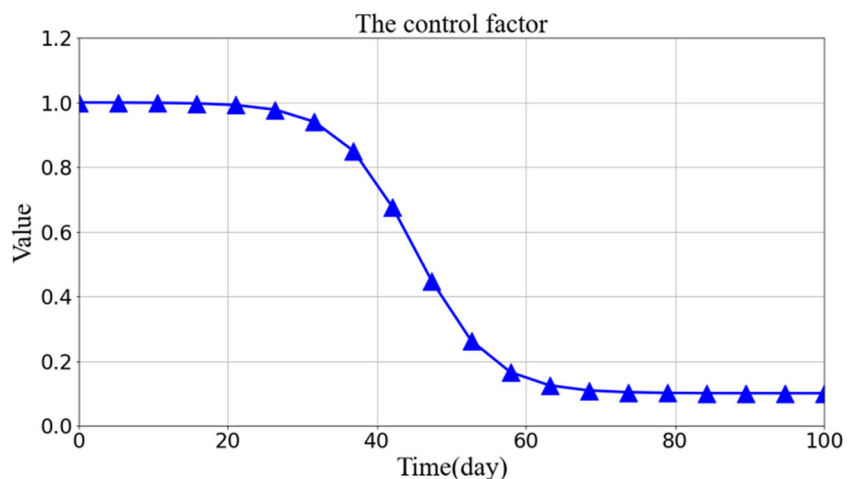
Figure 6 shows that the effective distance from China to Iran and Italy is relatively small, and the outbreak time of the two countries is relatively early. This is consistent with the fact that the destination of the One Belt, One Road Strategy via Iran is Italy, which validates the rationality of effective distance.

#### 3.2 | Model simulation

According to the actual situation of major countries in the world, the parameters of the model are estimated, as shown in Table 4.

The network dynamics epidemic transmission model based on effective distance was used to fit and compare the epidemic trend of major countries. Figure 7 presents the theoretical value of the infected population in major countries worldwide. The abscissa is the number of days after the event that China took the city closure

**FIGURE 4** At the time of  $t_0$ , the control measures have been taken, and the first turning point appears. With the improvement of epidemic prevention measures,  $\xi$  is decreasing gradually. The infection rate won't decline continuously when it decreases in a certain degree. The second turning point occurs at the time of  $t_0 + t_m$  and  $\xi$  reaches its minimum value  $\varepsilon$



**TABLE 2** Parameters' estimation of the control factor

Parameter	Definitions	Estimated mean value
$\epsilon$	The lowest infection rate	0.2–0.9
$t_0$	The initial time for control measures taken	10–100 days
$t_m$	The interval needed for stability after taking control measures	13–1000 days

**TABLE 3** Basic information of major countries

Country	Latitude	Longitude	Population	Date of first case
US	37.1	-95.7	328 802 000	2020.1.20
United Kingdom	55.4	-3.45	66 040 229	2020.1.31
France	46.2	2.21	65 273 512	2020.2.27
India	21	78	1 380 004 385	2020.1.30
Germany	51	9	83 783 945	2020.3.1
Italy	43	12	60 461 828	2020.2.21
China	30.6	114.3	1 404 676 330	2019.12.8
Iran	32	53	83 992 953	2020.2.19
Brazil	-14.2	-51.9	212 559 409	2020.2.26
Russia	60	90	145 934 460	2020.3.19

measures on January 23, and the ordinate is the infected population of different countries. The theoretical value of the model is basically consistent with the actual trend of each country. It can be seen that China, Italy, Germany, and other countries have taken strict measures to limit the spread of the epidemic and tend to be stable.

**Population Flow****FIGURE 5** The left map drawn in pie charts shows the geographical distance between China and other countries. Compared with it, the photograph on the right drawn in Gephi reflects the effective distance from China to other countries

The number of people infected in the main epidemic countries, mainly in the United States and Brazil, will also increase significantly.

