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Research article

Vietnamese sixth graders' mathematical communication competency developed by teaching fraction topics using the 5E model

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A R T I C L E I N F O

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A B S T R A C T

The 5E, an exploratory instructional model with growing popularity in current learner-centered educational practices, aims to develop learners' competencies. The study aims to evaluate the effectiveness of teaching fractions using the 5E instructional model to develop students' mathematical communication competency (MCC). The researchers used mixed methods with data triangulation in a 38-student experiment group (EG) who learned with the 5E model and a 41-student control group (CG) using traditional learning approaches to clarify the impact of the 5E model on the development of MCC, academic performance and learning attitude among learners. The research instruments include a pre-test, post-test, classroom observation, and a student survey. Subsequently, the data collected were analyzed qualitatively and quantitatively using IBM SPSS Statistics (Version 26) predictive analytics software. The independent *t*-test on the post-test scores of the two groups revealed that the academic performance of EG after the intervention was significantly better than that of CG with the significance level $\alpha = 0.05$ and degree of freedom $df = 77$, yielding the p-value (Sig. 2-tailed) = 0.001. Meanwhile, given the significance level $\alpha = 0.05$, the p-value (Sig. 2-tailed) = 0.001 and the Pearson correlation coefficient (r = 0.943), the result of the paired sample *t*-test implies that the mean post-test score of students in EG was significantly higher than that of their pre-test. Furthermore, with an effect size of approximately 0.88, these figures suggest that applying the 5E model in the experimental process positively affected the academic performance of the students in EG and the development of MCC. A qualitative analysis using classroom observations and student surveys revealed that the 5E model helped students in the EG develop their MCC and an increasingly positive attitude toward learning. In addition to these findings, it also reveals some limitations of the study and proposes recommendations for future research.

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1. Introduction

Mathematics is indispensable in life, helping humans solve daily problems [30]. Therefore, developing learners' mathematical competencies has always been considered the top priority of mathematics education. With the introduction of the 2018 General Education Curriculum, mathematics education in Vietnam has been dramatically transformed from a knowledge-based approach to a competency-based approach [26,40]. In particular, MCC is one of the core target mathematical competencies in the Mathematics Curriculum [26]. This competency is the ability to understand mathematical problems through communication with written symbols, oral signals, and graphics and to express mathematical ideas in various ways [29]. Teachers should provide opportunities for students to improve their written and oral communication skills [21]. In addition to written tasks, teachers should allow students to present their ideas, raise questions, and discuss in class [21] to help students comprehensively promote MCC, especially at the secondary school level, a critical stage in developing core competencies [26].

In light of this objective, student-centered teaching methods foster an environment conducive to active teaching and learning, potentially leading to increased efficacy in instruction for both teachers and learners [20]. The student-centered instructional model widely used in competency-based mathematics teaching is the 5E instructional model [38,43,46]. This model was presented by Ref. [4], in which teachers set up learning tasks to inspire the learner's curiosity and encourage them to explore different solutions together by proposing explanations, elaborating their understanding of concepts and evaluating their background knowledge [19]. However, math teachers have not widely exploited this 5E model to develop mathematical competencies in Vietnam (see Section 3.2). The 5E instructional model is still a new research area, involving mainly theoretical studies and applied research in teaching subjects from the natural sciences. Within mathematics education, there is limited research on applying the 5E model to mathematics teaching, especially in developing students' MCC.

In Vietnam, the sixth-grade algebra curriculum includes a variety of knowledge topics such as number sets, operations in a set of natural numbers, integers, exponentials, divisibility rules, divisors, multiples, etc. The concept of fraction is presented in this curriculum as an essential topic of algebra in the secondary school mathematics curriculum, which not only provides learners with the necessary calculating skills but also develops their logical thinking and applicability of the learned knowledge to solve real-world math problems and other topic-related issues [26]. However, despite earlier access to this topic at the primary school level, sixth-grade students may encounter difficulties in learning this topic [1,25,34], which may stem from the fact that they are expected to rebuild their perception of fractions with numerators and denominators as integers or performing operations with fractions, or expressing real-world math problems with natural language [26].

Meanwhile, the 5E model with exploratory educational characteristics has effectively supported students' knowledge acquisition and the development of MCC through the inquiry process [46]. Therefore, applying the 5E model to teach fractions could effectively develop the MCC of learners. Moreover, despite the considerable number of international research studies on applying the 5E model in teaching at multiple levels and topics, there has been no evidence of the effectiveness of this model in teaching fractions to develop sixth-grade students' communication competency. Given the reasons mentioned above, the authors focused on applying the 5E model in teaching fraction topics to develop the MCC of sixth-grade students in Vietnam to confirm the effectiveness and feasibility of the 5E model when they learn this topic.

2. Literature review

2.1. The 5E model

2.1.1. The concept of the 5E model

The 5E model, developed in 1987 by BSCS Science Learning, the American education and research institute, has grown in considerable popularity since the publication "The BSCS 5E Instructional Model: Origins and Effectiveness" [4], following a period of designing and piloting. The 5E model is an exploratory instructional model based on constructivism theory [7], including five stages: engagement, exploration, explanation, elaboration, and evaluation [4]. At each stage, the learners directly participate in the activity process, reason, take actions, and train a wide range of skills from simple ones such as listening, speaking, reading, writing, and teamwork to more complex ones such as purposeful observing, comparing, analyzing, synthesizing, practicing, experimenting, evaluating and self-evaluating, etc. [2,7]. As a result, they can form and develop the competencies of communication, collaboration, exploitation of scientific language, critical thinking, and problem-solving [4,32,35,37,39].

Stage 1: Engagement. This activity stimulates learners' interest in learning content and participation in the learning process [13]. It focusses on increasing the learners' awareness of why they learn the target topic [10]. Teachers attract the learner's attention to the learning content through storytelling, games, role-playing, situational tasks, relevant real-world exercises, real-life questions, images, and videos related to the lesson content [2,7].

Stage 2: Exploration. At this stage, students can thoroughly explore the learning content [7] through problem-solving, scientific discovery, simulation or experiments, and practice. At the same time, the teacher acts as an observer, facilitator and supporter in terms of time and space for the learning experience [2]. Consequently, students can synthesize the learned knowledge through problem-solving and critical thinking.

Stage 3: Explanation. In this stage, the teacher encourages the students to explain the concept and relate it to the ideas collected in the exploration stage [13]. Subsequently, the teacher supplements or adjusts the learner's explanation and directly presents the concept with precise terms, procedures, or practical skills [2,7,10], providing learners with feedback on their explanation as a reference to construct new knowledge [4].

Stage 4: Elaboration. In this stage, learners solve problematic situations in learning and their real lives [2]. The teacher creates favorable conditions for students to practice and apply what they have learned during exploration to master the knowledge and transform their new concepts and skills to solve similar or relevant problems.

Stage 5: Evaluation. The teacher evaluates how learners apply knowledge and skills and allows students to self-evaluate through mutual evaluation between groups of students [5,37]. Therefore, the inquiry session can be repeated at this stage so that learners can communicate and adjust their explanations [33]. The evaluation in this phase is performed in an inquiry-based procedure that differs from conventional lessons [2]. Evaluations, which can be formal or informal, reveal students' constructed knowledge in their responses to oral questions, writing short summaries, filling out empty maps, reading a graph, and evaluating tables [2]. Consequently, the teacher can evaluate the students' performance in learning new knowledge and their progress in adjusting the teaching and learning directions; simultaneously, the students can self-evaluate their understanding of the target concept [7].

In summary, the 5E instructional model presents learners with a learning procedure that includes explaining, constructing, applying, and communicating, followed by self-evaluation and self-adjusting their knowledge [37].

2.1.2. Advantages of applying the 5E model in teaching mathematics

Many studies have highlighted the effectiveness of the 5E model in developing students' competencies. Suckoo and Ishizaka [37] examined the application of the 5E model in combination with inquiry-based learning and asserted the positive impact of this model on the development of the learner's core mathematical competencies. Furthermore, other research studies have drawn the same conclusions on the improvement of critical thinking competency [32], problem-solving skills [39], and MCC [38,46].

Regarding the 5E impact of the instructional model on the teaching process, numerous studies have noted that it promotes student engagement [44], strengthens a positive learning attitude [2,6,17,24] and increases the self-confidence of students [2,39]. In mathematics education, many studies state that factors, including emotions and beliefs in mathematics, directly influence the effectiveness of teaching and learning [36]. As further supporting arguments for these findings [2], 17] [31], and [39] also confirmed that student academic performance was improved after learning with the 5E model. Furthermore, this model showed significant effectiveness in inquiry-based conceptual teaching [7].

