Normalized and Directional Interprety December and Interrogation of Normalized and
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Abstract

Experiment protectional generation, A proteoform is a specific form of a protein that includes variations arisin
From genetic changes, alternative RNA splicing, proteolytic processing, and PTMs. Genomic context-dependent h from genetic changes, alternative RNA splicing, proteolytic processing, and PTMs. Genomic context-dependent histone
proteoforms define the histone code, influencing cellular phenotype by dictating interactions with DNA and from genetic changes, and the histone code, influencing cellular phenotype by dictating interactions with DNA and chromatin-
associated proteins. Understanding the dynamics of histone proteoforms is essential for elucidati regulatory mechanisms. Advances in middle-down and top-down proteomics methods enable accurate identification
quantitation of hundreds to thousands of proteoforms in a single run. However, the resulting data complexity pre quantitation of hundreds to thousands of proteoforms in a single run. However, the resulting data complexity presents
significant challenges for analysis and visualization. Here, we introduce new computational methods to a quantitation of hundreds to thousands of protections in a single run. However, the resulting sind complete sing
significant challenges for analysis and visualization. Here, we introduce new computational methods to analyze shypamics of histone PTMs and demonstrate their use in mouse organs during aging. We have developed and
benchmarked two novel PTM crosstalk scores. The score that we term 'Normalized Interplay' addresses limitations
origin denchmarked two novel PTM crosstalk scores. The score that we term 'Normalized Interplay' addresses limite
original crosstalk score 'Interplay' providing a more complete and accurate measure of PTM crosstalk. The sec
'delt original crosstalk score 'Interplay' providing a more complete and accurate measure of PTM crosstalk. The second score,
'delta I' or Directional Interplay is an asymmetric measure quantifying the magnitude and directionali original crosstalk score 'Interplay' is an asymmetric measure quantifying the magnitude and directionality of crosstalk

between PTMs. Applying our two-stage scoring approach to data from CrosstalkDB, a community resource The I'm The Latin II or Directional Interpretional Interpretional Interpretional Interpretional Interpretional

Deltween PTMs. Applying our two-stage scoring approach to data from CrosstalkDB, a community resource that cur proteoform-level data, reveals the dynamics of histone H3 modifications during aging. The source code is available under
an Apache license at https://github.com/k-p4/ptm_interplay_scoring.
Introduction
As mass spectrometry

Introduction

protection-level data, reveals the dynamics of histone H3 modifications during aging. The source code is available under
an Apache license at https://github.com/k-p4/ptm_interplay_scoring.
As mass spectrometry has become m Introduction
As mass spectrometry has become more powerful proteomics experimexperimental and biological features. This includes analyzing proteome
biological replicates, across various conditions and temporal stages. Sur experimental and biological features. This includes analyzing proteomes from diverse tissues, using both technical and
biological replicates, across various conditions and temporal stages. Such large-scale proteomics effor experimental and biological replicates, across various conditions and temporal stages. Such large-scale proteomics efforts yield insights in
the system under study but pose significant challenges in data analysis and inter biological replications various various various conditions in the system under study but pose significant challenges in data analysis and interpretation. These challenges are
exacerbated when exploring protein post-transla exacerbated when exploring protein post-translational modifications (PTMs). The proteoform represents the tru
physiological state of proteins, including post-translational modifications co-occur in combination on single mo physiological state of proteins, including post-translational modifications co-occur in combination on single molecu
Multiple proteoforms of each protein in a sample exhibit unique sets of PTMs, each of which may present u physiological state of proteins, including post-translational modifications co-occur in combination on single molecules^{2,2}.
Multiple proteoforms of each protein in a sample exhibit unique sets of PTMs, each of which may functional effects³. This complexity is particularly noticeable in hyper-modified proteins such as histones, where a
array of histone proteoforms exist and are often interrelated in function⁴⁻⁷. The visualization of th functional effects³
array of histone pr
^{array} of histone pr $\frac{1}{2}$. The visualization of these highly oteoforms exist and are often interrelated in function⁴⁻⁷. The visualization of these highly array of histone proteoforms exist and are often interrelated in function⁺ $\dot{}$. The visualization of these highly
distribution of these highly the visualization of these highly experience of the visualization of t

functional and mechanistic biological insights. In this work, we introduce two new crosstalk scores to address the
shortcomings of existing scores, including a new metric that quantifies the directionality of histone PTM c

shortcomings of existing scores, including a new metric that quantifies the directionality of histone PTM crosstalk.
Histone proteins carry diverse epigenetic marks and play crucial roles in cellular processes like alterin shortcomings of existing scores, including a new metric that quantifies the directionality of histone proteins
Histone proteins carry diverse epigenetic marks and play crucial roles in cellular processes like altering chro Extructure, genome maintenance, and gene regulation. Enzymatic PTM readers recognize specific modifications, re
PTM writers and erasers to modify other sites, a process influenced by existing PTMs. This interplay between P Structure, genome maintenance, and gene regulation. Enzymatic University respectively between PTMs is
The provides and erasers to modify other sites, a process influenced by existing PTMs. This interplay between PTMs is
te Formed Crosstalk. Crosstalk can be either positive or negative and therefore we define this crosstalk as how the present
of one PTM can either potentiate or preclude the presence of another PTM as positive and negative cro the prior one PTM can either potentiate or preclude the presence of another PTM as positive and negative crosstalk,
respectively. One such canonical example is the binary mark of K9 methylation and S10 phosphorylation, whi of one PTM can either presence or presence or anomic PTM as points analyzed of another presence of the progres

important mechanism in the progression of mitosis (a 'methylation/phosphorylation switch').