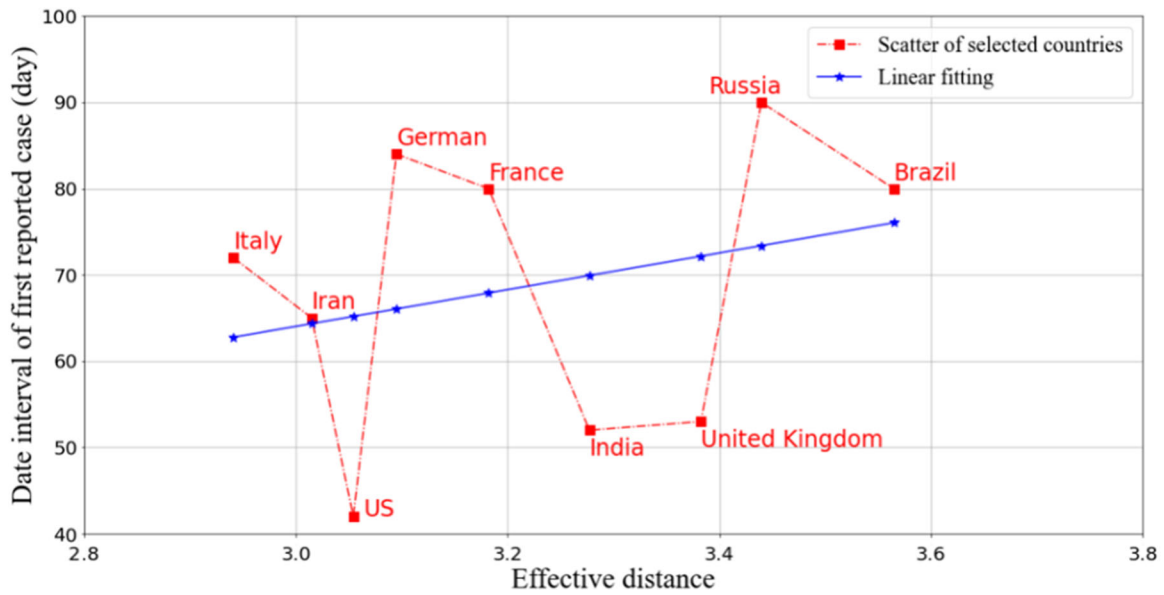
The epidemic situation in the world will be summarized to form the overall situation of the global epidemic situation. As shown in Figure 8, the global epidemic situation will continue for a long time to reach its peak, and the number of confirmed cases will reach nearly 20 million.

To illustrate the effectiveness of the improved model, it is necessary to compare it with the original model. The control measures in the original model are consistent. In the improved model, different countries adopt different levels of control measures, and the trend of epidemic situation will be different accordingly, as shown in Figure 9.

Figure 10 shows that there are obvious differences in the results of the comparison of the three situations of increasing and reducing the control strength of the United States by 20% and keeping the control strength unchanged. It can be seen from the comparison of the three situations that strengthening management and control can significantly reduce the number of infected people, the peak number of infected people will come ahead of time, and COVID-19 could be effectively prevented, which is in line with expectations and verifies the validity of the model. Therefore, all countries should strengthen their own epidemic prevention and resolutely resist the misconception of "mass immunization."

Considering the differences of epidemic control policies and public awareness of epidemic prevention in severe cases, the values of the heterogeneous control factor of the model were modified, as shown in Table 5.

As shown in Figure 11, the peak time of the epidemic situation in major countries in the world has been further delayed and improved. The United States, Brazil, and India will still be the main source of the epidemic development. The United States has always been the country with the most serious epidemic situation. Outbreaks in other countries will gradually stabilize. The global epidemic peak exceeds 80 million. The late stage of the epidemic shows a long tail shape, lasting more than one and a half

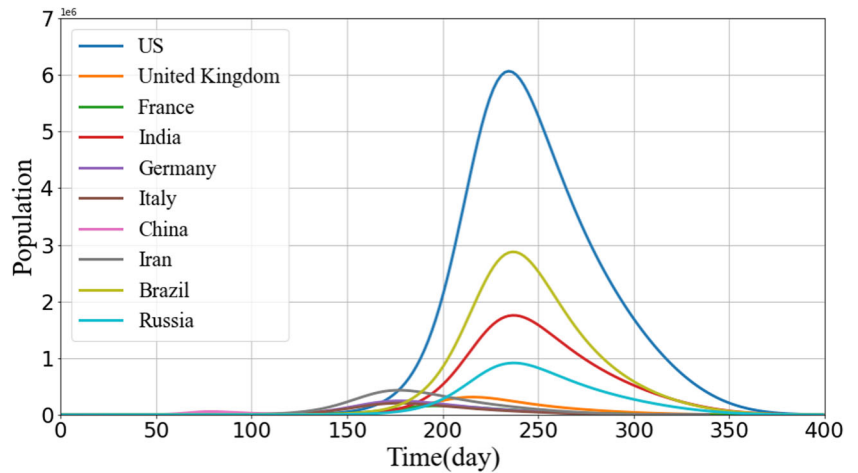


**FIGURE 6** There is a positive correlation between the dates of the first reported cases and the effective distance between China and others