2.1.3. Challenges of applying the 5E model in teaching mathematics

Various challenges in applying the 5E model have been mentioned in previous studies. In the systematic review by Turan and Matteson [44] with 16 qualitative research studies on challenges for teachers in the application of the 5E model, the teachers mentioned difficulties in finding relevant activities to the phases and in transitioning from a teacher-centered approach to a student-centered one, thereby struggling to identify their roles in implementing the 5E model.

Meanwhile, Enugu and Hakayem [13] and Turan and Matteson [44] agreed that teachers face many difficulties related to scientific content and teaching methods when applying the 5E model. Specifically, the challenges emerge mainly during the engagement stage [44] and the explanation and elaboration stages [13]. Additionally, teachers face challenges in time management when planning lessons and organizing teaching, evaluating individual student progress [13,44], and creating a correspondence between the lesson and the different phases of the model [13].

In the research on teachers, combining the 5E model with flipped classroom approaches with three phases: (1) pre-out-of-class phase to engage students, (2) in-class phase for student-centered learning activities, and (3) post-out-of-class phase for consolidation, Schallert et al. [33] pointed out that most teachers experienced difficulties in selecting the appropriate evaluation techniques. Furthermore, other factors, including teaching resources, teacher experience, beliefs, professional knowledge, and class size, affect teaching with the 5E model [44].

2.2. Mathematical communication competency

2.2.1. Definition

According to Niss and Højgaard [28], mathematical competencies consist of understanding, conducting, and using mathematical knowledge and activities to solve problems in different situations in which mathematics plays certain roles. As defined in the 2018 Mathematics General Education Curriculum, mathematical competencies (the most dominant expression of mathematical competencies) contain the following core components: mathematical thinking and reasoning competency; mathematical communication competency; mathematical modeling competency; mathematical problem-solving competency; MCC; mathematical aids and tools competency [26]. NCTM [27] states that mathematical communication is among the five core mathematical competencies: mathematical problem-solving competency, mathematical thinking and proving competency, and mathematical relating and representation competency. It is reasonable to conclude that mathematical communication is one of the core competencies that mathematics education seeks to develop in students.

According to PISA 2015, MCC is the ability to understand mathematical problems in communication through other written, oral, or visual mathematical statements, explanations, or texts and to express one's mathematical ideas differently [29]. This competency is significant in many countries' general education curricula [29]. More specifically, in mathematical communication, information is exchanged between subjects through a common system of symbols, signs, or behavior, in which the senders are generally teachers, textbook authors, or learners. At the same time, recipients are generally learners or teachers through auditory (e.g., speaking, listening) or physical (e.g., writing, gestures) channels of communication [3].

Languages are important in mathematics learning [14]. In order to communicate mathematically, the mathematical language and expressions are the two most vital elements apart from the common language. In a narrower sense, the mathematical language is specifically designed based on the system of mathematical symbols [28]. In a broader sense, mathematical language consists of mathematical terms, drawings, models, charts, graphs, etc., that are conventional in expressing mathematical content precisely, logically and concisely [28]. The mathematical language has two main functions, namely, communication and thinking. Mathematical representation is defined as a system of images, symbols (signals on paper, drawings, diagrams, graphs, charts, geometric sketches, equations), or specific objects with mathematical content to depict, symbolize or represent a mathematical object, relation or procedure, to address a question or draw a conclusion [30].

2.2.2. Indicators of MCC

In mathematics education, communication is an instrumental part of the educational process, in which active learning is created through conferences, discussions, exchange, identification and solving of problems, and collaborative knowledge exploration [46]. Firdaus et al. [15] also highlighted the positive impact of the problem-based learning approach using problematic learning situations, questions, and quizzes to encourage learners to participate in the learning process and promote the development of their mathematical competencies, including MCC.

The 2018 Vietnamese Mathematics General Education Curriculum clearly states the expressions and requirements for learners' MCC, including:

(1) Listening, reading, and taking note of necessary mathematical information from mathematical texts or other oral or written statements and expressions;
(2) Presenting and expressing (in written or oral form) mathematical content, ideas, and solutions while interacting with others (given appropriate requirements in terms of completion and precision).
(3) Effectively using mathematical language (numbers, letters, signs, graphs, charts, logical connections, etc.) with a common language or body language while presenting, explaining, and evaluating mathematical ideas during interaction (discussion and exchanges) with others.
(4) Demonstrating self-confidence when presenting, expressing ideas, raising questions, discussing, and arguing mathematical content and ideas [26].

The expressions mentioned above also comprise the learner's ability to construct new mathematical representations, flexibly combine different representations, evaluate the suitability of a representation to different tasks, and understand and explain the purposes of the mathematical representations [45]. Furthermore, Lin et al. [24] believe that another expression of MCC is the ability to interpret and use a variety of representations, including visualization, social interaction, and written prompts.

2.3. Applying the 5E model in MCC-based education

Theoretically, the fundamental ideas of mathematics have been rooted in the experiences and demands of humankind for the coherence, order, and predictability of these phenomena [30]. Then, mathematical representations allow humans to abstract a formula for a real-world problem that has been simplified or idealized. This resonates with the key principle of the 5E instructional model, in which the learner participates in the learning process with a real-world problem and subsequently undergoes phases of exploration and explanation to abstract the real-world problem, eventually using knowledge to elaborate and evaluate the constructed knowledge and solve real-world problems [41].

Many empirical studies have confirmed the benefits of the 5E model for learners to develop MCC, such as those of [38,46]. It can be seen that the 5E model could promote the development of MCCs of learners due to their intensive participation in the different stages of this model [38]. In particular, during the engagement phase, teachers ask questions to pique students' interest. During the exploration stage, students conduct group and individual inquiries; during the explanation phase, students can present mathematical concepts and participate in discussions with other groups and teachers. Additionally, during the elaboration phase, students hone their written communication skills in mathematics by taking on problem-solving tasks [2,7,38].

Furthermore, regarding the influence of the 5E model on the development of communication competence between different groups of learners, Zetriuslita and Uswatun [46] and Susanti et al. [38] showed that students of different academic levels mattered. According to Zetriuslita and Uswatun [46], the 5E model barely affects the development of MCC of students with high academic levels while positively influencing those with low and medium levels. The study also revealed that high-level students' MCC was superior to low- and medium-level students during the inquiry process, with an obvious inclination toward written communication. On the contrary, medium- and low-level students focused more on listening and taking notes.

To maximize the effectiveness of learners' MCC development when employing the 5E model, it is advisable for teachers to flexibly combine different approaches to train learners' skills to use mathematical language and representations throughout the phases of the model. In other words, teachers are recommended to fully exploit and create favorable conditions for learners to use a variety of mathematical languages and representations [18], including the following:

(1) Visual representations consisting of using the whiteboard and PowerPoint slideshow when presenting information, definitions, visualized graphs, videos, and images;
(2) Embodied representations use body language to represent or emphasize important information in the discussion, and embodied representations are generally expressed with visual, spatial, verbal, and motor congruence;
(3) Language representations are used to make statements.

(4) Questions including closed questions and open questions that require learners to explain, encouraging them to consolidate their knowledge, clarify the discussion topics, and improve their thinking skills;
(5) Encouragement of learners to maintain their learning motivation;
(6) Mediation comprising paraphrasing to support learners' understanding, asking questions, and introducing challenges to inspire learners' thinking.
(7) Maintenance language aiming to provide learners with the necessary resources [18].

3. Context of the study

3.1. The subject of fractions in the sixth-grade mathematics curriculum in Vietnam

The content of learning fractions in the sixth-grade mathematics curriculum in Vietnam consists of consolidating the knowledge of fractions in sets of natural numbers presented at the primary level and the new knowledge in sets of integers. These two main content sections and their expected learning outcomes are presented in Table 1.

3.2. Current situation of MCC-based teaching and application of the 5E model in mathematics teaching

To examine the current situation of MCC-based teaching and the extent of applying the 5E model in instructional practice in Vietnam, the research group surveyed 30 mathematics teachers from 02/2022 to 03/2022, including six teachers in some secondary schools in Ben Tre province, and Can Tho city, Vietnam, and some master candidates of the main methods and principles of mathematics education at Can Tho University in the academic year 2021–2022. To implement the survey, a questionnaire was designed to collect teachers' opinions on MCC-based teaching, the 5E model, and the current situation of MCC-based teaching of fraction topics in Vietnam's sixth-grade mathematics curriculum.