Accurately measur

important mechanism in the progression of mitosis (a 'methylation/phosphorylation switch').
Accurately measuring PTM crosstalk is challenging, necessitating detailed information on PTM presence across multip
specific amino important measuring PTM crosstalk is challenging, necessitating detailed information on PTM
specific amino acids on a histone. Traditional bottom-up mass spectrometry typically analyzes
chains, which most often does not ca Accuration, the accuration of the Hackenham cross of the mass spectrometry typically analyzes 7-20 aa long oligopeptide

chains, which most often does not capture this co-occurrence. Proteoform identification and quantitat specific aminost often does not capture this co-occurrence. Proteoform identification and quantitation is most directly
achieved by top-down proteomics^{4,6–10}. However, top-down approaches sometimes fail to distinguish is achieved by top-down proteomics^{4,6–10}. However, top-down approaches sometimes fail to distinguish isobaric PTM
combinations from each other. Notably, the intact analysis of physiological histone H3 is challenging due to achieved by top-down proteomics^{2,6}–²⁰. However, top-down approaches sometimes fail to distinguish isobaric PTM
combinations from each other. Notably, the intact analysis of physiological histone H3 is challenging due t isobaric species that are difficult to resolve chromatographically¹¹. Middle-down mass spectrometry overcomes these
limitations by analyzing longer histone segments such as the 1-50 aa N-terminal tail of Histone H3, ena isobaric species that are difficult to resolve chromatographically¹¹. Middle-down mass spectrometry overcomes these
limitations by analyzing longer histone segments such as the 1-50 aa N-terminal tail of Histone H3, enab

assessment of proteoform abundances and the relationships between PTM combinations^{12–15}.
Introduced in 2014, the interplay score was initially used to capture and quantify multilayered histone PTM inter
While Interplay c assessment of proteoform abundances and the relationships between PTM combinations²².
Introduced in 2014, the interplay score was initially used to capture and quantify multilayered l
While Interplay could only provide s Interplay could only provide snapshots of identified PTM pairs under isolated conditions, Abundance Corrected
Interplay (introduced in 2020) refined the interplay score to more precisely assess crosstalk, enabling comparis Interplay (introduced in 2020) refined the interplay score to more precisely assess crosstalk, enabling comparisons acredifferent conditions and temporal scales^{16,17}. However, both suffer from similar limitations that we Interpret conditions and temporal scales^{16,17}. However, both suffer from similar limitations that we address here with the inovel Normalized Interplay Score and Directional Interplay Score.
Materials & Methods
Data sourc different conditions and temporal scales^{20,27}. However, both suffer from similar limitations that we address here with the
novel Normalized Interplay Score and Directional Interplay Score.
Materials & Methods
Data source

novel non-play Score and Directional Interpretational Interpretational Interpretational Interpretational Inter
Data sources Materials & Methods

Data sources

Proteoform-level data used in this study was sourced from the CrosstalkDB database^{47,45}. Specifically, we focused on
datasets derived from histone H3 proteoforms quantitated from the brain, heart, liver, and kidneys of 3 24-month-old C57BL/6 mice. The dataset is a rich source of PTM co-occurrences across different biological contexts.
CrosstalkDB files CrDB000062 through CrDB000126 were downloaded in .CSV format for further processing. In 24-month-old CrosstalkDB files CrDB000062 through CrDB000126 were downloaded in .CSV format for further processing. In addititio the aging dataset, we also applied our analysis to a dataset from the Kelleher lab, which use Crossimal Entertainment was also applied our analysis to a dataset from the Kelleher lab, which used M4K – a method of total
Kinetic analysis to understand the mechanisms of how combinatorial methylation patterns are estab kinetic analysis to understand the mechanisms of how combinatorial methylation patterns are established on histone H3
K27 and K36 residues in myeloma cells expressing high (NTKO) or low (TKO) levels of MMSET/WHSC1/NSD2¹⁹ kinetic analysis to understand are included to understand in the mechanism pattern patterns are established the
K27 and K36 residues in myeloma cells expressing high (NTKO) or low (TKO) levels of MMSET/WHSC1/NSD2¹⁹. The
 K27 and K36 residues in myeloma cells expressing high (NTKO) or low (TKO) levels of MMSET/WHSC1/NSD2²². The
dataset was chosen as the effective rate constants determined were used to infer the crosstalk and its direction dataset was chosen as the effective rate constants determined were used to infer the canceler methylation states.
Formulation of Normalized Interplay
Definitions

ment, and the cases
Formulation of Nor
Definitions
Given two PTMs, *P1* Formulation of Normalized Interplay

Definitions

- P_{PTM1} is the probability of occurrence of PTM1.
- Given two PTMs, PI

 P_{PTM1} is the

 P_{PTM2} is the

 $P_{PTM1PTM2}$ i MT and PT
probability
probability
s the joint p μ_1 is the probability of occurrence of PT
 μ_2 is the probability of occurrence of PT
 μ_1 μ_1 μ_2 is the joint probability of co-occurrence of PT
Interplay (I) Schwammle et al. (2014) P_{PTM2} is the probability of occurrence of PTM2.
- MZ their associated probabilities of occurrence are defined as follows:
of occurrence of $PTM1$.
of occurrence of $PTM2$.
probability of co-occurrence of $PTM1$ and $PTM2$. M1.
M2.
Irren
Pen B \bullet P_{PTM1PT}

 μ_1 _{M1PTM2} is the joint probability of co-occurrence of μ_1
 μ_1 _{M1PTM2} is the joint probability of co-occurrence is al. (2014)

ay score quantifies the relationship betwe

coted co-occurrence if the two PTMs a *m* z.
irren
en P μ _{M2} is the joint probability of co-occurrence of *PT*
rplay (I) Schwammle et al. (2014)
ore quantifies the relationship between PTM1 and
co-occurrence if the two PTMs are independent, MT and PT
 H PTM2, cordefined as: mz.
npar The Interplay score quantifies the relationship betties the expected co-occurrence if the two PTMs and $I(PTM_1PTM_2)$ The Interpret co-occurrence if the two PTMs are independent, defined as:
 $I(PTM_1PTM_2) = \ln\left(\frac{p(PTM_1PTM_2)}{p(PTM_1) \cdot p(PTM_2)}\right)$

The log transformation ensures that:

$$
I(PTM_1PTM_2) = \ln\left(\frac{p(PTM_1PTM_2)}{p(PTM_1) \cdot p(PTM_2)}\right)
$$

The log transformation ensures that:
\n• $I > 0$ when PTM_1 and PTM_2 co-occur more often than expected (positive a
\n• $I = 0$ when PTM_1 and PTM_2 are independent.

-
-
- $I > 0$ when PTM_1 and PTM_2
 \bullet $I = 0$ when PTM_1 and PTM_2
 \bullet $I < 0$ when PTM_1 and PTM_2 • $I > 0$ when PT
• $I = 0$ when PT
• $I < 0$ when PT M_1 and PI
 M_1 and PT
 M_1 and PT m_2 co-occur more orten than expected (positive association).
 M_2 are independent.
 M_2 co-occur less frequently than expected (negative association). • $I = 0$ when PT
• $I < 0$ when PT M_1 and PI
 M_1 and PT m_2 are independent.
 M_2 co-occur less frequents • $I < 0$ when PT m_1 and PI m_2 co-occur less frequently than expected (negative association).
 m_2 co-occur less frequently than expected (negative association).

