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# Towards partial autonomy of operation and maintenance of unreliable equipment

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A R T I C L E I N F O

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A B S T R A C T

This study considers the case of unreliable equipment subjected to random failures that induce high maintenance and environmental costs. We consider situations where the equipment is located in remote areas, which are difficult to access, and situations where there could be confinement linked to a pandemic, making it impossible to perform the replacement of the failed equipment. In such situations, the objective is to explore the possibility of providing a system with self-maintenance capabilities to a certain extent by adding redundant (stand-by) identical modules. Both the designs (with and without passive redundancy) are considered. A mathematical cost model is developed for each alternative to help decide whether to adopt redundancy and determine the optimal number of redundant modules, which minimises the total expected cost. The latter includes the costs related to the acquisition, maintenance, and recycling of failed modules. A numerical example is presented, and a sensitivity study is performed to investigate the effect of variations in relevant input parameters on the optimal design.

## 1. Introduction

According to Ref. [1], the term "maintenance" refers to all planned and unplanned actions and activities taken to maintain the constant accessibility of working machinery in a manufacturing facility. In other words, it includes all strategies and operations used to ensure that the equipment operates as intended or that its predetermined functions are restored ([2]).

Analyzing the bibliography related to maintenance and reliability, it is easy to categorize maintenance actions based on the level to which the operational state of a system is restored by maintenance actions, into three different categories: (a) perfect maintenance, meaning that the equipment is returned to the “as good as new” state, in which case, the system has the same failure rate and time to failure functions as a novel equipment. (b) Minimal maintenance after which the equipment is restored to its failure rate level closely before failure. Barlow and Proschan [3] were the first authors introducing this kind of maintenance, for which the operational state is named “as bad as old.”

In the past decades, maintenance was viewed as difficult to manage but had to be done after such a failure ([4]). Recently, in Ref. [5] the authors mentioned in their study that machine tools, equipment, and machinery have evolved in response to technological advances and the scientific knowledge they have embedded. Regardless of their complexity, all machines and installations require repair work because of natural degradation or the effects of workwear. Eventually, this degradation results in system failure, which adversely affects safety, equipment quality, and unplanned machine downtime ([4]). Taking into and account maintenance strategy and cost can impact some industrial decisions. For example, Baklouti et al. ([6,7]) developed a mathematical model to help the

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decision maker decide about leasing or selling second-hand vehicles. They proved that the degradation state of a fleet of used vehicles and possible maintenance costs can impact the decision.

Throughout the last few years, manufacturing companies and enterprises in different countries worldwide have spent significant amounts of money, mainly on maintenance. Zhao et al. ([2]) mentioned in one of their recent research projects that maintaining industrial machines in China is estimated to represent more than 15% of the total cost of production. In Germany, the costs related to the maintenance represent 13–15% of the GDP, whereas it is 14% in the Netherlands,

Therefore, the research and scientific communities have made significant efforts and proposed many different cost-effective maintenance strategies. Peng et al. ([8]) discussed this to improve a component's reliability, increase its availability, and decrease its costs of maintenance.

According to the number of components, Cortés et al. ([9]) classified existing maintenance strategies into single-component maintenance strategies and multicomponent maintenance strategies.

Thus, single-component maintenance strategies can be separated into corrective maintenance CM, and predictive maintenance, whereas the maintenance strategies related to multicomponent systems are classified into batch maintenance, opportunity maintenance, and group maintenance. ([2]).

Systems with significant uncertainty require solutions that go beyond PM to prevent unnecessary maintenance and non-optimal costs.

Lee et al. in Ref. [10], stated that self-maintenance might be a good solution to this issue. This refers to a machine's capacity to perform routine quality and safety inspections by itself, identify anomalies, and perform emergency repairs when necessary, utilising spare parts that are kept in hand to prevent potentially catastrophic loss.

Sites that are difficult to access, such as offshore wind turbines, may benefit from self-maintenance, as stated by Singh et al. in Ref. [11].

If any form of breakdown or deterioration occurs, a self-maintaining machine may monitor and diagnose itself and continue to work for some time.

According to Ref. [10], adding intelligence is the only way to make a machine intelligent enough to perform functional maintenance. In other words, self-maintenance is a new embedded reasoning system that is added to existing machines.

Although the maintenance team still performs the task, the integration of machines, maintenance schedules, dispatch systems, and inventory management systems significantly reduces maintenance costs and increases customer satisfaction.

Despite the diversity of the proposed strategies, maintenance has a direct impact on system reliability, as previously stated. Endrenyi et al. in Ref. [12] explained that maintenance is a device used to increase the reliability of components and systems. Therefore, if frequently performed, the reliability may increase, but maintenance costs can rapidly increase. The reliability may decline because of an excessive number of expensive failures and poor system performance. These two expenses must be balanced in a cost-effective manner.

Therefore, reliability is a key design element for successful and efficient operation of modern technological systems. Tzafestas in Ref. [13] pointed out that the issue of utilising available resources in the most efficient manner to optimise the overall system reliability or minimise resource consumption while attaining specified reliability goals is of important in the planning and design of such multicomponent systems.

System theorists and practitioners have performed a significant amount of work over the past 20 years, covering the full range of optimal reliability design sub-problems, and a variety of methodologies have been employed.

According to Ref. [13], the primary system models utilised in reliability optimisation studies include series, parallel, and standby models. In the series model, there is a number of "n" stages that are statistically independent compared to the parallel model, in which there are a number "m" of statistically independent parallel redundant units. This means that the system is operating if at least one unit operates. However, a characteristic of the standby model is that not all parallel/series combinations or units are simultaneously active, but they are waiting for action.