First, about the frequency of activities to develop the MCC of the students, most of the teachers surveyed claimed that they frequently organized activities to develop the MCC of the learners. To be specific, 28 participating teachers, representing 93.4 % of all respondents, raised eliciting questions for students; 100 % of the teachers asked their students to read the textbooks, discuss, ask, and answer questions; 90 % usually encouraged students to listen and comment on their classmates' answers; 100 % required students to present their mathematical solutions in written or oral forms. However, 30 % of the respondents rarely used group work, and 20 % rarely used visual aids or real-world mathematical problems in their classes, which most teachers claimed to be frequently exploited in the survey.

Second, regarding teachers' interest in developing the MCC for students in the teaching of fraction subjects, most teachers said they were interested (accounting for 56.7 % of all respondents) or very interested (6.7 %). Only 3.3 % showed a negative or limited interest in this topic, underscoring the need for research given the demands and expectations of these secondary school teachers.

Third, regarding existing and potential challenges in teaching fractions in grade 6, most teachers explained their challenges with uneven background knowledge and students' skills (around 90 % of all respondents) and insufficient class time to implement their learning activities (approximately 83.3 %) effectively. Teachers themselves would hardly overcome these challenges in the short term. Furthermore, some teachers claimed that the mathematical exercises on this topic were very diverse, without a common theme (about 70 %) or that their students failed to distinguish different types of exercises (about 60 %), as well as other challenges (about 40 %). These findings highlight the importance of teacher training in the content and methodology of the competency-based education approach. However, there should be a continuous hierarchical preparation in terms of learners' necessary knowledge and skills in line with their education level, together with a correspondence between the lesson content and the class time allowance in the subject syllabus to guarantee the scale and effectiveness of different teaching approaches and instructional models.

Fourth, regarding teachers' understanding of the 5E instructional model, the survey result shows that most teachers (approximately 86.7 %) had neither heard nor learned about the model. Of the three teachers who claimed to know about the 5E model (accounting for 10 %), only one teacher understands this model.

Fifth, it is revealed that most of the teachers who participated in the survey (representing 83.3 % of all the respondents)

Table 1
Sections of knowledge content and expected learning outcomes of the fraction topic in the sixth-grade mathematics textbook [11,12].

Knowledge content	Expected learning outcomes
Fractions Basic characteristics of fractions Comparison of fractions	- Recognize fractions with numerator or denominators as negative integers; - Acquire the concept of equal fractions and their basic properties; - Compare two given fractions; - Identify the additive inverse of a fraction - Identify mixed positive numbers;
Operations on fractions	- Perform adding, subtracting, multiplying, and dividing fractions; - Apply the commutative, associative, and distributive laws of multiplication with addition, order of operations with brackets and fractions (for properly written, mental, or fast calculations); - Calculate the fraction value of a given number and the value of a number given its fraction value. - Solve some real-world problems related to fraction operations (for example, mathematical exercises related to movements in physics).

occasionally organized learning activities to motivate and engage their students in lessons, encouraged them to explore problems and proposed solutions, created opportunities for students to explain and clarify their ideas, thoughts, and solutions; they applied, advanced and consolidated the concepts learned. Additionally, these teachers evaluated the learners' performance in correspondence with the different stages of the 5E model. A small proportion of teachers rarely (approximately 3.3%–6.7 %) or usually (approximately 3.3%–10 %) designed the abovementioned activities.

Finally, regarding the teachers' opinion about applying modern teaching methods to develop mathematical competencies, most teachers agreed that active teaching methods would work effectively in their classes. Nevertheless, from the teacher's perspective, some updated teaching methods would be quite time-consuming regarding the time required to prepare and implement them, especially given the current class time allowance. From the student's perspective, learning with new methods is generally engaging, although it would take a considerable amount of time for learners to discover new knowledge themselves because of their uneven learning competence. Additionally, while all mathematical competencies must be developed equally for students, the MCC seems inadequate due to the lack of specific teaching approaches.

The survey results show that most teachers were interested in developing the MCC of student teaching fractions and looking for active teaching methods to improve instructional effectiveness. However, the teachers surveyed were restricted by their limited knowledge and investigation of the 5E model. Therefore, there is a research gap in the study of the organization of the teaching of this model, its effectiveness, and its feasibility in teaching fraction topics at the secondary school level in Vietnam.

3.3. Objectives and research questions

The study aims to evaluate the effectiveness of teaching fractions using the 5E instructional model to develop the MCC of the students. The experimental research addresses the following research questions:

(1) Is there a statistically significant difference in academic performance between students using the 5E instructional model (experimental group) and traditional methods (control group)?
(2) Is the difference in academic performance of the experimental group before and after the intervention statistically significant?
(3) Is there any progress in the MCC of the students in the experimental group?
(4) What is the learning attitude of the students in the experimental group towards the 5E instructional model?

4. Materials and methods

The study took nearly two months (from February 2022 to March 2022) in the second term of the academic year 2021–2022 of grade 6 in Vietnam. Seventy-nine student participants were divided into two groups: an experimental group (EG) that learned with the 5E model and a control group (CG) that learned with traditional teaching methods. Data were collected using a post-test with an MCC evaluation rubric, classroom observations with a checklist, and a student survey with a Likert scale [23].

The Council for Science and Education approved the study at Can Tho University and the School Boards (BQ2022-02/KSP). After being fully informed, all participants, in particular, as well as their parents, gave their approval to participate in the study. In addition, study participants demonstrated willingness and interest in participating in class activities. Furthermore, this study found that neither bias nor disrespect for the students was investigated, and neither negatively affected them.

4.1. Study design

An experimental study was conducted using the mixed-method approach to confirm the effectiveness of the 5E model in teaching fraction topics to develop students' MCC. Before the intervention, the participants' levels were assessed to ensure that the two groups had equal levels, so a pre-test was given to both groups. The EG was taught based on the 5E instructional model, while the CG participated in lessons with traditional teaching methods. In particular, the CG participants received conventional lectures. In other words, they did not benefit from learning using the 5E instructional model compared to the experimental class. Additionally, this group

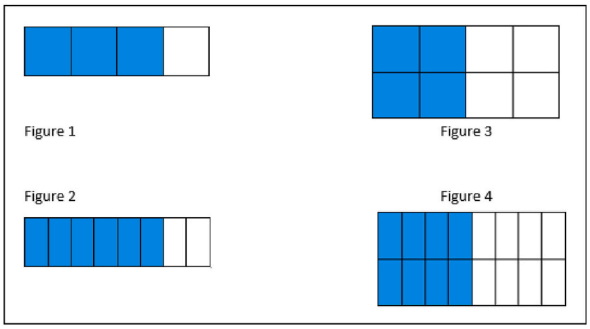

Fig. 1. Learning Activity: "Find the correct fraction."

of students does not know which topic will be covered. They were not encouraged to ask questions throughout the course, and the lectures were not divided into subtopics. No inquiry-based procedure was used to perform the evaluation. Subsequently, a post-test was implemented with both groups to measure the learners' academic performance after learning with the new methods [9,16]. This experimental research design has been used in previous studies on the effectiveness of the 5E model in mathematics education [2,17] and has been shown to fit the educational context in Vietnam. With the research design described, the experiment process included the following steps.

Instead of using a pre-test, the experimental and control groups were formed before the experiment based on the analysis results of the students' mathematics average grades in their first semester of the school year 2021–2022.

The researchers planned the lessons based on the participants' academic performance in the two groups, using the 5E model and traditional methods. The experimental group's lesson plan included six lessons, five incorporating new information created by the 5E model's criteria, and one lesson intended for evaluation after the intervention. In the lessons designed based on the 5E model, the teacher organized learning activities for each knowledge-building process according to the five stages of the 5E model, including engagement, exploration, explanation, elaboration, and evaluation. Below is an example of the activities designed.

Stage 1,2. Engagement and Exploration

The teacher organizes students to play the game "Find the correct fraction.": The teacher divides the class into four teams, each given four pictures and four fractions. The students take turns choosing the corresponding fraction for each picture (pasted on A2 paper). The team that finishes first will win.

The four fractions are as follows: $\frac{6}{8}; \frac{3}{4}; \frac{8}{16}; \frac{4}{8}$.

After completing the game, the teacher poses the problem: If we say, Fig. 1 shows the equivalence of two fractions $\frac{3}{4}$ and $\frac{6}{8}$ then two fractions $\frac{4}{8}$. Are they equivalent? So, what are the conditions for two fractions to be equivalent? We will decide this together in the next part.