While the Interprise provides integrals into the association scores for low-frequency outcomes: The Interplay score tends to inflate a scores for PTM pairs involving low-frequency events. As the marginal probabilities $p($ scores for PTM pairs involving low-frequency events. As the marginal probabilities $p(PTM_1)$ or $p(PTM_2)$
approach zero, the denominator becomes extremely small resulting in an Interplay score approaching negative
infinit scores for PTM pairs involving low-frequency events. As the marginal probabilities $p(PI)$
approach zero, the denominator becomes extremely small resulting in an Interplay score
infinity:
i. $\lim_{p(PTM_1)\to 0} I(PTM_1PTM_2) \to -\infty$ m_1) or $p(PI)$
e approaching
s to produce $m₂$)
g neg

i.
$$
\lim_{p(p \cap M_1) \to 0} I(PTM_1PTM_2) \to -\infty
$$

- infinity:
 $\lim_{p(p_{TM_1}) \to 0} I(PTM_1PTM_2) \to -\infty$

Low positive association scores for high-frequency outcomes: Conversely, Interplay tends to produce low

association scores for pairs involving high-frequency outcomes due to infinity
Low pos
associati
Unboun
- i. $\lim_{p(PT)}$
association
cores for pa
values: The
PTMs. This M_2) → $-\infty$
equency outerpread outer
frequency outerpread outer
t scale makes 2. Low position scores for pairs involving high-frequency outcomes due to the properties of the logarithmic fu
3. Unbounded values: The interplay score lacks fixed bounds in cases of perfect positive or negative associat
b association scores for pair interplay score lacks fixed bounds in cases of perfect positive or negative association
between two PTMs. This lack of consistent scale makes it difficult to compare interplay scores across mult 3. Between two PTMs. This lack of consistent scale makes it difficult to compare interplay scores across multiple
3. PTM pairs or datasets.
T Work: Abundance Corrected Interplay (ACI) Kirsch et al. (2020)
19. Property as

between the PTMs. This lack of consistent scale makes it annually scale makes it different scale makes it
price Soft. Abundance Corrected Interplay (ACI) Kirsch et al. (2020)
nce Corrected Interplay (ACI) was introduced to PHM PERMISSION
Drk: Abundance Corrected
Anterplay
bundance both relative Abundance Corrected Interplay (ACI) was introduced to address the life
PTM's abundance both relative to the other combination and the bina
 $\widehat{PTM}_1 = \frac{(p(PTM_1) - (p(PTM_1PT_1) \cdot (1 - p(PTM_2))p(PT_1 \cdot T_1)))}{(1 - p(PTM_2))(p(PT_1 \cdot T_1 \cdot T_2 \$

PTM's abundance both relative to the other combination and the binary combination:
\n
$$
\overline{PTM}_1 = \frac{(p(PTM_1) - (p(PTM_1PTM_2))(p(PTM_1))}{(1 - p(PTM_2)(p(PTM_1PTM_2))}
$$
\n
$$
\overline{PTM}_2 = \frac{(p(PTM_2) - (p(PTM_1PTM_2))(p(PTM_2))}{(1 - p(PTM_1)(p(PTM_1PTM_2))}
$$
\nYielding the ACI score as:

$$
ACI(PTM_1PTM_2) = \ln \left(\frac{p(PTM_1PTM_2)}{p(PTM_1) \cdot p(PTM_2)} \cdot \frac{(1 - p(PTM_1))(1 - p(PTM_2))(p(PTM_1PTM_2))}{(p(PTM_1) - p(PTM_1PTM_2)) \cdot (p(PTM_2) - p(PTM_1PTM_2))} \right)
$$
\nThis formulation was designed to provide better symmetry between positive and negative interplay.

Normalized Interplay (NI)

)
ay This formulated Interplay (NI)
Since the limitations of the raw interplay score can affect interpretability, we introduce here the Nori
Score. When two PTMs always co-occur, the probability of observing one given the prese score. When two PTMs always co-occur, the probability of observing one given the presence of the other equals the journalised interpretability of their co-occurrence. In this scenario the interplay score yields:
probabilit probability of their co-occurrence. In this scenario the interplay score yields:
probability of their co-occurrence. In this scenario the interplay score yields: probability of their co-occurrence. In this scenario the interplay score yields:

$$
I(PTM_1PTM_2) = -\ln (p(PTM_1) = -\ln (p(PTM_2)) = -\ln (p(PTM1PTM_2)))
$$

 $\frac{M_2}{}$
ural log
both the From probability is particularly appealing as it provides a symmetric normalization that adjusts both the upper and lot
bounds:
Normalization $Factor (PTM_1PTM_2) = -\ln(p(PTM_1PTM_2))$
The pormalization factor adjusts the interplay sco

$$
Normalization Factor (PTM_1PTM_2) = -\ln(p(PTM_1PTM_2))
$$

bounds:
 $Normalization Factor (PTM_1PTM_2) = -\ln(p(PTM_1PTM_2))$

The normalization factor adjusts the Interplay score based on the 'rarity' of the co-occurrence of the two PTMs.

Specifically as $n(PTM_1PTM_2)$ decreases, the value of $-\ln(n(PTM_1PTM_$ The norn
Specifica
odiusted $\frac{M_2}{J}$
rence
The In
a high Specifically, as $p(PTM_1PTM_2)$ decreases, the value of $-\ln(p(PTM_1PTM_2))$ increases. The Interplay score is the adjusted according to this scaling factor, such that more infrequent co-occurrences yield a higher (negative) ad Spec and log m_1 P T m_2) decreases, the value of $-$ In $(p$ (PT m_1 PT M_2) decreases, the value of $-$ In $(p|PI)$
scaling factor, such that more infrequer
 $NI(PTM_1PTM_2) = \frac{\ln(\frac{p}{p(I)})}{-\ln(\frac{p}{p(I)})}$ $\begin{aligned} &M_2$) J increases. The Interplay score is thus
ccurrences yield a higher (negative) adjustr $\frac{M_1PTM_2)}{M_1PTM_2)}\ &M_1PTM_2))\ \end{aligned}$

$$
NI(PTM_1PTM_2) = \frac{\ln(\frac{p(PTM_1PTM_2)}{p(PTM_1) \cdot p(PTM_2)})}{-\ln(p(PTM_1PTM_2))}
$$
\nThis normalization constrains the score within a bounded interval [–1, 1] offering a more intuitively interpretable metric.