In the same frame of reliability systems with different structures, in [ [14]; 13] it was proposed a new method based on non-probabilistic reliability bounds method for series structural systems as an active means for the evaluation of systems' non-probabilistic reliability. They considered redundant failure modes for series systems to ameliorate the efficiency and precision of the non-probabilistic reliability bounds method. They reached their goal by breaking down the system into several subsystems with two or three failure types, and by defining three identification criteria for redundant failure types.

Tillman et al. in Ref. [15], presented new directions for further reliability optimisation research, such as expanding the standard reliability optimisation problem to include defining the best level of component reliability and the number of redundancies in each stage simultaneously, or optimising the multistage system reliability by selecting a more reliable component at each stage.

In conclusion, various scientific papers and research projects related to maintenance, reliability, and the effect of maintenance on reliability have been published. However, few strategies exist for replacing the components and parts used throughout the various stages of maintenance. This is because none of the component manufacturers comprise technological processes or consider methods for recycling and discarding their own products (Mitrofanov et al., 2020).

Karavida and Nommik in Ref. [16] stated that metals, glass, plastic, and secondary raw materials continue to be lost from the material streams.

For instance, according to statistics from the European Commission, 6 of the 16 tons of materials consumed annually in Europe are wasted ([17]).

A recent method for recycling components, spare parts, and assemblies was presented by Ref. [17] to solve this problem, suggesting that manufacturers should incorporate reusing technology of materials/components during the initial steps of the design of their components.

## 2. Problem statement

### 2.1. Motivation and targeted contribution

Undeniably, a period of confinement linked to an epidemic or pandemic or any other reason can lead to major economic losses. To minimise these sudden deficits, progress is necessary to maintain important activities despite compulsory confinement ([18]). In addition, the excessive failure of some modules presents a real problem in terms of cost and availability. In the literature, we assist the studies of interest by proposing new maintenance strategies and/or optimising existing strategies. Generally, these studies focused on minimising the total maintenance costs or maximising the system availability. Despite the interesting results realised through these studies, we must improve the traditional system design by proposing a new design mode. This study investigated this framework. We propose a new strategy to address the problems related to several replacements of one component. In addition to the system of one module, we propose a new system composed of n identical models in parallel as a standby redundancy structure supervised by a switcher placed in series with n models. We aim to determine the possible and optimal economic gains realised according to this new system, considering system reliability, maintenance costs (preventive and corrective), acquisition cost, and recycling cost based on the replaced defect components. This problem is discussed in detail in the following section.

### 2.2. Problem description

We assume that we have a mission ensured by only one module over a finite horizon H. This module was subjected to random failure, which was characterised by a high constant failure rate. At every failures that occurred over the finite horizon, the module was replaced with a new one. From the viewpoint of cost, this replacement action incurs three costs: acquisition cost of the module, maintenance cost related to the intervention, and recycling cost related the old module replaced. To address these excessive costs, we focused on a solution related to system design. Precisely, in addition to the system with one module, we decided to create a new system composed of n identical models in parallel as a standby redundancy structure supervised by a switcher placed in series with the n models, considering a maximal number Nmax of components ($n \leq Nmax$). This new structure is subjected to failures, but fewer than the average number of failures that occur with a one-module structure. An optimised PM action age type was applied to this new system because the proposed system has an increased failure rate. A comparative study based on the total average cost integrating the acquisition, maintenance, and/or replacement, and recycling costs between the two proposed structures will be conducted. The goal was to determine the optimal number $n^*$ ($2 \leq n \leq Nmax$) adopted in the new structure to realise the maximum gain compared with the one-module structure. An analytical model was developed in the subsequent section to achieve this objective.

### 2.3. Problem assumptions

- o The duration of finite horizon is known and constant.
- o The maintenance staff is available for every maintenance action.
- o The preventive and corrective maintenance, recycling, and acquisition costs are known and constant.
- o The maximum number of components Nmax is known.

## 3. Analytical model

### 3.1. Analytical model development

Analytical models based on two strategies (i.e., single-component and redundant systems) are developed in this study. The total average cost integrates the costs related to acquisition, maintenance, and recycling over a finite horizon, H.

#### 3.1.1. Analytical model for the first strategy (single component)

The first strategy ensures a mission over a finite horizon H with only one component. The component is subjected to a random failure at a constant failure rate λ. The mission starts with a new component, and the failed component is substituted by a new component at every failure, considering maintenance and recycling costs based on the failure component. The total average cost of the mission's integrated acquisition, maintenance, and recycling over a finite horizon H for the first strategy (i.e., the system with one component) is expressed as

$$\boldsymbol{CTc} = \boldsymbol{Cc} + [(\boldsymbol{Cc} + \boldsymbol{Cm} + \boldsymbol{Crc}) \times \boldsymbol{\lambda}.\boldsymbol{H}]. \quad (1)$$

Proof:

For the component, we have a constant fair rate λ. Consequently, the reliability function can be expressed as

$$\boldsymbol{Rsys} = \boldsymbol{e}^{-\boldsymbol{\lambda}.\boldsymbol{t}}.$$

The average number of failures over the finite horizon H is given by

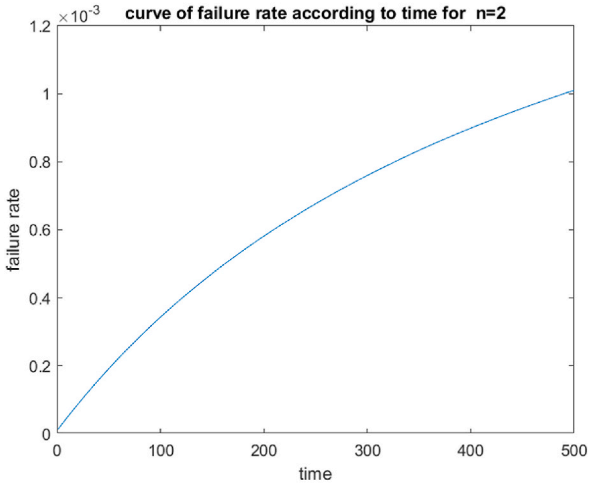


**Fig. 1.** Evolution of the failure rate according to time for n = 2.

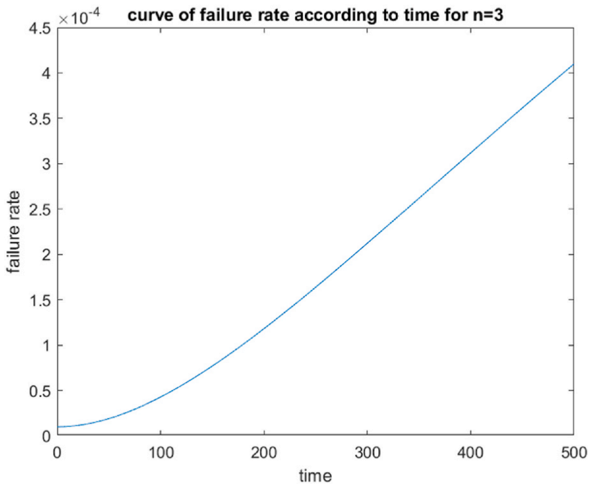


**Fig. 2.** Evolution of the failure rate according to time for n = 3.

$\lambda.H$.

The number of replacements over the finite horizon is estimated by λH. Every replacement induces a unit maintenance cost *Cm*, acquisition cost of one component *Cc*, and recycling cost Crc related to the failure component. The total intervention cost over the finite horizon H is expressed as follows: $(Cc + Cm + Crc) \times \lambda.\mathbf{H}$. Considering the acquisition cost of the first component at the beginning of the mission, the average total cost of the mission integrated acquisition, maintenance, and recycling cost over a finite horizon H is expressed as follows. Every replacement induces a unit maintenance cost *Cm*, acquisition cost of one component *Cc*, and recycling cost Crc related to the failure component. The total intervention cost over the finite horizon H is expressed as (*Cc* + *Cm* + Crc) × H. Considering the acquisition cost of the first component at the beginning of the mission, the total average cost of the mission's integrated acquisition, maintenance, and recycling costs over a finite horizon H is given by

$$CTc = Cc + [(Cc + Cm + Crc) \times \lambda.H].$$

End of the proof.

### 3.1.2. Analytical model for the second strategy (redundant system)

The second strategy, based on the redundancy concept, ensures a mission with a new system composed of n identical models in parallel as a standby redundancy structure supervised by a switcher with a constant failure rate placed in series. Note that ($2 \leq n <= Nmax$). (ajouter reference bibliographique)

Based on this structure, the reliability function of this system is defined by equation (2):

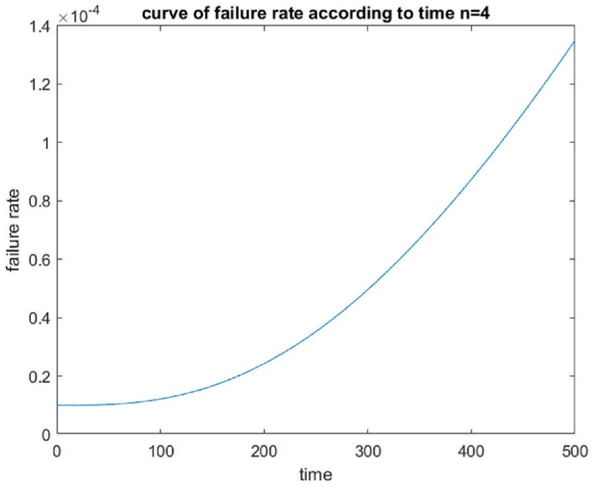


**Fig. 3.** Evolution of the failure rate according to time for n = 4.

$$Rsys(t) = e^{-(\lambda+\lambda sw).t}\sum_{i=2}^{n-1}\frac{(\lambda.t)^i}{i!}. \tag{2}$$

The failure rate of the system is expressed by the following equation (3):

$$\lambda sys(t) = \frac{-\frac{d\ Rsys}{dt}}{Rsys(t)} = \frac{-\frac{d\left(e^{-(\lambda+\lambda sw).t}\sum_{i=2}^{n-1}\frac{(\lambda.t)^i}{i!}\right)}{dt}}{e^{-(\lambda+\lambda sw).t}\sum_{i=2}^{n-1}\frac{(\lambda.t)^i}{i!}}. \tag{3}$$

For each value of n (n > 2), the failure rate increases. Figs. 1–3 below illustrate the evolution of the failure rate with time for different values of n (n = 2, 3, and 4). The curve of the failure rate of the system according to n is plotted using equation (3) expressing the failure rate according to time for ecah value of n.

In these figures, the failure rate increases with the number of components.