**TABLE 4** Estimation of epidemic parameters in major epidemic countries in the world

Parameter	US	United Kingdom	France	India	Germany	Italy	China	Iran	Brazil	Russia
$\lambda$	0.79	0.5	0.38	0.5	0.38	0.38	0.1	0.5	0.78	0.5
$\mu$	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
$\eta$	0.04	0.04	0.03	0.05	0.03	0.03	0.03	0.04	0.05	0.03
$\gamma$	0.16	0.15	0.15	0.13	0.16	0.16	0.16	0.12	0.12	0.13

**FIGURE 7** Theoretical value of infected population in major countries worldwide



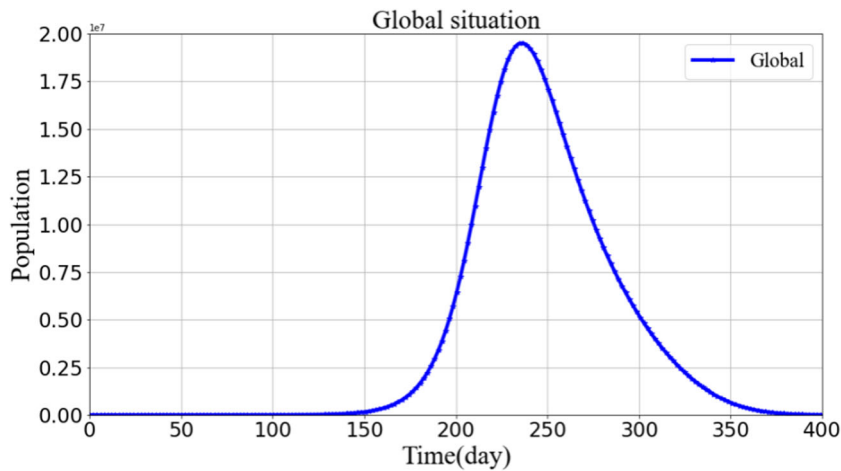
years. The COVID-19 is a major epidemic that is still spreading in summer and winter.

### 3.3 | Discussion and suggestions

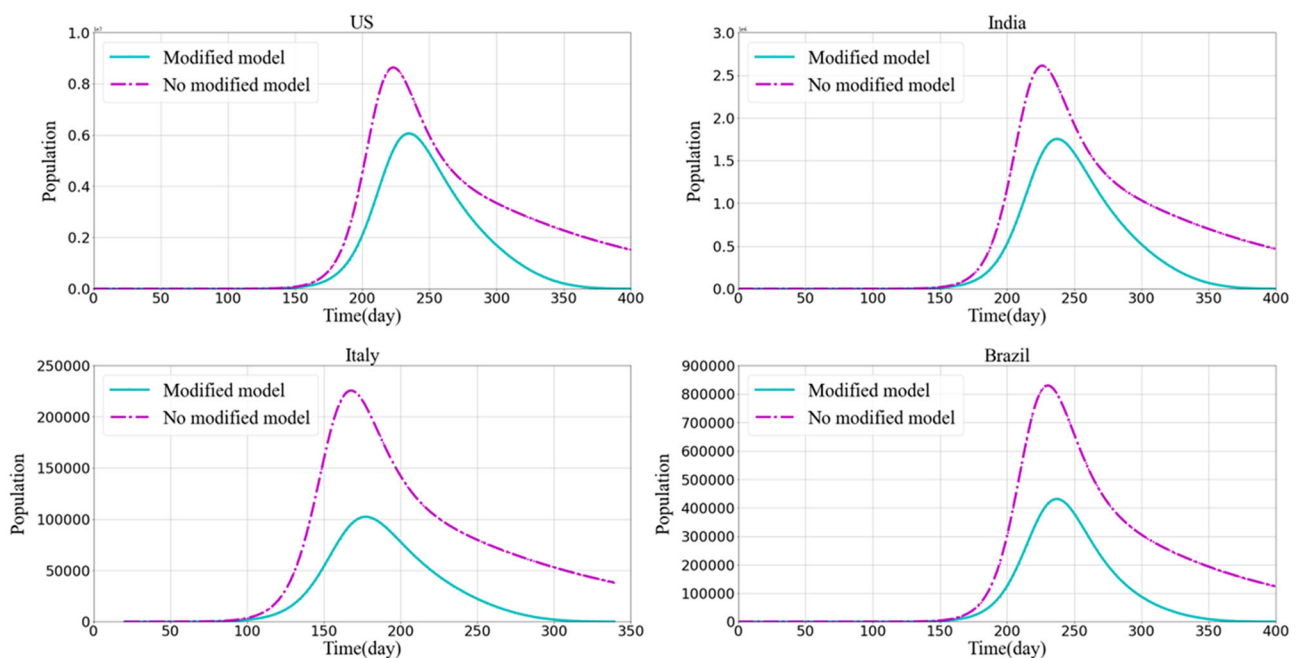
Different countries have different strengths in epidemic control, which is very important for epidemic analysis. So, it is