 M_2))
ering a

Formulation of the Directional Interplay Score (∆/)

Rationale for Introducing Directionality in PTM Crosstalk Analysis

This normalization constrains the score within a bounded interval $[-1, 1]$ offering a more intuitively interpretable metric:
Formulation of the Directional Interplay Score (ΔI)
Rationale for Introducing Directionality that the probability of PTM2 occurring given PTM1's presence is equal to the probability of PTM1 occurring given PTM:
presence. Mathematically, this is expressed as: $P(PTM2|PTM1) = P(PTM1|PTM2)$, where $P(PTM2|PTM1)$
represent t presence. Mathematically, this is expressed as: $P(PTM2|PTM1) = P(PTM1|PTM2)$, where $P(PTM2|PTM1)$
represent the probability of PTM1 given PTM2 and vice-versa. However, this assumption overlooks that PTM crosstalk is
often asymm presence. Mathematically, this is expressed as: $P(PI)$
represent the probability of PTM1 given PTM2 and vi
often asymmetric, where one PTM exerts a stronger i
measures of interplay collapse the distinct conditiona
introduc $MZ|PIM1$) = $P(PIM1|PIM2)$, where $P(PIMZ|PI)$ MZ), where $P(PT)$
mption overlooks
on-reciprocal man
 $M1)$ and $P(PTM1)$
regulatory systems *M* I)
M cro
³. Syn
2), w
M de often asymmetric, where one PTM exerts a stronger influence on the other in a non-reciprocal manner^{20–23}. Symmetric
measures of interplay collapse the distinct conditional probabilities $P(PTM2|PTM1)$ and $P(PTM1|PTM2)$, whi often asymmetric, where one PTM exerts a stronger influence on the other in a non-reciprocal manner²⁰–23. Symmetric
measures of interplay collapse the distinct conditional probabilities $P(PTM2|PTM1)$ and $P(PTM1|PTM2)$, whi measures of interplay collapse the distinct conditional probabilities $P(PI)$
introduces inaccuracies in interpretation, especially in hierarchical or sequent
metworks, where directionality is critical for understanding re $MZ|PIM1)$ and $P(PIM1|PI)$ *M2)*, which
PTM depend
ve introduce introduces interpretation, especially in interpretation of experimental regulation, experimentation interpretation,
networks, where directionality is critical for understanding regulatory interactions. To address this gap directional interplay score which separates these conditional probabilities by independently evaluating
 $P(PTM2|PTM1)$ and $P(PTM1|PTM2)$.

PTM Information Categorization and Definitions $P(PTM2|PTM1)$ and $P(PTM1|PTM2)$.
PTM Information Categorization and Definitions $P(PI | MZ | P I | M1)$ and $P(PI | M1 | P I)$

MZ).
Id Def PTM Information Categorization and Definitions

To derive the directional interplay score, we categorize the presence or absence of two PTMs, PTM1 and PTM2, into four

- a: Joint probability where both PTM1 and PTM2 are present $(P(PTM1 \cap PTM2))$
	- o calculated as: abundance of the binary combination PTM1PTM2 calculated as: abundance of the binary combination PTM1PTM2
- b: Joint probability where PTM1 is present and PTM2 is absent $(P(PT M1 \cap PT M2))$
	- \circ calculated as: normalized discrete abundance of PTM1 $-$ a

- \quad c: Joint probability where PTM2 is present and PTM1 is absent $\bigl(P(PTM2 \cap PTM1)\bigr)$
	- \circ calculated as: normalized discrete abundance of PTM2 a
- d : Joint probability where neither PTM1 nor PTM2 is present $(P(PTM1 \cap PTM2))$
	- o calculated as:

calculated as: $1 - (normalized discrete abundance PIM1 + normalized discrete abundance)$
 $1 - (normalized discrete)$

\overline{a} Bayesian Framework for Directional Interplay

The directional interplay score (ΔI) leverages conditional probabilities to quantify the influence one PTM exerts on the
other. By adopting a Bayesian framework, we treat the presence of PTM1 as information that updates o other. By a Bayesian framework, we treat the presence of P \geq PTM2's presence, akin to Bayesian updating24. For example, the conditional probability --2|-1 , representing the probability of PTM2 given PTM1, is computed as: $P(PTM2|PTM1) = \frac{P(PTM1|PTM1)}{P(PTM1)}$ $\frac{n \ln \ln P_1 m_2}{P(PTM1)} = \frac{a}{a+b}$

Here, $P(PIML|PIML)$ is the posterior probability of PTM2 updating our prior knowledge of $P(PIMZ)$ based on the $PIDIMZ$ σ occurrence of PTM1. Similarly, the conditional probability of $P(PIMZ|PIMI)$ reflects the likelihood of PTM2 when

$$
P(PTM2|PTM1) = \frac{P(PTM1 \cap PTM2)}{P(PTM1)} = \frac{c}{c+d}
$$

 T conditional probabilities quantify how the presence or absence of P M1 influences the likelihood of P

vice versa, allowing us to compute the directional interplay by comparing these probabilities.

Formulation of the Directional Interplay Score (ΔI)

The directional interplay score ∆I measures the difference between the conditional probabilities of one PTM depending on the presence or absence of another. Thus, quantifying whether the presence or absence of one PTM increases or decreases the likelihood of observing the other PTM:

$$
\Delta I(PTM2|PTM1) = P(PTM2|PTM1) - p(PTM2|PTM1) = \frac{a}{a+b} - \frac{c}{c+d}
$$

A positive value for ∆I(PTM2|PTM1) indicates that PTM2 is more likely to occur when PTM1 is present, suggesting a positive directional influence.

$$
\Delta I(PTM1|PTM2) = P(PTM1|PTM2) - p(PTM1|PTM2) = \frac{a}{a+c} - \frac{b}{b+d}
$$

 S_{max} , p and score quantifies how much PTM1 depends on the presence or absence or absence or p ΔI(PTM1∣PTM2) reflects a positive directional influence of PTM2 on PTM1. By independently evaluating $P(PI | M2 | PI | M1)$ and $P(PI | M1 | PI | M2)$, but captures the directional dependencies between PTMs, offering a more nuanced understanding of PTM crosstalk that cannot be quantified by symmetric measures. The cannot be quantified by symmetric measures. The contract of $\mathcal{P}(t)$

Data Processing Workflow

Jensen Aging dataset: Each run file from CrosstalkDB was divided into two H3 variant-specific files (H3.1/2 and H3.3). The percentage abundance of each proteoform was calculated by normalizing its intensity to the total intensity of the proteoform family, resulting in a per-family intensity of 100%. Proteoforms containing PTMs relevant to previous
analyses were retained. The PTM code column was aggregated to full proteoform sequences, including unmodified analyses were retained. The PTM code column was agregated to full protein protein protein \mathcal{L} residues. PTM combinations were generated using Python's iterations functions functions function for all possible PTM α configurations up to the proteoform level. Interplay scores were calculated for each binary PTM combination according to established formulae by mapping discrete PTM values to their constituent binary combinations. ΔI scores were computed by constructing contingency tables for each proteoform presence/absence pair. Normalized interplay scores were organized into matrices and visualized using the correlation package into source-target using the converted into source-target into source-target into source-target into source-target into source-target into source-ta

nodes for network analysis, with the antecedent as the source and the consequent as the target. Networks were constructed using Python's NetworkX library for further analysis^{es} fill.