Consequently, we applied a PM strategy to address the increasing failure rate. The maintenance strategy under consideration is a well-known age-based PM policy (Gertsbakh [19]). It consists of performing perfect PM at a certain age m or performing a perfect corrective maintenance CM action at failure if it occurs before the PM age. After each preventive or corrective maintenance CM action, the system is considered to be in a new state. Based on Gertsbakh in Ref. [19], for this maintenance strategy, the average total cost per unit time is defined as

$$CT(m) = \frac{R(m)Mp + (1 - R(m))Mc}{\int_0^m R(u)du}$$

where R represents the reliability function, and Mc and Mp represent the corrective and PM actions costs, respectively.

Applying this formula, we can establish the average total maintenance cost per time unit as

$$CT\ Ms(T,n) = \frac{R_{sys}(T)(Mp + Crpr) + \left(1 - R_{sys}(T)\right)(((n \times Cc) + Csw) + Cm + Crcr)}{\int_0^T R_{sys}(u)du}, \tag{4}$$

where Rsys is the reliability function of the system defined in (2), n is the number of components placed in parallel for the redundant system, and T is the PM age.

Proof:

The PM action induces a unit cost of the PM action, Mp, and a recycling cost based on the PM action, Crpr. However, the corrective maintenance CM action induces the acquisition of a new system ((n × *Cc*) + Csw)) added to the unit maintenance cost according to the replacement task CM and the recycling cost according to the PM action Crcr. Consequently, we can obtain the average total maintenance cost per unit time as

$$\mathrm{CT\ Ms}(\mathrm{T},\mathrm{n}) = \frac{R_{sys}(T)(Mp + Crpr) + \left(1 - R_{sys}(T)\right)(((n \times Cc) + Csw) + Cm + Crcr)}{\int_0^T R_{sys}(u)du}.$$

End of the proof.

If the preserved system has an increasing failure rate, then there is a unique optimal strategy T*, which minimises the average total maintenance cost CTMs for each value of n (n > 2) ([3]). The optimal value of the PM age noted T* is expressed as

$$\left.\frac{d(\boldsymbol{CTMs}(\boldsymbol{T},\boldsymbol{n}))}{\boldsymbol{dT}}\right|_{T*}=\mathbf{0}\forall \boldsymbol{n}\in\{\mathbf{2},\mathbf{3},\ldots\ldots\boldsymbol{Nmax}\}. \tag{5}$$

The minimal value CTMs noted CTMs* is defined by the following equation (6):

$$\boldsymbol{CTMs}^{*}=\boldsymbol{CT\ Ms}(\boldsymbol{T}^{*},\boldsymbol{n})\forall \boldsymbol{n}\in\{\mathbf{2},\mathbf{3},\ldots\ldots\boldsymbol{Nmax}\}. \tag{6}$$

Applying the optimised PMaction over a finite horizon H, we can obtain the total average cost of the mission integrated acquisition, maintenance, and recycling costs for the second strategy (i.e., the redundant system), as given by

$$\boldsymbol{CTreds}(\boldsymbol{n})=\begin{bmatrix}((\boldsymbol{n}\times\boldsymbol{Cc})+\boldsymbol{Csw})\\ +\boldsymbol{int}\left(\dfrac{\boldsymbol{H}}{\int_{0}^{T*}\boldsymbol{R_{sys}}(\boldsymbol{u})\boldsymbol{du}}\right)\times\left(\boldsymbol{R_{sys}}(\boldsymbol{T}^{*})(\boldsymbol{Mp}+\boldsymbol{Crpr})+\left(\mathbf{1}-\boldsymbol{R_{sys}}(\boldsymbol{T}^{*})\right)(((\boldsymbol{n}\times\boldsymbol{Cc})+\boldsymbol{Csw})+\boldsymbol{Cm}+\boldsymbol{Crcr})\right)\\ +\left(\mathbf{1}-\boldsymbol{R_{sys}}\left(\boldsymbol{H}-\left(\boldsymbol{int}\left(\dfrac{\boldsymbol{H}}{\int_{0}^{T*}\boldsymbol{R_{sys}}(\boldsymbol{u})\boldsymbol{du}}\right)\times\int_{0}^{T*}\boldsymbol{R_{sys}}(\boldsymbol{u})\boldsymbol{du}\right)\right)\right)\times(((\boldsymbol{n}\times\boldsymbol{Cc})+\boldsymbol{Csw})+\boldsymbol{Cm}+\boldsymbol{Crcr})\end{bmatrix}. \tag{7}$$

With.

$\boldsymbol{Rsys}(\boldsymbol{t})=\boldsymbol{e}^{-(\lambda+\lambda sw).t}\sum_{i=2}^{n-1}\frac{(\lambda.t)^{i}}{i!}$ [20]

$$\left.\frac{d(CTMs(T,n))}{dT}\right|_{T*}=0,$$

where int is the integer part.

Proof.

In the second strategy, based on the redundancy concept, we start the mission using a new system composed of n components and a switcher, meaning that the acquisition cost at the beginning is estimated by ((n × *Cc*) + Csw).

For this system, we applied a perfect PMstrategy at every T* time unit. In the case of failure before T*, CM is performed. A PMstrategy is developed. From (4), which expresses the maintenance cost, we note that the cycle length is estimated by

$$\int_{0}^{T}\boldsymbol{R_{sys}}(\boldsymbol{u})\boldsymbol{du}.$$

We can deduce that the number of cycles over the finite horizon H is

$$int\left(\frac{H}{\int_{0}^{T*}\boldsymbol{R_{sys}}(\boldsymbol{u})\boldsymbol{du}}\right)$$

For every cycle, we can take PM action to perform corrective maintenance CM action if a failure occurs before T*; otherwise, PM action is performed.