necessary to improve the original model. By setting appropriate parameters, the simulation results of the network dynamics epidemic transmission model introducing a heterogeneous control factor could fit well with the actual epidemic situation in major countries in the world, and the effectiveness of the improved model was obtained. If we only analyze the possible consequences of the epidemic, we can set other parameter values.





**FIGURE 8** Theoretical value of the total number of infected people in the world



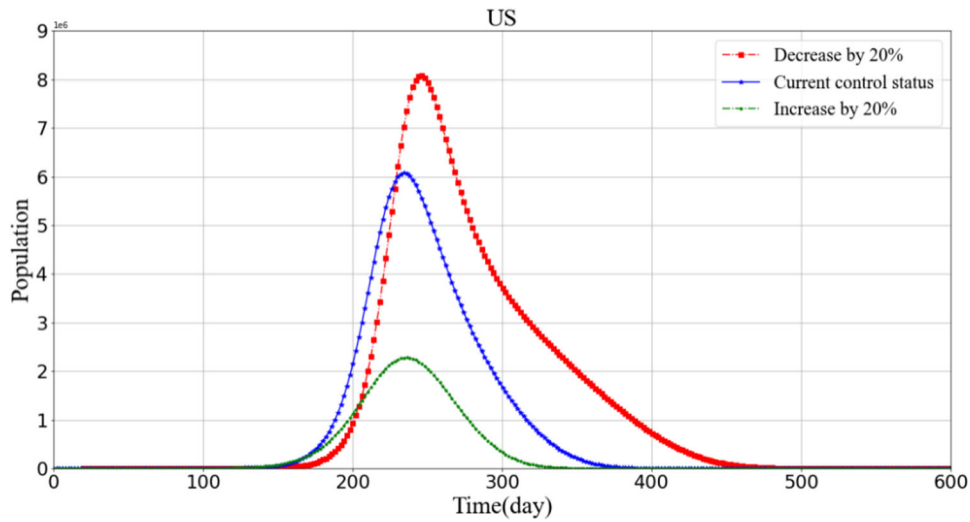
**FIGURE 9** The trend of the United States, India, Italy, and Brazil under different control measures is obviously different from that of the original model

Compared with the original model. The improved model is closer to the needs of reality and has higher credibility in comprehensive analysis. However, for a country, the influence of other countries on it is dynamic and cannot be described quantitatively. In this model, a comprehensive control factor is used instead of multiple factors related to specific measures, which can greatly reduce the difficulty of the model and has no impact on the analysis of the problem.

Now there is the risk of a multi-point outbreak in the global epidemic situation. The effective way to stop the global epidemic is to strengthen international cooperation and to impose international travel restrictions and other measures. The spread of the epidemic situation has brought huge losses to the economy of various countries. Whether a country with a controllable epidemic situation will break out for the second time is a question that needs to be studied next.

