Heliyon 10 (2024) e27886

Contents lists available at ScienceDirect

Heliyon

journal homepage: www.cell.com/heliyon

Research article

Cosine and cotangent similarity measures for intuitionistic fuzzy hypersoft sets with application in MADM problem

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A R T I C L E I N F O

Keywords:
Cosine
Cotangent
Soft sets (Ss)
Hypersoft sets (HSSs)
Intuitionistic fuzzy hypersoft sets (IFH0SSs)
Similarity measures (SMs)
Multi-attribute decision making (MADM)

A B S T R A C T

Intuitionistic fuzzy hypersoft sets (IFHSSs) are a novel model that is projected to address the limitations of Intuitionistic fuzzy soft sets (IFSSs) regarding the entitlement of a multi-argument domain for the approximation of parameters under consideration. It is more flexible and reliable as it considers the further classification of parameters into their relevant parametric valued sets. In this paper, we proposed some trigonometric (cosine and cotangent) similarity measures and their weighted trigonometric similarity measures (SMs). Trigonometric Similarity measures (SMs) for intuitionistic fuzzy hypersoft sets (IFHSSs) are significantly implied to check the similarity measures and help to determine the similarity between different factors. Also, in order to evaluate the validity of the significant study and apply the results to a daily life problem. We use them to solve problems involving the selection of renewable energy sources. According to several technical contributing factors, the analysis identifies the ideal location for the implementation of the energy production units. Future case studies with many features and additional bifurcation along with multiple decision-makers can use the suggested methodologies. Also, several existing structures, such as fuzzy, Pythagorean fuzzy, Neutrosophic theories, etc., can be utilized with the suggested method.

1. Introduction

Comprehending the interconnections and patterns present in datasets is crucial in the domain of data analysis. Many applications, including as categorizing text, clustering similar objects in recommendation systems, and identifying similarities in biological sequences, are based on quantifying the similarity or dissimilarity between data points. We would like to introduce you to the concepts of Distance Measures and Similarity Measures, which are fundamental to many analytical techniques and algorithms. The two compasses that direct an inquiry of a data landscape are similarity measures and distance measures. They let us to quantify the distances between objects, which aids in the discovery of undiscovered links and patterns in vast volumes of data. These measurements are essential in many fields, such as pattern recognition, machine learning, and information retrieval. They make it possible for us to analyze vast, complex databases and derive insightful conclusions and decisions. In this study, we explore the intricate world of Similarity and Distance Measures. We'll break down the core concepts, explore various measuring techniques, look at how they're used in a variety of

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https://doi.org/10.1016/j.heliyon.2024.e27886
Received 24 August 2023; Received in revised form 1 March 2024; Accepted 7 March 2024
Available online 21 March 2024

settings, and emphasize how significant an impact they have had on the development of modern data analysis. Join us as we examine the nuances and intricacies of these foundational concepts and demonstrate how they contribute to our comprehension of the underlying trends that influence our data-driven world (see Table 6).

Humanity has been perplexed by the relationship between accuracy and uncertainty for generations. When making decisions, it can be difficult to handle imprecise and unclear information. We must have to prefer the best multiple choices in our life to sort out which is suitable for us. In this way, MADM helps us to give information in a formulated manner and give results of contributing factors sequentially. To overcome obstacles in real life, decisions that contained uncertainty had to be addressed at various stages of life. Data uncertainty, ambiguity, and unreliability are the most important factors in resolving these issues. Many mathematical problems, such as Fuzzy Set theory (FST) [1]. interval probability theory [2], fuzzy groups [3,4] represents fuzzy sets and theory for possibilities and probability distribution. Afterward, the comparison of the interval value fuzzy set (IVFS) with others was discussed in above mentioned articles. The fuzzy set theory provided a specialized converter for analyzing the provided data and preferences in accordance with probability theory during group decision-making. It is a logistical technique for discussing problems including ambiguity, consistency, and inaccurate assessment. Atanassov presented the intuitionistic fuzzy set (IFS) theory in 1983 [5], which is an extension of the fuzzy set. Although it characterized the degree of satisfiability and non-satisfiability and offered an alternate method for addressing these ambiguities, vagueness, and fuzziness, it's important to note that single membership and non-membership degrees didn't adequately address these circumstances. Then, Liu et al. [6] introduced a vague set theory. This idea was similar to IFS, which is an extension of FS. Interval-based membership, which better captures the fuzziness of the data, is utilized in VS rather than point-based membership. Both IFS and VS are regarded as being equal in the literature. IFS is hence isomorphic to VS in this regard. Thus, Smarandache [7] first suggested the concept of the neutrosophic set (NS), which, from a philosophical perspective, deals with imprecise, ambiguous, and inconsistent information more adequately than the usual fuzzy set model and the IFS model. The neutrosophic set is defined by the truth function, the indeterminacy function, and the falsity functions which are all independent of each other. A Russian scientist, Molodtsov first formulated the notion of soft set theory (SST) [8] in 1999 which is an extended mathematical tool for handling unpredictability entities that are free from the foregoing complications

then define soft groups and determine some of their fundamental properties using Molodtsov's description of softsets. Soft set theory modifies the previous approaches, such as Probability theory, fuzzy set theory, and rough set theory (RST). In this regard, it is unaffected by the limitations of the parameterization methods of these theories. He effectively applied the soft set theory to a wide range of topics, including game theory, smoothness of functions, and many more. In recent years it has been seen a lot of interest in the algebraic structure of soft set theory, Aktas, and Cagman presented soft matrices [9], of which soft sets are characterized. Chen et al. [10] describe the parametrization of soft sets and new operations for soft set theory developed by Ali and Feng et al. [11]. Acar and Koyuncu [12] discussed some basic concepts of soft rings. Zou and Xiao [13,14] provided the data analysis approach of soft sets for incomplete information and compared the results with other approaches which are dealing with incomplete information. The idea of a Fuzzy soft set (FSS) the extended formulation of soft sets that gives more accurate information of data, was presented by Maji et al., in 2001 [14]. Further, fuzzy soft set in the generalized form presented by Majumdar et al. [15] by attached degree with fuzzy sets while dealing with a Fuzzy soft set.