The corrective maintenance CM action induces n components plus a switcher to replace $(n\times\boldsymbol{Cc})+\boldsymbol{Csw})$, maintenance action cost *Cm*, and recycling cost in the corrective maintenance CM action Crcr. This implies that the corrective maintenance CM action is estimated as

$$(((n\times\boldsymbol{Cc})+\boldsymbol{Csw})+\boldsymbol{Cm}+\mathrm{Crcr}).$$

The PM action induces maintenance action cost Mp and recycling cost in the case of PM action Crp. The PM action is estimated as

$$(\boldsymbol{Mp}+\mathrm{Crpr})$$

Recall that the probabilities of corrective and PM actions are estimated by.

$(1-R_{sys}(T^{*}))$ and $(R_{sys}(T^{*}))$, we can deduce that the cost of corrective and PM actions, considering the recycling costs, which is expressed as

$$R_{sys}(T^{*})(\boldsymbol{Mp}+\mathrm{Crpr})+\left(1-R_{sys}(T^{*})\right)(((n\times\boldsymbol{Cc})+\boldsymbol{Csw})+\boldsymbol{Cm}+\mathrm{Crcr}).$$

Considering the number of cycles over the finite horizon, the total average cost over all cycles is expressed as

$$\boldsymbol{int}\left(\frac{\boldsymbol{H}}{\int_{0}^{T*}\boldsymbol{R_{sys}}(\boldsymbol{u})\boldsymbol{du}}\right)\times\left(\boldsymbol{R_{sys}}(\boldsymbol{T}^{*})(\boldsymbol{Mp}+\boldsymbol{Crpr})+\left(\mathbf{1}-\boldsymbol{R_{sys}}(\boldsymbol{T}^{*})\right)(((\boldsymbol{n}\times\boldsymbol{Cc})+\boldsymbol{Csw})+\boldsymbol{Cm}+\boldsymbol{Crcr})\right).$$

We assist in the possible period between the end of the last cycle and the end of horizon H, as estimated by

$$H - \left( int\left( \frac{H}{\int_0^{T*} R_{sys}(u)du} \right) \times \int_0^{T*} R_{sys}(u)du \right).$$

A possible failure can be occurred with a probability is given by

$$1 - R_{sys}\left( H - \left( int\left( \frac{H}{\int_0^{T*} R_{sys}(u)du} \right) \times \int_0^{T*} R_{sys}(u)du \right) \right).$$

Finally, considering the acquisition cost at the beginning of the cycle, the maintenance and recycle costs over all cycles inside horizon H, and the final period between the end of the last cycle and horizon H, we can reduce the total average cost of the mission integrated acquisition, maintenance, and recycling costs for the second strategy (i.e., the redundant system) as

$$CTreds(n) = \begin{bmatrix} ((n \times Cc) + Csw) \\ +int\left( \frac{H}{\int_0^{T*} R_{sys}(u)du} \right) \times \left( R_{sys}(T^*)(Mp + Crpr) + \left(1 - R_{sys}(T^*)\right)(((n \times Cc) + Csw) + Cm + Crcr) \right) \\ +\left( 1 - R_{sys}\left( H - \left( int\left( \frac{H}{\int_0^{T*} R_{sys}(u)du} \right) \times \int_0^{T*} R_{sys}(u)du \right) \right) \right) \times (((n \times Cc) + Csw) + Cm + Crcr) \end{bmatrix}$$

End of the proof.

#### 3.1.3. Decision

Our goal is to move to a redundant system to address the problem of several maintenance tasks caused by the increased failures of the system of one component. However, an economic decision should be made based on the number of components n adopted in the redundant system. We must determine the optimal number n* of components to ensure the maximal gain based on the adoption of the redundant system. From G, we note that a possible gain is realised by adopting the second strategy.

$$(n) = CTreds - CTc$$

Using Equations (1) and (7), we can obtain the analytical expression of G according to n, for $n \in \{2, 3, .......\}$, illustrated by the following equation (8):

$$G(n) = \begin{bmatrix} ((n \times Cc) + Csw) \\ +int\left( \frac{H}{\int_0^{T*} R_{sys}(u)du} \right) \times \left( R_{sys}(T^*)(Mp + \mathrm{Crpr}) + \left(1 - R_{sys}(T^*)\right)(((n \times Cc) + Csw) + Cm + \mathrm{Crcr}) \right) \\ +\left( 1 - R_{sys}\left( H - \left( int\left( \frac{H}{\int_0^{T*} R_{sys}(u)du} \right) \times \int_0^{T*} R_{sys}(u)du \right) \right) \right) \times (((n \times Cc) + Csw) + Cm + \mathrm{Crcr}) \end{bmatrix}$$
$$- [Cc + [(Cc + Cm + Crc) \times \lambda.\mathrm{H}]]. \tag{8}$$

With:

$Rsys(t) = e^{-(\lambda + \lambda sw).t} \sum_{i=2}^{n-1} \frac{(\lambda.t)^i}{i!}$ [20]

$$\left. \frac{d(CTMs(T, n))}{dT} \right|_{T*} = 0$$

The problem to solve is expressed as

$$min(G(n))|_{n=n*} \,/\, G(n^*) \le 0 \text{ (P1)}.$$

Precisely, if gain is ensured, function G is negative. Consequently, we must minimise G to maximise the gain.
To solve this problem, a numerical solving will be adopted.