From the example that illustrates the equivalence of two fractions $\frac{3}{4}$ and $\frac{6}{8}$. The teacher guides students to compare the products of 3×8 and 4×6, and 4×16 and 8×8.

Students realize that $3 \times 8 = 4 \times 6$ and $4 \times 16 = 8 \times 8$.

From this, the teacher leads the students to derive the concept of two equivalent fractions. *Two fractions $\frac{a}{b}$ and $\frac{c}{d}$ are said to be equivalent, written as $\frac{a}{b} = \frac{c}{d}$, if $a \times d = b \times c$.*

Stage 3. Explanation

The teacher instructs the students to respond to questions about the assigned problem.

Question 1: Two fractions $\frac{4}{8}$, and $\frac{8}{16}$ are they equivalent?

Answer: $\frac{4}{8} = \frac{8}{16}$ because $4 \times 16 = 8 \times 8$.

Question 2: What are the conditions for two fractions to be equivalent?

Answer: $a \times d = b \times c$ is called the condition of two equivalent fractions $\frac{a}{b}$ and $\frac{c}{d}$.

Stage 4. Elaboration

The teacher helps students apply the conditions and concepts of two equivalent fractions by having students discuss in pairs and perform the following exercise.

Are the following pairs of fractions equivalent, and why?

a) $\frac{-8}{15}; \frac{16}{-30}$ b) $\frac{7}{15}; \frac{9}{-16}$

c) $\frac{-12}{16}; \frac{6}{-8}$ d) $\frac{-17}{76}; \frac{33}{88}$

Stage 5. Evaluation

After the lesson, the teacher asks the students to self-evaluate their knowledge. *What did you learn from today's lesson content?* In addition, the evaluation phase is carried out throughout the teaching process according to the model through comments on students' answers and exercise results.

To evaluate the MCC of the learners in learning fractions, the research team designed rubrics with different levels corresponding to the MCC related to this fraction topic, presented in Table 2. This is designed similarly in the study by Tong et al. [42] on teaching to

Table 2

Rubrics to evaluate MCC related to fraction topics.

Indicators of MCC	Mathematical language (words, terms)	Mathematical representations (symbols, number lines, models, drawings, graphs, charts, etc.)	Explanation (arguing, presenting)
Level 1	Fail to use the mathematical language.	Fail to recognize and interpret mathematical representations.	Fail to recognize or interpret mathematical problems.
Level 2	Incorrectly use mathematical language.	Misunderstand the requirements of the problem.	Provide an answer to the question without argument or explanation.
Level 3	Use a mathematical language with many errors.	Express only one aspect of the problem correctly.	Provide an answer to the question with explanations that may be insufficient and confusing.
Level 4	Use mathematical language fairly well.	Use one mathematical representation correctly as required in the mathematical problem.	Provide a correct answer to the question with sufficient explanations.
Level 5	Use mathematical language with precision with minimal errors.	Use many mathematical representations correctly as required in the mathematical exercise.	Precisely answer with well-organized, logical, coherent explanations, extensive details, and generalizations and extensions.

develop students' mathematical competence, which is also achieved through the design of scales when assessing the academic performance of students' abilities.

The experiment then proceeded with the designed lesson plans for the EG while the CG learned with the traditional instructions (nonintervention). During the intervention period, the researchers directly observed the lessons of the CG and implicitly observed the EG. The result of classroom observation was analyzed based on the following criteria: teaching methods, learning methods, skills achieved, classroom atmosphere, and particular expressions of MCC of learners in both groups before and after the intervention.

Finally, the students from both groups completed a post-test to evaluate the effectiveness of the 5E model in developing the students' MCC. Based on the current mathematics curriculum, a set of 15 questions was created, including 12 multiple-choice questions (MCQs) and three open-ended questions. The six MCQs aim to evaluate the ability and frequency of the students to use the other six MCQs to assess their mathematical representation ability, and the three open-ended questions required the students to present their solutions to each question.

Additionally, to examine the student's learning attitude, the research group conducted a student survey within the EG, consisting of 8 MCQs with a 5-point Likert scale ranging from strongly disagree - disagree - neutral - agree - strongly agree [23]. This questionnaire determines the learners' attitudes, motivation, interest and perceptions in fractions with the 5E model.

For the validity and reliability of the instruments, all the 5E model-based lesson plans were proofread by Can Tho University's experts in Methods and Principles of Mathematics Education. At the same time, other teachers from the schools surveyed validated the post-test to align the test with the lesson objectives stated in the current syllabus. Experts confirmed that the instruments were suitable for the experiment after being revised for academic content and MMC evaluation abilities. Furthermore, the Pearson correlation coefficient (r) was utilized to determine the correlation between the pre-test and post-test results for the EG.

4.2. Sample

The participants in this study were 79 sixth-grade students at Thanh Phu Secondary School, Thanh Phu district, Ben Tre province, Vietnam, who were divided into an experimental group (EG) with 38 students and a control group (CG) with 41 students. According to the math teachers at the selected school, the two groups were evaluated with equivalent levels of math proficiency and fairly good mathematics learning performance, making them suitable samples for such an experiment with sixth-grade students. The teaching strategies of the experiment were unknown to the students before the intervention. Table 3 shows the demographics of the students. Additionally, to form the EG and CG with equivalent levels, the research group analyzed the students' mathematics average grades in their first semester of the academic year 2021–2022 (see Section 5.1).

4.3. Data collection and analysis

The data were collected by analyzing the students' average academic performance in the first semester (instead of a pre-test), the post-test, classroom observation, and the EG student survey. The research design, as presented above, was validated in various previous studies, including [2,17], and [42]. The data collection procedure is presented in Table 4.

The researchers used qualitative analysis techniques with data collected from classroom observations, student surveys, and post-test results. In contrast, quantitative analysis methods were used with data from the pre-test, post-test, and student survey with the support of IBM SPSS Statistics (Version 26) predictive analytics software. More specifically, the scores of the experimental and control groups before and after the intervention were checked for normal distribution using the Shapiro-Wilk, normal Q-Q plot, and normal distribution curves. An independent *t*-test (2-tailed) with a 95 % confidence interval was used to determine whether the students' EG and CG levels were equivalent before the intervention and whether there were differences in their mean test scores.

Pearson correlation coefficient r was employed to identify the correlation between this group's mean pre-test and post-test scores, and the paired sample *t*-test (2-tailed) was used to compare the mean scores of the students in the EG before and after the intervention. Furthermore, the effect size was calculated using the mean standard deviation of Cohen et al. [8] to evaluate the effect size made in this study. Cronbach's Alpha was also used as a scale to test the reliability of the questionnaire in the post-test and student survey. The percentage findings of the student survey were examined using descriptive statistics.

5. Results

5.1. Pre-test results

In this study, the average math scores of both groups in the first semester were used to test the equivalence of the student's math proficiency levels. As the number of students in both groups was less than 50, the Shapiro-Wilk test was used to verify the normal distribution of the scores in the two groups. Table 5 shows that the mean scores of the two groups were normally distributed as the p-

Table 3
Demographic data of the students of the EG and CG.

Group	N	Age	GPA	Male	Female
EG	38	12	7.05/10	18	20
CG	41	12	6.85/10	19	22

Table 4
Data collection instruments.

Instruments	EG	CG
Pre-test	x	x
Post-test	x	x
Classroom Observation	x	x
Student survey	x	–

Table 5
Results of the Shapiro-Wilk test with pre-test.

	Statistic	Sig.
EG	0.954	0.124
CG	0.961	0.204

values of the EG and CG with the Shapiro-Wilk test were 0.124 and 0.204, respectively, both of which were >0.05. Furthermore, the Q-Q plot graphs present observed and expected values that fall close to the 45-degree reference line, and the normal distribution curve was bell-shaped symmetrically, implying that the pre-test data sets were normally distributed and eligible for the study.

An independent *t*-test (2-tailed) was used to test the null hypothesis that no statistically significant differences exist in the students' first-semester mathematics mean scores between the EG and the CG. Tables 6 and 7 indicate descriptive statistics and *t*-test results on the mean scores of the average mathematics scores of the two groups.

The descriptive statistics in Table 6 show that the mean scores of the first term of the EG and CG were nearly equal (6.979 and 6.784, respectively), with acceptable standard deviation values (1.877 and 1.782, respectively). The independent *t*-test was employed to check the significance of differences in the mean test scores between the EG and the CG to verify that the mean pre-test scores of the two groups were equal. According to Table 7, with the significance level $\alpha = 0.05$ and the degree of freedom $df = 77$, the p-value (Sig. 2-tailed) was 0.794 (>0.05). Therefore, the difference in mean test scores between the EG and CG was not statistically significant. In summary, based on the test results mentioned above, it can be concluded that the student level in the two groups, EG and CG, was equivalent and eligible for the experiment.