Intuitionistic and soft sets combine and indicate the idea of an intuitionistic fuzzy soft set (IFSS) [16]. It includes the parameters that measure the accuracy of the supplied data and guide us toward the finest option. Many ideas have also been put forth utilizing an intuitionistic set, such as similarity measure, pattern recognition, medical diagnosis, entropy measure, career prediction, and distance measure. By using IFS, Liang et al. [17] introduced the similarity measure with application in pattern recognition, ZS Xu et al. [18] given an overview of similarity and distance measures for intuitionistic fuzzy set and apply to medical diagnostics, by De. S. K. et al. [19]. Ejegwa et al. [20] developed an intuitionistic set for application in career determination, and Dengfeng et al. [21] presented the new similarity measures and applied them in pattern recognition for intuitionistic Fuzzy soft sets. Szmidt et al. [22,23] provided an approach to IFS in group decision-making and Wei C.P. et al. [24] presented entropy similarity measures using an Interval-valued Fuzzy set and its application. Jafar et al. [25] discussed IFSM and its application for the selection of a Laptop. Mitchell. H.B. et al. [26] defined the similarity measure for application in pattern recognition. The similarity index helps to determine the similarity between two constituents and the foundation for the similarity measurements is the direct operation of hesitation, membership, non-membership, and the upper bound of membership function. The measure of similarities is employed to extend the theories and recommend several practical applications, such as multi-attribute decision-making, pattern identification, physics teaching, and medical diagnosis. The improvement of similarity measure and application was proposed by Khorshidi et al. [27] and the suggested approach is also used for fuzzy risk analysis on similarity measures. The eigenvalue-based similarity measure and its application are

Table 1
The IFHSM of the IFHSS $(\acute{K}, \mathfrak{Q}_1^a \times \mathfrak{Q}_2^b \times \ldots \times \mathfrak{Q}_\beta^z)$

	$\mathfrak{Q}_1^a$	$\mathfrak{Q}_2^b$	...	$\mathfrak{Q}_\beta^z$
S^1	$\mathcal{X}_{p_\rho}(S^1, \mathfrak{Q}_1^a)$	$\mathcal{X}_{p_\rho}(S^1, \mathfrak{Q}_2^b)$	...	$\mathcal{X}_{p_\rho}(S^1, \mathfrak{Q}_\beta^z)$
S^2	$\mathcal{X}_{p_\rho}(S^2, \mathfrak{Q}_1^a)$	$\mathcal{X}_{p_\rho}(S^2, \mathfrak{Q}_2^b)$	...	$\mathcal{X}_{p_\rho}(S^2, \mathfrak{Q}_\beta^z)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
S^α	$\mathcal{X}_{p_\rho}(S^\alpha, \mathfrak{Q}_1^a)$	$\mathcal{X}_{p_\rho}(S^\alpha, \mathfrak{Q}_2^b)$	...	$\mathcal{X}_{p_\rho}(S^\alpha, \mathfrak{Q}_\beta^z)$

Table 2
Relationship between Geo-graphical region and Factors.

Geological regions ($\mathfrak{G}$)	$4.1h - 6h$	2001 m^3 − 4001m^3	$251mm - 501mm$	10 KM or Below	9% − 19%,
$\mathfrak{G}^1$	(0.6,0.7)	(0.2,0.3)	(0.2,0.4)	(0.8,0.8)	(0.7,0.8)
$\mathfrak{G}^2$	(0.9,0.2)	(0.6,0.4)	(0.6,0.3)	(0.4,0.3)	(0.3,0.4)
$\mathfrak{G}^3$	(0.2,0.1)	(0.6,0)	(0.2,0.2)	(0.4,0.5)	(0.2,0.5)
$\mathfrak{G}^4$	(0.4,0.7)	(0.3,0.2)	(0.2,0.5)	(0.4,0.7)	(0,0.2)
$\mathfrak{G}^5$	(0.6,0.4)	(0,0.7)	(0.3,0.4)	(0,0.5)	(0.2,0.6)

Table 3
Relationship between systems and factors.

Systems	$4.1h - 6h$	2001 m^3 − 4001m^3	$251mm - 501mm$	10 KM or Below	9% − 19%,
Solar	(0.4,0.1)	(0.0,0.3)	(0.1,0.7)	(0.8,0.1)	(0.2,0.2)
Wind	(0.3,0.9)	(0.1,0.4)	(0.1,0.4)	(0.5,0.0)	(0.1,0.3)
Hydro-electric	(0.0,0.2)	(0.1,0.5)	(0.7,0.6)	(0.3,0.7)	(0.3,0.7)
Geo-Thermal	(0.1,0.4)	(0.4,0.6)	(0.1,0.2)	(0.6,0.2)	(0.1,0.2)

Table 4
SMs using definition. 4.1 of $S^1_{IFHSS}(\mathscr{M},\mathscr{H})$:

Similarity measures	Geographical regions	Geo-Thermal	*Solar power*	Wind power	Hydro-electric Power
S^1_{IFHSS}	$\mathfrak{G}^1$	0.8767	0.8590	**0.8970**	0.8410
	$\mathfrak{G}^2$	0.7963	0.7564	0.7589	**0.8097**
	$\mathfrak{G}^3$	0.6824	0.6860	0.7208	**0.7981**
	$\mathfrak{G}^4$	0.7051	**0.8312**	0.8148	0.8044
	$\mathfrak{G}^5$	0.7674	0.7362	**0.8854**	0.7724

Table 5
SMs using definition. 4.2 $S^2_{IFHSS}(\mathscr{M},\mathscr{H})$:

Similarity measures	Geographical regions	Geo-Thermal	*Solar power*	Wind power	Hydro-electric Power
S^2_{IFHSS}	$\mathfrak{G}^1$	0.6946	0.7763	**0.7777**	0.7452
	$\mathfrak{G}^2$	0.7527	0.7423	0.6912	**0.7838**
	$\mathfrak{G}^3$	0.7894	0.7253	0.8436	**0.8498**
	$\mathfrak{G}^4$	0.7939	**0.8787**	0.8489	0.8741
	$\mathfrak{G}^5$	0.7421	0.8389	**0.8455**	0.7730

Table 6
SMs using definition. 4.3 $S^3_{IFHSS}(\mathscr{M},\mathscr{H})$:

Similarity measures	Geographical regions	Geo-Thermal	Solar power	Wind power	Hydro-electric Power
S^3_{IFHSS}	$\mathfrak{G}^1$	0.9499	0.9590	**0.9711**	0.9700
	$\mathfrak{G}^2$	0.97111	0.9695	0.9620	**0.9726**
	$\mathfrak{G}^3$	0.9754	0.9618	0.9810	**0.9825**
	$\mathfrak{G}^4$	0.9757	**0.9839**	0.9768	0.9735
	$\mathfrak{G}^5$	0.9690	0.9803	**0.9826**	0.9735

Table 7
SMs using definition. 4.4 $S^4_{IFHSS}(\mathscr{M},\mathscr{H})$:

Similarity measures	Geographical regions	Geo-Thermal	Solar power	Wind power	Hydro-electric Power
S^4_{IFHSS}	$\mathfrak{G}^1$	0.4458	0.5786	**0.5888**	0.4806
	$\mathfrak{G}^2$	0.4778	0.4815	0.4456	**0.6051**
	$\mathfrak{G}^3$	0.5175	0.4879	0.6015	**0.6034**
	$\mathfrak{G}^4$	0.5431	**0.7641**	0.6043	0.6781
	$\mathfrak{G}^5$	0.4805	0.6043	**0.6056**	0.4969

Table 8
SMs using definition. 4.5 $S^5_{IHSS}(\mathscr{M},\mathscr{H})$:

Similarity measures	Geographical regions	Geo-Thermal	*Solar power*	Wind power	Hydro-electric Power
S^5_{IFHSS}	$\mathfrak{G}^1$	0.7789	0.8337	**0.8942**	0.7949
	$\mathfrak{G}^2$	0.7942	0.7952	0.7795	**0.8442**
	$\mathfrak{G}^3$	0.8115	0.7969	0.8471	**0.8474**
	$\mathfrak{G}^4$	0.8215	**0.9073**	0.8475	0.8759
	$\mathfrak{G}^5$	0.7953	0.8475	**0.8479**	0.8027

presented by Tsai, D.M. et al. [28], and Strelkov, V.V. Strelkovet al. [29] presented the similarity measure for time series analysis and determining how similar two histograms are. The application of similarity metrics for IFS in medical diagnostic reasoning is covered by Szmidt et al. [23]. The similarity measures and their different operators for IFS were discussed by Baccour et al. in their article [30].