Kelleher M4K dataset: The observed levels of all 15 detectable combinatorial methylations of the H3K27-K36 peptide in
TKO and NTKO cells were downloaded from Table S1. Al scores were computed by constructing contingency ta $T_{\rm eff}$ such an and $T_{\rm eff}$ scores were computed by constructing contingency table since σ each PTM presence/absence pair in Excel (Microsoft Office 365 – Supplemental file 2).

Results & Discussion

Orientation Values for Interplay, Abundance Corrected Interplay, and Normalized Interplay

We examine the behavior of three PTM crosstalk scores across different scans of \mathcal{C} in Table 1).

• Interplay:
$$
I(PTM1PTM2) = \ln \left(\frac{PPTM1PTM2}{PPTM1 \cdot PPTM2} \right)
$$

• Abundance Corrected Interplay: $ACI(PTM1PTM2) = \ln \left(\frac{FPTM1PTM2}{P_{PTM1} \cdot P_{PTM}} \right)$ <u>PPTM1[,] Pptm2</u> , CONTENT (PPTM1) (1-PPTM2) PPTM1PTM2
Pptm₁, Pptm₂ (Pptm₁-Pptm₁ptm₂)(Pptm₂-Pptm₁p $\frac{1}{(PPTM_1 - PPTM_1PTM_2)(PPTM_2 - PPTM_1PTM_2)}$

• Normalized Interplay: $NI(PTM1PTM2) = \ln \left(\frac{FPTM1PTM2}{PPTM1 \cdot PPTM} \right)$ $\frac{1}{P_{PTM_1} \cdot P_{PTM_2}}$ // $-\ln(P_{PTM1PTM2})$

Perfect Positive Crosstalk

In the scenario of perfect positive crossing, where PTM1 and PTM2 always co-occur. Here, as the joint probability $r_{PTM1PTM2} - r_{PTM1} - r_{PTM2}$, the interplay simplifies to $-\text{m}$ ($r_{PTM1PTM2}$). For ACI, the correction factor introduces terms that adjust for the relative abundance or the PTMs. However, as the joint probability $r_{PTM1PTM2}$ approaches r_{PTM1} ui r , the decreases towards zero, causing the ACI to diverge towards positive infinity: $ACI \approx \ln\left(\frac{1}{0}\right) = +\infty$. Since NI normalizes the interplay score by the negative logarithm of the joint probability: $NI = \frac{-\ln (FPT M1PTM2)}{-\ln (PPTM1PTM2)}$ $-\ln (PPTM1PTM2) - 1.$

No Crosstalk

 W and W and probability, their joint probability equals the probability equals their individual probabilities: $p(PI M_1PI M_2) = p(PI M_1) \cdot p(PI M_2)$. In this case, all the scores essentially reduce down to In(1) = 0, accurately capturing the scenario of no crosstalk.

Perfect Negative Crosstalk

In the scenario of perfect negative crosstally, where PTM1 and PTM1 and PTM2, and Joint probability, $p(PTM_1PTM_2) = 0$. This results in: $I = \ln\left(\frac{0}{P_{PTM_1}P_{PTM_2}}\right) = \ln(0) = -\infty$. Similarly, for ACI, the numerator also becomes

zero leading to ACI diverging to negative infinity. However, for NI as the joint probability tends towards zero, NI converges

This bounded nature of NI ensures that even in cases of perfect positive or negative crosstalk, the score remains finite

and interpretable, offering a stable measure of interplay.

Table 2 Orientation values of PTM Crosstalk Scores

Condition of	Interplay	Abundance	Normalized
PTM co-occurrence	(1)	Corrected	Interplay
		Interplay (ACI)	(NI)
Perfect positive crosstalk	$-\ln(P_{PTM1PTM2})$	$+\infty$	
(When two PTMs always co-occur)			
No crosstalk	0	Ω	0
(When two PTM co-occur as expected			
under independence)			
Perfect negative crosstalk	$-\infty$	$-\infty$	-1
(When two PTMs never co-occur)			

Functional Behavior of Interplay Scores Across Co-occurrence Probabilities

Each score responds uniquely to changes in the joint probability of PTM co-occurrence, influenced by the terms in their respective formulas. Figures 1A and 1B illustrate the behavior of the Interplay (I), Abundance Corrected Interplay (ACI), and Normalized Interplay (NI) scores across different co-occurrence probabilities for two PTMs. In Figure 1A, both I and
ACI show non-linear responses. As the co-occurrence of PTMs approaches zero, both scores diverge towa ACI show non-linear responses. As the co-occurrence of PTMs approaches zero, both scores diverge towards negative infinity, indicating strong negative crosstalk. As the α move towards zero and the α diverges towards positive infinity, signaling strong positive crosstalk.

Figure 1B focuses on NI, which exhibits a linear and bounded response across the majority of the co-occurrence range.
Unlike I and ACI, NI scales proportionally with changes in co-occurrence, avoiding overemphasis on minor Unlike I and ACI, NI scales proportionally with changes in co-occurrence, avoiding over \mathcal{C} This linearity ensures that NI remains stable and interpretable, providing a consistent measure that transitions smoothly from -1 to 1 as the co-occurrence of two PTMs moves from mutual exclusivity to perfect co-occurrence.

Figure 1. Comparison of the three Crosstalk scores: (A) The Interplay score (I), Abundance Corrected Interplay (A ACI), and α functionalized interpray (NI) protted as a function of the co-occurrence of two PTMs (y-axis) with respect to their frequency α axis). The ACI (red) adjusts for PTM abundance, while the NI (blue) normalizes interplay scores within the bounds of [-1,1]. The original Interplay score (black) is shown for reference.