5.2. Quantitative analysis of post-test results

To test the reliability of the post-test, Cronbach's Alpha test was conducted with IBM SPSS Statistics (Version 26) predictive analytics software to validate the quality of the questionnaire following the designed lessons. It is shown that of all variances, the 12 MCQs have the eligible correct–total correlation (not <0.3), and Cronbach's Alpha = 0.888 (>0.7); three open-ended questions have the eligible correct–total correlation (not <0.3) and Cronbach's Alpha = 0.813 (>0.7). Therefore, it can be confirmed that the questions are eligible for internal reliability.

As the students in both groups were less than 50, the Shapiro-Wilk test was used to test the normal distribution of post-test scores in the two groups. Table 8 illustrates the p-values of the EG and CG in the Shapiro-Wilk test at 0.24 and 0.141, respectively; both were >0.05. In addition, the Q-Q plot graphs show that the observed and expected values fall close to the 45-degree reference line, and the normal distribution curve was symmetrically bell-shaped, implying that both groups' post-test results were normally distributed.

An independent *t*-test was used to test the null hypothesis that the mean post-test score of the EG was not different from that of the CG. The two groups ' descriptive statistics, independent *t*-test results, and post-test scores are shown in Tables 9 and 10.

It can be observed in Table 9 that the mean post-test score of the EG is higher than that of CG (at 7.375 and 6.013, respectively). Moreover, Table 10 shows that with the significance level $\alpha = 0.05$ and the degree of freedom $df = 77$, the p-value (Sig. 2-tailed) was 0.001 (<0.05). Therefore, the difference in the mean post-test scores between the EG and CG is statistically significant. From these analyses, it can be concluded that the results of the post-test of the EG and CG were significantly different. Specifically, the EG post-test result was better than the CG's. To assess the effect level of the intervention, the research team calculated the effect size (ES) based on the criteria of Cohen et al. [8], which was approximately 0.88, falling within the range of 0.8–1.0. Therefore, it can be concluded that the effect size after the intervention was significant. Furthermore, the research group used the paired *t*-test to compare the EG's mean pre-test and post-test scores and the Pearson correlation coefficient to determine the correlation between the EG's test scores.

Table 11 shows that the mean pre-test and post-test scores of the EG were significantly different. Specifically, the mean post-test score was higher than the mean pre-test score, implying the effectiveness of the 5E instructional model applied in the designed lessons. Given the significance level $\alpha = 0.05$, the p-value (Sig. 2-tailed) = 0.001 (<0.05) means that the result is statistically significant.

Table 6
Descriptive statistics of the pre-test scores before intervention.

	N	Mean	Median	Std Dev	Std Err
EG	38	6.979	7.05	1.877	0.304
CG	41	6.784	6.85	1.782	0.289

Table 7
Independent *t*-test results before intervention.

Df	t Stat	Sig. (2-tailed)	Mean Difference
77	0.262	0.794	0.108

Table 8
Results of the Shapiro-Wilk test with post-test scores.

	Statistics	Sig.
EG	0.963	0.24
CG	0.956	0.141

Table 9
Descriptive statistics of the post-test scores.

	N	Mean	Median	Std Dev	Std Err
EG	38	7.375	7.375	1.519	0.246
CG	41	6.013	6.25	1.545	0.251

Table 10
Results of the independent *t*-test on post-test scores.

df	t Stat	Sig. (2-tailed)	Mean Difference
77	3.603	0.001	1.265

Table 11
Result of the paired sample *t*-test on the pre-test and post-test scores of the EG.

	Mean	Std Dev	Std Err	Sig. 2-tailed	Correlation
Pre-test	6.979	1.877	0.304	0.001	0.943
Post-test	7.375	1.519	0.246		

Moreover, the Pearson correlation coefficient ($r = 0.943$) shows a strong correlation between the pre-test and post-test scores within the EG.

Furthermore, the visualization of the score distribution of the pre-test and post-test results, as illustrated in Fig. 2, shows the correlation between the two score datasets. It can be inferred that students with higher scores on the pre-test also achieved higher scores on the post-test.

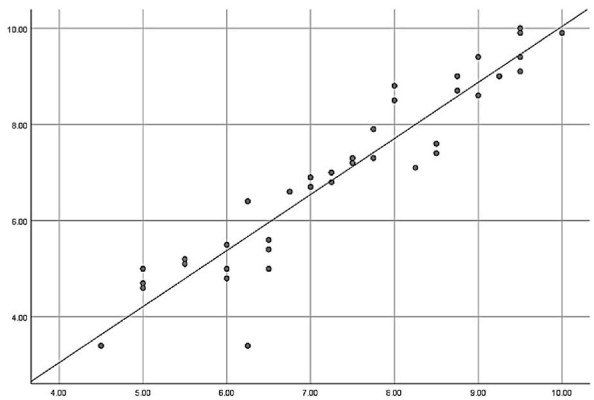

Fig. 2. Visualization of the score distribution of the pre-test and post-test results.

5.3. Qualitative analysis of post-test results

Table 12 presents the post-test score distribution for the EG and CG.

Table 12 shows that most of the EG students received a grade of 5 or higher (37/38 students), and none got below 4. 1/38 students received a grade [4; 5), 5/38 students grades [5; 6), 9/38 students grade [6; 7), 8/38 students grade [7; 8), 7/38 students grade [8; 9) and 8/38 students grade [9; 10]. Meanwhile, the overwhelming majority of students in the CG received grades ranging from 2 to less than 8.0 (37/41 students), among which only 4/41 students reached grade 8 and beyond, including a student with a grade [9; 10]. The two groups had a large difference in the distribution of post-test scores. Specifically, the EG witnessed an even score distribution, focussing mainly on the higher score ranges. On the contrary, post-test scores varied at different levels, with significant differences between score levels. In particular, the number of CG students who achieved grade 8 and above was relatively small (4/41 students).

It can be seen in Fig. 3 that in the EG, most of the student scores were above the average level (accounting for 97.37 %), with only one at the incompetent level (2.63 %), while in the CG, the proportion of scores at the incompetent and poor levels was fairly high, at 4.88 % and 14.63 %, respectively. Regarding the percentages of scores at fairly good and good levels, the figures for the EG were generally higher than those for the CG. In particular, the percentages of good scores in the EG were four times higher than those in the CG (at 39.47 % and 9.76 %, respectively). Thus, it can be inferred that the academic performance of the EG when they learned the fraction topic was considerably better than that of the CG.

In this study, a test was designed to evaluate different aspects of the MCC of students, including the use of mathematical language representations and the expression of mathematical ideas and concepts to solve a specific mathematical problem related to the content of fractions, as presented in Table 2. To be more specific, a level-based scale was also devised for the different expressions of MCC, as shown in Table 13.

As shown in Table 3, EG students exhibited more positive results than their peers in the CG regarding their achievements in different expressions of MCC. Specifically, EG students mostly reached levels 3 to 5, with none falling into levels 1 and 2, while in the CG, some students only reached level 2. In addition, the students' constructed response in the post-test was qualitatively analyzed. These questions required students to present their solutions, which helped the researchers evaluate their MCCs' ability to use mathematical language and representations and explain and argue in their responses. Below are some good and poor student responses from the EG and CG.

Fig. 4 shows that the EG student could use mathematical language with clear and well-organized presentations. However, there were some instances where students could interpret the math problem but could not properly present their solutions, indicating that they had some limitations on language and mathematical representations and their ability to present their solutions. Specifically, as shown in Fig. 5, the CG student S18 was aware of the order of operations but could not present the response correctly. Meanwhile, Fig. 6 illustrates that the EG student S8 did not draw the fraction bar, which implies insufficient mathematical representation competency.

Figs. 7 and 8 illustrate two good responses from student S14 of the EG and student S22 of the CG, respectively. Fig. 7 demonstrates the student's ability to interpret the mathematical language, utilize mathematical symbols correctly, and properly and coherently present the response with sufficient steps and conclusive answers to the mathematical exercise to find x. Furthermore, Fig. 8 shows another good response from the EG student S22. In addition to the good responses, most of the CG students did not succeed in handling this exercise. Figs. 9 and 10 demonstrate two typical errors committed by CG students.

Finally, the question assessed the student's ability to communicate mathematically through the ability to express, use mathematical language, and present the solution to a word problem. In general, EG students presented and argued clearly and correctly and used mathematical language following the requirements of the problem (see Figs. 11 and 12). However, some students in the CG still did not know how to present or understand the problem, so they could not come up with a solution.