#### 3.1.4. Numerical procedure

An iterative search procedure was developed to determine the optimal solution. This process comprises of four steps.

- o Step 1: Use (1) to compute the total average mission cost for the first strategy.
- o Step 2: Vary n ($n \in \{2, 3, .......Nmax\}$) and use Equations (4) and (5) to determine the optimal PM date T* for Strategy 2. The optimal PM cost related to T* is illustrated by equation (6). Then, use (8) to compute gain G.

**Table 1**
Nomenclature.

| Notation | Definition |
|---|---|
| n | Number of components of the redundant system (decision variable) |
| Nmax | Maximal number of possible components adopted in redundant system |
| H | Finite horizon |
| Csw | Cost of the switcher of the redundant system (stand-by system) |
| *Cc* | Unit component cost |
| *Cm* | Unit maintenance cost according to the replacement task |
| λ | Failure rate of one component |
| λsw | Failure rate of the switcher |
| Crc | Unit recycling cost of one component |
| Crcr | Unit recycling cost according to corrective maintenance CM action applied to redundant system |
| Crpr | Unit recycling cost according to PM action applied to redundant system |
| Mc | Unit cost of CM action for redundant system |
| Mp | Unit cost of PM action for redundant system |
| T* | Optimal date of PM action for redundant system |
| CTc | Total average cost of the mission with single component system |
| CTreds | Total average cost of the mission with redundant system |

**Table 2**
Nomenclature.

| Notation | Definition | Value |
|---|---|---|
| Nmax | Maximal number of possible components adopted in redundant system | 30 components |
| H | Finite horizon | 87000 tu (almost 10 years) |
| Csw | Cost of the switcher of redundant system (system stand by) | 200 mu |
| *Cc* | Unit component cost | 100 mu |
| *Cm* | Unit maintenance cost according to the replacement task | 90 mu |
| λ | Failure rate of one component | 0.005 |
| λsw | Failure rate of the switcher | 0.0003 |
| Crc | Unit recycling cost of one component | 25 mu |
| Crcr | Unit recycling cost according to corrective maintenance CM action applied to redundant system | (n* Crc+30) mu |
| Crpr | Unit recycling cost according to PM action applied to redundant system | Crcr/5 mu |
| Mc | Unit cost of corrective maintenance CM action for redundant system | n × cc + csw + cm + crcr |
| Mp | Unit cost of PM action for redundant system | Crpr + ((n × cc + csw + cm)/5) |

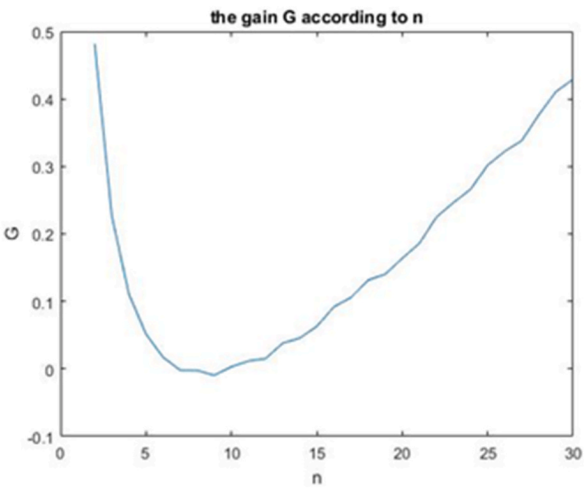


**Fig. 4.** The gain according to n.

- o Step 3: Determine [n1 n2] corresponding to $G \leq 0$, where [n1 n2] represents the interval of the number of components of the redundant system, ensuring a gain according to the adoption of the redundant system.
- o Step 4: Decision

If [n1 n2] = , the first strategy (one component) remains more economical.
If [n1 n2], determine n* ∈ [n1 n2] and minimise gain G.

**Table 3**
Sensitivity study according to the failure rate of the component.

| λ | MTTF (of component) | n* | [n1, n2] | T* |
|---|---|---|---|---|
| 0.0049 | 202 | 8 | 8 | 85257 |
| 0.005 | 200 | 7 | [7 9] | 73163 |
| 0.0055 | 181 | 5 | [5 13] | 458.72 |

**Table 4**
Sensitivity study according to the failure rate of the switcher.

| λsw | MUT (switcher) | n* | [n1, n2] | T* |
|---|---|---|---|---|
| 0.00028 | 3571 | 9 | [7 12] | 67883 |
| 0.00029 | 3448 | 8 | [7 11] | 43750 |
| 0.0003 | 3333 | 7 | [7 9] | 398 |
| 0.00031 | 3225 | ∅ | ∅ | ∅ |

## 4. Numerical study

### 4.1. Numerical example

A numerical example is presented in this section to illustrate the use of the analytical model developed in previous sections. The input data are summarised in Table 2 (mu and tu are monetary units (euros) and time units (hours)).

### 4.2. Numerical results

Matlab ®software was used to solve problem P(1) based on the numerical data cited in the last section. The curve of the gain G according to n is shown in Fig. 4 below.

It is clear that [n1 n2] = [7 9] and n* = 7, and the optimal gain obtained is G(n*) = −0.019. The numerical solution proved that the optimal PM date based on seven components adopted was T* = 70230. This means that the second strategy is more economical than the first when n ∈ [7 9], and the maximal gain is realised by adopting nine components and practising PM action at T* = 70230.