Then, Smarandache [31] proposed the notion of the hypersoft set (HSS) in 2018, as an extension of the soft set. It is helpful for tackling problems involving multiple objectives and multiple attributes with varied attribute values. Compared to the soft set, it is the theory with the most applicability that tackles ambiguity and helps us to obtain the best choice. Zulqarnain et al. [32] given a detailed analysis of the implications of intuitionistic hypersoft sets which are based on coefficients for correlation. Yolcu and Ozturk proposed fuzzy hypersoft sets and how to use them for decision-making [33]. Debnath [34] presented the fuzzy hypersoft sets and associated weighting operators for decision-making. Saeed et al. presented intuitionistic fuzzy hypersoft sets (IFHSS) [35]. Jafar et al. [36] proposed the aggregate operators of FHSS and Saeed et al. [35,37] developed the similarity measures for complex fuzzy Hypersoft set and discussed its basic operators with mathematical applications. N-soft sets, image fuzzy, interval-valued picture fuzzy, and picture fuzzy are a few additional definitions and operators for the set structures. Graphics on single-valued neutrosophic graphs and interval-valued Fermatean neutrosophic graphs have been proposed by Ref. [38].

IFHSSs are the result of combining the concepts of IFSs and HSSs. Let's first grasp each of these concepts separately before attempting to understand how they function collectively.

1. The IFSs extend conventional fuzzy sets by permitting multiple membership degrees. Rather than using a single membership value between 0 and 1, IFSs use two membership values: the degree of membership and the degree of non-membership, both of which run from 0 to 1. If the sum of these two values is greater than 1, it suggests that the membership assignment was made with hesitation or uncertainty.
2. HSSs generalize fuzzy sets by permitting greater latitude in membership assignments. Membership degrees in HSSs are not specified by a precise numerical value, but rather by linguistic phrases or gradations. This allows for a more complex representation of ambiguity or imprecision.
3. IFSs and HSSs are combined in IFSs to enhance the way uncertainty and ambiguity are represented in a given domain. By providing a structure for managing membership degrees that can be expressed in both verbal and numerical terms, they enable more precise and flexible modeling of fuzzy data.
4. The membership degrees in IFHSMs are represented by intuitionistic fuzzy numbers, which are composed of two membership values and a degree of hesitation. The degree of hesitancy is indicative of how unclear or questionable the membership categorization is.

1.1. Motivation of the proposed study

A new model called intuitionistic fuzzy hypersoft sets (IFHSSs) is proposed to overcome the drawbacks of intuitionistic fuzzy soft sets (IFSSs) with respect to the entitlement of a multi-argument domain for the approximation of parameters that are being considered. Because it takes into account the additional classification of parameters into their pertinent parametric valued sets, it is more dependable and versatile.

To evaluate strategies for dealing with the issue of uncertainty, intuitionistic fuzzy set theory and hypersoft set theory are integrated into this study. It aims to create a new terminology termed the "Intuitionistic fuzzy hypersoft set" by unifying these two theories.

1.2. Novelty of the study

In December 2018, Florence Samrandace unveiled the Hypersoft Set, a framework that works with multi-objective and multi-attributive decision-making structures. In the IFHSS structure due to the new structure a lot of work to do, so we developed some cosine and cotangent similarity measures that are missing in the literature. These definitions of SMs enable us to address multi-attributive, multi-objective, and issues involving both membership and non-membership and disjoint attributive value in the context of IFHSS.

The arrangement of the study is as in Sec-01 we reviewed some fundamental ideas about soft sets, HSSs, and IFHSSs in Sec-2. In Sec-3, we discussed the definition and example of Intuitionistic fuzzy hypersoft matrices with an example. In Sec.4 we provide five trigonometric similarity measurements for IFHSSs using the cosine and cotangent trigonometric functions. When it comes to these trigonometric similarity measurements, we provide operators, theorems, and assertions. Moreover, we provide weighted variations of these. In Sec-5, we apply them to the problem of choosing a renewable energy source to demonstrate how well the suggested similarity measurements work. The concluding section and plans for further research are covered in Section 6.

2. Preliminaries

We briefly describe SS, HSS and IFHS's in this section.

2.1. Definition

Molodtsov first formulated the idea of a SS in 1999 to overcome the problems of uncertain decision makings and defined as a family of parameterized subsets, where every element is contemplated as a collection of approximately similar elements. Let $\mathscr{J} = \{\tau_1, \tau_2, \tau_3, \ldots\tau_s\}$, be a universal set and $\wp$ indicates a set of attributes corresponding to $\mathscr{J}$. Let $\wp(\mathscr{J})$ indicate the collection of all subsets (Power Set) of $\mathscr{J}$ and $S^{'} \subset \wp$. A pair $(\mathscr{S}, S^{'})$ is called a SS over $\mathscr{J}$, where the mapp $\mathscr{S}$ is defined by

$$\mathscr{S} : S^{'} \rightarrow P(\mathscr{J}).$$

2.2. Definition

In December 2018 Florentine coined the emerging idea of HSS as an extension of SS to discuss multi-attributive disjoint problems. Suppose $\mathscr{J} = \{\tau_1, \tau_2, \tau_3, \ldots\tau_s\}$ is a set of alternatives and $\wp$ signifies a set of parameters. Let the collection of all subsets of $\mathscr{J}$ representing by $P(\mathscr{J})$. Suppose $\mathscr{L}^1, \mathscr{L}^2, \mathscr{L}^3, \ldots\mathscr{L}^m$ for $m \geq 1$ be distinct characteristics, and the sets $\mathscr{N}^1, \mathscr{N}^2, \mathscr{N}^3, \ldots\mathscr{N}^m$ of feature values that correlate to them with $\mathscr{N}^a \cap \mathscr{N}^b = \varnothing$ for $a \neq b$, *where* $a, b = 1, 2, 3, 4, \ldots m$, with respect to the features. So, the order pair $(\mathscr{S}, \mathscr{N}^1, \mathscr{N}^2, \mathscr{N}^3, \ldots\mathscr{N}^m)$ is called HHS on $\mathscr{J}$, where

$$\mathscr{S} : \mathscr{N}^1 \times \mathscr{N}^2 \times \mathscr{N}^3 \times \ldots \times \mathscr{N}^m \rightarrow \wp(\mathscr{J})$$