5.4. Observation results

Data collected from classroom observation sessions with some designed lessons on the fraction topic of EG and CG were analyzed

Table 12
Distribution of the post-test scores of the EG and CG.

Score interval	Frequency	
	EG	CG
[0; 1)	0	0
[1; 2)	0	0
[2; 3)	0	1
[3; 4)	0	3
[4; 5)	1	4
[5; 6)	5	7
[6; 7)	9	10
[7; 8)	8	12
[8; 9)	7	3
[9; 10]	8	1
Total	38	41

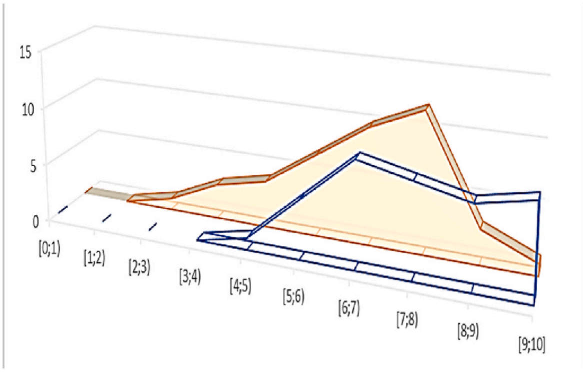

Fig. 3. Visualization of post-test score distributions of the EG (orange) and CG (blue).

Table 13
Results of evaluating the level-based test score according to different expressions of MCC.

		Level 1	Level 2	Level 3	Level 4	Level 5
Manifestations of mathematical language competence (words, terms)	EG	0	0	8	24	6
		0 %	0 %	21.05 %	63.16 %	15.79 %
	CG	0	4	18	16	3
		0 %	9.76 %	43.9 %	39.02 %	7.32 %
Mathematical representations (Symbols, number lines, models, drawings, graphs, and charts)	EG	0	0	7	17	14
		0 %	0 %	18.42 %	44.74 %	36.84 %
	CG	0	1	13	18	9
		0 %	2.44 %	31.71 %	43.9 %	21.95 %
Explanation (Argument, presentation)	EG	0	0	10	14	14
		0 %	0 %	26.32 %	36.84 %	36.84 %
	CG	0	2	15	21	3
		0 %	4.89 %	36.58 %	51.22 %	7.31 %

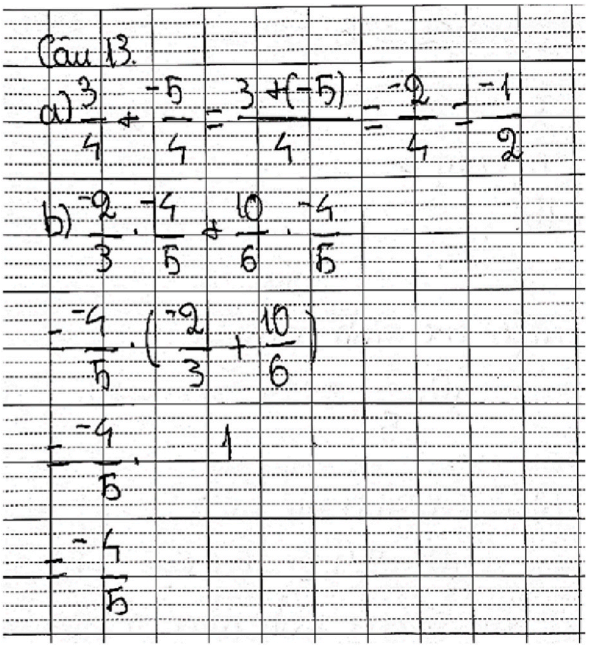

Fig. 4. Good response from student S21 from the EG.

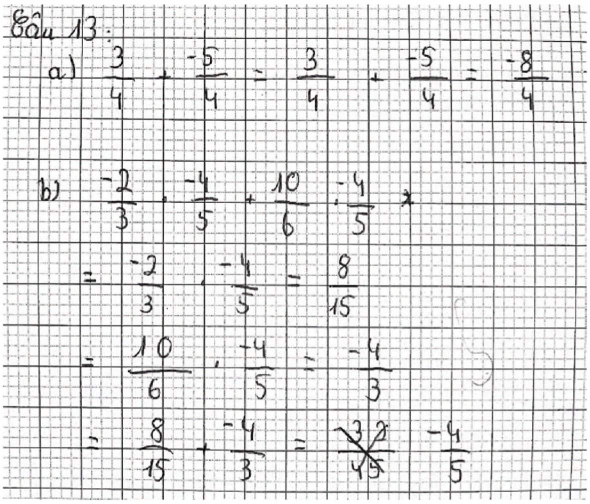

Fig. 5. Poor response from student S18 from the CG.

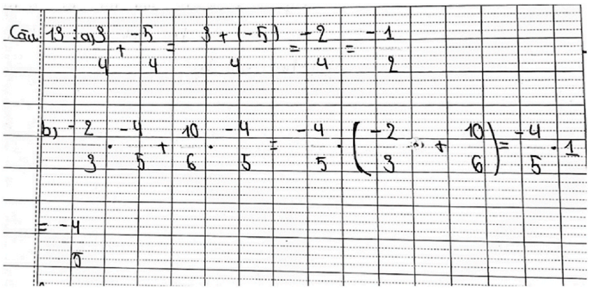

Fig. 6. Poor response of student S8 from the EG.

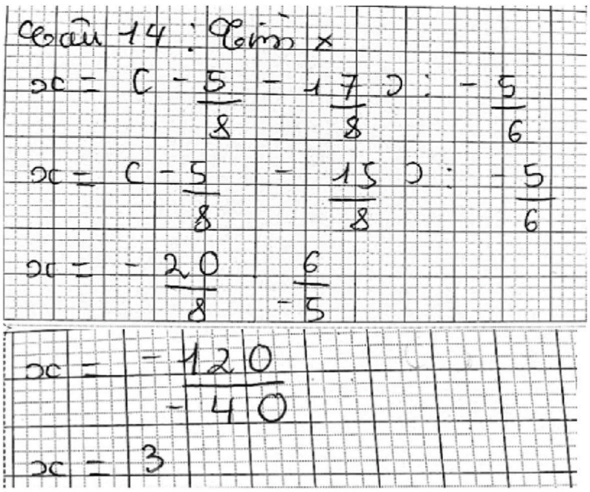

Fig. 7. Good response from student S14 from the EG.

and compared with the teaching methods of the teachers, the learner's learning methods, the skills achieved, and the students' learning engagement or the classroom atmosphere. As shown below, Table 14 displays the analysis results.

Furthermore, the research team observed and evaluated MCC expressions in students in both groups before and after the intervention. Before the intervention, those students (1) generally expressed their ideas lengthily, insufficiently, imprecisely, incoherently, or rarely presented their ideas orally and encountered multiple difficulties in doing this task. (2) Second, these students were unwilling

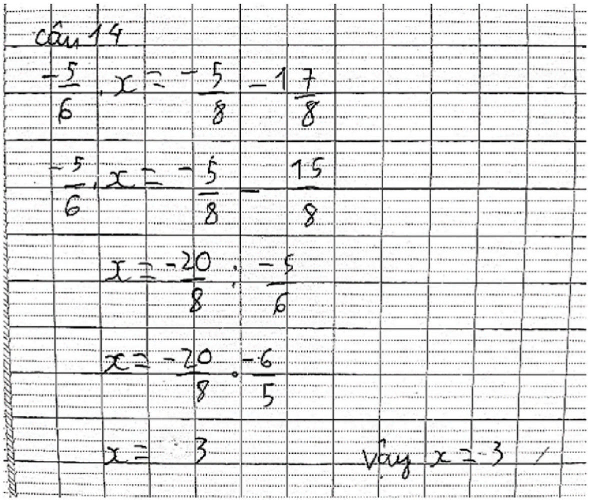

Fig. 8. Good response from student S22 from the EG.

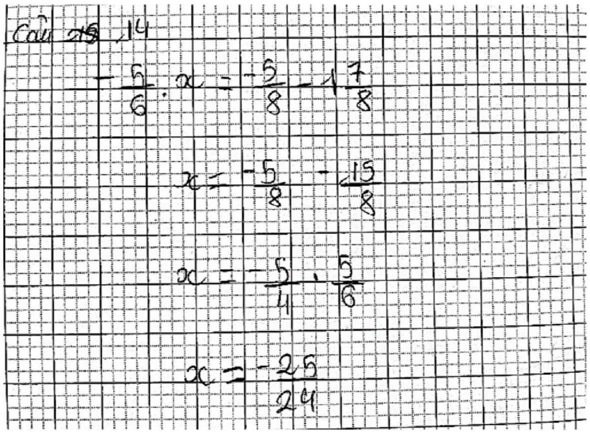

Fig. 9. Poor response from CG student S10.