(B) Detailed plot of Normalized Interplay (NI) highlighting the bounded range of [-1,1] as the co-occurrence of PT Ms

increases. This panel magnifies the NI transformation to clearly indicate how values differ at low co-occurrence rates. (C-E) Scatter plots of crosstalk between two PTMs quantified with each score across all organs, histones (H3.1/2 and H3.3) and ages. R° denotes the coefficient of determination between each pair of plotted points. (F) Percentage of 'Data
 Completeness' considering all quantifiable instances of crosstalk across histones H3.1/2 and H3.3 from different organs

Empirical Analysis of Interplay Scores in the Aging Dataset

Following the theoretical and functional comparison between the interplay scores We empirically analyze the Interplay
(I), Abundance Corrected Interplay (ACI), and Normalized Interplay (NI) scores using the aging dataset, (I), Abundance Corrected Interplay (ACI), and Normalized Interplay (NI) scores using the aging dataset, focusing on their pairwise correlations and data completeness across various organs and age groups.

Correlation between scores

and ages.

For PTM pairs where crosstalk is quantitated across all three scores, a high positive correlation is observed between all score comparisons. I and ACI exhibit a correlation of R°=0.90 (**Figure 1C**), indicating that while ACI follows the general
 trend of I, the deviations from the identity line show that ACI introduces specific abundance corrections that become
more pronounced at extreme positive values. The comparison between I and NI shows a slightly lower corre more pronounced at extreme positive values. The comparison between I and NI shows a slightly lower correlation (R°=0.84 **Figure 1D**) reflecting NI's normalization process. This, taken together with the high correlation between ACI and
م NI (R°=0.90, **Figure 1E**), indicates that NI incorporates the abundance corrections of ACI while introducing a
 normalization that differentiates its interpretation.

Data Completeness Across Organs and Ages

Notably, a number of PTM pairs are not quantitated across all three scores and therefore not reflected in the correlation
analysis between the scores. Across five organs and five ages, NI consistently demonstrates higher d analysis between the scores. Across five organs and five ages, NI consistently demonstrates higher data completeness due to its ability to quantitate perfect positive and perfect negative crosstalk (Figure 1F).

Theoretical Bounds and Simulation of Delta I

Theoretical Bounds of ∆I

The directional dependency between two post-translational modifications (PTMs) is quantified $\frac{1}{2}$, which is bounded by between -1 and 1. The extreme values of \mathcal{L} indicate specific types of relationships between PTM1 and PTM2. And PTM2. And PTM2. And PTM1 and PTM ∆I(PTM2 | PTM1) value of 1 corresponds to a perfect positive dependency, where PTM2 occurs exclusively when PTM1 is
present, i.e., P(PTM2|PTM1) = 1 and P(PTM2|–PTM1) = 0. Conversely, a ∆I(PTM2|PTM1) value of -1 indicates present, i.e., P(PTM2∣PTM2) = 1 and P(PTM2∣→PM1) = 0. Conversely, a ∆I(PTM2|PTM1) value of -1 indicates mutual exclusion, with P(PTM2|PTM1) = 0 and P(PTM2|¬PTM1) = 1, meaning PTM1 and PTM2 never co-occur. A ΔI(PTM2|PTM1)
value of 0 reflects independence. where P(PTM2|PTM1) = P(PTM2|¬PTM1). signifying that the occurrence of PTM1 h value of 0 reflects independence, where P(PTM2∣PTM1) = P(PTM2∣→PTM1), signifying that the occurrence of PTM1 has

Simulation of ∆I

To investigate the behavior of ∆I under different probabilistic conditions, we simulate values by fixing the probability of PTM1 (arbitrarily set at 0.5) and varying the conditional probabilities P(PTM2|PTM1 and P(PTM2|¬PTM1) across their full
range from 0 to 1. This allows us to assess how various combinations influence the magnitude and sig range from 0 to 1. This allows us to assess how various combinations influence the magnitude and sign of ∆I, capturing the directional dependency between PTM1 and PTM2. For each combination, the joint probabilities of PTM1 and PTM2
are computed and used to calculate ΔI , which quantifies the extent and direction of the dependency. The r are computed and used to calculate ∆I, which quantifies the extent and direction of the dependency. The results are visualized in a 3D scatter plot (Figure 2A), where the x-axis represents P(PTM2∣PTM1), the y-axis represents P(PTM2∣¬PTM1), and the z-axis corresponds to the calculated ∆I values. The surface formed by the plot represents all
possible theoretical values of ∆I, illustrating how the score smoothly transitions across varying condit possible theoretical values of ⊿I, illustrating how the score smoothly transitions across varying conditional probabilities.
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Crosstalk Between H3K27 and H3K36 Methylation States in TKO Cells

The relationship between H3K27 and H3K36 methylation has been extensively studied^{19,28–31}. Kelleher et al.'s M4K kinetic
model has provided detailed direct evidence of how methylation at one site influences the other. Th TKO and NTKO cells revealed strong antagonism in the rate constants for the formation of dimethylated species when the The nTKO cells revealed strong antagonism in the rate constants for the formation of dimethylated species when the formation of dimethylated species when the formation of dimethylated species when the formation of dimethy other site is di- or trimethylated—a phenomenon they termed "bidirectional antagonism." Conversely, unmethylated or monomethylated K36 facilitated faster methylation at K27 and vice versa.

Since no other directional scores exist for comparison, benchmarking the novel ΔI score is challenging. To demonstrate
its utility, we applied ΔI scoring to the Kelleher dataset. The M4K model's use of effective rate cons \mathbf{u} sure, \mathbf{y} scores \mathbf{y} and \mathbf{y} and \mathbf{y} model \mathbf{y} and $\mathbf{y$

directionality provides a distinct yet comparable framework for assessing how ΔI similarly captures the antagonis tic

interactions between H3K27 and H3K36 methylation states.

Figure 2. Simulation and benchmarking the directional Crosstalk score ∆I (delta I). Figure X: Simulation of ∆I va riation and kinetic rate comparison for K27-K36 PTM crosstalk.

(A) 3D Point cloud of theoretical ∆I (PTM2|PTM1) values, illustrating the relationship between conditional probab bilities of two post-translational modifications (PTM1 and PTM2) and their influence on directional interplay. The x-axis represents the probability of PTM2 occurring given PTM1 P(PTM2∣PTM1), while the y-axis shows the reverse probability P(PTM1∣PTM2). The z-axis corresponds to the ∆I score, with a color gradient indicating the degree of crosstalk ranging $\frac{1}{1000}$ T (strong negative or antagonistic crosstalk -- purple) to +1 (strong positive or cooperative crosstalk -- yellow). This plot emphasizes how the likelihood of PTM1 influences the modification at PTM2 and vice versa, providing insight into the full range of directional interplay observed between modifications. This theoretical model offers a robust fram mework for quantifying PTM crosstalk under varying probability conditions.