2.3. Definition

Suppose $\wp$ denotes a set of parameters and $\mathscr{J} = \{\tau_1, \tau_2, \tau_3, \ldots\tau_s\}$ is a set of alternatives. Let the collection of all subsets of $\mathscr{J}$ is indicated by $P(\mathscr{J})$. Suppose $\mathscr{L}^1, \mathscr{L}^2, \mathscr{L}^3, \ldots \mathscr{L}^m$ for $m \geq 1$ be m distinct and well-defined characteristics, and the feature values corresponding to them are sets $\mathscr{N}^1, \mathscr{N}^2, \mathscr{N}^3, \ldots\mathscr{N}^m$ with $\mathscr{N}^a \cap \mathscr{N}^b = \varnothing$ for $a \neq$ b, where a, b = 1, 2 … m, respectively. Then the pair is called IFHSS over $\mathscr{J}$, where $\mathscr{T} = \mathscr{N}^1 \times \mathscr{N}^2 \times \mathscr{N}^3 \times \ldots \times \mathscr{N}^n \rightarrow \wp(\mathscr{J})$:

$$\mathscr{S}\left(\mathscr{N}^1 \times \mathscr{N}^2 \times \mathscr{N}^3 \times \ldots \times \mathscr{N}^n\right) = \mathscr{S}(\mathscr{T})$$

$$= \{< \tau, T_{r}(\mathscr{S}(\mathscr{T})), F_{r}(\mathscr{S}(\mathscr{T})), \tau \in \mathscr{J} >\}$$

Where F denotes the falsity and T stands for the truthiness of respective values. $T_{r}, F_{r} : \mathscr{J} \rightarrow [0, 1]$ with $0 \leq T_{r}(\mathscr{S}(\mathscr{T})) + F_{r}(\mathscr{S}(\mathscr{T})) \leq 2$.:

3. Intuitionistic fuzzy hypersoft matrices (IFHSMs)

Decision-making in many different fields and industries depends heavily on matrices. They provide a rigorous, methodical approach to evaluating many choices according to various standards or features. We will expand on the idea of IFHSSs to IFHSMs in this part and go on to describe how they work. While PFHSSs provide more precise scenarios for decision-making, IFHSMs' matrix shape provides faster solutions. Next, we describe IFHSMs.

3.1. Definition

let $S = \{S_1, S_2, \ldots, S_\alpha\}$ be an universal set with α options, and let $\mathfrak{Q} = \{\mathfrak{Q}_1, \mathfrak{Q}_2, \ldots, \mathfrak{Q}_\beta\}$ be a set of disjoint β attributes with their corresponding attributive values of $\mathfrak{Q}_1^a, \mathfrak{Q}_2^b, \ldots, \mathfrak{Q}_\beta^z$ where a,b,c,…z = 1,2,3,…,n. Let $\mathscr{P}(S)$ denote the collection of all power sets of S. An IFHSS over S is defined as $(\acute{K}, \mathfrak{Q}_1^a \times \mathfrak{Q}_2^b \times \ldots \times \mathfrak{Q}_\beta^z)$ such that $\acute{K}, : \mathfrak{Q}_1^a \times \mathfrak{Q}_2^b \times \ldots \times \mathfrak{Q}_\beta^z \rightarrow \mathscr{P}(S)$ defined by $\acute{K}(\mathfrak{Q}_1^a \times \mathfrak{Q}_2^b \times \ldots \times \mathfrak{Q}_\beta^z) = \{\langle Ꝑ^j, (Ŀ_{Ꝑ}(Ꝑ^j, S_\tau), \eta_{Ꝑ}(Ꝑ^j, S_\tau)) \rangle, S_\tau \in S, Ꝑ \in \mathfrak{Q}_1^a \times \mathfrak{Q}_2^b \times \mathfrak{Q}_3^c \times \ldots \times \mathfrak{Q}_\beta^z\}$ where $0 \leq Ŀ_{Ꝑ}^2(S_\tau) + \eta_{Ꝑ}^2(S_\tau) \leq 1$. Thus, a IFHSM is defined in Table 1 with a matrix form as follows:

If $\zeta_{ij} = \mathscr{X}_{p_{Ꝑ}}(S^i, \mathfrak{Q}_j^k)$, where $i = 1, 2, 3\ldots\alpha, j = 1, 2, 3, \ldots\beta, k = a, b, c, \ldots z$, then a matrix is defined as

$$[\zeta_{ij}]_{\alpha\times\beta} = \begin{pmatrix} \zeta_{11} & \zeta_{12} & \cdots & \zeta_{1\beta} \\ \zeta_{21} & \zeta_{22} & \cdots & \zeta_{2\beta} \\ \vdots & \vdots & \ddots & \vdots \\ \zeta_{\alpha 1} & \zeta_{\alpha 2} & \cdots & \zeta_{\alpha\beta} \end{pmatrix}$$

where

$$\zeta_{ij} = \begin{pmatrix} \left(\mathtt{t}_{\mathfrak{Q}_j^k}(\mathrm{S}_i), \eta_{\mathfrak{Q}_j^k}(\mathrm{S}_i)\right), \mathrm{S}_i \in \mathrm{S} \\ , \mathfrak{Q}_j^k \in \left(\mathfrak{Q}_1^{\alpha} \times \mathfrak{Q}_2^{\ell} \times \ldots \times \mathfrak{Q}_{\beta}^{z}\right) \end{pmatrix} = \left(\mathtt{t}_{\mathfrak{Q}_j^k}(\mathrm{S}_i), \eta_{\mathfrak{Q}_j^k}(\mathrm{S}_i)\right).$$

For simplicity, we may assume that $\mathtt{t}_{\mathfrak{Q}_j^k}(\mathrm{S}_i) = \mathtt{t}_{ij}$ and $\eta_{\mathfrak{Q}_j^k}(\mathrm{S}_i) = \eta_{ij},$ where *i* represents the position of alternatives, *j* tells us about the attributes, hidden *k* tells us about sub-attributive value of the corresponding attribute, and $\mathfrak{Q}$ is the subset of the IFHSS. Thus, the matrix representation is as

$$M_{\alpha\times\beta} = \begin{bmatrix} (\mathtt{t}_{11}, \eta_{11}) & (\mathtt{t}_{12}, \eta_{12}) & \ldots & (\mathtt{t}_{1\beta}, \eta_{1\beta}) \\ (\mathtt{t}_{21}, \eta_{21}) & (\mathtt{t}_{22}, \eta_{22}) & \ldots & (\mathtt{t}_{2\beta}, \eta_{2\beta}) \\ \vdots & \vdots & & \vdots \;\; \vdots \\ (\mathtt{t}_{\alpha1}, \eta_{\alpha1}) & (\mathtt{t}_{\alpha2}, \eta_{\alpha2}) & \cdots & (\mathtt{t}_{\alpha\beta}, \eta_{\alpha\beta}) \end{bmatrix}$$

which is called a IFHSM of order $\alpha \times \beta$ over S.:

Example 1. Let $\mathrm{S} = \{\mathrm{S}_1, \mathrm{S}_2, \mathrm{S}_3, \mathrm{S}_4, \mathrm{S}_5\}$ be the set of five alternatives (mobiles) and $\mathfrak{Q} = \{\mathfrak{Q}_1, \mathfrak{Q}_2, \mathfrak{Q}_3\}$ be the set of attributes with $\mathfrak{Q}_1 =$ Battery $= \{4000\text{ mah}, 5000\text{ mah}\},\ \mathfrak{Q}_2 = \text{Ram} = \{6\text{ GB}, 8\text{ GB}, 10\text{ GB}\},\ \mathfrak{Q}_3 = \text{Display Size} = \{5, 6'', 7''\}$. Then, $\varrho(4000\text{mah}, 8\text{GB}, 6'') = \{\mathrm{S}_1, \mathrm{S}_2, \mathrm{S}_3\},$ where $\{(\mathrm{S}_1, 4000\text{mah}(0.5, 0.6)), 8\text{GB}(0.4, 0.5), 6''(0.8, 0.6)\},\ \{(\mathrm{S}_2, 4000\text{mah}(0.3, 0.7)), 8\text{GB}(0.4, 0.4), 6''(0.6, 0.7)\},$ and $\{(\mathrm{S}_3, 4000\text{mah}(0.7, 0.4)), 8\text{GB}(0.6, 0.5), 6''(0.6, 0.6)\}$. Thus, we have that

$$M = \begin{array}{c} \\ \mathrm{S}_1 \\ \mathrm{S}_2 \\ \mathrm{S}_3 \end{array} \begin{array}{c} \begin{array}{ccc} 4000\text{mah} & 8\text{GB} & 6'' \end{array} \\ \begin{bmatrix} (0.5, 0.6) & (0.4, 0.5) & (0.8, 0.6) \\ (0.3, 0.7) & (0.4, 0.4) & (0.6, 0.7) \\ (0.7, 0.4) & (0.6, 0.5) & (0.6, 0.6) \end{bmatrix} \end{array}$$

Each sum of square of the order pair of truthiness and falseness is always lying in the unit interval $[0, 1]$. The above-mentioned example showing the result of IFHSSs.

4. Cosine and cotangent SM of IFHSS's

In many domains, trigonometric similarity measures are significant because they offer vital instruments for resolving geometrical issues, traversing real-world environments, deciphering physical occurrences, and advancing science and technology. We are going to propose the SM's (Cosine and Cotangent) of IFHSS's.

4.1. Definition

Let $\mathscr{J}$ be a set of alternatives and $\mathscr{M} = \{< \tau, T_{\mathscr{M}}(\mathscr{S}(\mathscr{T})), F_{\mathscr{M}}(\mathscr{S}(\mathscr{T})), \tau \in \mathscr{J} >\}$ and $\mathscr{H} = \{< \tau, T_{\mathscr{H}}(\mathscr{S}(\mathscr{T})), F_{\mathscr{H}}(\mathscr{S}(\mathscr{T})), \tau \in \mathscr{J} >\}$ be the two IFHSSs for $\mathscr{S}(\mathscr{T})$. The cosine SMs between two sets $\mathscr{M}$ and $\mathscr{H}$ by using an arithmetic mean is given as

$$S^1_{IFHSS}(\mathscr{M}, \mathscr{H}) = \frac{1}{n}\sum_{r=1}^{n} \frac{(T_{\mathscr{M}}(\mathscr{S}(\mathscr{T})))_r \left(T_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))\right)_r + (\mathrm{F}_{\mathscr{M}}(\mathscr{S}(\mathscr{T})))_r (\mathrm{F}_{\mathscr{H}}(\mathscr{S}(\mathscr{T})))_r}{\sqrt{\left(\mathrm{T}^2_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))\right)_r + \left(\mathrm{F}^2_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))\right)_r}\sqrt{\left(\mathrm{T}^2_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))\right)_r + \left(\mathrm{F}^2_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))\right)_r}} \tag{4.1}$$

Proposition 1. The cosine SMs $\mathrm{S}^1_{\mathrm{IFHSS}}$ must holds the properties $(\mathrm{P}_1) -\ (\mathrm{P}_3)$

(P_1) $0 \le S^1_{IFHSS}\left(\mathscr{M}, \mathscr{H}\right) \le 1$

(P_2) $S^1_{IFHSS}\left(\mathscr{M}, \mathscr{H}\right) = S^1_{IFHSS}(\mathscr{H}, \mathscr{M})$

(P_3) if $\mathscr{M} = \mathscr{H}$ then $S^1_{IFHSS}\left(\mathscr{M}, \mathscr{H}\right) = 1$

Proof. (P_1), as given below

Property-1 (P1). $\frac{1}{n}\sum_{r=1}^{n} \frac{(\mathrm{T}_{\mathscr{M}}(\mathscr{S}(\mathscr{T})))_r (T_{\mathscr{H}}(\mathscr{S}(\mathscr{T})))_r + (\mathrm{F}_{\mathscr{M}}(\mathscr{S}(\mathscr{T})))_r (\mathrm{F}_{\mathscr{H}}(\mathscr{S}(\mathscr{T})))_r}{\sqrt{(\mathrm{T}^2_{\mathscr{M}}(\mathscr{S}(\mathscr{T})))_r + (\mathrm{F}^2_{\mathscr{M}}(\mathscr{S}(\mathscr{T})))_r}\sqrt{(\mathrm{T}^2_{\mathscr{H}}(\mathscr{S}(\mathscr{T})))_r + (\mathrm{F}^2_{\mathscr{H}}(\mathscr{S}(\mathscr{T})))_r}} = \cos\theta$

Remember that all the values must lying in closed interval [0, 1], therefore

$$0 \leq S^1_{IFHSS}(\mathscr{M},\mathscr{H}) \leq 1.$$

Property-2 (P2). Proof of (P_2) is very simple and easy.

Property-3 (P3). If $\mathscr{M} = \mathscr{H}$, then $T_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r = T_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r$ and $F_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r = F_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r$ for $r = 1,2,3,\ldots n$. consequently, we attained $S^1_{IFHSS}(\mathscr{M},\mathscr{H}) = 1$.

4.2. Definition

Suppose, $\mathscr{J}$ is a set of alternatives and $\mathscr{M} = \{<\gamma, T_{\mathscr{M}}(\mathscr{S}(\mathscr{T})), F_{\mathscr{M}}(\mathscr{S}(\mathscr{T})), \gamma \in \mathscr{J}>\}$ and $\mathscr{H} = \{<\gamma, T_{\mathscr{H}}(\mathscr{S}(\mathscr{T})), F_{\mathscr{H}}(\mathscr{S}(\mathscr{T})), \gamma \in \mathscr{J}>\}$ are two IFHSSs for $\mathscr{S}(\mathscr{T})$. A cosine SMs between $\mathscr{M}$ and $\mathscr{H}$ based on the function of cosine is given by

$$S^2_{IFHSS}(\mathscr{M},\mathscr{H}) = \frac{1}{m}\sum_{r=1}^{m} \cos\left[\frac{\pi}{2}\left(\left|T_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - T_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r\right| \vee \left|F_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - F_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r\right|\right)\right] \tag{4.2}$$

$$S^3_{IFHSS}(\mathscr{M},\mathscr{H}) = \frac{1}{m}\sum_{r=1}^{m} \cos\left[\frac{\pi}{6}\left(\left|T_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - T_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r\right| \vee \left|F_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - F_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r\right|\right)\right] \tag{4.3}$$

Proposition 2. The SMs cosine $S^t_{IFHSS}(\mathscr{M},\mathscr{H})$, ($t = 2$ and 3) accomplish P_1, P_2 and the following properties:

$(\mathbf{P}'_3)$: $\mathscr{M} = \mathscr{H}$ iff $S^t_{IFHSS}(\mathscr{M},\mathscr{H}) = 1$, ($t = 2$ and 3).