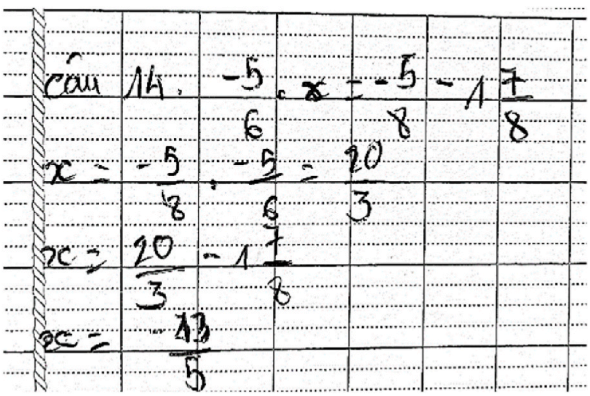

Fig. 10. Poor response from CG student S3.

or confident in sharing their ideas and knowledge. (3) Finally, many of them could understand mathematical problems but struggled to express themselves verbally in mathematical language. Alternatively, they could understand familiar mathematical representations such as symbols, drawings, common graphs, diagrams, etc. After the intervention, these difficulties were gradually overcome by the

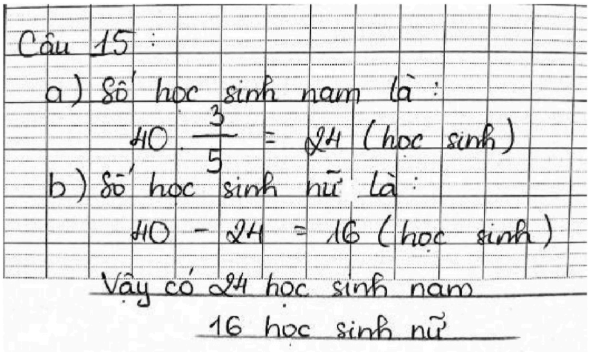

Fig. 11. Good response from student S10 from the EG.

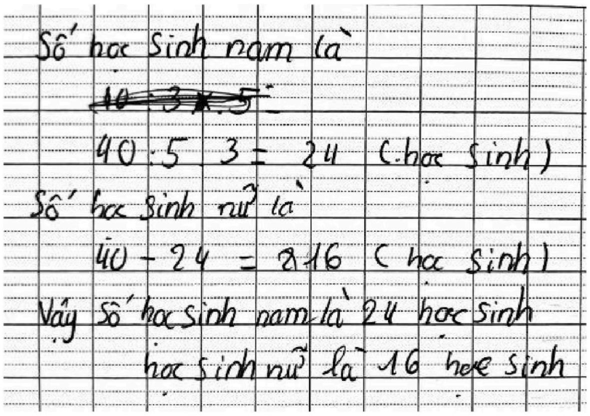

Fig. 12. Good response from student S3 of the EG.

students in the EG. Thanks to enhanced communicative learning activities, which encouraged students to express their ideas explicitly, EG students became more confident in using mathematical language to present ideas in oral and written form to solve mathematical problems more skilfully. Consequently, they trained their skills to use maps and models to summarize a mathematical problem, describe ideas, and solve the problem with a clearer and more reasonable presentation and explanation. In particular, the students were willing to share their understanding and were able to welcome their classmates' different opinions and solutions.

5.5. Student survey results

After learning the topic of fractions, 38 EG students completed a questionnaire consisting of eight survey statements and a 5-point Likert scale *(strongly disagree - neutral - agree - strongly agree)*. This survey collects students' opinions about the learning effectiveness and level of engagement of the designed lessons.

The Cronbach Alpha test was conducted using IBM SPSS Statistics (Version 26) predictive analytics software to test the reliability of the measure of student attitude toward the designed lessons. The test result shows that all variances (i.e., eight survey statements) had an eligible corrected item-total correlation (not <0.3) and Cronbach's Alpha $= 0.833$ (>0.7). Therefore, the questionnaire was acceptable in terms of internal reliability. The survey results were analyzed as follows.

As shown in Table 15, most of the EG students liked the lessons they took on the topic of fractions (63.2 % of all the EG students), while some were not interested in learning the content on this topic (34.2 %), and one student claimed a negative feeling about this topic (2.6 %). Table 15 shows that nearly 90 % of the students surveyed felt engaged in the designed lessons. Furthermore, none disagreed with the statement, consistent with the classroom observation results. Furthermore, it was observed that 90 % of all EG students claimed to be able to understand the designed lesson in class. This finding is significant for the investigation, which implies the effectiveness and feasibility of the measures and teaching models proposed and applied in the experiment. Item 4 aimed to determine the students' opinion on the effectiveness of group work activities and learning worksheets as a teaching aid. Table 15 shows that 36.8 % of the students surveyed strongly agreed that group work tasks involved them in the lessons and created a comfortable learning environment compared to previous mathematics lessons.

According to Table 15, visual aids enabled 100 % of the students to participate actively in the lessons. Through classroom observation, it can be observed that most of the students engaged in the lessons with many real-life images, videos, drawings, and mind

Table 14
Results of classroom observation in EG and CG.

Aspects	EG	CG
Teaching methods	- Introducing the problem by eliciting questions, discussion, explanation, suggestions, presentation, visualization - Creating opportunities for students to explore concepts or formulas. - Presenting on the board or projector screen based on different stages of the 5E model. - Using learning worksheets; - Designing the teaching procedure for new knowledge based on the phases of the 5E model.	- Lectures, main lectures, and explanations - Present the concept and formula first and then prove the formula. - Presenting on the board according to the lesson procedure in the textbook. - Explaining the concept and formula, then solving the sample and exercises in the textbook.
Learning methods	- Discussing, working in groups, collaborating, observing, predicting, exploring, presenting, critical thinking, visualizing - Working individually and in groups with worksheets; - Actively exploring knowledge with teachers' instruction; - Implementing activities with instructions step by step to memorize and perceive the learned knowledge	- Listening to the lecture and taking notes. - Raising ideas and working individually. - Acquiring new knowledge transferred by teachers. - Carrying out activities to memorize and perceive knowledge.
Achieved skills	- Teamwork skills, communication skills, and inquiry skills. - Ability to mobilize practical experience to explore new concepts; - Ability to categorize and relate to the characteristics and qualities of concepts. - Skills to construct new concepts with inductive methods (with instructions) - Problem-solving skills with deductive methods - Calculating skills - Ability to comment on and adjust the mathematical solution. - Skills to relate and apply knowledge to solving real-world problems. - Skills to systemize knowledge.	- Communication and inquiry skills. - Memorising skills - Problem-solving skills with deductive methods - Ability to prove a formula. - Calculating skill - Ability to comment on and adjust the mathematical solution.
Learning Engagement	- Students actively participated in exploratory activities to build new knowledge. - Learners actively participated in group work activities, teacher inquiry tasks, and ideas exchanges with peers. - The students participated in activities to construct and apply the new knowledge.	- The students passively acquire new knowledge transferred by teachers. - The students were not active in raising ideas in the lessons. - The students actively participated in activities to apply knowledge.

Table 15
Response statistics for items.