(B) Adapted from Figure S5C Zheng et al. (2012) Visual representation comparing directional interplay (∆I) with experimentally derived kinetic rate constants for K27 and K36 methylation crosstalk. Each row represents a differ ent

methylation state of K27 (K27 me2), K27 ma μ ₂, K27 me2, μ modifications influence the modifications influence the dimethylation of K36 (K36me2). For each state, the kinetic rate constant k(d−1) is displayed alongside the corresponding directional ∆I score, with green arrows denoting positive crosstalk (synergistic PTM interaction) and red arrows indicating negative crosstalk (antagonistic interaction). ∆I values correlate with experimentally derived kinetic parameters to reflect the dynamic interplay between these modifications.

ΔI and M4K Analysis in TKO Cells

As shown in Figure 2B, when K27 is unmethylated, ΔI(K36me2 |K27un) = 0.11, indicating that K36me2 is more likely to
occur in the absence of K27 methylation. This positive crosstalk correlates with a high rate constant for occur in the absence of K27 methylation. This positive crosstalk correlates with a high rate constant for the formation of K_1 and K_2 are flecting a cooperative relationship where the lack of K27 methylation promotes K36 methylation promotes K366 methylation promotes K37 methylation promotes K366 methylation promotes K37 methylation prom dimethylation. In the presence of K27 monomethylation, the directional crosstalk becomes even more pronounced, with Δ (K3 cme2 μ me2) μ and μ strong that K27me1 strongly favors the formation of K36me2. The rate constant for the μ transition, indicating active promotion of K36 dimethylation of K36 dimethylation by K36 dimethylation by K27me1. As K27 progresses \mathcal{L} to higher methylation states, the interplay shifts towards antagonism. When K27 is dimethylated, ΔI(K36me2|K27me2) = -0.37, indicating that K27me2 hinders K36 dimethylation. This antagonism aligns with the much lower rate constant, k = \sim suggesting that K36 dimethylation is not favored in K27me2-containing cells. A similar trend is observed for K^2 t_{max} , where Δ I(α 36me2|K27me2) = -0.27 and the transition rate constant is k α = 0.2, showing that K366 dimethylation is less likely when K27 is trimethylated.

 T shows that \tilde{G} shows that \tilde{G} are M4K-derived effective rate constants, or \tilde{G} reflection of the crosstalk between K27 and K36. ΔI provides a static measure of directional influence based on PTM abundance, indicating how the presence or absence or absence or absence or absence or absence or absence of α the other. Positive ΔI values correlate with higher transition rates from M4K, reflecting cooperative crosstalk. Negative ΔI values correspond to lower transition rates, highlighting antagonism between the two sites.

 $M_{\rm{max}}$ effective rate constants describe the rate at which specific PTM formations occur (e.g., κ and κ), but κ these values do not necessarily reflect the final abundance of these methylation states. While a high rate constant indicates rapid formation, factors such as demethylation, further methylation to higher states, or other regulatory \mathbf{a} impact the ultimate steady-state levels of these modifications. In contrast, the \mathbf{a} score captures the final

directional dependency between PTMs based on their observed abundance, rather than the kinetic process leading to \mathbf{f} formation. By \mathbf{f} interpretation, \mathbf{f} provides a static measure of the directional relationship of the directionship of the directional relationship of the directionship of the directionship of the dire between methylation states. Both metrics offer complementary perspectives: M4K provides insights into the dynamic
formation rates of PTMs, while AI quantifies the stable outcome of their interactions. Overall, their agreem that Δ I is a valuable tool for analyzing PTM dependencies, especially when kinetic data is unavailable. $\frac{1}{2}$ is a value $\frac{1}{2}$ is a value $\frac{1}{2}$ is unavailable . Especially when kinetic data is unavailable.

Normalized and Directional Interplay Scoring Histone Proteoforms During Aging

Normalized and directional interplay scoring delivers a more complete picture of the role of histone interplay in aging processes (**Figure 3**). During aging there is a progressive increase in positive crosstalk between K27me2 and K36me2 across multiple tissues and histone variants. In liver H3.2, the NI increases from 0.49 at 3 months to 0.57 at 5 months, reached 0.62 at 18 months, and climbed to 0.72 by 24 months. A similar pattern emerges in liver H3.3, where the NI increases from 0.45 at 3 months to 0.53 at 18 months and 0.57 by 24 months (**Supplemental File 1- S30-38**). Although the increase is more modest in brain H3.3, a comparable trend is evident during aging. Furthermore, these dynamics appear connected to acetylation states on histone tails, suggesting that these marks are associated with a more permissive and open chromatin structure. For instance, both K27me2 and H36me2 exhibit a moderately high NI of 0.11 with K14ac in the brain

(**Supplemental File 1- S6-8)**.

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Figure 3: Heatmap Analysis of Post-Translational Modification Patterns and Interplay in H3.3 Hist istones from Brain Samples. This figure presents a complex heatmap of post-translational modification dynamics on H3.3 from brain samples, illustrating from left to right the presence of PTM combinations, normalized interplay (NI) scores, binary combinations, and the discrete abundances of PTM1 and PTM2 across different aging samples. Each heatmap is aligned to highlight relationships between different representations of PTM modifications. Samples are categorized by brain region, age, and biological replicate. One-Hot PTM Heatmap **(heatmap #1 -- leftmost):** This heatmap displays the presence (black) or absence (white) of specific PTM combinations across all brain samples using one-hot encoding. Each row represents a PTM combination, and each column corresponds to constituent PTM status. **Normalized Interplay Heatmap (heatmap #2):** This heatmap presents the normalized interplay (NI) scores for PTM combinations, quantifying the positive and negative crosstalk of PTMs within H3.3 histones. Higher NI scores (darker shades of blue) indicate stronger interactions or dependencies between modification states. Binary Combination Heatmap (heatmap #3): This heatmap shows the abundance of each PTM combination across brain samples. The color intensity (blue) corresponds to relative abundance, with darker shades indicating higher values. This representation reveals the percentage abundance of all PTM combinations. **PTM1 Abundance Heatmap (heatmap #4):** This heatmap displays the discrete abundance of the first PTM (PTM1) in each pairing across samples, with color

intensity indicating relative abundance (darker shades represent higher levels). **PTM2 Abundance Heatmap (heatmap #5 --rightmost):** This heatmap illustrates the discrete abundance of the second PTM (PTM2) in each pairing across samples, with color intensity corresponding to relative abundance (darker shades indicate higher levels), similar to the PTM1 heatmap. All heatmaps are clustered based on the binary abundance of PTM combinations, revealing potential patterns and relationships between samples. Column names reflect specific sample characteristics, including brain region, age, and biological replicate. The heatmaps are scaled to emphasize differences in interplay and abundance across samples, offering insights into the combinatorial nature of PTMs in H3.3 histones during brain development and aging.