$(\mathbf{P}_4)$: If F is a IFHSSs and $\mathscr{M} \subset \mathscr{H} \subset F$ then $S^t_{IFHSS}(\mathscr{M},F) \leq S^t_{IFHSS}(\mathscr{M},\mathscr{H})$ and $S^t_{IFHSS}(\mathscr{M},F) \leq S^t_{IFHSS}(\mathscr{H},F)$.

Property-1 (P1). The SMs S^2_{IFHSS} and S^3_{IFHSS} are depending upon the cosine function's value and the membership and non-membership in the unit interval [0, 1] of IFHSSs. Therefore, $0 \leq S^t_{IFHSS}(\mathscr{M},\mathscr{H}) \leq 1$ for $t = 2$ and 3.

Property-2 (P2). Proof of (P_2) is very simple and easy.

Propert-3 (P3). In $\mathscr{J}$ suppose two IFHSSs $\mathscr{M},\mathscr{H}$, when $\mathscr{M} = \mathscr{H}$, then $T_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r = T_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r$ and $F_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r = F_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r$ for $r = 1,2,3,4,\ldots m$. so we get

$$\left|T_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - T_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r\right| = 0,$$

$$\left|F_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - F_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r\right| = 0.$$

Thus, the cosine SMs $S^k_{IFHSS}(\mathscr{M},\mathscr{H}) = 1$, for $t = 2$ and 3 conversely, let $S^t_{IFHSS}(\mathscr{M},\mathscr{H}) = 1$, for $t = 2$ and 3 because $\cos 0 = 1$, hence,

$$\left|T_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - T_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r\right| = 0,$$

$$\left|F_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - F_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r\right| = 0.$$

Hence, we have $T_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r = T_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r$ and $F_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r = F_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r$ for $r = 1,2,3,4\ldots m$. Hence $\mathscr{M} = \mathscr{H}$:

$(\mathbf{P}_4)$: If $\mathscr{M} \subset \mathscr{H} \subset F$, then $T_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r \leq T_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r \leq T_F(\mathscr{S}(\mathscr{T}))_r$ and $F_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r \geq F_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r \geq F_F(\mathscr{S}(\mathscr{T}))_r$ for $r = 1,2,3,4\ldots m$.

Therefore, we have

$$\left|T_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - T_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r\right| \leq \left|T_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - (T_F(\mathscr{S}(\mathscr{T}))_r\right|,$$

$$\left|T_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r - T_F(\mathscr{S}(\mathscr{T}))_r\right| \leq \left|T_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - (T_F(\mathscr{S}(\mathscr{T}))_r\right|,$$

$$\left|F_{\mathscr{M}}\mathscr{S}(\mathscr{T}))_r - F_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r\right| \geq \left|F_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - F_F(\mathscr{S}(\mathscr{T}))_r\right|,$$

$$\left|F_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r - F_F(\mathscr{S}(\mathscr{T}))_r\right| \geq \left|F_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - F_F(\mathscr{S}(\mathscr{T}))_r\right|.$$

Hence, $\mathscr{M} \subset \mathscr{H} \subset F$, then, $S^t_{IFHSS}(\mathscr{M},F) \leq S^t_{IFHSS}(\mathscr{M},\mathscr{H})$ and $S^t_{IFHSS}(\mathscr{M},F) \leq S^t_{IFHSS}(\mathscr{H},F)$. For $t = 2,3$, and we know that the cosine function is non-increasing in the interval $\left[0, \frac{\pi}{2}\right]$ so the proof is completed.

4.3. Definition

Let $\mathscr{J}$ be the set of alternatives and assume $\mathscr{M} = \{<\gamma, T_{\mathscr{M}}(\mathscr{S}(\mathscr{T})), F_{\mathscr{M}}(\mathscr{S}(\mathscr{T})), \gamma \in \mathscr{J}>\}$ and $\mathscr{H} = \{<\gamma, T_{\mathscr{H}}(\mathscr{S}(\mathscr{T})), F_{\mathscr{H}}(\mathscr{S}(\mathscr{T})), \gamma \in \mathscr{J}>\}$ are two IFHSSs of $\mathscr{S}(\mathscr{T})$. Then Cotangent SM's between $\mathscr{M}$ and D are defined as

$$S^4_{IFHSS}(\mathscr{M},\mathscr{H}) = \frac{1}{n}\sum_{r=1}^{n} cot\left[\frac{\pi}{4}+\frac{\pi}{4}\left(|T_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - T_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r| \vee |F_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - F_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r|\right)\right], \tag{4.4}$$

$$S^5_{IFHSS}(\mathscr{M},\mathscr{H}) = \frac{1}{n}\sum_{r=1}^{n} cot\left[\frac{\pi}{4}+\frac{\pi}{12}\left(|T_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - T_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r| \vee |F_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - F_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r|\right)\right]. \tag{4.5}$$

Where ∨ is a max-operator

Proposition- 3. Cotangent SM's S^t_{IFHSS}, ($t = 4$ and 5) satisfies P_1, P_2, P_3 and P_4.

Proof. By applying prop-2 we can prove it easily.

Weighted SM's of equations SMs (4.1) − (4.5) are given as following, Remember that the weights should follow the inequality relation

$$0 \le \mathscr{Q}_1, \mathscr{Q}_2, \mathscr{Q}_3, \ldots \mathscr{Q}_n \le 1 \text{ with } \sum_r^n \mathscr{Q}_r = 1.$$

$$wS^1_{IFHSS}(\mathscr{M},\mathscr{H}) = \frac{1}{n}\sum_{r=1}^{n}\mathscr{Q}_r \frac{(T_{\mathscr{M}}(\mathscr{S}(\mathscr{T})))_r(T_{\mathscr{H}}(\mathscr{S}(\mathscr{T})))_r + (F_{\mathscr{M}}(\mathscr{S}(\mathscr{T})))_r(F_{\mathscr{H}}(\mathscr{S}(\mathscr{T})))_r}{\sqrt{T^2_{\mathscr{M}}(\mathscr{S}(\mathscr{T})))_r + (F^2_{\mathscr{M}}(\mathscr{S}(\mathscr{T})))_r}\sqrt{(T^2_{\mathscr{H}}(\mathscr{S}(\mathscr{T})))_r + (F^2_{\mathscr{H}}(\mathscr{S}(\mathscr{T})))_r}}$$

$$wS^2_{IFHSS}(\mathscr{M},\mathscr{H}) = \frac{1}{n}\sum_{r=1}^{n}\mathscr{Q}_r \cos\left[\frac{\pi}{2}\left(|T_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - T_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r| \vee |F_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - F_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r|\right)\right]$$