Items	Strongly disagree	Disagree	Neutral	Agree	Strongly agree
1. I like fraction lessons.	0 0 %	1 2.6 %	13 34.2 %	12 31.6 %	12 31.6 %
2. I am interested in the designed lessons.	0 0 %	0 0 %	4 10.5 %	11 29 %	23 60.5 %
3. I can understand the lessons I learned.	0 0 %	0 0 %	7 18.4 %	15 39.5 %	16 42.1 %
4. I find that learning mathematics in groups and interacting with worksheets is helpful to improve my interest and effectiveness in learning mathematics.	0 0 %	0 0 %	4 10.5 %	14 36.8 %	20 52.6 %
5. The lessons use visuals (real-life images, videos, drawings, and mind maps) to engage the students.	0 0 %	0 0 %	7 18.4 %	17 44.7 %	14 36.8 %
6. Mathematics lessons that involve real-world problems promote my interest and affection for mathematics learning.	0 0 %	2 5.3 %	9 23.7 %	14 36.8 %	13 34.2 %
7. The lesson procedure in the experimental lessons helped me learn more effectively.	0 0 %	0 0 %	7 18.4 %	15 39.5 %	16 42.1 %
8. I would love to attend similar lessons on other topics.	20 0 %	0 0 %	7 18.4 %	14 36.8 %	17 44.7 %

maps. Further practice with real-world activities was indispensable in lessons with the 5E instructional model. Table 15 shows that 71 % of the students noticed increased learning interest and effectiveness in the lessons integrated with real-world mathematical problems, helping them relate their learned knowledge to real-world problems. However, a small proportion of students (2 students, representing 5.3 %) did not show enough interest in learning real-world mathematical problems, which required further measures to support their interest in mathematics. Responses to item 7, shown in Table 15, highlighted that most EG students found the lesson procedure in the designed lessons helpful in their learning effectiveness (claimed by 81.6 %). This finding suggests the effectiveness and feasibility of the lessons designed on the 5E model. The analysis results in Table 15 demonstrated the appreciation for the designed lessons of the students, which was also the underlying reason for their desire to attend similar lessons on other topics. According to the statistics, over 81 % of the participants agreed or strongly agreed to have similar lessons in the future, and none refused, again underlining the feasibility of future lessons with a similar design for further application to other mathematics topics.

6. Discussion and limitations

After analyzing the collected data, it was determined that, regarding MCC expressions and academic performance, the EG students outperformed the CG students. Specifically, EG students reached levels 3 to 5, while some CG students only reached level 2 on the MCC evaluation scale. Most EG students achieved average to good post-test scores, while their peers from the CG received unevenly distributed scores, focussing mainly on average and fairly good levels. Furthermore, classroom observation and student surveys revealed that most EG students demonstrated positive reactions toward the organization and procedure of teaching according to the 5E model in MCC-based education with a deeper understanding of mathematics. It should also be noted that the EG students became more confident and proactive in learning activities, overcoming their psychological obstacles such as timidity and fear. In other words, their activeness, initiative, and creativity in learning were improved considerably. Increased activities to support students in using mathematical language, including mathematical terms, symbols, and other expressions, facilitated coherent thinking, articulate expressions, and more effective perception of new mathematical concepts. This finding resonates with the results of Chen et al. [7].

Moreover, the quantitative analysis showed that the EG's post-test scores were higher than the CG's (Sig. 2-tailed = 0.001). There was an increase in the mean test score from the EG's pre-test to the post-test (Sig. 2-tailed = 0.001, r = 0.943), and the effect size of the intervention was considerable (0.88) [2,17,31], and [39] also verified that learners' academic performances were improved after they had learned based on the 5E instructional model. At the same time, these figures also affirm that learning with the 5E model benefits the MCC of the EG students, as the test questions were designed to evaluate the student's ability to use mathematical language and representations in arguing and presenting their solutions to a math problem. This finding is similar to that of Susanti et al. [38] and Zetriuslita and Uswatun [46]. Furthermore, the classroom observation and student survey pointed out that the 5E model exerted a great positive influence on the students' learning attitudes, interests and initiative, which resonates with the research conclusions in Refs. [2,17,24,39], and [44].

It is possible to conclude that the study's experimental findings effectively addressed the research questions. First, student academic performance was improved significantly with the 5E model compared to traditional instruction. Second, the student's academic performance was enhanced after taking 5E model-based designed lessons. Third, the students developed a positive learning attitude and participation in 5E model-based learning activities. Finally and most importantly, the 5E instructional model promoted the development of the MCC of the students, illustrated through increased communication skills in mathematics lessons and higher post-test scores.

In addition to the research findings, this study has inevitable limitations. First, due to public health security measures for COVID-19 protection, the study involved a relatively small sample population over a relatively short period (two months). If there had been a larger sample population and a longer duration of the experiment, the study's conclusions would have been more representative, and the study would have conducted a more thorough investigation of the experimental effects. Furthermore, despite offering sufficient evidence for the development of the MCC of the student with general comments in the classroom observation sessions, as well as the results of the post-test and student survey, the students' expressions of their MCC (in oral form) during their discussion and inquiry activities throughout the exploration phase of the 5E model were not presented. Third, although this experimental study did not elucidate teachers' and students' challenges when teaching and learning with the 5E model or MCC-based education, it may have contributed to the study's conclusions.

This study summarises the measures suggested to promote benefits and restrain the difficulties in applying the 5E model in mathematics teaching to develop MCC. To overcome the challenges in employing this model, as mentioned in the previous sections, there are several recommendations, including (1) training teachers on the 5E model [37], (2) improving teachers' classroom management and time management skills [13], (3) intensive and extensive training for teachers on subject content knowledge [13], (4) emphasizing connecting theory to practice in the teacher training curriculum with more priority for practical experiences [22] and authentic classroom observation to promote pre-service teachers' opportunities to interact and collaborate with in-service teachers in lesson planning, teaching, and experience sharing.

As a final comment, it is necessary that mathematical language training involves both semantic and syntactic aspects so that students can master mathematics knowledge and thus be capable of describing real-world situations precisely. In mathematics, symbols and symbolized objects exist as expressions and expressed objects. The syntactic aspect refers to looking at symbols and expressions, focussing on the structure and rules of their presentation to identify and transform them, whereas the semantic aspect is concerned with symbolized and expressed objects, focussing on their content or meaning. Therefore, when using these symbols and terms, teachers must explain their meanings to students and remind them of the possible changes in meaning when used in different domains.

7. Conclusions

The results of this mixed-methods experimental research confirm once again the effectiveness of the 5E instructional model on the

MCC of students when learning fraction topics. The results of the post-test, classroom observation sessions and student survey were in agreement, with 79 students divided into two groups: EG, 38 students, and CG, 41 students. The EG students who learned based on the 5E model had the opportunity to train and develop multiple expressions of MCC, improve their academic performance, and bring a positive learning attitude. Specifically, the independent *t*-test on the post-test scores of the two groups revealed that the academic performance of the EG after the intervention was significantly better than that of the CG with the significance level $\alpha = 0.05$ and degree of freedom $df = 77$, yielding the p-value (Sig. 2-tailed) = 0.001. Meanwhile, given the significance level $\alpha = 0.05$, the p-value (Sig. 2-tailed) = 0.001 and the Pearson correlation coefficient r = 0.943, the result of the paired sample *t*-test implies that the mean post-test score of the EG students was significantly higher than that of their pre-test. Furthermore, with an effect size of approximately 0.88, these figures suggest that applying the 5E instructional model in the experimental process positively affected the academic performance of the EG students and their MCC development.

This study focuses on developing sixth-grade students' mathematical communication skills, an important aspect of today's mathematical competencies. Furthermore, the study once again verified that applying the 5E instructional model (engagement, exploration, explanation, elaboration, evaluation) is an effective method to teach fractions, from which the research results bring new and profound insight into the process of learning this topic. The research results provide a strong theoretical basis and practical evidence that the 5E instructional model effectively impacts the development of students' mathematical communication skills. Finally, this study also provides practical guidance and suggestions for researchers, educators, and teachers on how to apply the 5E model in teaching fractions to increase students' MCC.

Regarding future research directions, the research group proposes some appropriate options, such as (1) developing the 5E model into the 6E, 7E or 9E model in teaching fraction topics; (2) applying the 5E model in teaching different mathematics topics to develop other mathematical competencies for learners; (3) employing the 5E model in combination with other active teaching methods such as problem-based learning [15] or blended learning [31,33]; (4) using the 5E model in teaching mathematics in integration with other natural science subjects in teaching STEM topics; (5) impacts of certain factors, such as learner's genders [21], academic levels [46], on MCC development; (6) long-term effects of the 5E model in the teacher training curriculum [18,44]. Regarding the research design, the research team would propose more studies with larger sample populations and longer observation durations to provide a more comprehensive evaluation of the impacts and challenges of the 5E model in competency-based education.

CRediT authorship contribution statement

Tien-Trung Nguyen: Writing – review & editing, Writing – original draft, Formal analysis, Data curation, Conceptualization. **Nam Danh Nguyen:** Software, Resources, Investigation, Data curation, Conceptualization. **Thao Phuong Thi Trinh:** Formal analysis, Data curation, Conceptualization. **Duong Huu Tong:** Writing – review & editing, Writing – original draft, Formal analysis, Conceptualization. **Bui Phuong Uyen:** Resources, Formal analysis, Data curation. **Nguyen Ngoc Han:** Methodology, Investigation, Data curation.

Data availability statement

Data included in article/supp. material/referenced in article.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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