## 4 | CONCLUSIONS

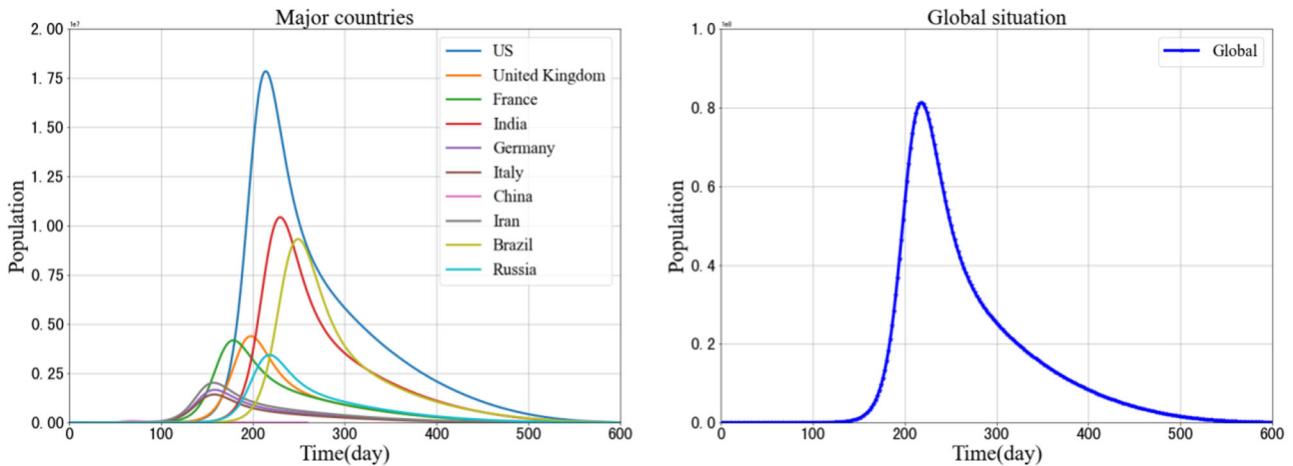
In this paper, we propose a network dynamics model of epidemic transmission introducing a heterogeneous control factor. The proposed model applied the classical SEIR model to the network based on effective distance and was modified by introducing a heterogeneous control factor with temporal and spatial characteristics. International aviation data were approximately used to estimate the flux fraction matrix, and the effective distance was calculated. Through parameter estimation and simulation, the theoretical values of the modified model fit well with practical values. By adjusting the parameters and observing the change of the results, we found that the modified model is more in line with the actual needs and has higher credibility in the comprehensive analysis. The assessment



**FIGURE 10** The three situations of increasing and reducing the control strength of the United States by 20% and keeping the control strength unchanged

**TABLE 5** The epidemic control factor in the worst case

Parameter	US	United Kingdom	France	India	Germany	Italy	China	Iran	Brazil	Russia
$t_0$	54	60	67	62	60	47	0	51	38	54
$t_m$	350	260	180	270	155	160	50	180	330	210
$\epsilon$	0.13	0.05	0.05	0.11	0.05	0.06	0.01	0.05	0.12	0.05



**FIGURE 11** Theoretical value of infected population in major countries and the global situation in the worst case

shows that the number of confirmed cases worldwide will reach about 20 million optimistically. In severe cases, the peak value will exceed 80 million, and the late stage of the epidemic shows a long tail shape, lasting more than one and a half years.

There are some limitations and improvements in this paper. Firstly, the proposed model is highly dependent on the correctness of the heterogeneous control factor. The factor is estimated values after comprehensive consideration of various control measures and there are no specific control strategies and evaluation methods, such

as a variety of isolation measures, setting the isolation period of floating population, nucleic acid testing, vaccination, and so forth, which need to be further studied in the future. Moreover, in reality, there may be many other situations, so the description of the control factors should be further modified and improved in combination with the actual situation. Secondly, the study of the model is based on a series of assumptions, such as ignoring the factors of seasonal climate, the second outbreak of global COVID-19, control of a country's relaxation of epidemic control for the sake of economy and



employment. Therefore, we can use other modeling methods such as complex networks based on multi-agents to carry out comprehensive research.

## CONFLICT OF INTERESTS

The authors declare that there are no conflict of interests.

## DATA AVAILABILITY STATEMENT

Some or all data, models, or codes generated or used during the study are available from the corresponding author by request.

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