$$wS^3_{IFHSS}(\mathscr{M},\mathscr{H}) = \frac{1}{n}\sum_{r=1}^{n}\mathscr{Q}_r\, cos\left[\frac{\pi}{6}\left(|T_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - T_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r| \vee |F_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - F_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r|\right)\right]$$

$$wS^4_{IFHSS}(\mathscr{M},\mathscr{H}) = \frac{1}{n}\sum_{r=1}^{n}\mathscr{Q}_r \cot\left[\frac{\pi}{4}+\frac{\pi}{4}\left[\left(\langle|T_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - T_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r| \vee |F_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - F_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r|\rangle\right)\right]\right]$$

$$wS^5_{IFHSS}(\mathscr{M},\mathscr{H}) = \frac{1}{n}\sum_{r=1}^{n}\mathscr{Q}_r\, cot\left[\frac{\pi}{4}+\frac{\pi}{12}\left(|T_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - T_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r| \vee |F_{\mathscr{M}}(\mathscr{S}(\mathscr{T}))_r - F_{\mathscr{H}}(\mathscr{S}(\mathscr{T}))_r|\right)\right]$$

5. Algorithm with application

In problem-solving and decision-making processes, similarity measurements and Multiple Attributive Decision Making (MADM) are related concepts. The study of decision-making when several, frequently conflicting variables need to be taken into account is the main emphasis of MADM. In the MADM framework, similarity measures play a critical role in assessing options or alternatives in light of these numerous criteria. Using similarity measurements, decision-makers can assess how close or similar alternatives are to one another, which facilitates assessing how well each option satisfies the necessary criteria. The selection of renewable energy sources involves assessing and comparing different options based on various criteria. Distance similarity measures can be linked to this problem by providing a quantitative way to compare the similarity or dissimilarity between different renewable energy sources.

This section is intended to address a methodology based on the proposed work then we will apply the proposed algorithm in Renewable Energy Source Selection.

5.1. Proposed algorithm

Let $\mathfrak{G}^1, \mathfrak{G}^2, \mathfrak{G}^3 \ldots \mathfrak{G}^n$ be the disjoint set of strategical regions of a country, �^{1}, �^{2}, �^{3}…�^{n} by the set of parameters for strategical regions and $\flat^1, \flat^2, \flat^3 \ldots \flat^n$ be the power set of different systems for each strategical region. The decision makers can evaluate $\flat$ power types and $\mathfrak{G}$ regions and under � norms by using a proposed algorithm. This analysis makes it possible to decide which environmentally friendly energy source should be used in which area. As a result, it can decide which geographic locations and sources of renewable energy are the best match.

Now, we give the implementation steps of the proposed cosine and cotangent SM's of IFHSS.

Step-1. It is essential to first choose which kind of renewable energy sources are appropriate for the various regions that will be evaluated. Consequently, it's important to establish the criteria and sources of energy for these regions. The relationship between geographical locations and parameters should be provided using a decision matrix of IFHSS (see, e.g., below Table 2).

Step-2. By using intuitionistic fuzzy hypersoft sets in decision matrix form, it is possible to figure out how standards and different types of renewable energy options relate to one another. (See, e.g., below Table 3).

Step-3. Using the proposed cosine and cotangent SM's for IFHSS, equations (4.1)-(4.5) in table 5 to table 8 are applied to figure out the connection between geographic locations and the alternatives. The highest value is chosen because it represents the finest option for the particular area, making it the best decision.

The proposed algorithm flowchart is shown below. in Fig. 1.

5.2. Application

Global warming has a negative impact on the environment, but countries are working to mitigate these effects. One of these initiatives is a development strategy that takes into account the government policy for renewable energy. The need for renewable energy sources has increased, and countries are starting to recognize the kinds and amounts of these resources they own. As a result, we try to design a mathematical technique for the problem of renewable energy sources, which is a problem of the environment. We emphasize appealing to all countries by keeping a high standard scale and broad lower standard ranges.

$$\mathfrak{G} = \{\mathfrak{G}^1, \mathfrak{G}^2, \mathfrak{G}^3, \ldots, \mathfrak{G}^{10}\}$$

Second, we determined the most extensively employed and recommended renewable energy sources globally, and we displayed them using the set E with.

$\wp$ = {Geothermal Power, Wind Power, Hydroelectric Power, Solar Power}.

We just choose most often used criteria and sub-criteria in order to assess these energy resources and geographic areas.

$$� = \left\{ \begin{array}{c} �^1 \text{ (average annual daily bath time in } (h/day)) \\ �^2 (\text{Intensity average flow of streams } (m^3/sec)) \\ �^3 (\text{Annually rainfall (average) in } (mm)) \\ �^4 (\text{Annually Average daily Wind Speed in } (km/h)) \\ �^5 \text{ (Geothermal Water Density (underground) } (\%)) \end{array} \right\}$$

Here little illustration of the above mentioned parameters

$$�^1 = \left\{ \begin{array}{c} \text{below } 1h, 1h - 4h, 4.1h - 6h \\ 6.1h - 8h, 8.1h - 10h, 10.1h - 14h \\ \text{above } 14\ h \end{array} \right\}$$

$$�^2 = \left\{ \begin{array}{c} \text{below } 501\ m^3, 501\ m^3 - 2001m^3 \\ 2001\ m^3 - 4001m^3, 4001 - 6001m^3 \\ 6001m^3 - 8001m^3, 8001m^3 - 10,001m^3 \\ 10,001m^3 - 20001m^3, 20,001m^3 - 40,001m^3 \\ \text{Above } 40,001m^3 \end{array} \right\}$$

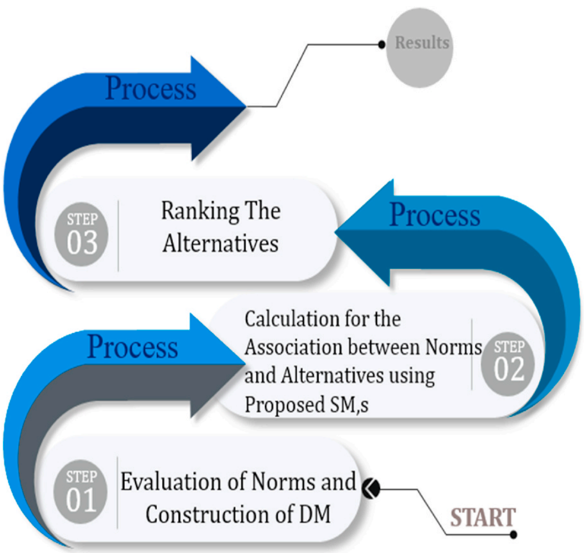

Fig. 1. Flow Chart of proposed Algorithm.