In liver H3.2, K9acK36me2 shows a positive NI of 0.38, and in liver H3.3, an NI of 0.13. These findings support the idea that K36me2 has a strong relationship with acetylation, contributing to the formation of permissive chromatin regions that are strengthened with age (**Supplemental File 1- S35**). Similarly, the crosstalk between K9me1 and K27me2 shows an upward trend, though with some variation across tissues and histone variants. In liver H3.2, the score starts at 0.46 at 3 months, peaks at 0.55 at 5 months, and then slightly declines to 0.47 by 18 months, remaining relatively stable at 0.45 at 24 months. This pattern suggests an early peak in positive crosstalk between these modifications, followed by a stabilization as the organism ages. In brain H3.2, the score increases steadily from 0.25 at 3 months to 0.34 by 18 months and continues to increase to 0.44 by 24 months, indicating a more sustained growth in positive interplay with age in neural tissues, where dynamic transcriptional regulation is critical (**Supplemental File 1- S5, 6**).

Several PTM pairs demonstrate increasingly negative crosstalk with age, reflecting growing mutual exclusion between modifications that mark distinct chromatin states. One of the most striking examples is the interplay between K9me2 and K27me2, which becomes progressively more negative as the organism ages. In liver H3.2, the score starts at -0.70 at 3 months, decreases to -0.88 by 5 months, and further drops to -0.91 at 18 months and -0.95 at 24 months. A similar trend is observed in liver H3.3, where the interplay score becomes more negative from -0.51 at 3 months to -0.66 by 24 months. This progressive decrease suggests that K9me2 and K27me2 become increasingly segregated, likely marking distinct repressive and permissive chromatin domains with greater precision as liver tissue ages. There is a less marked albeit consistently negative crosstalk between K9me2 and K27me2 in the brain consistently remaining at ~-0.48 across ages. Indicating

that K9me2 and K27me2K36me2 mark non-overlapping chromatin regions that are functionally distinct, repressive vs more permissive chromatin.

Interplay analysis also reveals organ-specific dynamics of aging. For instance, the mark K9me1K27me1 exhibits an interesting relationship. From 3 to 18 months, the crosstalk between K9me1 and K27me1 in heart tissue shows a clear trend of decreasing mutual exclusion. At 3 months, the average NI is strongly negative at - 0.82, indicating that these modifications are mostly mutually exclusive, marking distinct repressive chromatin regions. This exclusivity continues at 5 months, with a consistent NI of -1, showing that K9me1 and K27me1 rarely co-occur in the same regions. However, by 10 months, the mutual exclusion starts to weaken, as reflected by the average NI of -0.55. This change indicates that some chromatin regions start to show minor co-occurrence of K9me1 and K27me1, though exclusion still dominates. At 18 months, this trend becomes more pronounced, with the average NI shifting to 0.14, signaling a significant increase in co-occurrence. By this stage, K9me1 and K27me1 are no longer strictly exclusive, and they begin to mark overlapping chromatin regions, suggesting an evolving chromatin structure in aging heart tissue (**Supplemental File 1- S11-19**).

The value and limitations of interplay scoring

The unique value of top- and middle-down proteins is the ability to μ ability to μ effective use of proteoform information to derive fundamental mechanisms requires novel data analytics approaches. Recent work has shown that proteoform biology does not primarily operate at individual PTMs or single proteoforms but a regime between these extremes where proteoform families or themes of combinatorial PTM interplay dominate. Like PTMs, individual proteoforms mostly do not have an injective functional relationship. Thus, it is not sufficient to
quantitate proteoforms and connect them to function. Rather, we must endeavor to understand the interplay \mathbf{q} and connect them to function. Rather, we must end them to understand the interpretation. Rather, we must end combinatorial PTMs. This requires unique data analytics and visualization of the high dimensional combinatorial data of proteoforms to identify functional units shrouded within this data. Prior work has provided useful tools to derive this important biology; however, prior scores have seen and lack clear interpretations and lack clear interpretations and lack control \mathbb{R}^n . an improved method, Normalized Interplay, to improve symmetry between synergistic and antagonistic interplay and provide scores that are bounded. The resulting scoring function produces output that enables comparison between the
strength of negative and positive interplay and intuitive bounds, where 1 represents perfect synergy and perfect antagonism. These bounds also enhance data completeness because perfect synergy and antagonism are perfect antagonism. These bounds also enhance data completeness because perfect synergy and antagonism are

relatively common. Interplay is not limited to two PTMs and thus the measured relationship between two PTMs is likely influenced by other PTMs and the overall physiological state of the cell. The framework we present here is readily extended to higher dimension combinations, although we focus here on binary combinations. Binary combinatorial
interplay is not limited to mutual co-occurrence and directionality of these relationships is essential to unde proteoform mechanisms and can be readily inferred from quantitative proteoform data. We address this here with the proteoform mechanisms and can be readily inferred from quantitative proteoform data. We address this here with the $\overline{\text{int}}$ oduction of Directional Interplay Score (∆1). This scoring function provides the first systematic method to quantify the $\overline{\text{tr}}$ directionality of these relationships directly from proteoform data. The order of operations is fundamental to revealing
proteoform mechanisms. There are no known mechanisms where the writing or erasing of multiple PTMs is proteoform mechanisms. There are no known mechanisms where the writing or erasing of multiple PTMs is truly concomitant. Thus, there is no reason to expect that the biochemistry of a reversed order of operations should be similar. Indeed, hierarchical highly directional relationships are abundant in the currently limited proteoform biology

Conclusion

We present here two complementary novel scoring methods for the elucidation of proteoform biology directly from
quantitative proteoform data. The source code for these algorithms is made available to the top-down proteomic \mathbf{q} algorithms is made algorithms is made algorithms is made available to the top-down proteins is made available to the top-down proteins in the top-down proteins in the top-down proteins in the top-down proteins in community. These methods are convenient and intuitive for the deeper inquiry into proteoform data. It is important to recognize that, like any discovery tool, these scores only infer mechanistic relationships. Full elucidation of mechanism requires the positing and testing of a hypothesis often with complementary approaches^{24,33}. The methods presented are
 most useful for discovery of hidden functional relationships within quantitative protections with the \mathcal{H}_2 presented here can also be used effectively to test hypotheses when experiments are carefully designed for rigor.

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