### 4.3. Sensitivity study and discussion

#### 4.3.1. Sensitivity study according to the failure rate of the component

We varied the failure rate of the component to determine the evolution of the interval [n1 n2] and the optimal value n*. Table 3 summarised the results obtained.

The obtained results are logical. An increase in the failure rate (i.e., a decrease in the mean time to failure) results in a poor reliability situation for the system with one component. This situation allows the user to quickly move to the second strategy based on redundancy with a more favourable solution of n. The interpretations are presented in Table 1. In fact, when the failure rate of one component increased, the mean time to failure decreased, the interval [n1 n2] increased, and its lower boundary was quickly reached. Concerning the optimal value of n* ensuring maximum gain, increasing the failure rate of the component induces the realisation of the optimal gain according to Strategy 2, which is the redundant system, with a minimal number of components. Precisely, the optimal number of components n* decreases when the failure rate increases (Table 1). According to the values of T*, the value of T* also decreases when the optimal number of components n* ensuring maximal gain decreases. This is logical because when the number of components decreases, the failure rate of the redundant system increases (see figures in Section 3.2.2), and we have addressed the failure rate by PM action, indicating that we have to perform more PM actions. This is illustrated by the decrease in the optimal date of PM action T* based on the decrease in the number of components n*.

#### 4.3.2. Sensitivity study according to the failure rate of switcher

The failure rate of the switch varied. The results are summarised in Table 4.

The length of the interval [n1 n2] is reduced when the failure rate of the switchers increases, indicating that the number of solutions ensuring the efficiency of the redundancy strategy is reduced. In addition, we noted the optimal number n* decreased. For a large failure rate of the switcher, there is no possible solution, indicating that the first strategy of single component is the economic for every value of n. The results obtained are logical. In fact, the goal according to the redundancy strategy consists of building a system with more reliability; however, we automatically decrease the reliability of the redundancy and reduce the solution ([n1 n2]) when we decrease the reliability of the switcher of the redundant system, ensuring the efficiency of the second strategy. According to the optimal date of PM action T*, we have the same logical interpretation as the last sensitivity study. Notably, a decrease in the optimal date of PM action T* according to the decrease in the number of components n* is observed.

**Table 5**
Sensitivity study according the unit cost of replacement task.

| *Cm* | n* | [n1, n2] | T* |
|---|---|---|---|
| 80 | ∅ | ∅ | ∅ |
| 90 | 7 | [7 9] | 73163 |
| 200 | 9 | [2 30] | 67034 |

**Table 6**
Sensitivity study according to the unit cost of one component.

| *Cc* | n* | [n1, n2] | T* |
|---|---|---|---|
| 90 | 9 | [7 12] | 66141 |
| 100 | 7 | [7 9] | 73163 |
| 105 | 4 | 4 | 87000 = H |
| 115 | ∅ | ∅ | ∅ |

#### 4.3.3. *Sensitivity study according to the unit cost of replacement task*

We vary the unit cost of replacement task to see the evolution of the interval [n1 n2] and the optimal value n*. The results are summarised in Table 5.

For the first case in Table 5, which corresponds to a low value of the unit cost of the replacement task, we note that there is no solution for the number of the components needed for the redundancy strategy, meaning that the first strategy is more economic. However, from lines 2 and 3 in Table 5, an increase in the unit replacement cost induces the existence of a solution according to n*. In addition, increase in the length of the interval [n1 n2] was easily observed. We recall that the goal based on the redundancy strategy consists of facing the expensive maintenance costs related to the replacement action. When the unit replacement cost increases (i.e., length of [n1 n2] increased), the second strategy of redundancy is more solicited, which is why having more solutions is suggested. In addition, the optimal number of components n* increased with an increasing unit replacement cost. According to T*, the increasing in the unit replacement cost induces an increasing in the corrective maintenance CM action (see cost of corrective maintenance CM action in Table 2 ($Mc = n \times cc + csw + cm + crcr$)), which is why we have to perform more PM actions to face the increasing corrective maintenance CM action. This fact is illustrated in lines 2 and 3 in Table 5. Increasing the unit replacement cost induces a decreasing optimal date of PM action.

#### 4.3.4. *Sensitivity study according to the unit cost of one component*

We varied the unit cost of one component to determine the evolution of the interval [n1 n2] and the optimal value n*. The results are summarised Table 6.

The unit cost of one component affects the acquisition cost. This is why an increase in the unit cost of a component reduces the efficiency of the redundancy strategy because this strategy is closely related to the acquisition cost impacted by the unit cost of the component. In Table 4, when the unit component cost increases, the length of the interval [n1 n2] is reduced, and the optimal number n* decreases. In addition, for high value of a high-cost component, there is no possible solution, indicating that the first strategy of a single component is more economical for every value of n.

According to T*, the increase in the unit replacement cost induces an increasing PM action (see the cost of corrective maintenance CM action in Table 2 ($Mp = Crpr + ((n \times cc + csw + cm)/5))$), which is why we have to do less PM action to address the increasing corrective maintenance CM action. This fact is illustrated in lines 1, 2, and 3 in Table 5. Increasing the unit replacement cost induces an increasing in the optimal date of PM action.