$$�^3 = \left\{ \begin{array}{c} \text{below } 251mm, 251mm - 501mm \\ 501mm - 1001mm, 1001mm - 2001mm \\ 2001mm - 4001mm, 4001mm - 6001mm \\ 6001mm - 8001mm, 8001mm - 10,001mm \\ \text{above } 10,000mm \end{array} \right\}$$

$$�^4 = \left\{ \begin{array}{c} 10km \text{ or below}, 11km - 20km, 21km - 35km \\ 36km - 55km, 56km - 70km, 71km - 100km \\ \text{above } 100\ km \end{array} \right\}$$

$$�^5 = \left\{ \begin{array}{c} \text{below } 5\%, 5\% - 9\%, 9\% - 19\% \\ 19\% - 29\%, 29\% - 39\%, 39\% - 50\% \\ \text{above } 50\% \end{array} \right\}$$

The IFSSs are given as $\mathfrak{R} : (�^1 \times �^2 \times \ldots \times �^5) \rightarrow P(\mathfrak{G})$ and $\mathfrak{U} : (�^1 \times �^2 \times \ldots \times �^5) \rightarrow P(\flat)$. Let us assume that

$$\wp(\mathfrak{I}) = \left\{ \begin{array}{c} 6.1h - 8h, 501m^3 - 2001m^3, 501mm - 1001mm \\ 11km - 20km, 29\% - 39\% \end{array} \right\}$$

We evaluate $\{\mathfrak{G}^1, \mathfrak{G}^2, \mathfrak{G}^3, \mathfrak{G}^4, \mathfrak{G}^5\}$ and {Geothermal Power, Wind Power, Hydroelectric Power, Solar Power} (see Table 7). We construct the IFHSS to build the relation in $\{\mathfrak{G}^1, \mathfrak{G}^2, \mathfrak{G}^3, \mathfrak{G}^4, \mathfrak{G}^5\}$ and $\{6.1h - 8h, 501m^3 - 2001m^3, 501mm - 1001mm, 11km - 20km, 29\% - 39\%\}$. Now we construct the second association, as shown in Table 1. Then, we determine the association between $\flat =$ {Geothermal Power, Wind Power, Hydroelectric Power, and Solar Power} and $\{6.1h - 8h, 501m^3 - 2001m^3, 501mm - 1001mm, 11km - 20km, 29\% - 39\%\}$ According to Step 2, the association is given by IFHSSs decision matrix shown in Table 2. Now, we developed the relation between $\mathfrak{G} = \{\mathfrak{G}^1, \mathfrak{G}^2, \mathfrak{G}^3, \mathfrak{G}^4, \mathfrak{G}^5\}$ and $\flat =$ {Geothermal Power, Wind Power, Hydroelectric Power, Solar Power} using Step 3, the association is determined with the proposed cosine and cotangent SM's for IFHSSs by using equations (4.1)-(4.5), as shown in tables Table 4 and Table 8 (see Table 5).

6. Result discussion

Decision-making in a vague and imprecise environment is a crucial issue, we developed an IFHSS structure for such vague environments. In this article, we proposed three versions of cosine similarity and two versions of cotangent SMs. The results of all the definitions coincide which shows the accuracy of the definitions. In results G^1 and G^5 are selected for wind power, G^2 and G^3 are selected for Hydroelectric power and G^4 is selected for solar power according to the stability of the parameters with systems and geographical regions. Here we presented the

7. Comparison with existing techniques

Hypersoft sets offer a robust logical foundation for addressing the challenges inherent in multi-criteria decision-making. Their ability to represent granularity, flexibility in handling uncertainty, consideration of multiple criteria, and adaptability to decision-maker preferences make them a strong choice for modeling the complexities of real-world decision problems. The application of hypersoft sets in MCDM contributes to a more realistic and comprehensive approach to decision-making in the face of uncertainty and imprecision. Some of existing theories are shown in table below, due to sub-attributions of multiple attributes the decision making is more refined. We proposed IFHSS, IFHSM and their trigonometric SM's and compare it with existing techniques with the shortcomings and limitations of existing techniques which shows the novelity of the proposed study. Table 9.

8. Future directions and limitations

The suggested IFHSS-Similarity measures offer enormous potential for MADM difficulties in various fields, including supplier selection, manufacturing frameworks, and a variety of other management frameworks. There are many MADM techniques, such as TOPSIS, SAW, AHP, VIKOR, etc. In future works, we will re-construct these algorithms and apply them to these MCDM techniques of TOPSIS, SAW, AHP, VIKOR, etc. under the PFHSSs structure. The limitations of the proposed study is firstly it is dealing with only with disjoint attributes which is the first limitation of the proposed study. Secondly the trigonometric similarity measures have some limitations as dependency on the domain, trigonometric similarity measures may or may not be successful. Since they might work effectively in some situations but not others, they might not be as adaptive as they could be.

Table 9
Comparison of the Proposed Study with existing techniques

Researcher	Truthiness	Falseness	Attributes	Sub-attributes	Matrix Representaion	Trigonometric SMs
Z. Liang et al. [17]	✓	✓	✓	✘	✓	✘
M. Luo, R. Zhao [39]	✓	✓	✓	✘	✘	✘
F. Xiao [40]	✓	✓	✓	✘	✘	✘
M. Ram and S·B Singh [41]	✓	✓	✓	✘	✘	✘
S. Gogoi et al. [42]	✓	✓	✓	✘	✓	✘
Zulqarnain et al. [32]	✓	✓	✓	✓	✘	✘
Proposed	✓	✓	✓	✓	✓	✓

9. Conclusion

A novel model called the Intuitionistic Fuzzy Hypersoft Set (IFHSS) attempts to overcome the constraints of IFHSS regarding the use of a multi-argument domain for estimating the pertinent parameters. Because it accounts for the additional classification of parameters into their proper parametric valued sets, it is more dependable and versatile. For this investigation, we explored a number of other avenues. Several axiomatic findings, operational outcomes, and aggregation strategies were first presented. In this article we developed ten new similarity measures based on cosine and cotangent functions, in IFHSS environments. Several theorems, propositions, and results have been presented in this study using all proposed definitions. We developed algorithm to using proposed similarity measures to solve multi-attributive problems. Finally, we solved multi-attributive, multi-criteria decision-making problem of renewable energy source selection by using proposed technique. The suggested IFHSS-Similarity measures offer enormous potential for MADM difficulties in various fields, including supplier selection, manufacturing frameworks, and a variety of other management frameworks. There are many MADM techniques, such as TOPSIS, SAW, AHP, VIKOR, etc. In future works, we will re-construct these algorithms and apply them to these MCDM techniques of TOPSIS, SAW, AHP, VIKOR, etc. under the PFHSSs structure.

Data availability statement

No data was used for the research describle in the article.

CRediT authorship contribution statement

Muhammad Naveed Jafar: Writing – original draft, Conceptualization. **Muhammad Saeed:** Supervision. **Ayesha Saeed:** Methodology, Investigation, Formal analysis. **Aleen Ijaz:** Methodology. **Mobeen Ashraf:** Writing – review & editing, Visualization. **Fahd Jarad:** Funding acquisition.

Declaration of competing interest

The authors whose names are listed immediately below certify that they have NO affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers' bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements), or non-financial interest (such as personal or professional relationships, affiliations, knowledge or beliefs) in the subject matter or materials discussed in this manuscript.

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