## 5. Conclusion and prospects

In this study, in addition to a traditional concept based on a single-component structure, we proposed a new system based on the redundancy concept to stratify a mission over a finite horizon H. The component was subjected to random failure at a constant failure rate. Despite the constant failure rate, several failures uncured over the finite horizon induce a hard cost related to the replacement and recycling of the failed component. To address this problem, we focused on the design of a redundant system made up of n identical components on standby in series with a switcher which ensures the transition to operational components in case of failure. According to this strategy, consists of moving to a more reliable system to reduce the number of maintenance interventions and its hard costs over a finite horizon. An analytical model was developed to compare the two structures by proposing a gain function expressed according to the number of components used in the redundant structure. The obtained function is analyzed to determine if it exists, the interval [n1 n2] favouring the redundancy strategy, and the optimal value n*, representing the optimal number of components adopted in the redundancy structure, ensuring a maximal gain realised through the consideration of the redundant system. The proposed modelling approach was illustrated using a numerical example. In addition, a sensitivity study based on important data has been performed to proof the analytical model developed. Extensions of this study are under consideration. One consists of relaxing some hypotheses, for example, considering an increased failure rate for components in addition to the constant failure rate considered in this study. Another assumption related to a constant cost is adopted, which should be considered as a variable over a finite horizon. For example, the unit cost of an intervention should increase over a horizon. Other extensions can be investigated. We can focus on other systems that are not essentially redundant and study the possible gains.

## Author contribution statement

Khaled H. Almotairi: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

## Data availability statement

Data included in article/supp. material/referenced in article.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## References

[1] P. Gackowiec, General overview of maintenance strategies–concepts and approaches, Multidisciplinary Aspects of Production Engineering 2 (2019).
[2] J. Zhao, C. Gao, T. Tang, A review of sustainable maintenance strategies for single component and multicomponent equipment, Sustainability 14 (5) (2022) 2992.
[3] R.E. Barlow, F. Proschan, Mathematical Theory of Reliability, SIAM, 1996.
[4] B. Jonge, P.A. Scarf, A review on maintenance optimization, Eur. J. Oper. Res. 285 (3) (2020) 805–824.
[5] E. Pămînţaş, F. veronica Banciu, A brief overview of the evolution and trends in maintenance, in: MATEC Web of Conferences, vol. 184, EDP Sciences, 2018, 03003.
[6] A. Baklouti, J. Schutz, S. Dellagi, A. qnd Chelbi, Selling or leasing used vehicles considering their energetic type, the potential demand for leasing, and the expected maintenance costs, Energy Rep. 8 (2022) 1125–1135.
[7] A. Baklouti, Multiple-Attribute Decision Making Based on the Probabilistic Dominance Relationship with Fuzzy Algebras, Symmetry 15 (6) (2023), 1188.
[8] Y. Peng, M. Dong, M.J. Zuo, Current status of machine prognostics in condition-based maintenance: a review, Int. J. Adv. Des. Manuf. Technol. 50 (1) (2010) 297–313.
[9] Y.I. Cortés, D.C. Berry, K.M. Perreira, A. Stuebe, L. Stoner, C.W. Giscombé, M. Duran, A multi-component, community-engaged intervention to reduce cardiovascular disease risk in perimenopausal Latinas: pilot study protocol, Pilot and Feasibility Studies 7 (1) (2021) 1–16.
[10] J. Lee, M. Ghaffari, S. Elmeligy, Self-maintenance and engineering immune systems: towards smarter machines and manufacturing systems, Annu. Rev. Control 35 (1) (2011) 111–122.
[11] S. Singh, D. Galar, D. Baglee, S.E. Björling, Self-maintenance techniques: a smart approach towards self-maintenance system, IntJ Sys. Ass. Eng.Manag. 5 (1) (2014) 75–83.
[12] J. Endrenyi, S. Aboresheid, R.N. Allan, G.J. Anders, S. Asgarpoor, R. Billinton, C. Singh, The present status of maintenance strategies and the impact of maintenance on reliability, IEEE Trans. Power Syst. 16 (4) (2001) 638–646.
[13] S.G. Tzafestas, Optimization of system reliability: a survey of problems and techniques, Int. J. Syst. Sci. 11 (4) (1980) 455–486.
[14] X. Qiao, F. Zhang, J. Zhao, X. Fang, "Non-Probabilistic reliability bounds method for series StructuralSystems considering redundant failure modes" applied, Science 12 (24) (2022), 12584, https://doi.org/10.3390/app122412584, 2022.
[15] F.A. Tillman, C.L. Hwang, W. Kuo, Optimization techniques for system reliability with RedundancyꟸA review, IEEE Trans. Reliab. 26 (3) (1977) 148–155.
[16] S. Karavida, R. Nõmmik, Waste Management of End-Of-Service Wind Turbines, Aalborg University, Aalborg, Denmark, 2015.
[17] V. Mitrofanovs, I. Boiko, Ē. Geriņš, New Approach for Recycling Spare Parts, Components and Assemblies, 2020.
[18] Ben Mechlia, , et al.M. Ben Mechlia, J. Schutz, S. Dellagi, A. Chelbi, Quasi-optimal sizing of a vehicle fleet considering environmental impact, maintenance, and eventual containment measures, Sustainability 2021 13 (8) (2021) 4384, 10.3390/su13084384.
[19] I. Gertsbakh, Reliability Theory: with Applications to Preventive Maintenance, Springer, New York, NY, USA, 2013.
[20] M. Rausand, A. Høyland, System Reliability Theory: Models, Statistical Methods and Applications, Wiley-Interscience, Hoboken, NJ, 2004.